GYAN-VIGYAN SARITA: शिक्षा

A non-remunerative, non-commercial and non-political initiative to Democratize Education as a Personal Social Responsibility (PSR)

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Aim for the Best, but...

... start, without loosing time, with whatever is available.

संपादकीय

बालपन: अनुशासन सीखने का समय

बालपन यानि बचपन, जीवन का वह शुरूआती समय है, जब न कोई चिंता रहती है, न कोई जिम्मेदारी होती है। बस होती है, मौज ही मौज। सब लगे रहते हैं, तीमारदारी में। धीरे-धीरे जब उम्र बढ़ती जाती है, लोगों की आशायें बढ़ती जाती हैं, जिम्मेदारी मिलनी शुरू हो जाती है, और तब मेहनत करने की अपेक्षा की जाने लगती है।

नवम्बर महीना आते ही सबसे पहले 14 नवंबर याद आता है। चाचा नेहरू का जन्मदिन। इसे सब अपने बालपन से मनाते आ रहे हैं- बाल दिवस के रूप में। स्कूलों में दूकानें लगायी जाती थीं। सामान बेंचा जाता था, अपना सामान बेंचने के बाद अपने मित्रों की दूकानों से उनका सामान खरीदा जाता था, और सीखा जाता था- कैसे बेंचा जाये, क्या बेंचा जाये, कितने में बेंचा जाये, और कैसे पैसा बनाया जाये। सामान माता-पिता खरीदकर देते थे, और फायदा अपना हुआ करता था।

नेहरू जी का पूरा नाम था-पंडित जवाहरलाल नेहरू। अंग्रेजों की गुलामी के बाद भारत के स्वतंत्र होने पर नेहरू जी स्वतंत्र भारत के पहले प्रधानमंत्री बने थे। इनका जन्म 14 नवंबर 1889 को इलाहाबाद में हुआ था। नेहरू जी बच्चो से बहुत लगाव रखते थे। बच्चे उनको चाचा नेहरू कहकर पुकारते थे।

बच्चे देश के भविष्य होते हैं। जिस देश के बच्चों का बचपन जितना अधिक खुशहाल रहता है, उस देश का भविष्य भी उतना ही अधिक खुशहाल बनता है। खुशहाली एक सुगंध है, यह बहुत दूर तक जाती है, और यह सबको प्यारी लगती है।

बच्चे अगर शिक्षित हैं, संस्कारवान हैं, मानसिक और शारीरिक रूप से स्वस्थ हैं, तब परिवार के सभी सदस्य भी स्वस्थ रहते हैं। जब हमारा परिवार स्वस्थ रहता है, खुशहाल रहता है, तब हमारा समाज खुशहाल बनता है, और फिर देश अपने आप खुशहाल हो जाता है।

बच्चों की प्रतिभा में निखार आये और उनकी क्षमता बढ़े, जब यह हमारा ध्येय बन जाता है, तब बच्चे आगे बढते हैं। बच्चे बहुत नाजुक होते हैं। छोटी से छोटी बात भी उनके मन और मस्तिष्क पर तुरंत प्रभाव डालती है, गहरा प्रभाव डालती है और यह प्रभाव करीब करीब पूरी जिंदगी भर के लिये होता है।

चाहें शिक्षा किसी भी स्तर की क्यों न हो, देश वही आगे बढता है जिसकी शिक्षा में संस्कारवान बनाना, अनुशासित रहना, और देश के प्रति निष्ठावान होना सिखाया जाता है। जापान इस तरह की शिक्षा प्रणाली अपनाकर आगे बढ़ने वाला उदाहरणीय देश है। हम बहुत कुछ उससे सीख सकते हैं।

अभी अभी दीवाली का पर्व बीता है। बच्चों ने बहुत संतोष किया और अपने को पटाखों से दूर रखा। यह बताता है कि वे अपने चारों ओर के पर्यावरण के प्रति कितने जिम्मेदार हो गये हैं।

बच्चे आदतन पर्यावरण के प्रति बहुत सजग रहते हैं। वे प्रकृति के नजदीक रहने में आनंद उठाते हैं। जिम्मेदारी हमारी है कि हम शुरूआती दौर से ही बच्चों को प्यार देने के साथ-साथ अनुशासित रहना सिखायें। सभी काम समय पर और सलीके से करना सिखायें। जैसे-यदि हम बच्चों को अच्छी लिखावट के प्रति सजग कर दें तो उनकी पूरी जिंदगी में उनकी अच्छी लिखावट प्रशंसा पाती रहेगी और अच्छी लिखावट केवल अनुशासित तरीके से ही सीखी जा सकती है।

हमें याद रखना चाहिये कि बचपन इतना बेशकीमती होता है कि कोई भी इतना बड़ा धन नहीं बना जो बचपन को फिर से खरीद सके, वापिस ला सके। न बचपन फिर से वापिस आयेगा, और न ही बचपन में सीखने वाले गुण दुबारा सीखे जायेंगे।

14 दिसंबर 1954 को संयुक्त राष्ट्रसंघ ने बच्चों के कल्याण के प्रति संवेदनशीलता दिखाते हुये, अपने सामान्य अधिवेशन में निर्णय लिया था कि विश्वबालदिवस हर वर्ष 1956 से 20 नवम्बर को मनाया जायेगा। सोच थी कि इससे विश्व के बच्चों में आपसी तालमेल बढ़ेगा।

कनाडा, न्यूजीलैंड, ब्रिटेन आदि कुछ ऐसे देश हैं जो 20 नवंबर को बालदिवस मनाते हैं। आस्ट्रेलिया, अक्टूबर के चौथे बुधवार को बालदिवस मनाता है। अमेरिका में बालदिवस राष्ट्रीय स्तर पर जून के पहले रविवार को मनाया जाता है। भारत में बालदिवस मनाने की शुरूआत 14 नवंबर 1957 से हुयी।

अगर हम बच्चों की आंखों से विश्व को देखने का प्रयास करें तो वह बहुत ही खूबसूरत नजर आता है क्योंकि बच्चा बनावट से दूर होता है। बालपन जीवन का वह गुण है जो जीवन में ठीक उसी प्रकार से छिपा है जैसे बारिश की बूंदों में सोंधी महक छिपी रहती है, हवा में हल्की खुशबू छिपी रहती है। अलबर्ट आइंसटीन का मानना था कि यदि आप अपने बच्चे को बुद्धिमान बनाना चाहते हैं तो उसे परीकथायें पढ़ने दें। कल्पनायें ही नयी खोजों को जन्म देती हैं।

शिक्षा का उद्देश्य यह नहीं है कि हम दिमाग में ज्ञान ठूंस दें बल्कि यह है कि हम दिमाग को ऐसा विकसित कर दें कि वह नयी खोजें और आविष्कार की ओर चल पड़े।

वास्तविक बुद्धिमान वह है जो बच्चों के प्रश्नों से घबराये नहीं और उनकी जिज्ञासाओं को भिन्न-भिन्न प्रकार से शांत करे। बुद्धिमान लेखक वही है जो बच्चों के लिये रूचिकर पुस्तक लिखे।

अगर एक बच्चा अपने माता-पिता की इज्जत करना नहीं सीखा है तो वह कभी किसी और की इज्जत नहीं कर सकता। ज्ञानविज्ञानसरिता परिवार इसी विचारधारा को लेकर आगे बढ़ रहा है। हमारा मानना है कि एक किताब, एक कलम, एक बच्चा, और एक अध्यापक पूरे विश्व की तकदीर बदल सकते हैं।

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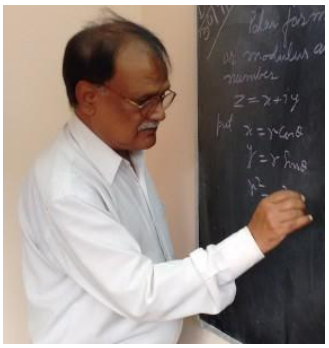
हमारा पंचवर्षीय प्रवास

Start: June-2012



April-2015

June-2016.....

पारम्परिक शैक्षणिक मार्दर्शन से प्रारम्भ कर आज हम तकनीकी-विकास के सहारे मूलभूत प्रासंगिकता को आगे बढ़ने में संलग्न हैं...

Philanthropy is a fashionable luxury for those who can afford it. But, it is a spiritual engagement for those who are passionately dedicated with Personal Social Responsibility (PSR).

An Appeal: Gyan Vigyan Sarita

A non-organizational initiative of a small set of Co-passionate Persons

Philosophy: Personal Social Responsibility (PSR)

Objective: Groom competence to Compete among unprivileged children from 9th-12th in Maths and Physics, leading to IIT-JEE.

Financial Model: ***Zero-&-Fund-Zero-Asset (ZFZA).*** It calls for promoters and facilitators to provide infrastructure for use to the extent they feel it is neither abused nor there is a breach of trust. And, reimbursement of operational expenses to the participators

Operation:

a. **Mode:** Online since July'16, using Google Hangouts, a free we-conferencing S/w, with connectivity upto 15 nodes.
b. **Participation:** Voluntary and Non-remunerative, Non-Commercial and Non-Political

Involvement:

a. **As Promoter** –
i. Initiate a Learning Center,
ii. Sponsor a Mentor who is willing to join on certain terms,
iii. sponsor cost of operation and up-gradation of infrastructure to voluntary mentors,
iv. Sponsor Website.
b. **As Facilitator** –
i. Provide space and infrastructure for **Interactive Online Mentoring Sessions (IOMS)**, which is generally available at most of schiils, with a marginal add-on,
ii. Garner support of elite persons to act as coordinators at a Learning Centre.
c. **As Participator** –
i. As a Mentor,
ii. As Coordinator,
iii. As Editor and or contributor of thought provoking articles for e-Bulletin, which are relevant to the initiative, and make it more purposeful and reachable to the target audience.
iv. As author of Chapters for Mentors' Manual, being uploaded as a Free Web Resource,
v. Anything else that you feel can add value to the mission and make it more purposeful.
vi. *Anything else that you consider to make this initiative to become more effective.*

Background: *The initiative had its offing in May'12, when its coordinator, a power engineer by profession, soonafter submission of Ph.D. Thesis in April'12, at IIT Roorkee, at the age of 61 years, decided to mentor unprivileged students.*

SARTHAK PRAYASH, a Ghaziabad based NGO, warmly accepted the proposition and created a facility to mentor students from 8+ to prepare in mathematics and physics and prepare them for engineering entrance tests. They warmly reciprocated and created a class room.

Experience in this selfless social work were used to navigate across without losing focus. He was associated with SUBODH FOUNDATION from Sept'15 to Sept'16 during which he published a monthly e-Bulletin ***SUBODH-पत्रिका*** *to create visibility across persons who could make a difference.*

In Sept'16, post transition, the mission has been continued as a non-organizational entity ***Gyan Vigyan Sarita****, with a set of Four persons, including retired* ***Prof. SB Dhar****, Alumnus-IIT Kanpur, a middle aged* ***Shri Shailendra Parolkar****, Alumnus-IIT Kharagpur, settled at Texas, US and* ***Smt. Kumud Bala****, Retired Principal, Govt. School Haryana. Earlier, they were complementing the IOMS. The initiative survived transition depicted @ http://gyanvigyansarita.in . It contains details of initiative with its Publication Menu* ***e-Bulletins****, and* ***Mentors' Manual****. You may like to read them.*

IOMS: It is operational at RKM School, Sitanagaram, Ditt Guntur, AP; Academy Home, Lucknow, UP; andSunshine Society, Sec 50, Noida, UP.

Actions Requested: May please like to ponder upon this initiative. **Queries**, *if any, are heartily welcome*. We would welcome your collective complementing in any of the areas listed at **Involvement**, above, to make the mission more purposeful and reachable to target children.

Coordinator's View

Business-Commerce-Economics: Education

Education has been drifting from state's domain to private domain with greater autonomy which started in India about 2-1/2 decades ago. It has been gaining mass like a rolling ice ball, despite survey and reports on pathetic condition of literacy, forget about education among larger demography. This drift retardation grew faster due to lack of ownership among various players in education system. Result is paucity of teachers, decline in quality of education, poor infrastructure at schools and density of school far below the requirement. Recent, reports and surveys on education in India have not shown signs of reversal of ground reality, despite a quarter century of growing privatisation in education. This created a necessity to analyse consequences of the drift in education in terms of creating a social capital in country where education and skill development was dominated by गुरु-शिष्य परंपरा. This analysis is based on experiences of Gyan Vigyan Sarita, an initiative on Education.

Business: Prevalent times are of a global competition and risk of war if, it breaks, God forbid!!, would have its genesis into imbalance of economy and availability of water. Trade & commerce is gaining central position in all political, diplomatic strategic and business deliberations. *Business is aimed at maximizing profits in shortest possible time*. Its emphasis is on creating demand to extract economic surplus of the target customers, and expand the base of target customers to multiply profits. In this pursuit effort is made to make business proposition and products attractive, fascinating and mesmerizing. With this reality it would be incorrect to expect business houses to enter into social cause unless it caters to their business needs. Statutory provision of CSR is a living example if its abuse, and there is enough experience on this.

In the process market promotion efforts going beyond the capability have not been uncommon. This called upon restraint on business advertisements and pro-action by consumer forum, but all this is consequential. *By the time such restraints come into play benefits are extracted by business players, as its short-term objectives, and the damage is done; while the business house moves on into new pastures.*

Commerce: Trade and commerce are closely coupled and focus upon availability points and consumption point of commodities, be it of any nature. Having identified such source and sinks, efforts are made to channelize flow of commodity to its destination, i.e. point of consumption, at the time of need as best as possible to fetch appropriate value of money and effort that is invested. Accordingly, time-frame of commercial propositions is much larger and objective is to assist producer or provider and the consumer. Thus, trade and commerce, while meeting vested interests, enhance the value of commodity to so as both, its producer and consumer.

Economics: It is about creating value of anything and everything through use of available resources. It makes resources to grow on themselves in a compounded manner. Among the resources of economic activity human capital is the foremost. *Economics is at the basic of trade-and-commerce and business as well. Economics influences demography at large and, therefore, social well-being is always at the back of the mind of thinkers of economics. Economic reforms are long-term measure and require perseverance, with continuity and commitment. Its first impact is on sociological environment.*

Economics is about utilization of resources so as to add a value to it. And value addition is a relative terms which is governed by Law of Diminishing Return. Thus economics is dynamic and it requires one to refrain from complacence and demands restless striving for realizing incremental value addition. Every growth increases expectations and greater efforts are required to maintain the rate of value addition for incremental gain. Thus in economic process there is no space for complacence. It demands seamless intensification of efforts creating more opportunity for competent persons to upkeep their competence; and creates more scope to grow competence to those with lesser of it. ***Education*** *plays a crucial role in adding value to the younger population of children, who have to carry the torch going forward.*

Education: *Recently* ***creating social capital*** *has become a common parlance in government policies. Role of education in creating social capital is crucial.* Investment in education is long term and its benefits are symptomatic and subjective. Against this, benefits in business are parametric and objective and measured in terms of an immediate gain. The two paradigms are diametrically opposite. Education leads to deep sociological reform a long term perspective and its effect can never be realized in immediate future. Japan is an excellent living example where commercial and industrial systems are woven with cultural ethos as its basic fabric, and education is its strong hold. This is the only reason that the country as a whole has survived despite devastations.

Thought Analysis: Thoughts of accomplished persons and social thinkers continue to remain sources of motivation. But, but they cannot be taken as a yardstick. Unless education system is resilient to contemporary circumstances it suffers from relevance. Making education

centred on earning livelihood leads to focus on skill development. Learn-by-doing is prima-facie logical and shall help to prepare child for his self-dependence. On this context, there seems to be nothing wrong with the child-labour. But, this argument is totally absurd, and is contradictory to wilfully crafted Right to Education (RTE). It grants a fundamental right through The Constitution (Eighty-sixth Amendment) Act, 2002 inserted Article 21-A.

Education must be able to transform thought process and just not producing run of the mill. Access to educational motivation and support has to be democratic and subject ascending the ladder beyond basic education based on ability to perform. In its absence natural instinct to grow will erode. *It is a sociological reform where interaction between teacher and students creates transfer of spirit, and subject matter serves as a carrier alongwith requisite set of knowledge or skill*. This process cannot be robotized, automated and measured in terms of eye-balls, foot-fall or ROI; a vocabulary in common parlance in prevalent highly commercial and business orientation.

Every statesman, philosopher, thinker and reformer has vehemently expressed his perspective on education among which thoughts of Confucius stretch beyond the present living to address its necessity for wellbeing of next generation. In Indian philosophy it is called पितृ-ऋण, limited to family perpetuation. It has been possible to address this need with a larger perspective in a form of a commitment "*Democratize education with a sense of personal social responsibility (PSR) to groom competence to compete among unprivileged children*". In its backdrop there is a Grandma's story *of a lean and thin fragile old man planting trees, with a hope that some pedestrian would get a shade in scorching heat or a cover in rains and get fruits to a hungry belly. The old man had all valid reasons to cry of his sufferings and reasons of retreat from any social obligations. But, he did not.*

This is a non-organizational initiative, being executed by a small set of co-passionate persons. It is on non-remunerative, non-commercial and non-political basis. This small group from elite class has assumed PSR and taken upon itself to mentor unprivileged children. This mentoring is neither teaching, nor coaching nor tuition; each of this in one form or the other involves financial transactions. It aims at to frame thought process of students to be able to think out-of-box, reason out every observation, belief and convention, analyse its consequences, and evolve feasible remedies for a more sustainable growth with peaceful coexistence. Subject is just a medium to evolve logical, analytical and consequential thought process. *Its financial model is Zero-Fund-&-Zero-Asset (ZFZA) where mentors do not accept anything in lieu of their efforts; neither in coin nor kind. But, welcomes individuals, institutions, organizations, NGOs and government to create, facilitate and maintain learning centres. At the learning centre a group of unprivileged students, without discrimination are encouraged to assemble and they are mentored. Any incremental cost on upgradation of setup or operation thereof incident on mentors, if required, is to be supported by those responsible for the learning centres.* It had started 5-1/2 years ago with Chalk-N-Talk. About 1/2 years ago it entered into a phase of web enabled Online Mentoring Session (OMS) using IT potential available in Digital India. And, just six months ago it got upgraded and stabilized fully into an interactive platform and is being called **Interactive Online Mentoring Sessions (IOMS)**.

