# GYAN-VIGYAN SARITA: शिक्षा

A non-remunerative, non-commercial and non-political initiative to Democratize Education as a Personal Social Responsibility (PSR) Supplement to Quarterly e-Bulletin No: 01-1, 2<sup>nd</sup> Oct,2016

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Our Website: <u>http://gyanvigyansarita.in/</u> e-Mail: <u>subhashjoshi2107@gmail.com</u> We wish you a very happy Deepawali with prayers to Lord Ganesh (God of wisdom), Maa Saraswati (Godess of knlowledge) and Devi Laxmi (Godess of Wealth) to inspire us to ignite every mind to acquire wisdom through education and enable them to Democratize Education with a sense of Personal Social Responsibility (PSR). This is sure to lead us to an era where educated individual grows preserving nature and humanity.

Fear is the main source of superstition, and one of the main source of cruelity. To conquer fear is the beginning of wisdom.

**Bertrand Russel** 





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### <u>Editorial</u>

## **Practice and Continuity in Education**

Brian Patrick Herbert is an American author. He believes that the capacity to learn is a gift; the ability to learn is a skill; the willingness to learn is a choice. Our objective should be always learning as life never stops teaching. It is a matter of great contentment for **Gyan Vigyan Sarita** that it has been able to awaken the students to foresee their superb future. Awakening is more important than teaching because if you are awakened, you will definitely learn.

Choice that our students have made for us; Chance that they have taken for us; and the change that they expected in their lives from us, keep us always busy to work for them. Our work has been to promote critical thinking to keep enthusiasm, passion, and influenced mind always charged for continuous study.

> करत करत अभ्यास के, जड़ मति होत सुजान; रसरी आवत जात से सिल पर पड़त निशान।

The normal issue of this Quarterly e-bulletin will come in January. The gap of 2 months between Two intervening Quarterly issues may affect continuity. The principle of learning is rigorous practicing. The practice leads to perfection. So, to keep our students well conversant with the concepts and equip them with updated material, it was inevitable for us to bring this supplement issue.

This supplement issue contains the regular material on:

- (a) Mechanics Part II covering Newton's Laws of Motion for Physics
- (b) Nomenclature of unsaturated hydrocarbons for Chemistry, and
- (c) Number Systems in Mathematics

Never forget that difficult roads often lead to beautiful destinations. Always remember the *success sutra*: Practice like you have never won and perform like you have never lost.

Coordinator's Views





## **Pollution Free Festivities**

We have witnessed Deepawali festivities where media did its job to highlight trends in pollution level to create an awareness among people to celebrate safe and pollution free Deepawali. This season in some parts of country witnesses worst during Deepawali. It is the thought process of the people that enjoyment is their right, because they can afford it. Its reflections are seen in match winning, political victories any such religious or family occasion or festival which deserves manifestation of joy.

I am a resident of a premier colony in NCR which is populated by elite and affluent families. After Laxmi Poojan I joined people enjoying Deepawali blowing crackers. The breathing experience was suffocating. Despite, people engaged in blowing crackers had justifications. Some said it is for an enjoyment, other claimed it to be on a reduced scale, few said a little of it will not make much difference. During these interaction, I had sense of guilt, since my son, living at home with environment friendly thought process and sense of Personal Social Responsibility, was busy blowing crackers with his children, with a purchase cost of aboutt Rs 10,000, a scale smallest in proportion to others was also doing same.

Unable to stand polluted environment, I came back and sleptover thinking – a) does affordability gives right to indiscriminate enjoyment? (there are many acts of enjoyment; but one does one start engaging in each of them everywhere with everyone} b) is buring cracker a necessity of survival, specially for those whse children are awardee for their thoughts on environment? c) are highly educated people wiser than illiterate saint Kabir? The Faquir had said –

> कबीरा खड़ा बाजार में सबकी मांगे खैर | ना काहू से दोस्ती ना काहू से बैर||

What a paradox ??

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#### **STRUGGLE OF SITA IN RAMAYANA**

The season of festivity, and change in weather starts from October. This is the month in which we celebrate Dusshera and Diwali. We all know the ancient epic story of Ramayana and the reason for celebration of Dusserra and Deepawali. The epic narrates the life of Ram; his banishment from the kingdom by Dasharath, his father and king of Ayodhya; his travels across forests in India with his wife Sita and brother Lakshmana; the kidnapping of his wife by Ravana, the demon king of Lanka, resulting in a war with him and Ram's return to Ayodhya to be crowned as a king.

In today's world, for a matrimony purpose, people generally say – what a Jodi, just like Ram and Sita! Mythologically, everyone knows that Ram had sent Sita to an exile because some people in Ayodhya did not accept her due to her long captivity under Ravana even after her Agnipariksha and Lord Brahma's declaration of her piousness. It is a fact that Ram's trust and affection for Sita never wavered and he knew that the allegations made on her were baseless even then, he sent Sita away and that too when she was pregnant. She didn't give up her life and gave birth to Luv and Kush in Valmiki's ashram.

Luv and Kush started growing in the ashram with unfamiliarity of the fact that their father was Ram, the king of Ayodhya. One fine day when they were listening to Ram-Sita katha by Valmiki, they were exposed to the whole story of Ramayana. They felt pity on Devi Sita and decided to fight for her justice. They stole the Horse Ashwamedha and declared war with Ram for the horse's conviction. The horse's presence was important for the yagya to be held in Ayodhya. When Ram came to the jungle for war, he got to know that Luv and Kush were his sons. Ramayana ends with the story of struggles faced by a mother in growing her kids. The epic shows that Sita lived her entire life in exile. She was strong to face any cruelty of life. She did not raise any voice even when her husband left her alone ignoring her full dedication of love and loyalty towards him. She always thought that wedding was a knot that tied the two souls to be always together in good or bad days. Sita played her role dedicatedly but what did Ram do? Just for the sake of society, he sent her to exile. He knew in the depth of his heart that Sita is pure but even after knowing this fact, he did so.

Even today a woman has to give tests of her purity. Woman is always treated weak, dependent and living on the mercy of her husband. But the time is changing. Now, the society talks more about women empowerment. Many new rules for her safety and rights have been framed. People accept a woman equal to a man. They treat her an independent, and as strong as a man. Saina Nehwal, Deepa Malik, and Sania Mirza, etc., are the examples where a woman can do what a man can.

Then why, every year on Dusshera, we accept the Ramayan's that story when the society wrongly decided about a woman's character?

Today, on Dusshera, I felt sad when I found that people still believe in the Ramanyan's story. From a woman's point of view, I still feel that our society decides a woman's character. How long shall we be living in the primitive days? Do we not need to change our mindsets in totality? Let us think to change our mindsets to make the society much better by giving a woman honour, grace and importance.



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## We want deeper sincerity of motive, a greater courage in speech and earnestness in action

Sarojini Naidu

#### Sandhya Tanwar

## **SPEAK UP FOR YOUR CHILD**

#### Aarti Sharma

Picture this - Its middle of August, and three straight holidays make it look like a long funfilled awesome weekend. Delhi Resident Tara Kapoor is wondering how to spend the weekend. There is fabulous Bharat Parv organised at Rajpath. She wanted to meet her nephew, an army officer, who had come back home after a long time. Her mother was not keeping well and she wanted to meet her. Alternately, an action thriller movie in the multiplex was also drawing rave reviews.

But, all her plans remained in thoughts only as she had to cope with weighty load of academics pressure of her two children. These three days of leave provided her an opportunity to practice and revise the lengthy chapters of science, maths, social science as well as that of other disciplines in the upcoming half yearly exams of her elder child aged fifteen and to re-explain hastily taught concepts of various subjects in the class of her younger child aged nine.

The story of Tara Kapoor is a typical one which reverberates in almost every urban Indian household. Parents send their children to public schools which they reckon as esteemed temples of learning. However, this coveted reputation which is manifested in high scores of the students in various academic disciplines proudly displayed on the school notice boards and sometimes even in print media comes at a heavy cost as the parents of the child are also pushed in to put all efforts in getting a high academic score.

This paper attempts to unfold the drawback of system generated acdemics pressure on the parent child relationship.

a. **Blissful Relationship Turning IntoSoured Chasing:** Readers would agree that the relationship between the child and the parents is undoubtedly the most beautiful relationship on the earth. Parents not nurture and care for the child but also assist ,counsel and teach the child in various phases of life .But the burden of heavy academics has crunched the most beautiful relationship on earth as the school faculty burden not only the child with mugging endless syllabus of various academic disciplines but also saddle the parents with this onerous task.

Holding the instructions given by the exalted educational institutions as supreme and in the best interest of their children, parents also start pressurising the child in the vicious cycle of mugging books, reproducing rotely learnt concepts in the exams, chasing the maddening rate race of getting score of 90<sup>s</sup> or 'o 'grades.

