GYAN-VIGYAN SARITA: शिक्षा

A non-remunerative, non-commercial and non-political initiative to Democratize Education as a Personal Social Responsibility (PSR)

2nd Supplement dt 1st Sept'17 to 4th Quarterly e-Bulletin

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e-Bulletin – Eco-friendly: It is a considered decision to make this communiqué an e-Bulletin, and thus save paper. May please like to share it, but please do not print it, unless it is a must.

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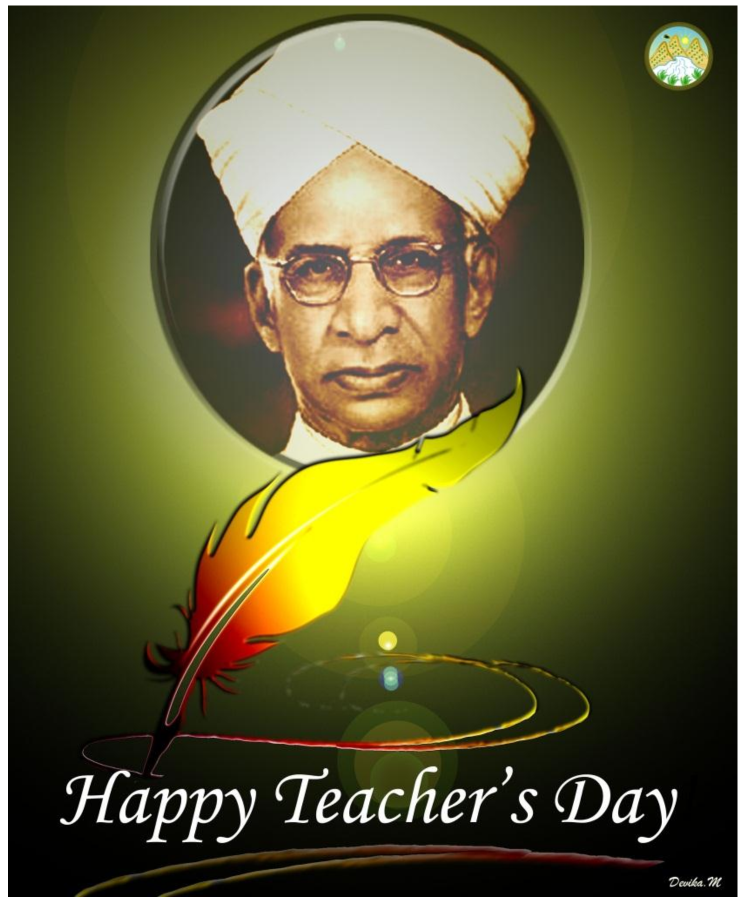


Oh my teacher! you taugh me,

Mentored me to think;

Mentored me to learn;

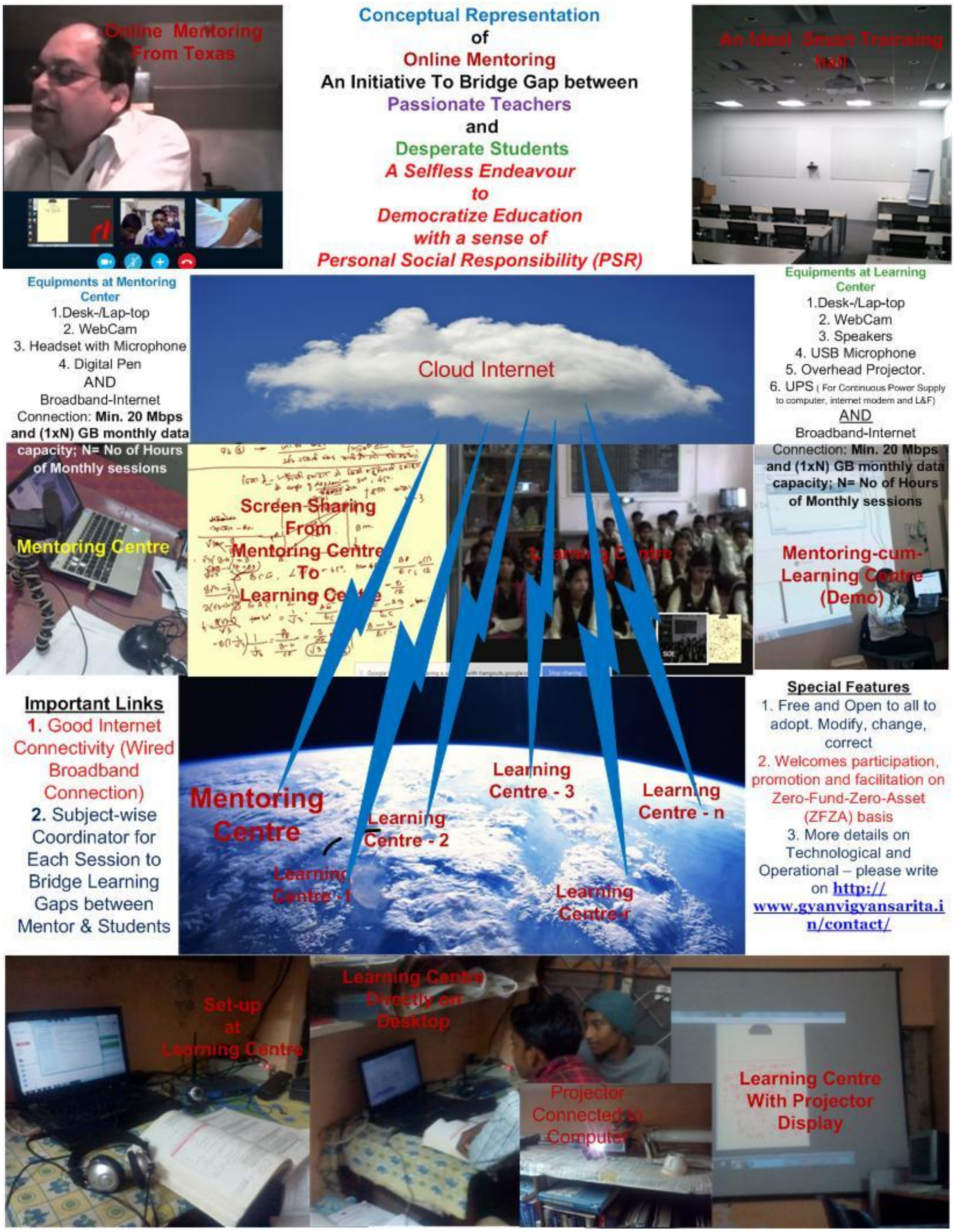
Mentored me to reason;

Without my realizing the change,

In what I think, see, observe and do,

I salute my teacher for me what I am.

Aim for the Best, but...



... start, without loosing time, with whatever is available.

संपादकीय

शिक्षा के स्तंभ: शिक्षण, शिक्षक और शिक्षार्थी

गुरू द्वारा शिष्य को ज्ञान देते समय, सीखने के रास्ते में आने वाली शिष्य की कमियों को दूर करना, और उसे सीखे गये ज्ञान की उपयोगिता समझाना शिक्षण कार्य कहलाता है। इसके लिये, गुरू सबसे पहले अपने शिष्य को उस वातावरण के अनुकूल बनाता है जिसमें रहकर वह गुरू द्वारा दिया जा रहा ज्ञान समझ कर ग्रहण कर सके, और वह ज्ञान लेते समय ही यह जान जाये कि सीखे ज्ञान की जीवन में वास्तविक उपयोगिता क्या है, कैसे है, और कहां है?

शिक्षा एक सजीव गतिशील प्रक्रिया है। इसमें शिक्षक और शिक्षार्थी के बीच अंतःक्रिया होती रहती है। यह अंतःक्रिया एक निश्चित लक्ष्य की ओर केंद्रित रहती है। शिक्षाशास्त्र की भाषा में, यह लक्ष्य केंद्रित अंतःक्रिया शिक्षण कहलाती है।

कल्पना करिये, आज क्या होता, अगर हमारे शिक्षक ने हमें प्रारंभिक अक्षर-ज्ञान नहीं दिया होता, अक्षरों को पढने, लिखने और पहचानने का अभ्यास नहीं कराया होता, अक्षरों से शब्द बनाने, और उनको मिलाकर वाक्य लिखना नहीं सिखाया होता, तो क्या हम यहां यह सब लिख पाते जो आप पढ़ रहे हैं?

शिक्षण-प्रक्रिया हर एक शिक्षक का मौलिक गुण होती है। यह एकरूप नहीं हो सकती है। कुछ खेल-खेल में, कुछ अभ्यास से, तो कुछ उदाहरण से, वह सब बता देते हैं, जो किताबों से पढ़कर नहीं जाना जा सकता है।

शिक्षक ज्ञात से अज्ञात की ओर, सरल से कठिन की ओर, स्थूल से सूक्ष्म की ओर, विशेष से सामान्य की ओर, अनुभव से तर्क की ओर, और तर्क से समझ की ओर चलते हुये अपने शिक्षार्थी को समझदार बनाता है। अच्छा शिक्षक अपने शिक्षार्थी के लिये सिखाते समय उसका सलाहकार बनकर, मित्र की भांति आचरण करता है, और पिता की भांति संरक्षण देता है।

एक अच्छा शिक्षक वह होता है जो शिक्षा देने कक्षा में आते ही बच्चों को मोहित कर ले। इसके लिये सबसे अधिक आवश्यक तत्व होता है - उसका समयानुकूल पहनावा और प्रफुल्लि चेहरा। रोनी सूरत और गंदा पहनावा किसी को पसंद नहीं आता है। एक अच्छा शिक्षक अपने शिक्षार्थियों से बात करते हुये कभी उन पर क्रोध नहीं करता है, और उनको नकारात्मक विचारों से दूर रखता है। वह अपने शिष्यों को कभी अपमानित नहीं करता है, बल्कि वह हमेशा उनको प्रेरित करने की कोशिश करता है।

सिखाते समय शिक्षक का आत्मविश्वास और उसकी प्रेरणा देने वाली कार्यशैली उसके शिक्षार्थी पर सबसे अधिक प्रभाव डालती है क्योंकि आत्मविश्वास का संबंध विषय की जानकारी और उस पर पूछे गये प्रश्नों के उत्तर से रहता है। हर शिक्षार्थी अपना आदर्श उस शिक्षक को बनाता है जो उसे सबकुछ सिखाकर अपने से भी उत्तम बना देता है, और सबके समक्ष यह उजागर कर देता है कि उसका शिष्य अतुलनीय है।

शिक्षण के छह महत्वपूर्ण चरण माने गये हैं: संलग्नता (Engagement), अन्वेषण (Exploration), व्याख्या (Explanation), विस्तार (Elaboration), मूल्यांकन (Evaluation), और, बढ़ोत्तरी (Extention). इनके प्रयोग की जिम्मेदारी शिक्षक पर रहती है, जो सीखने वाले की रूचि, क्षमता और उसकी आवश्यकता को ध्यान में रखकर करता है।

शिक्षार्थी समाज का एक अंग होता है। सीखने वाले के लिये वही शिक्षा उपयोगी होती है जो उसके रहन-सहन के वातावरण को ध्यान में रखकर दी जाती है। एक अच्छे शिक्षक को यह पता होता है कि उसका शिक्षार्थी समाज के किस भाग से आ रहा है, उसके समाज की सांस्कृतिक विरासत क्या है क्योंकि सांस्कृतिक विरासत, आस्था व परंपरा का मिश्रण होती है जो उसकी नसों में खून के साथ-साथ चलती है।

हमें नहीं भूलना होगा कि शिक्षा इसलिये दी जाती है कि व्यक्ति स्वयं एक आदर्श उदाहरण बने, अपनी व दूसरों की समस्याओं को समझने और उनका निराकरण करते समय व्यापक दृष्टिकोण

अपनाये, और अपने चारों ओर के समाज का समग्र विकास करे। प्रसिद्ध शिक्षा शास्त्री जान डीवी के अनुसार शिक्षा का अर्थ है-जीवन अथवा विकास, अर्थात एक पीढ़ी से अगली पीढ़ी को अपने संचित ज्ञान के स्थानांतरण का प्रयास शिक्षा है। स्वामी विवेकानंद के अनुसार, मनुष्य की अंतर्निहित पूर्णता को अभिव्यक्त करना शिक्षा है।

प्राचीन भारत में शिक्षा का उद्देश्य था-'सा विद्या या विमुक्तये' अर्थात विद्या उसे कहते हैं जो विमुक्त कर दे यानि अज्ञानता से ज्ञान की ओर ले जाये।

सीखते व सिखाते समय कई मौलिक समस्यायें आती हैं। इन्हें मौलिक इसलिये कहते हैं क्योंकि ये समस्यायें स्थान व समय के अनुसार उत्पन्न होती हैं। इन्हें कोई पहले से लिपिबद्ध नहीं कर सकता है।

यूनान के शिक्षाविद् और दार्शनिक प्लेटो की मान्यता थी कि परिपक्व बुद्धिवाला, ज्ञानी, व दार्शनिक ही सुयोग्य शासक बन सकता है, अर्थात उत्तम और पूर्ण शिक्षा युक्त व्यक्ति अच्छा शासक हो सकता है।

हम भारतीय सदा से शिक्षा को अपनी मूलभूत आवश्यकता समझते रहे हैं। हमारे समाज में हर व्यक्ति चाहें वह गरीब हो, अथवा अमीर, अपने बच्चे को शिक्षा दिलाने के प्रति सजग रहता है, वह बच्चे को विद्यालय भेजता है, और हर संभव प्रयास करता है कि उसका बच्चा उससे अधिक योग्य बने।

शिक्षकों का हर देश और काल में महत्व रहा है। शिक्षक का महत्व इसी बात से पता चलता है कि हर देश में वर्ष के किसी न किसी माह में वहां के शिक्षकों के प्रति सम्मान दिखाने के लिये, शिक्षा दिवस अथवा शिक्षक दिवस अवश्य मनाया जाता है। उदाहरण के लिये, विश्व स्तर पर, अंतराष्ट्रीय शिक्षक दिवस 5 अक्टूबर को मनाया जाता है। यह संयुक्त राष्ट्रसंघ की अगुवाई में मनाया जाता है। भारत में शिक्षक दिवस हर वर्ष 5 सितंबर को मनाया जाता है। **डा0 सर्वपल्ली राधाकृष्णन** भारत के द्वितीय राष्ट्रपति थे। वह एक प्रकाण्ड विद्वान व दार्शनिक थे। उनकी विद्वता का सम्मान करने के लिये हम उनका जन्मदिन **शिक्षक दिवस** के रूप में मनाते हैं। उनका मानना था कि शिक्षक वह नहीं है जो छात्र के दिमाग में तथ्यों को ठूंसे, बल्कि वास्तविक शिक्षक वह है जो उसे आने वाले कल की चुनौतियों के लिये तैयार करे।

ज्ञान विज्ञान सरिता परिवार एक आदर्श शिक्षक की भूमिका में अपना कर्तव्य निर्वहन करने के लिये समाज के समक्ष अपना योगदान निःस्वार्थ भाव से दे रहा है। निश्चय ही वह समय आयेगा जब धीरे धीरे ही सही, लोग इसके योगदान का महत्व समझेंगे।

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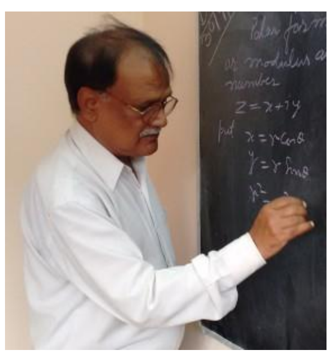
हमारा पंचवर्षीय प्रवास

Start: June-2012



April-2015

June-2016........

पारम्परिक शैक्षणिक मार्दर्शन से प्रारम्भ कर आज हम तकनीकी-विकास के सहारे मूलभूत प्रासंगिकता को आगे बढ़ने में संलग्न हैं...

An Appeal: Gyan Vigyan Sarita

A non-organizational initiative of a small set of Co-passionate Persons

Philosophy: Personal Social Responsibility (PSR)

Objective: Groom competence to Compete among unprivileged children from 9th-12th in Maths and Physics, leading to IIT-JEE.

Financial Model: *Zero-&-Fund-Zero-Asset (ZFZA).* It calls for promoters and facilitators to provide infrastructure for use to the extent they feel it is neither abused nor there is a breach of trust. And, reimbursement of operational expenses to the participators

Operation:

a. **Mode:** Online since July'16, using Google Hangouts, a free we-conferencing S/w, with connectivity upto 15 nodes.
b. **Participation:** Voluntary and Non-remunerative, Non-Commercial and Non-Political

Involvement:

a. **As Promoter** –
   i. Initiate a Learning Center,
   ii. Sponsor a Mentor who is willing to join on certain terms,
   iii. sponsor cost of operation and up-gradation of infrastructure to voluntary mentors,
   iv. Sponsor Website.
b. **As Facilitator** –
   i. Provide space and infrastructure for **Online Mentoring Sessions (OMS)**, which is generally available, with a marginal add-on,
   ii. Garner support of elite persons to act as coordinators at a Learning Centre.
c. **As Participator** –
   i. As a Mentor,
   ii. As Coordinator,
   iii. As Editor and or contributor of thought provoking articles for e-Bulletin, which are relevant to the initiative, and make it more purposeful and reachable to the target audience.
   iv. As author of Chapters for Mentors' Manual, being uploaded as a Free Web Resource,
   v. Anything else that you feel can add value to the mission and make it more purposeful.
   vi. *Anything else that you consider to make this initiative to become more effective.*

Background: *The initiative had its offing in May'12, when its coordinator, a power engineer by profession, soonafter submission of Ph.D. Thesis in April'12, at IIT Roorkee, at the age of 61 years, decided to mentor unprivileged students.*

SARTHAK PRAYASH, a Ghaziabad based NGO, warmly accepted the proposition and created a facility to mentor students from 8+ to prepare in mathematics and physics and prepare them for engineering entrance tests. They warmly reciprocated and created a class room.

Experience in this selfless social work were used to navigate across without losing focus. He was associated with SUBODH FOUNDATION from Sept'15 to Sept'16 during which he published a monthly e-Bulletin ***SUBODH-पत्रिका*** *to create visibility across persons who could make a difference.*

In Sept'16, post transition, the mission has been continued as a non-organizational entity ***Gyan Vigyan Sarita****, with a set of Four persons, including retired* ***Prof. SB Dhar****, Alumnus-IIT Kanpur, a middle aged* ***Shri Shailendra Parolkar****, Alumnus-IIT Kharagpur, settled at Texas, US and* ***Smt. Kumud Bala****, Retired Principal, Govt. School Haryana. Earlier, they were complementing the OMS. While, the initiative survived transition, a website: http://gyanvigyansarita.in has been launched. It contains under its* ***Menu: Publication>e-Bulletins****, and* ***>Mentors' Manual.*** *You may like to read them.*

Actions Requested: May please like to ponder upon this initiative. **Queries**, *if any, are heartily welcome*. We would welcome your collective complementing in any of the areas listed at **Involvement**, above, to make the mission more purposeful and reachable to target children.

Coordinator's Views

OMS – Challenges & Opportunities – Myths & Facts

Online Mentoring Sessions (OMS) for teaching is not a new proposition. It was started in 1960 by university of Illinois by linking computer systems. With the advent of technology, it has changed forms, quality and spread. But, the unprivileged students who are deprived of both opportunities and access to technology continue to remain at longer heads of education. In view of this Gyan Vigyan Sarita, a non-organizational initiative by Four co-passionate persons took up the cause to democratize education so as to groom competence to compete among unprivileged children with a ***sense of*** *Personal Social Responsibility* ***(PSR).*** *It is a totally selfless mission based on a financial model Zero-Fund-&-Zero-Asset (ZFZA) to implement OMS.*

OMS-An Overview: OMS is a proposition which aims at conceptual clarity in learning, a pre-requisite for competence to compete. It is aimed at complementing teaching efforts at school, which has its own constraints, and, therefore, runs into generating statistics irrespective of quality. It is seen that there are inquisitive students among target group, while others can be motivated, made to walk the talk by hand-holding and slowly and gradually made to reach a level of equality with their contemporary among elite class. This requires passionate, dedicated and committed persons, who can reach out to students and set the ball rolling. Increase in longevity, improvement in health conditions and financial wellbeing of persons in elite class offers an opportunity to connect target students with such passionate mentors.

I have become my own version of optimism. If I can't make it through one door, I will go through another door – or I'll make a door. Something terrific will come no matter how dark the present

- Rabindranath Tagore

With the increase in TV and internet network a good number of quality educational programmes are springing out. Despite, OMS is a proposition which provides bilateral connectivity to target students with the mentors. It neither monopolizes education nor makes teacher redundant. God forbid!, if teachers become liability, it would be a worst scenario where गुरु-शिष्य परंपरा would be lost in the oblivion.

OMS invokes schools, institutes, corporates, social groups and individuals with philanthropic intentions to come forward and adopt a school catering to target students. Such a school needs to be equipped with minimal IT infrastructure involving an incremental cost below Rupees Fifty to Sixty thousand, with nominal recurring expenses on a broadband internet connection. It also invokes elite groups to associate in the mission as a mentor, with only one request that since it is a free education it is not taken either as time pass activity or a matter of convenience. *In education continuity, consistency and commitment is most important*. Academic comfort at mentoring is only a matter of involvement and not a competition. Being a power sector engineer, it is vouched with Five Years' experience in this mission, that it highly tempting and satisfying. It is an opportunity of sharing competence, one of the most scarce yet ever growing resource, and that too to needy, without any perceivable cost. *This proposition is bound to grow not as a project but as a, much needed, social reform leaving a realization among participants of creating a legacy of living beyond self with a sense of PSR*.