This IOMS model banks many of social strengths:

a) Increase in longevity of Indians with growing financial and health independence among elites.

b) An opportunity to elite senior citizens to pay back their social dues by way of grooming competence to compete.

c) Such persons have a reasonable IT infrastructure available at their dwelling, it is just a matter of familiarizing with IOMS and getting started.

d) Growing IT network and easy accessibility provides an opportunity to connect passionate persons to desperate students who are otherwise deprived of deserving guidance by virtue of their circumstances.

e) Last but not the least it adds ***positivity to the living*** at the age and stage of life, a gift of GOD, when it matters most.

Mentoring, can neither be an occasional nor convenience driven activity. It has to be driven with passion and requires commitment with continuity and consistency to maintain learning curve of target students. Therefore it requires a restraint of self-discipline among mentors. Its immediate influence is on those who have rested hopes on our promises, and none to look toward as an alternative. Presently, the small group of Four persons are pursuing their passion to mentor with desperation. God willing, if the initiative grows, it would be an opportunity for countless number of competent and passionate persons to collectively complement in mentoring.

In this initiative conceptual understanding of subject and its relevance into surrounding of student, in a spirit of coexistence, is given highest priority. These concepts are being groomed in an interactive environment, a necessity in Mathematics and Science. It is followed with practice which helps to improve comprehension, accuracy and speed in their application with a clear distinction that there are no shortcuts to knowledge and excellence. Marks, selection, placement and package are consequential and given a rear seat.

This initiative is purely indiscriminative, and is also open to children from affluent families. But, it has been perpetually experienced that these students, and more so their parents, are too obsessed with their affluence to spend on most expensive and prestigious brands of coaching and tuition and to their convenience. In turn they tend to ignore requirement of discipline, continuity and consistency, the only demand posed by this mentoring initiative which is being extended to all free of cost. These experiences have compelled to create a barrier for students from privileged families. Unless such students and their parents demonstrate a proven commitment to abide by requirements of initiative, which are totally impersonal and non-financial, they would turn out to be only retardants to the mission.

Experience of implementing IOMS at **Ramkrishna Mission High School, Sitanagram, Guntur District**, AP, with a nearly 40 students, is thrilling. The initiative is deriving strength from spiritual environment of the Mission. It receives full attention of its dedicated management and group of teachers. ***IOMS it has been possible to ignite a fire of inquisitiveness and learning, aligning to the thoughts of William Butler Yeats***. Here, students are proactively participating with a thrill of learning much beyond and out-of-box. Here, IOMS has grown into a process and a model for replication. This was missing in all other places where this initiative was extended with the same selfless PSR. It is, therefore, appropriate to infer that in this remote IOMS the foremost and an essential requirement is motivation and acceptance of the model by management and teachers of the school. A similar model with a very small number of students at **Academy Home, Lucknow**,by an NGO, is operational. At both the places role of NGOs promoting the IOMS is short of their own commitment to facilitate IOMS. *Despite IOMS continues keep up hopes of target students so as to maintain their faiths in a selfless mission, to the extent possible*. Commitment of the IOMS is to none other than that the target students. The model envisages maximum 5-6 learning centres concurrently connected to a mentor at a time. The model is highly optimized with an upper limit if Rs 70 Thousands on capital cost in creating a learning centre, This shall *on one hand bridge connectivity between students and passionate mentors, a* ***dire necessity***. And on the *other hand teachers and coordinators at the learning centres would get ready to take upon IOMS, groom unprivileged children and grow it like a chain reaction,* ***to sustain the socio-economic transformation***. The model is apparently slow, but stable; it is sound and far reaching with persistence towards grooming competence to compete among unprivileged section a larger section of society. ***Thus, education serves as hairy roots to transfer nutrients from its surrounding and our rich cultural heritage, for a sound growth of socio-economic fundamentals***.

IOMS is a philosophy in action, open to all add, modify, change or take-away with a simple request to do that with a PSR to create a human capital out of unprivileged children who growing without hope or dream.

It is seen that potential of web-technology is being exploited, directly or indirectly, to monopolize educational environment by many. An obvious question that remains unanswered is -Why would one make huge investment in creating such an enterprise without implicit gains? It is observed that the environment is heading towards robotized education and make teachers redundant, **a biggest and irrecoverable damage to the society**. It deserves serious concern of elite and a caution by policy makers.

Conclusions: The manner and financial model with which IOMS is being pursued does not fit into either a business or commercial proposition. Therefore, in the race of business and commerce to make immediate fortune, education at its basics is being ignored. This is a biggest and unpardonable blunder. If we don't pro-act correctively, our own loving descendants would hate to remember us and our selfish motives.

But, elite audience may like to consider this proposition to create a social capital for a stable and sustainable future, in whatever best way they can. The IOMS model expects governmental attention to create an environment conducive to participation of a huge human resource of elite class. *They can perform to make a difference to create a social capital in true sense and strengthen* ***socio-economy reform*** *of the country*. It is pertinent to conclude in words of Albert Einstein – "***A hundred times a day I remind myself that my inner and outer life depends on the labours of other men, living and dead, and that I must exert myself in order to give in the measure as I have received and am still receiving***".

Education is a system like other where finance is required to implement projects. But, ***an essential and mammoth requirement is of passion and dedication, which is grossly missing at every level.***

We pray almighty that lest we not forget Grandma's story of old fragile man and keep working till last breadth to create social capital through education with PSR.

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Enlightening the Younger Generation Through Teachings of *Bhagavad-Gita*

Aarti Sharma

"When I read the Bhagavad-Gita and reflect about how God created this universe everything else seems so superfluous." Albert Einstein, Noble prize winner and father of modern physics who developed theory of relativity

"The Bhagavad-Gita has a profound influence on the spirit of mankind by its devotion to God which is manifested by actions." Dr. Albert Schweitzer, noble prize winning theologian, organist, writer, humanitarian, philosopher, and physician.

"Nurturing of The Bhagavad-Gita is the most systematic statement of spiritual evolution of endowing value to mankind. It is one of the most clear and comprehensive summaries of perennial philosophy ever revealed; hence its enduring value is subject not only to India but to all of humanity." Aldous Huxley, prominent English writer, philosopher and parapsychologist

"Now I am become Death, the destroyer of world"- Bhagwat Geeta verse quoted by Robert Oppenheimer, father of atomic bomb and professor of Physics at California University , upon detonation of the first atomic bomb in July 16 1945

"The Indian epics i.e. Poetic visions of Vyasa and Valmiki are long and complex narratives that speak to virtually every aspect of Human existence ." Anne E. Monius Professor Harvard University , upon inclusion of epics of Ramayana and Mahabharata in University Curriculum

The above quotes by eminent personalities highlight the importance of the teachings of Bhagwad Geeta literally translated as Song of the God in leading a life of wisdom and ethics.

Among the children, the onus of ingraining these values lies primarily on the parents and educational institutions. It is seen that children in contemporary society are brought up in an atmosphere of deficient value system or core beliefs and give undue importance to materialistic goals .They become highly self centred and egoistic which is the ultimate path of self destruction as well as society and nation.

Traditionally the imparting of the moral values has been the forte of the joint-family system where the grandparents and uncles and aunts used to narrate human value stories based on ancient scriptures which taught the best life skill lessons on how to become a good human being. However, this family structure has been fast disintegrating and is replaced by smaller nuclear family structure where parents are the central point who have to cope with official as well as domestic responsibilities. The child's outlook in a nuclear family set up is oriented towards targets and their achievement and in this mad rat race, the child is not able to gain apposite knowledge owing to lack of inculcation of ethical values.

The Bhagavad Gita is a perfect guide of imparting wisdom in meeting life with greater maturity and understanding. It is the real lifeline of our existence, teaching us not just how to lead our lives, but also what is the meaning of our existence. This treasure House has been a source of inspiration and strength to one and all. It is indeed the scripture of India.

From a student's perspective, it is of great assistance to young minds facing stiff competition and increasing consumerism in leading a meaningful life. Children can grow strong and wise if they are taught lessons from the Bhagwat Geeta. These teachings are also very helpful for kids to build up a moral and righteous character which is not envisaged in the current dispensation of education, that is focused entirely on academics for mental development of the child.

Key teachings of the Bhagwad Geeta from a student's perspective:

1. **Belief in Karma (selfless action):** The verses of Bhagwad Geeta ably enlighten about the importance of selfless action. Sri Krishna asks Arjuna to give up *Moha* or delusion and to perform his duty as a warrior to fight regardless of the consequences. This most important message for mankind is evident in one of most famous shlokas (2.47) *Karmany evadhikaras te, ma phalesuch kadacana /Ma karma-phala –hetur-bhur- ma te sango stv akarmani* //The verse explains that to work alone one has the competence and not to claim their fruits. Let not the longings for fruits be the motive force of the action. At the same time let not this attitude bear it out into indolent action.

   The Bhagwad Geeta further explains about importance of unperturbed sameness in all conditions. It advises to engage onself in action with a steadfast mind and to abandon attachment and be unperturbed in success and failure. It is the level headed man, the calm man of good judgment and cool nerves who does good work and so does good to himself.

   He also cautions about proper channelization of energy by explaining that all outgoing energy following a selfish motive is frittered away , but if restrained , it will result in the control of the senses and attainment of knowledge .

2. **Existence of the Soul:** The Geeta teaches about the existence of the soul or "Aatma" and its impenetrable and all pervading nature. Fear of death is absurd as for the soul there is neither death nor birth at any time. He has not come into being, does not come into being and will not come into being. He is unborn , eternal and indestructible (verse2.18). It is nitya (always)and shashwat (permanent) (verse 2.20) . It is the body that is perishable and transient. At the time of death of the soul, it leaves the body and enters the new one Hence , one should not grieve over death of near and dear ones.

   The soul is the highest eternal reality whereas the senses of mind are finite. Weapons cannot pierce it , fire cannot burn it and wind cannot dry it but this embodied soul is caught in the grip of desire ridden actions leading to impurities and are responsible for the bondage of the soul. It can escape from it only with the help of spiritual effort. At the time of death soul leaves body and depending upon the karma takes the next birth. Hence it is important to perform selfless action to liberate the soul from desires and attachments.

3. **Thinking Positive:** Positive thinking is a must for overall well-being of self and society. Not only does positive thinking impact the ability to cope with stress and depression but it boosts immunity by reducing risk of death from cardiovascular problems. According to Bhagwad Geeta, purity of heart, steadfastness in knowledge and control over senses can help to propel positive thinking and lead to freedom from vanity and self destructive negative forces

4. **Freedom from Restlessness** : Bhagwad Geeta puts a great importance on the control of the mind . The moving mind is explained as fickle and unstable and likened to wind which is turbulent, strong and obstinate. The restlessness of the mind is caused by the sense objects which keep it bound into unending vicious cycle of desires and attachments . Sri Krishna emphasizes on practice of dispassionate detachment to achieve stability of mind that is the state of *sthitaprajna* . Stability of mind means to remain same in all the condition by practicing control over senses. One who has stable mind shows no signs of rejoice when he achieves the objects of the desire nor agitation when unpleasant things happen. Sense objects will cease to torment once he starts practicing abstention . The mind is detached from the sense objects through cultivation of detachments.

5 **Taming body, Mind and Speech:** The chaste brain has tremendous energy and gigantic willpower which needs to be channelized in performing positive actions . Without chastity there can be no strength . This chastity can be brought about by observing regulation of mind, purity of heart and moderation in speech. Bhagwad Geeta talks about regulation of mind by observing moderation , detachment and self restraint in thoughts and actions . Temperance in Speech can be observed by speaking words that are inoffensive, true, pleasant and beneficial. The mind body and speech thus regulated leads to cessation of the travails of samsara (i.e. the problem of suffering)

6 **Control of Anger:** Bhagwad Geeta brilliantly teaches about overcoming anger by explaining the causes and ill effects of anger. According to Holy Book, anger generates delusion and delusion results in loss of memory . Loss of memory brings about the destruction of discriminative intelligence and loss of discriminative intelligence . If feeling of anger is avoided , the energy saved would be converted into positive power and work in our favour. A person of disciplined mind , who has his senses under his control attains tranquility resulting in end of all the sorrows .

7 **Concentration of mind :** Mind is unpredictable and indecisive and gets easily distracted towards objects of senses .But its fickle wanderings can be brought under control by dispassion and spiritual practice. One should practice tranquility bit by bit and abstain from wavering thoughts. This would help in the development of steadfast intellect and setting the mind firmly on the self.

Conclusion : Educationists , Leaders , philosophers , thinkers world wide have proclaimed The Bhagwad Geeta to be doctrine of universal truth as it sums up the fundamentals of the life and reality. The teachings of the Bhagwad Geeta are ultimate as they illuminate for all of humanity , the realization of the true nature of the divinity, dedication, strength and righteousness; and help the youth to shape their own destiny by performing selfless action as also control over mind, body and speech.

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Nature is an excellent example of unity in diversity. At its basic constituent level, atom, it is constituted by particles of different nature. Some of them are of opposite in nature, and experience a strong force of attraction, yet they continue to exist separately and individually; particles of similar nature, having stong force of repulsion continue to exist in vicinity. This has been there since beginning of nature, and shall continue to exist indefinitely. Any unregulated infringement on the other would be a disastrous. The secret of coexistence is respecting others position.

GROWING WITH CONCEPTS

Concepts of an expert are not like a static foundation of a huge structure; rather it is like blood flowing in a vibrant mind.

During growing into an expert, each one must have used best of the books available on subject and received guidance of best of the teachers. Authors might have had limitations to take every concept thread bare from first principle and so also must be the constraint of teacher while mentoring a class with a diversity of inquisitiveness and focus. As a result, there are instances when on a certain concept a discomfort remains. The only remedy is to live with the conceptual problem and continue to visualize it thread bare till it goes to bottom of heart and that is an ***ingenious illustration****.*

In this column an effort is being made to take one topic on Mathematics, Physics and Chemistry in each e-Bulletin and provide its illustration from First Principle. We invite all experts in these subjects to please mail us their ingenious illustrations and it would be our pleasure to include it in the column.

We hope this repository of ingenious illustrations, built over a period of time, would be helpful to ignite minds of children, particularly to aspiring unprivileged students, that we target in this initiative, and in general to all, as a free educational web resource.

This e-Bulletin covers – a) Mathematics, b) Physics , and c) Chemistry. This is just a beginning in this direction. These articles are not replacement of text books and reference books. These books provide a large number of solved examples, problems and objective questions, necessary to make the concepts intuitive, a journey of educational enlightenment.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics it is contemplated to come up representative problems from contemporary text books and Question papers from various competitive examinations and a guide to theirsolutions in a structured manner, as a dynamic exercise to catalyse the conceptual thought process.

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OUR MENTORING PHILOSOPHY: Mentoring is not teaching, neither tuition nor coaching. It is an activity driven by passion, and commerce has no place in it. In this effort is to caution students that -

- This place is not where they will be taught how to score marks and get higher ranks, but to conceptualize and visualize subject matter in their real life so that it becomes intuitive.
- This place is not to aim at solutions but inculcate competence to analyze a problem and evolve solution.
- This place does not extend selective and personalized attention, rather an opportunity to become a part of which is focused on learning and problem solving ability collectively.
- This place provides an opportunity to find students above and below one's own level of learning. Thus students develop not in isolation but learn from better ones and associate in problem solving to those who need help. This group dynamics while create a team spirit, an essential attribute of personality, while one learns more by teaching others.
- This place has strategically chosen Online Mentoring, so that those who are unprivileged can gather at one point and those who can facilitate learning of such students by creating, necessary IT setup. Aseperate **Mentor's Manual** is being developed to support the cause.

We are implementing this philosophy through **Online Mentoring**

INTEGRAL CALCULUS

Prof. SB DHAR

Definition

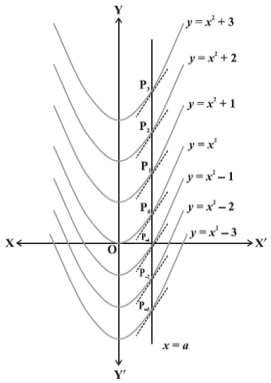
If $\frac{d}{dx}g(x) = f(x)$, then *f(x)* is called the **derivative** of g(x); g(x) is called **anti-derivative** or **primitive** or **integral** of f(x), and is written as $\int f(x)dx = g(x)$.

It is read as **"Integral of f(x) is equal to g(x)".** The symbol $\int$ denotes "Integral" and the process of evaluating g(x) is called **"Integration"**.

For example: $\frac{d}{dx}\sin x = \cos x \Rightarrow \int \cos x dx = \sin x$

Important Facts

1. If two primitives f_1 and f_2 exist for a function then they differ by a constant.
2. Geometrically, indefinite integral refers to family of curves parallel to a curve upward or downward. Example:



3. Derivative of a function is unique but anti-derivative is not unique.
4. A function is differentiable at a point but integrable on an interval.

Some Important Integrals

1. $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$
2. $\int dx = x$
3. $\int \frac{dx}{x} = \log_e x$
4. $\int a^x dx = a^x \log_a e$
5. $\int e^x dx = e^x$
6. $\int \sin x dx = -\cos x$
7. $\int \cos x dx = \sin x$
8. $\int \tan x dx = \log_e \sec x$
9. $\int \cot x dx = -\log_e \cos ecx$
10. $\int \sec x \tan x dx = \sec x$
11. $\int \cos ecx \cot x dx = -\cos ecx$
12. $\int \sec^2 x dx = \tan x$
13. $\int \cos ec^2 x dx = -\cot x$
14. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = -\cos^{-1} x$
15. $\int \frac{dx}{1+x^2} = \tan^{-1} x = -\cot^{-1} x$
16. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x = -\cos ec^{-1} x$
17. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} = -\cos^{-1}\frac{x}{a}$
18. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right|$
19. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right|$
20. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} = -\frac{1}{a}\cot^{-1}\frac{x}{a}$
21. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} = -\frac{1}{a}\cos ec^{-1}\frac{x}{a}$
22. $\int \sec x dx = \log(\sec x + \tan x) = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$
23. $\int \cos ecx dx = \log(\cos ecx - \cot x) = \log\tan\frac{x}{2}$

24. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a}$

25. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}$

26. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

27. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)$

28. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)$

29. $\int |x|dx = \frac{1}{2}x|x|$

Some Important Substitutions

(a) For integral of type $\int \sqrt{(x-\alpha)(\beta-x)}dx$, assume $x = \alpha\cos^2\theta + \beta\sin^2\theta$.

(b) For integral of type $\int \sqrt{\frac{x-a}{x-b}}dx ...or... \int \sqrt{(x-a)(x-b)}dx$, assume $x = a\sec^2\theta - b\tan^2\theta$.