As a child grows, most of his/her conversation with the parents revolves around endless list of *whats*, *whethers and whys* pertaining to his studies- what he has studied in the school on a particular day, whether he has completed his class assignments in time, what he home work he/she has got on a particular day, when is the next tution class, etc.

b. **Teaching Pragmatic Wisdom in Family:** The scope for realistic learning which make a child successful in life – life skill trainings, building strong personality traits, lessons in morality and ethics, the onus of which lies by and large on parents and the

family has taken a backseat as the parents are made to solely focus on studies.

As the cacophony of assignments and lessons resonates at home also, child becomes visibly irritated with the parents. A child overloaded with academics cannot vent his exasperation either on the curriculum setters or on the school authorities.

c. **Parent Teacher Interaction:** Every parents teacher interaction contains a lengthy list of dos and don'ts for the parents for improving the grades of the child. In fact Parent teachers interaction are generally are a one sided affair wherein entire onus of child's performance is thrust on the parents and they remain at the receiving end throughout the discussion.

Little does this beleaguered lot realise, that the primary responsibility of educating the child lies on the school authorities and not the parents. It is former which has the requisite infrastructure and qualified faculty to educate the child and not the parents. The teachers are paid adequate salaries for their services and the school management, whether private or government, has the primary responsibility of creating infrastructure and an enabling environment in which a child sans his/her IQ is able to learn the concepts in a meaningful and stress free atmosphere.

But, the reverse happens, the qualified faculty with all the teaching tools and methodologies pin the blame for the under performance of the child on the parents. In fact, it's the biggest travesty of our education system. Vexed parents are further asked to enrol the child into private tuitions if they cannot teach the child properly.

In this pathetic state of affairs, where parents are badly at the receiving end, brazenly exists despite the authorities acknowledging the importance of parents in educating the child in various orientation meetings and stressing that parents are an important stakeholder in this process.

d. **Vicious Cycle:** But, there is an apparent fear of being berated which does not let any stakeholder speak freely about the ills afflicting the education system. It is due to fear of victimisation of the child, the parents

are unable to express themselves openly regarding the constraints they face in educating the child, the teachers cannot express their constraints for fear of being reprimanded by the school management, the school management cannot take up the issue of excess curriculum with the state Board for the fear of being singled out and chastise.

The respective education Boards are not willing to assess the quantum of academics as pruning it down would imply that the students under the respective boards are academically less intelligent and hardworking compared to other boards In this vicious cycle of ostracizing, students are the worst sufferers.

- e. What Next: Given the prevailing scenario where pulls and pressure of the academics stress have sullied parent child association which cannot think beyond rote learning of concepts and chasing 90 plus score, thereby leaving the child wrenched with anxiety and fear, parents need to realise that they need to be assertive and protect the rights of their children to learn concepts in a stress free environment. Shunning the fear of victimisation of the child, they should speak up for this rightful cause so as to unburden the child and in turn de-stress their relationship of this burden. The following unwanted issues. should accordingly, be taken up by the parents at various forums to ease the child's life of the superfluous academic load -
  - 1. Vast and lenghty curriculum with theoretical concepts of various disciplines which have no co-relation with the real life world and which have been described in too much detail. Assessment of Quantum of academics load either by the teachers or the school authorities.
  - 2. Difficulty faced by the child in grasping and understanding the bookish concepts due to their being presented in a very staid manner which his mind is not able to capture.
- 3. Honing and refining of the teaching methodologies which are highly underdeveloped. The teachers just

read the lesson from the book and write the assignment on the Board. They need to elucidate the concepts in a simplified and creative manner to enable realistic learning by the child.

- 4. The written assignments given in the class are far too lengthy. The child writing these assignments has to sit continuously for hours either in the class or at home. Curriculum setters as well as teachers never bother to find out how the concept becomes absolutely dull and boring by writing lengthy assignments. Accordingly, they should seriously mull over assessing the quantum of written assignment, given the cpacaity of the child to grasp.
- Revision of a concept is a must for proper 5. understanding and indepth learning by the child but, in the rush of completing the vast curriculum, little or no attention is paid towards revising the concepts making the child feel if he is tied with а rope of as a galloping curriculum horse and dragged for miles and miles in the name of so called learning and education.
- 6. Existence of a feedback system as there is none in any school so that a forum can be provided for freely expressing constraints faced by both teachers and parents in educating the child and incorporating suitable changes in the curriculum and the teaching methodologies.

The absence of such a platform which can provide an opportunity for improving and reforming many aspects relating to schoolwork is a big drawback in the current Parents need to follow up the education system. above issues to lay a long way in reforming the education system and eventually enabling realistic learning for the child and de-stressing the parent child relationship, caused by unwanted academic burden. This would help to focus on other aspects in the child's growth and development.



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Man is condemned to be free; because once thrown into the world, he is responsible for everything he does

- Jean-Paul Sartre

## ABOUT US

This is an initiative, not an abrupt eruption, but driven by spirit of returning back to society with a spirit of Personal Social Responsibility (PSR) by a <u>team of co-passionate</u> <u>persons</u> who have survived many decades of rough weather conditions. It is not an organization, and it aims at Democratization of Education, in spiritual sense.

It works on non-remunerative, non-commercial and non-political manner. Its financial model is based on Zero-Fund-&-Zero-Asset, wherein participation is welcome from those who wish to contribute, with तन और मन. As and when the feel need of धन to supplement the initiative ownership of Funds and Assets is theirs, we are just user if it.

**OUR MENTORING PHILOSOPHY:** Mentoring is not teaching, neither tuition nor coaching. It is an activity driven by passion and commerce has no place in it. In this effort is to caution students that -

- This place is not where they will be taught how to score marks and get higher ranks, but to conceptualize and visualize subject matter in their real life so that it becomes intuitive.
- This place is not to aim at solutions but inculcate competence to analyze a problem and evolve solution.
- This place does not extend selective and personalized attention, rather an opportunity to become a part of which is focused on learning and problem solving ability collectively.
- This place provides an opportunity to find students above and below one's own level of learning. Thus students develop not in isolation but learn from better ones and associate in problem solving to those who need help. This group dynamics while create a team spirit, an essential attribute of personality, while one learns more by teaching others.
- This place has strategically chosen Online Mentoring, so that those who are unprivileged can gather at one point and those who can facilitate learning of such students by creating, necessary IT setup. Aseperate <u>Mentor's Manual</u> is being developed to support the cause.

## **GROWING WITH CONCEPTS**

## Concepts of an expert are not like a static foundation of a huge structure; rather it is like blood flowing in a vibrant mind.

During growing into an expert, each one must have used best of the books available on subject and received guidance of best of the teachers. Authors might have had limitations to take every concept thread bare from first principle and so also must be the constraint of teacher while mentoring a class with a diversity of inquisitiveness and focus. As a result, there are instances when on a certain concept a discomfort remains. The only remedy is to live with the conceptual problem and continue to visualize it thread bare till it goes to bottom of heart and that is an **ingenious illustration**.

In this column an effort is being made to take one topic on Mathematics, Physics and Chemistry in each e-Bulletin and provide its illustration from First Principle. We invite all experts in these subjects to please mail us their ingenious illustrations and it would be our pleasure to include it in the column.

We hope this repository of ingenious illustrations, built over a period of time, would be helpful to ignite minds of children, particularly to aspiring unprivileged students, that we target in this initiative, and in general to all, as a free educational web resource.

*This e-Bulletin covers – a)* <u>Mathematics</u>, b) <u>Physics</u>, and c) <u>Chemistry</u>. This is just a beginning in this direction. These articles are not replacement of text books and reference books. These books provide a large number of solved examples, problems and objective questions, necessary to make the concepts intuitive, a journey of educational enlightenment.

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**GROWING WITH CONCEPTS - Mathematics** 

## LET's LEARN OUR NUMBER SYSTEMS

#### **Prof. SB DHAR**

A **Number System** is a system of **Numeration**. It is a writing system for expressing numbers. In other words, it is a mathematical notation for representing numbers of a given set, using digits or symbols in a consistent manner.

It is a fact that everything around us is numbers. Numbers originated with life. We cannot think of life without numbers. Nature is full of numeric properties. Numbers speak. Mathematically, numbers make mathematics. Numbers were systematically studied firstly as abstractions by Greek philosophers **Pythagoras** and **Archimedes**. Let us learn about the different types of **NUMBERS**.

#### NUMBER AND NUMERAL

Number is defined as a *mathematical object* that is used to count.

The examples are: 1, 2, 3,...., and so forth.

A *notational symbol* that is used to denote a *number* is called a *numeral*.