Opportunities & Challenges: These are like two faces of the same coin and can't be thought of in isolation. Therefore, different aspects of opportunities and challenges are brought out below –

a. **IT Infrastructure:** In OMS critical requirement is of internet connectivity, while all other supporting setup for a smart class is readily available at reasonable cost. In digital India, gone are the days when internet connectivity is a bottleneck and ***a challenge*** to reckon with. Multiple options from high speed network to internet DTH are available with increasing speed, and data size. Moreover, service providers are exploring newer market potential. The only need is to move on, enquire and explore alternatives to overcome problems of low speed, discontinuity and data size, and utilize the ***available opportunity***. It is experienced that with audio, screen and video share data consumption is about 1 GB per hour per node, and data speed @ 8 mbps gives an acceptable ITC environment for OMS.

b. **Learning Centres:** Experience of working on the mission to democratize education over last five years poses a ***big challenge*** to get a learning centre, a pre-requisite for OMS. It is seen that in Government schools there is lack of ownership. In private schools motive has turned too commercial, right upto use of precious premises, made available at nominal cost for educational purposes. Institutes and organizations are too busy in making tall talks and brand building. Their engagements make render working on education at its basics, too trivial. Social organization and individuals who vouch for philanthropy are mesmerized with brand promotion and other compulsions of a competitive world. And at government level, it is neither a matter of charisma nor of immediate concern to their constituency. Accordingly, this selfless proposition involving a long term commitment has not been receiving an attention, that it deserves, either at the government or political level. *Thus, this motive to democratize education with PSR is of least fascination to any of them.*

The only real way to help a backward group is to give them opportunities of good education.
– Jawaharlal Nehru

Very recently an NGO promoted by NRIs was connected on a very high note with an ambitious plan to establish Online Mentoring for students in a rural area. They made an investment about 13-14 times of our estimate cost to establish facilities for OMS in school run by a presitigious mission in the country. During, first session, it was possible to arouse thrill and excitement among students, volunteer and management of the school. This needs to be sustained with passionate-committed mentor, school and promoters. Occasional events would be of cosmetic value, and in long run would turn into betrayal of trust of unprivileged children. Moreover, in a socially inspired and awakened society doing this with an *ability to raise the meagre funds, for such a cause, is believable.*

c. **Mentors:** The OMS proposition is an **opportunity** for elite and accomplished young and senior citizens to reach out to deprived section of students. Requirement of धन in mentoring is insignificant. All that is required is dedication of passion and commitment with तन और मन, a most scare resource at all times. Increase in longevity, improvement in health conditions and higher financial and intellectual competence of elite senior citizens, deserve to be used make their life purposeful and satisfying. *It is the time to return back to the society, all that they received, all along their fateful journey of life, which brought them a comfort at an advanced age.* As regards youngsters, it is only a matter of attitude to look beyond self. *In this all the strains and compulsions of personal and professional life would diffuse faster than that they experience during party-hard and days-out during long weekend.* Despite, it is a real **challenge** to change the mind-set of youngsters and senior citizen and to take them out-of-box of complacency, and excuses of occupation and that they left academics long ago. We can vouch with the personal experience that both the challenges are comfortably surmountable just by getting into it.

d. **Accessibility of Students and Mentors:** Despite increase in number of schools and rampant growth of coaching and tuition centres, mutual accessibility of desperate students and passionate teachers is confronted with multiple barriers viz. time, travel, geographical, physical, health barriers, associated cost etc. It is a **big challenge**. But, the technology of OMS which is easily accessible and affordable is a **great opportunity**, and its cost is insignificant as compared to benefits it can reap with involvement of passionate and committed people.

e. **Group Learning:** Whosoever and whatsoever best mentoring one receives, the real learning comes in helping colleagues. Even the best teachers admit that they are first a student. Chak-De-India movie, unlike Dangal movies, is an excellence example of group learning, where better ones complement their other, while others learn to respect and cooperate with the better ones. This in turn builds in them a team spirit and leadership, which goes with students, in rest of their life. In absence of this type of group dynamics, in OMS, inability of a mentor to monitor and guide each and every remote student is the **biggest challenge**, and it compounds exponentially, when multiple learning centres are catered by a mentor. In OMS, each student is encouraged and supported to identify two students, and help them in learning, and to keep connected with atleast one better student to seek guidance in solving difficulties.

This is seen as **an opportunity** to turn passive ones into better students, human being, a team player and a leader in a true sense. In addition, it complements mentor to monitor group of students at multiple learning centres, a potential of OMS, beyond all barriers.

f. **Target Students:** Drop out of target students in an educational initiative is due to multiple reasons is a real **challenge**. It is also an **opportunity** to *passionate persons and advocates of social reform to deploy their commitment for a real social reform at its roots.*

A businessman thinks of immediate returns, a politician acts for next election, and a statesman thinks and acts for next generation.

g. **Students from Elite Families:** Academic competence that they inherit is a **great opportunity** to integrate them into this OMS involving group dynamics, for an accelerated learning at each learning centre. In addition this would help to groom them into better and compassionate human being. But, a **real challenge** is to *mentor their elite parents who are obsessed with their affluence and ability to choose costliest tuition/ coaching/ school.* As against this quality education with unprivileged children, offered in OMS is perhaps below their dignity.

h. **Continuous Learning:** Learning is a process where continuity, consistency and commitment of mentor and students is extremely important. Making this process a periodical and a part-time activity is a fashionable luxury which most volunteers are not prepared to forego. This poses a **challenge** to ensure a desired regularity in mentoring. OMS is an **opportunity** to inculcate a self-discipline and selflessly engage in mentoring, right from home or place of work. Further, it offers a facility to collectively complement each other in this relay-marathon, in case occasional constraint of any individual. Elite who volunteers for such an initiative have been practicing this to ensure their survival and growth.

i. **Communication:** A webcam, in OMS, provides to mentor a visibility of response and participation of students, though it cost higher bandwidth and data consumption. But, it is affordable, at a marginal cost, as compared to its gains. In respect of audio, use of an external USB microphone is recommended to capture voice of student. In its absence internal microphone pick up all sound in vicinity and thus increases noise level which is both irritating and deafening to the mentor. The **only challenge** is interaction of student in posing problem or participating in evolving solution. A tablets with digital pen in range of Rs 5-7 thousands are available which facilitate interactive sharing of whiteboard available in video-conferencing applications.

Besides, **bigger challenge** is bridging communication gap in the event of any expectations, difficulties and problems remaining unresolved. Identifying a class representative and extending him e-mail facility is **an opportunity** to create a spirit of participation and pride among student, who are growing in a digital era. Transparency and speedy communication, which prevalent technology supports is a key to the success of OMS. Frequent and unpredictable interruption of power affecting continutity .has the only remedy in use of UPS.

j. **OMS Hours:** The OMS from class 9th onwards is a strategic decision and is based on an experience with students at that level. They develop a requisite maturity of remote motivation and mentoring. OMS during school hours faces a **stiff challenge** to match academic schedule while meeting its own objectives. It is always possible to hold motivational periodical OMS. But, the OMS proposition is an **excellent opportunity** to mentor motivated students who are always prepared to attend session outside school hours, given the facility of set-up created for the purpose. Democratization of opportunity is the foremost objective of OMS, but the pursuit of excellence has to come voluntarily from students, and organizations facilitating and prompting OMS. It would be unwise to make it mandatory for all and is fraught with the risk of retaliation or repulsion.

k. **Management of OMS:** A real problem is to ensure discipline and nearly distraction free environment during session. In social organizations engaged in multiple activities, it is **a challenge**. Therefore, a local coordinator, best would be a subject teacher, who would ensure required discipline, use of ICT resources, and act as a bridge between mentor and remote students. It is **an opportunity** for the coordinator to proactively participate and get groomed into an online mentor, to perpetuate the chain reaction.

l. **Resources:** There might be schools, organizations and institutions for whom even one-time cost and recurring cost of internet connectivity, for minimal infrastructure is **a challenge** to meet. This model is based on Zero-Fund-&-Zero-Asset (ZFZA) financial model. In this facilitators and promoters arrange requisite funds to create the infrastructure and maintain it. Motivational expenditure, if any, on local coordinators is desirable. The mentoring through OMS is offered free of cost, except incremental cost, if any, to the mentor during the association. It is **an opportunity** for one-and-all to make best of it, *can there be any proposal better and more selfless than this*?

m. **Verifiability to Stakeholders:** In any other model of right from school to tuition/coaching stakeholders, including parents of children remain disconnected, till results are available. It is a **real life challenge** for the stake holders. The OMS offers **an opportunity** to stake holders, who are at the command of learning centres, to monitor, and audit both methods and commitment of mentors that goes into OMS.

Myths & Facts: In prevalent scenario it is obvious to doubt any new selfless initiative, and is a must. But, a scientific way is to test a proposition before one bestows trust upon it. Nevertheless, *any effort to negate an initiative without either testing it, or to find an escape route by perpetuating myths is a professional dishonesty*. Some of the myths that have been confronted are brought out with the facts which were learnt in the process. *It is with a hope that readers shall ponder upon it and make a judicious call on the proposition.*

You must be the change you wish to see in the world.
– Mahatma Gandhi

a. **Loss of Connectivity:** Students at learning centres often complain loss of either of connectivity, screen sharing, audio or video. But, invariably this complaint comes from one of the learning centres and not all. We have connected maximum Five nodes during OMS, on Google Hangouts. Little of rejigging revealed that wifi-connection of the complaining centre node was either through dongle or it was in use in multiple video modes. In either case the available bandwidth was shared across multiple users, causing insufficient bandwidth for OMS, and hence interruptions. It is recommended that at learning centres internet connectivity is exclusive and through wired connection.

b. **Tolerance to Interruptions:** Interruption in top commercial and high powered/priority channels, where best of the IT resources are used, interruptions are experienced, and are hardly objected. Considering this situation interruptions, if any, *in OMS which is rendered with meagre resources, and that for a bigger cause, does it not deserve a rational tolerance*?

c. **Paucity of Mentors:** It is just unfortunate to observe a sharp decline in number and quality of passionate teachers and mentors. Nevertheless, a journey started alone has gained momentum and got complemented by more co-passionate mentors in a selfless manner. It is true that OMS is no substitute to Chalk-N-Talk, but it is an alternative to provide a wider accessibility. All that it requires is more torch bearers who are ready to complement collectively. *It is not far off when there is a fleet of passionate mentors among senior citizens and professionals, to remedy the paucity.*

d. **Superiority of Videos:** It is really heartening that very high quality videos are available, for free, on just click of a button. They are developed by professionals in a high-tech environment. But, for a student choosing a video to start with and a forward sequence, resolution of difficulties, if encountered, language constraints and response time in the event of any doubt are some of the considerations which deserve attention. Though, OMS is in its offing, it is a much needed reform in the education, driven with PSR. Sometimes, it might encounter constraints of ICT, capability in subject or mentoring. Basically, OMS got crystallized through passionate persons coming from diverse fields. But, in any mission, driving element is not capability but a passionate commitment; mentoring capability, is auto-generated by-product of the pursuit.

Summary: *It is seen that some schools, organizations and individuals proactively volunteered to take all the help openly extended by this initiative. Soonafter, efficacy of the model was established; abrupt disdain towards volunteer and loss of enthusiasm into non-communication is capricious. Nevertheless, it is important to remember that in this OMS mission, important is passionate desperation to ignite a flash, sustain the fire, and channelizing the energy purposefully. It is just not enough to acquire raise funds and requisite ICT setup. They are the passionate mentors who drive the OMS, and neither of the equipment or the resources. There are many fields open for trade and commerce. The least one can do is to is keep education out of it.*

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GROWING WITH CONCEPTS

Concepts of an expert are not like a static foundation of a huge structure; rather it is like blood flowing in a vibrant mind.

During growing into an expert, each one must have used best of the books available on subject and received guidance of best of the teachers. Authors might have had limitations to take every concept thread bare from first principle and so also must be the constraint of teacher while mentoring a class with a diversity of inquisitiveness and focus. As a result, there are instances when on a certain concept a discomfort remains. The only remedy is to live with the conceptual problem and continue to visualize it thread bare till it goes to bottom of heart and that is an ***ingenious illustration****.*

In this column an effort is being made to take one topic on Mathematics, Physics and Chemistry in each e-Bulletin and provide its illustration from First Principle. We invite all experts in these subjects to please mail us their ingenious illustrations and it would be our pleasure to include it in the column.

We hope this repository of ingenious illustrations, built over a period of time, would be helpful to ignite minds of children, particularly to aspiring unprivileged students, that we target in this initiative, and in general to all, as a free educational web resource.

This e-Bulletin covers – a) Mathematics, b) Physics , and c) Chemistry. This is just a beginning in this direction. These articles are not replacement of text books and reference books. These books provide a large number of solved examples, problems and objective questions, necessary to make the concepts intuitive, a journey of educational enlightenment.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics it is contemplated to come up representative problems from contemporary text books and Question papers from various competitive examinations and a guide to theirsolutions in a structured manner, as a dynamic exercise to catalyse the conceptual thought process.

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OUR MENTORING PHILOSOPHY: Mentoring is not teaching, neither tuition nor coaching. It is an activity driven by passion, and commerce has no place in it. In this effort is to caution students that -

- This place is not where they will be taught how to score marks and get higher ranks, but to conceptualize and visualize subject matter in their real life so that it becomes intuitive.
- This place is not to aim at solutions but inculcate competence to analyze a problem and evolve solution.
- This place does not extend selective and personalized attention, rather an opportunity to become a part of which is focused on learning and problem solving ability collectively.
- This place provides an opportunity to find students above and below one's own level of learning. Thus students develop not in isolation but learn from better ones and associate in problem solving to those who need help. This group dynamics while create a team spirit, an essential attribute of personality, while one learns more by teaching others.
- This place has strategically chosen Online Mentoring, so that those who are unprivileged can gather at one point and those who can facilitate learning of such students by creating, necessary IT setup. Aseperate **Mentor's Manual** is being developed to support the cause.

We are implementing this philosophy through **Online Mentoring**

GROWING WITH CONCEPTS - Mathematics

DIFFERENTIAL CALCULUS

Prof. SB DHAR

Differential Calculus is a branch of Mathematics where we study the determination, properties, and applications of derivatives and differentials. This is also defined as the sub-field of calculus where we study the rate of change of quantities.

For example: If the distance changes with respect to time, then the differential calculus helps in finding the velocity i.e. the rate of change of distance with respect to time at any instant.

Or, If some balloon is being filled with air, then we can find the change in the volume of the inflating balloon with respect to time at any moment with the help of differential calculus.

Or, If water is leaking from a tank and we want to find out the rate at which level of water is getting down, the differential calculus will help us in determining it.

Basic Terminology

Function: If X and Y are two non-empty sets and to each element of X there corresponds a unique element of Y, then we say that there exists a function from X to Y and is written as $f : X \rightarrow Y$ or $f(x)=y$ where $x \in X$ and $y \in Y$.

Or, *A function f is a set of ordered pairs (x, y) such that no two of which have the same first member.*

Note: The word ***"function"*** was introduced into Mathematics by ***Leibniz***, who used this term primarily to refer to certain kinds of Mathematical Formulas. It was later realized that Leibniz's idea of *function* was much too limited in its scope, and the meaning of the word has since undergone many stages of generalization.

Domain and Range:

The set of values taken by X is called the **domain** and is denoted by ***Df***.

The set of values taken by Y is called the **range** and is denoted by ***Rf***.

Or, ***Domain*** is the value of x for which the function remains defined. Hence ***domain*** is also named as ***domain of definition***.

Facts Relating To Domain

(a) Domain of expressions under even roots i.e. square root, fourth root etc., is always ≥ 0.

(b) Domain of $f(x) \pm g(x)$ or $f(x).g(x)$ or $f(x)/g(x)$ is given by $D_1 \cap D_2$ or $D_1 \cap D_2 - \{g(x)=0\}$ respectively.

Facts Relating To Range

(a) If $R_f \subseteq Y$, then the function is called **Into** Function.
(b) If $R_f = Y$, then the function is called **Onto** Function.
(c) Each of the two ***Into*** and ***Onto*** (**Surjective**) functions is either ***One-One*** or ***Many–One*** function.
(d) ***One-one Onto*** function is also called ***Bijective*** (or ***Injective and Surjective***) function.

Types of Function

There are a number of functions. Some of them, that are useful for the students of class XII and equivalents are being discussed here with their properties.

Inverse Function: If $f: X \rightarrow Y$ is a function defined, then f^{-1}, the inverse of f is defined as $f^{-1}: Y \rightarrow X$. Inverse of a function is defined only when it is one-one-onto. Domain is Y and range is X.

Constant Function: If $f(x) = c$ for all $x \in X$, then $f : X \rightarrow Y$ is called a constant function. Domain of constant function is X and range is {c}, a singleton set.

Identity Function: If $f(x) = x$ for all $x \in X$, then $f : X \rightarrow Y$ is called an identity function.

Domain and Range of this function is X or, $(-\infty, \infty)$, if it is defined on set of Real Numbers.

Power Function: If $f(x) = x^a$ for $a \in R$, then $f : R \rightarrow R$ is called Power function.

Domain of power function is $(-\infty,\infty)$ and the range is also $(-\infty, \infty)$.

Absolute value function or Modulus Function: Absolute value function is also called modulus function or numerical value function. It is defined as $f(x) = |x|$.

$|x|$ means it is + x if x > 0, -x if x < 0, and 0 if it is 0.

Domain of absolute value function is $(-\infty, \infty)$. The range of this function is $[0, \infty)$.

Signum function: It is defined as, $y = \text{sgn}(x) = \frac{|x|}{x}$.

Domain of signum function is R.

The range is {-1,1}.

Sometimes, it is defined as,

$$y = \begin{cases} \frac{|x|}{x} \; i.e., \\ +1 \; when \; x > 0 \\ -1 \; when \; x < 0 \\ 0 \; when \; x = 0 \end{cases}$$

In this case the domain is R, and the range is {-1,0,1}

Greatest Integer Function: The greatest integer function is also called the step up function. It is written as ***f(x) = [x].***

It means, f(0) = [0] = 0; f(3/2) = [3/2] = 1;

f(-1/2) = [-1/2] = -1.

It consists of two parts:

The ***Integral part*** and the ***fractional part***.

The fractional part is always greater than or equal to zero but less than 1.

It is also denoted as ***f(x)=[x]=x+{x}***.

Its Domain is R and the Range is the set of all integers.

Note:

(a) [x+k]=[x]+k, if k is any positive integer.
(b) [-x]=-[x], for all $x \in Z$.
(c) [-x]=-[x]-1, $x \notin Z$.
(d) $[x_1+x_2] \geq [x_1]+[x_2]$
(e) $\left[\frac{[x]}{n}\right] = \left[\frac{x}{n}\right], \forall n \in N$
(f) $[x] + \left[x + \frac{1}{n}\right] + \ldots + \left[x + \frac{n-1}{n}\right] = [nx]$

The fractional part function: The fractional part function

{x}	***= x,***	if $0 \leq x < 1$, and
	= 0	if $x \in Z$
{-x}	***= -1-{x}***	if $x \notin Z$

Domain is R, Range is [0,1).

Exponential function: Exponential function is defined as ***f(x)= a^x***.

It increases if x > 0 and decreases if x < 0 in case of *a>0*.

Domain of exponential function $f(x) = a^x$, $a > 0$ and $a \neq 1$ is set of Real Numbers.

Logarithmic function: Logarithmic function is defined as ***f(x)= $\log_a x$***.

It is defined only when ***a>0***. It is not defined when $a \leq 0$. It increases if $a > 1$ and decreases if $0<a<1$.

Domain of logarithmic function

$f(x) = \log_a x$; $(x, a > 0)$ and $a \neq 1$ is all real positive numbers.

Even function: A function f(x) is called an even function if ***f(-x) = f(x)*** for all x.

Odd function: A function f(x) is called an odd function if ***f(-x) = - f(x)*** for all x.

Note:

(a) The product of two even or two odd functions is always even function.
(b) The product of even and odd is always an odd function.
(c) Every function can be expressed as the sum of an even and an odd function.

For example

$$f(x) = \frac{1}{2}\left(f(x) + f(-x)\right) + \frac{1}{2}\left(f(x) - f(-x)\right)$$

Periodic function: A function f(x) is said to be periodic if there exists such an $T > 0$ for which $f(x + T) = f(x - T) = f(x)$ for all $x \in X$.

Note:

(a) There are infinitely many ***T*** satisfying the equality but the ***least positive*** is said to be ***the period***.
(b) Period of ***sinx, cosx, cosecx, secx is*** 2π and the period of ***tanx, cotx is*** π.

Monotonic Function: Monotonic functions are defined on interval. Monotonic functions are of following types:

(a) Monotonic increasing if for all $x_1, x_2 \in [a,b]$ and $x_1 < x_2$ there exists $f(x_1) \leq f(x_2)$.
(b) Monotonic decreasing if for all $x_1, x_2 \in [a,b]$ and $x_1 < x_2$ there exists $f(x_1) \geq f(x_2)$.

(c) Strictly monotonic increasing if for all $x_1, x_2 \in [a,b]$ and $x_1 < x_2$ there exists $f(x_1)<f(x_2)$.
(d) Strictly monotonic decreasing if for all $x_1, x_2 \in [a,b]$ and $x_1 < x_2$ there exists $f(x_1)>f(x_2)$.

Composite Function: If ***f(x) = y*** and ***g(y) = z*** then ***fog(y)*** is defined if the range of ***g*** $\subseteq$ ***domain of f*** ; and ***gof(x)*** is defined if range of ***f*** $\subseteq$ ***domain of g***.

