GYAN-VIGYAN SARITA: शिक्षा

A non-remunerative, non-commercial and non-political initiative to Democratize Education as a Personal Social Responsibility (PSR)

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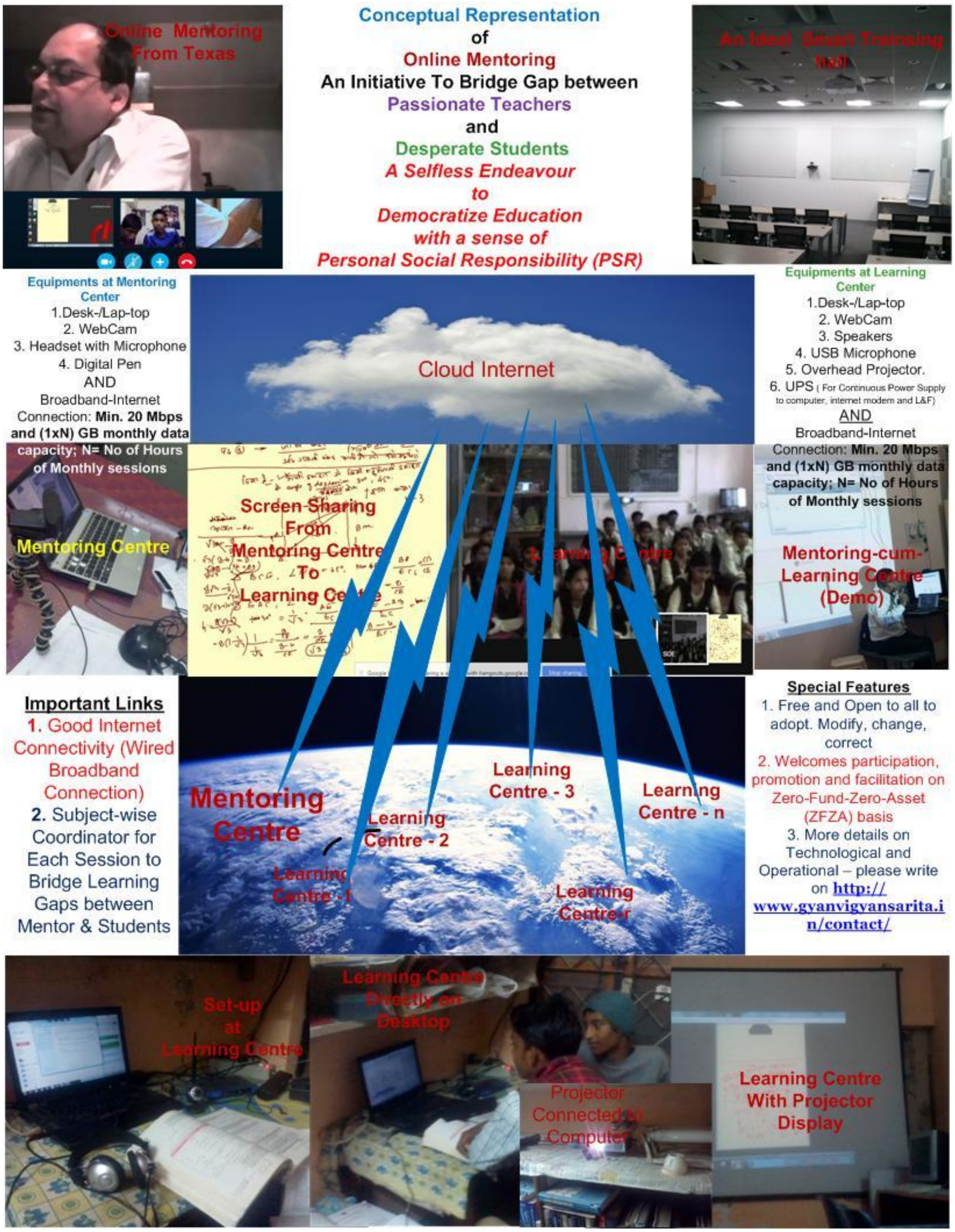
Aim for the Best, but…



… start, without loosing time, with whatever is available.

संपादकीय

अंतर्राष्ट्रीय योग दिवस

मनुष्य स्वभाव से दीर्घ जीवन जीने की चाह रखता रहा है। हमारे ऋषि-मुनियों ने दीर्घ जीवन की चाह को योग-क्रियाओं के अभ्यास से सफल सिद्ध किया हुआ है। अथर्ववेद के सूक्त इसी चाह के प्रमाण हैं जिनमें बताया गया है कि ***पश्येम शरदः शतम्*** अर्थात् हम सौ वर्षों तक देखें यानि हमारे आंखों की ज्योति स्पष्ट बनी रहे। ***जीवेम शरदः शतम्*** अर्थात् हम सौ वर्षों तक जीवित रहें। ***बुध्येम शरदः शतम्*** अर्थात् सौ वर्षों तक हमारी बुद्धि सक्षम बनी रहे।

21 जून वर्ष का सबसे बड़ा दिन होता है। संभवतया यही कारण रहा होगा जब 21 जून को अंतर्राष्ट्रीय स्तर पर योग दिवस मनाने के लिये संयुक्त राष्ट्र महासभा ने निर्णय लिया। पहली बार योग दिवस वर्ष 2015 में 21 जून को मनाया गया। संयुक्त राष्ट्र महासभा में भारत के प्रधानमंत्री श्री नरेन्द्र मोदी ने योग की महत्ता दर्शाते हुये कहा था कि योग भारत की प्राचीन परंपरा का विश्व के लिये एक अमूल्य उपहार है। योग से मनुष्य और प्रकृति के बीच सामंजस्य स्थापित होता है। योग केवल एक व्यायाम नहीं है बल्कि इससे हममें एकता की भावना पैदा होती है और हम प्रकृति से सीधे संपर्क साधने में सफल होते हैं।

योग की महत्ता इसी से प्रमाणित होती है कि कई बडे देशों के नेताओं ने खुलकर संयुक्त राष्ट्र महासभा में इसकी वकालत की। नेपाल के प्रधानमंत्री सुशील कोइराला, संयुक्त राज्य अमेरिका, कनाडा, चीन, मिस्र, आदि ने इसका समर्थन किया।

कोई भी संस्कृति तभी तक संरक्षित रहती है जब तक उसे राज्य का संरक्षण मिलता रहता है, अन्यथा वह अनाथ हो जाती है। संयुक्त राष्ट्र द्वारा संरक्षित होने से योग को समस्त विश्व में प्रसारित करने में अवश्य मदद मिलेगी। भारतीय सोच रही है कि हम सबके लिये हैं और हमारा जीवन सबके लिये है। हम दीर्घायु हों, स्वस्थ हों और हमारे साथ ही साथ अन्य लोग भी ऐसे ही हों।

यह मान्यता है कि जहां योग है वहां कमी, अशुद्धता, अज्ञानता अथवा अन्याय होने की संभावना घट जाती है। यह सच है कि किसी समय में योग साधु-संतो, ऋषि-मुनियों की साधना का एक अभिन्न हिस्सा होता था, परंतु आज वही योग सामान्य जन की जीवन जीने की कला का अंग हो गया है। योग करने के लिये किसी भाषा की आवश्यकता नहीं है। यही कारण है कि विश्व के देशों को भारत की भाषाओं अथवा संस्कृति की जानकारी न होते हुये भी योग करने में आनंद आता है और उन्हें स्वयं से यह अनुभूति करने में देर नहीं होती कि योग जीवन के लिये अति आवश्यक है, यह एक कला है और कोई कला किसी भाषा अथवा देश की परिधि की मोहताज नहीं होती है।

योग भारतीय संस्कृति की पहचान है। आदि देव शंकर को योग का प्रथम शिक्षक माना जाता है। बाद में कृष्ण ने इसे गीता में सामान्य जन के लिये प्रचारित किया। महावीर और बुद्ध ने इसे आगे बढाया। योगदर्शन के अनुसार योग, हमारी चित्त की वृत्तियों का निरोध करना भर है। यह एक शुद्ध विज्ञान है। भगवदगीता में कहा गया है कि जो सुख-दुःख, लाभ-हानि, शत्रु-मित्र, शीत-घाम आदि की लड़ाई में नहीं पड़ा और समभाव रहा, उसकी समझ से यह अर्थ निकाला जा सकता है कि वह योग जानता है।

योग के अभ्यास का प्रमाण लगभग 3000 ईसवी पूर्व सिंधु घाटी सभ्यता के समय की मोहरों और मूर्तियों में मिलता है। योग पर लिखा गया एक प्रामाणिक ग्रंथ योगसूत्र है जो 200 ईसवी पूर्व लिखा माना गया है ।

योग जीवन को खुशहाल रखने, बीमारियों से शरीर को बचाये रखने, और लंबी आयु पाने का आधार है। अगर हम योग को एक चिकित्सा पद्धति कहें तो यह अतिशयोक्ति नहीं होगी। योग के आसन हमें विभिन्न बीमारियों से छुटकारा पाने में मदद करते हैं। शवासन जहां हमें हाईब्लडप्रेशर से छुटकारा दिलाने में मदद करता है, वहीं भ्रामरी प्राणायाम, मन को शांति देता है, और मेरूदंडआसन, गठिया से मुक्ति दिलाता है।

योग से हमारे शरीर में रोग प्रतिरोधक क्षमता का विकास होता है और नियमित योग करने से हम तनावमुक्त हो सकते हैं। यही कारण है कि विश्व के विभिन्न देशों में इस पर शोध चल रहा है और इसकी उपयोगिता का वैज्ञानिक प्रमाण होने से इसे अपनाया जा रहा है।

प्रधानमंत्री श्री नरेन्द्र मोदी ने अपने मन की बात में 25 जून 2017 को बताया कि पूरा विश्व 21 जून 2017 को अंतर्राष्ट्रीय योग दिवस पर योगमय हो गया था। पानी से पर्वत तक लोगों ने सबेरे-सबेरे सूरज की किरणों का स्वागत योग के माध्यम से किया।

ब्रिटेन में टेफेलगर स्कवायर पर इकट्ठे होकर लोगों का योग दिवस मनाना, अथवा दक्षिण अफ्रीका, तथा मिस्र में योग दिवस का मनाया जाना बताता है कि धर्म से इसका कोई लेना देना नहीं है। चीन में लोगों ने वहां की विश्व प्रसिद्ध महान दीवार पर योग का अभ्यास किया। पेरू में विश्व की धरोहर माचू पिच्चू पर जो समुद्र तल से 2400 मीटर उपर है, योग का अभ्यास किया। फ्रांस में एफिल टावर के सामने लोगों ने योग का प्रदर्शन किया।

संयुक्त अमीरात के अबू धाबी में 4000 से अधिक लोगों ने सामूहिक योग किया। अफगानिस्तान में हेरात में सलमा बांध पर योग कर भारत से दोस्ती को एक नया आयाम दिया। सिंगापुर में करीब 70 स्थानों पर योग के कार्यक्रम हुये। संयुक्त राष्ट ने इस दिवस को महत्वपूर्ण बनाते हुये अंतर्राष्ट्रीय योग दिवस के 10 टिकट निकाले। संयुक्त राष्ट्र हेडक्वार्टर में योग दिवस पर वहां के स्टाफ और दुनियाभर के कूटनीतिज्ञ सम्मिलित हुये।

देश के अंदर भी, सभी ने चाहें वे समाज से जुड़े लोग हों, बच्चे हों अथवा बड़े, सुरक्षा से जुड़े सैनिक हों, अथवा व्यवसाय से जुड़े लोग हों, सभी ने इस दिवस पर योग का अभ्यास किया और यह बताने का संकेत दिया कि योग से हमारा जीवन सुखमय और बीमारी रहित हो सकता है। आवश्यकता है कि हम इसका नियमित अभ्यास करें।

योग हमें अपने पर विश्वास करना सिखाता है। योग का संदेश है कि आपमें जानने की क्षमता है, इसका उपयोग करो, अपने को सकारात्मक रखो, और सबकी भलाई के लिये उनके साथ चलो।

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INVITATION FOR CONTRIBUTION OF ARTICLES

*Your contribution in the form of an article, story poem or a narration of real life experience is of immense value to our students, the target audience, and elite readers of this Quarterly monthly e-Bulletin **Gyan-Vigyan Sarita: शिक्षा**, and thus create a visibility of the concerns of this initiative. It gives them a feel that you care for them, and they are anxiously awaiting to read your contributions. We request you to please feel free to send your creation, by **20th of this month** to enable us to incorporate your contribution in next bulletin, subhashjoshi2107@gmail.com.*

We will be pleased have your association in taking forward path our plans as under-

- ***1st Supplementary e- Bulletin of 4th Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा shall be brought out 1st August'17. It shall be dedicated to 15th August, to commomerate completion of 70 Years of Independence of our Country.***
- ***And this cycle monthly supplement to Quarterly e-Bulletin Gyan-Vigyan Sarita: शिक्षा aimed to continue endlessly***

We believe that this quarterly periodicity of e-Bulletins shall make it possible for our esteemed contributors to make contribution rich in content, diversity and based on their ground level work.

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An Appeal: Gyan Vigyan Sarita
A non-organizational initiative of a small set of Co-passionate Persons

Philosophy: Personal Social Responsibility (PSR)

Objective: Groom competence to Compete among unprivileged children from 9th-12th in Maths and Physics, leading to IIT-JEE.

Financial Model: ***Zero-&-Fund-Zero-Asset (ZFZA).*** It calls for promoters and facilitators to provide infrastructure for use to the extent they feel it is neither abused nor there is a breach of trust. And, reimbursement of operational expenses to the participators

Operation:

a. **Mode:** Online since July'16, using Google Hangouts, a free we-conferencing S/w, with connectivity upto 15 nodes.
b. **Participation:** Voluntary and Non-remunerative, Non-Commercial and Non-Political

Involvement:

a. **As Promoter** –
   i. Initiate a Learning Center,
   ii. Sponsor a Mentor who is willing to join on certain terms,
   iii. sponsor cost of operation and up-gradation of infrastructure to voluntary mentors,
   iv. Sponsor Website.
b. **As Facilitator** –
   i. Provide space and infrastructure for **Online Mentoring Sessions (OMS)**, which is generally available, with a marginal add-on,
   ii. Garner support of elite persons to act as coordinators at a Learning Centre.
c. **As Participator** –
   i. As a Mentor,
   ii. As Coordinator,
   iii. As Editor and or contributor of thought provoking articles for e-Bulletin, which are relevant to the initiative, and make it more purposeful and reachable to the target audience.
   iv. As author of Chapters for Mentors' Manual, being uploaded as a Free Web Resource,
   v. Anything else that you feel can add value to the mission and make it more purposeful.
   vi. *Anything else that you consider to make this initiative to become more effective.*

Background: *The initiative had its offing in May'12, when its coordinator, a power engineer by profession, soonafter submission of Ph.D. Thesis in April'12, at IIT Roorkee, at the age of 61 years, decided to mentor unprivileged students.*

SARTHAK PRAYASH, a Ghaziabad based NGO, warmly accepted the proposition and created a facility to mentor students from 8+ to prepare in mathematics and physics and prepare them for engineering entrance tests. They warmly reciprocated and created a class room.

*Experience in this selfless social work were used to navigate across without losing focus. He was associated with SUBODH FOUNDATION from Sept'15 to Sept'16 during which he published a monthly e-Bulletin **SUBODH-पत्रिका** to create visibility across persons who could make a difference.*

*In Sept'16, post transition, the mission has been continued as a non-organizational entity **Gyan Vigyan Sarita**, with a set of Four persons, including retired **Prof. SB Dhar**, Alumnus-IIT Kanpur, a middle aged **Shri Shailendra Parolkar**, Alumnus-IIT Kharagpur, settled at Texas, US and **Smt. Kumud Bala**, Retired Principal, Govt. School Haryana. Earlier, they were complementing the OMS. While, the initiative survived transition, a website: http://gyanvigyansarita.in has been launched. It contains under its **Menu: Publication>e-Bulletins**, and **>Mentors' Manual.** You may like to read them.*

Actions Requested: May please like to ponder upon this initiative. **Queries**, *if any, are heartily welcome*. We would welcome your collective complementing in any of the areas listed at **Involvement**, above, to make the mission more purposeful and reachable to target children.

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Coordinator's Views

Culture – Morality – Justice

In prevalent environment where most are in race for name, fame and wealth, social conscience is drifting in a direction where objective is self-gain come what may. In this context when world would be celebrating ***International Justice day*** *on 17th July, it was considered appropriate to review social perceptions on* ***Culture****,* ***Morality*** *and* ***Justice*** *which are seen to be closely entwined and cannot be seen in isolation.*

Culture : In Wikipedia culture is defined as "*the social behavior and norms found in human societies. Culture is a central concept in anthropology, encompassing the range of phenomena that are transmitted through social learning in human societies, what has been in practice and has a long history of sustenance*". It is about the way people coexist, right from predator stage. It is believes and practices that have survived over generations, not necessarily on logical and scientific backed with the reasoning of human experience. It is all about socio-economic-political-&-environmental compulsions driving coexistence, which has been getting transformed with passage of time, human experiences, changing believes and practices. These social practices have been deeply influenced by education, technological development, and access to it. It is observed that cultural variations not only in different parts of the world and even across adjoining villages bear a lot of diversity and at times contradictions. Despite, these cultures have survived without carrying bad omen in one culture, causing any harm in other culture, where it is not in vogue. There are many such examples viz. marriage institutions, child marriages, animal and human sacrifice, prostitution, sati, bonded labour, child labour etc. There were social thinkers who brought reforms to the cultural practices, in different parts of the world, and most of the above in India. These reforms become more sustainable when invoked by persons within the society. But, such reforms remains localized unless they are backed by law of land. Shaping reforms into law of land has a tedious journey involving social conscience called morality, which is capable of influencing law-makers. This is where the cultural aspects assume political importance.

Morality: In philosophy morality is defined as "*a system of behavior in regards to standards of right or wrong behavior. The word carries the concepts of: (1) moral standards, with regard to behavior; (2) moral responsibility, referring to our conscience; and (3) a moral identity, or one who is capable of right or wrong action. Common synonyms include ethics, principles, virtue, and goodness*". Like culture, morality is also highly subjective and prone to contradictions. Leave aside bondage between teacher and taught in the era of गुरु-शिष्य परंपरा in ancient India, a few decades ago students used to voluntarily bow to the teacher simply out of respect. But, in prevalent times of highly commercialized education it is a common observation that students either put their back or greet with Hi! to their teacher. Reason behind this transition is that the education has become a commodity which is priced for the results that one expects. Value of transforming thought process and with ability to evolve solution to unknown problems has taken a back seat. In this initiative recently, three promising students from highly qualified and highly placed parents were terminated from Online Mentoring Session, on the grounds of lack of response towards mentor who was delivering the best of the time, with a sense of Personal Social Responsibility (PSR). Parents, did plead for pardon, with a lucrative offer to support this mission. Perhaps their morality did not allow breach of protection to their beloved child, irrespective of quality and value of mentoring being rendered selflessly in this mission. The natural choices went for discriminating children from elite families having multiple options and dedicate time and efforts to unprivileged children who suffer from lack of vision, environment, guidance and persons to extend finger holding. The pleading of the respective parents and grandparents were extremely oppressive on the moral grounds. But, for sheer self-conviction about selfless passion, commitment, dedication and desperation for the cause, it would not have been possible to sustain the irreversibility of the decision. Ultimately, in an attempt to hope against hope, these parents were offered to create new learning center for unprivileged children, where their child could learn with the target students. Absence of any pro-active active reciprocation, by these parents, to fulfill their promise to support the mission vindicates the action.

Education as a process is the highest form of transforming morality of society, in which subject is only a carrier of the thought process to inculcate human values, preparedness to work hard and together as a team, while competing for competence. Once, education receives and honoured its deserving place all the issues of morality related to eating habits, addiction, family and social bondages, sex discrimination and crimes would automatically take a back seat. What is socially acceptable

Discipline, individually and collectively, is all about preserving one's own interest with due consideration, concern, sensibility, accountability and responsibility towards agony of others in fulfilling self-interests. Every successful person and leader is highly disciplined and, therefore, while growing he has been also able to take care of collective growth. Discipline is an important attribute of morality.

Justice: In dictionary Justice is defined as "*the quality of being just; righteousness, equitableness , or moral rightness".* Onus of extending justice lies with the individual's place in family, social, administrative, executive structure, and just not confined to four walls of a court-room. Therefore, it would not be inappropriate to say justice is a fabric whose spinning is at every home with its ends reaching each and every section of civilization.

In a recent past experience on this initiative of Online Mentoring Session (OMS) for students of a tribal district deserve consideration. This too was as a part of this initiative, driven with PSR, which was volunteered on non-remunerative, non-commercial and non-political basis in the spirit of this mission. In a school, specially government school, there were many distractions causing discontinuity, loss of focus, and consequent lack of consistency in mentoring. These factors are serious retardants in building competence which required intensive mentoring outside school curriculum to provide a vision, extend guidance and finger holding to develop competence to compete, among target students. Accordingly, it was suggested that these students could be mentored at Two hostels, separate for boys and girls, before and after school hours and even on holidays. This need had become essential based on experience of One Year of engagement with a school identified by district administration. Therefore, a One-O-One discussions were suggested with the incumbent, who himself has a brilliant career as an IITian and a young dynamic IAS, alongwith other stake holders, as deemed fit, to resolve the issues. Despite aggressive persuasion, for start and stabilization of OMS, since beginning inability of the incumbent to provide a time slot for Video Conference, for all valid reasons, even during a visit which was scheduled in consultation with him a dissatisfaction had to be expressed stating that- "*Unless educational issues, at its basics are addressed with deserving priority, such law-&-order situation would continue to engage all in fire-fighting AND provide fodder to trouble creators, and all efforts would entail nothing more than Window-Dressing./Para/ May please like to consider views of this non-entity person whose concern is out of Personal Social Responsibility and an effort to correct to the extent he is allowed.*" This communication had to made to the incumbent who had in the beginning had responded to the proposal of Online Mentoring as "***Excellent***".

This raises a serious question not on any individual but on the systemic issues to be answered by every elite person who can make a difference - ***is it not injudicious of a responsible person holding high public office to judge and welcome a proactive, voluntary and selfless participation in pursuance of his roles and duties?***

Despite continuous, consistent and desperate efforts to reach target students for the last five years, it is yet to reach a tipping point. It would be a dishonesty to deny a deep disappointment after every setback. But, a group of four, working selflessly on the mission, has been able to recover and emerge, in a short time, to retake a plunge into the cause with a doubly reinforced *resolve. It is believed that **Almighty** would definitely do his job, and someone like देवदूत would forward to carry this mission forward to connect to target children, and it continues…*

Anshuman Singh a young, dynamic, Supreme Court Lawyer has opined that - "In democracy, every power and every authority is conferred only for discharge of a public duty. If only the stakeholders of the country *could realize that power and authority are enjoined with responsibility to do public welfare, most conflicts of interest would end. Discretion in exercise of power is the corner stone of our democratic setup. Once this discretion is transgressed, democracy crumbles. It is for this reason the constitution of India makes clear separation of powers and duties among public functionaries."*

It is essential to appreciate that justice is all about striking a balance between expectations and responsibility of beneficiary and victim(s), which can be individuals or groups. It is not confined to any particular office rather sense of justice has to introspected by every individual. This reminds of a mythological story of Mahabharat, when prince Duryodhan and Yudhisthir were to be evaluated for succession of crown, Four persons, one Barhmin, One Khastriya, One Caishya and One Shoodra were presented grant of justice for jointly committing a dacoity and murder. Judgement passed on by Yudhistir of descending severity based on clan of the four criminals was upheld. Moral of the story is that - *none, specially elite who are blessed with competence to make a difference, cannot and should not be allowed to remain complacent. They have to pro-act, beyond their roles and duties with a sense of PSR.*

Inability to democratize education, is a matter of deep concern and root cause of all the problems. As per principles of Law *role of judiciary is implement law made by Lawmakers, and not to encroach upon their wisdom so long law is not made by incompetent legislator and there is no violation of constitution.* In democracy the law makers emerge from the polity, where every citizen is a stake holder. Unless, wisdom of justice begins at home, contradictions would prevail everywhere to engage everyone in fire-fighting and window dressing, keeping the injustice brewing within.

Society is as complex as human body, and every individual has his own set of competence with certain limitations and none replace the other in totality. Once a judicious introspection is made by everyone to review one role in pursuance of culture and morality. It needs to be believed that there is a role for everyone to collectively complement in furtherance of culture, morality and pursuit of justice. Methods, modes and extent can remain a matter of choice for every individual so long one abides to spirit of PSR.

Culture and morality of a society or a country need to pursue principles of natural justice to all and implement necessary changes on observation of any ill-effect. If this becomes a reality, a culture would emerge where descends would carry a pride of perpetuating a legacy of self-restraint, self-respect, harmony and peace, an endowment of mother-nature.

This initiative, at the outset is very open to all to know, change, modify or take away any and everything out of it with one simple request- "***please do not dump the proposition of the mission it blindly and deafly, because it is neither a philosophical statement, or sermons or preaching nor a talk of either of a frustrated mind or a person with his belly full. It is a desperation to act with a sense of PSR, to impart education in real sense to those whom it is a dream in luxury. It is totally selfless and devoid of expectation whatsoever.***"

If it really happens it would lead to रामराज्य. In this situation, persons occupying high positions whatsoever, might feel endangered of being rendered redundant. Though concept of रामराज्य is an ideal but, and as per laws of mathematics, science and human experience growth in demand and expectations of excellence in education, execution, administration and governance rise exponentially with the progress. Therefore persons perpetuating positive culture, morality and justice would always remain in high demand.

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A hundred times a day I remind myself that my inner and outer life are based on the labors of other men, living and dead, and that I must exert myself in order to give in the same measure as I have received and am still receiving.

-Albert Einstein

OVERCOMING FAILURES

Sandhya Tanwar

It is very important to encounter failures in life, they teach you the meaning of real success. One who has never failed in life, is actually unknown to the true meaning of success. Actually, failure is not the opposite of success, it is a part of success.

Only, when you fail in life, you get to know, what it is to be done differently, then you start applying your mind into a different direction with new permutations and combinations. Success is actually the outcome of hardships made into a project.

We should treat failures as another opportunity to give our 100% to a particular task, which was missed in the earlier attempt.

I have overcome failure by doing it again and trying not to make the same mistake. Recently, I felt like I have failed in my life and was totally devastated. Then, while introspection, I realized that this failure was so important for me, because, had this not come, I would have never realized that I need to adopt a different perspective towards life. I took that failure as an opportunity towards beginning of a new life. And I welcomed it with a feeling of starting a new life afresh and this time with a positive outlook.

Everyone was surprised to see a new me. The person who has never failed in life is happy with the new failure of his/her life.

It's all about our mindset, we become the way we think. It is very important to take a pause and think what actually we are doing, is it giving us mental peace and happiness or not.