Numerals are used for labels, or for ordering.

#### NUMBER LINE

A number line is a line that represents all the numbers. It is a straight line. Natural Numbers, Whole Numbers, and Integers are represented as dots. The Real numbers are represented by continuous line.

We shall study here:

- (a) Natural Numbers
- (b) Whole Numbers
- (c) Integers
- (d) Rational Numbers
- (e) Irrational Numbers
- (f) Real Numbers
- (g) Complex Numbers
- (h) Roman Numbers, and
- (i) Some especial numbers

#### NATURAL NUMBERS

The numbers that are used for counting or ordering the objects are called Natural Numbers. They are also called Cardinal numbers, or Positive Integers.

The primary method to represent a Natural Number was to put a mark for each object. *The Egyptians* developed system of numerals. *The Babylonians* developed place value system of the numbers. Italian Mathematician *Giuseppe Peano* studied Natural Numbers and laid down some axioms:

- (a) Each Natural Number has a successor.
- (b) Every Natural Number has a predecessor except 1.
- (c) Each Natural Number has unique predecessor.
- (d) Set of Natural Numbers is represented by N.

N={1,2,3,4,....}

On Number Line, Natural Numbers are denoted by the dark dots on the right of 0 at equal distance. There is no Natural Number in the **LEFT** of 1.



There are infinite Natural Numbers. The least Natural Number is 1 and the largest Natural Number is not defined. The right hand side number on the Number Line is always greater than any number on its left hand side.

Natural numbers satisfy:

- (a) Closure Property,
- (b) Commutative Property,
- (c) Associative Property, and
- (d) Distributive Property for Multiplication distributed over Addition only.

Note:

- (1) Natural Numbers are neither closed for Subtraction nor Division.
- (2) There exists Multiplicative Identity only.
- (3) There exists no Additive Identity in set of Natural Numbers.

#### **PROPERTIES OF ADDITION**

#### **Closure Property**

It states that if a and b are any two Natural Numbers, and their sum (a+b) is also a Natural Number, then it is said

that the set of Natural Numbers is closed for addition for all Natural Numbers.

Example: 2 and 5 are Natural Numbers. The sum (2+5)=7 is also a Natural Number. Hence the set of Natural Numbers is closed for Addition.

#### **Commutative Property**

If a and b are any two Natural Numbers and (a+b)=(b+a), then the set of Natural Numbers is called to obey Commutative Property of Addition for all Natural Numbers.

Example: 3+4= 4+3

#### **Associative Property**

If a, b, and c are any three Natural Numbers and a+(b+c)=(a+b)+c, then we say that the set of Natural Numbers follows Associative Property of Addition for all its members.

Example: 3+(4+5)=(3+4)+5

#### **Distributive Property**

If a, b, and c are any Natural Numbers and  $a \times (b+c)=axb + axc$ , then it is said that a is distributed multiplicatively over the addition of b and c.

Example: 4x(5+6)=4x5 + 4x6

#### **Existence of Additive Identity**

If there exists a number in the set of Natural Numbers such that its addition with any other Natural Number gives the Number itself, then the adding unique number is called Additive Identity. o is called the Additive Identity.

Example: 0+ 5=5, 0+9=9. But o is not a member of the set of Natural Numbers, hence there *exists no additive Identity* in the set of Natural Numbers.

#### **PROPERTIES OF MULTIPLICATION**

#### **Closure Property**

It states that if *a* and *b* are any two Natural Numbers, and their product *(axb)* is also a Natural Number, then it is said that they are Closed for Multiplication.

For example: 2 and 5 are Natural Numbers. The product (2x5)= 10 is also a Natural Number.

#### **Commutative Property**

If **a** and **b** are two Natural Numbers and **(axb)=(bxa)**, then they are called to obey Commutative Property for Multiplication.

Example: 3x4= 4x3

#### **Associative Property**

If **a**, **b**, and **c** are any three Natural Numbers and **a***x*(**b***x***c**)=(**a***x***b**)*x***c**, then we say that Natural Numbers follow Associative Property of Multiplication.

Example: 3x(4x5)=(3x4)x5

#### **Distributive Property**

If a, b, and c are Natural Numbers and ax(b+c)=axb+axc, then it is said that a is distributed multiplicatively over addition of b and c.

Example: 4x(5+6)=4x5+4x6

#### **Existence of Multiplicative Identity**

If there exists a number in the set of Natural Numbers such that its product with any other Natural Number gives the Number itself, then the multiplier unique number is called Multiplicative Identity. 1 is called the Multiplicative Identity.

Example: 1x 5=5, 1x9=9. 1 is in the set of Natural Numbers. So, multiplicative Identity exists in the set of Natural Numbers.

#### **IMPORTANCE OF NATURAL NUMBERS**

- (a) Natural numbers are the origin for making of other number sets.
- (b) Natural numbers give existence to Negative Integers through Additive Inverse.

(c) Negative Integers are mathematically additive inverse of natural numbers set. They correspond one to one, i.e., Additive Inverse is unique or in other words, it can be said that there is one and only one additive inverse for each number. Additive inverse means the number that makes the sum of self and the other number equal to zero. Additive Inverse is always a negative number of the number.

For example:8 is a Natural Number. Negative of (8) is (-8). The sum of (8) and (-8)=(8)+(-8)=0.

Accordingly, (-8) is called the Additive Inverse of (8).

(d) Natural Numbers give way for the existence of the Rational Numbers through Multiplicative Inverse. There exists multiplicative inverse for each of the natural numbers. They correspond one to one, or mathematically, we may say that there exists one and only one multiplicative inverse for each number.

Multiplicative inverse means the number that makes the product of any number with it equals to 1.

For example: 3 is a natural Number. (1/3) is called multiplicative inverse of 3 as the product of (3) and (1/3)=1.

#### Note:

- (1) Some mathematicians have started assuming 0 as a Natural Number under ISO 31-11. ISO 31-11 was the part of International Standard ISO 31 that defined mathematical signs and symbols in 1992. It was superseded in 2009 by ISO 80000-2. Its definition included 0 as a Natural Number.
- (2) Inclusion of o in the set of Natural Numbers gives rise to a new definition: "A Natural Number is either a Positive Integer (1,2,3,...) or a Non-negative Integer (0,1,2,3,...)".

First definition is used in Number Theory and the Second is used in Sets Theory and Computer Science.

(3) Computer scientists often start from zero for enumerating items like loop counters.

#### WHOLE NUMBERS

Set of Natural Numbers with Zero is called Whole Numbers. It is in general denoted by W.

W={0,1,2,3,....}

In Whole Numbers there is no fractional or decimal part and no negatives.

On a Number Line, Whole Numbers are denoted by the thick **BLUE** dots. There is no number in the LEFT of **o**.



There is no end of Whole Numbers. The smallest Whole Number is o and there exists no greatest Whole Number.

All right hand side numbers are greater than any number on the LEFT hand side of it.

**Note**: All Natural Numbers are Whole Numbers but all Whole Numbers are not Natural Numbers. Example: o is not a Natural Number but o is a Whole Number.

#### **PROPERTIES OF ADDITION**

The set of Whole Numbers follows:

- (a) Closure Property
- (b) Commutative Property
- (c) Associative Property
- (d) Distributive Property

Note: Additive Identity {0} exists in the set of Whole Numbers.

#### **PROPERTIES OF MULTIPLICATION**

The set of Whole Numbers follows:

- (a) Closure Property
- (b) Commutative Property
- (c) Associative Property
- (d) Distributive Property

#### NOTE:

- (1) Multiplicative Identity {1} exists in the set of Whole Numbers.
- (2) There exists no Additive Inverse in the set of Whole Numbers.
- (3) There exists no Multiplicative Inverse in the set of Whole Numbers.
- (4) Whole Numbers set is not closed for Subtraction and Division.

#### **INTEGERS**

Integer is defined as a Number that is not a fraction. It is a Whole Number. The name is derived from the Latin *Integer* which means "*whole*".

There are three types of Integers:

- (a) The Negative Integers
- (b) Zero Integer, and
- (c) The Positive Integers

Integers are represented by *I* or *Z*. *Z* is due to German word *Zahlen* meaning **numbers**.

Z= {....,-3,-2,-1, 0, 1, 2, 3, ....}

On Number Line, the Integers are described as the thick dots at equal distances.



There are infinite numbers of Integers. There exists neither any least Integer nor any greatest Integer.

Each number is greater than its left hand side numbers. Going Right is Ascending Order (i.e., in Increasing Order) and Going Left is Descending order (i.e., in Decreasing Order).

Integers contain all Natural numbers, all Whole Numbers, and Negative of all Natural Numbers.

Set of Natural Numbers is also called Positive Integers and is denoted by  $I^+$  or  $Z^+$ . They have been denoted by **BLUE** dots on the Number Line.