$fog \neq gof$ (in general).

Note:

(a) If ***f*** is one-one, ***gof*** is one-one.
(b) If ***f*** is onto, ***gof*** is onto.
(c) If ***f, g*** are one-one onto, ***gof*** is one-one onto.

Domains of some functions:

(a) $\sin^{-1} x$ = [-1,1]
(b) $\cos^{-1} x$ = [-1,1]
(c) $\tan^{-1} x$ = R
(d) $\cot^{-1} x$ = R
(e) $\text{cosec}^{-1} x$ = R-(-1,1)
(f) $\sec^{-1} x$ = R-(-1,1)
(g) $\sqrt{(a^2-x^2)}$ =[-a,a]
(h) $\sqrt{(x^2-a^2)}$ $=(-\infty,a]\cup[a,\infty)$
(i) $1/\sqrt{(a^2-x^2)}$ =(-a,a)
(j) $1/\sqrt{(x^2-a^2)}$ $= (-\infty,a)\cup(a,\infty)$
(k) $\sqrt{(x-a)(b-x)}$ = [a,b] iff a<b
(l) $1/\sqrt{(x-a)(b-x)}$ = (a,b) iff a<b
(m) $\sqrt{(x-a)(x-b)}$ $= (-\infty,a]\cup[b,\infty)$ if a<b
(n) $1/\sqrt{(x-a)(x-b)}$ $= (-\infty,a)\cup(b,\infty)$ if a<b
(o) $\sqrt{\frac{x-a}{x-b}}$ $= (-\infty,a]\cup(b, \infty)$ if a<b
(p) $\sqrt{\frac{x-a}{x-b}}$ $= (-\infty,b)\cup[a,b]$ if a>b
(q) $\sqrt{\frac{a-x}{b-x}}$ = [a,b] if a<b
(r) $\sqrt{\frac{a-x}{b-x}}$ = [b,a] if a>b
(s) log (x-a)(b-x) = (a,b) if a<b
(t) log (x-a)(x-b) $= (-\infty,a) \cup (b, \infty)$ if a<b

Ranges of some functions:

(a) $\sec^{-1}x$ $= [0,\pi]-\{\pi\}$
(b) $\text{cosec}^{-1}x$ $= [-\pi/2,\pi/2]-\{0\}$
(c) $\cot^{-1}x$ $= (0,\pi)$
(d) $\tan^{-1}x$ $= (-\pi/2,\pi/2)$
(e) $\sin^{-1}x$ $= [-\pi/2,\pi/2]$
(f) $\cos^{-1}x$ $= [0,\pi]$

Dirichlet function

$\lambda(x)$ = 1, if x is rational
= 0, if x is irrational.

It is discontinuous at each point.

Examples:

(a) $\lambda(0) = 1$
(b) $\lambda(-1/2) = 1$
(c) $\lambda(\sqrt{2}) = 0$
(d) $\lambda(\pi) = 0$

Note:

(a) If ***f(x)*** is polynomial function satisfying ***f(x)f(1/x)=f(x)+f(1/x)*** then the function should be assumed as ***f(x)= 1 ±*** x^n.
(b) $(x-1) < [x] \leq x$, $2x-1 < [2x] \leq 2x$.

The factorial function: This is defined as ***f(n)=*** n! = 1.2.3….. *n. for all positive integers.*

The domain of this function is the set of positive integers. The range increases so rapidly that it is more convenient to display this function in tabular form rather than as a graph. This is listed as the pairs *(n, n!)*.

Equal Functions: ***Two functions*** *f and g are equal if* and only if

(i) *f and g have the same domain, and*
(ii) *f(x) = g(x) for every x in the domain.*

Linear function: A function f defined for all real x by a formula of the form *f(x) = ax + b* , is called a linear function because its graph is *a straight line*.

Lattice Point: A point (x, y) in the plane is called a ***lattice point*** if both the co-ordinates x and y are integers.

Determinate Form: When a unique value of an expression ***f(x)*** at ***x = a*** is possible, it is said that it is of the determinate form.

For example: The expression f(x)= $\frac{x^2-4}{x-2}$ is determinate at ***x = 1*** as it has a unique value 3 at x=1.

Indeterminate Form: When a unique value of an expression ***f(x)*** at *x=a* is not possible, it is said that it is indeterminate.

For example: The expression f(x)= $\frac{x^2-4}{x-2}$ at x=2 becomes $\frac{0}{0}$ which will give no unique value at x=2, hence it is in Indeterminate form.

Some other Indeterminate forms

$\frac{\infty}{\infty}, 0\times\infty, \infty-\infty, 1^{\infty}, 0^0, \infty^0$ **etc.**

Note:

(a) $\log_a 0$ is not defined but $\log_a 0 \to -\infty$ for a > 1 and $+\infty$ for 0 < a < 1.

(b) In case of $1^{\infty}, 0^0, \infty^0$ take logarithm and then use the appropriate method to evaluate the limit.

Continuity: A function is said to be continuous when the value of the function is equal to its limit, i.e.

Value = Right Hand limit = Left Hand limit.

Facts regarding Continuity:

(a) A function is said to be continuous at a point x=c if f(c)= $\lim_{x\to c+} f(x) = \lim_{x\to c-} f(x)$

(b) If *RH Limit=LH Limit=value of the function*, the function is continuous otherwise discontinuous.

(c) If f(x) and g(x) are continuous then cf(x) is also continuous.

(d) f(x)±g(x), f(x).g(x), f(x)/g(x) are also continuous.

(e) If f(x) is defined on [a,b] then f(x) is said to be continuous at end points at x=a if $f(a) = \lim_{x\to a^+} f(x)$ and at x=b if $f(b) = \lim_{x\to b^-} f(x)$. At x=a, LHL and at x=b, RHL cannot be checked.

(f) A function is said to be continuous on its domain if it is continuous at the end points and at all points lying between a and b.

(g) If f(x) is defined on (a,b) then the function cannot be checked for continuity at end points as they are not included in the domain at all. In this case only at the interior point, the continuity may be checked.

(h) If f(x) is continuous on [a,b] such that f(a) and f(b) are of opposite signs, then there exists at least one solution of f(x)=0 in the open interval (a,b).

(i) Every polynomial is continuous at every point of the real line.
For example
$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n$
is continuous on R.

(j) Every Rational function is continuous at every point where the denominator is not zero.

(k) Logarithmic, Exponential, Trigonometric, Inverse Trigonometric, Modulus functions are continuous in their domain of definition.

(l) Point Function (i.e. domain and range containing only one point) is a discontinuous function.

Cauchy's Definition of Continuity: A real valued function f defined on an open interval I is said to be continuous at a $\in$ I iff for any arbitrarily chosen positive number ε, however small, we get a corresponding number $\delta > 0$ such that $| f(x) - f(a) | < \varepsilon$ for all values of x for which $| x - a | < \delta$.

Heine's Definition of Continuity: let a function f be defined on some neighbourhood of a point a, then f is said to be continuous at a iff for every sequence ***a_1, a_2, a_3,…., a_n,…***of real numbers for which

$\lim_{n\to\infty} a_n = a$, we have

$\lim_{n\to\infty} f(a_n) = \lim f(a)$

Discontinuity: The function is said to be discontinuous if either the limit does not exist or value is not equal to its limit.

Note:

(a) The discontinuity is said to be of ***first kind*** if both the limits (Right Hand Limit and Left Hand Limit) exist and are not equal. This is also called non-removable discontinuity of first kind.

(b) The discontinuity is said to be ***removable*** if $\lim_{x\to c+} f(x) = \lim_{x\to c-} f(x) \neq f(c)$

(c) The discontinuity is said to be of ***second kind*** if at least one of the limit does not exist. **Remember!** The limits are said to be existing if they are finite and definite. This is also called infinite discontinuity.

(d) The difference between RHL and LHL is called the ***jump*** discontinuity.

Properties of Limits

(a) If $\lim_{x\to a} f(x) = l, \lim_{x\to a} g(x) = m,$ then $\lim_{x\to a}\{f(x) \pm g(x)\} = l \pm m$; if l and m exist

(b) $\lim_{x\to a}\{f(x).g(x)\} = l.m$

(c) $\lim_{x\to a}\{f(x)\}^{g(x)} = l^m$

(d) $\lim_{x\to a}\{fog(x)\} = f\left(\lim_{x\to a} g(x)\right) = f(m)$

(e) In particular, $\lim_{x\to a} e^{f(x)} = e^{\lim_{x\to a} f(x)} = e^l$

(f) $\lim_{x\to a} \frac{1}{f(x)} = 0$ if $\lim_{x\to a} f(x)$ is + ∞ or - ∞.

(g) $\lim_{x\to 0^+}\left[\frac{\tan x}{x}\right] = 1^+$

(h) $\lim_{x\to 0^+}\left[\frac{\sin x}{x}\right] = 0$, as $\frac{\sin x}{x} < 1$.

(i) $\lim_{x\to 0^-}\left[\frac{\sin x}{x}\right] = 0$, as $\frac{\sin x}{x} < 1$

(j) $\lim_{x\to 0^+} \frac{\{x\}}{\tan\{x\}} = 1$, as $\{x\} \to 0$ when $x \to 0$.

(k) $\lim_{x\to\infty} x.\sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$

(l) $\lim_{x\to\infty} \cos\frac{1}{x} = 1$

(m) $\lim_{x\to\infty} x.\tan\frac{1}{x} = 1$

(n) If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ then $\lim_{x\to a}(1 + f(x))^{\frac{1}{g(x)}} = e^{\lim_{x\to a}\frac{f(x)}{g(x)}}$

(o) If $\lim_{x\to a} f(x) = 1, \lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a}\left(f(x)\right)^{g(x)} = \lim_{x\to a}(1 + f(x) - 1)^{g(x)}$ $= e^{\lim_{x\to a}(f(x)-1)g(x)}$

(p) Particularly: $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e; \lim_{x\to 0}(1+\lambda x)^{\frac{1}{x}} = e^{\lambda}$

L'Hospital Rule: This is applied if the function is differentiable and is of the form (0/0) or (∞/∞).

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}; g'(x) \neq 0$$

Example: $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$. This is of the form $\infty - \infty$.

Let us change to (0/0) form by simplifying as $\lim_{x\to 0}\left(\frac{\sin x - x}{x\sin x}\right)$

Apply L'Hospital Rule i.e. differentiate Numerator and Denominator separately equal number of times and when it is not of 0/0 form , put x=0 to find the limit = 0.

Methods of finding Limits

Factorization Method: When the expression is of the form (0/0). First the Numerator and the Denominator are factorised. Common factors are cancelled and then the value of x is put in the expression left after cancellation of the factors to find the required limit.

Example: $\lim_{x\to 2}\frac{x^2-4}{x-2}$ is of the form (0/0) when x=2 is put in the Numerator and Denominator. Hence first Factoise the Numerator and then proceed as below:

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=2+2=4$$

It is the limit of f(x) at x=2.

Rationalisation Method: When the expression is one of the Indeterminate forms and is not factorisable, first it is multiplied by its conjugate of the irrational part, in the Numerator and the Denominator so that its value does not change. This transforms the expression into the Factorization method where a common factor is generated to be cancelled and then by putting the value it gives the limit.

Example:

$\lim_{x\to\infty}\left(\sqrt{x^2+kx}-\sqrt{x^2-kx}\right)$ is of the form $\infty-\infty$ It cannot be factorized so multiplication by conjugate is done to find the limit as below:

$$\lim_{x\to\infty}\left(\sqrt{x^2+kx}-\sqrt{x^2-kx}\right)$$
$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+kx}-\sqrt{x^2-kx}\right)}{\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)}.\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)$$
$$=\lim_{x\to\infty}\frac{\left(x^2+kx\right)-\left(x^2-kx\right)}{\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)}$$
$$=\lim_{x\to\infty}\frac{2kx}{\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)},\left(\frac{\infty}{\infty}\right)\text{form.}$$

Divide the Numerator and the Denominator by the **highest power of x** i.e. x and evaluate the limit assuming $\lim_{x\to\infty}\frac{1}{x}\to 0$.

$$\lim_{x\to\infty}\frac{2kx}{\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)}$$
$$=\lim_{x\to\infty}\frac{2k}{\left(\sqrt{1+\frac{k}{x}}+\sqrt{1-\frac{k}{x}}\right)}=\frac{2k}{1+1}=k$$

Method for finding limit when x tends to infinity: When the expression is of the form $\left(\frac{\infty}{\infty}\right)$ then divide the Numerator and the Denominator by the highest power of the variable x present in the Numerator or denominator and then put x=∞ that leads to the required limit as (1/∞)→0.

Example:

$$\lim_{x\to\infty}\frac{2kx}{\left(\sqrt{x^2+kx}+\sqrt{x^2-kx}\right)}$$
$$=\lim_{x\to\infty}\frac{2k}{\left(\sqrt{1+\frac{k}{x}}+\sqrt{1-\frac{k}{x}}\right)}=\frac{2k}{1+1}=k$$

Important Note: Sometime the evaluation of the limit appears of no form. Then the exponential form of rewriting it helps in its evaluation.

For example: $\lim_{x\to 0}\left|\cot x\right|^{\sin x}=e^{\lim_{x\to 0}\sin x\log_e\left|\cot x\right|}$,as it is obvious that the use of exponential writing helps and changes one of the known forms as $e^{\log_e z}=z$

Evaluation of Trigonometric Functions: Trigonometric Functions are expandable. So using method of expansions the nature of Indeterminate is firstly removed and then the limit is evaluated. Remember Expansions of sin x, cos x, tan x, $\sin^{-1}$x, $\cos^{-1}$x, $\tan^{-1}$x etc.

Evaluation of Exponential Functions and Logarithmic functions are evaluated by the Method of expansion as they are also expandable. Remember the expansions of e^x, e^{-x}, a^x, log(1+x), log(1-x) etc.

Some limits that do not exist

$$\lim_{x\to 0}\left(\frac{1}{x}\right) \qquad \lim_{x\to 0}\cos\left(\frac{1}{x}\right)$$

$\lim_{x\to 0}\left(\frac{1}{x}\right)$ $\lim_{x\to 0} e^{\frac{1}{x}}$

$\lim_{x\to\infty}\cos x$ $\lim_{x\to\infty}\sec x$

$\lim_{x\to 0} x^{1/x}$ $\lim_{x\to a}\frac{|x-a|}{x-a}$

$\lim_{x\to\infty}\sin x$ $\lim_{x\to\infty}\cos ecx$

$\lim_{x\to\infty}\tan x$ $\lim_{x\to\infty}\cot x$

Useful Important Expansions for finding limits

(a) $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+......$ for x < 1

(b) $(1-x)^n = 1-nx+\frac{n(n-1)}{2!}x^2-......$

(c) $(1-x)^{-n} = 1+nx+\frac{n(n+1)}{2!}x^2+......$

(d) $\frac{x^n-a^n}{x-a} = x^{n-1}+ax^{n-2}+a^2x^{n-3}+.....+a^{n-1}$

(e) $\frac{x^n-a^n}{x+a} = x^{n-1}-ax^{n-2}+a^2x^{n-3}-...+(-)^{n-1}a^{n-1}$

(f) $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+......+\frac{x^r}{r!}+.......$

(g) $e^{-x} = 1-\frac{x}{1!}+\frac{x^2}{2!}-......+(-)^r\frac{x^r}{r!}+.......$

(h) $\log_e(1+x) = x-\frac{1}{2}x^2+\frac{1}{3}x^3-...........$

(i) $\log_e(1-x) = -x-\frac{1}{2}x^2-\frac{1}{3}x^3-...........$

(j) $\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-...........$

(k) $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-............$

(l) $\sec x = 1+\frac{x^2}{2}+\frac{5x^4}{24}+\frac{61x^6}{720}+...........$

(m) $x\cos ecx = 1+\frac{x^2}{6}+\frac{7x^4}{360}+\frac{31x^6}{15120}+........$

(n) $\sin^{-1}x = x+\frac{x^3}{3!}+\frac{1^2.3^2.x^5}{5!}+...........$

(o) $\cos^{-1}x = \frac{\pi}{2}-\left(x+\frac{x^3}{3!}+\frac{1^2.3^2.x^5}{5!}+.......\right)$

(p) $\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+...........$

(q) $\sec^{-1}x = 1+\frac{x^2}{2!}+\frac{5x^4}{4!}+\frac{61x^6}{6!}+.........$

(r) $x\cot x = 1-\frac{x^3}{3}+\frac{x^4}{45}-\frac{2x^6}{945}+...........$

DIFFERENTIATION

A function f(x) is said to be differentiable if ***RH derivative* = *LH derivative* = *finite*** , otherwise it is said to be not differentiable or,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f(x-h)-f(x)}{-h}$$

The Right hand derivative is also called Progressive derivative and the Left hand derivative is called the Regressive derivative

Properties:

(a) The derivatives should be finite.

(b) A function defined on open interval (a, b) is said to be differentiable in an open interval (a, b) if it is differentiable at each point of (a, b).

(c) A function defined on closed interval [a, b] is said to be differentiable at end points a and b, if it is differentiable from the right at a i.e.

$\lim\limits_{x \to a^+} \frac{f(x)-f(a)}{x-a}$ exists and is finite and is differentiable from the left at b i.e. $\lim\limits_{x \to b^-} \frac{f(x)-f(b)}{x-b}$ exists and is finite.

(d) A function is said to be differentiable function if it is differentiable at every point of its domain.

(e) If A function is differentiable in the open interval (a, b) and also at the end points a and b then it said to be differentiable in the closed interval [a, b].

(f) If a function is differentiable at a point, then it is necessarily continuous at that point but the converse is not true i.e. if it is continuous then it may or may not be differentiable at that point.

(g) If f(x) and g(x) are differentiable, then f(x) $\pm$ g(x) or f(x).g(x) are also differentiable.

(h) If f(x) is differentiable and g(x) is not differentiable then f(x) g(x) may be differentiable.

(i) If f(x) is not differentiable and g(x) is also not differentiable then f(x) g(x) may be differentiable.

(j) A function is not differentiable at ***kink (corner)*** as a unique tangent cannot be drawn at that point i.e. a function is derivable iff its graph is always smooth i.e. there exists no break or corner.

(k) The derivative of a Periodic Function is also a periodic function having the same fundamental period.

(l) The derivative of an even function is an odd function and the derivative of an odd function is an even function.

(m) Differentiability of a function at a point implies the continuity at that point only.

Note: The function may be continuous but not differentiable.

For example:

f(x) = x sin(1/x), $x \neq 0$

=0, otherwise

Is continuous at x=0 but not differentiable at x=0 as the limit does not exist.

Some Facts

(a) $\frac{dy}{dx}$ represents the derivative of ***y w.r.t. x*** and is also the rate of change of y with respect to x. This also represents the slope of the tangent to the curve at (x, y).

(b) If tangent is parallel to x-axis then $\frac{dy}{dx}$=0 and if it is perpendicular to the x-axis then $\frac{dy}{dx}=\frac{1}{0}$. The value 1/0 should not be written as ∞, as it is not a number but an assumption.

(c) A function is said to be increasing if f’(x) > 0 for all x in its domain.

(d) A function is said to be decreasing if f’(x) < 0 for all x in its domain.

(e) If f’(x) is possibly negative or positive then it is many-one.

(f) If f’(x) >0, for all real x then it in one-one onto.

(g) For comparison of two functions f(x) and g(x), we should check whether ***h(x)= f(x)-g(x)*** is increasing or decreasing.

(h) If a function is strictly increasing in closed interval [a, b] then f(a) is local minimum and f(b) is local maximum.

(i) If a function is strictly decreasing in closed interval [a, b] then f(a) is local maximum and f(b) is local minimum.

(j) In second derivative test for maximum and minimum values, one must note that this method cannot be applied at the points where f’(x) is undefined.

(k) For global maximum and minimum values in the closed interval [a,b] all values including at a and b of f(x) should be evaluated and then noted for maximum and minimum.

Leibnitz formula for successive differentiation of explicit functions:

$$(uv)^{(n)} = u^{(n)}v + {}^nC_1u^{(n-1)}v' + {}^nC_2u^{(n-2)}v'' + ... + {}^nC_nuv^{(n)}$$

Use of Newton-Leibnitz formula for definite integral: if

$$I = \int_{\psi(x)}^{\phi(x)} f(x)dx \Rightarrow \frac{dI}{dx} = f\{\phi(x)\}\frac{d\phi(x)}{dx} - f\{\psi(x)\}\frac{d\psi(x)}{dx}$$

Some derivatives

$$\frac{d}{dx}(constt) = 0 \qquad \frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$$

$$\frac{d}{dx}([x]) = 0 \qquad \frac{d}{dx}(x^n) = n.x^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\cos ec^2 x \qquad \frac{d}{dx}(\sec x) = \sec x.\tan x$$

$$\frac{d}{dx}(\cos ecx) = -\cos ecx.\cot x$$

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos ec^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(vers^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{2ax-x^2}}$$

where ***vers x* = *1-cosx*** and ***1-sinx* = *covers x***

$$\frac{d}{dx}\{f(x) \pm g(x)\} = \frac{d}{dx}\{f(x)\} \pm \frac{d}{dx}\{g(x)\}$$

$$\frac{d}{dx}\{f(x).g(x)\} = g(x).\frac{d}{dx}\{f(x)\} + f(x)\frac{d}{dx}\{g(x)\}$$

$$\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\left(\frac{d}{dx}Nr\right)Dr - \left(\frac{d}{dx}Dr\right)Nr}{Dr^2}$$

Note:

(a) In differentiation of inverse Trigonometric functions if no branch is mentioned then, then the Principal branch should be taken in consideration.