The reason for writing about this topic "Overcoming Failures" is that I want to convey a message to all the students, that never ever feel sad about your failures be it the failure to attain good marks, to secure first position in the class etc. You will not realize when this sadness would turn into depression and ruin your peace of mind and will ultimately kill your happiness. In fact, treat failures as an opportunity to perform better next time. The true success is overcoming the fear of being unsuccessful. To share another instance of my recent failure, I had participated in a dance competition. For just a 3 minutes performance, I have practiced multiple number of times. I was so confident that I will surely get through this selection process and will perform in round 2. Unfortunately, I wasn't get selected. Any human being who face the failure will become upset but I was happy that I wasn't selected, because somewhere I knew that I have missed giving my 100% percent. I clearly remembered, where I became totally blank in the third turn of performance, maybe because the whole focus was on me and I got little conscious. So I accepted the decisions of judges with full respect.

What I have learnt from this failure is that we all know where we lacked the excellence. Sometimes, we feel that we are outstanding but from the point of view of judges, someone else has done better than us without missing anything. So we should respect the decision and strive to work hard to do excellent by learning from our previous mistakes. Failure is simply the opportunity to begin again, this time more intelligently and more wisely.

At 30 years old, Steve Jobs was left devastated and depressed after being removed from the company he started. He overcame all obstacles to become successful and Apple products speaks the rest.

Walt Disney, was fired by the editor of a newspaper because he lacked imagination and had no good ideas. Today, he is one of most successful people in this world.

You try you fail, you again try, you again fail, the real failure is when you stop trying. So try until you succeed. Never, Give Up. Failure come to make us a better person. Just believe in yourself and all that you are. Know that there is something inside you that is GREATER than any obstacle. Always remember, if you have never failed, you have never tried anything new.

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GROWING WITH CONCEPTS

Concepts of an expert are not like a static foundation of a huge structure; rather it is like blood flowing in a vibrant mind.

*During growing into an expert, each one must have used best of the books available on subject and received guidance of best of the teachers. Authors might have had limitations to take every concept thread bare from first principle and so also must be the constraint of teacher while mentoring a class with a diversity of inquisitiveness and focus. As a result, there are instances when on a certain concept a discomfort remains. The only remedy is to live with the conceptual problem and continue to visualize it thread bare till it goes to bottom of heart and that is an **ingenious illustration**.*

In this column an effort is being made to take one topic on Mathematics, Physics and Chemistry in each e-Bulletin and provide its illustration from First Principle. We invite all experts in these subjects to please mail us their ingenious illustrations and it would be our pleasure to include it in the column.

We hope this repository of ingenious illustrations, built over a period of time, would be helpful to ignite minds of children, particularly to aspiring unprivileged students, that we target in this initiative, and in general to all, as a free educational web resource.

This e-Bulletin covers – a) Mathematics, b) Physics , and c) Chemistry. This is just a beginning in this direction. These articles are not replacement of text books and reference books. These books provide a large number of solved examples, problems and objective questions, necessary to make the concepts intuitive, a journey of educational enlightenment.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics it is contemplated to come up representative problems from contemporary text books and Question papers from various competitive examinations and a guide to theirsolutions in a structured manner, as a dynamic exercise to catalyse the conceptual thought process.

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OUR MENTORING PHILOSOPHY: Mentoring is not teaching, neither tuition nor coaching. It is an activity driven by passion, and commerce has no place in it. In this effort is to caution students that -

- This place is not where they will be taught how to score marks and get higher ranks, but to conceptualize and visualize subject matter in their real life so that it becomes intuitive.
- This place is not to aim at solutions but inculcate competence to analyze a problem and evolve solution.
- This place does not extend selective and personalized attention, rather an opportunity to become a part of which is focused on learning and problem solving ability collectively.
- This place provides an opportunity to find students above and below one's own level of learning. Thus students develop not in isolation but learn from better ones and associate in problem solving to those who need help. This group dynamics while create a team spirit, an essential attribute of personality, while one learns more by teaching others.
- This place has strategically chosen Online Mentoring, so that those who are unprivileged can gather at one point and those who can facilitate learning of such students by creating, necessary IT setup. Aseperate **Mentor's Manual** is being developed to support the cause.

We are implementing this philosophy through **Online Mentoring**

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GROWING WITH CONCEPTS – Mathematic

MATRICES AND DETERMINANTS

Prof. SB DHAR

Matrix is a simple method to solve equations involving many variables. Matrix is a method to write data or symbols in rows and columns. It has no numerical value. It is simply an arrangement of data. A British Mathematician Cayley is the Inventor of Matrix Algebra.

Way of writing a matrix

If we have some data ***a,b,c,d,e,f,g,h,i***, we can arrange them as below in matrix form:

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

The matrix is denoted by capital letters as ***A*** and (a b c), (d e f), (g h i) i.e. the horizontals are called ***rows*** and verticals $\begin{pmatrix} a \\ d \\ g \end{pmatrix}, \begin{pmatrix} b \\ e \\ h \end{pmatrix}, \begin{pmatrix} c \\ f \\ i \end{pmatrix}$ are called the ***columns***.

Order of a Matrix

If in a matrix there are m rows and n columns then its order is $m \times n$ and is read as ***m by n***.

i.e. ***#Rows × #Columns*** (where # means number of). The value of the product ***(m × n)*** is equal to the number of elements in the matrix.

For example, the above matrix A has 4 rows and 4 columns and has total 4 × 4 i.e. 16 elements.

Notes:

(i) Matrix has no arithmetical value; it is simply an arrangement of elements. The elements may be arithmetical numbers but matrix as whole will remain an arrangement.

(ii) If the matrix has all its elements as 0 and is of order 4 × 4, the number of elements shall be 16.

Equality of Two Matrices

(i) Two matrices are said to be equal if they are of the same order and have the same corresponding elements for example:

(ii) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ are two matrices and **A=B** then **a=1, b=2, c=3** and **d=4**

(iii) Different orders matrices cannot be equal.

Types of Matrix

(i) **Row matrix:** Matrix having only one row as $A = (a_{11} \quad a_{12} \quad a_{13})$

Or, $A = \lfloor a_{ij} \rfloor_{m \times n}$ is a row matrix if m=1

(ii) **Column matrix**: Matrix having only one column as $B = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}$ Or, $A = \lfloor a_{ij} \rfloor_{m \times n}$ is a column matrix if n=1

(iii) **Horizontal Matrix**: Matrix where # rows < # columns as

$$C = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

(iv) **Vertical Matrix**: Matrix where # columns < # rows

$$D = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}$$

(v) **Singleton Matrix**:

Matrix having only one element i.e. $E = (a_{11})$ or a matrix of order 1×1.

(vi) **Null or Zero Matrix**: Matrix whose all elements are zero i.e. $O=\begin{pmatrix}0 & 0\\0 & 0\\0 & 0\end{pmatrix}$

A null matrix may be square or non-square i.e. it may be of any order.

(vii) **Square Matrix**: The matrix that has equal number of rows and columns.

i.e. 1×1, 2×2, 3×3, 4×4 etc.

(viii) **Upper Triangular Matrix**: A square matrix that has non - zero elements in principal diagonal and above it and all other elements are zero i.e. $F=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\0 & a_{22} & a_{23}\\0 & 0 & a_{33}\end{pmatrix}$

(ix) **Lower Triangular Matrix**: A square matrix that has all zero elements above principal diagonal i.e. $G=\begin{pmatrix}a_{11} & 0 & 0\\a_{21} & a_{22} & 0\\a_{31} & a_{32} & a_{33}\end{pmatrix}$

(x) **Diagonal Matrix**: A square matrix in which all the elements are zero except the principal diagonal elements i.e. $H=\begin{pmatrix}a_{11} & 0 & 0\\0 & a_{22} & 0\\0 & 0 & a_{33}\end{pmatrix}$

(xi) **Scalar Matrix**: A diagonal matrix whose all elements are equal to some scalar k i.e.

$$J=\begin{pmatrix}k & 0 & 0\\0 & k & 0\\0 & 0 & k\end{pmatrix}$$

(xii) **Identity Matrix**: A scalar matrix that has all its elements as 1 i.e. $I=\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$

It is always denoted by I. It is always a square matrix.

Important Terms relating to Matrix

(a) **Principal Diagonal**: Only a square matrix has principal diagonal. It is the diagonal stretched from top left to down right. For example in matrix H elements $(a_{11} \quad a_{22} \quad a_{33})$ form principal diagonal.

(b) **Trace of Matrix**: The arithmetical sum of the elements of the principal diagonal is called the trace of a matrix and is denoted by Trace of A.

For example:

The trace of the matrix A is written as **Tr (A)=** $a_{11}+a_{22}+a_{33}+a_{44}$.

(1) The trace may be any data.

(2) Trace (A+B) = Tr A + Tr B

(3) Tr (kA)= k (Tr A)

(4) Tr A'= Tr A

(5) Tr I_n = n

(6) Tr O = 0.

(7) Tr (AB) ≠ (Tr A). (Tr B) (in general)

Operation on Matrix

(i) **Addition of Matrices:** The matrices whose sum is being done must be of same order. The sum is the sum of corresponding places elements. For example: if

$A=\begin{pmatrix}a & b\\c & d\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}$ then the sum of the two matrices shall be

$$A+B=\begin{pmatrix}a+1 & b+2\\c+3 & d+4\end{pmatrix}$$

Properties of Addition:

(a)A+B=B+A i.e. commutative

(b) A+ (B+C) = (A+B) +C i.e. associative

(ii) **Difference of two matrices:** Same as in addition where corresponding elements are added, here the corresponding elements are subtracted from A's to B if A-B is required i.e.

$$A-B=\begin{pmatrix}a-1 & b-2\\c-3 & d-4\end{pmatrix}$$

(iii) **Multiplication of a matrix by a scalar:** If k is a scalar then kB of the above matrix will be

$kB = \begin{pmatrix} 1k & 2k \\ 3k & 4k \end{pmatrix}$ i.e. all the elements of the original matrix are multiplied by the same scalar k.

(iv) **Multiplication of a matrix by another matrix**: If A is a matrix of order m×n and B is a matrix of order n×p then product AB is possible i.e. the product is possible only when the #columns in 1st matrix=# rows in 2nd matrix and the resultant i.e. product is of order m×p i.e. #row of 1st × # columns of 2nd. It is done as follows:

$A = \begin{pmatrix} a & b & c \\ e & f & g \end{pmatrix}$ And $B = \begin{pmatrix} x & y \\ z & u \\ v & w \end{pmatrix}$ then AB's order will be 2×2 and AB is possible as A is of order 2×3 and B is of order 3×2 hence the products order will be 2×2.

$$AB = \begin{pmatrix} ax+bz+cv & ay+bu+cw \\ ex+fz+gv & ey+fu+gw \end{pmatrix}$$

Note:

In matrix multiplication,

(a) if AB is possible then BA may or may not be possible.

(b) If AB and BA are possible, even then AB and BA may or may not be equal.

(c) If AB=0 then it is possible that either A=0 or B=0 or both may not be zero separately.

(v) Since matrix has no arithmetical value, hence division of one matrix by another matrix is meaningless.

Some Special matrices

1. **Transpose of a Matrix :**A transpose matrix is a matrix obtained by changing the rows into columns of the original matrix. It is denoted by $A' or A^T$.

For example: the transpose of $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}_{2\times3}$ is $A' or A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}_{3\times2}$

Properties of a Transpose Matrix:

(i) (A')'= A

(ii) (kA)'=kA'

(iii) (A+B)'=A'+B'

(iv) (AB)'=B'A'

2. **Adjoint (or Adjugate) of a Matrix:** Adjoint of a matrix is the matrix obtained by the transpose of the matrix of the cofactors of the original matrix. It is always a square matrix.

If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, then $AdjA = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$

where, C_{ij} are the cofactors of a_{ij}.

Example:

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then $C_{11} = 4, C_{12} = -3, C_{21} = -2, C_{22} = 1$

The matrix with cofactors

$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix}$ Then transpose of C is the Adjoint of A i.e.

$$Adjoint(A) = C^T = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

3. ***Properties of Adjoint matrix:***

(a) $|AdjA| = |A|^{n-1}$ Where n is the order of the matrix.

(b) $|Adj(AdjA)| = |A|^{(n-1)^2}$

(c) Adj (AB) = (AdjB) (AdjA)

(d) $AdjA^m = (AdjA)^m$

(e) $Adj(kA) = k^{n-1}(AdjA)$

(f) Adj I=I

(g) If A is singular then $|AdjA| = 0$

(h) $Adj(AdjA) = |A|^{n-2} A$

(i) $AdjA' = (AdjA)'$

4. **Singular Matrix:** The square matrix, whose determinant is zero, is called Singular matrix.

   For example: $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ is a singular matrix as $|A| = 0$

5. **Non-singular Matrix:** The square matrix whose determinant is not zero is called non-singular matrix. I.e. if $|B| \neq 0$

6. **Symmetric Matrix:** A square matrix is said to be a symmetric matrix if A'=A.

7. **Skew Symmetric Matrix**

   A square matrix is said to be skew-symmetric if $A' = -A$

   (a) Diagonal elements of a skew-symmetric matrix are zero.

   (b) Every square matrix can be written as a sum of a symmetric and a skew-symmetric matrix. i.e.
   $$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

   (c) If A, B are symmetric matrices of same order then AB is also a symmetric matrix.

   (d) B'AB is a symmetric or skew-symmetric as A is symmetric or skew-symmetric.

   (e) All integral powers of a symmetric matrix are symmetric.

8. **Conjugate of a Matrix:** A conjugate of a matrix is a matrix formed with the conjugates of the corresponding elements of the original matrix. For example,

   If $A = \begin{pmatrix} a - ib & 3i \\ a & -4i \end{pmatrix}$,then $\overline{A} = \begin{pmatrix} a + ib & -3i \\ a & +4i \end{pmatrix}$

9. **Conjugate Transpose of a Mtrix:** Is a matrix obtained by the transpose of the conjugate of the original matrix i.e?

   $\overline{A}' = \begin{pmatrix} a + ib & a \\ -3i & 4i \end{pmatrix}$. It is also denoted by A^{θ}.

10. **Hermitian Matrix:** If $A^{\theta} = A$, the matrix A is called a Hermitian matrix.

11. **Skew-Hermitian Matrix:** If $A^{\theta} = -A$, the matrix A is called a skew-Hermitian matrix.

    Note: Every square matrix is uniquely expressible as the sum of a Hermitian and a skew-Hermitian matrix.

12. **Orthogonal Matrix:** If $AA' = I$ then the matrix A is called an orthogonal matrix.

13. **Idempotent Matrix:** If $A^2 = A$, the matrix A is called an Idempotent matrix.

14. **Involuntary Matrix:** If $A^2 = I$ then the matrix A is called an involuntary matrix

15. **Nilpotent Matrix:** If $A^p = 0$ (but $A^{p-1} \neq 0$) then the square matrix A is called the Nilpotent matrix of nil potency p (or order p).The order of nilpotency is different from the order of the matrix A.

16. **Unitary Matrix:** If $A^{\theta} A = I$, the matrix A is called a Unitary matrix.

17. **Periodic Matrix:** If $A^{k+1} = A$ then the matrix A is called the periodic matrix of period k.

18. **Inverse of a Matrix:** Inverse of a matrix is possible only when it is a square matrix and non-singular i.e. determinant of the matrix is not zero.

    It is given by $A^{-1} = \frac{AdjA}{|A|}$

    (a) Inverse is unique, i.e. if A is an inverse of B then B is the inverse of A.

    (b) If A and B are invertible matrices of same order then $(AB)^{-1} = B^{-1}A^{-1}$

(c) If A is invertible then transpose of A is also invertible i.e. $(A^T)=(A^{-1})^T$

19. Rank of a matrix

(a) Rank of a matrix is the order of the matrix or the sub matrix formed with the elements of the original matrix that is non-singular.

(b) It cannot be less than 1. i.e. it is always positive integer.

(c) Order of the null matrix is not defined as there exists no sub matrix which is non-singular. But some writer presume it to be 0 which is wrong.

(d) It is denoted by ρ(A).

(e) If ρ(A)= ρ(B)=n then ρ(AB)=n where A and B are square matrices of order n.

(f) Rank of a matrix whose all elements are unity is 1.

(g) Every skew symmetric matrix of odd order has rank less than its order.

(h) The rank of a non-null matrix is always greater than or equal to 1.

(i) Elementary transformations do not change its rank.

(j) **Echelon Form of a Matrix:** A non-zero matrix A is said to be in Echelon form if A performs good in following tests:

(a) All the non-zero rows of A, if any precede the zero – row.

(b) The number of zeros preceding the first non-zero element in a row is less than the number of such zeros in the succeeding row.

(c) The first non-zero element in a row is unity

Note1: The number of non-zero rows of a matrix given in the Echelon form is its rank.

For example: The matrix , $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, by the definition it is obvious there are 3 non-zero rows that precede the 4th row.

Number of zeros in R2, R3, and R4 are 1, 2, and 4 that are in ascending order.

The first non-zero element is unity.

Rank= three non-zero rows

Note2: The matrix B obtained from matrix A by a finite number of elementary row (or column) operations is called a matrix **equivalent** to the matrix A and is written as B ~ A.

Note3:A matrix obtained by the application of any one of the elementary row (or column) operation to the identity matrix is called an **elementary** row (or column) matrix.

(d) Solution of system of linear equations by Rank Method: If AX=B is a system of linear equations in n variables. Then the following working rule is adopted to find solutions:

(e) If ρ(A) ≠ρ(A B), then the system of equations is inconsistent.

(f) If ρ(A) = ρ(A B) = the number of unknowns, then system of equations is consistent and has a unique solution.

(g) If ρ(A) = ρ(A B) < the number of unknowns, then the system of equations is consistent and has infinitely many solutions.

(h) In case of homogeneous system of equations AX=O

(i) If ρ(A) = number of variables, then it has a trivial solution.

(j) If ρ(A) < number of variables, then it has a non-trivial solution, i.e. infinitely many solutions.

(k) Uses of matrix in solving linear equations: homogeneous and non-homogeneous

Uses of Matrix in solving Linear equations: Linear Equations are of two types –

i. Homogeneous

ii. Non-homogeneous

Let us consider the set of equations:

$$a_{11}x + a_{12}y = b_{11}$$

$$a_{21}x + a_{22}y = b_{21}$$

1. If all of b_{11}, b_{21} are zero, the system of equations is called **homogeneous**.

   $a_{11}x + a_{12}y = 0$

   $a_{21}x + a_{22}y = 0$

   For homogeneous system of equations D must be zero because D1, D2 are already zero.

   $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, D_1 = \begin{vmatrix} 0 & a_{12} \\ 0 & a_{22} \end{vmatrix} = 0,$

   $D_2 = \begin{vmatrix} a_{11} & 0 \\ a_{21} & 0 \end{vmatrix} = 0$

2. Matrix use-Solution of the equation is written as

   AX=B where $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is the matrix formed by the coefficients of x and y.

   $X = \begin{pmatrix} x \\ y \end{pmatrix}$ is a matrix formed with variables.

   And $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is a matrix of numeral on right sides of equations.

3. **Consistency and Inconsistency**

   (a) The solution of the equations is given by $X = \frac{AdjA}{|A|}B$

   (b) The system of equations is consistent and has infinitely many solutions if $(AdjA)B = 0$ and $|A| = 0$

   (c) The system of equations is inconsistent if $(AdjA)B \neq 0$ and $|A| = 0$

   (d) If $|A| \neq 0$, then the homogeneous system (B=O always), AX=O has only the trivial solution i.e. x=0, y=0.

   (e) For homogeneous system AX=O, to have a non-trivial solution i.e. not all zero, $|A| = 0$ (must)

   (f) For non-homogeneous system of equations i.e. AX=B, (B≠0) has a unique solution if $|A| \neq 0$ and the solution is given by $X = A^{-1}B$.

   Notes:

   (i) **Consistent**: if solution exists whether unique or many

   (ii) **Inconsistent**: if the solution does not exist.

   (g) Non-homogeneous system of equations:

   If at least one of b_{11}, b_{21} is not zero, then it is called **non-homogeneous** equations.

4. **Characteristic equation of a matrix**

   (a) If A is a square matrix then |A-x **I**|=0 is the characteristic equation of A.

   For example:

   If $x^3 - 4x^2 - 5x - 7 = 0$ is the characteristic equation of A then $A^3 - 4A^2 - 5A - 7I = O$.

   (b) The roots of this equation are called the characteristic roots or characteristic values or Eigen values or latent roots of A.

   (c) The set of the Eigen values of the matrix A is called the spectrum of the matrix A.

   (d) Any matrix A and its transpose both have the same eigen values.

   (e) The trace of the matrix is always equal to the sum of the eigen values of a matrix.

   (f) The determinant of the matrix A is equal to the product of the eigen values of A.

   (g) If $\lambda_1, \lambda_2, \lambda_3, \ldots.\lambda_n$ are the n-eigen values of A, then eigen values of kA are $k\lambda_1, k\lambda_2, k\lambda_3, \ldots., k\lambda_n$.

   (h) If $\lambda_1, \lambda_2, \lambda_3, \ldots.\lambda_n$ are the n-eigen values of A, then eigen values of A^m are $\lambda^m_1, \lambda^m_2, \lambda^m_3, \ldots., \lambda^m_n$.

(i) If A and P are square matrices and if P is invertible then matrices A and $P^{-1}AP$ both have the same characteristic roots.

(j) 0 is the characteristic root of a matrix if and only if the matrix is singular.

(k) If A and B are two square invertible matrices then AB and BA have the same characteristic roots.

(l) The characteristic roots of a triangular matrix are just the diagonal elements of the matrix.

(m) All the characteristic roots of a Hermitian matrix are real.

(n) Characteristic roots of a real symmetric matrix are all real.

(o) Characteristic roots of a skew Hermitian matrix is either zero or a pure imaginary number.

(p) If λ is an eigen value of an orthogonal matrix, then (1/λ) is also an eigen value.

(q) Every square matrix satisfies its characteristic equation. (**Cauchy-Hamilton Theorem**)

(r) If λ is a root of A then λ is also root of A^{-1}.

(s) If λ is a root of A then λ^{-1} is characteristic root of A^{-1}.

Note 1: There is no name of the matrix: $\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$ nether it is unit nor unitary.

Note 2: If A is a square matrix of order n, then Maximum number of distinct entries if A is a triangular matrix= {n(n+1)/2} +1.

Minimum number of zeros if A is triangular matrix=n(n-1)/2

Maximum number of distinct entries if A is a diagonal matrix=n+1

Minimum number of zeros if A is a diagonal matrix=n(n-1)

DETERMINANTS

(1) Gottfied Wilhelm Leibnitz is treated as the Inventor of Determinant.

(2) Determinant is a method of writing n x n quantities in an array form. It has a fixed value.

(3) It is represented by a Capital letter of English Alphabet.

(4) If ax + by =0, cx + dy =0 then after elimination of x and y or writing the coefficients of x and y in the following form is called a determinant: $\begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad - bc$

(5) Order of a determinant= (number of rows) x (number of columns). For example if it has 2 rows and 2 columns then the order = 2 x 2 .

(6) The elements forming parallel lines are called Rows and the vertical lines are called the columns.

(7) The determinant of a lesser order is called the Minor. Minor is a determinant formed by excluding the row and column of the element in which it exists.

(8) The Principal diagonal is the line joining the elements a_{11} to a_{nn} if the determinant has n rows and n columns.

(9) The value of the determinant remains unchanged if the rows are changed into columns or vice-versa.

(10) If the two rows or columns are interchanged, the determinant is multiplied by (-). If the same process is repeated again, the determinant is multiplied by (-) (-) i.e. $(-)^2$.

(11) If two rows or columns of a determinant are identical i.e. all constituents are same, the value of that determinant is zero.

(12) Determinant of a skew symmetric matrix of odd order is always zero.

(13) If all the constituents of a row or column of a determinant are zero then the value of that determinant is zero.

(14) If all the constituents of a determinant above or below the principal diagonal are zero then the value of the determinant is equal to the product of the principal diagonal constituents.

(15) Determinant of a diagonal matrix is product of their diagonal elements.

(16) $|I_n| = 1$

(17) $|O_n| = 0$

(18) Non- square matrix has no determinant.

(19) Conjugate of a determinant is the determinant formed by the conjugates of all constituents.

(20) If A and B are square matrices of same order then $|AB|= |A|\,|B|$ and $|A^n| = |A|^n$

(21) If one row or one column is multiplied by some scalar quantity k, the value of the determinant gets multiplied by k i.e. if each row or column of a matrix of order 3 x 3 is multiplied by a scalar k then the value shall be multiplied by k.k.k i.e. k^3.

(22) If one row or column is sum of difference of two terms then the determinant can be represented as the sum or difference of two determinants as follows:

$$\begin{vmatrix} a_{11} \pm \alpha & a_{12} \pm \beta \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \pm \begin{vmatrix} \alpha & \beta \\ a_{21} & a_{22} \end{vmatrix}$$

(23) The determinant may be of any order but the number of rows must be equal to the number of columns.

(24) The determinant has a fixed order of + and – signs as below: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$ i.e. alternate (+) and (-) starting from the upper extreme left.

(25) The determinant is written between two parallel lines.

(26) The value of a determinant is calculated by expanding it along a row or column according to Laplace Law as below: if $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

then

$$A = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$= a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}$

$= a_{11}A_{11}-a_{12}A_{12}+a_{13}A_{13}$.

(27) C_{11}, C_{12}, C_{13} are called the Cofactors and A_{11}, A_{12}, A_{13} are called the Minors. Minors are all with plus sign while cofactors are always with proper corresponding signs of the elements.

(28) A common factor of any row (or column) may be taken outside of the determinant. In other words if all the elements of one row (or column) is multiplied by a non-zero number; 'K' then the value of new determinant is K times the value of original determinant.

(29) If determinant becomes zero on putting x = α, then we say that (x-α) is a factor of the determinant.

(30) The value of determinant is unaffected when any row (or column) is multiplied by a number or any expression and then added or subtracted from any other row (or column).

(31) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

(32) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(33) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^4 & b^4 & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)$

$= \{a^2+b^2+c^2+ab+bc+ac\}$

(34) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$.

$= (a-b)(b-c)(c-a)(ab+bc+ac)$

(35) $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

$= (\alpha+\beta+\gamma)$

$$\left\{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\right\}$$

(36) To differentiate a determinant, we differentiate one row (or column) at a time, keeping other unchanged.

Multiplication of two determinants can be done in 4 ways.

(i) Row by row multiplication

(ii) Row by column multiplication

(iii) Column by column multiplication

(iv) Column by row multiplication

(37) To express a determinant as product of two determinants, you have to require a lot of practice. This can be done only by inspection.

(38) System of linear equations is said to be consistent if it has at least one solution.

(39) System of linear equations is in consistent if it has no solution.

(40) System of linear equation in two variable x and y

$$a_1x+b_1x+c_1=0,$$

$$a_2x+b_2x+c_2=0,$$

$$a_3x+b_3x+c_3=0$$

is consistent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

(41) System of linear equations in 3 variables x, y, & z are -

$$a_1x+b_1y+c_1z=d_1,$$

$$a_2x+b_2y+c_2z=d_2,$$

$$a_3x+b_3y+c_3z=d_3,$$

if $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$

$$\Delta_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},$$

if $\Delta\neq 0$, then, $x=\frac{\Delta_x}{\Delta}, y=\frac{\Delta_y}{\Delta}, z=\frac{\Delta_z}{\Delta}$ and system of equations is called consistent and has unique solution.