**Zero** is neither Positive nor Negative Integer. It is denoted by *I*<sup>o</sup> or *Z*<sup>o</sup>. It has been denoted by **GREEN** dot.

The **Olmec** and **Maya** civilizations used **o** as a separate number as early as 1<sup>st</sup> Century BC, but the usage could not spread beyond Mesoamerica. The use of numeral **o** in modern times is the work of Indian Mathematician **Brahmgupta** in 628. Romans do not have any symbol for zero. They used word "**nulla**" meaning **none**.

Negative Integers have been denoted by **RED** dots on the Number Line. It is also denoted by *I* or *Z*.

Positive Integers with Zero are also called Non-negative Integers. Negative Integers with Zero are called Non-Positive Integers.

#### **PROPERTIES OF ADDITION**

The set of Integers obey

- (a) Closure Property for Addition, Subtraction and Multiplication
- (b) Commutative Property for Addition and Multiplication
- (c) Associative Property for Addition and Multiplication
- (d) Distributive Property of Multiplication over Addition and Subtraction
- (a) Additive Identity {0}, and Multiplicative Identity {1}, both exist in the set of Integers.
- (b) Additive Inverse exists in the set of Integers as there is negative of all positive Integers and there is positive of all negative integers.
- (a) Set of Integers is not closed for Division.
- (b) The set of Integers has Additive Inverses but does not have Multiplicative Inverses.

#### Examples:

 2 is an integer. (1/2) is its multiplicative inverse as (2)x(1/2)=1. But (1/2) is not a member of the set of Integers. Hence multiplicative Inverse does not exist.

#### **RATIONAL NUMBERS**

A Rational Number is a number in mathematics that represents a comparison of two numbers. The number that can be expressed as a ratio is called a Rational Number. In other words, the numbers that can be expressed in the form of p/q, where p and q are integers and especially q is not zero are called Rational Numbers.

Example: 1/2, 7/3, 0/1, , 1/1, etc.

Rational Numbers set is represented by Q. it was first denoted by *Giuseppe Peano* after *quoziente*, an Italian word for "*quotient*".

Rational Numbers are of two types:

- (a) Terminating decimal expression. Example  $\frac{1}{2}$  =0.5, and
- (b) Non-terminating or recurring decimal expression. Example: 1/3=0.333333

#### Note:

- (1) Sum, difference, product and division of two nonzero rational numbers is always a rational number.
- (2) Every Natural Number is a Rational Number.
- (3) Every Whole Number is a Rational Number.
- (4) Every Integer is a Rational Number.
- (5) There are infinite Rational Numbers.
- (6) There are infinite Rational Numbers between any two Rational Numbers.

#### PROPERTIES

- (a) Rational Numbers set is closed for Addition, Subtraction, Multiplication and Division by a nonzero number as division by zero is NOT defined.
- (b) Rational Numbers set obeys commutative, and associative properties for addition, and Multiplication.
- (c) It does not obey Commutative, and Associative Properties for Subtraction and Division.
- (d) It obeys the Distributive Property of Multiplication over Addition and Subtraction both.
- (e) There exists Additive Identity and Multiplicative Identity in the set of Rational Numbers.
- (f) There exists Additive and Multiplicative Inverses.
- (g) Set of Rational Numbers cannot be expressed in Roster form i.e., we cannot make the list of all rational numbers as there are infinite rational numbers between any two numbers.
- (h) Set of Rational numbers can be expressed on Number line as a non-stop continuous line.

#### **IRRATIONAL NUMBERS**

The numbers that cannot be expressed in the form of p/q are called Irrational Numbers. The numbers that are not Rational are called Irrational Numbers. The numbers that are neither terminating nor recurring are called Irrational Numbers.



 $\sqrt{2}$  is an irrational number.

Set of Irrational Numbers is denoted by **Q**<sup>c</sup>.

Example:  $\sqrt{2}$ ,  $\sqrt{3}$ ,... are Irrational Numbers but  $\sqrt{4}$  is not an Irrational Number as it terminates to 2.

Irrational Numbers can also be expressed on Number Line though they do not have a fixed numerical value. This is done by using **Pythagoras Theorem** that says that the square of the hypotenuse in the right angle triangle is equal to the sum of the squares of the other two sides.

The sum, difference, product and division of two Irrational Numbers is NOT always an Irrational Number.

Examples:

- (a)  $2+\sqrt{3}$ , and  $2-\sqrt{3}$  are two different Irrational numbers. The sum of these two numbers is 2, which is not an Irrational number.
- (b) For the irrational numbers  $2+\sqrt{3}$ , and  $2-\sqrt{3}$ , the product is 1 which is not an irrational number.
- (c) For the irrational numbers  $\sqrt{3-2}$ , and  $\sqrt{3+2}$ , the difference is also not an irrational number as it is 4, a rational number.
- (d) If  $2\sqrt{3}$ , and  $5\sqrt{3}$  are two irrational numbers then their quotient is 2/5, a rational number and not an irrational number.

#### Note:

- (1) There are some especial Irrational numbers: e,  $\pi$ , golden ratio (1+ $\sqrt{5}$ )/2, log<sub>2</sub>3, e<sup> $\pi$ </sup>,  $\pi$ <sup>e</sup>, etc
- (2) Greek Mathematician Pythagoras believed that all numbers were rational.
- (3) **Hippasus**, a student of Pythagoras proved using Geometry that square root of 2 cannot be written as a fraction and so it was not a rational number.

The followers of Pythagoras could not accept it and Hippasus was drowned at sea as a punishment from the gods.

#### SURDS

When it is not possible to remove the radical sign ( $\sqrt{}$ ) from the number, the number is called a surd. Surd is, infact, another name for Irrational Number.

Examples:

(a)  $\sqrt{2}$  (square root of 2) cannot be simplified further, hence it is a surd.

(b) (8)<sup>1/3</sup> (cube root of 8) can be simplified as 2. Hence it is not a surd.

#### Note:

- (1) Around 820 AD *al-Khwarismi* (a Persian mathematician from whom we got name "Algorithm" called Irrational Numbers "Inaudible" which was later translated to the Latin *surdus* (meaning deaf or mute).
- (2) Surds are simplified by rationalizing the denominator of the expression.

#### **CONJUGATE OF SURDS**

If x and y are two surds and their sum is a rational number, then both surds are called conjugate of each other.

Examples:

- (a)  $4 + \sqrt{5}$  is a conjugate of  $4 \sqrt{5}$ .
- (b)  $-4 + \sqrt{5}$  and  $-4 \sqrt{5}$  are conjugates of each other.

## FINDING SQUARE ROOTS OF A SURD $\sqrt{2+\sqrt{3}}$

#### Steps:

- (a) Assume  $\sqrt{2+\sqrt{3}} = \sqrt{a} + \sqrt{b}$  where **a**, and **b** are positive rational numbers.
- (b) Square both sides and equate rational part to rational part and the irrational part to irrational part.

$$2 + \sqrt{3} = a + b + 2\sqrt{ab}$$
$$a + b = 2 \qquad \dots (i)$$
$$2\sqrt{ab} = \sqrt{3} \qquad \dots (ii)$$

(c) Find out (a-b) from the formula (a-b)<sup>2</sup>=(a+b)<sup>2</sup>-4ab.

$$a-b=1$$
 (iii)

$$a = \frac{3}{2}, b = \frac{1}{2}$$

(e) The required square root is 
$$\sqrt{\frac{3}{2}} + \sqrt{\frac{1}{2}}$$

Note:

Assume  $\sqrt{a} - \sqrt{b}$  for the square root of  $\sqrt{2 - \sqrt{3}}$ .

#### **REAL NUMBERS**

The collection of all Rational Numbers and Irrational Numbers together is termed as the set of Real Number. It is represented by R. Real numbers can be represented on Number Line.

Real numbers set contains infinite numbers. All Natural Numbers, o, Negatives of all Natural Numbers, all Rational Numbers, and all Irrational Numbers are Real Numbers.

The sum, difference, product and quotient of two Real Numbers is always a Real Number. The sum, difference, product or quotient of a Rational and an Irrational Number is always an Irrational Number.

#### PROPERTIES

- (a) Set of Real Numbers is closed for Addition, Subtraction, Multiplication and Division by a non-zero number as division by zero is NOT defined.
- (b) Set of Real Numbers obeys commutative, and associative properties for addition, and Multiplication.
- (c) Set of Real Numbers does not obey Commutative, and Associative Properties for Subtraction and Division.
- (d) Set of Real Numbers obeys the Distributive Property of Multiplication over Addition and Subtraction both.
- (e) Set of Real Numbers contains Additive Identity {0}, and Multiplicative Identity {1}.
- (f) Set of Real Numbers contains Additive and Multiplicative Inverses.
- (g) Set of Real Numbers cannot be expressed in Roster form i.e., we cannot make the list of all Real Numbers.
- (h) Set of Real Numbers can be expressed on Number Line as a non-stop continuous line.
- (i) Set of Real Numbers are also represented in Interval forms as (2,4), [2,4], (2,4], [2,4) etc.
- (j) Real numbers are uncountable.