(b) The differentiation of a determinant is the sum of the differentiations of the determinants of their rows in order, one at a time, i.e.,

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & f(x) \\ h(x) & f(x) & g(x) \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ g(x) & h(x) & f(x) \\ h(x) & f(x) & g(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g'(x) & h'(x) & f'(x) \\ h(x) & f(x) & g(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & f(x) \\ h'(x) & f'(x) & g'(x) \end{vmatrix}$$

(c) $$\frac{d}{dx}\begin{pmatrix} f(x) & g(x) & h(x) \\ g(x) & h(x) & f(x) \\ h(x) & f(x) & g(x) \end{pmatrix} = \begin{pmatrix} f'(x) & g'(x) & h'(x) \\ g'(x) & h'(x) & f'(x) \\ h'(x) & f'(x) & g'(x) \end{pmatrix}$$

(c) The chain rule is expressed as: $\frac{dy}{dx}=\frac{dy}{ds}\cdot\frac{ds}{dt}\cdot\frac{dt}{dx}$

n^{th} Derivatives Of Functions:

(a) $\frac{d^n}{dx^n}x^m=m(m-1)(m-2)..(m-n+1)x^{m-n}$

(b) $\frac{d^n}{dx^n}e^{ax}=a^n e^{ax}$

(c) $\frac{d^n}{dx^n}a^{bx}=b^n a^{bx}(\log_e a)^n$.

(d) $\frac{d^n}{dx^n}\sin(ax+b)=a^n\sin\left(ax+b+\frac{n\pi}{2}\right)$

(e) $\frac{d^n}{dx^n}\cos(ax+b)=a^n\cos\left(ax+b+\frac{n\pi}{2}\right)$.

(f) $\frac{d^n}{dx^n}e^{ax}\cos(bx+c)\ =\left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\cos(bx+c+n\phi)$

where $\tan\phi=\frac{b}{a}$

(g) $\frac{d^n}{dx^n}e^{ax}\sin(bx+c)$

$=\left(a^2+b^2\right)^{\frac{n}{2}}e^{ax}\sin(bx+c+n\phi)$

where $\tan\phi=\frac{b}{a}$

Important Formulae

(a) The ***slope*** of the tangent for the function y=f(x) at point (x_1,y_1) is given by $\tan\psi=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$ = tangent of the angle between the positive direction of x-axis and the tangent.

(b) The slope of the normal is given by $-\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$

(c) ***Tangent*** at (x_1, y_1) is written as
$y-y_1=(dy/dx)(x-x_1)$

(d) ***Normal*** at (x_1, y_1) is written as
$y-y_1=-(dx/dy)(x-x_1)$

(e) If the line is parallel to x-axis
dy/dx=0

(f) If the line is perpendicular to x-axis
dy/dx= 1/0

(g) ***Length of tangent*** $=\frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}}$

(h) ***Length of normal*** $=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

(i) ***Length of sub-tangent*** $=\frac{y}{\left(\frac{dy}{dx}\right)}$

(j) Length of ***sub-normal*** = $y.\frac{dy}{dx}$

(k) Intercept of tangent on x-axis=$\left|x-y.\left(\frac{dy}{dx}\right)\right|$

(l) Intercept of tangent on y-axis=$\left|y-x.\left(\frac{dy}{dx}\right)\right|$

(m) Two curves touch each other if at the point of contact $m_1 = m_2$

(n) Two curves cut each other orthogonally if $m_1 m_2 = -1$

(o) If function f(x) is continuous on [a,b] such that f'(c) ≥0, or f'(c)>0 for each c ∈(a,b) then f(x) is said to be monotonically (strictly)increasing function on [a,b]

(p) If function f(x) is continuous on [a,b] such that f'(c) ≤0, or f'(c)<0 for each c ∈(a,b) then f(x) is said to be monotonically (strictly) decreasing function on [a,b]

(q) If f(x) and g(x) are monotonically (or strictly) increasing (or decreasing) functions on [a,b] then gof (x) is a monotonically (strictly) increasing function on [a,b]

(r) If one of the function f(x) and g(x) is montonically (or strictly) increasing and other amonotically (or strictly) decreasing , then gof(x) is monotonically (or strictly) decreasing on (a,b)

(s) If f(x) is an ***increasing function***[1] on (a,b) then tangent makes an acute angle with +ive direction of x-axis ie dy/dx > 0.

(t) If f(x) is ***decreasing function***[2] on (a,b) then tangent makes an obtuse angle with the +ive direction of x-axis ie dy/dx<0.

(u) The sign of the derivative gives a **sufficient condition** for the function to be increasing or decreasing but this condition is by means **necessary** . The function $f(x)=x^3$ produces a counter example as this is differentiable and increasing on (-1,1) and everywhere else except at x=0 where it is 0.

Important Theorems

Fermat Theorem : Let a function ***y=f(x)*** be defined on a certain interval and have a maximum or a minimum value at an interior point x_0 of the interval.

If there exists a derivative $f'(x_0)$ at the point x_0 then $f'(x_0)=0$.

Rolle's Theorem : ***f(x)*** is continuous on ***[a,b]***, derivable in ***(a,b)*** and ***f(a) = f(b)*** then there exists atleast one point ***c*** $\in$***(a,b)*** such that $f'(c)=0$.

Lagrange's Mean Value Theorem : If ***f(x)*** is continuous on ***[a, b]***, derivable in ***(a, b)*** then there exists atleast one point ***c*** $\in$ ***(a,b)*** such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

Cauchy's Theorem : Let ***f(x)*** and ***g(x)*** be two functions continuous in the interval ***[a,b]*** and have finite derivatives at all interior points of the interval. If these derivatives do not vanish simultaneously and $g(a)\neq g(b)$, then there exists $\varepsilon\in(a,b)$ such that $\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(\varepsilon)}{g'(\varepsilon)}$

Sandwich Theorem (Squeeze Theorem) : This is sometimes also called Pinching Theorem. It states that if ***g(x)*** is squeezed between ***f(x)*** and ***h(x)*** at ***x = a***,

i.e. if $f(x)\le g(x)\le h(x), \forall x\in(a-\delta,a+\delta)$

and

$$\lim_{x\to a} f(x)=\lim_{x\to a} h(x)=l_1$$

Then

$$\lim_{x\to a} g(x)=l$$

Memorable facts :

(a) If δx is an error in the variable then $\frac{\delta x}{x}.100$ is called the percentage error in x.

(b) A function f(x) is said to have a local maximum value at x=a if there exists a nbd $(a-\delta,a+\delta)$ of **a** such that $f(x)<f(a)$ for all $x\in(a-\delta, a+\delta)$, $[x\neq a]$ or $f(x)-f(a)<0$ for all $x\in(a-\delta,a+\delta)$, $[x\neq a]$. f(a) is called the local maximum value of f(x) at x=a.

(c) A function f(x) is said to have a local minimum value at x=a if there exists a nbd $(a-\delta,a+\delta)$ of **a** such that $f(x)>f(a)$ for all $x\in(a-\delta, a+\delta)$, $[x\neq a]$ or $f(x)-f(a)>0$ for all $x\in(a-\delta,a+\delta)$, $[x\neq a]$. f(a) is called the local minimum value of f(x) at x=a.

(d) The points at which the function has either the local maxima or minima are called extreme values of f(x).

(e) The values of x for which f'(x)=0 are called stationary values or critical values of x and the corresponding values of f(x) are called the stationary or turning values of f(x). The points at which f'(x) does not exist are also called critical points.In nutshell the ***critical*** points are the values of x for which **f(x) is undefined**, **f'(x)=0** and/or **f'(x) does not exist**.

(f) Point of ***Inflexion*** is a point where $d^2y/dx^2=0$ but d^3y/dx^3 is not zero.

(g) First derivative test
   i. If f(x) is differentiable at x=a and f'(a)=0 and f'(x) changes sign from + to – as x passes through then f(x) is said to have the local ***maximum*** value at x=a.

   ii. If f(x) is differentiable at x=a and f'(a)=0 and f'(x) changes sign from - to + as x passes through then f(x) is said to have the local ***minimum*** value at x=a.

(h) If y is maximum or minimum then log y is also maximum or minimum provided y >0.

(i) n^{th} derivative test for Relative Extrema

Find the critical number for $x=x_0$.

Find also $f'(x_0)$.

If $f''(x_0)>0$, f(x) is minimum at $x=x_0$

If $f''(x_0)<0$, f(x) is maximum at $x=x_0$

If $f''(x_0)=0$, neither maxima nor minima but this point is called the point of inflexion if $f'''(x_0)\neq 0$.

Repeat this process till we obtain $f^n(x_0)\neq 0$.

If n is odd f(x) has neither maxima nor minima.

If n is even and $f^n(x_0)>0$, f(x) is minimum at $x=x_0$

If n is even and $f^n(x_0)<0$, f(x) is maximum at $x=x_0$.

Examples:

(a) $f(x) = x^4$

$f'(0)=0, f''(0)=0, f'''(0)=0$ and $f^4(0)>0$

Hence minimum at x=0

(b) $f(x) = - x^4$

$f'(0)=0, f''(0)=0, f'''(0)=0$ and

$f^4(0) < 0$;

Hence maximum at x=0

(c) $f(x)=x^3$

$f'(0)=0, f''(0)=0$ and

$f^3(0)=6$;

Hence neither maxima nor minima at x=0

Note For Greatest And Least Values : A ***maximum*** value of ***f(x)*** at $x=x_0$ in an interval ***[a,b]*** does not mean that it is the greatest value of ***f(x)*** in that interval. There may be a value of the function greater than a maxi mum value. As a matter of fact there may exist a minimum value of the function which is greater than or equal to some maximum value of the function in ***[a,b]***.

L'Hospital Rule: If the function f(x) and g(x) are differentiable in the certain neighborhood of the point **a** , except, may be, at the point **a** itself, and $g'(x)\neq 0$ and if $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0..or..\infty$ then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ provided $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ exists. The point ***a*** may be either finite or improper i.e. $+\infty$ or $-\infty$.

Note : If a function is defined and continuous in some interval, and if this interval is not a closed one then it can have neither the greatest nor the least value.

Questions on Limits

1. ***If f(x) is an odd function and*** $\lim_{x\to 0} f(x)$ ***exists, then find the limit.***

Hint: Given f(-x)= -f(x) as f(x) is an odd function.

Limit exists, i.e. LHL=RHL

$\Rightarrow f(0+h) = f(0-h)$

$\Rightarrow f(h) = f(-h) = -f(h)$

$\Rightarrow 2f(h) = 0$

$\Rightarrow .f(h)=0$

2. ***Evaluate:*** $\lim_{x\to 0}\frac{\sin[\cos x]}{1+[\cos x]}$.

Hint: As $x\to 0$, $\cos x\to 1$

$\Rightarrow [\cos x] = [<1] = 0$

3. ***Evaluate:*** $\lim_{x\to 0}\frac{2^x+2^{3-x}-6}{2^{-x/2}-2^{1-x}}$.

Hint: Factorize the Numerator after simplification as $(2^{x/2}-2)(2^{x/2}+2)(2^x-2)$ and proceed as usual.

4. ***Evaluate:*** $\underset{x\to\infty}{Lim}\left(\frac{3x^2+2x+1}{x^2+x+1}\right)^{\frac{6x+1}{3x+2}}$

Hint: Rewrite the given expression as

$$\underset{x\to\infty}{Lim}\left(\frac{3+\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x}+\frac{1}{x^2}}\right)^{\lim_{x\to\infty}\frac{6+\frac{1}{x}}{3+\frac{2}{x}}} = \left(\frac{3}{1}\right)^{\frac{6}{3}}.$$

5. ***Evaluate:*** $\underset{n\to\infty}{Lim}\left(\frac{n!}{(n+1)!-n!}\right)$.

Hint: Simplify to $\frac{1}{(n+1)-1}=\frac{1}{n}$ and evaluate the limit.

6. Evaluate: $\lim_{x\to\infty} 3^x \sin\left(\frac{4}{3^x}\right)$.

Hint: Use $\lim_{x\to 0}\frac{\sin mx}{mx}=1$

Rewrite the expression as below:

$$\lim_{x\to\infty}\frac{\sin\left(\frac{4}{3^x}\right)}{\frac{4}{3^x}}.4=4$$

7. Evaluate: $\lim_{x\to\infty}\left(\frac{\log x}{x^m}\right)$ if m>0.

Hint: Use L'Hospital Rule to get the Limit=0

8. Evaluate: $\lim_{x\to\infty}\left(\frac{x^6}{6^x}\right)$.

Hint: Rewrite the given expression as

$$\left(\frac{x^6}{e^{x\log_e 6}}\right)=\frac{x^6}{1+(x\log_e 6)+\frac{(x\log_e 6)^2}{2!}+...}$$

Divide N^r and D^r by x^6 and evaluate.

9. Evaluate: $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$.

Hint: Assume $y = (1+x)^{1/x}$

$\Rightarrow \log_e y=(1/x)\log(1+x)$

or,

$y= e^{(1/x)\log(1+x)}$

$$=e^{1-\frac{x}{2}+\frac{x^2}{3}-...}$$

$$=e.e^{\left(-\frac{x}{2}+\frac{x^2}{3}-\right)}$$

$$=e.\left\{1+\left(-\frac{x}{2}+\frac{x^2}{3}-...\right)+\frac{1}{2!}\left(-\frac{x}{2}+\frac{x^2}{3}-..\right)^2+...\right\}$$

$$\Rightarrow y=e.\left\{1-\frac{x}{2}+\frac{x^2}{3}+\frac{x^2}{8}+\frac{x^4}{18}-\frac{x^3}{6}+...\right\}$$

$$=e.\left\{1-\frac{x}{2}+\frac{11x^2}{24}-\frac{5x^3}{12}+...\right\}$$

On putting this value in the original expression,

$$\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$$

$$=\lim_{x\to 0}\frac{e-\frac{ex}{2}+\frac{11ex^2}{24}-\frac{5ex^3}{12}-e+\frac{1}{2}ex}{x^2}$$

$$=\lim_{x\to 0}\frac{+\frac{11ex^2}{24}-\frac{5ex^3}{12}}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{11e}{24}-\frac{5ex}{12}+g(x)\right)$$

where g(x) is the expression containing x.

$$=\frac{11e}{24}$$

10. Evaluate: $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$.

Hint: Rewrite the limit as below:

$$\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}=\lim_{x\to 1}(1+(1-x))^{\tan\frac{\pi x}{2}}$$

$$=\lim_{x\to 1}(1+(1-x))^{\frac{\tan\frac{\pi x}{2}}{1-x}}=e^{2/\pi}$$

11. Evaluate: $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$.

Hint: Rewrite as below:

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$$

$$=\lim_{x\to 0}\left(1+\frac{\sin x-x}{x}\right)^{\frac{x}{\sin x-x}\cdot\frac{\sin x}{x-\sin x}}=e^{-1}$$

12. Evaluate: $\lim_{x\to 0} x^x$

Hint:

$$\lim_{x\to 0} x^x = e^{\lim_{x\to 0} x\log x} = e^{\lim_{x\to 0}\frac{\log x}{1/x}} = e^{\lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}} = e^{-x} = e^0 = 1$$

13. Evaluate: $\lim_{x\to\infty} x^{\frac{1}{x}}$

Hint:

$$\lim_{x\to\infty} x^{\frac{1}{x}} = e^{\lim_{x\to\infty}\frac{1}{x}\log x} = e^{\lim_{x\to\infty}\frac{\log x}{x}} = e^{\lim_{x\to\infty}\frac{\frac{1}{x}}{1}} = e^{\frac{1}{\infty}} = e^0 = 1$$

14. Find the value of a if $\lim_{x\to a}\frac{a^x - x^a}{x^x - a^a} = -1$ ***and*** *a>0.*

Hint: Use L'Hospital Rule.

Differentiate Numerator and Denominator separately once.

Put $x=a$

have $\frac{\log a - 1}{1+\log a} = -1$ and hence $a=1$.

15. Evaluate: $\lim_{m\to\infty}\lim_{n\to\infty}\{1+\cos^{2m}(n!\pi x)\}, \forall x\in R$

Hint: Assume 2 cases

Case I : x=p/q , (i.e.a rational number)

$\Rightarrow$ (n!πx) = n! π. (p/q)

= an integral multiple of π as n→∞

$\Rightarrow$cos^2(n!πx) =1 and hence cos^{2m}(n!πx)=1

$\Rightarrow$ the required limit (1+1)=2

Case II : x an irrational number.

Then (n!πx)= an irrational number, i.e. not an integral multiple of π

$\Rightarrow$ -1 < cos(n! πx) < 1

$\Rightarrow$ 0 < cos^2 (n! πx) < 1

$\Rightarrow$ cos^{2m} (n! πx)→0 as m→∞

Hence the required limit= 1+0=1

16. Evaluate: $\lim_{x\to[a]}\frac{e^{\{x\}}-\{x\}-1}{\{x\}^2}$ ***where {x} represents the fractional part and [x] represents the integral part of a.***

Hint: RHL at *a*

$$= \lim_{x\to[a]+}\frac{e^{\{[a]+h\}}-\{[a]+h\}-1}{\{[a]+h\}^2}$$

$$= \lim_{h\to 0}\frac{e^h - h - 1}{h^2} = \frac{1}{2}$$

As

{[a]+h}=[a]+h-[[a]+h]=[a]+h-[a]=h

LHL at *a*

$$= \lim_{x\to[a]-}\frac{e^{\{[a]-h\}}-\{[a]-h\}-1}{\{[a]-h\}^2}$$

$$= \lim_{h\to 0}\frac{e^{1-h}-(1-h)-1}{(1-h)^2} = e-2$$

As

{[a]-h}=[a]-h-[[a]-h]=[a]-h-[a]+1=1-h

17. Find h(x) in terms of f(x) and g(x) if h(x) = $\lim_{n\to\infty}\frac{x^{2n}f(x)+g(x)}{1+x^{2n}}$

Hint: Assume 3 cases

Case I: x^2<1

The expression $h(x) = \frac{0.f(x)+g(x)}{1+0} = g(x)$

Case II: x^2>1

The expression h(x)

$$= \lim_{n\to\infty}\frac{f(x)+\frac{1}{x^{2n}}g(x)}{\frac{1}{x^{2n}}+1} = \frac{f(x)+0}{0+1} = f(x)$$

Case III : x^2=1

The expression h(x)

$$= \lim_{n\to\infty} \frac{1.f(x)+g(x)}{1+1} = \frac{f(x)+g(x)}{2}$$

18. Evaluate: $\lim_{x\to 0}\left[\frac{a.\sin x}{x}\right]+\left[\frac{b.\tan x}{x}\right]$ ***where a,b are integers and [.] denotes the greatest integer function.***

Hint: $\frac{\sin x}{x}<1$, and, $\frac{\tan x}{x}>1$

Hence the required limit = *a-1+b*

19. Prove that $e^{2x} + e^{x} + 2\sin^{-1}x + x - \pi = 0$ has atleast one real solution in [0,1].

Hint: Note that

f(0)= 2-π < 0 and

f(1)= e^2 + e -1 >0

Hence there exists one solution f(c)=0 of f(x)=0 between the given limit as the function f(x) is continuous between 0 and 1.

20. If f(x) = x-[x], $k \le x < k+0.5$
= [x], $k+0.5 \le x < k+1, k\in I$

and g(x) = $\sin^4 x + \cos^4 x$,

then find the value of f(g(x)).

Hint: Obviously,

g(x)= 1- (1/2) $\sin^2$ 2x.

⇒ (1/2) ≤ g(x) ≤ 1

⇒ g(x)=1 if sin2x=0

⇒x = nπ/2

Hence,

f(g(x)) = f(1) = 1-[1] = 0

⇒f(g(x)) = 0 for all x∈ R.

21. Show that the equation x=1+sin x has a root.

Hint: Assume a function

f(x)= x-1-sin x .

Check the continuity of the function.

It is continuous for all x ∈ R.

Also f(π/2) < 0 and f(π) > 0,

Hence there exists certainly a value of x between π/2 and π such that ***f(x=c)=0*** .

Hence, at least one solution exists.

Note: This question can be done by the **Graph Method also.**

Draw the graphs of ***y = x-1*** and ***y = sin x***. If they intersect each other, then there exists a solution otherwise there is no solution.

22. ***Show that f(x) is a constant function if f(x)satisfies*** $f:R\to R; |f(x)-f(y)|\le|x-y|^3$.

Hint: $|f(x)-f(y)|\le|x-y|^3, x\ne y$

$$\Rightarrow\left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|^2$$

$$\Rightarrow\lim_{y\to x}\left|\frac{f(x)-f(y)}{x-y}\right|\le\lim_{y\to x}|x-y|^2=0$$

$$\Rightarrow|f'(x)|=0\Rightarrow f(x)=c,$$

i.e. a constant function.

23. Find dy/dx when y=f(x) where $f\left(x+\frac{1}{x}\right)=x^4+\frac{1}{x^4}$

Hint: Rewrite the function as below:

$$f\left(x+\frac{1}{x}\right)=x^4+\frac{1}{x^4}$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2=\left[\left(x+\frac{1}{x}\right)^2-2\right]^2-2$$

Replace the expression $\left(x+\frac{1}{x}\right)$ by z and rewrite as $f(z)=(z^2-2)^2-2$ and then the function becomes

$f(x)=(x^2-2)^2-2 = x^4- 4x^2 + 2$

Now differentiate to find the required derivative.

24. If $f\left(\frac{x+2y}{3}\right)=\frac{f(x)+2f(y)}{3}, \forall x,y\in R$ ***and f'(0) exists and is finite, then show that f(x) is continuous on the whole Number Line.***

Hint: It is given that f(x) is differentiable at x=0

$$\Rightarrow f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x}$$

It is given that this is finite and also exists.