(c) For integral of type $\int \frac{1}{\sqrt{(x-a)(x-b)}}dx$ assume $x-a = t^2$.

(d) For integral of type $\int \sqrt{2ax-x^2}\,dx$, assume $x = a(1-\cos\theta)$.

Methods To Find Integrals

(a) ILATE Rule: $\int uvdx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx$.

u is named the first function and v the second function. The First function is selected through ILATE (order should be Inverse, Logarithmic, Algebraic, Trigonometric, and Exponential).

(b) General formulae for Integration by parts

$\int uvdx = uv_1 - u'v_2 + u''v_3 - ... + (-1)^{n-1}u^{n-1}v_n - (-1)^{n-1}\int u^n v_n dx$

(c) In the Integrals of Type $\int e^{ax}\cos bx.dx$ any one of the two can be taken as the First Function.

(d) In the Integral of Type $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \sqrt{ax^2+bx+c}dx$ the Denominators/Numerator should be made perfect square and then (x+ b/2a) should be put equal to t and proceed further.

(e) In the Integral of Type $\int \frac{Q(x)dx}{ax^2+bx+c}$, First the Numerator should be made polynomial of lesser than the Denominator by dividing the Denominator and then proper substitution should be made to start.

(f) In the Integral of Type $\int \frac{dx}{a\cos^2 x + b\sin^2 x}$, $\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{(a\sin x + b\cos x)^2}$, $\int \frac{dx}{a+b\cos^2 x + c\sin^2 x}$, the numerator and the denominator should be multiplied by sec² x and then in the denominator *tan x* should be put = t and start doing integrations after proper substitutions.

(g) In the integrals of Type $\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a+b\cos x}$, $\int \frac{dx}{a\sin x + b\cos x}$, $\int \frac{dx}{a+b\sin x + c\cos x}$, the proper way is to use $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and after putting tan x/2 = t the sum can be done.

(h) In the integral of the Type $\int \frac{(p\sin x + q\cos x + r)dx}{(a\sin x + b\cos x + c)}$, the proper substitution is to put Numerator= λ+ μ(differential coefficient of denominator)+ν. And then after finding the numeric values for the

assumed arbitrary constants, the Integrals can be done.

(i) In the integral of Type $\int \frac{Q(x)dx}{P(x)}$, the Numerator should be first made of lesser degree than the Denominator and then if the Denominator is decomposable to factors, the method of partial fractions should be used to split into different fractions and then one of the proper methods can be used to start for Integration.

(j) In the integral of Type $\int \frac{dx}{(a+bx)\sqrt{cx+d}}$, put $cx+d=t^2$ and proceed using the method as required.

(k) In the integral of Type $\int \frac{dx}{(a+bx+cx^2)\sqrt{px+q}}$, put $px+q=t^2$ and proceed using the method as required.

(l) In the integral of Type $\int \frac{dx}{(a+bx)\sqrt{px^2+qx+r}}$, put $a+bx=\frac{1}{t}$ and proceed using the method as required.

(m) In the integral of Type $\int \frac{dx}{(a+bx^2)\sqrt{cx^2+d}}$, put $x=\frac{1}{t}$ and proceed using the method as required.

(n) For the integral of type $\int \frac{\cos 2\alpha x-\cos 2\beta x}{1+2\cos 2\gamma x}dx$ where α, β, γ are such that 3γ= α+β multiply the Numerator and the Denominator by sin(γ/2) and then use 2sinAcosB or 2sinAsinB formulae as required.

Example:

$$I=\int \frac{\cos 13x-\cos 14x}{1+2\cos 9x}dx$$

$$=\int \frac{(\cos 13x-\cos 14x)\sin\frac{9x}{2}}{(1+2\cos 9x)\sin\frac{9x}{2}}dx$$

$$=\int \frac{2\sin\frac{27x}{2}.\sin\frac{x}{2}.\sin\frac{9x}{2}}{\sin\frac{27x}{2}}dx$$

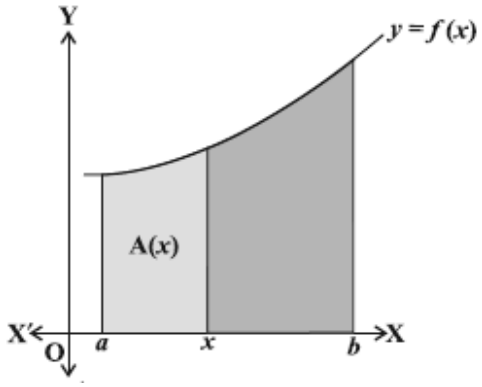
$$=\int 2\sin\frac{x}{2}.\sin\frac{9x}{2}dx=\int(\cos 4x-\cos 5x)dx$$

Definite integrals

Definition

Definite integral is represented as $\int_{x=a}^{x=b} f(x)dx$. It is read as integration from *x=a to x=b*. ***a*** is called the *lower limit* and ***b*** is called the *upper limit*. This integral represents the bounded region i.e., the area between *x=a* and *x=b*.

The figure represents the integral from *x=a* to *x=b* of function *y=f(x)*. The value of this integral is nothing but the area from *x=a* to *x=b* of the function *y=f(x)* bounded between the regions made by ordinates *x=a, x=b* and above *x-axis*. It is Area function as *ydx* represents a rectangle of sides *dx* and *y*.



Properties of Definite Integrals

(a) $\int_a^b f(x)dx=\int_a^b f(t)dt$

(b) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(c) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, a < c < b$

(d) $\int_0^a f(x)dx = \int_0^{a/2} f(x)dx + \int_0^{a/2} f(a-x)dx$

(e) $\int_a^b f(x)dx = 0, if\ ..\ f(a+x) = -f(b-x)$

(f) $\int_a^b f(x)dx = 2\int_a^{\frac{a+b}{2}} f(x)dx, if\ ..\ f(a+x) = f(b-x)$

(g) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(h) $\int_{-a}^a f(x)dx = 0, if\ ..\ f(-x) = -f(x)$

(i) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx, if\ ..\ f(-x) = f(x)$

(j) $\int_0^{2a} f(x)dx = 0, if\ ..\ f(2a-x) = -f(x)$

(k) $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, if\ ..\ f(2a-x) = f(x)$

(l) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(m) $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$ if f(x+T) = f(x)

(n) $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$ if n is an Integer

(o) $\int_0^{a+T} f(x)dx = \int_0^T f(x)dx$ if n=1.

(p) $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$

(q) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$ where n is an Integer.

(r) $\int_a^{a+T} f(x)dx$ is independent of a.

Mean value theorem of Integral Calculus

If a function *f* is continuous on *[a,b]*, it assumes its mean value in *[a,b]*, that is $\frac{1}{b-a}\int_a^b f(x)dx = f(c)$ for some *c* such that $a \le c \le b$.

Some Inequlaities

(a) Schwarz-BunyanKowsky Inequality

If f(x) and g(x) are integral on (a,b) then $\left|\int_a^b f(x)g(x)dx\right| \le \sqrt{\left(\int_a^b f^2(x)dx\right)\left(\int_a^b f^2(x)dx\right)}$ where $f^2(x)=\{f(x)\}^2$.

(b) If f(x) ≥ g(x) on [a, b] then $\int_a^b f(x)dx \ge \int_a^b g(x)dx$

(c) If f(x) is increasing and has a concave graph in [a,b] then

$$(b-a)f(a) < \int_a^b f(x)dx < (b-a)\frac{f(a)+f(b)}{2}$$

(c) If f(x) is increasing and has a convex graph in [a,b] then

$$(b-a)\frac{f(a)+f(b)}{2} < \int_a^b f(x)dx < (b-a)f(b)$$

(d) If m and M be global minimum and global maximum of f(x) respectively in [a,b] then m(b-a) ≤ $\int_a^b f(x)dx \le$ M(b-a)

(e) $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$.

(f) If the integral of any function $\int_a^b f(x)dx$ that is continuous on [a,b] and it is not possible to evaluate this Integrand then we use sandwich formula after finding two continuous functions $f_1(x)$ and $f_2(x)$ on [a,b] such that $f_1(x) \le f(x) \le f_2(x)$; $\forall$ x $\in$ [a,b] then $\int_a^b f_1(x)dx \le \int_a^b f(x)dx \le \int_a^b f_2(x)dx$

(g) If f(t) is an odd function then $\phi(x)=\int_a^x f(t)dt$ is an even function.

(h) If f(t) is an even function then $\phi(x)=\int_a^x f(t)dt$ is an odd function.

(i) Definite integral as the limit of a sum:

$$\int_a^b f(x)dx = \lim_{h\to 0, n\to\infty} h[f(a)+f(a+h)+f(a+2h)+\ldots + f(a+(n-1)h)]$$ where b-a=nh

(j) Find the r^{th} term and write it as $\lim_{n\to\infty}\sum \frac{1}{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$

Some useful Functions

Gamma function:

$\int_0^\infty e^{-x}x^{n-1}dx = \Gamma n$ where x $\in$ Q^+ and n is a positive number. This is also called the Eulerian Integral of 2nd Kind.

Properties

(a) Γn is pronunciated as Gamma n, and, it is denoted as $\Gamma n = (n-1)\Gamma(n-1)$.

(b) For example: $\Gamma 4 = 3.2.1.$; $\Gamma 1 = 1$; $\Gamma 0 = \infty$; $\Gamma(-n) = \infty$ if n is positive integer ; $\Gamma(1/2) = \sqrt{\pi}$

(c) If n is a natural number then $\Gamma(n+1) = n!$ and $\Gamma(1/2) = \sqrt{\pi}$.

(d) $\int_0^{\frac{\pi}{2}} \sin^m x.\cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$ for all m> -1 and n > -1.

Beta Function:

$\int_0^1 x^{m-1}.(1-x)^{n-1}dx$, where m,n>0 is called the Beta Function and is denoted by B(m,n). This is also called ***Eulerian integral of Ist kind***.

Properties

(a) B(m,n)=B(n,m)

(b) B(m,1)= 1/m

(c) $\frac{B(m+1,n)}{B(m,n)} = \frac{m}{m+n}$.

(d) B(m,n)= $\frac{n-1}{m}B(m+1,n-1)$, n > 1

(e) B(m,n)= $\frac{(m-1)!}{n(n+1)(n+2)\ldots(n+m-1)}$ if m is a positive integer.

(f) If m,n are positive integer then B(m,n)= $\frac{(m-1)!(n-1)!}{(m+n-1)!}$.

(g) Another form of Beta Function is given by B(m,n)= $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ where m,n>0. This form is obtained by replacing x=1/(1+y) in the original format.

Relation between Gamma and Beta Function:

(a) B(m,n)= $\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$, m,n>0

(b) $\overline{n}\overline{1-n} = \frac{\pi}{\sin n\pi}$, where 0<n<1

(c) $\int_0^\infty \frac{x^{n-1}}{(1+x)} dx = \frac{\pi}{\sin n\pi}$.

(d) $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

Newton-Leibnitz Formula

$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$ where F(x) is one of the anti-derivative of f(x). This is called as ***Newton-Leibnitz formula***.

Note:

This formula is true to compute the definite integral of a function that is continuous on [a,b].

Integrals with Infinite Limits

If a function f(x) is continuous for a≤ x<∞ then by definition $\int_a^\infty f(x)dx = \lim_{b\to\infty} \int_a^b f(x)dx$. If there exists a finite limit on the right hand side, then the improper integral is said to be convergent otherwise it is divergent.

Area Function

A(x)= $\int_a^x f(x)dx$, if x is a point in [a,b].

Facts to remember

(a) The area of the region bounded by y²=4ax, x=c, c>0,a>0 = $\frac{8c\sqrt{ac}}{3}$.

(b) The area of the region bounded by x²=4ay, y=c, c>0,a>0 = $\frac{8c\sqrt{ac}}{3}$.

(c) The area of the region bounded by y²=4ax and its latus rectum(x=a)= $\frac{8a^2}{3}$.

(d) The area of the region bounded by x²=4ay and its latus rectum(y=a)= $\frac{8a^2}{3}$.

(e) The area of the region bounded by y²=4ax and y=mx, = $\frac{8a^2}{3m^3}$.

(f) The area of the region bounded by x²=4ay and x=my, = $\frac{8a^2}{3m^3}$.

(g) The area of the region bounded by y²=4ax and x²=4ay, = $\frac{16a^2}{3}$.

(h) The area of the region bounded by y²=4ax and x²=4by, = $\frac{16ab}{3}$.

(i) The area of the region bounded by y²=4ax and x²=4ay, and x=a, = $\frac{5a^2}{4}$.

Fundamental Theorems of Integral Calculus

(a) First fundamental theorem of integral calculus:

If Area function, A(x)= $\int_a^x f(x)dx$ for all x ≥ a, & f is continuous on [a,b].Then $A'(x) = f(x)$ for all x ∈ [a, b].

(b) Second fundamental theorem of integral calculus:

Let f be a continuous function of x in the closed interval $[a, b]$ and let F be another function such that $\frac{d}{dx}F(x) = f(x)$ for all x in domain of f, then

$\int_a^b f(x)dx = [F(x)+c]_a^b = F(b) - F(a)$

Some Typical Results:

(a) $\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} = \pi, if, \beta > \alpha$

Ex. $\int_2^3 \frac{dx}{\sqrt{(x-2)(3-x)}} = \pi$

(b) $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}dx = \frac{\pi}{8}(\beta-\alpha)^2$

(c) $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}}dx = \frac{\pi}{2}(\beta-\alpha)$

(d) $\lim_{x\to 0}\left|\frac{\int_0^x f(x)dx}{x}\right| = f(0)$

Ex. $\lim_{x\to 0}\left|\frac{\int_0^x e^x dx}{x}\right| = e^0 = 1$

(e) $\int_a^b f(x)dx = \frac{1}{n}\int_{na}^{nb} f(x)dx$

(f) $\int_{a-c}^{b-c} f(x+c)dx = \int_a^b f(x)dx$

(g) $\int_a^b f(x)dx = (b-a)\int_0^1 f[(b-a)t+a]dt$

(h) $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{1}{2}(b-a)$

(i) $\int_0^{\pi/2} \frac{a.\sin x + b.\cos x}{\sin x + \cos x}dx = \int_0^{\pi/2} \frac{a.\sec x + b.\cos ecx}{\sec x + \cos ecx}dx$

$= \int_0^{\pi/2} \frac{a.\tan x + b.\cot x}{\tan x + \cot x}dx$

(j) $\int_a^b (|x-a|+|x-b|)dx = (b-a)^2$ Ll

(k) $\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \frac{(n-1)(n-3)....4.2}{n(n-2)....3.1}$, when n is odd

$= \frac{(n-1)(n-3)....3.1}{n(n-2)....4.2}$ when n is even

(l) $\int_0^a \frac{dx}{1+e^{f(x)}} = \frac{a}{2}$, if f(a-x)=f(x)

Ex. $\int_0^\pi \frac{dx}{1+e^{\sin x}} = \frac{\pi}{2}$

(m) $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)dx = \frac{1}{n-1}$

(n) $\int_0^{\pi/4} (\cot^n x + \cot^{n-2} x)dx = \frac{1}{n-1}$

(o) Let a function f(x,α) be a continuous for a≤ x≤ b and c ≤α≤ d . Then for any α∈ [c,d], if **I**(α)= $\int_a^b f(x,\alpha)dx$ then **I**'(α)= $\int_a^b f'(x,\alpha)dx$ where I'(α)is the derivative of **I**(α) wrt α and f'(x,α) is the derivative of f(x,α) wrt α keeping x constant.

(p) $\int_a^b [x]dx = \frac{(b-a)(b+a-1)}{2}$ where a,b are integers and [x] is a greatest integer function

(q) $\int_0^n [x]dx = \frac{n(n-1)}{2}$ where n is a positive integer.

(r) $\int_0^n \{x\}dx = \frac{n}{2}$ where n is a positive integer.

(s) $\int_0^n [x]dx = [n].\left(\frac{n+\{n\}-1}{2}\right)$ where n is a real number.

(t) $\int_0^n \{x\}dx = \frac{(b-a)}{2}$ where a ,b are integers.

(u) $\int_0^n [kx]dx = \frac{nk(nk-1)}{2k}$ where n,k are positive integers.

(v) $\int_0^n [x^2]dx = -\left(1+\sqrt{2}+\sqrt{3}+....+\sqrt{n^2-1}+(n^2-1)n\right)$ where n is an integer.

(w) $\int_0^n [x^k]dx = -\left(1+2^{\frac{1}{k}}+3^{\frac{1}{k}}+....+(n^k-1)^{\frac{1}{k}}+(n^k-1)n\right)$

where n,k are positive integers.

(x) $\int_0^{\frac{2\pi}{k}} [\cos kx]dx = -\frac{\pi}{k}$ where k is a positive integer.

(y) $\int_0^{\frac{2\pi}{k}} [\sin kx]dx = -\frac{\pi}{k}$ where k is a positive integer.

(z) $\int_a^b [x]dx = \int_a^{[a]+1} [x]dx + \int_{[a]+1}^{[b]} [x]dx + \int_{[b]}^b [x]dx$ where a, b are any real numbers.

(aa) $\int_a^b \{x\}dx = \int_a^{[a]+1} \{x\}dx + \int_{[a]+1}^{[b]} \{x\}dx + \int_{[b]}^b \{x\}dx$ where a,b are any real numbers.

(bb) $\int_0^n \left[\frac{x}{k}\right]dx = pn - \frac{p(p+1)k}{2}$ where p=[n/k] and n,k are positive integers.

(cc) $\int_0^n \{kx\}dx = \frac{n}{2}$ where n is a positive integer.

(dd) $\int_0^n \left\{\frac{x}{k}\right\}dx = \frac{n^2}{2k} - pn + \frac{p(p+1)k}{2}$ where p=[n/k] and n,k are positive integers.

Partial fraction methods are as under:

(a) $\frac{1}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)}$,

(b) $\frac{1}{(x-a)(x-b)^2} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-b)^2}$

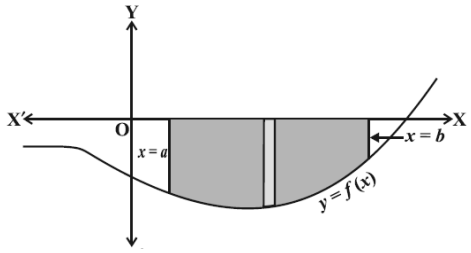
(c) $\frac{1}{(x-a)(x^2+b)} = \frac{A}{(x-a)} + \frac{Bx+C}{(x^2+b)}$

(d) $\frac{1}{(x-a)(x^2+b)^2} = \frac{A}{(x-a)} + \frac{Bx+C}{(x^2+b)} + \frac{Dx+E}{(x^2+b)^2}$

Application of Integration

Integration is very useful in finding the area of a curve bounded by either ordinates or abscissas. The area is always a positive quantity, hence we denote the Area Function as $\left|\int_{x=a}^{x=b} f(x)dx\right|$ or $\left|\int_{y=a}^{y=b} f(y)dy\right|$.