**Note:** The adjective Real was introduced in 17<sup>th</sup> Century by French Mathematician *Rene Descartese*, by distinguishing Real and Imaginary roots of polynomials.

#### KNOW MORE ABOUT NUMBERS

- (1) The usage or study of numbers is called arithmetic.
- (2) 0,1,2,3,4,5,6,7,8,9 are called **digits**.
- (3) 10, 11, 54,.., etc are called **numbers**.
- (4) A number divisible by 2 is called an **even** number.
- (5) A number not divisible by 2 is called an **odd** number.
- (6) Numbers greater than 1 and **not prime** are called **composite numbers**.
- (7) A number greater than 1 and having exactly two factors are called **prime numbers**.
- (8) Two numbers are called **co primes** if their HCF is1.
- (9) Prime numbers that differ by 2 are called **twin primes**.

(3,5),(5,7),(11,13),(17,19),(29,31),

(41,43),(59,61),(59,61),(71,73)

(10) The actual value of the number is called face value. The product of the number with the value of its place (Ones, Tens, Hundreds ...) is called place value of the number.

#### **DIVISIBILITY RULES**

**Divisibility by 2:** A number is divisible by 2 if its unit's digit is any one of 0,2,4,6, and 8.

**Divisibility by 3:** A number is divisible by 3 if the total of its digits is divisible by 3.

**Divisibility by 4:** A number is divisible by 4 if the number formed by the last 2-digits is divisible by 4.

**Divisibility by 5:** A number is divisible by 5 if its unit's digit is either 0 or 5.

**Divisibility by 6:** A number is divisible by 6 if it is divisible by both 2 and 3.

**Divisibility by 7:** A number is divisible by 7 if the difference of the double of the last digit and the number formed by rest of the digits is divisible by 7.

Example: 679.

Double of 9=18; Difference of 67 and 18= 49,

49 is divisible by 7.

Hence 679 is divisible by 7

**Divisibility by 8:** A number is divisible by 8 if the number formed by last three digits is divisible by 8.

**Divisibility by 9:** A number is divisible by 9 if the sum of all its digits is divisible by 9.

**Divisibility by 10:** A number is divisible by 10 if its unit digit is 0.

**Divisibility by 11:** A number is divisible by 11 if the difference of the total of digits at odd places and the total of digits at even places is divisible by 11.

**Divisibility by 12:** A number is divisible by 12 if it is divisible by 3 and 4 both.

**Divisibility by 14:** A number is divisible by 14 if it is divisible by 2 and 7 both.

**Divisibility by 16:** A number is divisible by 16 if the number formed by last 4 digits is divisible by 16.

#### **COMPLEX NUMBERS**

The numbers in the form of x + iy where x and y are real

numbers and  $i = \sqrt{-1}$  are called complex numbers. Complex Numbers may be purely real or purely imaginary. The set of complex numbers is represented by C.

Set of complex Numbers is closed for Addition, Subtraction, Multiplication and Division by a non-zero number. Set of complex Numbers follows Commutative, Associative, and Distributive Properties. Set of Complex Numbers contains Additive Identity, Additive Inverses, Multiplicative Identity and Multiplicative Inverses.

Complex Numbers are represented on the **Argand Plane**. This plane is also called **Gaussian Plane** or **Complex Plane**. In this plane, the ordinate or y-axis of Descartes Plane becomes imaginary. When a single letter is used to denote a Complex Number, it is sometimes called an **affix**. Complex Number x + iy, where x and y are real numbers, are represented in the following forms:

- (a) Cartesian Form : z = x + iy
- (b) Polar Form:  $z = r(\cos \theta + i \sin \theta)$
- (c) *Eulerian* Form:  $z = re^{i\theta}$  where r is the modulus and  $\theta$  is the principal argument.
- (d) Ordered Pair Form: z = (x, y) where x is real part, and y is imaginary part.

#### Note:

- (1) x is called the real part of z.
- (2) y is called the imaginary part of z
- (3) x iy is called the conjugate of x + iy
- (4) |x + iy| is called the modulus or magnitude of complex number x + iy and its value is  $\sqrt{x^2 + y^2}$
- (5)  $\tan^{-1} \frac{y}{x}$  is called the argument or amplitude of the complex number x + iy
- (6) o is a complex number that is purely real and purely imaginary.

#### **ROMAN NUMERALS**

Roman Numerals originated in ancient Rome. Roman Numerals use *seven symbols* (I, V, X, L, C, D, M) for representing numbers.

I for 1	II for 2	III for 3
V for 5	X for 10	L for 50
C for 100	D for 500	M for 1000

#### **Rules for using Roman Numerals:**

- (a) Any numeral can be repeated maximum up to 3 times.
- (b) Repetition means addition. Example: III means 1+1+1=3, X+X=20
- (c) Only I, X, C and M can be repeated. Examples: CC=200, MMM=3000
- (d) V, L, and D cannot be repeated. Examples: VV≠
   10, it is not allowed; LL≠100, it is not allowed; DDD≠1500, it is not allowed.

- (e) When a numeral of lower value is written to the right of a numeral of higher value, the value of all the numerals are added. Example: DCLVIII=500+100+50+5+1+1+1=658
- (f) When a numeral of lower value is written to the left of a numeral of higher value, then the value of lower numeral is subtracted from the value of higher numeral. Example: XL=50-10=40, CLIX=100+50-1+10=159
- (g) V is never written to the left of X.
- (h) If a horizontal line is drawn over the numerals then their value becomes 1000 times. Example:  $XV=15, \overline{XV} = 15000$

#### Note:

- (1) Romans did not use any symbol for o (zero).
- (2) 4000 or more than 4000 cannot be written in Roman Numerals without using bar on the numerals because none of the numerals can be repeated more than 3 times.

#### WRITING NUMERALS IN WORDS

There are two ways of writing and reading numbers in mathematics:

- (a) Indian Numbering System or Indian Place Value Chart
- (b) International Place Value Chart

#### **Indian Place Value Chart**

#### **Periods Consisting Of Two Places**

- PADMA TEN PADMA, PADMA
- NEEL TEN NEEL, NEEL
- KHARAB TEN KHARAB, KHARAB
- ARAB TEN ARAB, ARAB
- CRORE TEN CRORE, CRORE
- LAKH TEN LAKH, LAKH

THOUSAND TENTHOUSAND, THOUSAND

#### **Period Consisting Of Three Places**

ONE HUNDRED, TEN, ONE

#### Separators in Indian System

CRORE	LAKH	THOUSAND	ONE
00,	00,	00,	000
12,	34,	56,	789

#### Correct Way of Writing Numbers in Words

TWELVE CRORE THIRTY FOUR LAKH FIFTY SIX THOUSAND SEVEN HUNDRED EIGHTY NINE

#### Wrong Way of Writing Number:

TWELVE CRORES THIRTY FOUR LAKHS FIFTY SIX THOUSANDS SEVEN HUNDRED EIGHTY NINE

TWELVE CRORE, THIRTY FOUR LAKH FIFTY SIX THOUSAND, SEVEN HUNDRED EIGHTY NINE

TWELVE CRORE, THIRTY FOUR LAKH, FIFTY SIX THOUSAND, AND SEVEN HUNDRED EIGHTY NINE

Note: No **COMMA** between the periods, no **PLURALS** of the period, and nowhere **AND** is used in writing and reading the big numbers in words.

#### **International Place Value Chart**

All the Periods Consist Of Three Places as

- (a) **TRILLIONS** consists of Hundred Trillions, Ten Trillions, Trillions
- (b) **BILLIONS** consists of Hundred Billions, Ten Billions, Billions
- (c) **MILLIONS** consists of Hundred Millions, Ten Millions, Millions
- (d) **THOUSANDS** consists of Hundred Thousands, Ten Thousands, Thousands
- (e) ONES consists of Hundreds, Tens, Ones

#### Separators in International System

MILLIONS	S THOUSAN	NDS	ONES	
000,	000,		000	123,
45	6,	789		

#### **Correct Way Of Writing Numbers In Words**

ONE HUNDRED TWENTY THREE MILLION FOUR HUNDRED FIFTY SIX THOUSAND SEVEN HUNDRED EIGHTY NINE. Note: No **COMMA** between the periods, no **PLURALS** of the period, and nowhere **AND** is used in writing and reading the big numbers in words.