$\Rightarrow$ by definition of differentiability $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{f\left(\frac{3x+3h}{3}\right) - f\left(\frac{3x+0}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{f\left(\frac{3x+2.\frac{3h}{2}}{3}\right) - f\left(\frac{3x+2.0}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{f(3x)+2f\left(\frac{3h}{2}\right)}{3} - \frac{f(3x)+2f(0)}{3}}{h}$$

$$= \lim_{h\to 0} \frac{2f\left(\frac{3h}{2}\right) - 2f(0)}{3h}$$

$$= \lim_{h\to 0} \frac{f\left(\frac{3h}{2}\right) - f(0)}{\frac{3h}{2}} = f'(0) = c$$

(say a constant)

On Integration of both sides

f(x)=cx + d where d is also another arbitrary constant. It is linear, hence it is continuous.

25. ***Find the equations of all tangents to the curve y = cos(x+y); ($-2\pi \le x \le 2\pi$) that are parallel to the line x+2y=0***

Hint: Differentiate the curve and find (dy/dx). Equate it the slope of the given line i.e. (– ½).

$$\Rightarrow \frac{dy}{dx} = -\sin(x+y).\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)} = -1/2$$

$\Rightarrow$sin(x+y)=1

$\Rightarrow$cos(x+y)=0

$\Rightarrow$y=0 from the equation of the given curve and hence sinx=1

$\Rightarrow$x= ($-3\pi/2$), ($\pi/2$)

Write the points P($\pi/2$,0) and Q($-3\pi/2$,0) and then the equations of the tangents using one point and slope formula i.e. $y-y_1 = m_T (x-x_1)$

26. ***Assume that a raindrop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of radius of the rain drop.***

Hint: Given $\frac{dV}{dt} = -kS$

where

V=Volume of the spherical raindrop

=$(4/3)\pi R^3$,

S =Surface area of the raindrop =$4\pi R^2$ and

K= an arbitrary constant.

Differentiate V w.r.t to t and get the required equation: dR/dt=-k.

27. ***Find the intervals in which the function f(x)= sin 3x, $x \in [0, \pi/2]$ is increasing or decreasing.***

Hint: f'(x)= 3 cos 3x

for increasing or decreasing f'(x) ≥ 0, or, ≤ 0

f'(x)=0

since x $\in$ [0, $\pi/2$]

$\Rightarrow$ 3x= $\pi/2$, $3\pi/2$ $\Rightarrow$ x = $\pi/6$, $\pi/2$

$\Rightarrow$ x = $\pi/6$ divides the interval [0, $\pi/2$] into two disjoint intervals [0, $\pi/6$) and ($\pi/6$, $\pi/2$]

Obviously,

f'(x)>0 for [0, $\pi/6$) i.e. increasing and f'(x)<0 for ($\pi/6$, $\pi/2$] i.e. decreasing.

28. ***Find the interval in which the function $f(x)= \sin x + \cos x$, $0 \le x \le 2\pi$,is strictly increasing or strictly decreasing.***

Hint: $f'(x)= \cos x - \sin x$

$\Rightarrow f'(x)=0$

$\Rightarrow \cos x- \sin x=0$

$\Rightarrow x = \pi/4$, $5\pi/4$ as $x \in[0, 2\pi]$

$\Rightarrow$ these values of x divide the given interval into 3 disjoint intervals

i.e.$[0, \pi/4)$, $(\pi/4, 5\pi/4)$, $(5\pi/4, 2\pi]$

obviously,

$f'(x) >0$ for $[0, \pi/4) \cup (5\pi/4, 2\pi]$ i.e increasing

and

$f'(x)<0$ for $(\pi/4, 5\pi/4)$ i.e decreasing.

29. ***Show that f(x)= [x] on $(1,\infty)$ is monotonic increasing but not strictly increasing.***

Hint: Assume $x_1, x_2 \in(1,\infty)$ such that $x_1 < x_2$. Obviously there exists n_1 and n_2 s.t. $n_1 < n_2$ for $x_1 \in[n_1,n_1+1)$ and $x_2 \in[n_2,n_2+1)$.

Certainly f(x) is monotonic increasing but not strictly increasing.

But f(5/4)=[5/4]=1 and f(3/2)=[3/2]=1

Here (5/4)< (3/2) but f(5/4)=f(3/2).

30. ***Find the Absolute maximum and the absolute minimum values of $f(x)=2x^3-5x^2+4x-1$ on [-1,2].***

Hint: Find the critical points by solving f'(x)=0 or at the x where f'(x) does not exist.

Obviously x=2/3,1 .

It lies in (-1,2).

So find out all the values of f(x) at x=-1, 2/3, 1, 2.

The smallest of these will give the absolute minimum and the greatest will be the absolute maximum one.

31. ***Find the interval in which λ should lie so that $f(x)= \sin^3 x + \lambda \sin^2 x$, where $-\pi/2 < x < \pi/2$, has exactly one maxima and exactly one minima.***

Hint: Find, $f'(x) = 3\sin^2 x \cos x + \lambda 2 \sin x \cos x$

$= \sin x \cos x (3\sin x +2\lambda)$

For maxima or minima f'(x)=0,

hence

Either sinx=0 or cosx=0 or $3\sin x+2\lambda =0$

Since $-\pi/2 < x < \pi/2$ hence $\cos x \neq 0$

Then sinx=0 or $\sin x =- 2\lambda /3$

$\Rightarrow x=0$ or $x= \sin^{-1}(-2\lambda /3)$

$\Rightarrow x=0$ or $-1 < - 2\lambda /3 < 1$

$\Rightarrow \lambda \in (-3/2, 3/2)$

But if $\lambda=0$ then x=0 and then only one solution will be there hence $\lambda \neq 0$

Therefore $\lambda \in (-3/2, 0) \cup (0, 3/2)$

For two solutions.

32. ***Show that $|\sin u - \sin v| \le |u-v|$.***

Hint: Assume f(x)=sin x.

Mean Value Theorem's essentials exist as it is continuous and differentiable in[u,v] for all x in the specified interval.

Hence there exists some $c\in (u,v)$ such that $f'(c)=\{f(u)-f(v)\}/(u-v)$

As $|\cos c| \le 1$ hence $\{f(u)-f(v)\} \le |u-v|$

33. ***Show that $2 \sin x + \tan x \ge 3x$ where $0 \le x < \pi/2$***

Hint: Assume $f(x)= 2 \sin x + \tan x - 3 x$

Prove f(x) , an increasing function in the given interval. If it comes true then it is well otherwise assumption is False.

$$f'(x)= \frac{(\sec x-1)^2(\sec x+2)}{\sec x}$$

for $0 < x < \pi/2$, $f'(x) > 0 \Rightarrow f(x)$ is an increasing function in $(0, \pi/2)$.

For x=0, f(x)=0 hence on combining the above two results, f(x) results true.

34. ***If $x = a \cos^3\theta$, $y= a \sin^3\theta$ then evaluate $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$***

Hint: Use differentiation method for parametric form.

Find $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

And simplify to get the answer.

35. If $x = t^2$, $y = t^3$, then find the value of $\frac{d^2y}{dx^2}$

Hint: note $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

36. Evaluate $\frac{d^{20}}{dx^{20}}(2\cos x\cos 3x)$

Hint: Change the given expression as below

2cosAcosB=cos(A-B)+cos(A+B)

And then use formula for n^{th} derivative of cos x as

$$\frac{d^n}{dx^n}\cos x = \cos\left(\frac{n\pi}{2} + x\right)$$

37. If f(x)= $\int_{-1}^{x}|t|dt, x \geq -1$, then show that f and f' are continuous for x+1>0.

Hint: Note $\int_{-1}^{x}|t|dt = \left[\frac{t|t|}{2}\right]_{-1}^{x}$

Hence f(x)= $\frac{x|x|}{2} + \frac{1}{2}$

And f'(x)=|x| using Newton-Leibnitz formula of definite integral.

Obviously f(x) and f'(x) are continuous for x > -1.

38. Let f(x) = x^p cos (1/x), when x≠ 0 and f(x)=0 when x =0. Then show that f(x) will be differentiable at x=0 if p>1.

Hint: Use

$$f'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0}\frac{h^p\cos\frac{1}{h}}{h} = \lim_{h\to 0} h^{p-1}\cos\frac{1}{h}$$

It is obvious that if p-1 ≤ 0 then the limit will not exist finitely. Hence for existence of the limit , p must be >1.

39. If f(x) = 1+ 2 sin x + 3 $\cos^2$ x, 0 ≤ x ≤ 2π/3 then show that it is minimum at x =π/2

Hint: Differentiate once and find f'(x). find value(s) of x. find d^2y/dx^2 at x and then show that it is positive for this x. (i.e. use second derivative test for maxima and minima)

40. ***Find the value of n^{th} differential coefficient of $\frac{x^3}{x^2-1}$ for x=0, if n is even and also if n is odd and greater than 1.***

Hint: Rewrite the given expression as

$$y= \frac{x^3}{x^2-1} = x + \frac{x}{x^2-1} = x + \frac{A}{x-1} + \frac{B}{x+1}$$

use method of partial fraction to decompose into two fractions and then evaluate A and B.

the n^{th} derivative is

$$y_n = \frac{1}{2}(-1)^n(n!)(x-1)^{-n-1} + \frac{1}{2}(-1)^n(n!)(x+1)^{-n-1}$$

$$\Rightarrow (y_n)_{x=0} = \frac{1}{2}(-1)^n(n!)(-1)^{-n-1} + \frac{1}{2}(-1)^n(n!)$$

Two cases arise

Case I: when n is even i.e. n=2m ⇒y_n at x=0 is 0

Case II: when n is odd i.e. n=2m+1⇒ y_n at x=0 is – (n!)

41. ***Show that there exists a function f(x) satisfying f(0)=1, f'(0)=-1,f'(x)>0 for all x and f''(x)<0 for all x.***

Hint: f(x) is continuous and differentiable under given conditions for all x , then f(x) is decreasing for x>0 and concave down. Therefore f''(x)<0.

42. Find the maximum value of f(x)=2x^3-15x^2+36x-48 on set A= { x: x^2+20≤ 9x }.

Hint: From x^2+20 ≤ 9x ⇒ x∈[4,5]

f'(x) is decreasing as

f''(x)= - sin x –(6/π) is negative i.e. < 0, for all x∈[0,π/2]

f'(x)>0 as x< π/2.

Since f'(x) is decreasing,

hence f'(x)> f'(π/2)⇒f(x) is increasing. i.e.

For x≥0, f(x) ≥f(0) ⇒ sinx + 2x- 3x(x+1)/π≥0.

Dr S.B. Dhar, is **Editor of this Quartrerly e-Bulletin**. He is an eminent mentor, analyst and connoisseur of Mathematics from IIT for preparing aspirants of Competitive Examinations for Services & Admissions to different streams of study at Undergraduate and Graduate levels using formal methods of teaching shared with technological aids to keep learning at par with escalating standards of scholars and learners. He has authored numerous books – Handbook of Mathematics for IIT JEE, A Textbook on Engineering Mathematics, Reasoning Ability, Lateral Wisdom, Progress in Mathematics (series for Beginner to Class VIII), Target PSA (series for class VI to class XII) and many more.
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INVITATION FOR CONTRIBUTION OF ARTICLES

Your contribution in the form of an article, story poem or a narration of real life experience is of immense value to our students, the target audience, and elite readers of this Quarterly monthly e-Bulletin ***Gyan-Vigyan Sarita: शिक्षा****, and thus create a visibility of the concerns of this initiative. It gives them a feel that you care for them, and they are anxiously awaiting to read your contributions. We request you to please feel free to send your creation, by* ***20th of this month*** *to enable us to incorporate your contribution in next bulletin,* *subhashjoshi2107@gmail.com*.

We will be pleased have your association in taking forward path our plans as under-

- ***With the start of Second year of operation, 1st4th Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा shall be brought out 1st October'17.***
- ***And this cycle monthly supplement to Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा aimed to continue endlessly***

We believe that this monthly supplements to quarterly periodicity of e-Bulletins shall make it possible for our esteemed contributors to make contribution rich in content, diversity and based on their ground level work.

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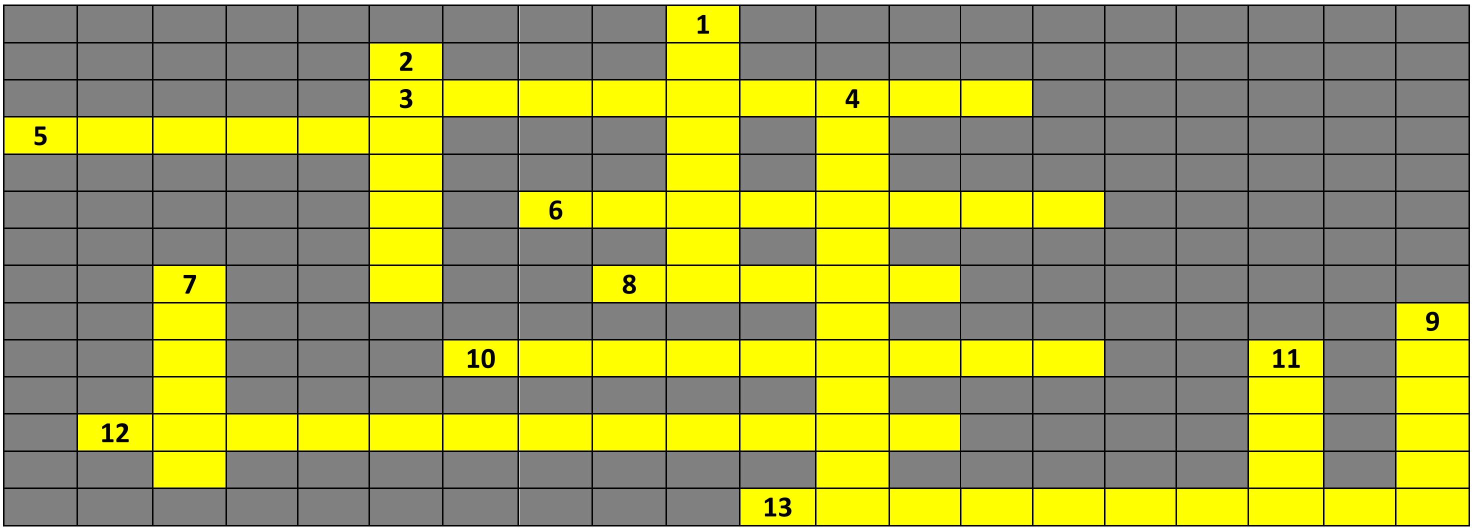
Nature is an excellent example of unity in diversity. At its basic constituent level, atom, it is constituted by particles of different nature. Some of them are of opposite in nature, and experience a strong force of attraction, yet they continue to exist separately and individually; particles of similar nature, having stong force of repulsion continue to exist in vicinity. This has been there since beginning of nature, and shall continue to exist indefinitely. Any unregulated infringement on the other would be a disastrous. The secret of coexistence is respecting others position.

CROSS WORD PUZZLE Sept'17: EDUCATION

S.B. Dhar



ACROSS

3 Removing a student from a school
5 Set of standards linked to learning
6 Scientific and cultural community
8 Young human
10 Theory of adult education
12 Application of coercive technique to change belief
13 Person's subjective appraisal of himself

DOWN

1 Person who excels in multiple fields
2 Oral presentation to teach people
4 Mental capacity to reason
7 Ability of brain to retain
9 Ability to make correct Judgment
11 Person incapable of learning

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ANSWER: CROSSWORD PUZZLE Aug'17: SET THEORY

Prof. S.B. Dhar

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Growing with Concepts : Physics

Modern Physics: Part I: Thought Experiment and Special Theory of Relativity

Knowledge is limited, but imagination is not.

- Albert Einstein

*Every innovation is a result of an out-of-box thoughts and Theory of Relativity, propounded by **Albert Einstein** is one of the brilliant examples. It came up at a time when whole school of scientific pursuit was on modernization. It is just an imagination, called thought experiment, without any experimental verification, whose results revolutionized subsequent course of science. This theory in original text of Einstein might be difficult for students at school level, and it is in general with research papers. Yet it is very simple and understandable with the concepts developed at this stage of this Mentors' Manual. This illustration is limited to motion along X-axis, while Einstein developed general purpose equation in XYZ coordinates. It is an effort to take out phobia of theory of relativity from a students, who is already hypnotized with the magnanimity of the theory. Therefore, a supporting illustration of the theory upto most famous equation* $\boldsymbol{E = mc^2}$ *is covered here for familiarization of the concept. It is an effort to open up the thought process, an aim of this endeavor, in which this theory forms a best case study on strength and potential of an imagination and encourage students to think out-of-box. Though, it does not form part of course content of competitive exams for students of 12th class, it likens to an essay recommended for a pleasure time reading, with mathematical alertness.*

This is more of a structured compilation and interlacing of the context from different sources; thanks to Google web resource which was extremely useful in presenting this case study on Thought Experiment - Theory of Relativity.

Thought Experiment: When an idea is taken as an hypothesis, theory or principle to analyze its consequence without either experimental verification or a proposition of experiment which many not be possible to performs is called a *Thought Experiment*. Main objective of a thought experiment is to explore potential consequence of the idea in question by performing an intentional and structured process of intellectual deliberation in order to speculate potential consequence of a designed condition within the specified problem domain. History of science, philosophy, psychology, law, mathematics and all fields of abstract knowledge, is full of such though experiments predating to Socrates, 400 BC.

Contribution of every thinker, philosopher and scientist is an outcome of thought experiment, which has continued irrespective of social, theological and political responses. Nearing the end of 19th century, there was a sudden spike in Thought Experiment, and in this contributions of *Albert Einstein* in 1905, changed the perspective of scientific community through *Special Theory of Relativity*.

BACKGROUND: Electromagnetic Wave Equation by James Clerk Marx, little before the birth of Einstein had predicted velocity of light in vacuum $\approx \mathbf{3 \times 10^8}\ \mathrm{ms^{-1}}$. Einstein, an unusual child, at the age of 16 around 1995, had entered into thought experiments to examine relevance of mechanics, assuming himself riding on the light wave, and as he grew, he became increasingly restless to reconcile laws of classical mechanics with the laws of Electromagnetic Field Theory established by Maxwell.

Albert Michelson and Edward Morley in 1887, failed to determine speed of earth's revolution in Luminiferous Aether, an absolute medium, through a specially designed experiment based on principles of *classical mechanics*. Negative results of the experiment became matter of debate and deliberation among contemporary scientist. It shook faith of scientific community on Classical Mechanics, which believed that they were close to a complete description of the universe. This dilemma seems to have encouraged *Though Experiments* of Einstein, to conclude that ***velocity of light (c) is absolute***. In 1887 and 1895, Vogit and Lorentz, respectively, published their early approximation of correlating coordinates of a point, in frame moving with a constant velocity, w.r.t a stationary frame. These transformations were later brought to modern form by Jules Henri Poincaré in 1905, giving it a name Lorentz Transformation. These transformations find place in the landmark paper "Special Theory of Relativity" in 1905 by Einstein. He showed that these transformations follow the principles of relativity; and clarified at the note-1, in the paper, that *he was unaware of transformation propounded by Lorentz* . There are reasons to believe Einstein in light of his pursuance of the Theory of Relativity and multiple cases of identical discoveries, in history of science, concurrently done by different scientists, who were unaware of each other's work. *Despite, failure of the Michelson's experiment, in its objective, it is a classic case of breakthrough of scientific discoveries*. **His theory made velocity of light** c **is a universal constant while time and space are relative to each observer, and thus abandoned idea of absolute time and absolute rest. Stephen Hawking** in his Brief History of Relativity has said that "*The equivalence of mass and energy is summed up in Einstein's famous equation* $E = mc^2$, *probably the only physics equation to have recognition on the street*".

In the following section, efforts have been made to collate texts and derivations of various associated concepts *starting with Michelson & Morley's Experiment upto Relativistic relation of Energy-Momentum, based on concepts developed upto class 12th*, the target of this manual. It is little short of ***General Theory of Relativity***, and would be supplemented separately, at an appropriate time.

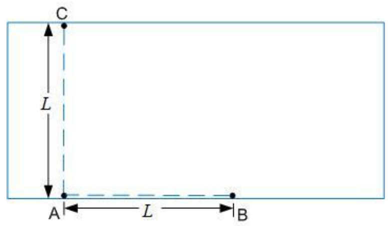
This *Special Theory of Relativity* is based on ***Two Postulates***:

1. *The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.*

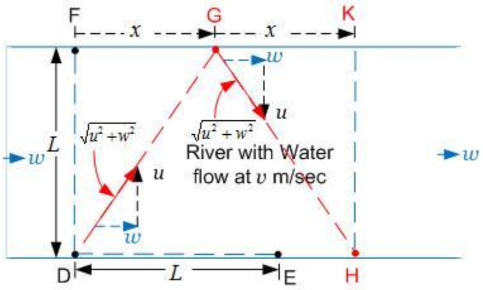
2. *Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity* ***c***, *whether the ray be emitted by a stationary or by a moving body.*

After the theory of relativity was accepted, *Einstein recalled his imagination at the age of 16 years and how important role the thought experiment played in establishing Special Relativity*. Here it is important to quote Einstein "***The object of all science, whether natural science or psychology, is to co-ordinate our experiences and to bring them into a logical System***".