(42) System of linear equations in two variables x and y

$$a_1x+b_1x+c_1=0,$$

$$a_2x+b_2x+c_2=0,$$

$$a_3x+b_3x+c_3=0$$

is consistent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

(43) System of linear equations in 3 variables x, y, & z is -

$$a_1x+b_1y+c_1z=d_1,$$

$$a_2x+b_2y+c_2z=d_2,$$

$$a_3x+b_3y+c_3z=d_3$$

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

(i) if $\Delta\neq 0$,) then, $x=\frac{\Delta_x}{\Delta}, y=\frac{\Delta_y}{\Delta}, z=\frac{\Delta_z}{\Delta}$ is called consistent and has unique solution

(ii) if $\Delta=0$ but at least $\Delta_x, \Delta_y, \Delta_z\neq 0$. Then the system of equations has no solution, (Inconsistent solution).

(iv) if $\Delta=0=\Delta_x=\Delta_y=\Delta_z$. then the system of equations is consistent and has infinitely many solutions.

Value of Determinant:

(i) The value of determinant made by the matrix of order 1 x 1, A= [a] is denoted by

|A|= a.

(ii) The value of determinant made by the matrix of order 2 x 2 is-

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Cofactors and Minors

Minors: In a determinant like A given under

$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, the value is given by -

$A = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$, then

$$A = a_{11}A_{11} - a_{12}A_{12} + a_{13}A_{13}$$

$$= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$0 = a_{11}A_{21} - a_{12}A_{22} + a_{13}A_{23}$$

$$= a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23}$$

i.e. the sum of the products of the elements of a row and the minors or cofactors of another row is always zero.

Note 1: A11, A12,A13,..are called Minors and C11,C12,C13 ...are called Cofactors.Cofactors are the minors with proper signs.

Note 2: cofactor $C_{ij} = (-1)^{i+j} A_{ij}$ where A_{ij} is the minor of the element of i^{th} row and j^{th} column.

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$,and $\Delta' = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ is a determinant of corresponding cofactors then $\Delta\Delta' = \Delta^3$ i.e. $\Delta' = \Delta^2$

Note: $\Delta' = \Delta^{n-1}$ where n is the order of the determinant i.e. if the determinant is of order 2 x 2 , then $\Delta' = \Delta$ and if it is of order 3 x 3, then $\Delta' = \Delta^2$.

Uses of determinant

(1) For solution of simultaneous linear equations (Cramer's rule)

Consider the set of Linear equations

$$a_{11}x + a_{12}y = b_{11}$$

$$a_{21}x + a_{22}y = b_{21}$$

If all of b_{11}, b_{21} are zero, the system of equations is called homogeneous. i.e.

$$a_{11}x + a_{12}y = 0$$

$$a_{21}x + a_{22}y = 0$$

For homogeneous system of equations where D_1, D_2 are zero.

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},\ D_1 = \begin{vmatrix} 0 & a_{12} \\ 0 & a_{22} \end{vmatrix} = 0,\ D_2 = \begin{vmatrix} a_{11} & 0 \\ a_{21} & 0 \end{vmatrix} = 0$$

(a) The solution is trivial i.e. x=y=z=0 if D≠0

(b) and non-trivial i.e. infinite solutions if D=0

(2) Area of triangles having vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

Is given by $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

(3) The three points are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(4) Equation of a line passing through two given points (x1, y1) and (x2, y2) is given $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

(5) Differentiation and Integration of a Determinant

If $F(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ d & e & f \end{vmatrix}$ is a determinant where f, g, h are functions or x and a, b, c, d, e, f are all constants then the derivative of F(x) is given by

$$\frac{d}{dx}F(x)=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ a & b & c \\ d & e & f \end{vmatrix}$$

And if the other rows are also functions of x then the required determinant will be the sum of the differentiated determinants of the rows keeping other constants. As under:

If $F(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ \phi(x) & \varphi(x) & \gamma(x) \\ d & e & f \end{vmatrix}$ then derivative of F(x) is given by, $\frac{d}{dx}F(x)=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ \phi(x) & \varphi(x) & \gamma(x) \\ d & e & f \end{vmatrix}$

$$+\begin{vmatrix} f(x) & g(x) & h(x) \\ \phi'(x) & \varphi'(x) & \gamma'(x) \\ d & e & f \end{vmatrix}$$

(6) Similarly, the Integration of F(x) of a single row variable is given by

$$\int F(x)dx=\begin{vmatrix} \int f(x)dx & \int g(x)dx & \int h(x)dx \\ a & b & c \\ d & e & f \end{vmatrix}$$

(7) Summation of determinant

If $F(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ d & e & f \end{vmatrix}$ then

$$\sum_{r=1}^{n}F(x)=\begin{vmatrix} \sum_{r=1}^{n} f(x) & \sum_{r=1}^{n} g(x) & \sum_{r=1}^{n} h(x) \\ a & b & c \\ d & e & f \end{vmatrix}$$

(8) **Symmetric determinant:** In which the elements situated at equal distance from the principal diagonal are equal in sign and magnitude. For example:

$$A=\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(9) Skew-symmetric determinant

Whose principal diagonal elements are all zero and elements at equal distance from the principal diagonal are equal in magnitude but different in sign.

Like, $A=\begin{vmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{vmatrix}$

(10) **Circulant**

$$A=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(11) Non-homogeneous system of equations

$D\neq 0$ $\Rightarrow$ unique solution, consistent

$D=0$, $D_1=D_2=D_3=0$ $\Rightarrow$ many solutions, consistent

$D=0$, at least one of D_1, D_2, $D_3\neq 0$ $\Rightarrow$ no solution, inconsistent

(12) **Homogeneous system of equations**:

$D_1=D_2=D_3=0$ (always)

$D=0$ $\Rightarrow$ many solutions, consistent

$D\neq 0$ $\Rightarrow$ unique solution, trivial solution, consistent

Solved Examples

1. Let a,b,c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

Solution:

Simplify the determinant

$=(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$

$= -(a+b+c)[(a-b)^2 + (b-c)^2+(c-a)^2]$

Since the expression in 2nd bracket is always positive being square and the first bracket is also positive hence the sign of the expression will be negative.

2. Find the number of solutions of the system of equations

$2x + y - z = 7$,

$x - 3y + 2z = 1$

$x + 4y - 3z = 5$

Solution:

Note the value of the determinant made by the coefficients $= \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{vmatrix} = 0$

And at least one of the $D_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ 5 & 4 & -3 \end{vmatrix} \neq 0$, Hence the given system has no solution.

3. If $p \neq a$, $q \neq b$, $r \neq c$ and $\begin{vmatrix} p & b & c \\ p+a & q+b & 2c \\ a & b & r \end{vmatrix} = 0$, then find the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$.

Solution: After applying R_2 by $R_2 - R_1$,

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

Now apply R_1 by $R_1 - R_3$, R_2 by $R_2 - R_3$,

$$\begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0$$

Taking (p-a)(q-b)(c-r) common from First, Second and Third columns respectively,

$$(p-a)(q-b)(c-r)\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{c-r} \end{vmatrix} = 0$$

Or, $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{a}{p-a} & \frac{b}{q-b} & \frac{r}{c-r} \end{vmatrix} = 0$, on expanding, we get the required value 2.

4. If $A = \begin{pmatrix} 2 & 2 \\ -3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ then find the value of $(B^{-1}A^{-1})^{-1}$.

Hint for the Solution:

Note the given expression is AB as $(AB)^{-1} = B^{-1}A^{-1}$ and $(A^{-1})^{-1} = A$, so $(B^{-1}A^{-1})^{-1} = AB$.

So just write the product of A and B matrices.

5. If A is an Invertible matrix, then evaluate the value of det (A^{-1}).

Solution: We know that $AA^{-1} = A^{-1}A = I$

$\Rightarrow |AA^{-1}| = |A^{-1}A| = |I| = 1 \Rightarrow |A||A^{-1}| = 1$

$\Rightarrow |A^{-1}| = 1/(|A|)$

6. For what conditions we can conclude B=C from the matrix equation AB = AC ?

Solution: We know that AB=AC

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C$

$\Rightarrow IB = IC \Rightarrow B = C$

But this is possible only when A is Invertible i.e. A is also a non-singular matrix i.e. its determinant is not zero.

7. Find a root of the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

Solution:

Obviously, x=0 is a root as it makes the given determinant a skew-symmetric matrix of odd order and the determinant of a skew symmetric matrix of odd order is zero.

$$\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}. \text{ Hence } \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Note: A skew-symmetric matrix is a matrix whose all the principal diagonal elements are necessarily zero and A'=-A

8. If A is a n x n matrix with real entries, then prove that the value of det($A^2 + I_n$)≥0.

Hint:

$A^2+I_n = (A+iI_n)(A-iI_n)$

Assume $\lambda_1,\lambda_2,\lambda_3,..,\lambda_n$ Eigen values of A then $(A+I_n)$ will have its Eigen values as λ_1+i, λ_2+i, $\lambda_3+i,....\lambda_n+i$.

And similarly, Eigen values of $(A-I_n)$ will be λ_1-i, λ_2-i, $\lambda_3-i,....,\lambda_n-i$.

$\Rightarrow$det $(A + I_n)= (\lambda_1+i)(\lambda_2+i)(\lambda_3+i)....(\lambda_n+i)$ and det $(A - I_n)= (\lambda_1-i)(\lambda_2-i)(\lambda_3-i)....(\lambda_n-i)$

Since A has real entries, its roots (i.e. eigen values) come in conjugate pairs.

Obviously $(\lambda_1-i)(\lambda_1+i)= \lambda_1^2+1=$ a real positive number. Hence the required value is always ≥ 0.

9. If $A = \begin{pmatrix} 1 & \frac{\alpha}{n} \\ -\frac{\alpha}{n} & 1 \end{pmatrix}$, then evaluate $\lim_{n\to\infty} \frac{1}{n} A^n$

Solution:

Rewrite the given matrix A as

$$A= \frac{1}{n}\begin{pmatrix} n & \alpha \\ -\alpha & n \end{pmatrix}$$

Assume for simplification purposes:

$n=r\cos\theta$, $\alpha= r\sin\theta$

$\Rightarrow n^2+\alpha^2= r^2$ and $\theta= \tan^{-1}(\alpha/n)$.

$$\Rightarrow A= \frac{1}{n}\begin{pmatrix} r\cos\theta & r\sin\theta \\ -r\sin\theta & r\cos\theta \end{pmatrix} = \frac{r}{n}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\Rightarrow A^n= \frac{r^n}{n^n}\begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}.$$

$$= \left(\frac{n^2+\alpha^2}{n^2}\right)^{n/2}\begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$$

$$\Rightarrow \frac{A^n}{n} = \left(1+\frac{\alpha^2}{n^2}\right)^{n/2}\begin{pmatrix} \frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ -\frac{\sin n\theta}{n} & \frac{\cos n\theta}{n} \end{pmatrix}.$$

Hence $\lim_{n\to\infty} \frac{1}{n} A^n =$

$$\lim_{n\to\infty}\left(1+\frac{\alpha^2}{n^2}\right)^{n/2} \lim_{n\to\infty}\begin{pmatrix} \frac{\cos n\theta}{n} & \frac{\sin n\theta}{n} \\ -\frac{\sin n\theta}{n} & \frac{\cos n\theta}{n} \end{pmatrix}.$$

$= 1.\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ =O, a null matrix.

As, $\lim_{n\to\infty}\left(1+\frac{\alpha^2}{n^2}\right)^{n/2} = 1$, and

$$\lim_{n\to\infty}\frac{\cos n\theta}{n} = \lim_{n\to\infty}\frac{\sin n\theta}{n} = 0.$$

10. If abc=p and $A = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$ and AA'=I then find the equation whose roots are a,b,c.

Solution:

Obviously,

$$AA'=\begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}\begin{pmatrix} a & c & b \\ b & a & c \\ c & b & a \end{pmatrix} =$$

$$\begin{pmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{pmatrix} = I\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

i.e. $a^2+b^2+c^2=1$, $ab+bc+ca=0$

since $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (a+b+c)=+1$ or -1

also $abc=p$

so the required equation is

$x^3-(a+b+c)x^2+(ab+bc+ca)x-abc=0$

i.e. $x^3 \pm x^2+0.x-p=0$

11. Find the coefficient of x in the expansion of $\begin{vmatrix} (1+x)^{22} & (1+x)^{44} & (1+x)^{66} \\ (1+x)^{33} & (1+x)^{66} & (1+x)^{99} \\ (1+x)^{44} & (1+x)^{88} & (1+x)^{144} \end{vmatrix}$.

Hint:

Obviously the determinant is a polynomial of degree 22+66+144=232

i.e. $f(x)= A_0 + A_1x + A_2 x^2 + A_3 x^3 +....+ A_{232} x^{232}$.

and A_1= coefficient of x = $f'(0)$, hence,

$$f'(x)=\begin{vmatrix} 22(1+x)^{21} & 44(1+x)^{43} & 66(1+x)^{65} \\ (1+x)^{33} & (1+x)^{66} & (1+x)^{99} \\ (1+x)^{44} & (1+x)^{88} & (1+x)^{144} \end{vmatrix}+$$

$$\begin{vmatrix} (1+x)^{22} & (1+x)^{44} & (1+x)^{66} \\ 33(1+x)^{32} & 66(1+x)^{65} & 99(1+x)^{98} \\ (1+x)^{44} & (1+x)^{88} & (1+x)^{144} \end{vmatrix}+$$

$$\begin{vmatrix} (1+x)^{22} & (1+x)^{44} & (1+x)^{66} \\ (1+x)^{33} & (1+x)^{66} & (1+x)^{99} \\ 44(1+x)^{43} & 88(1+x)^{87} & 144(1+x)^{143} \end{vmatrix}.$$

$$\Rightarrow f'(0)=\begin{vmatrix} 22 & 44 & 66 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}+\begin{vmatrix} 1 & 1 & 1 \\ 33 & 66 & 199 \\ 1 & 1 & 1 \end{vmatrix}+\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 44 & 88 & 144 \end{vmatrix}$$

=0

12. If, $\begin{vmatrix} a & \cot\frac{A}{2} & l \\ b & \cot\frac{B}{2} & m \\ c & \cot\frac{C}{2} & n \end{vmatrix}=0$ where a,b,c, A,B,C are the elements of a triangle ABC with usual meaning, then find the value of a(m-n) + b(n-l)+c(l-m).

Hint:

Use cot (A/2)=s(s-a)/Δ, cot (B/2)= s(s-b)/Δ, and cot (C/2)=s(s-c)/Δ, 2s=a+b+c, r=Δ/s, the usual notations.

Simplify to $\begin{vmatrix} a & s-a & l \\ b & s-b & m \\ c & s-c & n \end{vmatrix}=0=\begin{vmatrix} a & s & l \\ b & s & m \\ c & s & n \end{vmatrix}$.

And the required value of the expression after the expansion=0.

13. If a,b,c are real then find the interval in which, $f(x)=\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$ is decreasing.

Solution:

Simplify $f(x) = x^2(x+a^2+b^2+c^2)$

$\Rightarrow f'(x)=2x(x+a^2+b^2+c^2)+ x^2$

$\Rightarrow f'(x) < 0$ if $x\in (-2/3(a^2+b^2+c^2),0)$

14. If f(x) satisfies the equation $\begin{vmatrix} f(x-3) & f(x+4) & f((x+1)(x+2)-(x-1)^2) \\ 5 & 4 & -5 \\ 5 & 6 & 15 \end{vmatrix}=0$ for all x, then test whether it is a periodic function or not and if it is a periodic function , find its period.

Solution:

Note $(x+1)(x+2)-(x-1)^2=(x-3)$

Simplify the determinant to

$100f(x-3)-100f(x+4)=0 \Rightarrow f(x-3)=f(x+4)$

Replace x by x+3

$\Rightarrow f(x)=f(x+7)$ hence it is periodic with period 7

15. If $g(x)=\begin{vmatrix} a^{-x} & e^{x\log_e a} & x^2 \\ a^{-3x} & e^{3x\log_e a} & x^4 \\ a^{-5x} & e^{5x\log_e a} & 1 \end{vmatrix}$, then show that it is an odd function.

Hint:

Rewrite g(x)= $\begin{vmatrix} a^x & a^x & x^2 \\ a^{3x} & a^{3x} & x^4 \\ a^{5x} & a^{5x} & 1 \end{vmatrix}$.

Obviously g(-x)=-g(x) hence an odd function.

16. A skew symmetric matrics S satisfies the relation $S^2+I=0$, where I is an Identity matrics then show that S is an orthogonal matrix.

Hint: Note S is a skew symmetric matrix

$\Rightarrow S'=-S$, Also $S^2 = - I \Rightarrow S.S'=-I$

$\Rightarrow S.S.S'=-I.S'=I.S=S$

$\Rightarrow S^{-1}.S.S.S'=S^{-1}.S \Rightarrow (S^{-1}.S).S.S'=I$

$\Rightarrow I.S.S'=I \Rightarrow S.S'=I \Rightarrow$ S an orthogonal matrix.

17. If, f(x)= $\begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 f(x)dx = -16$, where a,b,c, d are in AP then find the common difference of the AP.

Hint: Obviously common difference is b-a=c-b=d-c= k (say)

Apply operators R3-R2,R2-R1, to find b-a,c-b,d-c

f(x)= $\begin{vmatrix} x+a & x+b & x+a-c \\ k & k & 2k-1 \\ k & k & 1+2k \end{vmatrix} = \begin{vmatrix} x+a & k & -c \\ k & 0 & k-1 \\ k & 0 & k+1 \end{vmatrix}$.

$=-2k^2$

$$\Rightarrow \int_0^2 f(x)dx = -16 \Rightarrow \int_0^2 -2k^2 dx = -16 \Rightarrow k = \pm 2$$

18. Express the determinant Δ= $\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix}$ as the product of two determinants and hence show that Δ=0.

Hint: Rewrite the given determinant as below:

$$\Delta = \begin{vmatrix} \cos^2\alpha+\sin^2\alpha & \cos\beta\cos\alpha+\sin\beta\sin\alpha & \cos\gamma\cos\alpha+\sin\gamma\sin\alpha \\ \cos\alpha\cos\beta+\sin\alpha\sin\beta & \cos^2\beta+\sin^2\beta & \cos\gamma\cos\beta+\sin\gamma\sin\beta \\ \cos\alpha\cos\gamma+\sin\alpha\sin\gamma & \cos\beta\cos\gamma+\sin\beta\sin\gamma & \cos^2\gamma+\sin^2\gamma \end{vmatrix}$$

$$= \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix} \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{vmatrix} = 0$$

19. A triangle has its three sides equal to a,b,c. If the coordinates of its vertices are $A(x_1,y_1)$, $B(x_2,y_2)$and $C(x_3,y_3)$, then show that $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$ = (a+b+c)(b+c-a)(c+a-b)(a+b-c).

Solution: Assume the area of the triangle = Δ= $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$\Rightarrow 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 4\Delta = 2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}$$

$$\Rightarrow 16\Delta^2 = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$$

Also note

$\Delta^2=s(s-a)(s-b)(s-c)$ and s=(a+b+c)/2= half of the perimeter. On simplification the required result can be evaluated.

20. If $\bar{a},\bar{b},\bar{c}$ are coplanar vectors, then show that $\begin{vmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \end{vmatrix} = \bar{0}$.

Hint:

$\bar{a},\bar{b},\bar{c}$ are coplanar vectors hence there exist non-zero x,y,z such that x$\bar{a}$ +y$\bar{b}$ +z$\bar{c}$ =0

On applying $xC_1+yC_2+zC_3$ to the given determinant, we may get

$$\frac{1}{x}\begin{vmatrix} x\bar{a}+y\bar{b}+z\bar{c} & \bar{b} & \bar{c} \\ x\bar{a}.\bar{a}+y\bar{a}.\bar{b}+z\bar{a}.\bar{c} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ x\bar{b}.\bar{a}+y\bar{b}.\bar{b}+z\bar{b}.\bar{c} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \end{vmatrix} =$$

$$\frac{1}{x}\begin{vmatrix} x\bar{a}+y\bar{b}+z\bar{c} & \bar{b} & \bar{c} \\ \bar{a}.(x\bar{a}+y\bar{b}+z\bar{c}) & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{b}.(x\bar{a}+y\bar{b}+z\bar{c}) & \bar{b}.\bar{b} & \bar{b}.\bar{c} \end{vmatrix} = \frac{1}{x}\begin{vmatrix} \bar{0} & \bar{b} & \bar{c} \\ \bar{0} & \bar{a}.\bar{b} & \bar{a}.\bar{c} \\ \bar{0} & \bar{b}.\bar{b} & \bar{b}.\bar{c} \end{vmatrix} = \bar{0}.$$

21. If l_1 , m_1 , n_1 ; l_2 , m_2 , n_2 *and* l_3 , m_3 , n_3 are the direction cosines of three mutually perpendicular lines, then prove that $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = \pm 1$.

Solution:

We know that when the three lines are mutually perpendiculars, then

$l_1^2 + m_1^2 + n_1^2 = 1;$

$l_2^2 + m_2^2 + n_2^2 = 1;$

$l_3^2 + m_3^2 + n_3^2 = 1$

and

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$;

$l_1 l_3 + m_1 m_3 + n_1 n_3 = 0$;

$l_3 l_2 + m_3 m_2 + n_3 n_2 = 0$

Let us assume the given determinant = Δ

then,

$$\Delta^2 = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} . \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} =$$

$$\begin{vmatrix} l_1^2+m_1^2+n_1^2 & l_1l_2+m_1m_2+n_1n_2 & l_1l_3+m_1m_3+n_1n_3 \\ l_1l_2+m_1m_2+n_1n_2 & l_2^2+m_2^2+n_2^2 & l_3l_2+m_3m_2+n_3n_2 \\ l_1l_3+m_1m_3+n_1n_3 & l_3l_2+m_3m_2+n_3n_2 & l_3^2+m_3^2+n_3^2 \end{vmatrix}$$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$ and hence the required result is obtained after taking the square root of both sides.

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—oo—

First they will ignore you,

then they laugh at you,

then they fight you,

then you win (truth follows…)

\- **Mahatma Gandhi**

GROWING WITH CONCEPTS – Mathematic

COMPLEX NUMBER

Prof. SB Dhar

*(This is a supplement to article on **Theory of Equations** brought out in 2nd Supplement dt 1st June'17 to Third Quarterly e-Bulletin).*

(a) We know that (-1)×(-1)= +1 or $\sqrt{+1}=\pm 1$, meaning thereby that the product of two Positive numbers yields a Positive Number and the Product of two Negative numbers also yields a positive number.

(b) But there is no number whose square is negative. In other words we may say that if a negative number comes under a Square root sign (√), it becomes meaningless so for the real number is concerned, i.e., $\sqrt{-1}$ is neither +1 nor -1.

(c) To avoid this obstacle in regular calculation, the need of introduction of a number iota (i) was necessary such that $i^2=-1$ or $\sqrt{-1}=\pm i$ and $i^3=-i$ or $i^4=1$. i is not a real number. It is the imaginary unit.

Note: Leonhard Euler introduced first time the symbol i in 1748 with property $i^2=-1$.

Fallacy:

$$\left(\sqrt{-1}\right)\left(\sqrt{-1}\right)=\sqrt{(-1).(-1)}=\sqrt{+1}=1.$$

Because

$$\left(\sqrt{-1}\right)\left(\sqrt{-1}\right)=\sqrt{i^2.i^2}=\sqrt{i^4}=i^2=-1.$$

Definition

Complex number may be defined as a number consisting of two parts: one a real part and the other an Imaginary Part, or it may be defined as an ordered pair of two real numbers and is denoted by (x,y).

This can also written as Z= (x, y) = x + iy, where x , y ∊ R ; x is called the Real part and y is called the Imaginary part. Complex number can be purely real or purely imaginary. When real part is zero, complex number is purely imaginary and when imaginary part is zero, complex number is purely real.

The real part is written as Re(z)= x, and Imaginary Part as Im(z)=y.

Note:

(i) x and y are real but not necessarily a rational number.
(ii) 0 is the only Number that is both purely real and purely imaginary.

Important Results

1. Algebraic operations such as Addition, Subtraction, Multiplication and Division are possible with complex numbers.
2. If z_1 and z_2 are two complex numbers then the complex number $z=\frac{mz_2+nz_1}{m+n}$ represents the point dividing the line segment joining z_1 and z_2 in the ratio m : n.
3. If the vertices of a triangle are z_1,z_2,z_3 then its centroid is given by $(z_1+z_2+z_3)/3$.
4. If z_1 and z_2 are two complex numbers then the distance between z_1 and z_2 is given by $|z_1-z_2|$.
5. Square root of a negative real number is an imaginary number.
6. i is neither negative , zero, nor positive. Hence ordered relations are not defined in imaginary numbers.
7. Complex number x+iy < or > a + ib is meaningless.
8. Only equality is defined if real part is equal to real part and imaginary part is equal to imaginary part.

9. If a and b are non-negative then $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$
10. $\sqrt{-a}\times\sqrt{-b}\neq\sqrt{ab}$
11. $|z_1 z_2| = |z_1| . |z_2|$
12. $\left|\frac{z_1}{z_2}\right|=\left|\frac{z_1}{z_2}\right|$
13. $|z_1+z_2|\le|z_1|+|z_2|$.
14. $\big||z_1|-|z_2|\big|\le|z_1+z_2|\le|z_1|+|z_2|$.
15. arg (z) = 0 or π => z is purely real.
16. arg $(z)=\pm\frac{\pi}{2}$ => z is purely imaginary.
17. $\arg(\bar{z})=-\arg(z)=\arg\left(\frac{1}{z}\right)$.
18. $\arg(z-\bar{z})=\pm\frac{\pi}{2}$
19. arg $(z)+\arg(\bar{z})=0$
20. arg $(z)+\arg(-\bar{z})=\pi$
21. arg(any real positive number)=0
22. arg(any real negative number)= π
23. Sum of pth powers of nth roots of unity
    = n when p is a multiple of n
    = 0 when p is not a multiple of n

24. If x= cosα + i sinα,
    y= cosβ + i cosβ ,
    z= cosγ + i cosγ , then

    yz+zx+xy=0,

    $x^2 + y^2 + z^2 = 0$,

    $x^3 + y^3 + z^3 = 3xyz$,

    (1/x) + (1/y) + (1/z)=0

25. Cube roots of unity:
    $z=(1)^{1/3}$ are given by 1, ,ω, ω² where $\omega=e^{i2\pi/3}$

26. If 1,ω, ω² are cubic roots of unity, then a + bω + cω² =0 ⇔ a=b=c iff a,b,c are real.

27. $1+\omega^n+\omega^{2n}$ =3 or 0 according as n is a multiple of 3 or not.

28. $\omega^{3m}+\omega^{3n+1}+\omega^{3p+2}=0$ if m,n,p >0.

29. When 1, ω, ω² are represented on Argand plane, they lie on the vertices of an Equilateral triangle inscribed in a unit circle whose center is at origin and one of its vertices is at positive real axis.

30. If p is cube root of a number then its other roots are pω , pω² .

31. Sum of the roots of unity is always zero but the product is either +1 or -1. The Product is +1 when the number of roots is odd and the product of roots is -1 when the number of roots is even.