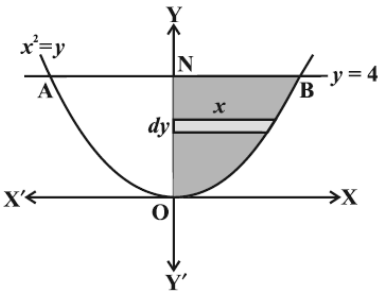
The value of integral is negative when the curve lies below x-axis.



Example:

Find the area of the region bounded by the curve $y=x^2$ and the line $y=4$.

Solution:



The figure shows the required area as

$$A=\left|\int_{y=0}^{y=4} f(y)dy\right| = \left|\int_{y=0}^{y=4} \sqrt{y}dy\right| = \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_{y=0}^{y=4} = \frac{16}{3}$$

Some Illustrations

1. Find the integral of $\frac{1-\sin x}{\cos^2 x}$.

Solution:

$$\int \frac{1-\sin x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx - \int \frac{\sin x}{\cos^2 x}dx$$

$$= \int \sec^2 x dx - \int \sec x \tan x dx$$

=tanx-secx+C,

where C is a constant of Integration.

2. Evaluate: $\int \frac{\sin(\tan^{-1} x)}{1+x^2}dx$

Solution:

Assume $\tan^{-1}x=t$.

On differentiating both sides, we get

$$\frac{1}{1+x^2}dx = dt$$

Hence

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2}dx = \int \sin t dt = -\cos t + C$$

3. Evaluate: $\int \frac{dx}{1+\tan x}$

Solution:

$$\int \frac{dx}{1+\tan x} = \int \frac{dx}{1+\frac{\sin x}{\cos x}} = \int \frac{\cos x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{\cos x+\sin x+\cos x-\sin x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \left(\frac{\cos x+\sin x}{\sin x+\cos x}+\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$$

$$= \frac{1}{2}\int \left(1+\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$$

$$= \frac{1}{2}\left(x+\log(\sin x+\cos x)\right)+C$$

4. Evaluate: $\int \sin 2x \cos 3x dx$

Solution:

Let us use the Identity

2sinA cosB=sin(A+B)+sin(A-B)

Hence

$$\int \sin 2x \cos 3x dx = \frac{1}{2}\int \left[\sin(2x+3x)+\sin(2x-3x)\right]dx$$

$$= \frac{1}{2}\int \left[\sin 5x - \sin x\right]dx$$

$$= \frac{1}{2}\left(-\frac{\cos 5x}{5}+\cos x\right)+C$$

5. Evaluate: $\int \frac{dx}{\sqrt{5x^2-2x}}$

Solution:

We know that

$$5x^2-2x = 5\left(x^2-\frac{2}{5}x\right) = 5\left(x^2-\frac{2}{5}x+\frac{1}{25}-\frac{1}{25}\right)$$

$$= 5\left(\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2\right)$$

Therefore,

$$\int \frac{dx}{\sqrt{5x^2-2x}} = \int \frac{dx}{\sqrt{5\left[\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2\right]}} = \frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{t^2-a^2}}$$

where $t = x-\frac{1}{5}, a = \frac{1}{5}$

$$= \frac{1}{2a\sqrt{5}}\log\frac{t-a}{t+a}+C$$

Replace t and a by its values.

6. Find $\int \frac{dx}{(x+1)(x+2)}$.

Solution:

Write $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)}+\frac{B}{(x+2)}$

On solving the equation, A=1, B=-1

Hence $\frac{1}{(x+1)(x+2)}=\frac{1}{(x+1)}-\frac{1}{(x+2)}$

Now $\int\frac{dx}{(x+1)(x+2)}=\int\frac{dx}{(x+1)}-\int\frac{dx}{(x+2)}$

$=\log_e(x+1)-\log_e(x+2)+C$

7. Find $\int x\cos x dx$

Solution:

Use ILATE rule

$$\int x\cos xdx=x\int\cos xdx-\int\left(\frac{d}{dx}x\right)\left(\int\cos xdx\right)dx$$

=x(sinx)- $\int 1.(\sin x)dx$ =x(sinx)+cosx+C

8. Find $\int_0^2 x^2dx$ as the limit of a sum.

Solution:

We know that

$$\int_{x=a}^{x=b}f(x)dx=(b-a)\underset{n\to\infty}{Lim}\frac{1}{n}\{f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h\}$$

Where, $h=\frac{b-a}{n}$

a=0, b=2, therefore nh=2

$$\int_{x=0}^{x=2}x^2dx=2\underset{n\to\infty}{Lim}\frac{1}{n}\{f(0)+f(0+h)+f(0+2h)+...+f(0+(n-1)h\}$$

$$=2\underset{n\to\infty}{Lim}\frac{1}{n}\{0+h^2+4h^2+...+(n-1)^2h^2\}$$

$$=2\underset{n\to\infty}{Lim}\frac{1}{n}\sum(n-1)^2h^2$$

g

$$=2\underset{n\to\infty}{Lim}\frac{1}{n}\sum(n^2-2n+1)h^2$$

$$=2\underset{n\to\infty}{Lim}\frac{1}{n}\left(\sum n^2-2\sum n+\sum 1\right)h^2$$

$$=2\underset{n\to\infty}{Lim}\frac{1}{n}\left(\frac{n(n+1)(2n+1)}{6}-2\frac{n(n+1)}{2}+n\right)h^2$$

$$=2\underset{n\to\infty}{Lim}\left(\frac{(n+1)(2n+1)}{6}-2\frac{(n+1)}{2}+1\right)h^2$$

$$=\frac{2}{6}\underset{n\to\infty}{Lim}((n+1)(2n+1)-6(n+1)+6)h^2$$

$$=\frac{2}{6}\underset{n\to\infty}{Lim}(2n^2-3n+1)h^2$$

$$=\frac{1}{3}\underset{n\to\infty}{Lim}(2n^2h^2-3nh^2+h^2)$$

$$=\frac{1}{3}(2\times4)=\frac{8}{3}$$

9. Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2x}dx$

Solution:

Let , $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2x}dx$ ....(i)

Applying property $I=\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2x}dx \quad ....(ii)$$

By (i)+(ii)

$$2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2x}dx\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2x}dx$$

$$\Rightarrow I=-\frac{\pi}{2}\int_1^{-1}\frac{dt}{1+t^2}dx=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}dx$$

$$\Rightarrow I=\frac{\pi}{2}\left(\tan^{-1}t\right)_{-1}^{1}=\frac{\pi}{2}\left(\tan^{-1}1-\tan^{-1}(-1)\right)$$

$$\Rightarrow I = \frac{\pi}{2}\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = \frac{\pi^2}{4}$$

10. Evaluate: $\int_0^{\frac{\pi}{2}} \log\sin x dx$

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \log\sin x dx$ .....(i)

Using Property $I = \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2}-x\right)dx = \int_0^{\frac{\pi}{2}} \log\cos x dx \text{ ...(ii)}$$

From (i)+(ii),

$$2I = \int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log(\sin x\cos x)dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\frac{2\sin x\cos x}{2}dx$$

$$2I = \int_0^{\frac{\pi}{2}} (\log\sin 2x - \log_e 2)dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log\sin 2x dx - \int_0^{\frac{\pi}{2}} \log_e 2 dx$$

$$2I = \frac{1}{2}\int_0^{\pi} \log\sin t dt - (\log_e 2)\frac{\pi}{2}$$

$$2I = \int_0^{\pi/2} \log\sin t dt - (\log_e 2)\frac{\pi}{2}$$

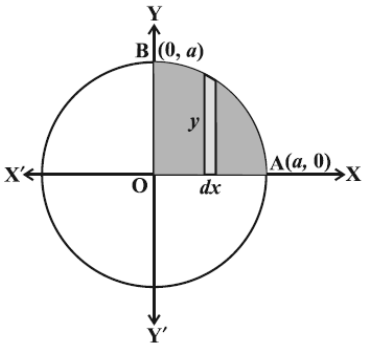
$$2I = \int_0^{\pi/2} \log\sin x dx - (\log_e 2)\frac{\pi}{2}$$

$$2I = I - (\log_e 2)\frac{\pi}{2}$$

$$\therefore I = -(\log_e 2)\frac{\pi}{2} = \frac{\pi}{2}\log_e\frac{1}{2}$$

11. Find the area enclosed by the circle $x^2+y^2=a^2$.

Solution:



The required area is $A = 4\int_0^a ydx$

$$A = 4\int_0^a \sqrt{a^2-x^2}dx$$

Using standard Integral

$$\int_0^a \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$$

$$A = 4\left(\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)_0^a = \pi a^2$$

12. If {x} represents the fractional part of x, then evaluate $\int_0^{100} \{\sqrt{x}\}dx$.

Hint: Write {x}=x-[x]

And hence the integrand can be written as $\int_0^{100}\left(\sqrt{x}-\left[\sqrt{x}\right]\right)dx=\int_0^{100}\sqrt{x}dx-\int_0^{100}\left[\sqrt{x}\right]dx$

Rewrite the second integrand as $\int_0^1\left[\sqrt{x}\right]dx+\int_1^4\left[\sqrt{x}\right]dx+\int_4^9\left[\sqrt{x}\right]dx+....+\int_{81}^{100}\left[\sqrt{x}\right]dx$

Or $\int_0^1 0.dx+\int_1^4 1.dx+\int_4^9 2.dx+....+\int_{81}^{100}9.dx$

Solve and find the required value.

13. Evaluate $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=1}^{n}r.e^{\frac{r}{n}}$

Hint: Rewrite the expression as

$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}.e^{\frac{r}{n}}$. Replace (r/n) by x and (1/n) by dx and put limit x=0 as lower limit and x=1 as the upper limit and integrate.

14. Evaluate $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{2n}\right)$

Hint: Find out the rth term and convert it into (r/n) and (1/n) form to be replaced by x and dx respectively and integrate between limits x=0 to x=1.

15. If F(x)= $\frac{1}{x^2}\int_4^x\{14t^2-2F'(t)\}dt$ then find F(4).

Hint: Obviously if x=4, the integrand becomes from x=4 to x=4 i.e. 0

16. Find the value of $\int_0^1\frac{d}{dx}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right)dx$.

Hint: Use Newton-Leibnitz formula for definite integral treating upper limit x and then putting x=1 to have the value π/2, as below:

$=\sin^{-1}\left(\frac{2x}{1+x^2}\right)-0=\sin^{-1}\left(\frac{2.1}{1+1}\right)$

Newton-Leibnitz formula for differentiation of a definite integral:

$\frac{d}{dx}\int_{f(x)}^{g(x)}F(x)dx=F(g(x))d(g(x))-F(f(x))d(f(x))$

17. Find the mistake in the following evaluation of the integral

$\int_0^{\pi}\frac{dx}{1+2\sin^2 x}=\int_0^{\pi}\frac{\sec^2 xdx}{1+3\tan^2 x}=\frac{1}{\sqrt{3}}\int_0^{\pi}\frac{d(\sqrt{3}\tan x)}{1+(\sqrt{3}\tan x)^2}$

$=\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi}=0.$

The integral of a function positive everywhere turns out to be zero.

Hint: Note that the function $\tan^{-1}(\sqrt{3}\ \tan x)$ is discontinuous at (x = π/2) in the interval [0,π].

As LHL ≠ RHL at x=π/2.

Hence the correct result can be evaluated as below:

$\int_0^{\pi}\frac{dx}{1+2\sin^2 x}=\int_0^{\pi}\frac{\sec^2 xdx}{1+3\tan^2 x}=2\int_0^{\pi/2}\frac{\sec^2 x}{1+(\sqrt{3}\tan x)^2}dx$

using the property of Definite Integral for even function.

$=2\int_0^{\pi/2}\frac{\sec^2 x}{1+(\sqrt{3}\tan x)^2}dx=\frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_0^{\pi/2}$

$=\pi/\sqrt{3}$

18. Evaluate $\int_0^{\infty}\frac{e^{-ax}\sin bx}{x}dx$ and then deduce the value of $\int_0^{\infty}\frac{\sin bx}{x}dx$.

Hint: Assume a variable function of b

g(b)= $\int_0^{\infty}\frac{e^{-ax}\sin bx}{x}dx$

differentiate both sides w.r.t b

$$\frac{dg(b)}{db}=\int_0^\infty \frac{x.e^{-ax}\cos bx}{x}dx=\int_0^\infty e^{-ax}\cos bx.dx$$

On integration by Parts, the RHS becomes

$$\left[e^{-ax}.\frac{\sin bx}{b}\right]_0^\infty+\frac{a}{b}.\int_0^\infty e^{-ax}\sin bx.dx$$

On again integration by parts, the Intrgrand becomes

$$\frac{a}{b^2}-\frac{a^2}{b^2}\frac{dg(b)}{db}$$

i.e. $\frac{dg(b)}{db}=\frac{a}{a^2+b^2}$ .since b is a variable and a is a constant hence

g(b)= $\tan^{-1}(b/a)+c$

when b=o, g(b)=o and hence c=o

therefore,

g(b)= $\tan^{-1}(b/a)$.

by putting a=o,

the other result can be evaluated as below

$$\int_0^\infty \frac{e^{-ax}\sin bx}{x}dx=\tan^{-1}\frac{b}{a}$$

$$\int_0^\infty \frac{\sin bx}{x}dx=\lim_{a\to 0}\tan^{-1}\frac{b}{a}=\frac{\pi}{2}.$$

19. Evaluate $\int_0^\infty \frac{\tan^{-1}(bx)}{x(1+x^2)}dx$.

Hint: Assume a variable function of b

g(b)= $\int_0^\infty \frac{\tan^{-1}(bx)}{x(1+x^2)}dx$.

differentiate w.r.t b

$$\frac{dg(b)}{db}=\int_0^\infty \frac{1}{(1+b^2x^2)(1+x^2)}dx$$

.

Use partial fraction method to evaluate the Integral as $\frac{dg(b)}{db}=\frac{\pi}{2(1+b)}$.

Now integrate both sides to get g(b)= $\frac{\pi}{2}\log(1+b)+c$

and c=o when b=o so the function is $\frac{\pi}{2}\log(1+b)$

20. Show that $\left|\int_{10}^{19}\frac{\sin x}{1+x^8}dx\right|<\frac{1}{10^7}$

Hint: This problem is related to Inequality

$$\left|\int_a^b f(x)dx\right|\le\int_a^b|f(x)|dx$$

$$\left|\int_{10}^{19}\frac{\sin x}{1+x^8}dx\right|\le\int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx=\int_{10}^{19}\frac{|\sin x|}{1+x^8}dx\le\int_{10}^{19}\frac{1}{1+x^8}dx...as..|\sin x|\le 1$$

$\le\int_{10}^{19}\frac{1}{x^8}dx$ and this can be done now easily.

21. Evaluate: $\int_0^{\pi/2}\log_e\left(\alpha^2\cos^2\theta+\beta^2\sin^2\theta\right)d\theta$

Hint: Assume the Integrand

I= $\int_0^{\pi/2}\log_e\left(\alpha^2\cos^2\theta+\beta^2\sin^2\theta\right)d\theta$

Differentiate w.r.t α under integral sign

$$\frac{dI}{d\alpha}=\int_0^{\pi/2}\frac{2\alpha\cos^2\theta}{\left(\alpha^2\cos^2\theta+\beta^2\sin^2\theta\right)}d\theta$$

$$=\frac{2\alpha}{\left(\alpha^2-\beta^2\right)}\int_0^{\pi/2}\left\{1-\frac{\beta^2}{\left(\alpha^2-\beta^2\right)\cos^2\theta+\beta^2}\right\}d\theta$$

Integrate and Simplify using traditional method of multiplying the second part of the Integrand's Nr and Dr by $\sec^2\theta$.

$$\frac{dI}{d\alpha}=\frac{\pi}{\alpha+\beta}$$

Now Integrate,

I= $\pi\log_e(\alpha+\beta)+c$

Put $\alpha=\beta$ and evaluate $I=\int_0^{\pi/2}\log_e\alpha^2\left(\cos^2\theta+\sin^2\theta\right)d\theta=\pi\log\alpha$

$\Rightarrow\pi.\log_e\alpha=\pi.\log_e 2\alpha+c \Rightarrow c=\pi\log_e(1/2)$.

$\Rightarrow I=\pi\log_e(\alpha+\beta)/2$.

22. Let a, b, c be non-zero real numbers such that,

$$\int_0^1\left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

$$=\int_0^2\left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

then show that the quadratic equation $ax^2+bx+c=0$ has at least one root in (1,2).

Hint: Let a function

$$f(x)=\int_0^x\left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

It is continuous on[1,2] and differentiable on(1,2).

Also note f(1)=f(2)

Then there exists a k such that f'(k)=0 by Rolle's Theorem.

$f'(x)=\left(1+\cos^8 x\right)\left(ax^2+bx+c\right)$ where $(1+\cos^8 x)\neq 0$ hence $ax^2+bx+c=0$

And $k\in(1,2)$.

23. Form the differential equation of the family of all parabolas with focus at the origin and the x-axis as the axis.

Hint: Assume the parabola $y^2=4a(x+a)$ and eliminate the arbitrary constant a by twice differentiating both sides.

24. Express in Beta function: $\int_0^1\frac{x^2}{\sqrt{1-x^5}}dx$.

Hint: Rewrite the given integrand as below:

$$\int_0^1 x^2.(1-x^5)^{-1/2}dx=\int_0^1 x^{-2}.(1-x^5)^{-1/2}.x^4dx$$

.

$$=\frac{1}{5}\int_0^1 y^{-2/5}.(1-y)^{-1/2}dy \text{ on putting } x^5=y$$

$= (1/5)\,B(3/5, 1/2)$

25. Prove: $\int_0^a(a-x)^{m-1}.x^{n-1}dx=a^{m+n-1}B(m,n)$.

Hint: Assume x=ay and get the required answer.

26. Let f be an odd function defined and integrable everywhere and also periodic with period 2 as below: $g(x)=\int_0^x f(t)dt$

then

(a) Find the value of f(4).