Michelson & Morely's Experiment: This experiment has played a key role in advancement and establishing theory of relativity. It derives inspiration of an observation of difference in swimming time swimming across and along the river and water pool. It is based on relative velocity of the swimmer with respect to bank of river and pool, and is in accordance with classical mechanics, as shown in the figure. Taking two points B and C, in steady pool, which are equidistant from a swimmer, but in perpendicular direction from a swimmer A. The swimmer separately touches the Two points and return back to original position, swimming at a speed u. In this experiment, time taken to swim for to-&-fro between points A and C shall be $2t_1 = \frac{2L}{u}$. Since conditions are same for to-&-fro swimming, time would be same. Likewise, in another attempt to swim to-&-fro between points A and B shall be $2t_2 = \frac{2L}{u} = 2t_1$. In this direction of swimming is perpendicular to that while swimming along A and C, but neither the velocity of swimmer nor the velocity of water in the pool, which is steady, has changed, and hence both the timing are same. Thus, $\Delta t = t_2 - t_1 = 0$.

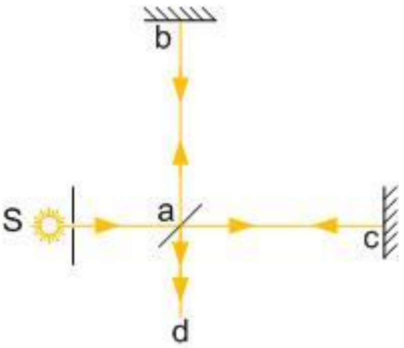


Taking another case of attempt to swim along and across the river, over a same distance L, in a river which is flowing with velocity w. In this case while swimming along DE, the relative velocity of the swimmer w.r.t. to ground shall be $u + w$, and swimmer shall reach faster, but during return along ED, the relative velocity shall be $u - w$. And thus time taken in to-&-fro journey between D and E shall be $t_1' = \frac{L}{u+w} + \frac{L}{u-w} = \frac{2Lu}{u^2-w^2}$.

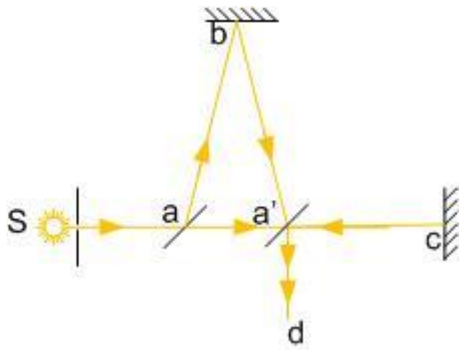


But, in another attempt to swim along DF, due to river current path of swim is DG, maintaining same swimming velocity u during. Thus the swimmer ends up at G a distance x along the bank he is heading to. Let t_2' is time taken by swimmer to cover DG, then $x = v \times t_2'$. Effective velocity of swimmer perpendicular to the river current i.e. along DF is $\sqrt{u^2 - w^2}$. Hence, time taken by the swimmer to reach H via G is $2t_2' = \frac{2L}{\sqrt{u^2-w^2}}$.

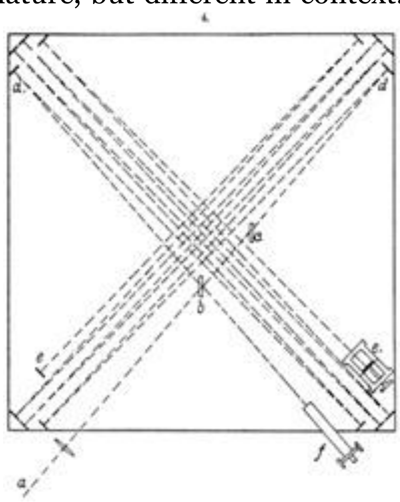
*In this experiment **u** and **v** are comparable. Thus, the relative difference in time of travel $2\Delta t' = 2(t_1' - t_2')$ which is perceivable. This experiment was extended by Michelson and Morley in a separately designed experiment as shown in the figure below to verify orbital speed of earth. It was visualized by the duo that velocity of light ($c = 3 \times 10^8\,\mathrm{ms}^{-1}$), is very high as compared with orbital speed of earth ($v = 3 \times 10^4\,\mathrm{ms}^{-1}$). This relative motion between the earth and luminiferous aether*, which was *considered to be a medium of transmission of light, an absolute and stationary frame of reference. It is akin to the river bank, in the above example, and considers earth to be moving at an orbital while rotation at a speed relative to the eather. The experiment with a diagram is illustrated below.*



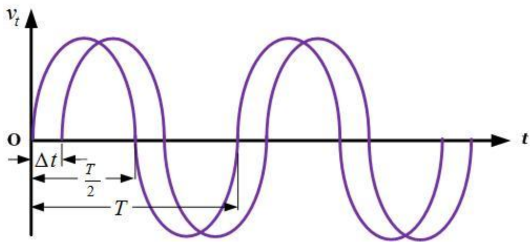
This a conceptual diagram of the experiment based on original paper, with an assumption there is no relative motion between eather and experimental setup on the earth's surface. A ray **Sa** from a monochromatic source of light 'S' is incident on 'a' half-silvered glass played at an angle 45° to the incident ray. Out of this, part of light passes through the glass as ray **ac**. After reflection of ray ac by mirror at 'c', placed perpendicular to the direction of ray, it is reflected as ray **ca**. This ray **ca** is again reflected at 'a' and returns to the observer as ray **ad**. Another part of the ray reflected at 'a' becomes a ray **ab**, at 90° to the incident ray **Sa**, and after reflection by mirror at 'b', perpendicular to the incident ray, is returned along line **ba**. Again a part of this ray passes through glass as ray along 'ad' to interfere with the reflected ray **ad**, cited above. This interference is noticed by an observer through a telescope at d. In this distance of mirrors at 'b' and 'c' from the half-silvered glass is at 'a' is equal. Discrepancy in the optical length of the ray **ab**, which before reflection is refracted twice in glass sheet at 'a', with that of the ray **ac** is corrected by placing a glass sheet of same refractive index and thickness as that at 'a', but it is not shown in the diagram for simplicity. If the assumption were true, there would both the rays to the observer at 'd' shall be in the phase and there would not be any interference.



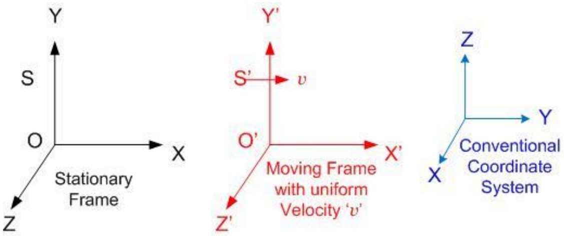
Comparing the two experiments, it leads to the fact that they are identical in nature, but different in context. Accordingly, replacing the variables $u \to c$ and $w \to v$, $\Delta t = t_1' - t_2' = \frac{2Lc}{c^2-v^2} - \frac{2L}{\sqrt{c^2-v^2}} = \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$. This is simplified using binomial theorem, with an approximation that all terms of higher order of $\frac{v^2}{c^2} \to 0$. This is a valid approximation since $v \ll c$. Accordingly, $\Delta t == \frac{2L}{c}\left(\left(1+\frac{v^2}{c^2}\right) - \left(1+\frac{1}{2}\cdot\frac{v^2}{c^2}\right)\right) = L\frac{v^2}{c^3}$. Thus path difference to the observer is $\Delta l = c \cdot \Delta t$, while $\lambda = c \cdot T = \frac{c}{\upsilon}$, here, λ is wavelength, and υ is the frequency of light. In interference for first fringe to occur $\Delta t = \frac{T}{2}$, first coincidence of (+)ve and (-)ve peaks coincide to

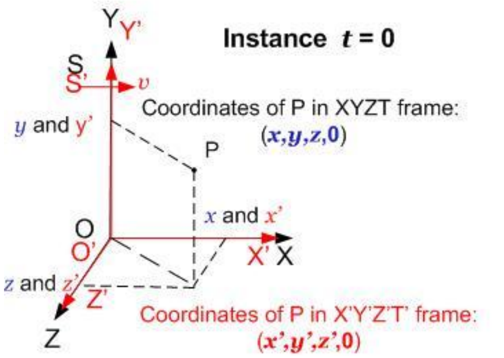


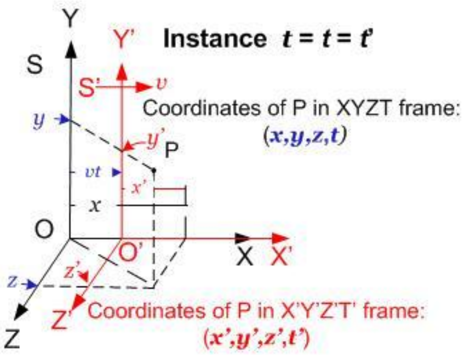
create a dark patch. Accordingly, fringe number is expressed as $n = \frac{\Delta t}{\frac{T}{2}} = \frac{2\Delta t}{T}$. Further, wavelength $\lambda = \frac{c}{\upsilon} = c \cdot T$, or, $T = \frac{\lambda}{c}$ therefore, $n = \frac{2c \cdot \Delta t}{\lambda} = \frac{2c}{\lambda} \cdot L\frac{v^2}{c^3} = \frac{2Lv^2}{\lambda c^2}$. Since this fringe number is directly proportional L, perceivable fringe number, length was elongated by multiple reflections creating a folded path as shown in the figure. In experiment for $L = 10\,m$ and $\lambda = 4 \times 10^{-7} m$, calculated fringe number is arrived at $n = 0.4$, with their experimental having an accuracy of 0.01. This experiment was repeated by rotation the instrument by 90^0, i.e. virtually interchanging the two paths, at different location and in different seasons. But, the experimental results showed null fringe shift. *This resulted into – **a)** negation of the premise of earth, and **b)** orbital velocity of earth aimed at could not be determined.* This controversy was explained by *Einstein through his second postulate according to which **velocity of light in vacuum is a universal constant**.*



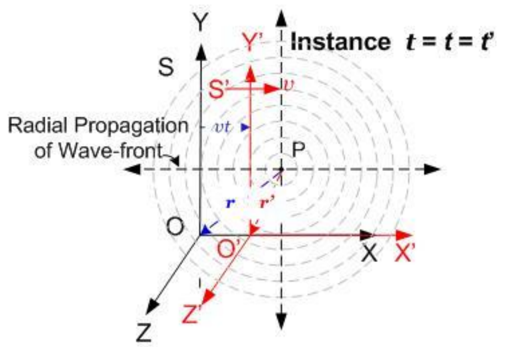
Lorentz Transformation: A beginning of ***Lorentz Transformation*** is made by analyzing coordinates of a fixed point in a stationary frame S, and frame S' which is moving with a constant velocity ($\boldsymbol{v}$) along X axis. The three reference coordinates of S and S' viz-a-viz conventional frame of reference is shown in the figure. Two instances are identified one at $\boldsymbol{t}$=**0,** when **O** and **O'** coincide, and all the three corresponding axes **X-X', Y-Y'** and **Z-Z'** also coincide. Time is taken to be absolute and hence corresponding time $\boldsymbol{t = t'}$ for S and S'. Thus as per Galilean **Transformation**: $\boldsymbol{x' = x - vt,\ y' = y,\ z' = z}$ and $\boldsymbol{t' = t}$. Accordingly, $r^2 = x^2 + y^2 + z^2 = c^2t^2$, in frame S; here, $\boldsymbol{t}$ is the time taken by wave-front of light to reach O, as per *Huygens Wave Theory, and shown in the Figure.* Alternately, it can be written as, $x^2 + y^2 + z^2 - c^2t^2 = 0$. While, in frame S' $r'^2 = x'^2 + y^2 + z'^2 = c^2t'^2$, which works out to $x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$. Here, $\boldsymbol{t'}$ is the time taken by wave-front to reach O'. Thus, combining the Two equation of spherical wave-fronts leads to $x'^2 + y^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2$. It satisfies the Two observers at O and O' in frames S and S'. Using Galilean Transforms, it reduces to m $x^2 - x'^2 = 0$, or $x^2 = x'^2$. It, further, simplifies to $x^2 = (x - vt)^2 => x^2 = x^2 - 2vtx + v^2t^2 => x = vt$. But, from the basic premise, of **classical mechanics** as shown on the figure $x \neq x'$ and hence $x \neq vt$, and it is a contradiction in the premise.







These contradictions were used to modify Galilean Transformation in to a set of linear equations, contemplating Time-Space coordinate system. Accordingly, a new set of transformations are: $\boldsymbol{x' = a_1x + a_2t}$, $\boldsymbol{y' = y,\ z' = z}$ and $\boldsymbol{t' = b_1x + b_2t}$, where values of coefficients $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{b_1}$ and $\boldsymbol{b_2}$ are to be determined. From equation of $\boldsymbol{x'}$, it works out to $\boldsymbol{x' = a_1x + a_2t = 0}$, it leads to $x = -\frac{a_2}{a_1}t$. In this case, $x = vt$. Equivalence between these Two values of x leads to $v = -\frac{a_2}{a_1}$. Further, a sequence of mathematical manipulation are carried out, starting with $\boldsymbol{x'}$. Thus, $\boldsymbol{x' = a_1\left(x + \frac{a_2}{a_1}t\right) = a_1(x - vt)}$. Substituting, this $\boldsymbol{x'}$



together with the t', in combined equation of spherical of wave-fronts can be rewritten as $a_1^2(x-vt)^2+y'^2+z'^2-c^2(b_1x+b_2t)^2=x^2+y^2+z^2-c^2t^2$. This expression leads to $a_1^2(x-vt)^2-c^2(b_1x+b_2t)^2=x^2-c^2t^2 => (a_1^2-c^2b_1^2-1)x^2-(2a_1^2v-2c^2b_1b_2)tx+(a_1^2v^2-b_2^2c^2-c^2)t^2=0$.

Equating coefficients of x^2, t^2 and tx, it leads to a set of Three equations : **i)** $a_1^2-c^2b_1^2=1 => b_1^2c^2=a_1^2-1$, **ii)** $b_2^2c^2=c^2+a_1^2v^2$, and **iii)** $2a_1^2v-2c^2b_1b_2=0 => c^2b_1b_2=a_1^2v$. Further, multiplying equations (i) and (ii) together, $b_1^2b_2^2c^4=(a_1^2-1)(c^2+a_1^2v^2)$. The Left Hand Side of this product equation is square of the LHS of equation **iii).** Thus these three equations combine into $a_1^4v^2=a_1^4v^2-a_1^2v^2+a_1^2c^2-c^2$. This resolves into $a_1^2c^2-a_1^2v^2=c^2$, it further leads to $a_1^2=\frac{c^2}{c^2-v^2}$, and in turn one of the coefficients $a_1=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. This together with value of $v\left(=-\frac{a_2}{a_1}\right)$ leads to value of another coefficient $a_2=-\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$. Likewise from (i) $\boldsymbol{b_1^2}=\frac{a_1^2-1}{c^2} => b_1^2=\frac{\frac{c^2}{c^2-v^2}-1}{c^2}=\frac{\frac{v^2}{c^2-v^2}}{c^2}=\frac{v^2}{c^2}\cdot\frac{1}{c^2-v^2}=\frac{v^2}{c^4}\cdot\frac{\mathbf{1}}{\mathbf{1-\frac{v^2}{c^2}}}$. Taking square-root of the final form, $b_1=\pm\frac{v}{c^2}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and choice was made of -ve value as $b_1=-\frac{v}{c^2}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ to arrive at uniformity of pattern in the new set of transformations, and thus Third coefficient I determined. In respect if Fourth coefficient, using (ii), $\boldsymbol{b_2^2}=\frac{c^2+a_1^2v^2}{c^2}$. It, further, leads to $\boldsymbol{b_2^2}=1+a_1^2\frac{v^2}{c^2}=1+\frac{c^2}{c^2-v^2}\cdot\frac{v^2}{c^2}=1+\frac{v^2}{c^2-v^2}=\frac{c^2}{c^2-v^2}=\frac{\mathbf{1}}{\mathbf{1-\frac{v^2}{c^2}}}$.This closes on $b_2=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=a_1$. Use of a factor $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ in all the Four coefficients, has be summarized as Lorentz Factor $\lambda=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. Accordingly, the set of coefficients in the new transformations work out to : $\boldsymbol{a_1=\lambda}$, $\boldsymbol{a_1=-\lambda v}$, $\boldsymbol{b_1}=-\frac{\boldsymbol{v}}{\boldsymbol{c^2}}\cdot\boldsymbol{\lambda}$ and $b_2=\boldsymbol{\lambda}$. Thus there is a new set of transformations are : **a)** $\boldsymbol{x'=\lambda(x-vt)}$, **b)** $\boldsymbol{y'=y}$, **c)** $\boldsymbol{z'=z}$, and **d)** $\boldsymbol{t'=\lambda\left(t-\frac{v}{c^2}x\right)}$. These were propounded by Hendrik Lorentz in 1889 and are known as Lorentz Transformation. Effect of Lorentz Transformation is strange and yet not realizable in real life since fastest travelling object that can be realized has $v\ll c$ and this leads to $\lambda\to 1$, as much as, $\frac{v}{c^2}\to 1$where Gailean Transformation is valid. This is where Theory of Realtivity becomes essential to look beyond classical mechanics.

It is obvious to question that *if transformation of coordinates of S → S' is it exists there should also be a correspondence between coordinates of S' → S, and* ***it does****, which is called* ***Inverse Lorentz Transformation*** and is as under.

Taking transformation of $t'\to t,x$, it comes to $\frac{t'}{\lambda}=t-\frac{v}{c^2}\cdot x => t=\frac{t'}{\lambda}+\frac{v}{c^2}\cdot x$, likewise from transformation of $x'\to x,t$, it arrives at $x'=\lambda\left(x-v\left(\frac{t'}{\lambda}+\frac{v}{c^2}\cdot x\right)\right)=\lambda\left(1-\frac{v^2}{c^2}\right)x-vt' => x=\frac{x'+vt'}{\lambda\left(1-\frac{v^2}{c^2}\right)}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\cdot(x'+vt')$. It eventually comes to $\boldsymbol{x=\lambda(x'+vt')}$.

In a nested manner, using this equation of $\boldsymbol{x}$, into equation of $\boldsymbol{t}$ used above $t=\frac{t'}{\lambda}+\frac{v}{c^2}\cdot\boldsymbol{\lambda(x'+vt')}$. It further resolves into $\boldsymbol{t}=\frac{t'}{\lambda}+\frac{v}{c^2}\cdot\lambda(x'+vt')=\left(\frac{1}{\lambda}+\frac{\lambda v^2}{c^2}\right)t'+\frac{v}{c^2}\cdot\lambda x'=\left(\frac{\mathbf{1+\lambda^2\cdot\frac{v^2}{c^2}}}{\boldsymbol{\lambda}}\right)\boldsymbol{t'}+\frac{\boldsymbol{v}}{\boldsymbol{c^2}}\cdot\boldsymbol{\lambda x'}$. In order to simplify the solution $\frac{\mathbf{1+\lambda^2\cdot\frac{v^2}{c^2}}}{\boldsymbol{\lambda}}=\frac{1+\frac{1}{1-\frac{v^2}{c^2}}\cdot\frac{v^2}{c^2}}{\lambda}=\frac{\frac{1}{1-\frac{v^2}{c^2}}}{\lambda}=\frac{\lambda^2}{\lambda}=\boldsymbol{\lambda}$, Thus, $\boldsymbol{t=\lambda\left(t'+\frac{v}{c^2}\cdot x\right)}$. The Lorentz Transformations and their inverses are summarized in a table here and are extremely useful in pursuit of *Theory of Relativity*.

Implication of Lorentz Transformation: These implication are being analyzed, through a set of ***Thought Experiments***, taking Two non-inertial frames of reference as **S** and **S'**, such that at time measured $\boldsymbol{t=0}$ origins of both the frames coincide and that **S** is stationary while **S'** is moving with a uniform velocity $\boldsymbol{v}$

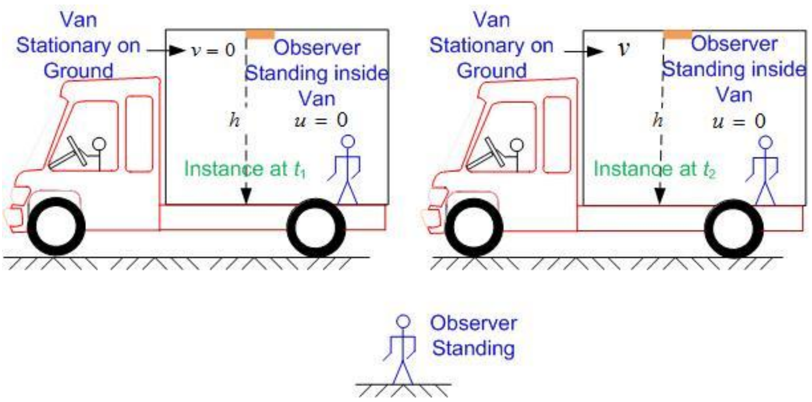
along X-axis. This Model shall be used throughout the illustration. Alignment of X-axis of both the frames is for convenience. In this sequence of coordinates is similar to the convention with only one difference that coordinates have been rotated by one position in the same sequence, and is shown earlier in the figure.

Lorentz Transformation	Inverse Lorentz Transformation
$x' = \lambda(x - vt)$	$x = \lambda(x' \mp vt')$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = \lambda\left(t - \frac{vx}{c^2}\right)$	$t = \lambda\left(t' + \frac{vx'}{c^2}\right)$
Here, $\lambda = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is called **Lorentz Factor**	

Principle of Simultaneity: Two events, simultaneous for one observer, may not be simultaneous for another observer if the observers are in uniform relative motion. But, it is no longer satisfactory when events are connected in time series occurring at different places.