32. If z = x + iy is complex number then conjugate of z, $\bar{z}=x-iy$.

33. $\bar{\bar{z}}=z$

34. $|z|=|-z|=|\bar{z}|=|-\bar{z}|=|iz|=|i\bar{z}|$

35. $z\bar{z}=|z|^2=(\mathrm{Re}(z))^2+(\mathrm{Im}(z))^2$

36. $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$
37. $\overline{z_1-z_2}=\overline{z_1}-\overline{z_2}$
38. $\overline{z_1.z_2}=\overline{z_1}.\overline{z_2}$
39. $\left(\overline{z^n}\right)=\left(\bar{z}\right)^n$
40. $\overline{\left(\frac{z_1}{z}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$

41. Mid point of the line segment joining z_1 and z_2 is given by $\frac{z_1+z_2}{2}$.

Important Loci

(a) The Locus of a point z satisfying $|z-z_1|=|z-z_2|$ is the perpendicular bisector of the line joining points z_1 and z_2.

(b) The locus of z satisfying the condition $|z-z_1|+|z-z_2|=|z_1-z_2|$ is the line segment joining z_1 and z_2.

(c) The locus of z satisfying the condition $|z-z_1|-|z-z_2|=|z_1-z_2|$ is also a straight line joining z_1 and z_2 but z does not lie between z_1 and z_2.

(d) The Locus of z such that $|z-z_1|+|z-z_2|=2a$ where 2a> $|z_1-z_2|$ is an Ellipse and its Foci will be at z_1 and z_2 and a is a real positive number.

(e) The Locus of z such that $|z-z_1|-|z-z_2|=2a$ where 2a< $|z_1-z_2|$ is a Hyperbola and its Foci will be at z_1 and z_2 and a is a real positive number.

(f) The locus of z satisfying $\left|\frac{z-z_1}{z-z_2}\right|$ = k (if k is not equal to 1) is a circle.

42. Conjugate of a complex number is again a complex number but with a negative i.

43. The value of Argument of a complex number is not unique.

44. The Principal value of the Amplitude or Argument is the angle θ that satisfies the inequality $-\pi < \theta \leq \pi$.

45. The three cube roots of unity are the vertices of an equilateral triangle inscribed in the circle |z| = 1.

46. The fourth roots of unity are 1, -1, -i, i.

47. Argument of the complex number 0 is not defined.

De Moivre's Theorem

One form

$$(\cos\theta_1+i\sin\theta_1)(\cos\theta_2+i\sin\theta_2).....(\cos\theta_n+i\sin\theta_n)$$
$$=\cos(\theta_1+\theta_2+....+\theta_n)+i\sin(\theta_1+\theta_2+....+\theta_n)$$

Second form $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

Third Form $\frac{1}{\cos n\theta+i\sin n\theta}=\cos n\theta-i\sin n\theta$

θ can take any real value but n cannot take all real values. It can take only non-irrational values or in other words De Moiver's Theorem is true for all n except irrational numbers.

Note: The theorem was named after the French statistician and mathematician Abraham de Moivre (1667-1754).

Some Illustrations

1. **If f(z) is divided by (z-i) and (z+i), the remainders are respectively i and 1+i. Find the remainder when f(z) is divided by z^2+1.**

Hint: Obviously f(z)= (z-i) g(z) + i ...(i)

And f(z)=(z+i) h(z) + (1+i) .... (ii)

Where g(z) and h(z) will be quotients.

And if f(z) is divided by z^2 +1 it will be

f(z)=(z^2+1) u(z)+ pz + q ....(iii) where p, q are complex numbers.

Now from (i) f(i)=i and from (ii)f(-i)=1+i

And from (iii) f(i) gives pi+q=i

And f(-i) gives –pi+q=1+i

On solving p=i/2 and q=(1+2i)/2

And hence the remainder can be evaluated.

2. **Find the locus of z=x+iy if the amplitude of (z-2-3i) is π/4.**

Hint: Write (z-2-3i) in CiS form i.e. rcosθ + irsinθ
Find out x and y.
Eliminate r as is θ given equal to π/4.

3. **If w is the n^{th} root of unity, then find the value of $(1 + w + w^2 + \ldots + w^{n-1})$.**

Hint: Given w is the nth root of unity

$\Rightarrow w^n=1$

$\Rightarrow w^n-1=0$ or $(w-1)(1+w+w^2+\ldots+w^{n-1})=0$

$\Rightarrow w=1$ or $(1+w+w^2+\ldots+w^{n-1})=0$ the required result.

4. **Find the number of solutions of the equation $z^2+|z|=0$, where z is a complex number.**

Hint: Assume z=x+iy and put value in the given equation.
Find $2x^2+2ixy=0$
i.e. real part $x^2=0$ and Imaginary $xy=0$
$\Rightarrow x=0$ and y= any real value
$\Rightarrow z=iy$ i.e. infinite number of solutions

5. **If α is a complex number such that $\alpha^2+\alpha+1=0$, then find the value of α^{31}.**

Hint: Find out $\alpha = \dfrac{-1\pm i\sqrt{3}}{2} = \omega, \omega^2$ from the given relation and proceed with value.

6. **If z is a complex number then find a relation between the square of the mod of z and the square of z.**

Hint: Assume z=x+iy and note that

$z^2= x^2-y^2+2ixy$

$\Rightarrow |z^2| = \sqrt{(x^2-y^2)^2+4x^2y^2} = (x^2+y^2) = |z|^2$

7. **If z_1, z_2, z_3 are three complex numbers in AP, then find the curve on which they lie.**

Hint: The given numbers are in AP , i.e. $z_2= (z_1+z_3)/2$

$\Rightarrow$ the required locus is the mid- point of the line joining z_1 and z_2, i.e. a straight line.

8. **For all complex number z_1, z_2 satisfying $|z_1|=12$, $|z_2-3-4i|=5$, find the minimum value of $|z_1-z_2|$.**

Hint: Given $|z_1|=12$ i.e. z_1 is on the circle whose centre is origin and radius os 12.

And $|z_2-3-4i|=5$ means that z_2 lies on the circle whose centre is (3,4) and radius is 5 units.

The diameter of this circle =10.

Hence $|z_1-z_2|$ will be least only when they lie on the line joining the two centres= 12-10=2 units.

9. **If $\left|z-\dfrac{4}{z}\right|=2$ then find the greatest value of $|z|$.**

Hint: Rewrite the given equation as below:

$$|z| = \left|z-\frac{4}{z}+\frac{4}{z}\right| \le \left|z-\frac{4}{z}\right| + \left|\frac{4}{z}\right| \le \left|z-\frac{4}{z}\right| + \frac{4}{|z|}$$

$$\Rightarrow |z| \le 2+\frac{4}{|z|} \Rightarrow |z|^2 \le 2|z|+4 \Rightarrow (|z|-1)^2 \le 5$$

$\Rightarrow |z| \le \sqrt{5}+1$

10. **If z_1, z_2 are two complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$ then find the $\arg(z_1)-\arg(z_2)$.**

Hint: Assume

$z_1= r_1(\cos\theta_1 + i\sin\theta_1)$ and
$z_2= r_2(\cos\theta_2 + i\sin\theta_2)$.

on putting values in the given relation $|z_1+z_2| = |z_1| + |z_2|$

get, $\cos(\theta_1 - \theta_2)=1 \Rightarrow \theta_1-\theta_2=0$

11. **If the equation $z\bar{z}+(4-3i)z+(4+3i)\bar{z}+5=0$ represents a circle then find the radius of it.**

Hint: Assume z=x+iy and simplify the given relation to

$x^2+y^2+8x+6y+5=0$

use formulae for radius= $\sqrt{(g^2+f^2-c)}$, g, f and c have their usual meanings in $x^2+y^2+2gx+2fy+c=0$.

12. If w is a complex cube root of unity then find the value of $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$.

Hint: Multiply the Numerators by w^3 (i.e. by 1, value will not change)

The expression becomes

$$\frac{a\omega^3+b\omega^4+c\omega^5}{c+a\omega+b\omega^2}+\frac{a\omega^3+b\omega^4+c\omega^5}{b+c\omega+a\omega^2}$$

$$=\frac{\omega^2(a\omega+b\omega^2+c\omega)}{c+a\omega+b\omega^2}+\frac{\omega(a\omega^2+b+c\omega)}{b+c\omega+a\omega^2}.$$

$=w^2+w=-1$

as $1+w+w^2=0$

13. Find the locus of z which satisfies the inequality $\log_{0.3}|z-1| > \log_{0.3}|z-i|$.

Hint: Note: As the bases of the logarithm are same and less than unity hence the sign of inequality will change as below:

$|z-1| < |z-i|$

Assume z=x+iy and use modulii

$(x-1)^2+y^2 < x^2 + (y-1)^2$

i.e. $x-y > 0$.

14. If $\frac{5z_2}{7z_1}=ia(a\neq 0)$ **is a purely imaginary number, then find the value of** $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$

Hint: Assume $\frac{5z_2}{7z_1}=ia(a\neq 0, a\in R)$.

Rewrite the given expression as below:

$$\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|=\left|\frac{\frac{2z_1}{3z_2}+1}{\frac{2z_1}{3z_2}-1}\right|=\left|\frac{10+21ia}{10-21ia}\right|=1.$$

As the modulii of number and its conjugate are equal.

15. Evaluate: $(1+i)^6+(1-i)^3$

Solution: $(1+i)^6+(1-i)^3$

$=\{(1+i)^2\}^3+(1-i)^3$

$=\{1+i^2+2i\}^3+(1-i^3-3i+3i^2)$

$=\{1-1+2i\}^3+(1+i-3i-3)$

$=8i^3+(-2-2i)$

$=-8i-2-2i$

$=-2-10i$

Solution of a quadratic equation

$ax^2+bx+c=0$, when a ,b,c are real and discriminant $b^2-4ac<0$ is given by $x=\frac{-b\pm i\sqrt{b^2-4ac}}{2a}$

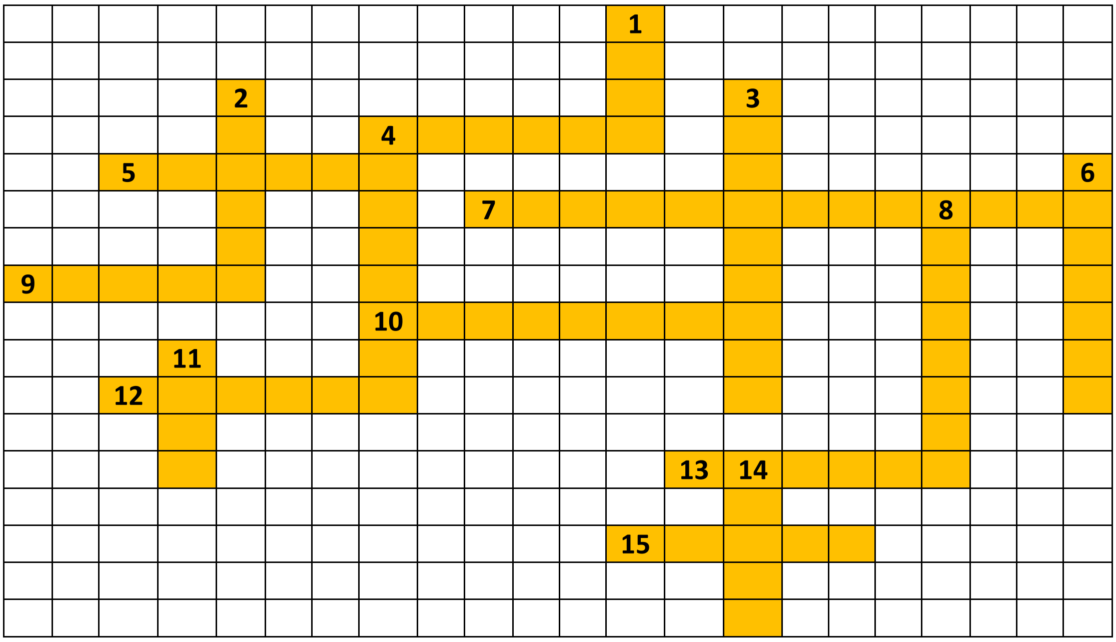
16. Solve for x: $21x^2-28x+10=0$

Solution: Here a=21, b=-28, c=10 and $b^2-4ac=(-28)^2-4(21)(10)=-56$, negative. Hence the solution will not be real. The solution can be written as, $x=\frac{-b\pm i\sqrt{b^2-4ac}}{2a}=\frac{28\pm i\sqrt{56}}{42}=\frac{28\pm i2\sqrt{14}}{42}=\frac{14\pm i\sqrt{14}}{21}$.

--oo--

CROSSWORD PUZZLE July'17: YOGA

Prof. SB. Dhar



Across

4 Female Energy

5 Energy Centre

7 Sun Salutations

9 A hand gesture

10 The ancient Indian science of Health

12 An enlightened one

13 The person who performs yoga of Union

15 Life energy

Down

1 A male practioner of Yoga

2 The teachings of Buddha

3 Breath control

4 Corpse pose

6 Gazing point used during asana practice

8 Enlightenment

11 A spiritual mentor

14 Yoga posture

—oo—

(Answer to this Crossword Puzzle shall be provided in Supplementary e-Bulletin Dt. 1st August'17)

GROWING WITH CONCEPTS- Physics

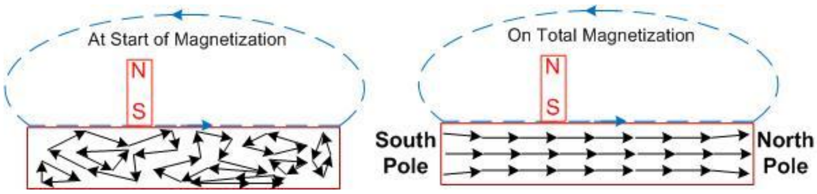
ELECTROMAGNETISM – Part II: Magnetic Effects of Current

Learning of electromagnetism is an excellent example of spiral growth of knowledge starting at observation, which in turn invokes experience; exploration of the experience leads to discovery and innovation. Every fresh discovery creates a new set of observations and the cycles has grown like a chain reaction. Basic idea behind this manual is not to reach out readers with a fresh set of discoveries, but to make learning exciting through making the subject matter contextual to human observations and experience. In this back drop this set of chapters, constituting Mentors' Manual would be found different from most of the text books and reference books, and is believed to be considered helpful in igniting fire of learning and exploration.

Discovery of magnetism in ancient times was linked to current electricity by Hans Christian Ørsted *in 1819 when he accidently discovered sudden trembling of compass needle near a current carrying wire. This event was an opening of a gateway to Electromagnetism. Series of discoveries by Ampere, Biot, Savart. Faraday and Maxwell, complemented efforts by adding new dimensions to understanding of Electromagnetism. Though discovery of magnetism is ancient but, magnetism and electricity are not only inseparable but interactive in nature. The only difference is that in electricity (+)ve and (-)ve electric charges can exist in isolation but in magnetism two poles always exist in pair called dipole. An increasing understanding of atomic structure and association of magnetic field to electric current revolutionized the theory of magnetism leading to electromagnetism. Nevertheless, elaboration of the concepts has been sequenced considering their interdependency.*
During development of this text, few figures had to be reduced in size to facilitate space management while placing them along the relevant elaborations. Readers if find that details in figure are not readable, they are requested to zoom it, an advantage with e-Manual.

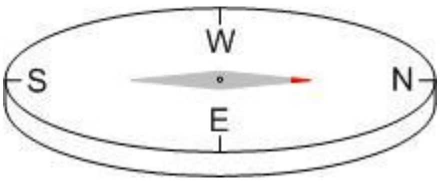
Magnetism: It was discovered in ancient times based on stones called Lodestone found in the region Magnesia, which could attract iron. These stones were called Magnetite, signifying the place where it was found, and the property to attract iron pieces was called Magnetism. While Aristotle was the First to engage in discussion on magnetism, while around the same era, Sushruta, an ancient Indian surgeon is stated to have used magnets for surgical purposes. It is around 11th century, Shen Kuo, a Chinese scientist described use of magnetic needle for navigation. It was believed that magnetic property of certain material was associated with tiny magnetic particles, called dipoles, lying in scattered manner, as shown in the figure, such that net magnetic effect is cancelled. It is only under influence of magnetic field that these dipoles get aligned whereby the material starts behaving like magnets. A magnetic piece when suspended, takes a free position such that its one part is in north direction and is called **North Pole**, and other part in south direction is called ***South Pole***. This orientation of the magnetic piece is independent of place of suspension as long as it is in free state.

If two magnets with their North Poles so identified, when brought together, they repel each other, and so also South Poles. But, when North Pole and south Poles are brought together they attract each



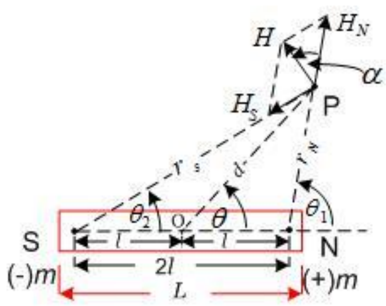
other. This combination in a formation of closed chain, as shown in the figure above, magnetic property of the magnetic material disappears. Nevertheless, if such material is rubbed with a magnet, repetitively, these dipoles start getting oriented and the magnetic material starts demonstrating magnetic properties. It may be observed in the figure, that in the state of total magnetization – **a)** all dipoles are aligned in parallel open chains, **b)** interface of North Pole and South Pole of dipoles does nor exhibit magnetization, which is experienced inly at ends; One end is North Pole, while the other is South Pole, **c)** Point where magnet leaves contact of the material to be magnetized becomes North Pole; this is attributed to attraction of north poles of the dipoles which aligns them in the direction of motion of magnet , **d)** at ends of the solid magnetized bar, repulsion of like poles of dipoles causes slight dispersion, **e)** North Pole and South Poles cannot exist in isolation, they exist only in pairs and is unlike electric charges. Since, all materials are not magnetic, and hence this experience occurs only with materials which are inherently magnetic. Elaboration of magnetic property of material and its reasons shall be elaborated separately. Nevertheless, inquisitive readers are welcome to write through *Contact Us*.

Magnetic Compass *is a light needle suspended over a pivot to move free to position itself in any direction along 360 degrees, and is enclosed in a circular box of non-magnetic material. Dial inside the box is marked with N, S, E, W indicating North, South, East and West directions, having a graduation marking of degrees. Box of the needle is gently turned, without disturbing the needle so as to align distinctly marked end to North on the dial, as shown in the figure. This indicates indicating North Direction and other directions and their angles with respect to North Direction. This instrument is called Magnetic Compass and has been in use since ancient times.*



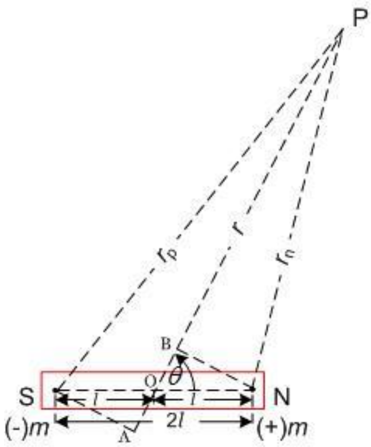
In **magneto-statistics Coulomb's Law** is magnetic equivalent of Coulomb's Law of Electrostatic Forces. Since, magnetic poles do not exist in isolation; net magnetic force at a point is combined effect of forces exerted by both the poles of dipoles. It, therefore, creates difficulties of experimental verification. *Coulomb, in fact pronounced Laws of Force between Electrostatic Charges. Observation and its similarity to magneto-static forces between magnetic poles, except for the proportionality constant, this law is also known as* **Coulomb's Inverse Square Law of Magnetostatic Forces** and is mathematically expressed as $\overline{F} = \frac{\mu}{4\pi} \cdot \frac{m_1 m_2}{r^2} \hat{r}$. Here, $\overline{F}$ is the force vector having unit Newton, m_1 and m_2 are *strengths magnetic poles also referred to as magnetic charge*, $\hat{r}$ is unit vector of displacement between two magnetic poles, r is the magnitude of displacement between the Two poles, and $\mu = \mu_0 \mu_r$ is permeability of medium filling the gap between the two magnetic poles, while $\mu_0 = 4\pi 10^{-7}$ is permeability of free space and μ_r is relative permeability of the medium and for vacuum or air having a value its value is 1. *Here,* ***unit of*** μ_0 ***and Magnetic Pole strength*** *would be progressively defined as elaboration of interaction between magnetic field and current proceeds, which would make their definition more realistic and scientific. For the present,* ***Unit strength of magnetic pole*** *is defined as that, when Two Like magnetic poles of equal strength are separated by 1m experience a force of repulsion equal to* 10^{-7} *Newton*.

In this series documents efforts, while elaborating concepts basics including its source and history, to arouse enthusiasm of readers to the fact that the knowledge which is so readily available has to us has taken restless efforts of many scientists, who never had facilities that are privileged to us right from birth. Author has not been able to corroborate association of Law Magnetostatic Forces to Coulomb, just beyond its nature analogous to the law of Electrostatic Forces propounded by Coulomb, readers are requested to update if they have any information on the source of the Inverse law through *Contact Us*. We would gratefully acknowledge the contribution of readers, with its inclusion in this document in furtherance of knowledge.



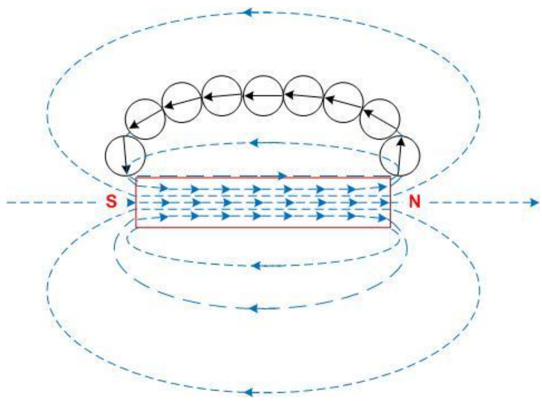
Magnetic Field Intensity (B) produced by a magnetic pole is defined as Force experienced by a Unit Magnetic Pole, is determined like that of electric field due to electric charge(s); its unit is Newton/Unit_Pole. It is also referred to as ***Magnetic Field Intensity (MFI)*** at a point. Since magnetic pole does not exist in isolation and hence determination of $\boldsymbol{B}$ due to magnet is driven by vectors $\overline{\boldsymbol{B_N}}$ and $\overline{\boldsymbol{B_s}}$ as shown in the figure. It may be noted from the figure that poles of a magnet lie within its geometry and hence M**agnetic Length (2*l*)**, distance between North Pole and South Pole of a magnet is shorter than its geometric length ($L < 2l$). Ratio of Magnetic Length to Geometrical Length is generally $\left(\frac{2l}{L}\right)$ found to be 0.84. In this analysis medium is considered to be air such that $\boldsymbol{\mu_r} = \mathbf{1}$**.** It gives rise to three cases.

Case I: If $\theta = 0$ or π net value of $B = \frac{\mu_0}{4\pi}\cdot\left(\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}\right) = \frac{\mu_0(4ml)}{2\pi}\cdot\frac{d}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi}\cdot\frac{2Md}{(d^2-l^2)^2}$ and in a direction e^{io} for $\theta = 0$ and i.e. $e^{i\pi}$ for $\theta = \pi$, repulsive and attractive is direction of force is attributed to vicinity of North or South Pole. **Case II**: If $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ then, $= \theta_2$ $B = \frac{\mu_0}{4\pi}\cdot\frac{2m\cos\alpha}{d^2+l^2}$. Geometrically, $\cos\alpha = \frac{l}{\sqrt{d^2+l^2}}$, hence $B = \frac{\mu_0}{4\pi}\cdot\frac{2m}{d^2+l^2}\cdot\frac{l}{\sqrt{d^2+l^2}} = \frac{\mu_0}{4\pi}\cdot\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$ in a direction $e^{i\pi}$. This formulation introduces a new term ***Magnetic Dipole Moment (M)*** of a magnet, which always exists like a dipole, $M = 2ml$. **Case III**: Taking a generic case where point P is at an angle $\pm\theta$ with the principal axis of magnet. This will create $\bar{B} = \overline{B_N} + \overline{B_s}$,. Taking $d \gg l$, a fair approximation, it would yield $\theta \cong \theta_1 \cong \theta_1$. Accordingly, $\text{NP} \cong d - l\cos\theta$ and $SP \cong d + l\cos\theta$ and, further, at point P magnetic field intensity would be $B = \frac{\mu_0}{4\pi}\cdot\left(\frac{m}{(d-l\cos\theta)^2} - \frac{m}{(d+l\cos\theta)^2}\right) = \frac{\mu_0}{4\pi}\cdot\frac{4mdl\cos\theta}{(d^2-l^2\cos^2\theta)^2}$. This equation satisfies Case I for$\theta \to 0$, and also for the Case II when in addition to $d \gg l$ another condition $\theta \to \frac{\pi}{2}$ or $\to \frac{3\pi}{2}$ is satisfied and $B \to 0$, i.e. $\overline{B_N} = -\overline{\boldsymbol{B_N}}$.