#### SOME SPECIAL NUMBERS

#### **Perfect numbers**

A perfect number is a Positive Integer whose twice is equal to the sum of its all divisors.

(a) 6 is a Perfect Number. Its divisors are: 1, 2, 3, and6.

2x6=1+2+3+6

- (b) The next Perfect Numbers are: 28, 496, ...
- (c) The general formula for the  $n^{th}$  Perfect Number is  $(2^{n}-1)2^{n-1}$  where n=2, 3,...

Mathematics makes life. Everyone knows mathematics. Even dogs know counting. If someone thinks it is not true then he should put three dog biscuits in his pocket for three dogs, and then give one each to only two of them. Watch what happens...ha ha ha...

## Numbers are very interesting. We shall continue with some new number systems like Binary, Octal, Hexadecimal, etc and their inter-relationships in the next e-bulletin....



Dr S.B. Dhar, is **Editor of this Quartrerly e-Bulletin**. He is an eminent mentor, analyst and connoisseur of Mathematics from IIT for preparing aspirants of Competitive Examinations for Services & Admissions to different streams of study at Undergraduate and Graduate levels using formal methods of teaching shared with technological aids to keep learning at par with escalating standards of scholars and learners. He has authored numerous books – Handbook of Mathematics for IIT JEE, A Textbook on Engineering Mathematics, Reasoning Ability, Lateral Wisdom, Progress in Mathematics (series for Beginner to Class VIII), Target PSA (series for class VI to class XII) and many more.

Narcissistic numbers

 $153=1^3+5^3+3^3$ 

 $370=3^3+7^3+0^3$ 

 $371=3^3+7^3+1^3$ 

 $407 = 4^3 + 0^3 + 7^3$ 

1634 = 14 + 64 + 34 + 44

Taxi cab number

Perfect digital invariant

It is equal to the sum of the cubes of its digits.

 $1729=1^3+12^3=10^3+9^3=7 \times 13 \times 19 = 19 \times 91.$ 

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## **INVITATION FOR CONTRIBUTION OF ARTICLES**

Your contribution in the form of an article, story poem or a narration of real life experience is of immense value to our students, the target audience, and elite readers of this Quarterly monthly e-Bulletin **Gyan-Vigyan Sarita**: शिक्षा, and thus create a visibility of the concerns of this initiative. It gives them a feel that you care for them, and they are anxiously awaiting to read your contributions. We request you to please feel free to send your creation, by <u>20<sup>th</sup> of this</u> <u>month</u> to enable us to incorporate your contribution in next bulletin, <u>subhashjoshi2107@gmail.com</u>.

We will be pleased have your association in taking forward path our plans as under-

- Next monthly Supplement to Quarterly e-Bulletin <u>Gyan-Vigyan Sarita</u>: Mail be brought out 1<sup>st</sup> Dec'16
- > And this cycle monthly supplement to Quarterly e-Bulletin <u>Gyan-Vigyan Sarita</u>: शिक्षा shall continue endlessly

We believe that this quarterly periodicity of e-Bulletins shall make it possible for our esteemed contributors to make contribution rich in content, diversity and based on their ground level work.

**GROWING WITH CONCEPTS - Physics** 

## **MECHANICS – Part II: NEWTON'S LAWS OF MOTION**

#### Dr. Subhash Joshi

Some real life situations are being brought out to relate concepts Newton's Laws of Motion to happening around us. *First situation* is of glass fill of water is kept on the table, when we move close to it want to drink we lift it. The urge of thrust is so strong that we do not realize that some effort is being made to lift the glass.

In *Second situation* a box of instrument is kept on the floor, near teacher's table in a class. These instruments are to be used by Physics teacher to explain concepts of mechanics to the class. A teacher enters the class and calls Two of the students in the class. The one of the student is unenthusiastic about it unwillingly makes effort to lift the box and place it on teacher's table. The other student being enthusiastic to learn, happily joins hands to lift the box and place it on table. Expression of both the students clearly depict relative difference in their efforts to do the same work, together.

In *Third Situation* a man steals your bag on a railway platform and tries to run away. As soon as notice it, you will first race to reach the thief. A soon as you catch hold of the thief he tries to run faster. In an effort to be successful have to apply extra force so that the thief does not escape.

In *Fourth Situation* an object is released on a smooth inclined surface joining a smooth horizontal surface. The object continues to slide unless it is obstructed by another object. Interaction between the sliding object and obstruction involves forces.

*Fifth Situation* calls for visualization of ride Sky Wheel, one feels of weightlessness when the cradle in which we are sitting descends on the periphery of the sky wheel, while feel gaining wait when the wheel ascends.

More of such situations can be observed in day to day experiences to visualize as to how does mass, force and acceleration are coming into play. Discussion to follow start with Newton's Laws of Motion, a subject matter of *classical mechanics or Newtonian Mechanics*, and is the basic concept behind these observations. It recognizes existence of an external force that can change state of rest or motion of an object. It is a set of Three Laws : Newton's First Law of Motion – also known as *law on inertia*, Newton's Second Law of Motion – also known as *law of acceleration*, Newton's Third Law of Motion – also known as *law of reaction or Cause and Effect*. Each of these law and associated mathematical concepts are elaborate below.

**Newton's First Law of Motion (NFLM) :** *A body in an inertial frame of reference continues to be in a state of rest or motion with a constant velocity so long it is in a <u>state of equilibrium</u>. Every body in this universe is experiencing force of one or the other kind from its surrounding. Such a situation cannot be called a state of No force or Zero Force. But, when all the forces ( a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is ( a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is Zero and is like a state of No Force.* 

Forces acting at any point on an object shown can be conceptually shown in a manner where end of a vector is beginning of another vector. In case of equilibrium, it is a closed polygon, having zero resultant of all the forces. While in case of inequilibrium it is an open polygon, having resultant of all the forces, having a resultant i.e., an equivalent of all the forces represented by a vector joining the starting point of the first vector and end point of the last vector in the sequence of open polygon.

All the forces acting on the object, as shown in the figure below, can also be represented in star like formation. Mathematically, resultant of these forces is :

$$\bar{F}_{eq} = F_x \hat{\iota} + F_y \hat{j} = \sum_{k=1}^{k=n} \left( F_{xk} \hat{\iota} + F_{yk} \hat{j} \right);$$

Here,  $F_{xk}$  is the x-component of  $k^{\text{th}}$  force,  $F_{yk}$  is the y-component of  $k^{\text{th}}$ . In case of equilibrium  $F_{xk} = F_{xk} = 0$  or  $|F_{eq}| = 0$ , while in case of in-equilibrium either  $F_{xk}$  may be zero or  $F_{xk}$  may be zero, but both cannot be zero, i.e.  $|F_{eq}| \neq 0$ .



**Newton's Second Law Of Motion (NSLM):** The conditions of in-equilibrium in the above illustration gives rise to proposition of Newton's Second Law of Motion, to provoke a thought as to what would happen. Newton propounded the effect of in-equilibrium of forces acting on an object as Second Law of Motion which states that : *in an inertial frame of reference net force acting on an object causes an acceleration of the object in the direction of the force, such that Force is equal to product to the mass of the object and the acceleration.* Mathematically, it is expressed as:  $\bar{F} = m\bar{a}$ . This has led to evolution of a new term *Momentum* ( $\bar{p} = m\bar{v}$ ) and rate of change of momentum:  $\bar{F} = m\bar{a} = m\frac{d\bar{v}}{dt} = \frac{d}{dt}(\bar{p})$ . This definition of rate of change of momentum becomes very useful in analysing propulsion of rockets which eject mass during their motion and will be analysed as we proceed in study of dynamics.

**Newton's Third Law of Motion (NTLM):** In fourth situation, visualized in the beginning of this, the sliding object exerts a force (Cause) on the obstructing object. But, what happen to the sliding object (Effect) as a consequence. This was analysed by Newton and propended as **Third Law** which states that : *when an object exerts a force (action or cause) on another object, the second object simultaneously exerts an equal-and-opposite-force (reaction or effect) on the first object.* 

Newton's Third Law of motion has its manifestations in Two forms, first is in Inertial Frame of Reference (IFOR) w.r.t which object is in state of rest. And second is Non-inertial Frame of Reference (NFOR) where frame of reference is itself accelerating.

#### Newton's Third Law in IFOR:



A Ball of Mass *m* is experiencing a gravitational acceleration  $\bar{g}$ . But, the acceleration is stopped by the table top on which it is kept . As a result, as per NSLM, the table top exerts a force  $R_I$  on the ball to stop the acceleration of the ball under gravity and thus ball stays at rest on the table top. It is a case of equilibrium in IFRM. Accordingly, the ball is in a state of equilibrium such that  $m\bar{g} = \bar{R}_1$ .