Relativity of Space – Length Contraction: In this **Thought Experiment** a rod is taken, in frame S, having its two ends at x_1, y_1, z_1, t and x_2, y_2, z_2, t. Thus length of the rod observed is a state of rest in S shall be $l_0 = x_2 - x_1$. The same rod observed by an observer in frame S' which is moving with a velocity v will depend upon position coordinates of ends of the rod x_1', y_1', z_1', t' and x_2', y_1', z_1', t' observed in S'. According to Inverse Lorentz Transformation, for an observer w.r.t. S, $x_1' = \frac{(x_1 - vt)}{\sqrt{1-\frac{v^2}{c^2}}}$ and $x_2' = \frac{(x_2 - vt)}{\sqrt{1-\frac{v^2}{c^2}}}$. Thus, length of the rod observed in frame S' shall be $l_0' = x_2' - x_1' = \frac{(x_2 - vt)}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{(x_1 - vt)}{\sqrt{1-\frac{v^2}{c^2}}}$. This reduces to $l_0' = \frac{(x_2 - x_1)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{l_0}{\sqrt{1-\frac{v^2}{c^2}}} = \lambda l_0$, where λ is Lorentz Factor. It is independent of direction wither along X-axis or in a direction on negative of the axis, since this term appears as square. Thus, $l_0 = l_0'\sqrt{1 - \frac{v^2}{c^2}}$. As long as $v \neq 0$, the factor $0 < \sqrt{1 - \frac{v^2}{c^2}} < 1$ and hence $l_0 < l_0'$, length observed in S is less than that observed in S' and it is ***Relativistic Length Contraction***.

Relativity of Time – Time Dilation: It refers to the time duration of a physical process in which time taken by object in a moving frame appears to be longer than that when viewed from the stationary reference frame. This is another consequence of relativistic mechanics which is important to arrive at relativistic mass, in furtherance of Special Theory of relativity. This can be proved using **Inverse Lorentz Transformation,** by taking Two instances $\boldsymbol{t_1'}$ and $\boldsymbol{t_2'}$, when an object is stationary in a moving frame, it implies $\boldsymbol{x_1' = x_2'}$. Thus corresponding time in stationary frame of reference shall be $t_1 = \frac{t_1' + vx_1'}{\sqrt{1-\frac{v^2}{c^2}}}$ and likewise, $t_2 = \frac{t_2' + vx_1'}{\sqrt{1-\frac{v^2}{c^2}}}$. Therefore, time duration in stationary frame $\Delta\boldsymbol{t} = t_{21} - t_1 = \frac{t_2' + vx_1'}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1' + vx_1'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \lambda\Delta\boldsymbol{t'}$. Since, the range of



Lorentz Factor (λ) is $0 \leq \lambda \to \infty$, as velocity of the moving frame of reference ranges $0 \ll v \to c$, therefore, $\Delta t'$ to an observer in a moving frame would appear to be longer by a factor λ to a stationary observer in a stationary frame.

This is proved with another **thought experiment** in Two stages. In First Stage - van having a light source at its roof, has its walls are transparent to an observer standing on the ground; ground is a stationary frame of reference. This generates Two observation brought as Set 1 below. In second stage of experiment, the van is moving with constant velocity $\boldsymbol{v}$ w.r.t ground, and an observer is standing on the ground. In this stage also Two Observations brought out in Set 2, below. This makes visible to the observer Four instances, Two sets of Two instances each as brought out here under –

Set 1:

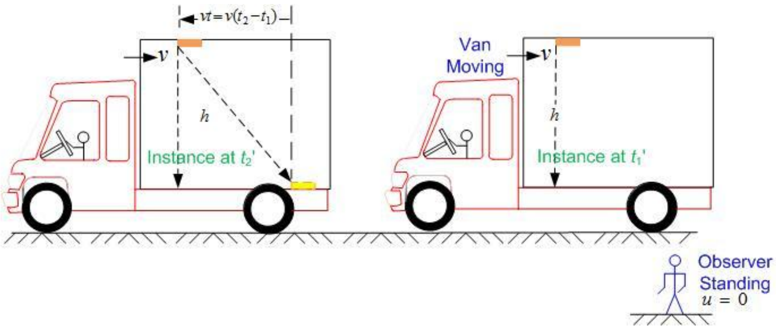
At instance $\boldsymbol{t_1}$: A light beam emanates from the source in the roof of the van.

At instance $\boldsymbol{t_2}$: The light beam emanated at t_1 the light beam reached floor of the van.

Set 2:

At instance $\boldsymbol{t'_1}$: A light beam emanates from the source in the roof of the van already moving with a velocity $\boldsymbol{v}$.

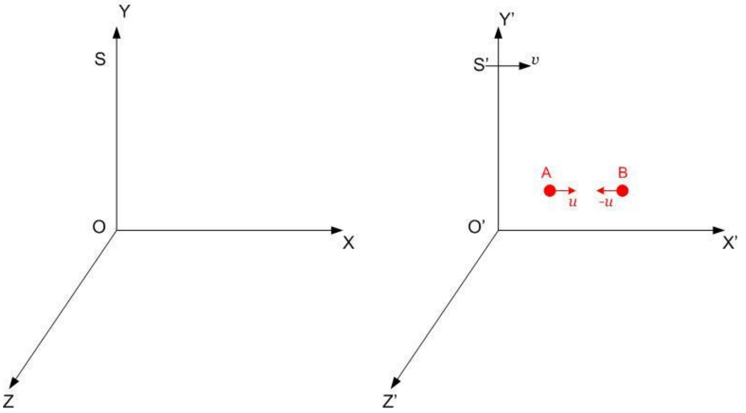
At instance $\boldsymbol{t'_2}$: The light beam emanated at $\boldsymbol{t'_1}$ the light beam reached floor of the van, which continues to move with velocity $\boldsymbol{v}$.



Thus time taken by the light beam to cover a distance $\boldsymbol{h}$, height of the van, at its velocity $\boldsymbol{c}$ is $\boldsymbol{t = t_2 - t_1 = \frac{h}{c}}$. Whereas, time taken by the light beam in the van which continues to move from time 0^- with a constant velocity $\boldsymbol{v}$ is $(\boldsymbol{t' = t'_2 - t'_1})$**.** But, in this case the light beam covers a diagonal distance $ct' = \sqrt{(vt')^2 + h^2}$. This with algebraic manipulations lead to $c^2{t'}^2 = v^2{t'}^2 + h^2$, or $(c^2 - v^2){t'}^2 = c^2t^2 => {t'}^2 = \frac{c^2t^2}{c^2-v^2}$. In the standard form it is $t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} = \lambda t$, where λ is Lorentz Factor. This can be produced in the standard form of relativistic equations $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where $t_0 = t$, and relativistic time is represented as t and not t'. Since, magnitude of velocity can be anything in the range $0 \le |v| \le c$, therefore, corresponding value of time observed by an stationary shall be such that $\boldsymbol{t' > t}$, i.e. time elongates and is called **Dilation of Time.**

Relativistic Composition of Velocity – Velocity Addition: In this **Thought Experiment c**onsider an object moving in S' such that $u' = \frac{dx\prime}{dt\prime}$. Here, x' is coordinate of the object in S' and t' is the instance at which object is observed by an observer in S'. As per Galilean Mechanics velocity of the object in S should be $u = \frac{dx}{dt} = v + u'$. Here, , x and t are the coordinate and the instance at which the object is observed by an observer in S. Therefore, $u = v + \frac{dx\prime}{dt\prime}$. Since, u is in S and. therefore, x' and t' in shall have to be transformed in x and t pertaining to the frame. Using partial derivatives of inverse Lorentz Transformation of x, $\boldsymbol{dx} = \lambda(\boldsymbol{dx' \mp vdt'})$ and likewise, $\boldsymbol{dt} = \lambda\left(\boldsymbol{dt' \mp \frac{v}{c^2}dx'}\right)$**.** Therefore, $\boldsymbol{u = \frac{dx}{dt} = \frac{\lambda(dx'\mp vdt\prime)}{\lambda\left(dt'\mp\frac{v}{c^2}dx\prime\right)} = \frac{dx'\mp vdt\prime}{dt'\mp\frac{v}{c^2}dx\prime}}$**.** Dividing, numerator and denominator by $\boldsymbol{dt'}$**,** it leads to $u = \frac{\frac{dx\prime}{dt\prime}+v}{1+\frac{v}{c^2}\cdot\frac{dx\prime}{dt\prime}}$. It finally resolves into $u = \frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$. This equation is called relativistic addition of velocities where $< (v + u')$, since denominator. In limiting condition of $u' \to c$, it leads to $u \to \frac{v+c}{1+\frac{v}{c}}$, or $u \to c$. It leads to an important conclusion that any velocity added to **c** tends to be **c**, and is a ***mathematical confirmation of the postulate that no object can be seen to be travelling faster than velocity of light. In other words c is the maximum velocity.***

Relativity of Mass – Variation of Mass with Velocity: In this **Thought Experiment** there be two balls A and B, having mass **m**, are moving towards each other in frame S', parallel to X'-axis, as shown in the figure. In collision the two masses coalesce into one body. Therefore, in frame S' as per law of conservation of momentum $mu + m(-u) = 0$. Accordingly velocity of coalesced mass $(2m)$ is Zero, i.e. in state of rest, in frame S'.



Now analyzing the collision phenomenon in frame S, where velocities of the ball A would be $u_1 = \frac{v+u}{1+\frac{vu}{c^2}}$ and that of ball B would be $u_2 = \frac{v-u}{1-\frac{vu}{c^2}}$. Likewise, masses of the balls A and B w.r.t frame S be m_1 and m_2. Then Law of conservation of momentum in S would lead to $m_1u_1 + m_2u_2 = (m_1 + m_2)v$.

Using values of m_1 and m_2, obtained above, $m_1 \cdot \frac{v+u}{1+\frac{vu}{c^2}} + m_2 \cdot \frac{v-u}{1-\frac{vu}{c^2}} = (m_1 + m_2)v$. This on separating variables m_1 and m_2 leads to $m_1 \cdot \left(\frac{v+u}{1+\frac{vu}{c^2}} - v\right) = m_2 \cdot \left(v - \frac{v-u}{1-\frac{vu}{c^2}}\right) => m_1 \cdot \left(\frac{v+u-v-\frac{v^2u}{c^2}}{1+\frac{vu}{c^2}}\right) = m_2 \cdot \left(\frac{v-\frac{v^2u}{c^2}-v+u}{1+\frac{vu}{c^2}}\right)$. It resolves into a form where, $m_1 \cdot \left(\frac{u-\frac{v^2u}{c^2}}{1+\frac{vu}{c^2}}\right) = m_2 \cdot \left(\frac{u-\frac{v^2u}{c^2}}{1-\frac{vu}{c^2}}\right) \to m_1 \cdot u \left(\frac{1-\frac{v^2}{c^2}}{1+\frac{vu}{c^2}}\right) = m_2 \cdot u \left(\frac{1-\frac{v^2}{c^2}}{1-\frac{vu}{c^2}}\right)$, or a ratio $\frac{m_1}{m_2} = \left(\frac{1-\frac{v^2}{c^2}}{1-\frac{vu}{c^2}}\right)\left(\frac{1+\frac{vu}{c^2}}{1-\frac{v^2}{c^2}}\right) = \frac{1+\frac{vu}{c^2}}{1-\frac{vu}{c^2}}$.

At this point a close examination of value of variables u_1 and u_2 derived earlier in this section, leads to a simpler yet effective algebraic manipulation n algebraic manipulation as under –

Manipulation of u_1:

$$1-\frac{u_1^2}{c^2} = 1-\frac{(v+u)^2}{c^2\cdot\left(1+\frac{vu}{c^2}\right)^2} = 1-\frac{\frac{(v+u)^2}{c^2}}{\left(1+\frac{vu}{c^2}\right)^2} = \frac{1+2\cdot\frac{vu}{c^2}+\frac{u^2v^2}{c^4}-\frac{v^2}{c^2}-2\cdot\frac{uv}{c^2}-\frac{u^2}{c^2}}{\left(1+\frac{vu}{c^2}\right)^2}$$

$$=> \frac{1+\frac{u^2v^2}{c^4}-\frac{v^2}{c^2}-\frac{u^2}{c^2}}{\left(1+\frac{vu}{c^2}\right)^2} => \frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{vu}{c^2}\right)^2}$$

$$=> \frac{\left(1-\frac{v^2}{c^2}\right)\cdot\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{vu}{c^2}\right)^2}$$

Manipulation of u_2:

$$1-\frac{u_2^2}{c^2} = 1-\frac{(v-u)^2}{c^2\cdot\left(1-\frac{vu}{c^2}\right)^2} = 1-\frac{\frac{(v-u)^2}{c^2}}{\left(1-\frac{vu}{c^2}\right)^2} = \frac{1-2\cdot\frac{vu}{c^2}+\frac{u^2v^2}{c^4}-\frac{v^2}{c^2}+2\cdot\frac{uv}{c^2}-\frac{u^2}{c^2}}{\left(1-\frac{vu}{c^2}\right)^2}$$

$$=> \frac{1+\frac{u^2v^2}{c^4}-\frac{v^2}{c^2}-\frac{u^2}{c^2}}{\left(1-\frac{vu}{c^2}\right)^2} => \frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vu}{c^2}\right)^2}$$

$$=> \frac{\left(1-\frac{v^2}{c^2}\right)\cdot\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{vu}{c^2}\right)^2}$$

Taking ratios of the above two manipulations leads to $\frac{1-\frac{u_1^2}{c^2}}{1-\frac{u_2^2}{c^2}} = \frac{\frac{\left(1-\frac{v^2}{c^2}\right)\cdot\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{vu}{c^2}\right)^2}}{\frac{\left(1-\frac{v^2}{c^2}\right)\cdot\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{vu}{c^2}\right)^2}} => \frac{\left(1-\frac{vu}{c^2}\right)^2}{\left(1+\frac{vu}{c^2}\right)^2}$. Taking square roots of the inverse of both the sides $\frac{1+\frac{vu}{c^2}}{1-\frac{vu}{c^2}} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}$. This equation together with ratio $\frac{m_1}{m_2}$ has consecutive equivalence and hence, $\frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} => m_1 = m_2 \cdot \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}$. This result is simplified by taking Ball B in a state of rest w.r.t S before collision i.e. as $u_2 \to 0$, $\sqrt{1-\frac{u_2^2}{c^2}} \to 1$, and $m_2 \to m_0$, here, m_0 is the rest mass of the ball B. Thus a generic formula is arrived where $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

Another *interesting conclusion* of this relativistic mass is that **as $v \to c$, mass of the object $m \to \infty$,** i.e. a body travelling at velocity of light shall have infinite mass which is improbable. Therefore, for an object to attain velocity of light greater than c, which can happen only if velocity is continuously increased, it shall have to pass through a state where $v = c$, which itself is impossible, and hence no object can travel at velocity greater than velocity of light ($v > c$).

Mass Energy Equivalence: As per classical mechanics momentum of a moving object is $\boldsymbol{p = mv}$, here $\boldsymbol{m, v}$ and $\boldsymbol{p}$ are mass, velocity and momentum of an object in a stationary frame S **.** Since, force on an object as per laws of mechanics is $F = \frac{dp}{dt} = m\frac{dv}{dt} + \frac{dm}{dt}v$, and work done by the force in moving the object through a distance dx is $dW = Fdx = m\frac{dv}{dt}dx + \frac{dm}{dt}vdx$. This equation can be manipulated as $dW = m \cdot dv \cdot \frac{dx}{dt} + dm \cdot v \cdot \frac{dx}{dt}$. Since, $v = \frac{dx}{dt}$

it can be written as $dE = mv \cdot dv + v^2dm$. Using relativistic mass $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} => m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$. It leads to $m^2c^2 = m_0^2c^2 + m^2v^2$. Taking partial derivative of this equation $c^2 \cdot 2mdm = m^2 \cdot 2vdv + v^2 \cdot 2mdm$, since both c and m_0 are constant. It leads to $2mc^2dm = 2m(mvdv + v^2dm) => c^2dm = mvdv + v^2dm$. Using equation derived from classical mechanics, and as per Einstein's First Postulate in Special Theory of Relativity, all laws of mechanics are equally valid in any frame which is in a state of rest or uniform motion. Thus $c^2dm = dE$, and on integration it leads to $\boldsymbol{E = mc^2}$, regarded as ***world's most important equation*** and *was contributed by Einstein through his thought experiments leading to the Theory of Relativity.*

Energy and Momentum Relation: Extending relativistic definition of mass $\left(m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\right)$ to momentum in classical mechanics ($p = mv$) and, mass-energy equivalence classical mechanics ($E = mc^2$) , another interesting result is obtained. Taking square of mass-energy equivalence equation and subtracting from it c^2p^2, another mathematical acumen, using above equations it leads to $E^2 - c^2p^2 = m^2c^4 - c^2m^2v^2 => \frac{m_0^2}{1-\frac{v^2}{c^2}}c^4 - c^2 \cdot \frac{m_0^2}{1-\frac{v^2}{c^2}} \cdot v^2$.

It leads to $E^2 - c^2p^2 = m_0^2c^4\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{1-\frac{v^2}{c^2}} \cdot \frac{v^2}{c^2}\right) = m_0^2c^4\left(\frac{1-\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) = m_0^2c^4$. Accordingly, $\boldsymbol{E^2 = m_0^2c^4 + c^2p^2}$ it implies that ***total energy of a mass (m) moving with a velocity (v) mass is equal to equal square-root of the sum of square of equivalent energy of the rest mass*** ($\boldsymbol{m_0}$) ***and square of product momentum with velocity of light***.

Summary: Einstein after having stirred the scientific community with his Special Theory of relativity, for inertial frames of reference, did not stop at that. He extended his imagination to accelerated frames of reference and contributed to General Theory of Relativity in 2015. He continued his imagination till his last breadth to discover a single theory which could explain the entire phenomenon governing universe. In this pursuit. imagination and mathematics have crossed boundaries of physical observations which were verifiable. At this point it is worth sharing that -

> ***There is no idea which is obscure, trivial, ridiculous or obnoxious. All that is needed is to think, imagine and meditate. Pursue the idea relentlessly. In the process, it shall undergo refinement and auto correction and then emerge in a final form, the NEED.***

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Education is just not training;
It is about ability to think;
It is about ability to reason;
It is about ability to choose;
It is to develop a faith in self,
And, a passion to apply.

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Rich- Poor Paradox

Parameter	***On Barometer Scale***	
	Rich	***Poor***
Resources	*No time to use abundance*	*Lest than necessity*
Quest for resources	*Never Ending*	*Prepared to share*
?Laughter	*Needs laughing club*	*Singnies his presence*
Apetite	*Needs apetiser*	*Good and Healthy*
Sleep	*Needs Trainqulizer*	*Sound and Refreshing*

A point of consideration?

GROWING WITH CONCEPTS - Chemistry

ENTROPY AND GIBB'S FREE ENERGY

Kumud Bala

'Entropy is a measure of randomness or disorder of the system'. The greater the randomness, the higher is the entropy. The crystalline solid state has the lowest entropy. The gaseous state has the highest entropy. The liquid state has the entropy in-between the two. It is represented by 'S'. It is a state function. The change in its value during a process i.e the entropy change (ΔS) from initial state to final state, is given by

$$\Delta S = S_{\text{final state}} - S_{\text{initial state}} .$$

For chemical reactions: - $\Delta S = \Sigma S$ (products) - ΣS (reactants)

For a reversible process at equilibrium, the change in entropy may be expressed as:

$$\Delta S = \frac{q_{rev,iso}}{T}$$

" So we can express ΔS as the entropy change during a process and is defined as the amount of heat (q) absorbed isothermally and reversibly divided by the absolute temperature (T) at which the heat is absorbed."

The quantity $\frac{q}{T}$ is a measure of disorder. It is justified on the basis of the following two reasons,

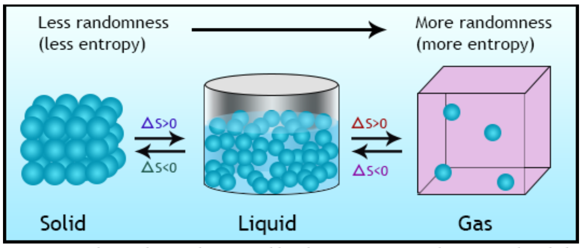
(i) When a system absorbs heat, the molecules start moving faster because kinetic energy increases. Hence, the disorder increases. More the heat absorbed, greater is the disorder.

(ii) For the same amount of heat absorbed at low temperature, the disorder is more than at higher temperature. This shows that entropy change is inversely proportional to temperature. Thus, there is large increase in entropy when a lot of energy is transferred to a system at a low temperature.

Units of entropy change:- As $\Delta S = \frac{q}{T}$, it is an extensive property. In C.G.S. system the unit of entropy are calories/K/mol and in S.I units are Joules/K/mol.

The physical significance of the entropy:- The physical meaning of entropy is the measure of degree of disorder or randomness of a system. This concept may be further understood with the help of the following examples:

(i) Melting of ice



(ii) In a school, when all classes are being held, all students are sitting in their respective class rooms and the disorder is minimum. As soon as the bell goes, the students of different classes come out to go to other rooms and thus get mixed up. In other words, the disorder or the randomness increases.

(iii) In the game of hockey or football, to start with all the players take up some definite positions and are thus said to be in order. As soon as the game starts, the players start running and thus they are said to have an increased randomness which increases further as the game catches more and more momentum.