Work done in Moving a Magnet in Magnetic Field: This derivation is analogues to that done in electric field. Accordingly, a point P is at $r\angle\theta$ from midpoint of a magnet. Despite the fact that in magnets North and South poles coexist, but for the effect of magnetic field each of the end pole was considered in isolation and same is being extended for determining magnetic potential at point P. Accordingly, magnetic potential at point P due to north pole would be $V_n = \int_{\infty}^{r_n}\frac{\mu_0}{4\pi}.\frac{m}{x^2}.(-dx) = \frac{\mu_0 m}{4\pi r_n}$ and due to south pole $V_s = -\frac{\mu_0 m}{4\pi r_s}$. Thus net magnetic potential at point P is $V = \frac{\mu_0 m}{4\pi r_n} - \frac{\mu_0 m}{4\pi r_s}$. In case point P is so located that $r \gg 2l$, then $V = \frac{\mu_0 m}{4\pi}\left(\frac{1}{r_n} - \frac{1}{r_s}\right) \approx \frac{\mu_0 m}{4\pi}\left(\frac{1}{r-l\cos\theta} - \frac{1}{r+l\cos\theta}\right)$. It leads to $V = \frac{\mu_0 m}{4\pi}\left(\frac{1}{r_n} - \frac{1}{r_s}\right) \approx \frac{\mu_0 m}{4\pi}\left(\frac{2l\cos\theta}{r^2-l^2\cos^2\theta}\right) \approx \frac{\mu_0}{4\pi}\cdot\frac{M\cos\theta}{r^2}$. This is called ***Scalar Magnetic Potential***. As studies are advanced in Electromagnetism, which is outside the domain of this Manual, *Vector Magnetic Potential* shall be encountered. Nevertheless, inquisitive readers are welcome to write us through *Contact Us.*

Magnetic Lines of Force: These *are imaginary lines representing direction of magnetostatic force at any point*. There are two methods of drawing ***Magnetic Lines of Force (MLF)***: **a) Analytical Method** - it uses above mathematical formation to determine direction of B and at different points and with best-fit curve joining these points to draw unidirectional lines from north pole to south pole of the magnet. Access of computer has made it much easier and faster. **b) Experimental Method** - it uses magnetic compass to which is placed near North pole of the magnet and two ends of the compass needle, when reached stationary, are marked. Next compass needle is so positioned that, in stationary state, its south pole coincides with marking of north pole; in this new position North Pole of the compass is marked. This process is continued till South Pole of the magnet is reached, which makes One lines of Force. This process is repeated for many points near the North Pole of magnet and for every point a new MLF shall appear, as shown in the figure. It will be observed that MLF have following properties – **a)** Outside magnet they start at North and terminate at South Pole. **b)** Direction of magnetic field (B) at any point on MLFTangent at any point on MLF indicates direction of B at that point. lines of force, **c)** none of the MLF intersect each other, **d)**Density of MLF at any point is indicative of MFI (B) at that point, and is highest near magnetic poles, **e)** Two MLF never intersect each other, **f)** MLF,

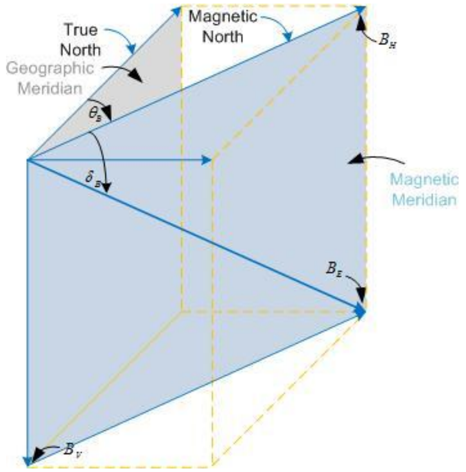


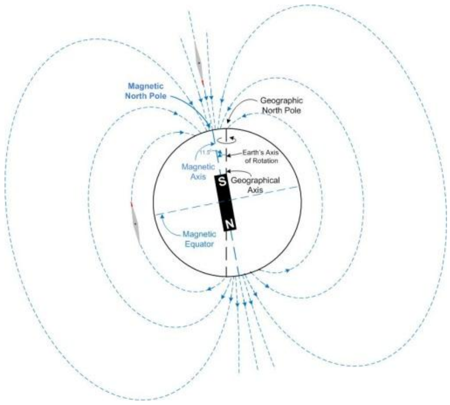
in any region, are parallel and equi-spaced then MFI in the region is uniform, **g)** MLF repel each other in a direction perpendicular to them, and is in accordance with repulsion between like polarity, **h)** MLF along length are like stretched strings and is due to force of attraction between like polarity, and **i)** MLF form a closed loop which outside magnet emerges from North Pole and terminates at South Pole, while inside magnet, it closes from South Pole to North Pole. This differentiates MLF with to Electrostatic Lines of Force. Since, and is attributed to existences of dipoles as against electric charges which can exist separately as (+)ve and (-)ve charges.

Terrestrial Magnetism: Use of magnetic compass for navigation was based on observation that *magnetic compass, irrespective of position any where on earth always settles in a direction pointing towards Geographical North. But, **why?** And answer to this question lead to discovery of **Terrestrial Magnetism**.* Magnetic needle always settling in north direction indicates that it is line with the direction of magnetic field as seen during elaboration of MLF. This can happen only when it is behind south of emerging lines of force. All these MLF converge on Magnetic North Pole (MNP) of the Earth and accordingly Magnetic Needle becomes perpendicular to the earth's surface at MNP. Thus if seen that internal magnetism of the earth also acts at as a dipole with its north pole aligned towards Magnetic South Pole (MSP) and south pole aligned to MNP. The magnetic axis of earth is tilted at an angle of about 11.5° with geographic axis i.e. axis of rotation. Analysis of cause of earth's magnetism would throw discussions out of context that has been built so far, and is refrained at this stage. However, inquisitive readers are requested to write us through *Contact Us*.

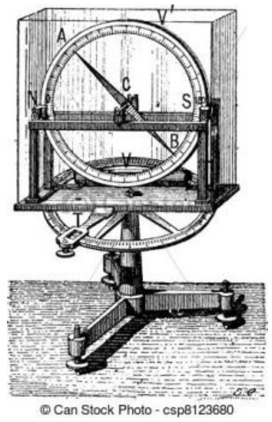
There are certain, terms used in context of terrestrial magnetism and are defined for a ready reference.

Angle of Dip or Inclination ($\boldsymbol{\delta_B}$): This the angle made by earth's magnetic field with horizontal direction.



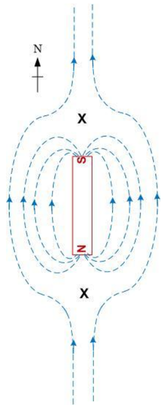


Accordingly, Lines which join points having identical angle of dip are called Isogenic Lines. Further, Geographical Meridian is a plane passing through geographical poles of earth and a point under consideration. Likewise, Magnetic Meridian is a plane passing through magnetic poles of earth and a point under consideration And, line joining points having $\boldsymbol{\delta_B = 0}$ is called Magnetic Equator. Likewise, angle between geographical axis and magnetic axis of the earth is called Angle of Declination ($\boldsymbol{\theta_B}$). The principle behind two angles is shown in the figure. Horizontal Component of Magnetic Field $B_H = B_E \cos \delta_E$ and Vertical Component of Magnetic Field $B_V = B_E \sin \delta_E$. Accordingly, $\tan \delta_E = \frac{B_V}{B_H}$. An instrument called Dip Circle is used for measuring this angle of dip ($\boldsymbol{\delta_B}$) and is shown in the figure (Source of Picture: http://comps.canstockphoto.com/can-stock-photo_csp8123680.jpg). In horizontal turn table is levelled using spirit level and vertical frame housing magnetic needle is tuned till it is becomes perfectly vertical and its two ends point (+)90° and (-)90° on the vertical scale. At this position forces due to B_H forming a couple on the needle are zero and accordingly under influence of vertical magnetic field settles vertically. Now, the vertical scale is turned through 90° on the horizontal scale, at this position torques produced by B_H and B_V are in equilibrium and the needle shall settle at an angle be $\boldsymbol{\delta_B}$ as shown in the figure.



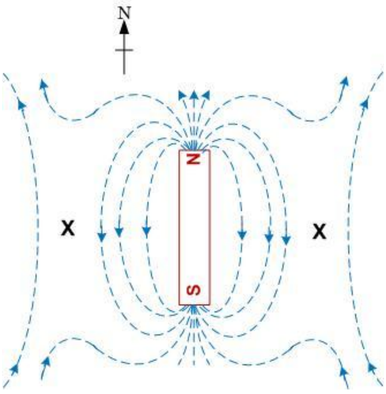
Magnetic Lines of force of Permanent Magnet placed in Earth's Magnetic Field : Four cases can be created for MLF of a magnet placed in magnetic field of the earth. **Case I:** North Pole of Magnet aligned towards North Direction. **Case II:** North Pole of Magnet aligned towards North Direction. **Case III:** North Pole of Magnet aligned towards East Direction and **Case IV**: North Pole of Magnet aligned towards West

Direction. Case IV is diagonal image of Case III. Generally, plotting of MLF is done on a horizontal plane and there invariably interacting magnetic field intensities of permanent magnet and the earth are horizontal components, and are so considered in the following analysis.

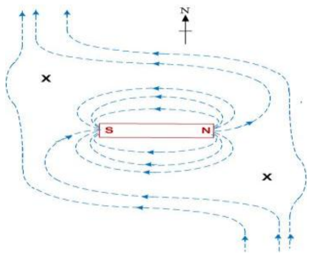


Case I: Qualitatively it is seen from the plot of MLF as shown in the figure (it is experimentally verifiable) in the region around points X magnetic field intensities due to magnet $B_M \left(= \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2-l^2)^2}\right)$ and horizontal component of earth's magnetic field intensity B_H tend to be in opposite direction. Since, MLF have a property that they do not cross each other, they choose an alternative path. Nevertheless, point at which $\bar{B}_M = -\bar{B}_H$, net B shall be zero or a **NULL POINT** of magnetic field intensity and is also called **Neutral Point**. Distance (***d***) of Null Point from mid-point of the permanent magnet along its axis would depends upon its dipole moment (***M***), length of magnet (***l***) and $\boldsymbol{B_H}$. In case length of magnet is small while pole strength of permanent (***m***) is sufficiently large, Null Point would occur such that $\boldsymbol{d \gg l}$. Accordingly, $d \cong \left[\left(\frac{\mu_0 M}{2\pi}\right) \cdot \frac{1}{B_H}\right]^{\frac{1}{3}}$.

Case II: In this case $\boldsymbol{B_M} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2+l^2)^{\frac{3}{2}}}$ and NULL POINT shall be on line perpendicular to axis of

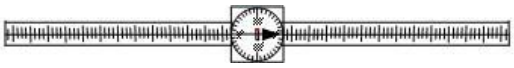


the permanent magnet on it's the midpoint own in the figure. In this case also distance (***d***) of Null Point from mid-point of the permanent magnet along its axis would depends upon its dipole moment (***M***), length of magnet (***l***) and $\boldsymbol{B_H}$. If length of magnet is small while pole strength of permanent (***m***) is sufficiently large Null Point would occur such that $\boldsymbol{d \gg l}$. Accordingly, $d \cong \left[\left(\frac{\mu_0 M}{\$\pi}\right) \cdot \frac{1}{B_H}\right]^{\frac{1}{3}}$.



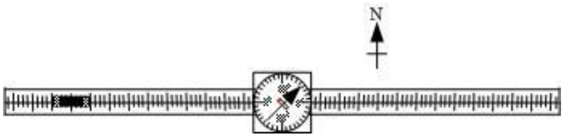
Case III and IV: These cases are diagonal images of each other and can be derived from $B_H = \frac{\mu_0}{4\pi} \cdot \frac{4mdl\cos\theta}{(d^2-l^2\cos^2\theta)^2}$ both analytically and experimental plot of lines MLF and is left for reader as an exercise. A conceptual plot of MLF for a permanent magnet with its north pole aligned to east direction is shown in the figure for convenience. Nevertheless, inquisitive readers are welcome to write through [Contact Us](#).

Deflection Magnetometer: This is an instrument used to determine NULL POINT of a permanent bar



magnet. It has a small magnetic compass placed on a pivots at the centre of a wooden box. This box is covered with glass and a mirror below the compass. A light non-magnetic pointer is so fixed to the compass that it is perpendicular, and its centre as the pivot. The mirror is useful to remove parallax error in measuring angular position of the needle. This box is placed centrally on a wooden scale.

Use if this instrument in Two positions helps to determine ratio $\left(\frac{M}{B_H}\right)$ of Dipole Moment (M) and horizontal component of earth's magnetic field (B_H).

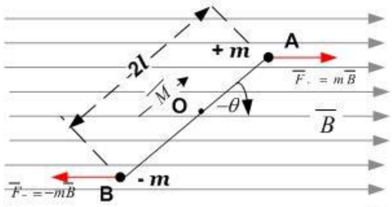


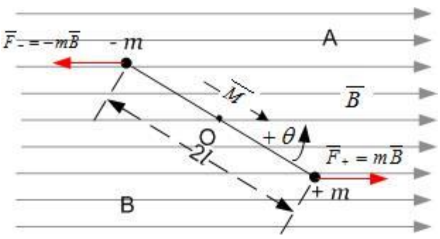
The two positions are called - a) *tan-A* Position and *tan-B* Position. Principally, these uses are based on analysis of Magnetic Field Intensity (***B***) of a permanent Magnet done for Cases I & II above, for which magnetic lines of forces were conceptualized in presence of horizontal component of earth's magnetic field ($\boldsymbol{B_H}$) at Cases I & II above. Thus combining the two cases the ratio for the Two positions is summarized below, for verification by the readers. Here , θ is the angular position of the needle in equilibrium of torque produced by ***M*** and $\boldsymbol{B_H}$ on the magnetic needle. Thus value of ***M*** and $\boldsymbol{B_H}$ determined with this apparatus are relative to each other, and not absolute.

.*tan*-A Position	***tan-B* Position**

$\frac{M}{B_H}=\frac{2\pi}{\mu_0}\cdot\frac{(d^2-l^2)^2}{d}\cdot\tan\theta$	$\frac{M}{B_H}=\frac{4\pi}{\mu_0}\cdot(d^2+l^2)^{\frac{3}{2}}\cdot\tan\theta$

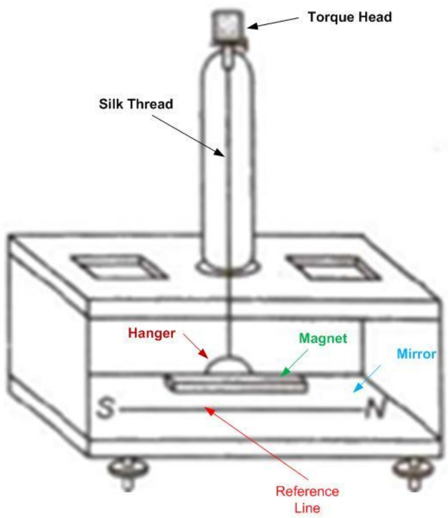
Torque on a Magnet placed in Uniform Magnetic Field: In the dipole shown below, turning moment on North Pole of strength $+\boldsymbol{m}$ is $\bar{\Gamma}_+=\overline{OA}\times\bar{F}_+=\bar{l}\times m\bar{B}=mlB\sin\theta(-\hat{z})$. It is similar to the torque on an electric dipole placed in uniform electric field discussed in Electrostatics. Likewise, turning moment on South Pole ($+\boldsymbol{m}$) of the dipole would be $\bar{\Gamma}_-=\overline{OA}\times\bar{F}_+=(-\bar{l})\times(-m)\bar{B}=lmB\sin\theta(-\hat{z})$. Here, $\bar{l}$ and $-\bar{l}$ are distances of the poles from the midpoint of the dipole. Thus, **Torque** on the dipole placed in a uniform magnetic field ($\bar{B}$) is $\Gamma=\bar{\Gamma}_-+\bar{\Gamma}_-=-2mlB\sin\theta\,\hat{z}=-MB\sin\theta\,\hat{z}$ and it tends reduce the angle of the magnet with the direction of Magnetic Field. Here, $\bar{M}=2lm\hat{l}=M\hat{l}$. Likewise, when magnetic field is at an angle $+\theta$ with respect to the pole, as shown in the latter of the Two cases above, $\Gamma=MB\sin\theta\,\hat{z}$ along $+\hat{z}$ direction, still it tends to the angle. This finds extensive application in Oscillation Magnetometer that follows.





***Potential energy** (PE) of a magnet at a particular angular position (θ) w.r.t. uniform magnetic field ($\bar{B}$), earlier of the above Two cases, is* $U_\theta=\int_0^\theta\Gamma(-d\alpha)$. Since, angle is α is the angle of magnet at an intermediate position, and turning magnet against torque through an angle $\Delta\alpha$ would resolve to $U_\theta=\int_0^\theta(MB\sin\alpha)(-d\alpha)$. Accordingly, it would lead to $U_\theta=MB\int_0^{-\theta}\sin\theta\,d\theta=MB[-\cos\theta]_0^\theta=MB(1-\cos\theta)$. Thus potential energy of a magnet in a position perpendicular to uniform magnetic field is MB Joule. Same is true in latter of the above Two cases. Accordingly, a slight perturbation in magnet would set it into oscillation.

.**Oscillation Magnetometer:** Limitation of deflection magnetometer to determine independent values of $\boldsymbol{M}$ and $\boldsymbol{B_H}$ lead to evolution of Oscillation Magnetometer to measure $\boldsymbol{MB_H}$ as shown in the Figure [Source: http://ncerthelp.com/ncertimages/Class12/physics/ch5/ch5physicsn09.jpg]. It has magnet freely suspended from a silk thread, in box openable to place a magnet on a light non-magnetic hanger. Inner bottom of the box has a mirror with a central line marked on it. The box has levelling screw to ensure that suspended magnet in normal state is on the central line marked N-S called Reference Line . A magnet is placed on the hanger, with untwisted thread. It is aligned to magnetic north, which in turn is aligned to the reference line. In this mean position suspended magnet is deflected about the vertical axis i.e. silk thread with the help of an external magnet. Torsion Head is at the top to which one end of the silk thread is fixed and at its other end hanger is fixed. This sets the suspended magnet into oscillation about the vertical axis. In any position, deflected through an angle θ, it experiences a torque the magnet $\Gamma=\left|(2l\hat{l})\times(m\bar{B}_H)\right|=(2ml)B_H\sin\theta=MB_H\sin\theta\approx MB_H\theta$. Further, as per principles of mechanics, under the influence of the torque (Γ) magnet would experience an angular acceleration such that magnitude of the torque is $\Gamma=I\alpha\rightarrow I\alpha=MB_H\theta\rightarrow\alpha=\frac{MB_H}{I}\theta$. This is a *valid case of SHM, which satisfies two basic conditions*: **a)** Torque is proportional to the displacement from mean position and **b)** The torque is always directed towards its mean position. Accordingly, angular acceleration (α) and angular velocity (ω)of the oscillating magnet shall be such that $\alpha=\omega^2\theta$. Thus, $\omega^2=\frac{MB_H}{I}\rightarrow\omega=\sqrt{\frac{MB_H}{I}}$. In this, $I\left(=W\frac{a^2+b^2}{12}\right)$ is Moment of Inertia of the Magnet is its geometric property, where W is the weight, **a** (=*2l*) is its length and ***b*** is its width. Time Period of Oscillation (***T***) for a magnet is measurable



through experiment. Accordingly, to achieve the objective of inventing this instrument the relationship derived above can be written as $MB_H = \frac{4\pi^2}{T^2} I$.

Having determined $\frac{M}{B_H}$ with the help of deflection magnetometer, whichever position, and MB_H with the help of deflection magnetometer, it is possible to determine values of M and B_H separately, in terms of geometric properties with only value of absolute permittivity (μ_0) question. This involves elaboration of electromagnetism to follow.

Dip Circle together with **Deflection Magnetometer** and **Oscillation Magnetometer** work on the basic principle of mechanics i.e. position of least potential under influence of torque couples on a dipole is a state of equilibrium. Accordingly, *measurements through these instruments is susceptible to **errors**, which largely common in nature* and are tabulated below -

Type of Error	**Dip Circle**	**Oscillation Magnetometer**	**Deflection Magnetometer**
Off-Centers of Needle and Vertical Scale	Possible	Possible, *but off-center could be with horizontal circular scale*	Not Applicable
Offset in Geometrical and Magnetic Axis of Magnet	Possible	Possible	Possible
$0^0 - 0^0$ of vertical scale is not horizontal	Possible	Not Applicable	Not Applicable
Centre of Mass of the needle does not coincide with the pivot	Possible	Not Applicable	Possible, *but off-center could be is with Centre of Mass of Magnet and silk-thread.*
Offset in Magnetic and Geometric Centers	Not Applicable	Possible	Possible
Offset in Zero of Linear Scale a center of Circular Scale	Not Applicable	Possible	Not Applicable

Details of the above three instrument involve concepts elaborated earlier. Tangent Galvanometer is an important **Electromagnetic Instrument** used in magnetism. But, it involves concepts of electromagnetism which shall be elaborated a little later.

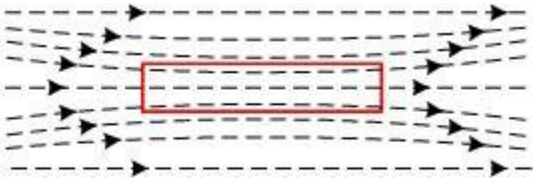
Gauss's Law in Magnetism: In electrostatics, where (+)q and (-)q electric charges can exist in isolation, $\oint \bar{E}.\overline{ds} = \frac{q}{\epsilon_0}$, here q is the electric charge in space. But, in magnetism, howsoever thin a magnet is North Pole and South Pole cannot exist in isolation, rather they are complementary to each other. It shall be soon elaborated in *Electromagnetism*. Hence, in magnetism Gauss's Law reduces to $\oint \bar{E}.\overline{ds} = 0$. It implies that *in any closed surface in a magnetic field, be it uniform or non-uniform the MLF entering the surface is equal to MLF leaving the surface.*

Types of Magnetic Material: There are materials which are non-responsive to static magnetic field and are called **Non-Magnetic** materials. There are certain materials which have qualitatively distinct behavior when placed in static magnetic field and are accordingly classified, as under –

Ferro Magnetic Material: There are materials like iron have relative permeability $\boldsymbol{\mu_r} \gg \mathbf{1}$ and this causes heavy concentration of MLF within it, when placed in magnetic field as shown in the figure. Ferromagnetic materials, also get aligned in the direction parallel to external magnetic field as it happen to

magnetic compass needle. They are strongly attracted by a magnet.

Paramagnetic Material: There are the material whose relative permeability is just $\mu_r > 1$ and this causes feeble concentration of MLF within it, when placed in magnetic field as shown in the figure. Paramagnetic material has a sharp qualitative difference with Ferromagnetic Material, while qualitatively it is same. Like Ferromagnetic materials, these also get aligned in the direction parallel to the external magnetic field. They are weakly attracted by a magnet.

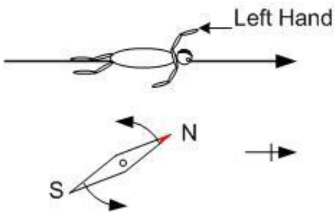


Diamagnetic Material: These materials are in contrast to Ferromagnetic and Diamagnetic Materials, since they distract MLF unlike these materials. This attributed to its characteristic difference where $\mu_r < 1$. Unlike Ferromagnetic and Paramagnetic material they get aligned in a direction perpendicular to the external magnetic field.

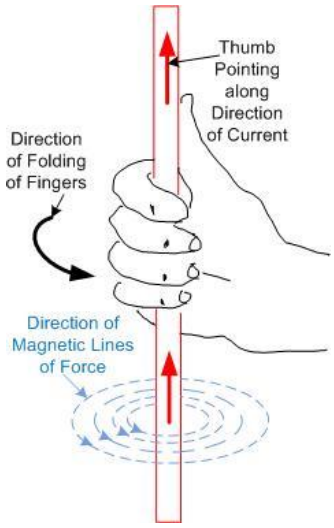
Electromagnetism: Observation of deflection of a magnetic needle when brought near a current carrying current by **Hans Christian Oersted** (pronounced as Ørsted) , in 1819, was a *great beginning to discovery of inseparable and interactive nature of electric current and magnetic field and known as **Electromagnetism***. This would lead to redefining of unit and dimensions of strength of magnetic pole (**m**) and absolute permeability (μ_0) in terms of Fundamental dimensions. It is the most awaited and most happening topic in physics which was deliberately kept in abeyance till proper context and concepts are built.

Ørsted Experiment: An accidental observation of jerks in magnetic near a current carrying conductor by Hans Christian Ørsted in 1819. By then, sufficient knowledge had been gained about magnetism. Accordingly, Ørsted's observation provided a good reason to explore *relationship between electric current and magnetism and a new stream of discoveries called **Electromagnetism** Started.*

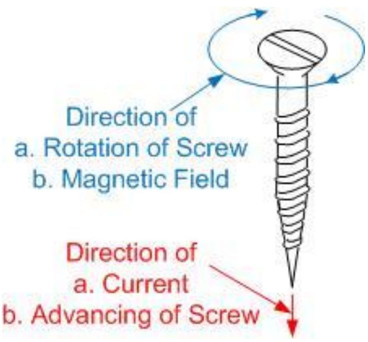
Ørsted found that, for a straight wire carrying a steady (DC) current -



- The magnetic field lines encircle the current-carrying wire
- The magnetic field lines lie in a plane perpendicular to the wire
- If the direction of the current is reversed, the direction of the magnetic force reverses.
- The strength of the field is directly proportional to the magnitude of the current.
- The strength of the field at any point is inversely proportional to the distance of the point from the wire.

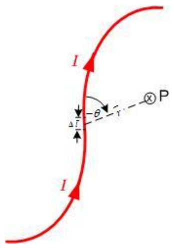


Ampere's Swimming Rule : Investigation into magnetic behavior of current lead to discovery of **André-Marie Ampère**, in 1820 about the direction of magnetic field created by a current carrying conductor. It states that= "*if we imagine a man is swimming along the wire in the direction of current with his face always turned towards the needle ,so that the current enters through his feet and leaves at his head, then the north pole of magnetic needle will be deflected towards his left hand*". It is graphically shown in the figure. It is also known as **SNOW Rule**. It is graphically shown in the figure. It states that in a conductor carrying electric current flowing from **S**outh to **N**orth and it is placed **o**ver the conductor then north pole of the magnetic needle would align along **W**est. A simplification to this rule was made by **James Clerk Maxwell** and is known as **Maxwell's Right Hand Thumb (or Grip) Rule** according to it if *a current carrying conductor is held in right with its thumb pointing towards the direction of current then the folding of fingers is in the direction of magnetic field created by the current*, as shown in the figure. A another version of it, a **Cork Screw Rule** according which



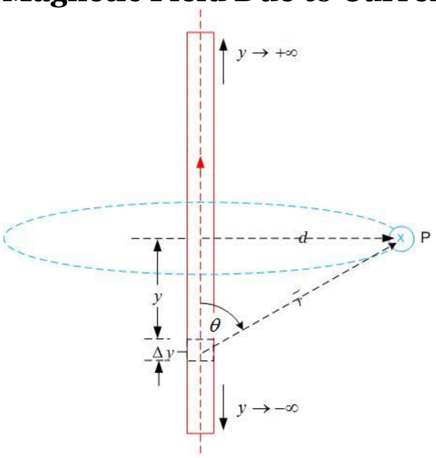
states when *a screw is rotated in cork, direction of rotation of screw and magnetic field is same, while direction of travel of screw is the direction.* Maxwell's Right Hand Screw Rule is considered to be simplest to apply.

Biot-Savart's Law: A major step towards quantification of magnetic field at a point near a current carrying conductor was made , through a mathematical statement, by **Jean-Baptiste Biot and Félix Savart** and is known as **Biot-Savart's Law** in 1820. Accordingly, magnetic field intensity at a point P due to a current ($\boldsymbol{I}$) through an element of infinitesimal length ($\boldsymbol{dl}$) is given by $\boldsymbol{d\bar{B}} = \frac{\mu_0}{4\pi} I \frac{d\bar{l}\times\hat{r}}{r^2}$, **in vector form**. Here, $d\bar{B}$ is the elemental **Magnetic Field Intensity (MFI)** Vector, $d\bar{l}$ is the element vector in the direction of the current ($\boldsymbol{I}$) through the conductor, **r** is distance of the point under consideration from the element of the conductor under consideration, and $\hat{\boldsymbol{r}}$ is the direction vector of the point w.r.t. the conductor element.. It resolves into $d\bar{B} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin\theta(-\hat{k})$. Here, *direction vector* $(-\hat{k})$ *is in accordance with the rule of cross-product of vectors and is in conformance with the Maxwell's Right-Hand Thumb Rule.*

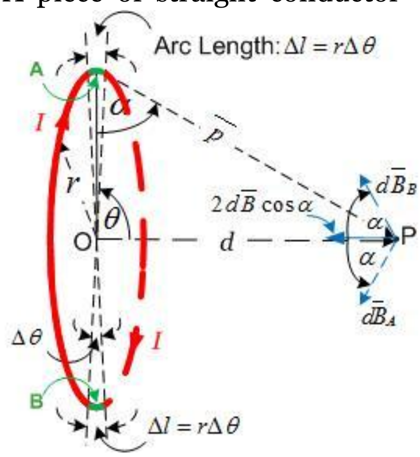


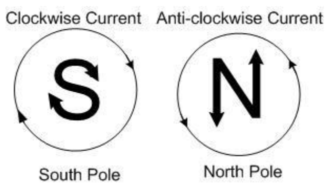
This Law has been extended to Two cases **a)** determination of magnetic field at any point across a long straight conductor and **b)** along axis of a coil. Both the cases are elaborated below.