Hint: f(x) is odd function and defined everywhere, hence f(0)=0 and so f(0)=f(2)=f(4)=0 because it is periodic also with period 2.

(b) Find the value of g(4).

Hint: $g(x)=\int_0^x f(t)dt \Rightarrow g(4)=\int_0^4 f(t)dt$

assume t=u+2.

So dt=du and $g(4)=\int_{-2}^2 f(u+2)du=\int_{-2}^2 f(u)du$

Because f(u+2)=f(u) as it is periodic with period 2 and is also an odd function so by the property of definite integral, its value is zero.

(c) Find the value of g(x+2).

Hint: g(x+2)

$$= \int_0^{x+2} f(t)dt = \int_0^{x} f(t)dt + \int_x^{x+2} f(t)dt$$

$$= g(x) + \int_0^{2} f(t)dt = g(x) + g(2)$$

(d) If f'(-2)= -2 then find the value of f'(2).

Hint: f is an odd function, it will be symmetric about origin specially in the domain (-2,0) and (0,2). Hence the slope at x=2 and x=-2 will be same.

i.e. f'(-2)=f'(2)=-2 as it is given.

(e) If $g(x^2)= x^2(1+x)$ then find the roots of the equation $x^2- f(x^2)=0$.

Hint: Differentiate w.r.t x

$2x\, g'(x^2)=2x(1+x)+x^2$

Or, $g'(x^2)= 1+(3/2)x$

Also g'(x)=f(x) by using Newton-Leibnitz formula in the given relation.

So, the given equation becomes:

$x^2-1-(3/2)x=0$ and hence the roots may be evaluated.

Dr S.B. Dhar, is **Editor of this Quartrerly e-Bulletin**. He is an eminent mentor, analyst and connoisseur of Mathematics from IIT for preparing aspirants of Competitive Examinations for Services & Admissions to different streams of study at Undergraduate and Graduate levels using formal methods of teaching shared with technological aids to keep learning at par with escalating standards of scholars and learners. He has authored numerous books – Handbook of Mathematics for IIT JEE, A Textbook on Engineering Mathematics, Reasoning Ability, Lateral Wisdom, Progress in Mathematics (series for Beginner to Class VIII), Target PSA (series for class VI to class XII) and many more.
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INVITATION FOR CONTRIBUTION OF ARTICLES

Your contribution in the form of an article, story poem or a narration of real life experience is of immense value to our students, the target audience, and elite readers of this Quarterly monthly e-Bulletin ***Gyan-Vigyan Sarita: शिक्षा****; and thus create a visibility of the concerns of this initiative. It gives them a feel that you care for them, and they are anxiously awaiting to read your contributions. We request you to please feel free to send your creation, by* ***20th of this month*** *to enable us to incorporate your contribution in next bulletin, subhashjoshi2107@gmail.com.*

We will be pleased have your association in taking forward path our plans as under-

- ***With the start of Second year of operation, 1st4th Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा shall be brought out 1st October'17.***
- ***And this cycle monthly supplement to Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा aimed to continue endlessly***

We believe that this monthly supplements to quarterly periodicity of e-Bulletins shall make it possible for our esteemed contributors to make contribution rich in content, diversity and based on their ground level work.

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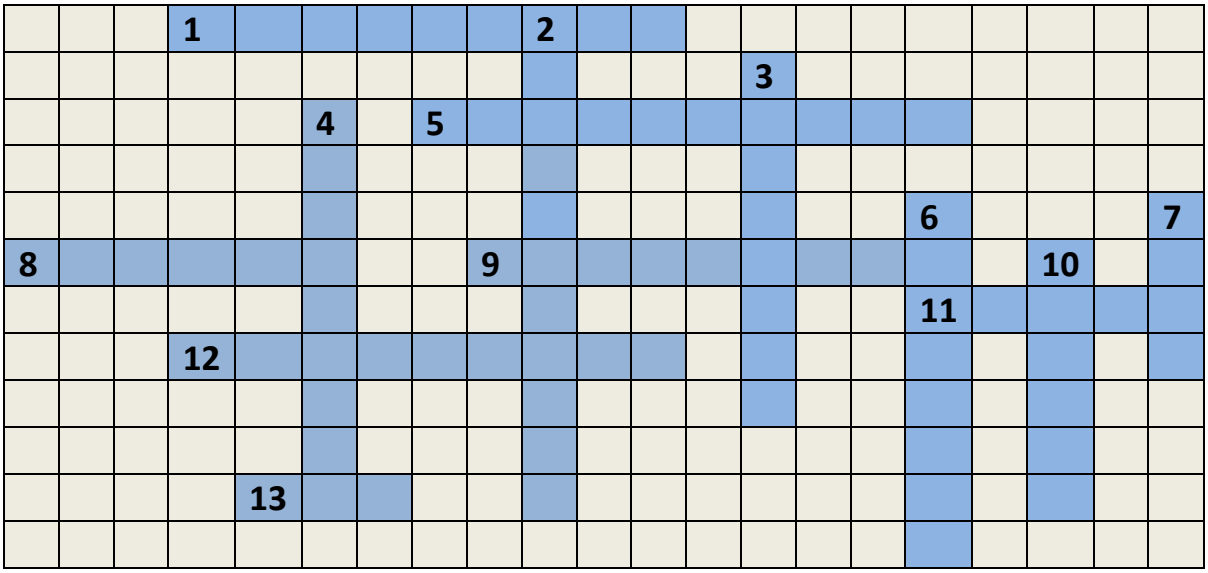
CROSS WORD PUZZLE Nov'17: INTEGRATION

S.B. Dhar



Across

1 Name of One-One-Onto Function

5 Two functions ⊥ to each other

8 Derivative of a function

9 Name of Integral

11 Rule followed in integration by parts

12 Constant of Integration is also called

13 Name of Function f(x) when $\int_{-a}^{a} f(x)dx = 0$

Down

2 Inverse process of Differentiation

3 Integrals of same function differ by

4 Function that is integrable is called

6 Mathematician who introduced function

7 Name of Function represented by $\int_{a}^{x} f(x)dx$

10 Indefinite Integral of a function represents

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ANSWER: CROSSWORD PUZZLE Oct'17: World Days

Prof. S.B. Dhar

		1P																	
	2C	O	L	U	M	B	3U	S											
		S					N												
4A	R	T	H	R	I	T	I	S				5S			6M	A	7T	H	S
							T					T					E		
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							D					N		9M			C		
		10D		11H			N					D		O			H		
12A	N	I	M	A	L		A				13H	A	L	L	O	W	E	E	N
		W		B			T					R		E			R		
14S	T	A	T	I	S	T	I	C	S			D					S		
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There is no idea which is obscure, trivial, ridiculous or obnoxious. All that is needed is to think, imagine and meditate. Pursue the idea relentlessly. In the process, it shall undergo refinement and auto correction and then emerge in a final form, the NEED.

—00—

Education is just not training;
It is about ability to think;
It is about ability to reason;
It is about ability to choose;
It is to develop a faith in self,
And, a passion to apply.

—00—

Growing with Concepts : Physics

Modern Physics: Part II: Atomic Structure

This world comprises of matter and energy; matter is defined with its smallest constituents called Atom which is electrically neutral and exhibits uniform chemical property. The concept of atom is ancient and was evolved philosophically. In 1804 John Dalton explained chemical reactions involving elements in ratios of whole number with the concept of atom. In 1897 when J J Thomson discovered electrons, understanding of atoms took a new turn and this continued with a series of independent hypotheses and experimental discoveries. These discoveries form another set of revolutionary thought experiments, in the form of fantasy based on scientific analysis and understanding of known practical observations. Later, the study of atom took two different directions. One of the directions is into physics which involves organization of elementary particles in atoms alongwith charges, forces and energies associated with them. The other direction is into chemistry which focuses on the impact of structure of atoms into their chemical affinity causing composition and decomposition into molecules. Both of them are complementary to each other.

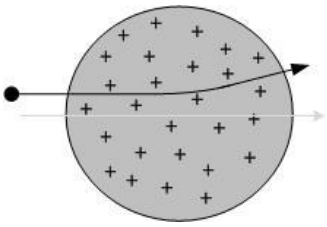
In this manual, upto this point, generally it has been possible to relate how? and why? of illustrations of subject matter using concepts covered in previous chapters of this manual. But, modern physics is a very complex matrix to be handled sequentially in a similar manner. Nevertheless, it is our endeavor to continue with the convention in this section too. However, resolving conceptual notches in content and scope wherever found to be making the trajectory too far, they have been supplemented in Appendix on Quantum Mechanics, an integral part of this section. Desperate readers, needing more details are requested to please come up with their inquisitiveness through CONTACT US. We are committed to pacify them suitably.

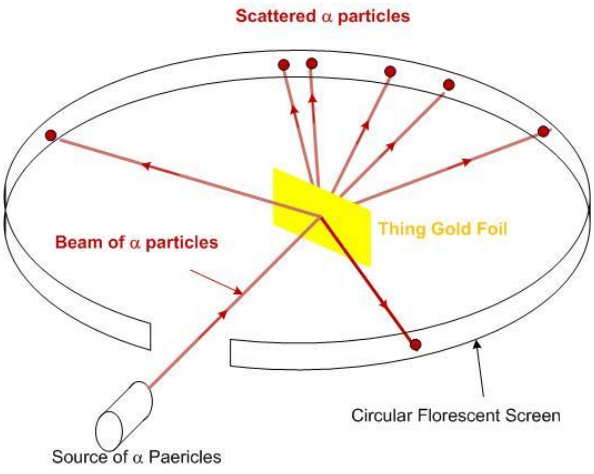
Introduction: Departure in understanding of physics, from classical approach, started in late Nineteenth Century and it took shape with Six revolutionary hypotheses – Quantization of radiation by Max Planck in 1900, Special Theory of Relativity with Photo Electric Effect by Albert Einstein in 1905, Louis de Broglie in 1924 with duality of particle and wave, Weiner Heisenberg on 1927 with Principle of Uncertainty and Erwin Schrödinger with wave equation in 1926. There were a galaxy of scientists who made invaluable contributions through experimental verification of these hypotheses to unfold arrogance of ignorance and thus creating new horizons for imagination. Accordingly, journey into the subject shall start with ***Structure of Atom***, with an appendix on **Quantum Mechanics**. It would leave much more to imagine and explore as one move forward in the journey.

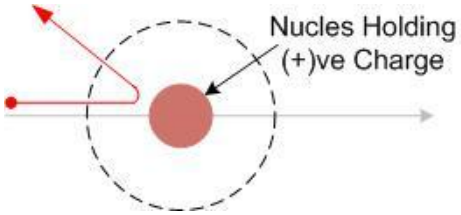
Structure of Atom: Matter is composed of tiny particles अणु परमाणु, in era of science being called molecule and atom, is available in ancient scriptures. A scientific understanding started with study of behavior of gases by **Robert Boyle** in 17th century where he explained compressibility of gases with its tiny particles filling available volume with lot of spacing between them. **John Dalton** in 1800 proposed concept of atom to explain why element in fixed proportion whole number participate in reactions. It was in 1827, **Robert Brown** microscopically studied erratic motion of dust particles in water, in 1827, known as **Brownian Motion** forming basis of *Kinetic Theory of Gases*. Beyond this, *the Brown's theory validated the Dalton's proposition of atom*.

In 1898, **J.J. Thomson**, while conducting experiments with cathode rays proposed structure of atom as a solid sphere carrying (+)ve charge with electron holding (-)ve charge embedded in it to exhibit electrically neutral charge. This model could explain emission of electrons as well as ionization and formation of chemical

compounds. **Philipp Lenard** in 1903 while experimenting effect of cathode rays directed towards a thin film. He observed that some of electrons were passing through the film and thus proposed empty space in atoms allowing passage to the electrons. **Ernest Rutherford**, while bombarding **α-particles,** nucleus of Helium atoms, on gold foil, observed that most of alpha particles either passed through the foil or with small deviation as shown on the figure. Then an understanding of α-particles was incomplete except that they are naturally emitted by radioactive material, and that they carry (+) charge and are ejected by unstable atoms at a speed of about 10^7 m/sec. These particles could travel several cm in air and upto 0.1 mm thin solid material. ***Protons and Neutrons***, a constituent **α-particles** was discovered later in 1918 and 1930, respectively. Probability of straight reflection of **α-particles** was about 1 in 1800. And, in 1911, Rutherford estimated that the **α-particles** is about 7350 times heavier than electron. Thus neither the electron nor (+)ve charged particles could cause large scale deflection. But, the direct rebounding at an angle of deflection 180^0 was indicative of head-on collision with much heavier mass carrying (+) charge which occupy small space at the center of atoms of the material of the foil. Further, he estimated that the size of this central mass, and was later called as **Nucleus** of size 10 Fermi (1 Fermi=1 Fermi = 10^{-15}m) and about 10^{-5} of the size of the atom, and thus empty space in atom around its nucleus was about 10^{-15} times of its size, by simple cubic proportion as per geometry.

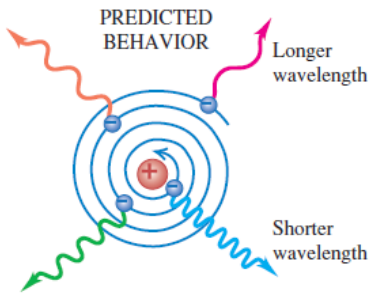






This formed the basis of Nuclear Model of atom, according to which: atom comprises of a concentrated mass called Nucleus having (+)ve charge. Remaining volume of the atom outside nucleus is occupied with electrons at a separation revolving around the nucleus. Number of electrons is such that it equalizes charge on nucleus so as to maintain each atom to be electrically neutral. This provides an explanation of easy emission of electron. Moreover, electron revolving around nucleus in circular orbit provides stability of the atomic structure, caused by equilibrium of electrostatic (centripetal) forces and centrifugal force caused by circular motion.

Limitations of Rutherford's Model: These experimental observations of Rutherford contradicted Thomson's model of distributed mass within the volume of atom. But, this model again did not survive the logical argument: **a)** What prevents electron to fall into positively charged nucleus, under the action of persistent electrostatic force of attraction*;* **b)** If electron are revolving around nucleus, it is possible only on account of persistent centripetal acceleration, and an accelerated charged particle must emit radiation in accordance with theory of electromagnetic field. *But, no such radiation coming out of stable atoms is observed;* **c)** This radiation of energy should lead to loss of kinetic energy of revolving electrons and eventually decrease in radius of the orbit of revolution till it collapses into nucleus. *This does not happen and the atom continues to be stable;* **d)** Reduction in radius of orbit should produce a continuous

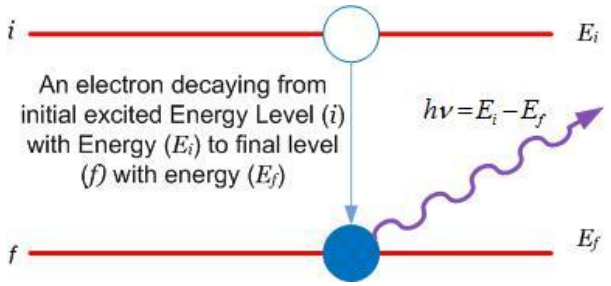


spectrum of radiation and reduction in volume of atom, with the passage of time, creating high density material, *which does not happen*.

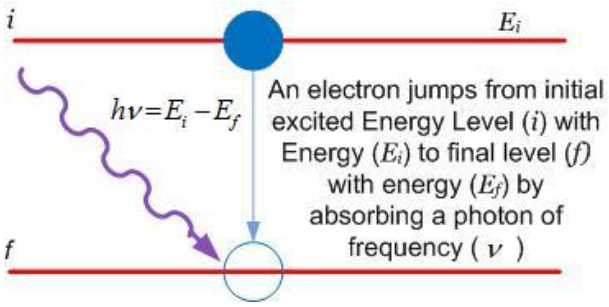
Spectral Lines: A heated metal radiating a continuous spectrum, is an observation since immemorial times. Experimenting with heating of hydrogen in sealed tube, **Johann Balmer**, in 1885 observed discrete emission spectral line, each corresponding to a particular wavelength, known as **Balmer Series**. Later in 1906, **Theodore Lyman** observed another set of spectral lines. Soonafter, in 1908, **Friedrich Paschen**, discovered another set of spectral lines. Wavelengths were fitted into an empirical formula $\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\ m^{-1}$, where, $R = 1.0973 \times 10^7\ m^{-1}$ is known as Rydberg's constant. In similar experiments in **Sumner Brackett** and **August Herman Pfund** discovered new set of spectral lines in 1922 1nd 1924, respectively. Until Bohr could make a sense of these spectral lines to propose structure of atom, discussed below, they remained mystery and a subject of experimental curiosity. Accordingly, elaboration of these spectral lines is deferred until Boh'r is discussed.

Spectral Lines	m	n
Balmer Series	2	1
Lyman Series	>2	2
Paschan Series	>3	3

Boh'r Model: Neil Bohr, who was working with Rutherford, advanced a hypothesis, in 1913, which made a sharp turn from pure classical mechanics with the amalgamation of quantum mechanics into it. The hypothesis was based on a premise that – "*energy of an atom can have only discrete values*". This hypothesis had bearing on spectral lines discovered upto Lyman Series. Accordingly he suggested that :

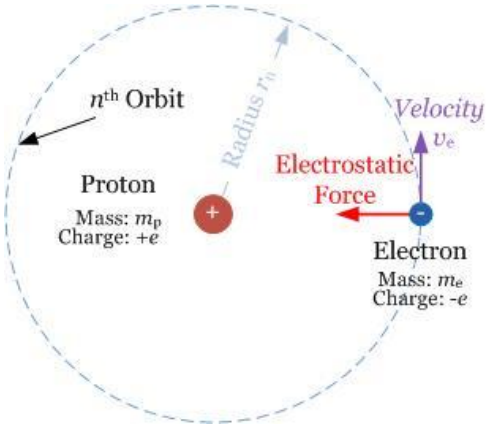


- **a.** Sharp observation of spectral lines on hydrogen is attributed to photon of energy $\boldsymbol{E = h\nu}$ emitted/absorbed by hydrogen atom.
- **b.** During emission of a photon internal energy of an atom is reduced by a quantum equivalent to E.
- **c.** Each atom must be able to exist in states of specific values of internal energy.
- **d.** There are a set of possible energy levels and internal energy of atoms corresponding to each of the discrete energy level occupied by electrons in an isolated atom. No electron can stay in any position which is intermediate to discrete Two energy levels.
- **e.** All isolated atoms of any element have some set of energy levels, but different elements have different set of energy levels.
- **f.** An atom can be excited from one energy level (E_i) to another level (E_f) by imparting energy through by heating, collision of an atom by an accelerated particle. This excited atom makes transition to lower discrete energy level by emitting a photon of wavelength corresponding to difference of energy between two energy levels such that $\boldsymbol{h\nu = E_i - E_f}$.