Entropy as a state function:- Consider a system consisting of a cylinder containing a gas and fitted with frictionless and weightless piston and placed in contact with a large heat reservoir. Suppose the system absorbs heat 'q' isothermally and reversibly at temperature T and expands from volume V_1 to V_2, then the change in entropy of the system is given by $\Delta S_{sys} = \frac{q_{rev}}{T}$

As equivalent amount of heat is lost by the reservoir, change in entropy of the reservoir, $\Delta S_{res} = -\frac{q_{rev}}{T}$ $\therefore$ Total change in entropy $\Delta S_1 = \Delta S_{sys} + \Delta S_{res} = \frac{q_{rev}}{T} + [-\frac{q_{rev}}{T}] = 0$

Now, if we compress the gas isothermally from volume V_2 to V_1, heat q_{rev} will be given out by the system and absorbed by the reservoir so that $\Delta S_{sys} = -\frac{q_{rev}}{T}$ and $\Delta S_{res} = \frac{q_{rev}}{T}$ $\therefore$ Total entropy, $\Delta S_2 = -\frac{q_{rev}}{T} + \frac{q_{rev}}{T} = 0$. Total change in entropy for the complete cycle = $\Delta S_1 + \Delta S_2 = 0$. Thus, at the end of the cyclic process, the entropy of the system remains the same as it originally had. This proves that entropy is a state function.

Spontaneity in terms of entropy change:- Consider the following spontaneous process in an isolated system: (i) Mixing of the two gases on opening the stopcock (ii) spreading of a drop of ink in a beaker filled with water. These processes do not involve any exchange of matter and energy with the surroundings. Hence, these are isolated systems. Further, these processes are accompanied by increase of randomness and hence increase of entropy, i.e., for these processes, entropy change (ΔS) is positive. Hence , it may be concluded that for spontaneous processes in the isolated systems, the entropy change is positive ($\Delta S > 0$).

Now, let us consider the following spontaneous processes in an open vessel: (i) Cooling down of a cup of tea (ii) reaction taking place between a piece of marble ($CaCo_3$) or sodium hydroxide ($NaOH$) and hydrochloric acid in an open vessel. These are not isolated systems because they involve exchange of matter and energy with the surroundings. Hence, for these processes, we have to consider the total entropy change of the system and the surroundings. i.e., $\Delta S_{total} = \Delta S_{system} + \Delta S_{surroundings}$. For the process to be spontaneous, ΔS_{total} must be positive. Hence, it can be generalized that "for all spontaneous processes the total entropy change (ΔS_{total}) must be positive" i.e.,

$$\Delta S_{total} = \Delta S_{system} + \Delta S_{surr} > 0.$$

Further, in all the above spontaneous processes, there is randomness. Hence, the entropy keeps on increasing till ultimately an equilibrium is reached, e.g., uniform distribution of gases after mixing or uniform distribution of ink in water is a stage of equilibrium. Thus, the entropy of the system at equilibrium is maximum and there is no further change of entropy, i.e., $\Delta S = 0$. Hence, it may be concluded that "for a process in equilibrium, $\Delta S = 0$". Arguing in a similar manner, it can be proved that if ΔS_{total} is negative, the direct process is non-spontaneous whereas the reverse process may be spontaneous.

Combining all the results discussed above, it may be concluded that the criterion for the spontaneity in terms of entropy change is as follows:

(i) If ΔS_{total} is positive, the process is spontaneous.

(ii) If ΔS_{total} is negative, the direct process is non-spontaneous; the reverse process may be spontaneous.

(iii) If ΔS_{total} is zero, the process is in equilibrium.

Apply the entropy criterion: for example, consider conversion of water into ice at 1 atmosphere pressure $H_2O\,(l) \rightarrow H_2O\,(s)$

The total entropy change for three different temperatures:

I. At 272K (-1°C) $\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} = -21.85 + 21.93 = +0.08\ Jk^{-1}\,mol^{-1}$. So ΔS_{total} is positive at 272K. And hence freezing of water into ice is spontaneous.

II. At 273K (0°C) $\Delta S_{total} = \Delta S_{sym} + \Delta S_{surr} = -21.99 + 21.99 = 0$. Since ΔS_{total} is zero, process is at equilibrium i.e., neither freezing nor melting is spontaneous. At this temperature water and ice are in equilibrium at 273K. No net change is observed.

III. At 274K (+1°C) $\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} = -22.13 + 22.05 = -0.08Jk^{-1}mol^{-1}$. Since ΔS_{total} is negative. So the process is non-spontaneous at 274K. But for the reverse process ΔS_{total} will be positive. $\Delta S_{total} = +0.08\ Jk^{-1}mol^{-1}$. Melting of ice is spontaneous at 274K.

Thus, we have observed ΔS_{total} is a criterion for spontaneity of a change.

CHARACTERISTICS OF ENTROPY

(i) Entropy is an extensive property (Its value depends upon the amount of the substance present in the system).

(ii) Entropy of a system is a state function, $\Delta S = \Delta S_{final\ state} - \Delta S_{initial\ state}$ (depends upon the state variables T, P, V, n) (Independent of the path).

(iii) The change in entropy for a cyclic process is always zero.

(iv) The total entropy change of an isolated system is equal to the entropy change of system and entropy change of surrounding. $\Delta S_{universe} = \Delta S_{sys} + \Delta S_{surr}$. The sum is called entropy change of universe.

(v) In a reversible process $\Delta S_{universe} = 0$ therefore $\Delta S_{sys} = -\Delta S_{surr}$

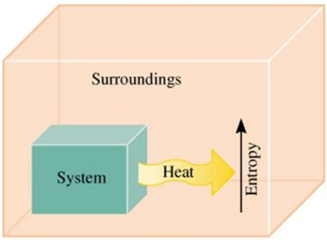
(vi) In a irreversible process, $\Delta S_{universe} > 0$ spontaneous change

In term of entropy the definition of the second law may be given as follows: All spontaneous processes are accompanied by a net increase of entropy, i.e., for all the spontaneous processes, the total entropy change (sum of the entropy changes of the system and the surroundings) is positive.

Entropy changes in system and surrounding and total entropy change for exothermic and endothermic reactions:-

1. **For exothermic reactions:**

(a) In exothermic reactions, the randomness and hence entropy increases so that ΔS_{sys} is positive. The heat given out by the reaction is absorbed by the surrounding. Hence the disorder of the surrounding also increases. The total entropy change is highly positive. Hence, the reactions are highly spontaneous.



Exothermic Process
$\Delta S_{surr} > 0$

(b) In some exothermic reactions, the entropy of the system may decrease for example- in the condensation of gas or solidification of a liquid. Heat evolved is very high that it greatly increases the disorder of the surrounding and the overall entropy change (ΔS_{total}) is again positive.

In oxidation of magnesium:- $2Mg + O_2(g) \rightarrow 2MgO$, $\Delta S_{reaction} = -217 Jk^{-1}mol^{-1}$

$\Delta H_{reaction} = -1202K\ Jmol^{-1}$, this heat is absorbed by the surrounding at 25°C .

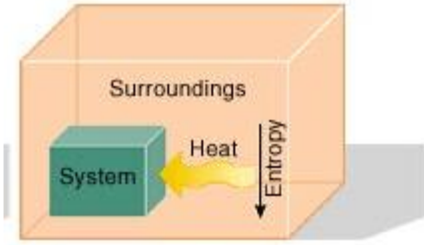
$\Delta S_{surrounding} = -\frac{-1202 \times 1000\ J\ mol^{-1}}{298K} = 4034 JK^{-1}mol^{-1}$

$\Delta S_{total} = -217 JK^{-1}mol^{-1} + 4034 JK^{-1}mol^{-1} = 3817\ JK^{-1}mol^{-1}$

ΔS_{total} is positive, the reaction is spontaneous.

2. **For endothermic reactions:**

In endothermic reactions, heat is absorbed by the reactants while it is lost by the surroundings. The disorder of the surrounding decreases i.e., (ΔS_{surr} is negative).The disorder of the reactants may increase. When the entropy of the system increases enough to overcome the decrease in entropy of the surrounding then overall entropy change is positive. Hence the reaction will be spontaneous.



Entropy change during phase transition: The change of matter from one state to another state is called phase transition. (i) Solid to liquid (melting point at definite temp.) (ii) Liquid to vapour (boiling point at definite temp.)

The entropy change $\Delta S = \frac{q\ rev.}{T}$, q_{rev} = heat absorbed or evolved, T= Temperature

Entropy of fusion: It may be defined as: "The entropy change when 1 mole of the solid substance changes into liquid form at its melting point". For example: when ice melts i.e Water (s) → water (l)

$S_{water} - S_{ice} = \Delta S_{fus} = \Delta_{fus}H^o / T_f$, where $\Delta_{fus} H^o$ = enthalpy of fusion and T_f = fusion temperature (Melting point)

Numerical: The standard enthalpy of fusion $\Delta_{fus}H°$ at 273K and 1 bar pressure for water is 6.0 $KJmol^{-1}$. Calculate the entropy of fusion of ice.

Solution: $H_2O(s) \rightarrow H_2O(l)$

$\Delta_{fus} H° = +6.0\ KJmol^{-1}$ T = 273K

$\Delta S_{fus} = \frac{q\ fus}{T} = \frac{\Delta H°}{T} = \frac{6.0 \times 1000\ J\ mol^{-1}}{273}$

$= 21.98\ JK^{-1}mol^{-1}$

Entropy of vaporization: It is defined as "The entropy change when 1 mole of a liquid changes into vapours at the boiling point." The entropy of vaporization for a liquid at its boiling point is:

$\Delta_{vap} S = \Delta_{vap} H / T_b$, $\Delta_{vap} H°$ is the standard enthalpy of vaporization and T_b is the boiling point.

Numerical: Calculate the entropy of vaporization for water. The enthalpy change for the transition of liquid water to steam is 40.8KJ mol^{-1} at 373K.

Solution: $\Delta_{vap} S° = \frac{\Delta vap\ H}{T}$

$\Delta_{vap} H° = 40.8 KJ\ mol^{-1}$ and T = 373K

$\Delta_{vap}S° = \frac{40.8 \times 1000\ J\ mol^{-1}}{373K} = 109.4 JK^{-1}mol^{-1}$

Entropy of sublimation: Sublimation is a direct conversion of a solid into its vapour.

Entropy of sublimation may be defined as: "The entropy chane when 1 mole of a solid changes into vapour at a particular temperature."

$$\Delta_{Sub} S° = \frac{\Delta\, Sub\, H}{T}$$

Δ_{Sub} H = Standard enthalpy of sublimation

T = Temperature

$\Delta_{Sub} H = (\Delta_{fus}H + \Delta_{vap}H)$

Entropy change in processes not involving any phase transformation: In these processes entropy also increases when the number of molecules of products is greater than the number of molecules of reactants.

e.g.

$$2\, SO_3\, (g) \rightarrow 2SO_2\, (g) + O_2\, (g)$$
$$PCl_5\, (g) \rightarrow PCl_3\, (g) + Cl_2\, (g)$$
$$N_2O_4 \rightarrow 2NO_2\, (g)$$

GIBBS FREE ENERGY: This is another thermodynamic quantity that helps in predicting the spontaneity of a process. Gibbs energy of the system is defined as "the maximum amount of energy available to a system during a process that can be converted into useful work". It is a measure of capacity of a system to do useful work. It is denoted by G .

Mathematically: $G = H - TS$, H = heat content, T = absolute temperature, S = entropy of the system. For isothermal process we have: $G_1 = H_1 - TS_1$ (for the initial state), $G_2 = H_2 - TS_2$ (for the final state)

$\therefore\ G_2 - G_1 = (H_2 - H_2) - T\,(S_2 - S_1)$, $\Delta G = G_2 - G_1$ is the change in Gibbs free energy of the system. $\Delta H = H_2 - H_1$ is the enthalpy change, $\Delta S = S_2 - S_1$ is the entropy change,

$\therefore\ \Delta G = \Delta H - T\Delta S$

This equation is known as Gibbs Helmholtz equation or Gibbs energy equation.

Free energy change and non- mechanical work (useful work): In addition to predicting the spontaneity of a process, another important aspect of Gibbs energy is its relationship to the useful work (other than pressure- volume work) that can be obtained from the system. Hence, Gibb's free energy is defined as "The thermodynamic quantity of a system the decrease in whose value during a process is equal to the maximum possible useful work that can be obtained from the system." This result may be derived as follows:

The relationship between heat absorbed by a system q, the change in its internal energy ΔU, and the work done by the system is given by the equation of the first law of thermodynamic, i.e.,

$$q = \Delta U + w_{\text{expansion}} + w_{\text{non- expansion}} \text{ ....(i)}$$

Under constant pressure condition, the expansion work is given by PΔV

$$\therefore\ q = \Delta U + P\Delta V + w_{\text{non-expansion}}$$

$$= \Delta H + w_{\text{non-expansion}} \quad (\text{because } \Delta U + P\Delta V = \Delta H) \text{ .... (ii)}$$

For a reversible change taking place at a constant temperature,

$$\Delta S = \frac{q\ rev}{T} \text{ or } q_{rev} = T\Delta S \text{ ....(iii)}$$

Substituting the value of q from equation (iii) in equation (ii)

$$T\Delta S = \Delta H + w_{\text{non-expansion}}$$

$$\text{Or, } \Delta H - T\Delta S = -w_{\text{non-expansion}} \text{ .... (iv)}$$

For a change taking place under conditions of constant temperature and pressure,

$$\Delta G = \Delta H - T\Delta S\ ,$$

substituting this value in eqn. (iv) above, we get

$$\Delta G = -w_{\text{non-expansion}} \text{ ....(v)}$$

Thus, free energy change can be taken as a measure of work other than the work of expansion. For most changes, the work of expansion cannot be converted to other useful work, whereas the non-expansion work is convertible to useful work.

Rearranging equation (v), we may write :

$$-\Delta G = w_{\text{non-expansion}} = w_{\text{useful}} \text{ .... (vi)}$$

Hence, the decrease in free energy of the system during any change, ΔG, is a measure of the useful or net work derived during the change. It may, therefore, be generalized that the free energy, G, of a system is a measure of its capacity to do useful work. It is a part of the energy of the system which is free for conversion to useful work and is, therefore, called free energy. It can be shown that the free energy change is equal to the maximum possible useful work that can be obtained from a process (for a reversible change at constant T and P). Hence, we can write

$$-\Delta G = w_{\max} \text{ .... (vii)}$$

Thus, a spontaneous process can be utilized to do useful work. The greater the change in Gibbs energy, greater is the amount of work that can be obtained from the process. If the work involved is the electrical work as in the case of galvanic cells, then as electrical work =n FE, the above relationship is written as $-\Delta G = n\,FE$, where n = number of electrons involve in the cell reaction, E = EMF of the cell , F = Faraday If all the reactants and products of the cell reaction are in their standard states, i.e., 298K and 1 atm pressure, the above relationship is written as

$$-\Delta G^\circ = n\,FE^\circ \quad\text{........(viii)}$$

Where, ΔG° = standard free energy change and E° = standard EMF of the cell.

In a fuel cell, it is found that the heat evolved (ΔH) is not completely converted into energy, i.e., useful work (given by ΔG).The ratio $\Delta G/\Delta H$ is called the efficiency of the fuel cell. Thus, Efficiency of a fuel cell = $\frac{\Delta G}{\Delta H}$.

Assignment :

(1) When potassium chloride is dissolved in water (A) entropy increases (B) entropy decreases (C) entropy increases and then decreases (D) free energy increases.

(2) For a reversible process at equilibrium, the change in entropy may be expressed as:
(A) $\Delta S = Tq_{rev}$ (B) $\Delta S = \frac{q\,rev}{T}$ (C) $\Delta S = \frac{\Delta H}{T}$ (D) $\Delta S = \Delta G$.

(3) For a spontaneous reaction, ΔG should be: (A) positive (B) negative (C) equal to zero (D) may be positive or negative.

(4) Which of the following processes is accompanied by decrease in entropy?
(A) Evaporation of water (B) Sublimation of dry ice (C) Melting of ice (D) Condensing steam.

(5) Free energy is related to enthalpy and changes in entropy as: (A) $\Delta G = \Delta H - T\Delta S$
(B) $\Delta G = T\Delta S - \Delta H$ (C) $\Delta G = \frac{\Delta H - \Delta S}{T}$ (D) $\Delta G = \Delta H + T\Delta S$

Answer 1. (A) 2. (B) 3. (B) 4. (D) 5. (A)

Author is M.Sc. (Chem.), M.Ed. and Advanced Diploma in German Language (Gold Medallist). She retired as a Principal, Govt. School Haryana, has 3-1/2 years' experience in teaching Chemistry and distance teaching through lectures on Radio and Videos. She has volunteered to complement mentoring of students for Chemistry through Online Web-enabled Classes of this initiative.
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Answers to Science Quiz in Aug'17

Kumud Bala

ANSWERS : 1. (A) 2. (D) 3. (C) 4. (C) 5. (C) 6. (C) 7. (A) 8. (A) 9. (B) 10. (B) 11. (B) 12. (B) 13. (A) 14. (B) 15. (C) 16. (C) 17. (A) 18. (B) 19. (A) 20. (D)

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SCIENCE QUIZ : Sept'17

Kumud Bala

1. Neutrons are present in all atoms except :
   (A) He (B) C (C) H (D) Ne
2. When α- particles are passed through a thin metal foil, most of them go straight through the foil because:
   (A) α- particles are lighter than electrons
   (B) α-particles are positively charged and nucleus also has same charge
   (C) most part of the atom is empty space
   (D) α-particles move with high velocity.
3. Neutron was discovered by:
   (A) J.J Thomson (B) Chadwick
   (C) Rutherford (D) Millikan
4. The spectral line obtained when an electron jumps from n = 6 to n = 3 energy level belongs to:
   (A) Lyman series (B) Paschen series
   (C) Balmer series (D) Pfund series
5. The electron level which allows the hydrogen to absorb photons but not to emit is :
   (A) 3s (B) 2p (C) 1s (D) 3d
6. Bohr's model can explain:
   (A) the spectrum of hydrogen atom only
   (B) the spectrum of the atom or ion containing one electron only
   (C) the spectrum of hydrogen molecule
   (D) the solar spectrum
7. Which of the following has more electrons than neutrons?
   (A) Al^{+3} (B) C (C) O^{-2} (D) F
8. The nitrogen atom has 7 protons and 7 electrons. The nitride ion (N^{-3}) will have
   (A) 7 protons and 10 electrons
   (B) 4 protons and 7 electrons
   (C) 4 protons and 10 electrons
   (D) 10 protons and 7 electrons.
9. The electronic configuration of an element with atomic number 26 is:
   (A) [Ar] $4s^2\ 3d^6$ (B) [Ar] $4s^1\ 3d^5$
   (C) [Ar] $3d^8$ (D) [Ar] $4s^2\ 3d^4$
10. Which of the following does not make sense?
    (A) 7p (B) 5g (C) 4f (D) 2d
11. An electron having an Azimuthal quantum number l=3 is:
    (A) s-electron (B) p-electron
    (C) d-electron (D) f-electron
12. Which of the following are iso electronic pairs?
    (A) K^+, O (B) Ne, O (C) Na^+, F^- (D) Mg^{2+}, Cl^-
13. The splitting of the spectral lines under the influence of magnetic field is called:
    (A) Zeeman effect (B) Compton effect
    (C) Photoelectric effect (D) Diffraction
14. The quantum number not obtained from Schrodinger wave equation is:
    (A) n (B) l (C) m (D) s
15. The radius of an atomic nucleus is of the order of:
    (A) 10^{-10} (B) 10^{-15} (C) 10^{-8} (D) 10^{-13}
16. Which orbital has two angular nodal planes?
    (A) 2s (B) 2p (C) 3d (D) 4f
17. The electronic configuration of an element is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5\ 4s^1$. This represents its :
    (A) excited state (B) ground state
    (C) cationic form (D) anionic form
18. The number of d-electrons remained in Fe^{2+} ion (At. No. of Fe = 26) is:
    (A) 4 (B) 5 (C) 6 (D) 3
19. The orientation of an atomic orbital is governed by:
    (A) Spin quantum number
    (B) Magnetic quantum number
    (C) Principal quantum number
    (D) Azimuthal quantum number
20. In an atom, the signs of lobes indicate the:
    (A) sign of charges
    (B) sign of probability distribution
    (C) sign of the wave function
    (D) presence or absence of electron.

(Answers to this Science Quiz – Sept'17 shall be provided in Quarterly e-Bulletin dt 2nd October'17)

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Theme Song :

PREMISE: ***We are pleased to adopt a song*** ***" इतनी शक्ति हमें देना दाता....." from a old Hindi Movie Do Aankhen Barah Haath दो आँखें बारह हाथ of year 1957, directed by The Late V. Shantaram. The lyrics are by Shri Bharat Vyas, singer Melody Queen Sushri Lata Mangeshkar, and Music Direction by Vasant Desai. It has become a widely accepted inspirational song and/or prayer in many educational institutions and socially inspired initiatives engaged in mentoring of unprivileged children. This newly formed non-organizational initiative, being selflessly operated by a small set of compassionate persons, finds its philosophy in tune with the song and conveys its gratitude to all he eminent persons who brought out the song in a manner that it has attained an epitome of popularity. While working its mission and passion, the group invites one and all to collectively complement in grooming competence to compete among unprivileged children. The song/prayer goes as under -***

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

दूर अज्ञान के हो अंधेरे, तू हमें ज्ञान की रोशनी दे
हर बुराई से बचते रहें हम, जितनी भी दे भली ज़िन्दगी दे
बैर हो ना किसी का किसी से, भावना मन में बदले की हो ना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

हम ना सोचें हमें क्या मिला है, हम ये सोचे किया क्या है अर्पण
फूल खुशियों के बाँटे सभी को, सब का जीवन ही बन जाए मधुबन
अपनी करुणा का जल तू बहा के, कर दे पावन हर एक मन का कोना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

Together **E**ach **A**chieves **M**ore
(**TEAM**)

Every end, so also end of this e-Bulletin, is a pause for a review, before re-continuing of a journey far beyond ...