It is also observed that, despite the ball on the rectangular Table Top, placed at its centre, which is exerting an upward force  $\bar{R}_1$ , the table top remain at a state of rest on the earth's surface, the IFRM. As per NFLM this is possible only when  $\bar{R}_1 = 2\bar{R}_2$ ; here,  $\bar{R}_2$  is the force exerted on pair of table legs, one behind the other.

Despite  $2\bar{R}_2$  force on the table legs, their motion is stopped by the earth's surface, an IFRM, on which the table is kept. This can only happen in a state of equilibrium i.e.  $2\bar{R}_2 = 2\bar{R}_3$ . Here,  $\bar{R}_3$  is force exerted by the earth's surface on each pair of legs. Thus reaction (effect) of the table top, and the earth's surface in IFRM is equal to action (cause) i.e. weight of the ball i.e.  $m\bar{g} = 2\bar{R}_2 = 2\bar{R}_3$ .

Here, for a moment discussions on  $2\bar{R}_4$  shown in the figure are put on hold till discussions on NTLM in NFOR, to follow, are completed. But, it can be realized from the impression of legs of the table, that it leaves, on bare ground surface. These

impressions are visible when table is removed, and is indicative of force causing depression on the surface in the form of the impressions.

**Newton's Third Law in NFOR:** It is an excellent example of out of box visualization of scientific principles in surrounding. Those living in rural background must have experienced that when a bucket is released in a well, its weight for a moment apparently decreases. On the contrary, when bucket is pulled out of well it requires more force than that required to hold it stationary in hair. Similar experience one gets when during a sky-wheel ride.



When bucket is released in the well, it is descending down with an acceleration a w.r.t. earth, the IFRM. Thus, the bucket NFRM. Now, if bucket, which is experiencing g has to be transformed into IFRM, it shall have to be subjected to a retardation a w.r.t. to itself. Thus Ma, is virtual force shall have to be assumed. This will lead to T = Mg - Ma = M(g - a). This virtual force is called *pseudo force is against the direction of acceleration of NFRM*, which in the instant case is bucket, and hence bucket is experienced to be lighter.

Likewise, when bucket is pulled out of well with an acceleration a' w.r.t earth the IFRM, the pseudo force would add to gravitational pull and thus T' = Mg + Ma' = M(fg + a'). This is conformance with experience of additional force, or effort, to pull out the bucket, as compared to that of holding it in place in the well.

This stipulation of Pseudo Force is transformation of Non-inertial Frame into state of virtual equilibrium like that of IFRM, where problem is transformed into application of NFLM.

This is now appropriate stage to examine what happens to effect  $2\bar{R}_4$  shown in the figure having a ball kept on the table. Earth surface was considered to be IFRM. The  $2\bar{R}_4$  effect of  $2\bar{R}_3$  should be causing acceleration earth. Accordingly, this problem also should have been analysed on the lines of NTLM in NFRM But,  $M_e$  us very large as compared to M  $(M_e, >>M)$  such that acceleration of earth  $(a_e)$  as per NSLM would be  $a_e = \frac{Mg}{M_e} \rightarrow 0$ . This is the reason that despites earth being n constant acceleration, due to rotatory and revolving motion, which again has much smaller angular speed that the physical objects being observed, is taken as IFRM. Circular and rotatory motion shall be discussed little later.

Next comes sky wheel where we find that cradles are radial when they are at top of the wheel or on the bottom of the wheel. Otherwise, they remain suspended with a tilt outwards. Understanding, the cause of this observation requires concepts of circular motion.

**Uniform Circular motion:** In this a particles is taken to be *revolving around a fixed point with a constant radius and* with a constant angular speed  $\overline{\omega} = \frac{d\overline{\theta}}{dt}$ , a vector quantity, here  $\overline{\theta}$  is the angular displacement on a plane a vector quantity. Likewise, there exists angular acceleration  $\overline{\alpha} = \frac{d\overline{\omega}}{dt}$ , and its will be used while working with rotational motion. Since angles are measured in clockwise direction and hence  $+\overline{\omega}$  is upwards, coming out of a surface, and  $-\overline{\omega}$  is inwards, entering a surface. This circular motion is analysed below.



Linear velocity at any instant is  $\bar{v} = \bar{\omega} \times \bar{r}$ , a vector cross product.. Since, axis of rotation ( $\hat{\omega}$ ) and radius of rotation ( $\hat{r}$ ) are perpendicular to each other hence. Accordingly,  $v = |\bar{v}| = |\bar{\omega}||\bar{r}|$ ,  $v = r\omega$ ; since *r* and  $\omega$ *are constant, basic premise of circular motion, hence v is also constant.* Let,  $\theta_t$  be the displacement of radial vector during time  $\Delta t$  when velocity vector of the particle changes from  $\bar{v}_0$  to  $\bar{v}_t$ . Accordingly,  $\Delta \bar{v} = \bar{v}_t - \bar{v}_0$ . Since, *v* is constant in the vector diagram for  $\Delta \bar{v}$ , both  $v_t$  and  $v_0$ forming an isosceles triangle are also constant at an angle  $\theta_t = \omega \Delta t$  and geometrically length of the third side  $\Delta v = r\theta_t = 2v \sin \frac{\theta_t}{2}$ .

As such, acceleration of the article performing circular motion shall be:  $a = \frac{\Delta v}{\Delta t}\Big|_{\Delta t \to 0} = \frac{2v \sin \frac{\theta_t}{2}}{\Delta t}$ . Substituting v and  $\Delta t$  from

the above:  $a = \frac{2r\omega\sin\frac{\theta_t}{2}}{\frac{\theta_t}{\omega}} = r\omega^2 \frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}$ . Further, geometrically  $\frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}\Big|_{\Delta\theta_t \to 0} = 1$  and hence,  $a = r\omega^2$ . A close observation of the

vector diagram reveals that,  $\Delta \theta_t \rightarrow 0$ ,  $\Delta \bar{v}$  tends to become perpendicular to instantaneous velocity of the particle, performing circular motion, i.e. radially inwards called **centripetal acceleration a vector**  $\bar{a}$ . This centripetal acceleration is keeping the particle perform uniform circular motion, else the particle will run away. This experience can be obtained with water soaked in a wet handkerchief, when it is rotated holding its one end.

Review of NTLM in NFOR : This, is the point to discuss why cradles of sky wheel get automatically tilted outwards and

observe as to how NTLM in NFOR automatically comes into play. The cradle, being fixed on to rim at point P of the sky-wheel is performing circular motion. Accordingly, it will experience a constant acceleration  $\bar{a}$ , and thus the cradle becomes a NRFM. When, the forces on the cradle are transformed to IFRM, an observer on the ground, an IFRM, this centripetal acceleration is considered as causing a pseudo force as shown in the diagram below. Resultant of the pseudo force and gravitational pull, by IFRM, is outwards and it causes tilting of cradle outwards, depending upon its magnitude and direction, except on the highest and lowest points where pseudo and gravitational force are collinear. If accidently cradle gets unhinged to the rim it would run away outwards, with instantaneous velocity at the time of release and would perform projectile motion



under gravity, with no role of centripetal acceleration. There are numerous situations encountered in daily life, involving circular motion on IFRM and NFRM. Thus pseudo force is called *Centrifugal Force*.

There is another situation when a particle is experiencing **non-uniform circular motion**, it can happen when velocity of particle performing circular motion has its velocity v is either accelerating or retarding in which case the trace of

particle would be growing spiral or collapsing spiral, respectively. In this case net **tangential acceleration** of the particle shall be not focussed towards centre of the spiral, unlike uniform circular motion. The net acceleration shall be drifted forward from the centre of the spiral, i.e. in the direction of velocity in case of tangential acceleration; and backward, i.e. against the velocity, in case of tangential retardation. Mathematically, this is elaborated as a =

$$\overline{r} + a_t^2 = \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2}$$
 and angle of drift is  $\alpha = dr$ 

 $\sqrt{a_r^2}$ 



 $\tan^{-1} \frac{dv/dt}{r\omega^2}$ , and supported with necessary illustration diagram. A similar effect of drift in gravitational pull is experienced when one moves from equator towards poles. But, it is not due to non-uniform circular motion, rather it is due to pseudo force on an object which remains un-displaced due to predominant gravitational force by the earth, while centrifugal force caused by rotation of the earth which is radial to the axis of rotation but not the gravitational force which is along the radial joining the object and centre of the earth. Thus as one questions the observations, more of integration of different concepts is involved, and such problems and articles would be found in references cited below.