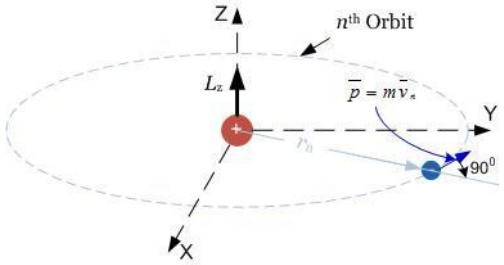


Bohr hypothesis while relating atomic spectra to energy levels of Hydrogen, the simplest atom, he combined quantum mechanics with classical electromechanics. The force of attraction between electron revolving, around nucleus having a proton, in n^{th} orbit of radius r_n in accordance with *Coulomb's Law* is $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r_n^2}$ N, and acts as

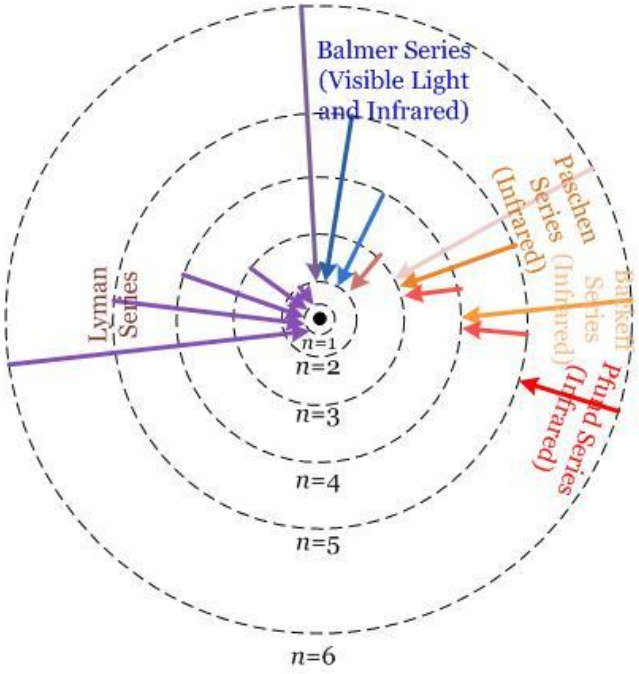
centripetal force. While performing circular motion the electron experiences a constant acceleration $\frac{v_n^2}{r}$ radially directed towards center of the orbit. Accordingly, as per Newton's Second Law of motion, a centrifugal force comes into play which creates equilibrium for electron to continue to perform circular motion. Thus equation of equilibrium of electron having mass m, is $\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{r_n^2}=\frac{mv_n^2}{r_n}\rightarrow r_n=\frac{4\pi\epsilon_0 m}{e^2}\cdot(v_n r_n)^2$.



As per quantum mechanics, Bohr quantized angular momentum of electron as $L_n=mv_nr_n=n\frac{h}{2\pi}\rightarrow v_nr_n=n\frac{h}{2\pi m}$. Combining this with the equation of force, $r_n=\frac{4\pi\epsilon_0 m}{e^2}\cdot\left(n\frac{h}{2\pi m}\right)^2=\epsilon_0\frac{n^2h^2}{\pi me^2}$, is the radius of the n^{th} orbit such that $r_n\propto n^2$ and velocity on electron in the orbit would be $v_n=\frac{1}{\epsilon_0}\cdot\frac{e^2}{2nh}$. Radius of 1st orbit is called Bohr's radius $a_0=5.29\times10^{-11}m$, using predefined values of $\epsilon_0=8.854\times10^{-12}\mathrm{C^2N.m^2}$, $h=6.624\times10^{-34}\mathrm{J.s}$, $m=9.109\times10^{-31}\mathrm{Kg}$, and $e=1.6028.854\times10^{-12}C^2\mathrm{N.m^2}$ Thus permitted radii for orbiting electron are $r_1=a_0$, $r_2=4a_0$, $r_3=9a_0$, and so on.

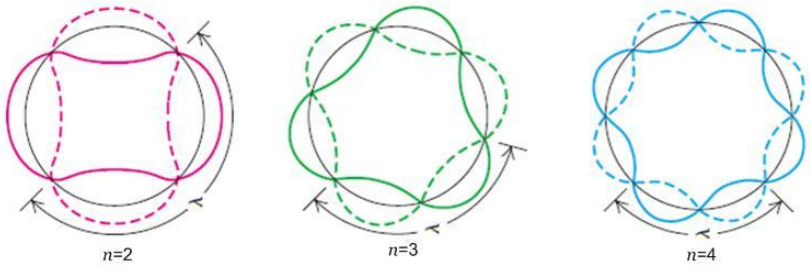


Now, extending *Bohr's Model* to the analysis of energy (E_n) of an electron at n^{th} energy level. It has Two components, Potential Energy $E_{Pn}=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r_n}=-\frac{1}{\epsilon_0^2}\cdot\frac{me^4}{4n^2h^2}$ and Kinetic Energy $E_{Kn}=\frac{1}{2}mv^2=\frac{1}{\epsilon_0^2}\cdot\frac{me^4}{8n^2h^2}$. Thus total energy of the electron $E_n=E_{Pn}+E_{Kn}=-\frac{1}{\epsilon_0^2}\cdot\frac{me^4}{8n^2h^2}=-\frac{hcR}{n^2}$, here, $R=\frac{me^4}{8\epsilon_0^2}$ is an analytically determined constant is conforming to empirically determined Rydberg's constant. Accordingly, as postulated by Bohr, $h\nu=\frac{hc}{\lambda}=E_n-E_m=hcR\left(\frac{1}{n^2}-\frac{1}{m^2}\right)\rightarrow\frac{1}{\lambda}=R\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$, here λ is the wavelength of radiation emitted by an electron decaying from mth orbit to nth orbit. This striking coincidence in empirical constant and analytically determined constant is amazing. Accordingly, sets of spectral lines in isolated hydrogen atoms emitted by electron decaying from higher energy level through intermediate levels to ground level are shown in the figure below. Similar spectral lines were observed in hydrogen like atom having one electron in outermost orbit.



Quantization of angular momentum by Bohr as an integral multiple of $\frac{h}{2\pi}$, which makes a way to wave nature of electron predicted by Broglie, discussed in Appendix, according to which electrons instead of moving strictly in circular orbit perform sinusoidal motion as a standing wave having integral wavelengths such that $2\pi r_n=n\lambda_n$.

Thus, revisiting the angular momentum leads to wavelength of an electron in n^{th} orbit as $mv_n r_n = n\frac{h}{2\pi} \rightarrow 2\pi r_n = \frac{nh}{mv_n} = n\lambda_n \rightarrow \lambda_n = \frac{h}{mv_n}$. Wave nature of electrons in orbit is shown in the figure. Together with this *Heisenberg's Uncertainty,* elaborated at Appendix, and *Schrödinger Wave Equation, introduced at Appendix,* detailed description of atomic structure runs outside the scope of this manual. Bohr's model though good enough to explain hydrogen atom is insufficient to explain structure of atoms having more than one electron.



Limitation of Bohr's Model: Bohr's model together with wavelength of spectral lines provided by the model is reasonably accurate and is a good justification for preliminary acceptance of Bohr's model even today. But, it suffers from following limitation for a universal acceptance :

- **a.** It fails to explain why does orbiting electron experiencing constant centripetal acceleration does not decay into nucleus.
- **b.** Wavelengths of spectral lines for hydrogen are very close to actual as shown earlier. But, prediction of spectral lines for larger atoms is very poor,
- **c.** It assumes orbital position of electron, its velocity and momentum to be certain, which is impossible as per *Heisenberg's Uncertainty Principle*.
- **d.** It fails to predict Zeeman Effect and Stark Effect causing splitting of spectral lines in presence of magnetic field and electric field, respectively. Both these effects are excluded in present elaborations, being outside the domain of this manual.

Illustration of ***Zeeman Effect*** and ***Stark Effect*** together with the reason as to why stable atoms do not emit radiation is beyond the scope of this manual. Nevertheless, quarries of inquisitive readers are invited through *CONTACT US.*

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APPENDIX – Quantum Mechanics

Aim of this Appendix is to make concepts of Quantum Mechanics contextual for elaboration of modern physics. While evolving this appendix care has been taken to limit the discussions within the scope and concepts covered in this manual. It is requested that this may not be treated as complete text of the concepts, which are much beyond.

In 1900, hypothesis of **Max Karl Ernst Ludwig Planck,** also referred as ***Max Planck*** and ***Karl Planck***, in which he quantized radiation of energy, be it emission or absorption, is in the form packets calling them **Quantum**. This concept with contribution of many scientists developed into **Quantum Mechanics**, a branch of Physics. Prior to Planck, in 1887, **Heinrich Hertz** experimentally demonstrated emission of electrons from metal surface on incidence of light, and it was christened as **Photo Electric Effect**. *This would be dealt with separately in the next chapter*, nevertheless it is essential to make its relevance into motivation to Planck to propound his hypothesis of Quantum of Radiation.

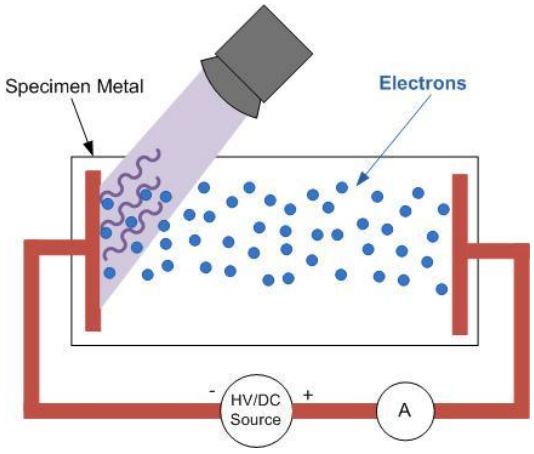
It was observed by Hertz that when light falls on a metal surface, under certain conditions, there is emission of electrons. It had following characteristics –**a)** Photo electric effect occurs only on incidence of light above a fixed frequency, called *Threshold Frequency* (v_0), and not below that; **b)** The emission of electrons is **instantaneous** on incidence of light; **c)** Minimum Kinetic Energy is independent of intensity of light. These observations contradict inferences of classical mechanics and also Maxwell's Electromagnetic Field Theory which propounds that – **a)** Transition from of state occurs on gain or loss of sufficient energy, called *Threshold Energy* (∅), either in impulse or a gradual and incremental process, **b)** The gradual process shall take some time to attain the threshold energy and cannot be instantaneous, **c)** Decrease in intensity of light with increased time of incidence will not cause accumulation of light energy to initiate emission of electron. Elaboration of Photo-Electric Effect here is contextual to build premise of Quantum Mechanics, crucial in understanding of atomic structure. Nevertheless, details of this effect shall be dealt with in next chapter.

These contradictions were motivation to ***Planck*** to advance hypothesis of Quantization of Electromagnetic Radiation and light is part of its spectrum, while he was studying **black body radiation**. He stated that "A black body radiation chamber can be supposed to be filled up with harmonic oscillators each with a characteristic frequency (known as Planck's Oscillators). Energy of each oscillator cannot change continuously but is limited to a discrete set of values which are integral multiple of small unit of energy called quantum of radiation. Accordingly, plank proposed a constant ***h*** known as Planck's Constant.

In 1897, J.J. Thomson during his experimented with cathode rays under combined influenced of magnetic and electric field, using Coulomb's Law and Ampere's Force Law. He discovered negatively charged particles calling them corpuscles having a mass 1840 times a Hydrogen atom, then available knowledge of electro-magnetic interaction. At that time, presence of subatomic particles was not accepted. Further, he determined ratio of charge to mass of these corpuscles $\frac{e}{m} = 1.7588196 \times 10^{11}\ \mathrm{C.kg^{-1}}$. It was only in 1909 when Robert Millikan measured charge of an electron and currently accepted value **e= -**1.6021773×10^{-19} C through a famous Oil Drop Experiment. This helped to accurately determine value of $h = 6.626096896(33) \times 10^{-34} 4\ \mathrm{J.s.}$

Understanding of interference, diffraction and polarization is well substantiated by wave theory. But, inability of the wave theory to explain scattering effect and contradictions surfaced in Photo-Electric Effect, and associated experimental observations were a beginning attributed to Planck, with a distinguished hounour of being called ***Father of Quantum Mechanics***.

Discovery of Photon: Photo-Electric Effect is being briefly visited to take forward the concepts of quantum mechanics. An evacuated tube with two separated electrode when electric potential difference is applied across them no current is established. But, as soon as the electrode at (-)ve potential is illuminated, small current established across the electrodes. Stopping of this current, requires a minimum reverse voltage called **stopping voltage** ($\boldsymbol{V_0}$), such that $\frac{1}{2}m_e v_{max}^2 - eV_0 = 0$, or $v_{max} = \sqrt{\frac{2eV_0}{m_e}}$. Here, $\boldsymbol{m_e}$ is mass of electron, $\boldsymbol{e}$ is charge of electrons and v_{max} is the maximum velocity attained by electron by incidence of light.

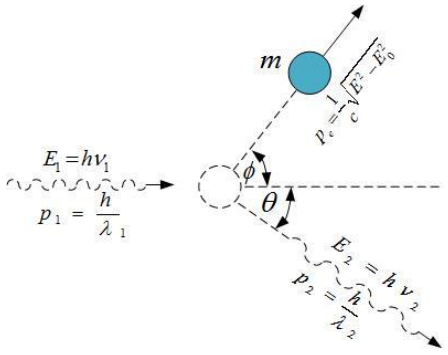


Albert Einstein in 1905 used Planck's hypothesis to explain photo electric effect and suggested that every cycle of an electromagnetic

radiation is an energy packet with energy content $E = h\nu$; here E is the energy of the packet, and this equation is ***Planck-Einstein Relation***. This energy packet was termed as ***Photon***, after about Two decades on 1926 by **Frithiof Wolfers** and **Gilbert N. Lewis** . Ever since Arthur H Crompton was awarded Nobel prize on 1927, for his studies on scattering of light independent existence of photon, a packet of electromagnetic energy. According to Einstein's theory individual photon is either fully absorbed by a single electron or nothing; there is neither accumulation nor sharing as per wave theory. An electron to escape requires minimum energy called Work Function ($\emptyset$) and is different for each metal, Later in this chapter difference of $\emptyset$ across different metals shall be seen to be on account of their atomic structure. Thus energy balance equation $h\nu = \emptyset + \frac{1}{2}m_e v^2$ represents effect of an incident photon. This provides answer to minimum stopping potential as $V_0 = \frac{\frac{1}{2}m_e v_{max}^2}{e} = \frac{h\nu - \emptyset}{e}$. In this equation driving factor for photo-electric effect is frequency of radiation and not its intensity. This concept of photon is applicable over complete spectrum of electro-magnetic radiation. As per special theory of relativity, a particle with Zero Rest Mass has eventually velocity $\boldsymbol{c}$, its momentum-energy correlation is $E = pc$, where E and p are energy and momentum of photon. Since, $E = h\nu$ it leads to $h\nu = pc \rightarrow h\frac{c}{\lambda} = pc \rightarrow p = \frac{h}{\lambda}$. Here, momentum ($p$) being a vector has same direction as that of its velocity. *This concept shall go a long way into understanding of atomic structure.*

Understanding of Photon Effect created an obvious intuitive corollary among the contemporary scientists, as to can energy imparted metal cause emission of Photon. **Wilhelm Röntgen**, while experimenting with Cathode Rays observed in 1895, observed that a cathode when provided with thermal energy equal to its work function ($\emptyset$), a thermionic emission of electrons takes place. These electrons, under vacuum and under high potential difference, bombard on anode with high velocity and a radiation is produced in support of the corollary. Since, characteristics of this radiation was not known, it was called X-Rays. In this appendix efforts is to build context of Quantum Mechanics, more elaboration of X-rays would follow in the next section.

Particle Nature of Photon: In 1922 **Arthur H Crompton** in an experiment aimed X-rays beam at a solid target recoded lower frequency radiation at points with angular displacement with the path of X-ray. This was in contradiction to wave theory which propounds absorption of radiation by an electron sets it in oscillation in response to electric field of EM radiation. The oscillating electron in turn radiates energy in all directions, like an antenna, in the form of EM waves of same frequency. Crompton observed that for an incident radiation of wavelength $\boldsymbol{\lambda}$ the scattered radiation at an angle $\emptyset$ with the line of incidence having wavelength $\boldsymbol{\lambda'}$ satisfied a relation: $\lambda - \lambda' = \frac{h}{mc}(1 - \cos\emptyset)$. This equation is mathematically sustainable using principles of elastic collision and relativistic mechanics. Taking $\bar{\boldsymbol{p}}$ and $\bar{\boldsymbol{p}}'$as momentum vectors of incident and scattered photons having magnitudes $\boldsymbol{p}$ and $\boldsymbol{p}'$, respectively. Electron, which was initially at rest with rest mass $\boldsymbol{m}$, is like a packet of energy $\boldsymbol{E_0 = mc^2}$. After impinging of photon the electron gains a momentum vector $\overline{p_e}$ having magnitude $\boldsymbol{p_e}$ and an energy $= \boldsymbol{p_e c}$, as per relativistic principles. As per conservation of momentum. $\bar{p} = \bar{\bar{p}}' + \overline{p_e}$. This as per vector algebra $\overline{p_e} = \bar{p} - \bar{\bar{p}}'$ and $p_e = \sqrt{p^2 + P'^2 - 2p \cdot p_e \cdot \cos\emptyset}$. As per conservation of energy, total energy of photon-electron system is $pc + mc^2 = p'c + E_c$. Here, the energy of electron post collision is: $E_c = \sqrt{(E_0)^2 + (p_e c)^2}$, in accordance with



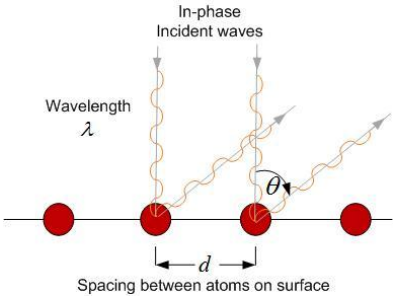
relativistic energy-momentum relation. It leads to $E_c^2 = (E_0)^2 + (p_e c)^2 \rightarrow (mc^2)^2 + (p_e c)^2 = (pc + mc^2 - p'c)^2$. Substituting value of p, above in this equation and squaring it square of terms pc, $p'c$ and mc^2 would cancel out, leaving behind: $pmc^3 - p'mc^3 = pp'c^2 - pp'c^2 \cos\emptyset$. Dividing this equation in reduced form with $pp'c^2$, final form of this equation is $\frac{mc}{p} - \frac{mc}{p} = 1 - \cos\emptyset$. Further, multiplying by $\frac{h}{mc}$ to this final form $\frac{h}{p} - \frac{h}{p'} = \frac{h}{mc}(1 - \cos\emptyset)$. Using the relativistic definition of wavelengths the equation becomes $\lambda - \lambda' = \frac{h}{mc}(1 - \cos\emptyset)$, and this is same as concluded by Crompton.