Magnetic Field Due to Current in a Straight Wire: Conceptually, this analysis is similar to that done for determining electric field intensity E due to a conductor carrying uniform charge density, but difference is in mathematical formation, due to nature of the problem. Accordingly, extending Biot-Savart's Law, MFI due to a small conductor element of Δx as shown in the figure $d\bar{B} = \frac{\mu_0}{4\pi} I \frac{dy\times\hat{r}}{r^2}$. It leads to magnitude $= \frac{\mu_0}{4\pi}\cdot\frac{I}{r^2} dy\sin\theta$. Therefore integrated effect of a straight long conductor $B = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}\sin\theta dy$. It involves three variables, which is simplified by substituting $-y = d\cot\theta \rightarrow dy = (d\,\mathrm{cosec}^2\,\theta)d\theta$. Using the same trigonometric analogy, $r = d\,\mathrm{cosec}\,\theta$, and $B = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\cdot\frac{I}{r^2}\sin\theta dy$. Making substitutions, $B = \int_{-\infty}^{\infty}\frac{\mu_0}{4\pi}\cdot\frac{I}{d^2\,\mathrm{cosec}^2\,\theta}\sin\theta(d\,\mathrm{cosec}^2\,\theta)d\theta$. Resolving this with substitutions of limits, as $y\rightarrow-\infty = \theta\rightarrow 0$, and as $y\rightarrow\infty = \theta\rightarrow-\pi$, the equation becomes $B = \int_0^{-\pi}\frac{\mu_0 I}{4\pi d}\sin\theta\,d\theta \rightarrow \frac{\mu_0 I}{4\pi d}[-\cos\theta]_0^{\pi} = \frac{\mu_0 I}{2\pi d}$.



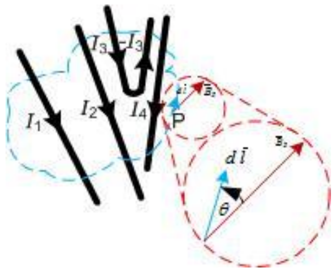
Magnetic Field Due to Current in a Circular Loop along its Axis: A piece of straight conductor carrying current ($\boldsymbol{I}$), in clockwise direction, when shaped into a circular loop field is required to be determined at a point P located at a distance (d) from thee entre of the loop along its axis. Each of the element A and B, of the circular current carrying loop, of length $\Delta l = r\Delta\theta$, has carrying current flowing through it in opposite din opposite directions. Since, these elements A and B are placed diametrically placed opposite they will produce MFIs $d\bar{B}_A$ and $d\bar{B}_B$ at point P which symmetrically displaced by an angle α w.r.t. axis OP, Thus components of the MFIs perpendicular to axis OP would cancel out and net field would be directed towards center O in accordance with *Maxwell's Right Hand Thumb Rule and magnitude* $dB' = 2dB\cos\alpha = 2\cdot\frac{\mu_0}{4\pi}\cdot\frac{Idl}{p^2}\cos\alpha$ is as per *Biot-Savart's Law. Accordingly,* $B = \int_0^{\pi}\frac{\mu_0 I}{2\pi p^2}\cdot rd\theta\cdot\cos\alpha = \frac{\mu_0 Ir}{2p^2}\cdot\cos\alpha$. It leads to Magnetic Field Intensity at point P, $B = \frac{\mu_0 Ir}{2(r^2+d^2)}\cdot\frac{r}{\sqrt{r^2+d^2}} = \frac{\mu_0 Ir^2}{2(r^2+d^2)^{\frac{3}{2}}}$. Thus MFI at point O, when $d\rightarrow 0$, $B = \frac{\mu_0 I}{2r}$ and shall be along $-\hat{\boldsymbol{\imath}}$ or it will act like south pole with MLF entering the loop. Determination B at any point off center of the loop becomes a complex mathematical formation, nevertheless inquisitive readers are welcome to write through *Contact Us.*



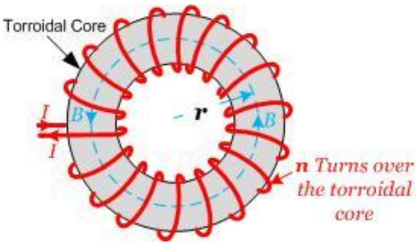


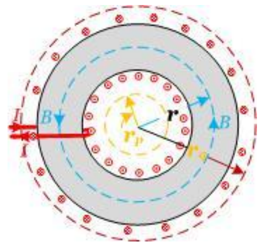
Likewise for current in clockwise direction shall be along $\hat{\boldsymbol{\imath}}$ i.e. it will act like north pole with MLF leaving the loop. In this derivation width of loop has no significance and is in ***conformity with the premise, that North-South pole always coexist***. An easy to apply anecdote to remember the magnetic pole created by a loop with current seen to be flowing in the coil, either clockwise and anticlockwise direction of current, is shown in the Figure.

Ampere's Circuital Law : It is an extension of *Bio-Savart's Lawm propounded by Ampere in 1823 to* quantitatively relates MFI (B) around a current source(s). It states that ***line integeral of Magnetic Flux Density in a closed path is proportional to current encircled inside the closed path.*** *It is mathematically expressed as* $\oint_C \bar{B}\cdot d\bar{l} = \mu_0 I_{enc}$. A closed path is traced around a system of current carrying conductor, as shown in the figure, such that net current inside the loop is $I = I_1 + I_2 + I_3 - I_3 + I_4 = I_1 + I_2 + I_4$. Taking a small element of the loop of length vector $d\bar{l}$ at point P where let MFI is $\overline{B_p}$, then as per Ampere's Circuital Law $\oint_C \overline{B_p}\cdot d\bar{l} = \oint_C B_p \cos\theta\, dl = \mu_0 I$. As per *Biot-Savart's Law* MFI at a distance r from the axis of a long conductor carrying current (I) is $B = \frac{\mu_0 I}{2\pi r}$. This B is always tangential to the circular path of length $2\pi r$ and satisfies *Ampere's Circuital Law*.

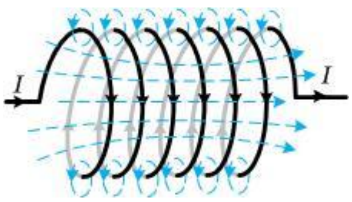


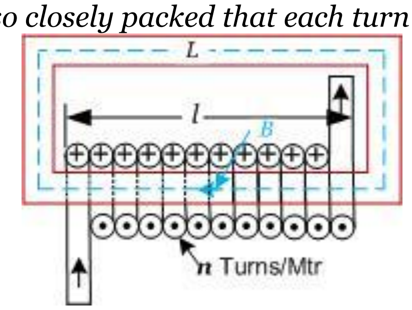
Extension of *Ampere's Circuital Law* (**ACL**) practical applications has been through **Torroids and Sloenoid** and is elaboreated below. **Torroid** is a circular core of Ferromagnetic material having relative ermeability μ_0 over it circular loops in series are wound with one inlet and outlet for current $\boldsymbol{I}$. A circular path of radius $\boldsymbol{r}$ is taken inside to the torroid. Then as per ACL, $\oint_C \bar{B}\cdot d\bar{l} = 2\pi r B = \mu(NI) = \mu_0\mu_r(2\pi r n)I$. It leads to $\oint_C \bar{B}\cdot d\bar{l} = (2\pi r)\boldsymbol{B} = \boldsymbol{\mu_0\mu_r N I}$, here total number are turns $\boldsymbol{N} = \boldsymbol{2\pi r n}$, where $\boldsymbol{r}$ is the radius of the circuital path chosen and $\boldsymbol{n}$ is number of turns per-unit length on the toroid. Accordingly, **MFI inside the toroid** $\boldsymbol{B} = \frac{\boldsymbol{\mu N I}}{\boldsymbol{2\pi r}}$.





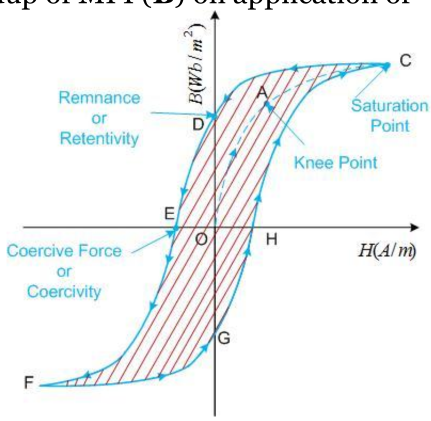
Now, taking a path of radius $\boldsymbol{r_p}$ in air inside the inner radius of the toroid, applying ACL to determine B_p along the path $\oint_C \overline{B_p}\cdot d\bar{l} = 2\pi r_p B_p = \mu_0(0\cdot I) = 0$, it leads to $B_p = 0$, since $r_p \neq 0$. Likewise, for a path of radius $\boldsymbol{r_q}$ outside the torroid, $\oint_C \overline{B_q}\cdot d\bar{l} = 2\pi r_q B_q = \mu_0 N\cdot(I - I) = 0$, or $B_q = 0$. **Solenoid** is a like a straightened torroid cut at between points of injection and exit of current as shown in the figure. Such a formation of coil is like a helix and has fringing of MLFs at ends is seen due to resultant magnetic field created by gaps between adjoining turns as shown in the figure. *In solenoid turns are thin conducting loops, insulated from each pther, are so closely packed that each turn can be considered to be perpendicular to the axis of the solenoid.* In absence of this assumption there is fringing of B at ends of a solenoid, which would make it quite complicated for evolving solution at this stage, and hence the assumotion is carried through. In analysis of the idealized solenoid having $\boldsymbol{n}$ turns per unit length is idealized such that it is placed on a ferromagnetic core of permeability $(\mu = \mu_0\mu_r)$ having uniform cross-section ($\boldsymbol{A}$) and mean circutal length ($\boldsymbol{L}$) but solenoid having $\boldsymbol{n}$ turns per meter length spread over a length ($\boldsymbol{l}$) such that total number of $N\ (= nl)$, applying ACL $\oint_C \bar{B}\cdot d\bar{l} = \overline{\boldsymbol{B}}\cdot\overline{\boldsymbol{L}} = \boldsymbol{BL} = \boldsymbol{\mu}(\boldsymbol{nl})\boldsymbol{I} = \boldsymbol{\mu N I}$.



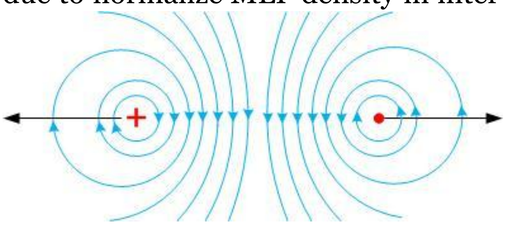


It leads to total flux in the magnetic corecreated by the solenoid is $BA = \phi = \frac{NI}{\frac{l}{\mu A}}$. In electromagnetism force driving flux is called **Magneto-Motive Force (MMF)** *which is mathematically expressed as* $NI = \emptyset\left(\frac{l}{\mu A}\right)$, *is anlogus to Ohms Law in Current Eletricity*; here MMF (NI) corresponds to EMF, Magnetic Flux $(\emptyset)$corresponds to Electric Current, Reluctance $(\Re = \frac{l}{\mu A})$ corresponds to Resistance (R), and permittivity (μ) is analogus to conductivity (σ) which is reciprocal of resitivity $\left(\rho = \frac{1}{\sigma}\right)$. *Solenoid has extensive application in electromagnets, electrical devices and machines, subject matter that shall be introduced a little later and advanced studies in engineering.* ***Unit of MMF, generally*** *expressed as* ***H, is A-m-1***. **Magnetic Susceptibility of Magnetic Materials:** In **magnetic circuits** presence of magnetic material influences its behaviour. The permeability (μ) has two components – **a)** absolute permeability (μ_0), already defined earlier and **b) susceptibility** (χ)**.** These are related through an equation which defines characteristic of material and it is $\mu = \mu_0(1 + \chi)$. Value of χ for three basic class of magnetic materials classified earlier are- **i)** Ferromagnetic Materials $\chi \gg \mathbf{1}$**, ii)** Diamagnetic materials $\chi > \mathbf{1}$**,** for **iii)** Diamagnetic Materials $\chi < \mathbf{0}$**.**

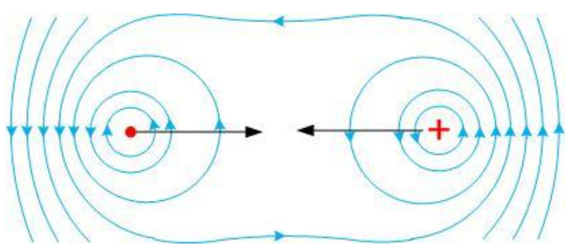
Magnetizing Characteristics of Ferro-magnetic Material: It is the buildup of MFI (**B**) on application of MMF (**H**), a typical plot of the magnetizing characteristic is shown in figure. It depends upon point of start of magnetization based on residual magnetism in the magnetic material. This residual magnetism depends upon magnetic history of the magnetic specimen and can be better understood taking magnetization of a totally demagnetized ferromagnetic material shown by Point 'O' on B-H curve, in figure. As MMF (***H***) is increased, magnetic induction increases almost linearly upto point 'B', it is known as **Knee Point**. Beyond Knee Point incremental increase in ***H*** for increase in ***B*** is non-linearly high. Beyond Point 'C' practically there is no increase in **B**, despite increase in ***H***. On reaching point 'C' decreasing ***H*** takes a path above the curve OAC, such that when $\boldsymbol{H} = \mathbf{0}$, there is magnetic induction, called ***Residual Magnetism, Remnance or Retentivity*** identified as 'D' a point of intersection of magnetization curve with ordinate i.e. ***B***-axis. Moving forward on the (–)ve *H*-axis, it is at point 'E', ***B*** reduces to Zero, and is called ***Coercive Force or Coercivity.*** Saturation point 'F' on (-)ve induction is shown in figure, after reaching that increase in ***H*** traces magnetization curve a paths FGHC, and this cycles repeats, unless magnetization is abruptly changed, which make the retracing of magnetizing characteristic plot different from the earlier. The loop CDEFGHC is called ***Hysteresis Loop*** and are under the loop is called ***Hysteresis Loss.***



Force Between Two Current Carrying Conductors: Force between Two current carrying conductors can be classified in Two categories, First when conductors are parallel and Second when there is angular shift between the Two conductors. This again leads to Two cases – a) Current in the Two conductor is in same direction and b) Current in Two Conductors is in opposite direction. A qualitative analysis reveals that the parallel conductors carrying current in opposite direction have unidirectional flux in the inter-conductor region and thus tends to increase the MLF, which have as per their property do not intersect each other. Thus in order to adjust to uniform density of MLF, they act as tight stings of a bow and trying to push the conductors away like arrows. This results into a force of repulsion on the conductors as shown in the figure. Extending the same logic to conductors carrying current in same direction it is seen that conductors experience of attraction due to normalize MLF density in inter-conductor region is feeble due cancellation MFI due to Two conductors



in opposite directions. *Quantitative analysis of this observation was made by Ampere in 1825 and Gauss in 1833 and to calculate force between two parallel conductors carrying current. As known as* ***Ampere's Force Law***. According to the Law ***force per unit length experienced by two parallel conductors, [1]carrying currents*** $\boldsymbol{I_1}$ ***and*** $\boldsymbol{I_2}$***, separated by a distance r is equal to*** $\frac{\boldsymbol{F}}{\boldsymbol{L}} = \boldsymbol{B} \cdot \boldsymbol{I_2} - \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2}{r}$ **N-m-1.** Explanation for the interrelation between force and two current conductors is due interactive nature of magnetic field and current. It has been derived from Biot-Savrat's Law that MFI at a distance ***r*** from a straight conductor carrying current $\boldsymbol{I_1}$ is $B_1 = \frac{\mu_0 I_1}{2\pi r}$ and force experience by another conductor carrying current $\boldsymbol{I_2}$, placed in this magnetic field, force experienced by the conductor per unit length is $F/l = B_1 I_3$ **and in vector form** $\frac{\overline{\boldsymbol{F}}}{\boldsymbol{l}} = \overline{\boldsymbol{I}} \times \overline{\boldsymbol{B}}$. *It was used to define unit of current* ***Ampere.*** *In SI unit* $\mu_0 = 4\pi 10^{-7}$ *N-A-2.*

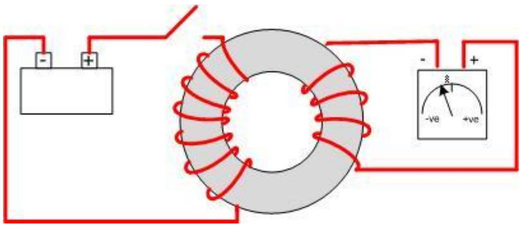


Accordingly, in SI unit when Two conductors each carrying ***One unit Current****, causes force between the conductors is* 10^{-7} *N, with dimension* ***I as one of the Fundamental Dimensions.***

Further, once unit of current is defined in measurable units of mechanics, units equivalent of Charge (Q) which is One Coulomb equal to One Ampere-Second having Dimension $[IT]$, where $[I]$ is one of the fundamental dimensions; Flux Density *(B)* is Tesla having dimension $[MI^{-1}T^{-2}]$and is equal to Weber/per m²; Magnetic Flux (ϕ) has a unit Weber having dimension $[ML^2I^{-1}T^{-2}]$; Magnetic Pole Strength (***m***) as Ampere-Meter having dimension $[IM]$, Permittivity of space (ε_0) has a unit Coulomb/Newton/Meter² ($C^2N^{-1}M^{-2}$) having dimension $[M^{-1}L^{-3}T^4I^2]$. Maxwell through his Electromagnetic Equations unified permittivity (μ_0) and permeability (ϵ_0) into velocity of light such that $\mu_0\epsilon_0 = \frac{1}{c^2}$. Since Maxwell's Electromagnetic theory is outside domain of this manual, therefore any inquisitiveness of the readers together verify dimension of each of the above electro-magnetic quantity are welcome through *Contact Us.*

Force On a Moving Charge: This is a stage where independent interaction of magnetic field and electrical charges has been established and a new point of exploration is created to know behaviour of a moving charged particle in presence of both the magnetic field and electrostatic field. This integrated behaviour was propounded by **Hendrik Lorentz** in 1896 and is known as **Lorentz Force Equation** and it is $\bar{F} = q\bar{E} + q\bar{v} \times \bar{B}$. The Force (in Newton) *has two components –* ***a)*** *component of Electrostatic Force* ***(= qE)*** *pronounced by Coulomb's Inverse Square Law, where E is the electric field intensity created by a system of charges, and* ***b)*** *component of magnetostatic force created by magnetic field intensity on a moving charge* ($= q\bar{v} \times \bar{B}$) *as per Ampere's Force Law, here* ***q*** is the charge on the moving particle (in Coulomb) and $\bar{v}$ is the velocity vector of the charged particle in m/sec. In this component of magnetostatic force definition of current ($\bar{I} = q\bar{v}$) is used. This magnetic field can be due to permanent magnets, terrestrial magnetism, electromagnets or a current carrying conductor as discussed earlier. *Thus this development is attributed to cumulative contributions of many scientists.* This integration by Lorentz created a path for invention of Cyclotron by Ernest O. Lawrence in 1932 to invent cyclotron to accelerate charged particles and a new era in nuclear physics.

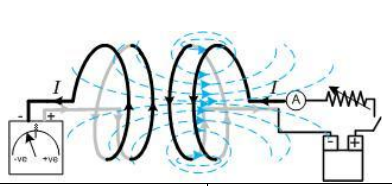
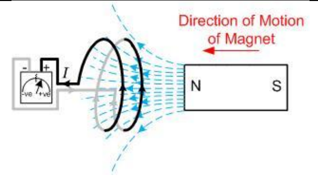
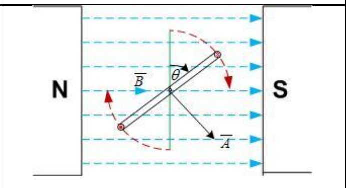
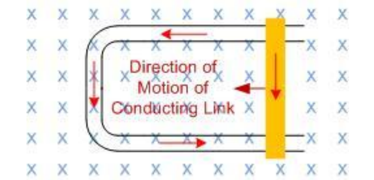
Electromagnetic Induction: Another big leap in Electromagnetism was provided by ***Michael Faraday*** through experimental verification of induction of voltage and in turn current through relative change in linkage of flux with a coil in 1831 through an experiment as shown in the figure. Whenever, circuit supplying current to one coil on a toroid was closed or interrupted, current was induced current in the other coil wound



on the toroid. Direction of the current induced in second coil while closing switch supplying current to the first coil was opposite to that while interrupting. Incidentally, in 1832 **Joseph Henry** independently discovered the phenomenon but, publication of observation by Faraday precedes that of Henry and accordingly it is known as **Faraday's Laws of Induction** which states that whenever magnetic field through closed circuit changes an ***electromotive force (EMF) is induced. in any closed circuit is equal to negative rate of change of the magnetic flux enclosed by the circuit***. It is mathematically expressed as $E = \left|\frac{d\Psi}{dt}\right| = \left|\frac{d(N\emptyset)}{dt}\right|$, here $\emptyset$ is the flux linking ***N*** turns of the coil. This law was silent in respect of direction of EMF which was complemented by ***Emil Lenz*** in, 1834, stating that ***direction of induced in coil by changing flux, called flux linkage, as per Faraday's Law of Induction, is such that it will create a magnetic field that opposes the change****. De facto it is electrical versions of Newton's Third Law of Motion reaction is equal and opposite to the action. Thus* ***complete version of Faraday's Law is*** $E = -\frac{d\Psi}{dt} = -\frac{d(N\emptyset)}{dt} = \frac{d}{dt}\oint N\bar{B} \cdot d\bar{S}$***, here, (−)ve sign is attributed to Lenz and thus it can also be called Faraday-Lenz's Law, with a necessary condition that flux linkage must be time variant.*** This law became an integral part of Electromagnetic Theory propounded by Maxwell, much talked about in this text, but in a different form which is outside scope of this manual and inquisitive readers are welcome to write through *Contact Us*.

This principle with above broad classification has found widest application in growth of science. It is pertinent to recall a conversation of Faraday, when he demonstrated **Dynamo**, a device to convert mechanical energy into electrical energy, based on this principle of electromagnetic induction, he was asked by a curious person – "***What is use of this new device*** *(dynamo)*". Faraday, very humbly replied – "***it has the same use as that of a new born child". That dynamo has tuned out to be (Great)^n Grandfather of the energy sources that are supplying electricity to support the technological development on this earth.***

Faraday's Laws are applicable to changing magnetic field w.r.t. time for whatever reason be it – **Case I:** changing MMF, **Case II:** shifting coil in magnetic field, **Case III:** rotation of coils causing angular displacement between coil surface and /or, **Case IV:** changing of area intercepting magnetic field and has found application over a very wide range. This broad classification is conceptualized in table below, with analytical elaboration to follow.

Case I		**Case II**		**Case III**		**Case IV**	
							
Cause	**Effect**	**Cause**	**Effect**	**Cause**	**Effect**	**Cause**	**Effect**
Current in Coil on the right is increasing through rheostat. It tends to increase flux linkage of coil.	Induced current in coil on the left is tending to compensate increase in flux-linkage of coil.	Motion of Coil away from magnet, tends to decrease in flux linkage of coil.	Induced current in coil is clockwise, tending to compensate decrease in flux-linkage of coil.	Rotational motion of coil in uniform magnetic field, tends to decrease flux linkage of coil.	Induced current in coil tending to compensate decrease in flux linkage of coil.	Translational motion of conducting link in uniform magnetic field, tending to decrease flux linkage of loop.	Induced current in loop tends to compensate decrease in flux linkage of loop.

Inductance: *Biot-Savart-Ampere's Law* together with *Faraday-Lenz's Law* has introduced **Inductance** a Third property of electrical element, after *Resistance and Capacitance*. Inductance is classified in Two Categories – **a) Self-inductance** of an electrical due to flux created by the current flowing through itself, and **b) Mutual Inductance** of an electrical element is due linkage of in another electrical element. This *Inductance in accordance with Faraday-Lenz law come into play only when current causing the flux changes*, which states that $E = \frac{d\Psi}{dt} = \frac{dLi}{dt} = L\frac{di}{dt}$. *This inductance for an electrical element depends upon the geometry of the element is fixed, as long as geometry of the element remains unchanged*. Determination of inductance is outside domain of this manual, nevertheless for simple geometry its being elaborated to strengthen basic concept.

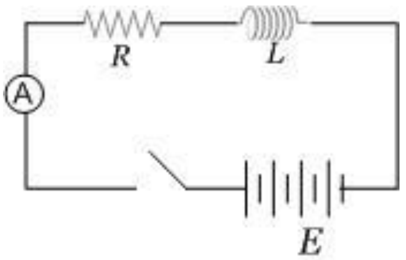
In an idealized solenoid on a toroid $\boldsymbol{B} = \frac{\boldsymbol{\mu NI}}{\boldsymbol{2\pi r}}$, therefore total flux linking the N turns of the toroid $\emptyset = \int \bar{B} \cdot d\bar{A}$. Thus $\Psi = N\int \bar{B} \cdot d\bar{A} = N\left(\frac{\boldsymbol{\mu NI}}{2\pi r}\right)A = (n\boldsymbol{2\pi r})\left(\frac{\boldsymbol{\mu(n2\pi r)I}}{2\pi r}\right)A = \mu n^2 lAI$. Here, $\boldsymbol{n}$ is number of turns of solenoid spread over per unit length of toroid, $\boldsymbol{l}$ is the mean length of the toroid and $\boldsymbol{A}$ is the area of cross-section of toroid such that magnitude $\boldsymbol{A}>>\boldsymbol{l}$. Accordingly, flux linkage per ampere of current, is called $\boldsymbol{L} = \frac{\Psi}{\boldsymbol{I}} = \boldsymbol{\mu n^2 lA}$. Since, this inductance of solenoid on toroid is due to flux created by solenoid, when current flows through it, is linking it's own turns and hence it is called **Self-inductance**. Likewise, when flux produced by one coil called ***primary coil***, links other coil, called **secondary coil**, *as shown in Case I in elaboration of Faraday's Laws of Induction*. In this case $\Psi = N_1k_1\emptyset_2 = k_1N_1(k_2N_2I_2) = kN_1N_2I_2$, here k_1 and k_2 depend upon geometry of primary and secondary coils and properties of magnetic circuit, while N_1 and N_2 number of turns of the respective coils. Thus, $M = \frac{\Psi}{I} = \boldsymbol{kN_1N_2}$, where $\boldsymbol{k = k_1k_2}$ a geometrical-cum-magnetic constant of pair of the Two coils, and M is called their ***Mutual inductance***. Unit of Inductance is Henry and its dimension is $[ML^2I^{-2}T^{-2}]$.