Thus Crompton combined Einstein' Theory with classical mechanics. *This turned out to be first step to propound* ***dual nature of Photon*** *i.e. a particle as well as a wave.*

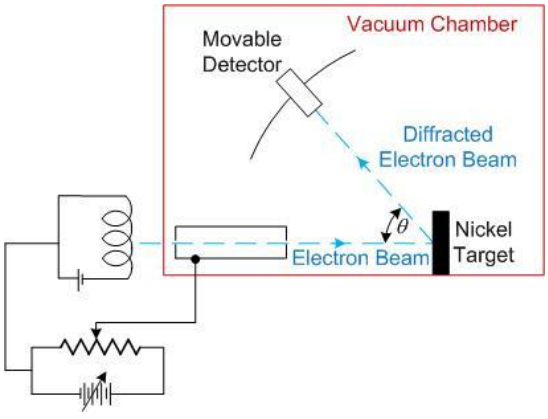
In 1933, **Patrick Blackett** and **Giuseppe Occhialini** observed that when Gamma radiation, which constitutes highest frequency of radiation, when is incident on a target, it may not scatter, instead it may disappear completely and a new pair particles are created an electron and a positron both having equal mass but with opposite charges. Thus the pair maintains electrical neutrality of the process. Thus, minimum frequency of radiation required to produce the pair conforms to equation: $E_{min} = h\nu_{min} = 2\left(\frac{1}{2}m_e c^2\right)$. This process is reversible, as much as collision of electron and proton causes decay of the two particles with emergence of a photon gamma radiation. Though, this phenomenon is unexplained in photo-electric effect, it is in conformance with center of momentum, where total momentum remains Zero.

Particle Behaving as a Wave: A natural consequence of above discoveries another obvious question was: can a particle behave like a wave? How an atom does remain stable despite ever accelerating electrons, due to centripetal force caused by their orbital motion? Does it not radiate energy like an accelerating charged particle?

In 1924, **Prince Louis de Broglie** (last name pronounced as **Broy**) made an hypothesis that: nature loves symmetry. Accordingly, light has a dualistic nature whereby in some situation light behaves like wave and in other like particle. It is in accordance with symmetry of nature of particle as wave, which is bound to have velocity, wavelength and frequency, essential parameters of a wave. Broglie postulated that a particle with rest mass ***m*** moving with a velocity ***v*** has a momentum has a momentum $p = \boldsymbol{mv}$. As per stipulations of quantum mechanics the particle should exhibit a wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$ and this is called **Broglie's Wavelength.** In the event of the particle attaining a velocity comparable to velocity of light (**c**), relativistic mass of the particle would come into play and thus momentum would be $p = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}v = \chi mv$, here $\chi = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, is the Lorentz factor. Further, relativistic principle stipulates that $E = h\nu$, here, ν is the frequency, Thus, Broglie related momentum and frequency to the energy of the particle in the same way as that of the photon.



Three years later in 1927 **C.J. Davisson and L.H. Germer** during their studies with electron beam directed on a piece of metal observed strong maxima and minima at specific angles in conformity with diffraction pattern realized with a EM beam passed through a grating.

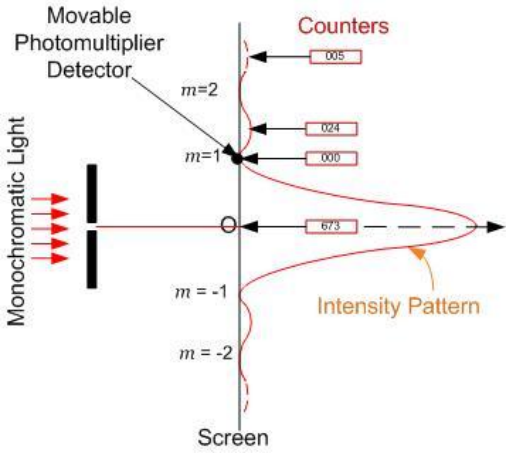


The speed of electron was determined from accelerating voltage $\boldsymbol{V_{ba}}$ using principles of classical mechanics and electro-statistics, kinetic energy of the electron is $\boldsymbol{KE = eV_{ba} = \frac{1}{2}m_e v_e^2 = \frac{p_e^2}{2m_e}}$. Here, $\boldsymbol{e}$ is the charge of electron. Accordingly, $\boldsymbol{p_e = \sqrt{2m_e eV_{ba}}}$. Thus, in accordance with Broglie's Wavelength, wavelength of the accelerated electron shall be $\boldsymbol{\lambda_e = \frac{h}{\sqrt{2m_e eV_{ba}}}}$. This clearly indicates that with the increase in accelerating voltage ($\boldsymbol{V_{ba}}$) wavelength exhibited shorter wavelength on electron wave, and is in conformity with the diffraction pattern. It in turn revealed that atoms near surface were arranged in row with a distance ($\boldsymbol{d}$)between them. Accordingly, occurrence of strong reflections $\boldsymbol{d \sin\theta = m\lambda}$ (m-1,2,3…) are identical to those with gratings with center-to-center spacing as $\boldsymbol{d}$**.** This diffraction pattern was the first evidence of Broglie's Hypothesis to be regarded as **Broglie's Principle**. In 1928, **G.P. Thomson** carried out different experiment on electron diffraction using polycrystalline metallic foil. While the electron beam passed through the foil it depicted a circular diffraction pattern around the beam reconfirming the Broglie's Hypothesis.

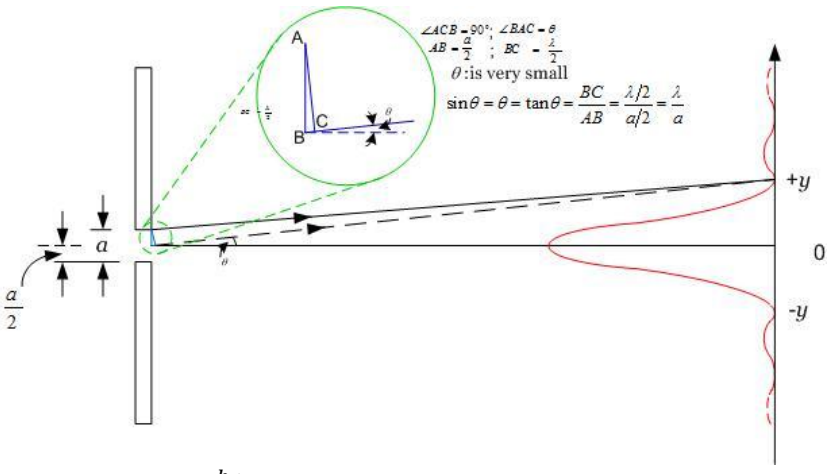
Electron Microscope: Using Broglie's Principle first electron microscope was invented, by **Max Knoll** and **Ernst Ruska** in 1931, to resolve objects smaller than possible with perfectly designed optical microscope, using wavelength of 500 nm. Since then there have been many developments in design to improve the resolution. An accelerated electron beam can reach wavelength many thousand times shorter than the optical limit. Thus electron microscope renders many thousand times larger magnification than that available with optical microscope. It is important to note that trajectory of electron can be determined based on classical principles of physics involving charged particles governed by electric and magnetic forces. It is only when resolution is the concern wave properties of electron are involved.

Wave Particle Duality, Probability and Uncertainty: The foregoing discussions open a new curiosity as to how can EM radiation can behave as a particle and a wave too. **Neils Bohr** in 1928 advanced a **Principle of Complementarity** which stipulates that: *in EM radiation both particle and wave models exist such that in any phenomenon of the radiation either of the Two model is operative, and both of them in concurrence*. This was demonstrated with single-slit diffraction pattern placing photon counters or alternatively photographic plate in place of screen used by **Fraunhoffer**; the wave model elaborated in section on Optics. It was seen that pattern of distribution of photons correspond to the intensity of light. Similar observation was obtained with double slit interference pattern in conformity wave model justifying the complementarity of the Two models.

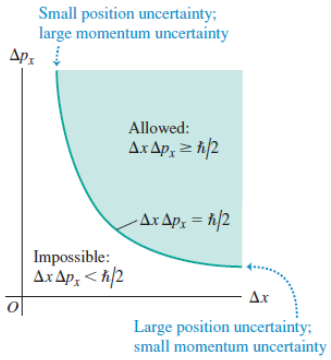
In classical mechanics an object is treated as a point mass which has energy and momentum, which makes it possible to describe motion of the particle at any time in space (X,Y and Z coordinates). But, Photon being considered to have a Zero rest mass, it cannot be treated as a point mass. This introduced uncertainty into path of photons having same initial state. This uncertainty is demonstrated both in position and momentum of photon and, therefore, an inseparable probability to the Two variables. In the single slit diffraction of a wave with wavelength λ through a slit of width a, first minima occurs at an angle θ_1 with the central line. At a sufficiently long distance from a narrow slit, it is seen that $\sin\theta_1 = \theta_1 = \frac{\lambda/2}{a/2} = \frac{\lambda}{a}$. Looking at this phenomenon, a photon striking at edge of first fringe, probability to its momentum along X-axis and Y-axis are assigned as p_x and p_y. Probability of a photon striking at edge of first fringe and central



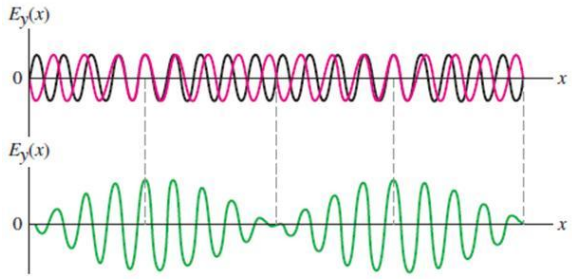
maxima and accordingly, $\tan\theta_1 = \theta_1 = \frac{p_y}{p_x} \rightarrow \frac{\lambda}{a} = \frac{p_y}{p_x}$. Photons striking detector within First minima around central maxima are within angle $(+)\frac{\lambda}{a}$ to $(-)\frac{\lambda}{a}$ are 85 %,such that all photons leaving the slit reach detector either above or below the central maxima and therefore $p_y\big|_{average} = 0$. Accordingly, uncertainty of a photon striking away from central maxima is $:\Delta p_y = p_x\frac{\lambda}{a}$. Relation between momentum of a photon and its wavelength is $\lambda = \frac{h}{p}$. Therefore, along the central line $\Delta p_y = p_x\frac{h/p_x}{a} = \frac{h}{a}$. This leads to an evident inference is narrower the slit larger is the probability of a photon striking away from central maxima; accordingly, a wider diffraction pattern.



Uncertainty Principle: In statistics uncertainty of a quantity is defined in terms of standard deviation. In 1927, **Weiner Heisenberg** postulated uncertainty in momentum and position of a photon to be $\Delta x\Delta p \geq \frac{\hbar}{2}$. Here, $\hbar$ (pronounced "h-bar") *is called* **reduced Planck constant** or **Dirac constant such that** $\hbar = \frac{h}{2\pi}$. It was another milestone in Quantum Mechanics which states that position and momentum of a particle are complementary and it is impossible to either with certainty. This is valid with any equipment howsoever sophisticated it may be. This was explained with a simple argument that any detector to be able to function has to interact with the particle. In the process there is moderation in state of the particle, and thus uncertainty becomes fundamental and intrinsic. This uncertainty is just not to be treated specific to X-axis, it is also valid in space and hence $\Delta y\Delta p \geq \frac{\hbar}{2}$ and $\Delta z\Delta p \geq \frac{\hbar}{2}$, as shown in the figure.



Wave Uncertainty: Equation of an EM wave propagating along X-axis with its electric field polarized along Y-axis as a function of position and time given by Maxwell is $E_y(x,t) = A\sin(kx - \omega t)$, already elaborated in Chapter on Waves and Motions. Here, $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$. In quantum mechanics frequency is invariable represented with symbol ν instead of f, unit being the same Cycles-per-second. These two parameters k can be expressed in terms of momentum of photon it leads to $p_x = \frac{h}{\lambda} = \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \hbar\cdot k$. Likewise, another parameter ω can be expressed as energy of the photon, leading to $E = h\nu = \frac{h}{2\pi}\cdot 2\pi\nu = \hbar\omega$. Thus the wave equation gets transformed into $E_y(x,t) = A\sin\left(\frac{(p_x x - Et)}{\hbar}\right)$. If a definite value us assigned to p_x, it leads to Zero uncertainty, expressed mathematically as $\Delta p_x = 0$. This is where Uncertainty Principle comes into play and probability of position of a photon



$\Delta x = \infty$. This leads to loss of nature of a wave where amplitude is uniform throughout the length of propagation, a total loss of information.

In diffraction pattern it has been observed that there is a known probability of a photon being found in a certain position. This calls for rewriting wave equation as with Two waves with slightly different wavelengths which can cause beats as elaborated in chapter on Waves and Motion, as: $E_y(x,t) = A_1 \sin\left(\frac{(p_{x1}\cdot x - E_1 t)}{\hbar}\right) + A_2 \sin\left(\frac{(p_{x2}\cdot x - E_2 t)}{\hbar}\right)$. Thus, at certain time say $t = 0$, $E_y(x, t=0) = A_1 \sin\left(\frac{p_{x1}\cdot x}{\hbar}\right) + A_2 \sin\left(\frac{p_{x2}\cdot x}{\hbar}\right)$. When $A_1 = A_2$, the nature of beats can be expressed as: $E_y(x, t=0) = 2A \sin\left(\left(\frac{p_{x1}+p_{x2}}{2}\right)\cdot\frac{x}{\hbar}\right)\cdot\cos\left(\left(\frac{p_{x1}-p_{x2}}{2}\right)\cdot\frac{x}{\hbar}\right)$. It leads to the probability of a photon being found at a place is maximum where amplitude is maximum and can be ascertained only by assigning probability to p_{x1} and p_{x2} in conformance with the Heisenberg's Uncertainty Principle $\Delta p_x . \Delta p_x \geq \frac{\hbar}{2}$.

Energy Uncertainty: The wave equation at $x = 0$, becomes $E_y(x=0,t) = A_1 \sin\left(-\frac{E_1 t}{\hbar}\right) + A_2 \sin\left(\frac{E_2 t}{\hbar}\right)$. It implies that: $E_y(x=0,t) = -A_1 \sin\left(\frac{E_1 t}{\hbar}\right) + A_2 \sin\left(\frac{E_2 t}{\hbar}\right)$. Thus electric field at a point is combination of Two oscillating fields with $\omega_1 = \frac{E_1}{\hbar}$ and $\omega_2 = \frac{E_2}{\hbar}$, is synonymous to beats referred to above. This equation also equally attracts Uncertainty Principle where $\Delta E \Delta t \geq \frac{\hbar}{2}$. Most likely a photon can be found at a time when amplitude is largest; the price of localizing photon in time is uncertainty of energy of wave.

Schrödinger Wave Equation: Around the same time when Heisenberg advanced Uncertainty Principle, In 1926 Erwin Schrödinger published a wave equation considering wave-particle duality equation and probability of finding a particle at a certain position. The Schrödinger Wave Equation is a mathematical model of quantum mechanical behaiour of sub-atomic particles. It is used to find allowed energy levels of atoms transistors. Elaboration of the equation involves partial derivatives along with vector calculus, which is beyond the scope of this manual. Nevertheless, inquisitive readers are requested to raise their quarries through *CONTACT US.*

It is an important inference of quantum mechanics that, despite wave-particle duality at a time only either particle nature can be realized or the wave nature, simultaneous realization of both the nature is not possible. In this effort to make Quantum Mechanics contextual to understanding of atomic structure, involving elaboration in previous chapter, there might abridging of concepts, inadvertently some questions on How? and Why? of involved concepts might have been left unanswered. Such questions of inquisitive readers are invited through *CONTACT US.*

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GROWING WITH CONCEPTS – Chemistry

CHEMICAL BONDING AND MOLECULAR STRUCTURE

Kumud Bala

Molecules and chemical bond: Atoms are usually not capable of free existence but groups of atoms of the same or different elements exist as one species, e.g., H_2, O_2, H_2O, P_4

"A group of atoms existing together as one species and having characteristic properties is called a molecule." Obviously, there must be some force which holds these atoms together within the molecules. "This force which holds the atoms together within a molecule is called a chemical bond.

A number of questions now arise: (i) Why do atoms combine? (ii) Why are only certain combinations possible, e.g., hydrogen exists as H_2 and not as H_3? (iii) Why do some atoms combine while certain others do not, e.g., two H-atoms combine to form H_2 but two helium atoms do not combine to form He_2? (iv) Why do molecules possess definite shape, e.g., CO_2 is linear but H_2O is a bent molecule (V-shape)? Similarly, BF_3 is planar but NH_3 is pyramidal. To answer such questions different theories and concepts have been put forward from time to time. These are:

1. Kossel-Lewis approach
2. Valence shell electron pair repulsion (VSEPR) theory
3. Valence bond theory
4. Molecular orbital theory

Why do atoms combine?-Kossel-Lewis approach to chemical bonding: In order to explain the formation of chemical bond in terms of electrons, a number of attempts were made, but it was only in 1916 when Kossel and Lewis succeeded independently in giving a satisfactory explanation. They were the first to provide some logical explanation of valence which was based on the inertness of noble gases.