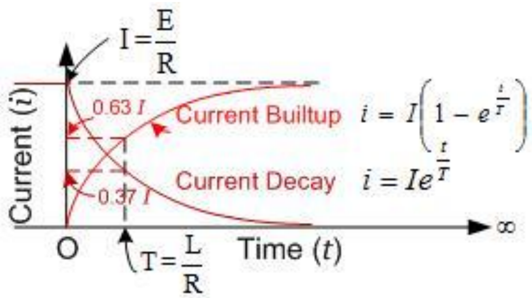
With this definition of inductance, voltage across an inductance is $V_L = L\frac{dI_L}{dt}$, accordingly power in inductor at any point of time is $p_L = V_L I_L = \left(L\frac{dI_L}{dt}\right)I_L = L\left(I_L\frac{dI_L}{dt}\right)$, and **energy in inductor** $\boldsymbol{E = \int p_L dt = L\int I_L dI_L = \frac{1}{2}LI_L^2}$. It is to be noted that *energy in the inductor is stored in the form of magnetic field and is attributed to current through the inductor. This is analogous to energy in capacitor* $\left(=\frac{1}{2}CV^2\right)$ *where energy stored in the form of electric field and is attributed to voltage across the capacitor*. Accordingly a comparison of inductor and capacitor is as under –

Parameter	**Inductor**	**Capacitor**
Constant Value	**L :** depends upon geometry of electrical element	**C:** depends upon geometry of the electrical element
Unit	Henry (H)	Farad (F)
Dimension	$[ML^2I^{-2}T^{-2}]$	$[I^2T^4IM^{-1}L^2]$
Voltage across the element	$E = L\frac{dI}{dt}$	$V = \frac{Q}{C}$
Current across the element	I	$I = \frac{dQ}{dt} = C\frac{dv}{dt}$
Energy across the element	$E = \frac{1}{2}LI^2$	$E = \frac{1}{2}CV^2$

Combination of resistance (R), inductance (L) and capacitance (C) has found wide application in electrical systems and hence influence of these three elements, in various combinations, is extremely useful in analysis and their application. Accordingly, three generic combinations are **a)** *RL* circuits, **b)** *RC* circuits and **c)** *RLC* circuits when supplied from battery are being taken up initially.

RL Circuits: There are two cases in this circuit - **a)** inductance is fully energized, i.e. no current is flowing through and **b)** inductance is energized i.e. current is flowing through it. Taking **First Case**, when switch is closed, as per Kirchhoff's Voltage Law (KVL) $= iR + L\frac{di}{dt}$. Accordingly, $\int_{0+}^{i}\frac{1}{E-iR}di = \int_0^t\frac{dt}{L}$. Taking an intermediate variable $u = E - iR$ it leads to $du = -Rdi$. Substituting u it resolves into $-\frac{1}{R}\int_{0+}^{i}\frac{R}{u}du = \frac{1}{L}\int_0^t dt$. It leads to $\int_{0+}^{i}\frac{1}{u}du = -\frac{R}{L}\int_0^t dt$, which on integration is $[\log(E - iR)]_0^i = -\frac{R}{L}t$, or $\log\frac{E-iR}{E} = -\frac{R}{L}t \rightarrow \frac{E-iR}{E} = e^{-\frac{R}{L}t}$. It simplifies into $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$. Thus, at $t = 0$, $e^{-\frac{R}{L}t} = 1$ and hence, $i = 0$. While, as $t \rightarrow \infty, e^{-\frac{R}{L}t} \rightarrow 0$, accordingly, $I = \frac{E}{R}$. Thus the expression of current reduces to $i = I\left(1 - e^{-\frac{t}{T}}\right)$, where $\boldsymbol{T = \frac{L}{R}}$ is the ***Time Constant of RL circuit***.

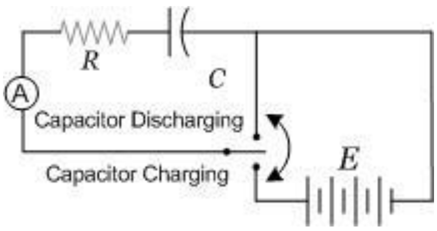




Likewise, when current (I_0) flowing through ***inductor is interrupted*** accordingly voltage equation would be $0 = iR + L\frac{di}{dt}$, since there is no voltage source in the circuit. Further, set different set of boundary conditions would exist, such that at $t = 0_+$ current through the circuit is I, and as $t \rightarrow \infty$, $i \rightarrow 0$, since after switching of the circuit there is no source of current. Accordingly, $\int_{I_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt$. On integration in leads to $\log\left(\frac{i}{I_0}\right) = -\frac{R}{L}t \rightarrow i = I_0e^{-\frac{R}{L}t}$ and in its standard form $i = I_0e^{-\frac{t}{T}}$. In the ***i-t curve*** shown in the figure $I_0 = I$ has been taken. A comparison of the current through RL circuit when switched ON and OFF is shown in a table below.

RC Circuits: There are two cases in this circuit - **a)** a discharged capacitor is charged, i.e. no voltage across the capacitor, and **b)** an energized capacitor is discharged. Taking **First Case**, when switch is closed, as per Kirchhoff's Voltage Law (KVL) $E = iR + \frac{q}{C} = R\frac{dq}{dt} + \frac{q}{C} \rightarrow \frac{EC-q}{C} = R\frac{dq}{dt} \rightarrow \frac{1}{RC}dt = \frac{1}{EC-q}dq$. Integrating both sides,

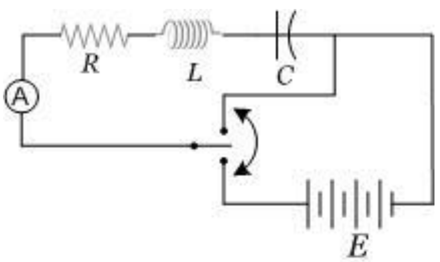
$\log(EC-q)=K-\frac{1}{RC}t$, here **K** is *integration constant*. It leads to $EC-q=e^{K-\frac{1}{RC}t}$. At $t=0_+, q=0,\ EC-0=e^K$ since capacitor is fully discharged, It transforms the equation into $EC-q=ECe^{-\frac{1}{RC}t}$, or, $q=EC\left(1-e^{-\frac{1}{RC}t}\right)=EC\left(1-e^{-\frac{t}{T}}\right)$, here $\boldsymbol{T}$ is the **Time Constant of RC circuit** is $T=RC$. This equation is analogues into when inductor is switched in the first case, and so will be the charge build-up curve w.r.t time ($\boldsymbol{t}$). Accordingly, a capacitor in RC circuit would be 63% charged at its Time Constant. Likewise, when Capacitor as per KVL, $0=iR+\frac{q}{C}\to R\frac{dq}{dt}=-\frac{q}{C}$. Accordingly, $\log q=-\frac{t}{RC}+K$, or $q=e^{K-\frac{t}{RC}}$. Let, at $t=0$, charge on capacitor is Q_0, then $Q_0=e^K$ accordingly the capacitor discharge equation transforms into $q=Q_0e^{-\frac{t}{RC}}$. This equation is also analogues to current discharge equation of RL circuit, and capacitor of a RC circuit shall retain 37% charge at its time constant.



The behavior of RL and RC circuits, elaborated above is summarized below.

Behaviour of RL and RC Circuit When Switched ON and OFF					
Parameter		**RL Circuit**		**RC Circuit**	
		Switch ON	**Switch OFF**	**Switch ON**	**Switch OFF**
Initial Current at $(t=0_+)$		$i=0$	$i=I_0$	$q=0$	$q=Q_0$
Instantaneous Current		$i=I\left(1-e^{-\frac{t}{T}}\right)$	$i=I_0e^{-\frac{t}{T}}$	$q=EC\left(1-e^{-\frac{t}{T}}\right)$	$q=Q_0\left(1-e^{-\frac{t}{T}}\right)$
At Saturation i.e. at $t\to\infty$		$I=\frac{E}{R}$	$I=0$	$Q=EC$	$Q=0$
Behaviour	at $t=0_+$	Open Circuit	Short Circuit	Short Circuit	Open Circuit $(i=0)$
	at $t\to\infty$	Short Circuit	Open Circuit $(i=0)$	Open Circuit $(i=0)$	Short Circuit
Time Constant		$T=\frac{L}{R}$		$T=RC$	
Current at $t=T$		$i_T=0.63\ I$	$i_T=0.37\ I_0$	$q_T=0.63\ Q$	$q_T=0.37\ Q_0$

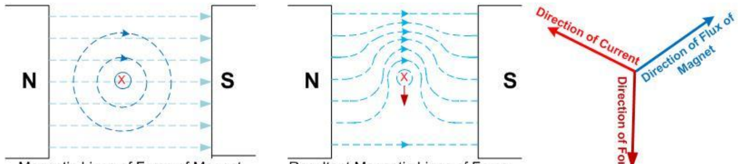
RLC Circuits: Having learnt about three basic electrical elements R, L and C and RL, RC circuit, there shall be an obvious curiosity to know about behavior of a circuit having all the three elements are there in the circuit forming an RLC circuit. This also has Two possibilities – **a)** Switching On of the RLC Circuit and **b)** Switching of RLC Circuit. According to KVL, voltage equation in **First Case** would be $E=iR+L\frac{di}{dt}+\frac{q}{C}\to R\frac{dq}{dt}+L\frac{d^2q}{dt^2}+\frac{q}{C}$. Rearranging the voltage equation, $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E$ is a Second Order Differential Equation and is outside the scope of this manual. Likewise, in **Second Case**, the voltage equation is of the form $\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$, and is again a Second Order Differential Equation. Nevertheless, inquisitive readers are welcome to write through Contact Us. *RLC Circuit in Second case, by ignoring resistance takes a form* $\frac{d^2q}{dt^2}=-\frac{1}{LC}q=0$,



and is of oscillatory nature where charge on capacitor and in turn energy stored in it in the form of electric field reversibly transferred to inductor in the form of magnetic field and this cycle continues at a frequency $f = \frac{1}{2\pi\sqrt{LC}}$, *and is called natural frequency of the oscillatory LC circuit. This is analogous to SHM motion where interchange of potential energy into kinetic energy and vice-versa continues. In LC circuit, resistance R acts, like friction in SHM, to damp the oscillation magnitude, without affecting its natural frequency* (f).

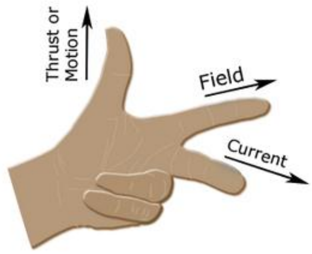
Application of Electromagnetism: Electromagnetism has found widest application from industry, domestic, medical, communication and name the field it is there. Nevertheless, limiting these applications to the scope of this manual three basic applications are being elaborated **a) *Electric Motors*** for conversion of electrical energy to mechanical energy, **b) *Generator*** for conversion of mechanical energy to electrical energy, **c) *Galvanometer*** for measurement of electrical quantities, **d) *Eddy Currents*** and **e**) *Induction Coil*. These three applications have found utility with many of its variants which are again outside scope of this manual, which is aimed at reinforcing basic concepts.

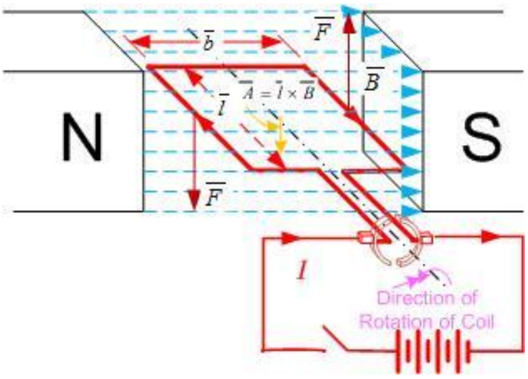
Electric Motor: Basic laws of electromagnetism discussed so far are being extended to application Ampere's Force Law seen qualitatively has been brought out in figure. This was generalized by **John Ambrose Fleming** in late 19th Century as **Fleming's Left Hand Rule (FLHR)** and is quite explicitly shown in the figure. *FLHR is anecdote directional relevance of Ampere's Force Law, applicable to force on current carrying conductor placed in a magnetic field*. This rule is used to explain direction of rotation Electric Motor. A conceptual representation of electric motor is also shown in the figure.



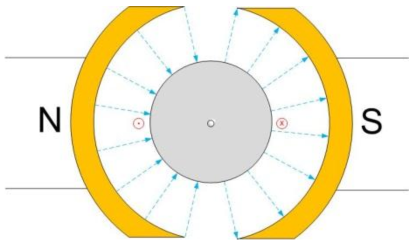
Magnetic Lines of Force of Magnet and Current Carrying Conductor shown Independently

Resultant Magnetic Lines of Force Caused byof Magnet and Current Carrying Conductor

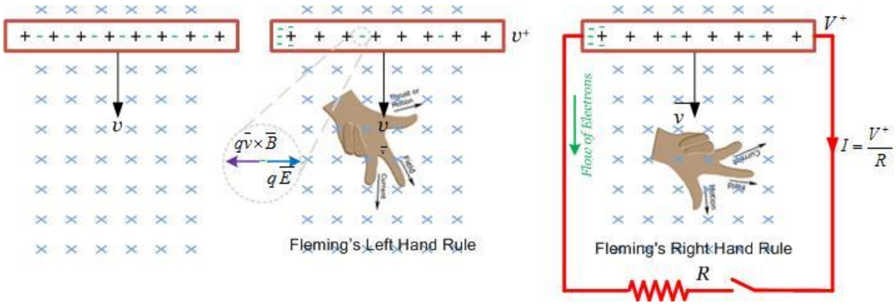




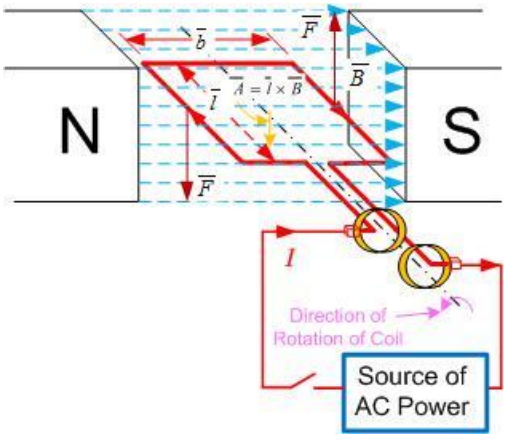
Force on both the section of coil each conductor of length $\bar{l}$ *are carrying equal current I and in uniform magnetic field* $\bar{B}$, *Accordingly, force on the conductor as per Amperes Force Law* $\frac{\bar{F}}{l} = \bar{I} \times \bar{B} \rightarrow \bar{F} = l\bar{I} \times \bar{B} = I\bar{l} \times \bar{B}$, here interchange of scalar and vector quantity $l \leftrightarrow \bar{l}$ is valid because current is along the conductor and there is no change in vector form of equation. Accordingly, Two equal, opposite and parallel forces $\bar{F}$ separated by a distance $\bar{b}$, width of the coil form a couple, and similar to the electric and magnetic dipole and thus, $\bar{\Gamma} = \bar{b} \times \bar{F} = \bar{b} \times I\bar{l} \times \bar{B} = I\left((\bar{b} \times \bar{l}) \times \bar{B}\right)$. Here, area of the coil $\bar{A} = \bar{l} \times \bar{B}$ and accordingly torque equation transforms into $\bar{\Gamma} = (I\bar{A}) \times \bar{B} = \bar{\mu} \times \bar{B}$, where $\bar{\mu} = I\bar{A}$ is called ***Magnetic Dipole Moment*** or ***Magnetic Moment*** *of current loop. In case coil has multiple turns* (***n***) *then* $\bar{\mu} = nI\bar{A}$. Since, Torque $\bar{\Gamma}$ is a vector product, as coil turns, torque on the coil, which is maximum when face of the coil is along the magnetic field, which in turn has an angle θ between vectors $\bar{A}$ and $\bar{B}$ equal to 90^0 since $\sin\theta|_{\theta=90^0} = 1$. But, as coils turns, in every rotation θ varies $0^0 \le \theta \le 360^0$. This sinusoidal variation, similar to magnitude of a wave, is smoothened by making a magnetic flux radial with innovation as shown in the figure. Here, poles are provided with identical arc shaped cover with a circular core, to make the flus concentric and perpendicular to both the magnetic surfaces. Inside the core, MLFs complete there path from north pole to south pole. Coil on the core are embedded into it in motors, unlike that shown in the figure, to make the core rotate about its axis, and transfer mechanical energy through shaft at the axis of the core like axle of the paddle of a bicycle. While in ***galvanometer***, used for measurement, to be elaborated a little later, the core is fixed, while coils made of fine wire or mounted on a former and detached from the core.

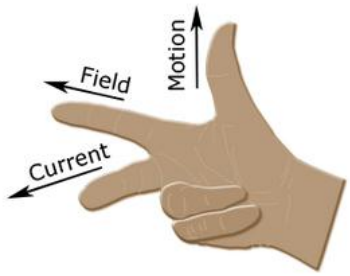


Generator: It is an extension of Case 2, 3 and 4 of Faraday's Law of Electromagnetic Induction. It is a device used to convert mechanical energy into electric energy by changing linkage of magnetic flux w.r.t. a coil mounted on a shaft, coupled to a mechanical device called ***prime mover*** not shown in the figure.

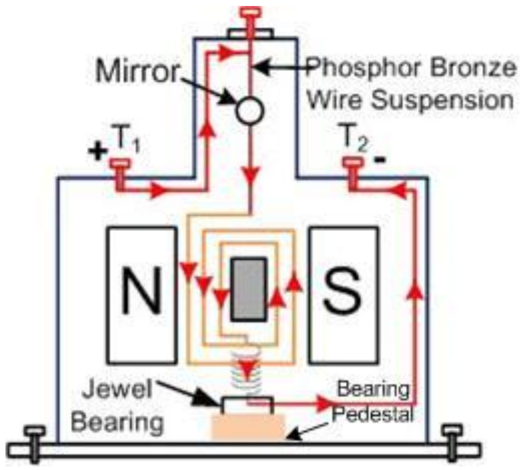


Conceptually, *it is similar to that in case of electric motor, with only one difference that in motor rotation of coil is produced when current is passed through, while* ***in generator, rotation of coil produces an EMF capable of driving current in the circuit closing the coil,*** *as shown in the figure*. Conductor, as discussed in current electricity consists of (+)ve charges due to protons embedded in nucleus and electrons as (-)ve charges some of which keep revolving around nucleus as bound charges and some of it are like ***free electron*** cloud performing Brownian motion inside volume of the conductor. Thus when conductor is moved in the magnetic field it experiences electrostatic and electromagnetic forces as *per* ***Lorentz's Force Equation***. *Such drift of electrons continues till an equilibrium is produced between Electrostatic Force and Electromagnetic force and thus potential difference between two ends of the conductor is called EMF. When, two ends of the conductor are closed through a resistance, the EMF drives current through circuit, which in turns equilibrium of constituents of the Two Forces. This in turns induces drift of electrons to maintain the EMF corresponding to B, angular velocity of coil, area of cross-section of coil and its instantaneous position of coil, in accordance with Faraday's Law of Induction. It is to be noted that forces on electrons (-)ve charge and their drift is opposite to the notional direction of forces and current, which is defined for (+)ve charges. Accordingly, correlation between, motion, flux and current is defined by Fleming known as* **Fleming's Right Hand Rule (FRHR);** this in fact is a colloloray of FLHR. *A generator producing alternating current of sinusoidal waveform, known as* ***Alternator****, is consptualized in the figure. Aleternating Current and circuits shall be elaborated in next section.*

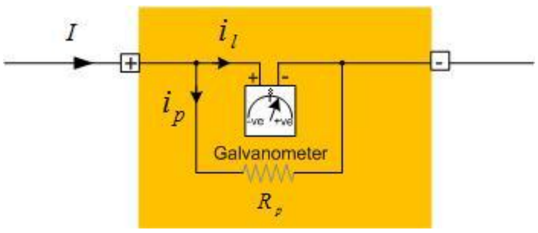
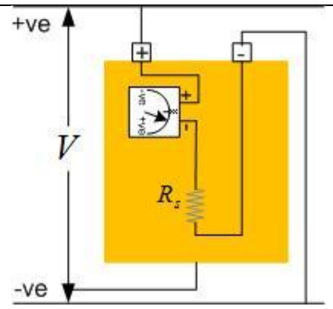
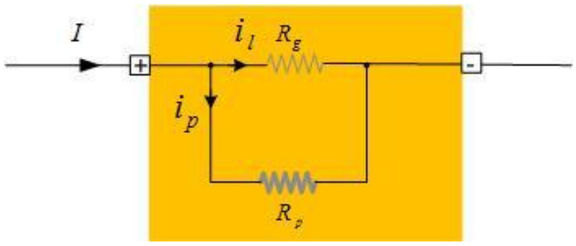
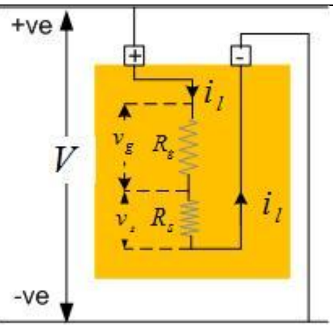




Galvanometer: Galvanometer is a sensitive device used for measurement of electrical quantitities based on principle of electric motor, which produces mechanical torque when electric current is supplied through it.Vibration Moving Coil Galvanometer, shown in the figure. Considering the requirement of sensitivity, which is totally an application area is left out while elaborating basic concepts of physics. In brief, it has a light moving coil assembly, made of thin wire mounted on a former, between narrow space magnetic poles, in the shape of circular, and a fixed magnetic core at its center. The coil suspended with the help of an elastic phosphor bronze wire and a spring of conducting material resting on a jewel bearing which is nearly frictionless, to keep it suspended concentrically along the axis of magnetic field. Whenever, current is supplied through the coil, having a rating of a few milli-amperes, it experiences a deflecting torque($\tau_d \propto i$), as a motor, while the phosphor bronze wire and the spring exert a controlling torque($\tau_c \propto \theta$). This leads to eventually, $i \propto \theta$, and current through the galvanometer is calibrated with the angle of deflection, determined geometrically, by observing shift in image of a light beam caused by reflection of mirror, as shown in the figure. This instrument being

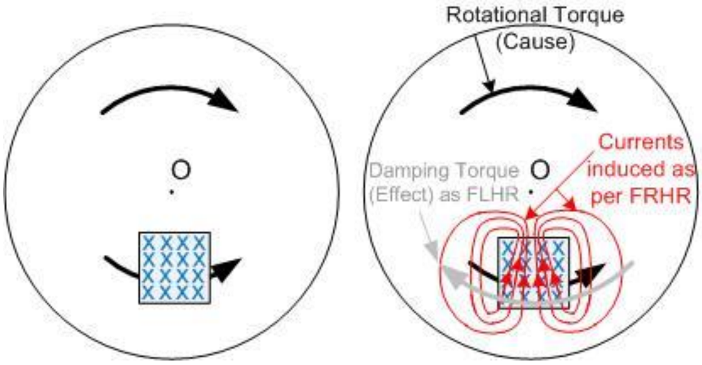


very delicate cannot handle current and voltages of the order experienced in scientific experiments and various application. In view of this extending range of galvanometer to enable its use as **a) Ammeter**, for measurement of current in circuit, and **b) Voltmeter**, for measurement of voltage across two terminals is being consolidated in a comparative manner.

Particulars	**Ammeter**	**Voltmeter**
Purpose	Measure Current (*I*) in amperes in a circuit	Measure Potential Difference (*V*) in volts across two terminals in a circuit
Constraint	$\boldsymbol{I \gg I_g}$, here, $\boldsymbol{I_g}$ rated current of galvanometer	Current through Galvanometer on direct connection $\boldsymbol{\frac{V}{R_g} \gg I_g}$, here, $\boldsymbol{R_g}$ is resistance of galvanometer
Problem Statement	Reduce current through galvanometer $i_l \le I_g$, where i_l is the limiting value of current for desired range of instrument.	Reduce current through galvanometer $i_l < I_g$, where i_l is the limiting value of current for desired range of instrument.
Remedy	Use a low resistance $\boldsymbol{R_p}$ in parallel to galvanometer such that $\boldsymbol{R_p \ll R_g}$	Use a low resistance $\boldsymbol{R_s}$ in series galvanometer such that $\boldsymbol{R_s \gg R_g}$
Schematic Diagram		
Equivalent Circuit		
Calculation on Current Limiting Resistance	• It is solving simple parallel combination of resistances for the given condition $i_l \le I_g$ $$\frac{i_L}{I} = \frac{R_p}{R_p + R_g}$$ • Hence, for Rated Current (I_R), $$i_L = \frac{R_p I_R}{R_p + R_g} \le I_g$$ • Equivalent resistance of Ammeter $$R_A = \frac{R_p R_g}{R_p + R_g}$$	• It is solving simple series combination of resistances for the given condition $i_l \le I_g$ $$i_L = \frac{V}{R_s + R_g}$$ • Hence, for Rated Current (I_R), $$v_L = \frac{R_s I_R}{R_s + R_g} \le I_g R_g$$ • Equivalent resistance of Voltmeter $$R_V = R_s + R_g$$

Tangent Galvanometer is a special purpose variant of *Galvanometer* shall be discussed at the end in Section on Instrumentation.

Eddy Currents: A time varying magnetic field ($\Phi(t)$) of any current carrying coil when intercepted by any external conductive material, it acts like virtual conductive rings forming secondary coil. As per *Faraday's Laws of Induction current produced in these secondary coils is called **Eddy Current***. This eddy current is a disadvantage due additional loss of energy and consequent heating and is pertinent to AC circuits, which shall be elaborated in a separate part of this Chapter on Electromagnetics. Determination of eddy current losses, is also beyond scope of this text. The eddy currents have been effectively used in electrical instruments for damping a qualitative illustration of this is shown in the figure. A conductive disc, rotating at its passing through center 'O', has one magnet placed away from the axis with its field perpendicular to the plane of the disc. When disc is rotated in clockwise direction (cause of action), virtual radial stands of the disc intercept magnetic field and produce an emf across it as per FRHR. The current so produced, in turn intercepts magnetic field, producing a torque on each of the stand as per FLHR.



Induction Coil: Is a simple device of a solenoid wound over a closed magnetic circuit to act as source of high voltage when circuit is interrupted $\boldsymbol{e = L\frac{di}{dt}}$ and act as short-circuit when current through it becomes steady $\boldsymbol{e = 0|_{i=\text{constant}}}$. Further, the Eddy current effect enhances it utility in creating heating. In alternating circuits, inductance has a different behavior which shall be elaborated in next section.

Comparison of Electrostatics, Magnetostatics and Magnetism: It is time to vouch that these three fields are so interconnected that it is impossible to think of either of them in isolation. Nevertheless, an effort has been made to elaborate these topics in rhythmic manner to bring them to a point where, with understanding of vector calculus it shall be possible to explore Electromagnetic Wave Theory and Maxwell's Wave Equations, which are outside scope of this manual. Nevertheless, inquisitive readers are welcome to write through Contact Us.

Electrostatics	Magnetostatics	Electromagnetism
$\bar{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}$ **N** (Coulomb's Law)	$\bar{F} = \frac{\mu_0}{4\pi}\cdot\frac{m_1 m_2}{r^2}\hat{r}$ **N** (Coulomb's Law)	$\bar{F} = q\bar{E} + q\bar{v}\times\bar{B}$ **N** (Lorentz's Force Equation)
Permittivity of vacuum: $\varepsilon_0 = 8.854\ldots\times 10^{-12}$ **$C^2N^{-1}m^{-2}$**	Permeability of vacuum: $\mu_0 = 4\pi\times 10^{-7}$ **NA^{-2}**	$\mu_0\varepsilon_0 = \frac{1}{c^2}$ c=2.99792458 ms^{-1} (Velocity of Light) (Maxwell's Wave Equation)

Summary: *As one advances into journey into Physics, he finds increasing integration of Mathematics into Physics. The advent of current electricity has opened a new era of not only nature but transforming world through technological developments.*

Understandings of Electrostatics, and Current Electricity, in earlier three parts, has been extended into basic concepts of Electromagnetism. Understanding Electromagnetism is slightly different from other topics, where one could observe the phenomenon. In this topic everything happen, but nothing is visible. But, verification of phenomenon of current electricity is through observations of its effects. At this point, some of the inter-related topics are referred to but their elaboration has been deferred, till related concepts are covered. Nevertheless, readers are welcome to raise their inquisitiveness, beyond the contents, through Contact Us. Likewise, any suggestion or correction considered essential by the readers are welcome; it would be gratefully acknowledged and incorporated suitably

Solving of problems, is an integral part of a deeper journey to make integration and application of concepts intuitive. This is absolutely true for any real life situations, which requires multi-disciplinary knowledge, in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person. References cited below provide an excellent repository of problems. Readers are welcome to pose their difficulties to solve any-problem from anywhere, but only after two attempts to solve. It is our endeavour to stand by upcoming student in their journey to become a scientist, engineer and professional, whatever they choose to be.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics, representative problems from contemporary text books and Question papers from various competitive examinations shall be drawn and supported with necessary guidance to evolve solutions as a dynamic exercise which is contemplated to accelerate the conceptual thought process.

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SOLUTION TO THE PUZZLE-June'17: Mother

Prof. S.B. Dhar

					1M	A									
					A										
2A	H	3M			T			4A	M	5M	A		6M		7M
M		8A	A	Y	I					O		9M	A	N	A
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GROWING WITH CONCEPTS - Chemistry

HEAT CAPACITY AND HEAT OF REACTION

Kumud Bala

Heat capacity: The heat capacity of a system is defined as the amount of heat required to raise the temperature of the system through 1°C.

Thus, if q is the amount of heat supplied to a system and as a result, if the temperature of system rises from T_1 to T_2 then the heat capacity (C) of the system is given by

$$C = \frac{q}{T2-T1} = \frac{q}{\Delta T} \qquad \text{(i)}$$

However, since the heat capacity varies with temperature, therefore, the value of C has to be considered over a very narrow temperature range. Thus, if δq is small amount of heat absorbed by a system which raises the temperature of the system by a small amount dT (say from T to T+dT) , then the heat capacity of the system will be given by

$$C = \frac{\delta q}{dT} \qquad \text{(ii)}$$

The specific heat capacity, C_s, of a substance is defined as the amount of heat required to raise the temperature of 1 gram of the substance through 1°C. If instead of 1g, 1 mole of the substance is taken, the term used is called molar heat capacity, C_m. Thus, Molar heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through 1°C. i.e., $C_m = \frac{C}{n}$ where n is number of moles.

Example – To understand the difference between heat capacity, specific heat capacity and molar heat capacity, let us take the following example: A piece of Al metal weighing 3g requires 5.4 J of heat to raise the temperature from 298K to 300K.

Solution – Heat capacity of the piece of Al = $\frac{5.4\,J}{2K}$ = 2.7 JK^{-1}

Specific heat capacity of Al = $\frac{5.4\,J}{3g \times 2K}$ = 0.9$Jg^{-1}K^{-1}$

Molar heat capacity of Al = $\frac{5.4J}{3g\times 2K}\times 27$ = 24.3$Jmol^{-1}K^{-1}$ (1 mole of Al =27g)

Evidently , the amount of heat, q, required to raise the temperature from T_1 to T_2 of mass m gram of a sample and having specific heat c, can be calculated from the expression q =m×c×(T_2-T_1) = m×c×ΔT

Or using the heat capacity(C), we have q=C×ΔT It is useful to remember that the specific heat capacity of water is 1cal g^{-1} K^{-1}

Types of heat capacities or molar heat capacities: Since q is not a state function and depends upon the path followed; therefore C is also not a state function. Hence, to know the value of C, the condition such as constant volume or constant pressure have to be specified which define the path. Thus, there are two types of heat capacities: (i) heat capacity at constant volume (represented by C_v), (ii) heat capacity at constant pressure (represented by C_p). The heat supplied to a system to raise its temperature through 1°C keeping the volume of the system constant is called heat capacity at constant volume. Similarly, the heat supplied to a system to raise its temperature through 1°C keeping the external pressure constant is called heat capacity at constant pressure. Now, according to first law of thermodynamic,

we know that $\delta q = dU + PdV$ (iii)

$$C = \frac{dU+PdV}{dT} \qquad \text{(iv)}$$

When the volume is kept constant dV=0 and therefore equation (iv) becomes

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v \qquad \text{(v)}$$

Or for an ideal gas, this equation may simply be written as

$$C_v = \frac{dU}{dT} \qquad \text{(vi)}$$

Thus, the heat capacity at constant volume may be defined as the rate of change of internal energy with temperature at constant volume.

When pressure is constant during the absorption of heat, equation (iv) becomes

$$C_p = \left(\frac{\partial U}{\partial T}\right)_p + P\left(\frac{\partial V}{\partial T}\right)_p \qquad \text{(vii)}$$

Also, we know that the heat content or enthalpy of a system is given by H=U+PV. Differentiating w.r.t T at constant P, we get

$$\left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_p + P\left(\frac{\partial V}{\partial T}\right)_p \qquad \text{(viii)}$$

Combining equation (vii) and (viii), we get

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p \qquad \text{(ix)}$$

Or for an ideal gas, the equation may simply be put in the form

$$C_p = \frac{dH}{dT} \qquad \text{(x)}$$

Thus, the heat capacity at constant pressure may be defined as rate of change of enthalpy with temperature at constant pressure.

Relationship between C_p and C_v : If the volume of the system is kept constant and heat is added to a system, then no work is done by the system. Thus, the heat absorbed by the system is used up completely to increase the internal energy of the system. Again if the pressure of the system is kept constant and heat is supplied to the system, then some work of expansion is also done by the system in addition to the increase in internal energy. Thus, if at constant pressure, the temperature of the system is to be raised through the same value as at constant volume, then some extra heat is required for doing the work of expansion. Hence $C_p > C_v$. The difference between the heat capacities of an ideal gas can be obtained by subtracting equation (vi) from (x) so we have

$$C_p - C_v = \frac{dH}{dT} - \frac{dU}{dT} \qquad \text{(xi)}$$

But H=U+PV (by definition)

And PV= RT (for 1 mole of an ideal gas) ∴ H = U + RT differentiating this equation w.r.t T, we get

$$\frac{dH}{dT} = \frac{dU}{dT} + R \qquad \text{(xii)} \quad \text{or}$$

$$\frac{dH}{dT} - \frac{dU}{dT} = R \qquad \text{(xiii)}$$

Combining equation (xi) and (xiii)

$C_p - C_v = R$ (for 1 mole of an ideal gas)

Thus, C_p is greater than C_v by the gas constant R i.e., approximately 2 calories or 8.314 joules.

$C_p - C_v = nR$ (for n mole of an ideal gas)

Thermo Chemical Equation : In a thermo chemical equation, the physical states of reactants and products along with heat energy evolved or absorbed are indicated. For example-

$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O\ (l)$, $\Delta H = -286kJ$

Heat of Reaction or Enthalpy of Reaction: "The amount of heat evolved or absorbed in a chemical reaction when the number of moles of the reactants as represented by the chemical equation have completely reacted is called the heat of reaction." For example:

$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$, $\Delta H = -393.5$ k J mol^{-1}

$C_{(s)} + H_2O_{(g)} \rightarrow CO_{(g)} + H_{2(g)}$, $\Delta H = +131.4$kJ mol^{-1}

Different Types of the Enthalpies of Reaction:

Enthalpy of Formation is - "The enthalpy of formation of a substance is defined as the heat change i.e., heat evolved or absorbed when 1 mole of the substance is formed from its elements under given conditions of temperature and pressure." It is usually represented by $\Delta_f H$

For example, $C_{(s)} + 2H_{2(g)} \rightarrow CH_{4(g)}$ $\Delta_f H = -74.8$ kJ mol^{-1}

Standard enthalpy of formation of a substance is defined as the enthalpy change accompanying the formation of 1 mole of the substance in the standard state form its elements, also taken in the standard state (i.e., 298K and1 bar pressure). It is usually represented by $\Delta_f H^\circ$ (enthalpies of formation of all free elements in their standard state taken to be zero)

Standard enthalpy of reaction = standard enthalpies of formation of products – standard enthalpies of formation of reactants

Enthalpy of Combustion is - "The enthalpy of combustion of a substance is defined as the heat change (usually the heat evolved) when 1 mole of substance is completely burnt or oxidized in oxygen." For example

$CH_4 + 2O_2 \rightarrow CO_{2(g)} + H_2O_{(l)}$ $\Delta H = -890.3 kJ\ mol^{-1}$

Combustion is always an exothermic reaction. Therefore $\Delta_c H$ must be negative for the combustion reactions. If the combustion of a substance is incomplete or is carried in the limited supply of air or oxygen, then the energy evolved does not represent enthalpy of combustion. For example,

$C_{(s)} + \frac{1}{2} O_{2(g)} \rightarrow CO_{(g)}$ $\Delta H = -108.7 kJ\ mol^{-1}$

$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$ $\Delta H = -393.5 kJmol^{-1}$

Complete oxidation means oxidation to $CO_2$and not to CO. Hence, heat of combustion of carbon is 393.5KJ mol^{-1}. Standard enthalpy of combustion is the amount of heat evolved when one mole of the substance under standard condition (298K, 1 bar pressure) is completely burnt to form the products also under standard condition. It is represented by $\Delta_c H^\circ$

Combustion of Foods and Fuels: We know that a fuel such as wood, petrol and kerosene etc., undergoes combustion in a machine or furnace to produce heat energy for the running of machines. Similarly for the working of human machines, we eat carbohydrates, fats etc. in the form of food. The carbohydrates are first decomposed in our body by the enzymes to form glucose which then undergoes oxidation by the oxygen that we inhale to produce energy.

$C_6H_{12}O_{6(s)} + 6O_{2(g)} \rightarrow 6CO_{2(g)} + 6H_2O_{(g)}$ $\Delta H^\circ = -2802.0 kJmol^{-1}$

This oxidation reaction is usually called combustion of food. Different fuels and foods produce different amounts of heat on combustion. These are usually expressed in terms of their calorific values. These are defined as "the calorific value of a fuel or food is the amount of heat in calories or joules produced from the complete combustion of one gram of the fuel or the food". Thus, according to the above reaction, 1 mole of glucose, i.e., 180g of glucose produce heat = 2802kJmol-1. Hence, calorific value of glucose=$\frac{2802}{180}$ =15.56kJg-1 A normal person needs about 3000kcal (about 12000 – 13000 kJ) per day. S.I. unit is kJg-1

Calorific Values of Food and Fuel

Food	Calorific value (kJg^{-1})	Fuel	Calorific value (kJg^{-1})
Curd	2.5	Wood	17
Milk	3.2	Charcoal	33
Egg	7.3	Kerosene oil	48
Meat	12.0	Fuel oil	45
Honey	13.3	Butane (LPG)	55
Ghee	37.6	Hydrogen	150

Enthalpy of solution: It may be define as "The change in enthalpy or heat energy when one mole of a substance is dissolved in such a large excess of the solvent at a given temperature that the further addition of the solvent does not produce any more heat energy change." For example,

$NaCl_{(s)} + aq \rightarrow NaCl(aq)$ $\Delta_{sol} H = +5.35 kJmol^{-1}$

$KCl_{(s)} + aq \rightarrow KCl_{(aq)}$ $\Delta_{sol} H = +18.56 kJmol^{-1}$ (aq. Aqueous means large excess of water)

Enthalpy of Hydration: "The change in enthalpy or heat energy when one mole of anhydrous salt changes to a hydrated salt by combining with specified number of moles of water." For example,

$CuSO_{4(s)} + 5H_2O_{(l)} \rightarrow CuSO_4.5H_2O$ $\Delta_{hyd} H = -78.2 kJmol^{-1}$

Phase Transition: Solid, liquid and gas are the three phase of a substance. When substance changes from one phase to another phase, it evolves or absorbs energy due to inter particle forces of attraction. For example, Solid→liquid→gas

Enthalpy Changes of the Different Phases :

Enthalpy of fusion: "The change in enthalpy or heat energy when one mole of a solid at its melting point change to its liquid state." For example,

$H_2O_{(s)} \rightarrow H_2O_{(l)}$ at 273K $\Delta_{fus} H = + 6.0 kJmol^{-}$
(Ice) (water)

$NaCl_{(s)} \rightarrow NaCl_{(l)}$ $\Delta_{fus} H = + 29.0 kJ\ mol^{-1}$

Enthalpy of fusion of NaCl is more than $\Delta_{fus} H$ of ice due to more attractive forces in $NaCl_{(s)}$

Enthalpy of Vaporization: "The change in enthalpy or heat energy when one mole of a liquid at its boiling point changes to its gaseous state." For example,

$H_2O_{(l)} \rightarrow H_2O_{(g)}$ $\Delta_{vap} H = +40.7 kJ\ mol^{-1}$

As condensation is reverse of vaporization, the enthalpy of condensation has the same value as the enthalpy of vaporization but has opposite sign. Thus,

$H_2O_{(g)} \rightarrow H_2O_{(l)}$, $\Delta_{cond} H = -40.7 kJ\ mol^{-1}$

Enthalpy of Sublimation: Enthalpy of sublimation of a substance is the enthalpy change accompanying the conversion of one mole of a solid directly into vapour phase at a given temperature below its melting point. For example,

$I_{2(s)} \rightarrow I_{2(g)}$, $\Delta_{sub} H = + 62.39 kJ\ mol^{-1}$

It may be pointed out that sublimation is nothing but fusion and vaporization carried out in one step, i.e., $\Delta_{sub} H = \Delta_{fus} H + \Delta_{vap} H$

Enthalpy of Atomization (bond enthalpies): When a bond is formed between two atoms, a certain amount of energy is needed to break the bond. This is known as bond dissociation energy or bond enthalpy or enthalpy of atomization. It may be defined as the amount of energy required to break one mole bonds of a particular type between two atoms in the gaseous state of a substance. For example,

$H_{2(g)} \rightarrow 2H_{(g)}$, $\Delta_a H = 435.0 kJ\ mol^{-1}$

Hess's Law of Heat Summation: G.H. Hess, a Russian chemist, in1840, gave a law about the heat of reaction on the basis of experimental observation. This law is known after his name as Hess's law. It states as the total amount of heat evolved or absorbed in a reaction is the same whether the reaction takes place in one step or in a number of steps.

Basis of Hess's law: Hess's law follows from the fact that enthalpy is a state function, i.e., enthalpy change depends only on the initial state (i.e., enthalpy of the reactants) and the final state (i.e., enthalpy of the products) and does not depend upon the path followed. For example, when carbon(graphite) burns to carbon dioxide directly in one step, 393.5kJ mol^{-1} of heat produced, i.e.,

$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$, $\Delta_r H° = -393.5 kJ\ mol^{-1}$

If carbon burns to form carbon monoxide first which then burns to form carbon dioxide, the heats evolved in the two steps are as follows:-

(i) $C_{(s)} + \frac{1}{2} O_{2(g)} \rightarrow CO_{(g)}$, $\Delta_r H_1° = -110.5 kJ\ mol^{-1}$

(ii) $CO_{(g)} + \frac{1}{2} O_{2(g)} \rightarrow CO_{2(g)}$, $\Delta_r H_2° = -283.0 kJ\ mol^{-1}$

Thus, the total heat evolved in the two steps will be $\Delta H° = (-110.5) + (-283.0) = -393.5 kJ\ mol^{-1}$ which is the same when the reaction takes place directly in one step.

Theoretical Proof of Hess's law : Consider the general reaction A→D. Suppose the heat evolved in this reaction is Q joules. Now, if the same reaction takes place in three steps as follows :- A → B → C→ D, and the heats evolved in these three steps are q_1 , q_2 , q_3 joules respectively. Then, the total heat evolved = $q_1 + q_2 + q_3$ = Q' joules (say). According to Hess's law, we must have Q =Q'. If Hess's law were not correct, then either Q' < Q or Q' > Q . Suppose Q' > Q, this means that if we go from A to D in a number of steps, the heat evolved is more than the heat absorbed when we return from D to A directly in one step. Thus, when the cyclic process is completed, Q-

Q' joules of heat is produced. Thus, by repeating the cyclic process a number of times, a large amount of heat can be created. This is, however, against the law of conservation of energy. Hence, Q must be equal to Q', i.e., Hess's law must be correct.

Application of Hess's law: The most important application of Hess's Law is in the calculation of heat changes for those reactions for which experimental determination is not possible. The calculations are based upon the following consequence of Hess's law:-

The thermo chemical equation can be treated as algebraic equations which can be added, subtracted, multiplied or divided. A few important applications of Hess's law are given below:-

1 .Calculation of Enthalpy of Formation:- the enthalpies of formation of many compounds cannot be determined experimentally. These are calculated by the application of Hess's law.

Example – Calculate the enthalpy of formation of methane (CH_4) from the following data:

(i) $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$ $\Delta_r H° = -393.5$kJ mol^{-1}

(ii) $H_{2(g)} + \frac{1}{2} O_{2(g)} \rightarrow H_2O_{(l)}$ $\Delta_r H° = -285.8$kJ mol^{-1}

(iii) $CH_{4(g)} + 2O_{2(g)} \rightarrow CO_{2(g)}$ $\Delta_r H° = -890.3$kJ mol^{-1}

Solution : we aim at $C_{(s)} + 2H_{2(g)} \rightarrow CH_{4(g)}$; $\Delta_f H° = ?$

Multiplying equation (ii) with 2, adding to (i) and then subtracting equation (iii) from the sum, we get

$C_{(s)} + 2H_{2(g)} \rightarrow CH_{4(g)}$, $\Delta_r H° = -393.5 + 2(-285.8) - (-890.3) = -74.8$ kJ mol^{-1}

Hence , enthalpy of formation of methane is $\Delta_f H° = -74.8$kJ mol^{-1}

2. Calculation of Enthalpy of Allotropic Transformation: Elements like carbon and sulphur exit in different allotropic forms. The change of one form to the other involves a very small amount of heat and is a very slow process. Hence, the experimental determination of heat changes for such transformations is very difficult. These are calculated by the application of Hess's law as illustrated by the example given below:-

Example Calculate the enthalpy change accompanying the transformation of C (graphite) to C (diamond). Given that the enthalpies of combustion of graphite and diamond are 393.5 and 395.4kJmol^{-1} respectively.

Solution: We are given

(i) C(graphite) + $O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta_C H° = -393.5$kJ mol^{-1}

(ii) C(diamond) + $O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta_c H° = -395.4$kJ mol^{-1}

We aim at C(graphite) → C(diamond), $\Delta_{trans} H° = ?$

Subtracting equation (ii) from (i), we get C(graphite) – C(diamond) → 0;

$\Delta_r H° = -393.5 - (-395.4) = +1.9$kJ mol^{-1} or

C(graphite) → C(diamond); $\Delta_{trans} H° = +1.9$kJ mol^{-1}

3. Calculation of the Enthalpy of Hydration: The experimental determination of the enthalpy of hydration is almost impossible. However, it can be easily calculated using Hess's law as illustrated by the following example:

Example: Calculate the enthalpy of hydration of anhydrous copper sulphate ($CuSO_4$) into hydrated copper sulphate ($CuSO_4.5H_2O$). Given that the enthalpies of solutions of anhydrous copper sulphate and hydrated copper sulphate are -66.5 and +11.7kJ mol^{-1} respectively.

Solution: We are given

(i) $CuSO_4(s)$ + aq → $CuSO_4(aq)$; $\Delta_{sol} H = -66.5$kJ mol^{-1}

(ii) $CuSO_4.5H_2O_{(s)}$ + aq → $CuSO_4(aq)$; $\Delta_{sol} H = +11.7$kJ mol^{-1}

We aim at $CuSO_{4(s)} + 5H_2O_{(l)} \rightarrow CuSO_4.5H_2O_{(s)}$; $\Delta_{hyd} H = ?$

Equation (i) can be written in two steps as

(iii) $CuSO_{4(s)} + 5H_2O_{(l)} \rightarrow CuSO_4.5H_2O_{(s)}$; $\Delta H = \Delta H_1 kJmol^{-1}$

(iv) $CuSO_4.5H_2O_{(s)} + aq \rightarrow CuSO_{4(aq)}$; $\Delta H = \Delta H_2 kJmol^{-1}$

According to Hess's law $\Delta H_1 + \Delta H_2 = -66.5$ $kJmol^{-1}$

Further, equation (ii) and (iv) are same. ∴ $\Delta H_2 = +11.7$ $kJmol^{-1}$

Putting this value above, we get $\Delta H_1 + 11.7 = -66.5 kJmol^{-1}$ or

$\Delta H_1 = -66.5 - 11.7 = -78.2 kJ\ mol^{-1}$ Thus, equation (iii) may be written as

$CuSO_{4(s)} + 5H_2O_{(l)} \rightarrow CuSO_4.5H_2O_{(s)}$; $\Delta_{hyd} H = -78.2 kJmol^{-1}$ This is what we aimed at. Hence, the

required value of the enthalpy of hydration is $\Delta_{hyd} H = -78.2\ kJ\ mol^{-1}$.

5. **Predicting the Enthalpy Change for any Reaction**: Hess's law can be applied to predict the enthalpy change for any reaction from the enthalpy changes of certain other reactions.

Author is M.Sc. (Chem.), M.Ed. and Advanced Diploma in German Language (Gold Medallist). She retired as a Principal, Govt. School Haryana, has 3-1/2 years' experience in teaching Chemistry and distance teaching through lectures on Radio and Videos. She has volunteered to complement mentoring of students for Chemistry through Online Web-enabled Classes of this initiative.
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Answers to Science Quiz in June'17

Kumud Bala

1. (A) 2. (C) 3. (B) 4. (C) 5. (A) 6. (A) 7 (A) 8. (B) 9. (C) 10. (A)
11 (A) 12. (B) 13. (C) 14. (D) 15. (C) 16. (C) 17. (A) 18. (D) 19. (C)

—oo—

I am impressed with the urgency of doing. Knowing is not enough, use it; being willing is not enough, do it.

\- **Leonardo da Vinci**

SCIENCE QUIZ- July'17

Kumud Bala

1. Agent used as 'seed' in artificial rain is
   (a) Silver chloride (b) Silver iodide
   (c) Sodium chloride (d) Calcium chloride.
2. Barium hydroxide is also known as
   (a) Baryta water (b) Lime water
   (c) Flood water (d) Soda water.
3. The important metal used with iron to produce stainless steel is
   (a) Chromium (b) Nickel
   (c) Magnesium (d) Potassium.
4. The gas used in fire extinguisher is
   (a) CO_2 (b) SO_2
   (c) NO_2 (d) SiO_2.
5. Solder is an alloy of
   (a) Tin and Lead (b) Zinc and Lead
   (c) Aluminium and Magnesium
   (d) Aluminium and Copper.
6. Oil of vitriol is
   (a) H_2SO_4 (b) HNO_3
   (c) HCl (d) H_2CO_3.
7. Metal used in incandescent lamps is
   (a) Molybdenum (b) Tungsten
   (c) Zirconium (d) Copper.
8. Main ore of mercury is
   (a) Cinnabar (b) Hematite
   (c) Copper pyrite (d) Bauxite.
9. Lobe of human brain associated with hearing is -
   (a) Temporal lobe (b) Frontal lobe
   (c) Cerebellum (d) Pons.
10. Name the largest gland in the human body is -
    (a) Kidney (b) Heart
    (c) Liver (d) Brain.
11. Mosquito the carrier of Zika virus is -
    (a) Aedes (b) Culex
    (d) Anopheles (d) House fly.
12. Metal present in insulin is
    (a) Sodium (b) Potassium
    (c) Zinc (d) Magnesium.
13. Graveyard of RBC's is
    (a) Liver (b) Spleen
    (c) Gall balder (d) Intestine.
14. The name of the scientist who stated the matter convertible into energy is
    (a) G.H.Hess (b) Einstein
    (c) de-Broglie (d) Neils Bohr.
15. The direction of the magnetic field within a magnet is
    (a) North to South (b) South to North
    (c) East to West (d) West to East.
16. The waves used in sonography are
    (a) Ultrasonic waves (b) Microwaves
    (c) Radio waves (d)Eelectromagnetic waves.
17. The hydraulic brake used in automobiles is a direct application of
    (a) Pascal's law (b) Hess's law
    (c) Boyle's law (d) Charles law.
18. The principle used in designing ships and submarines is
    (a) Heisenberg's uncertainty principle
    (b) Archimedes principle
    (c) Pauli Exclusion principle
    (d) None of these
19. The forest in India that has the most extensive mangrove vegetation is -
    (a) Sunder bans (b) Tropical deciduous forest
    (c) Alpine (d) Tropical thorn forest.
20. The most abundant element in the earth's crust is -
    (a) Aluminium (b) Silicon
    (c) Oxygen (d) Carbon.

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(Answers to this Science Quiz – July'17 shall be provided in Supplementary e-Bulletin dt 1st August'17)

—00—

Theme Song :

PREMISE: ***We are pleased to adopt a song*** **" इतनी शक्ति हमें देना दाता....."** ***from a old Hindi Movie Do Aankhen Barah Haath दो आँखें बारह हाथ of year 1957, directed by The Late V. Shantaram. The lyrics are by Shri Bharat Vyas, singer Melody Queen Sushri Lata Mangeshkar, and Music Direction by Vasant Desai. It has become a widely accepted inspirational song and/or prayer in many educational institutions and socially inspired initiatives engaged in mentoring of unprivileged children. This newly formed non-organizational initiative, being selflessly operated by a small set of compassionate persons, finds its philosophy in tune with the song and conveys its gratitude to all he eminent persons who brought out the song in a manner that it has attained an epitome of popularity. While working its mission and passion, the group invites one and all to collectively complement in grooming competence to compete among unprivileged children. The song/prayer goes as under -***

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

दूर अज्ञान के हो अंधेरे, तू हमें ज्ञान की रोशनी दे
हर बुराई से बचते रहें हम, जितनी भी दे भली ज़िन्दगी दे
बैर हो ना किसी का किसी से, भावना मन में बदले की हो ना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

हम ना सोचें हमें क्या मिला है, हम ये सोचे किया क्या है अर्पण
फूल खुशियों के बाँटे सभी को, सब का जीवन ही बन जाए मधुबन
अपनी करुणा का जल तू बहा के, कर दे पावन हर एक मन का कोना ||

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना
हम चले नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना ||

Together **E**ach **A**chieves **M**ore
(**TEAM**)

Every end, so also end of this e-Bulletin, is a pause for a review, before re-continuing of a journey far beyond ...