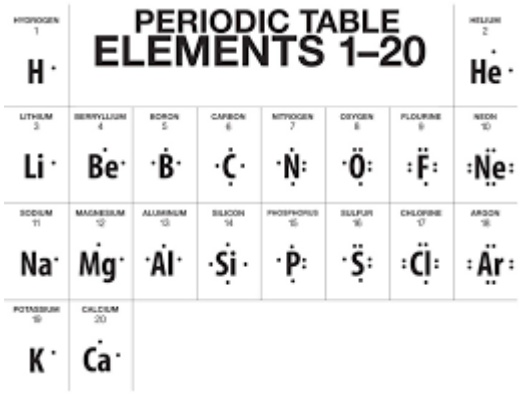
The study of noble gases, earlier called inert gases, (group 18 elements) suggests that neither they combine chemically with any other element nor among themselves, i.e., they are chemically inactive. Further, their electronic configurations are as follows:

From the study of electronic configurations of the noble gases, it is clear that they have 8 electrons in their outermost orbit except in case of helium which has 2. Thus, noble gases are inactive or stable because they have 8 electrons in the outermost shell (called octet) or 2 electrons in case of helium (called duplet). Hence, it was suggested that they possess stable electronic configurations. In case of all other elements, the number of electrons in their outermost shell is less than 8 and hence they are chemically reactive. This led to the following conclusion, called octet rule: The atoms of different elements combine with each other in order outermost shell having 2 electrons) in case of H, Li, and Be to attain stable nearest noble gas configuration.

Noble Gas	Atomic Number	Electronic Configuration
Helium	2	2
Neon	10	2, 8
Argon	18	2, 8, 8
Krypton	36	2, 8, 18, 8
Xenon	54	2, 8, 18, 18, 8
Radon	86	2, 8, 18, 32, 18, 8

Lewis Symbols- Representing the valence electrons: In the formation of a molecule, only the outer shell electrons are involved and they are known as valence electrons. The inner shell electrons are well protected and are generally not involved in the combination process. It is, therefore, quite reasonable to consider the outer shell electrons, i.e., valence shell electrons while discussing chemical bonds. G.N. Lewis

introduced simple symbols to denote the valence shell electrons in an atom. The outer shell electrons are shown as dots surrounding the symbol of the atom. These symbols are known as Lewis symbols or electron dot symbols. These symbols ignore the inner shell electrons. A few examples are given below:



Significance of Lewis symbols: The number of dots around the symbol gives the number of electrons present in the outermost shell. This number of electrons helps to calculate the common valency of the element. That is why these electrons are called valence shell electrons. The common valency of the element is either equal to the number of dots in the Lewis symbol (if these are ≤ 4) or 8 minus the number of dots (if these are > 4). For example, Li, Be, B and C have valencies 1, 2, 3, and 4 respectively, i.e., equal to the number of dots whereas valencies of N, O, F and Ne are 3, 2, 1 and 0 respectively, i.e., 8 minus the number of dots.

How do atoms combination? (Modes of chemical combination): As discussed above, atoms combine together in order to complete their respective octets so as to acquire the stable inert gas configuration. This can occur in two ways:

1. By complete transference of one or more electrons from one atom to another. This process is referred to as electrovalency and the chemical bond formed is termed as electrovalent bond or ionic bond.
2. By sharing of electrons. This can occur in two ways as follows: (a) When the shared electrons are contributed by the two combining atoms equally, the bond formed is called covalent bond. (b) When these electrons are contributed entirely by one of the atoms but shared by both, the bond formed is known as a coordinate bond, also called dative bond.

Electrovalent or Ionic Bond: " When a bond is formed by complete transference of electrons from one atom to another so as to complete their outermost orbits by acquiring 8 electrons (i.e., octet) or 2 electrons (i.e., duplet) in case of hydrogen, lithium etc. and hence acquire the stable nearest noble gas configuration, the bond formed is called ionic bond or electrovalent bond".

Explanation of the formation of ionic bond- Atoms are electrically neutral. Therefore, they possess equal number of protons and electrons. On losing an electron, an atom becomes positively charged since now the number of protons exceeds the number of electrons.

$$A \rightarrow A^{+} + e^{-}$$

On the other hand, in case of atom, gaining the electron, the number of electrons exceeds the number of protons and thus the atom becomes negatively charged.

$$B + e^{-} \rightarrow B^{-}$$

The oppositely charged particles formed above attract each other by electrostatic forces of attraction. The bond thus formed is known as electrovalent or ionic bond. Such a type of bond is formed only when one of the atoms can easily lose electrons while the other can gain electrons and thus each acquires the stable electronic arrangement of the nearest noble gas.

Examples:

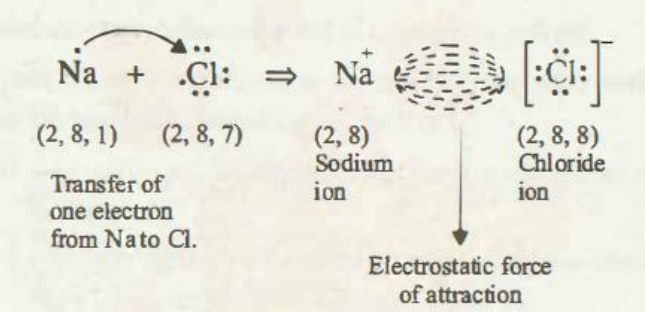
1. Formation of sodium chloride. Sodium (atomic number = 11) has electronic configuration 2, 8, 1. By losing one electron of its outermost shell it acquires the inert gas configuration of neon and changes into ion.

$Na \rightarrow Na^+ + e^-$
2,8, 1 2, 8

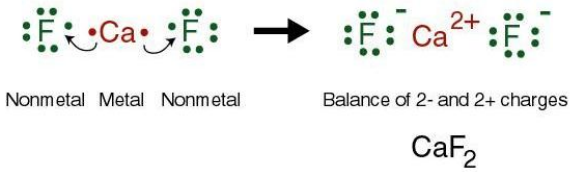
On the other hand, chlorine (atomic number = 17) having electronic configuration 2, 8, 7 , accepts one electron released by sodium to complete its octet by attaining stable configuration of argon. In this process, chlorine is converted into chloride ion.

$Cl + e^- \rightarrow Cl^-$
(2, 8, 7) (2, 8, 8)

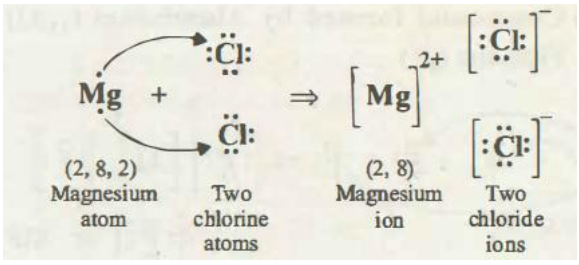
Now, we have two species, one is positively charged sodium ion and the other is negatively charged chloride ion. As they approach each other, they are held together by strong electrostatic forces of attraction. Thus formation of sodium chloride takes place.



2. Formation of calcium fluoride (CaF_2)



3. Formation of magnesium chloride ($MgCl_2$)



When the structures of atoms or ions are written in such a way that the electrons present in the outermost shell are represented by dots (.) around the symbol of the element, as in the example above, these structures are called Lewis dot structures.

"The number of electrons lost or gained during the formation of an electrovalent linkage is termed as the electrovalency of the element."

For example, sodium and calcium lose 1 and 2 electron respectively and so their valencies are 1 and 2. Similarly, chlorine and two fluorine atoms gain 1 and 2 electrons respectively, so they

Factors governing the formation of ionic bonds: The formation of ionic bond involves, (i) the formation of a positive ion by loss of electrons from one kind of atoms. (ii) The formation of a negative ion by gain of electrons from another kind of atoms. (ii) Holding the positive and negative ions by electrostatic forces of attraction.

The formation of ionic bond depends upon the following factors:

(i) Ionization Enthalpy: Ionization enthalpy of any element is the amount of energy required to remove an electron from the outermost shell of an isolated atom in gaseous phase so as to convert it into a gaseous positive ion.
It is clear that lesser the ionization enthalpy, easier will be the removal of an electron, i.e., formation of a positive ion and hence greater the chances of formation of an ionic bond. Ionization enthalpy of alkali metals is low, hence they have more tendency to form to positive ions. $Na\ (g) \rightarrow Na^+\ (g) + e^-$ I.E $= -495 kJ\ mol^{-1}$

(ii)Electron Gain Enthalpy:" Electron gain enthalpy (electron affinity) of an element is the enthalpy change that takes place when an extra electron is added to an isolated atom in the gaseous phase to form a gaseous negative ion.
Higher is the electron affinity, more is the energy released and stable will be the negative ion

produced. Consequently, the probability of formation of ionic bond will be enhanced. Halogens possess high electron affinity. So the formation of their negative ions is very common, e.g., in case of chlorine, electron affinity is +348 $kJmol^{-1}$.

(iii) Lattice enthalpy: In the formation of ionic compounds, the positively charged ions combine with negatively charged ions to form the compound.

$$A^+ (g) + B^- (g) \rightarrow A^+B^- (s)$$

The energy released when the requisite numbers of gaseous positive and negative ions combine to form one mole of the ionic compound is called lattice enthalpy.

The higher the value of lattice enthalpy of the resulting ionic compound, the greater will be the stability of the compound and hence greater will be the ease of its formation. We know that the force of attraction between the oppositely charged ions is directly proportional to the magnitude of the charges (q_1, q_2) and inversely proportional to the square of the distance 'd' between them, i.e., force of attraction $\alpha\ q_1 q_2 / d^2$

Hence, the value of lattice enthalpy depends upon the following two factors:

(a) *Charge on the ions:* The higher the charge on the ions, greater is the force of attraction and hence larger is the amount of energy released. For example, lattice enthalpies of some ionic compounds are in the order:

NaCl < $CaCl_2$ < MgO

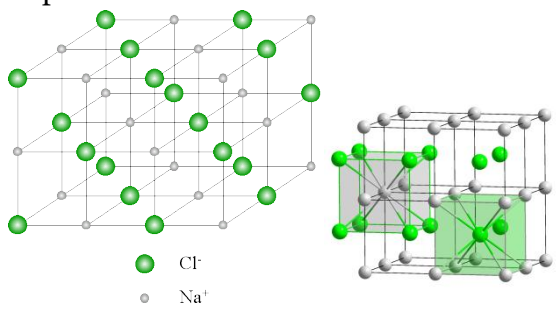
(– 758kJ mol^{-1}) (– 2254kJ mol^{-1}) (– 3932kJ mol^{-1})

(b) *Size of the ions:* Smaller the size of the ions, lesser is the internuclear distance. Consequently, the interionic attractions will be high and the lattice enthalpy will also be large. For example, ionic radius of K^+ (133 pm) is larger than that of Na^+ (95 pm), therefore, the lattice enthalpy of NaCl (758.7 kJ mol^{-1}) is greater than that of KCl (681.4kJ mol^{-1}).

Net effect: If lattice enthalpy + electron gain enthalpy > ionization enthalpy, the net effect will be the release of energy and hence an ionic bond is formed.

General characteristics of ionic compounds:

1. *Physical State:* These compounds usually exist in the solid state.
2. *Crystal Structure:* X- ray analysis of the ionic compounds shows that they exist as ions and not as molecules. These ions are arranged in a regular pattern in the three dimensional space to form a lattice. The pattern of arrangement, however, depends upon the size and charges of the ions. For example, in case of sodium chloride, each sodium ion is surrounded by six chloride ions and each chloride ion by six sodium ions, thus giving rise to a three dimensional octahedral crystal structure. The formula of an ionic compound merely indicates the relative number of ions present.



Crystal structure of NaCl

3. *High Melting and Boiling Point:* Ionic compounds possess high melting and boiling points. This is because ions are tightly held together by strong electrostatic force of attraction and hence a huge amount of energy is required to break the crystal lattice.
4. *Solubility:* Electrovalent compounds are soluble in solvents like water which are polar in nature and have high dielectric constant. It is due to the reason that the polar solvent interacts with the ions of the crystals and further the high dielectric constant of the solvent (i.e., capacity of the solvent to weaken the forces of attraction) cuts off the force of attraction between these ions.

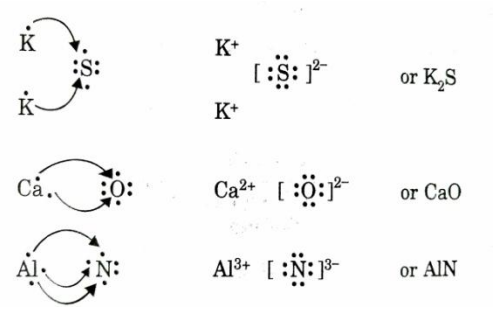
Furthermore, the ions may combine with the solvent to liberate energy called the hydration enthalpy which is sufficient to overcome the attractive forces between the ions. Non – polar solvents like carbon tetrachloride, benzene etc. having low dielectric constants are not capable of dissolving ionic solids. Hence, ionic solids are soluble in polar solvents and insoluble in non-polar solvents.

5. *Electrical Conductivity:* Ionic compounds are good conductors of electricity in solution or in the molten state. In solution or molten state, their ions are free to move. As the ions are charged, they are attracted towards electrodes and thus act as carrier of electric current.
6. *Ionic reaction:* The reactions of the ionic compounds are, in fact, the reactions between the ions produced in solution. As the oppositely charged ions combine quickly, these reactions are, therefore, quite fast.

$Na^+ Cl^- (aq) + Ag^+ NO_3^- (aq) \rightarrow AgCl (s) + NaNO_3 (aq)$

Example: Use of Lewis symbols to show electron transfer between the following atoms to form cations and anions.(i) K and S (ii) Ca and O (iii) Al and N

Solution:



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Answers to Science Quiz in Oct'17

Kumud Bala

1. (B); 2. (B); 3. (A) ; 4. (A); 5. (B); 6. (A); 7. (C); 8. (A); 9. (D); 10. (A)

11. (A); 12. (B); 13. (B); 14. (B); 15. (D); 16. (D); 17. (D); 18. (B); 19. (D); 20. (D)

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SCIENCE QUIZ : Nov'17

Kumud Bala

1. Brass gets discolored in air because of the presence of which of the following gases in air?
   (A) Oxygen (B) Hydrogen sulphide
   (C) Carbon dioxide (D) Nitrogen
2. Which of the following is a non metal that remains liquid at room temperature?
   (A) Phosphorous (B) Bromine
   (C) Chlorine (D) Helium
3. Chlorophyll is a naturally occurring chelate compound in which central metal is :
   (A) Copper (B) Magnesium
   (C) Iron (D) Calcium
4. Which of the following metals forms an amalgam with other metals?
   (A) Tin (B) Mercury
   (C) Lead (D) Zinc
5. The gas usually filled in the electric bulb is:
   (A) Nitrogen (B) Hydrogen
   (C) Carbon dioxide (D) Oxygen
6. Quartz crystals normally used in quartz clocks etc. is chemically
   (A) Silicon dioxide
   (B) Germanium oxide
   (C) A mixture of germanium oxide and silicon oxide
   (D) Sodium silicate
7. Which of the gas is not known as green house gas?
   (A) Methane (B) Nitrous oxide
   (C) Carbon dioxide (D) Hydrogen
8. Tetraethyl lead is used as –
   (A) Pain killer
   (B) Fire extinguisher
   (C) Mosquito repellant
   (D) Petrol additive.
9. The average salinity of sea water is:
   (A) 3% (B) 3.5%
   (C) 2.5% (D) 2%
10. Monazite is an ore of
    (A) titanium (B) zirconium
    (C) iron (D) thorium
11. The important sugar in honey is
    (A) Lactose (B) Fructose
    (C) Maltose (D) Sucrose
12. Viticulture is related with
    (A) Grapes (B) Pine apple
    (C) Orange (D) Strawberry
13. Which one of the following waves is used by the common TV remote control?
    (A) Radio waves (B) Lasers
    (C) Infrared waves (D) Ultrasonic waves
14. How will you define the process of vulcanization?
    (A) Sample of butane mixed with sulphur and litharge
    (B) Sample of propane mixed with sulphur and litharge
    (C) Sample of plastic formed carbon with sulphur and litharge
    (D) Sample of rubber mixed with sulphur and litharge
15. In which form the natural petroleum is found?
    (A) Gas (B) Kerosene
    (C) Crude oil (D) Tar
16. Which country consumes maximum petroleum?
    (A) Saudi Arabia (B) India
    (C) UK (D) US
17. Which of the following is the heaviest metal?
    (A) Osmium (B) Mercury
    (C) Iron (D) Nickel
18. Which of the following treatment is used for removed of biological impurities?
    (A) Sedimentation (B) Boiling
    (C) Sterilization (D) Distillation
19. In bacteria name the colour of light which is responsible for photosynthesis
    (A) Ultra-violet (B) Blue
    (C) Red (D) None of the above
20. The process of getting back a full organism from its body part is called
    (A) Spore formation (B) Budding
    (C) Regeneration (D) Fragmentation

(Answers to this Science Quiz shall be provided in 2nd Supplementdt 1st Dec'17 of 5th Quarterly e-Bulleti)

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Theme Song :

PREMISE: ***We are pleased to adopt a song*** **" इतनी शक्ति हमें देना दाता....."** ***from a old Hindi Movie Do Aankhen Barah Haath दो आँखें बारह हाथ of year 1957, directed by The Late V. Shantaram. The lyrics are by Shri Bharat Vyas, singer Melody Queen Sushri Lata Mangeshkar, and Music Direction by Vasant Desai. It has become a widely accepted inspirational song and/or prayer in many educational institutions and socially inspired initiatives engaged in mentoring of unprivileged children. This newly formed non-organizational initiative, being selflessly operated by a small set of compassionate persons, finds its philosophy in tune with the song and conveys its gratitude to all he eminent persons who brought out the song in a manner that it has attained an epitome of popularity. While working its mission and passion, the group invites one and all to collectively complement in grooming competence to compete among unprivileged children. The song/prayer goes as under -***

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना

हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

दूर अज्ञान के हो अंधेरे, तू हमें ज्ञान की रोशनी दे

हर बुराई से बचते रहें हम, जितनी भी दे भली ज़िन्दगी दे

बैर हो ना किसी का किसी से, भावना मन में बदले की हो ना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना

हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

हम ना सोचें हमें क्या मिला है, हम ये सोचे किया क्या है अर्पण

फूल खुशियों के बाँटे सभी को, सब का जीवन ही बन जाए मधुबन

अपनी करुणा का जल तू बहा के, कर दे पावन हर एक मन का कोना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना

हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

Together **E**ach **A**chieves **M**ore
(**TEAM**)

Every end, so also end of this e-Bulletin, is a pause for a review, before re-continuing of a journey far beyond ...