Work, Power and Energy: A person lifting an object placed on floor and placing on a raised platform does a work. But,

displacing an object displaced on a smooth horizontal surface is not. This requires to understanding **definition of work** (*W*) in physics according to it *work is the product of Force and displacement caused by it in the direction of the force.* Mathematically, it is expressed as DOT product of force and displacement, both of which are vectors, while work is scalar.  $W = \overline{F} \cdot \overline{D} = FD \cos \theta$ , where *W* is work,  $\overline{F}$  is



force acting on an object,  $\overline{D}$  is the displacement of the object under influence of force  $\overline{F}$  and  $\theta$  is the angle between vectors  $\overline{F}$  and  $\overline{D}$ . Thus, alternatively, work is product of displacement and force in the direction of displacement, a mathematical equivalent. The SI unit of Force in Newton, Distance us Meter and accordingly unit of Work is Newton Meter and also called Joule, which is more widely used in Heat. Dimensionally,  $[W] = [F][D] = [MLT^{-2}][L]$ . In the expression of work both  $\theta$  and  $\cos \theta$  are dimensionless, and accordingly dimension of work reduces to  $[W] = [ML^2T^{-2}]$ .

While **power (P) is rate of doing work**. It can be compared with two vehicles climbing on an inclined road starting from same point. Time taken vehicle A to reach destination is 2 Hours, while time taken by vehicle B to reach destination is 3 hours. Then in common parlance it is said that vehicle A has more power than the B. Accordingly, **Work** is mathematically expressed as  $P = \frac{W}{T}$ , and thus unit is Joules per Second or Joule/Sec. and dimensionally it reduces to  $[P] = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$ .f

In another, situation vehicle A after one full tank filling in it makes 5 trips, but the vehicle B after full tank filling makes 7 trips than in common parlance *energy* of vehicle B is greater than that of A, and in this power has no consideration. Mathematically energy (E) is expressed as  $E = P \times T$ , and accordingly unit of energy is [(Joules/Sec) X (Sec) = (Joule)]. Energy in classical mechanics is considered to be conservative. The *Law of Conservation of Energy (LCE)* states that *energy can neither be created nor destroyed, it can be transformed from one form to the other*. In mechanics energy is considered to be two form; one is *Potential Energy*, *due to position of an object* and the other is *Kinetic energy*, *due to velocity of an object*. Here, discussion is limited to these two forms.

This is the time to review concept of work when  $0^0 \le \theta < 90^0$  and  $90^0 < \theta \le 180^0$ . In the earlier case  $0 < \cos \theta \le 1$ , force  $\overline{F}$  has a component in the direction of  $\overline{D}$ , and mathematically net work done is +ve. While in latter case  $-1 \le \cos \theta < 0$  and force  $\overline{F}$  has a component against the direction of  $\overline{D}$ , and mathematically net work done is -ve. Thus +ve work done by external agent exerting the force stores energy in the object. Here, it is to be noted that when an external force is so exerted that displacement is slow without causing any acceleration. Accordingly, as per NSLM, it is a case NTLM in IFOR, and thus there will be an equal and opposite force of reaction. Therefore, in terms of reaction or restraining force, work done by external force causing change of position is stored as energy in the object in IFRM; this is called potential energy. Two typical examples of potential energy are being elaborated here under:

Potential in Gravitational Field: Work done in moving a unit mass from Earth's surface, against the gravitational

pull, up to a point P at a radial distance  $\mathbf{r} : PE = \sum \Delta w = \int_{R_e}^{r} \left( G \frac{M_e}{x^2}(-\hat{x}) \right) \cdot d\bar{x} =$ 

$$GM_e \left[\frac{1}{x}\right]_{R_e} = GM_e \left[\frac{1}{r} - \frac{1}{R_e}\right] = \frac{GM_e}{r}\Big|_{taking, \frac{GM_e}{R_e} = 0}$$
. Here, gravitational force, is in direction

 $-\hat{x}$ ; while displacement  $\Delta x$  is also in direction  $\hat{x}$  and shall be discussed, a little later in this series. In present context, till **Law of Gravitation** is discussed, in the



formulation of PE acceleration due to gravity (*g*) as per GEM and NSLM is used instead and thus formulation of PE becomes:  $PE = \sum \Delta w = \int_0^h m\bar{g} \cdot d\bar{h} = [mgh]_0^h = mgh$ . Further, in above derivation, it is assumed that Potential at Earth's surface is Zero, instead at infinity, -ve sign is not used with  $d\bar{h}$ . Hence the PE at point P for a unit mass, calculated above, is called as relative **Potential** w.r.t. Earth's surface. The moment mass of the object being moved is considered, other than unity, it becomes **Potential Energy**.

**Potential in Gravitational Field: Potential Energy:** When spring is stretched/ compressed by length *x* it requires a force in the direction of push/pull= kx. An incremental pull/push over an infinitesimal length  $\Delta x$  would call upon external work, stored in the form of energy in the system:  $\Delta W = -k\bar{x} \cdot \Delta \bar{x}$  external work, stored in the form of energy in the system:

 $\Delta W = -k\bar{x} \cdot \Delta \bar{x}$ . Hence,  $W = PE = \int_{x=0}^{l} -k\bar{x} \cdot d\bar{x} = -\frac{1}{2}k[x^2]_0^l = -\frac{1}{2}kl^2$ . This is the absolute Potential Energy of the spring when stretched by length *l*. When *l=o*, its PE=o. It is to be noted that in the above derivation of **Potential Energy** of spring, primary parameter is spring constant k which control restraining force. This restraining force is being overcome by external force to cause displacement without acceleration, and there is no role of mass in it. This is equally valid when spring is compressed by an external force. Thus, in case of spring its elongation or compression change in PE remains uninfluenced, while in case of Gravitational field of change in PE of an object while descending reverses to that while ascending.



As per law of conservation of energy in classical mechanics, an object from a height shall while losing PE should get KE equivalent to the loss of PE and in turn velocity. But, a question may arise how to relate velocity component of KE to the PE. Here, TGEM comes into play. In simple case a ball of mass m, at a height has PE = mgh. And for a fall through height h, gain in velocity from an initial velocity u = 0, as per TGEM,  $v^2 = 2gh$ . Accordingly, *equivalence of the two form of energy*  $KE = PE = m\left(\frac{v^2}{2}\right) = \frac{1}{2}mv^2$ .

**Momentum-Impulse-Collision:** The concept of momentum was introduced during discussions on NSLM, which is being extended into *Law of Conservation of Momentum (LCM)*. Together with this, concept Law of Conservation of energy will be helpful to conceptualize Impact or impulse and Collision.

An experiment is suggested, where a tightly closed box, with air inside, has a centrally placed lid. The box, is placed in sunlight and gets heated developing an internal force causing self-opening of the box lid; there is no external force. It is seen that the lid, having mass  $\boldsymbol{m}$ , moves with a high velocity, and the box, having mass  $\boldsymbol{M}$ , is pushed in a direction opposite to the lid with a velocity relatively much smaller than that of the lid. This follows laws of momentum according to which external force  $\bar{F} = 0 = \frac{d}{dt}\bar{p} = \frac{d}{dt}((M + m)\bar{v}_0)$ , and  $\bar{P}=0$ . Initial velocity  $\bar{v}_0$  of the combined mass  $M + m \neq 0$  and hence  $\bar{v}_0 = 0$ . When the lid open let the lid has velocity  $\bar{v}_m$  and the container has velocity  $\bar{v}_M$ . Therefore, in absence of external force,  $\bar{P}$  would continue to be Zero i.e.  $\bar{P} = 0 = M\bar{v}_M + m\bar{v}_m$ , or  $\bar{v}_M = -\frac{m}{M}\bar{v}_m$ . This mathematical analysis is in conformance with the example experiment cited above. This experiment can be tried and if need be some stove can be used in open, to heat the box, with a care to avoid any accident.

Nevertheless, in presence of an external force or no force , as per NSLM,  $\bar{F} = \frac{d}{dt}\bar{p}$  and hence  $\bar{J} = \int_{\bar{p}_i}^{\bar{p}_f} d\bar{p} = \int_{t_i}^{t_f} \bar{F} dt$ ; or  $\bar{J} = \bar{p}_f - \bar{p}_i$ . This simplifies the analysis to initial and final momentum or in turn velocities of each of the interacting components of the object on which force is applied.

**Elastic Collision:** It is pertinent to understand that in the case of elastic collisions on a horizontal plane essential condition for collision are – (a) Both the initial velocity vector should be on same plane and anti-parallel, and (b) Precollision positions of the object such that trace of displacements has a point of convergence called collision.

During impact there would be elastic deformation, but during deformation it would store energy, and release the stored energy during restoration of shape prior to collision. This is similar to the conversion of  $PE \leftrightarrow KE$  in case of spring

discussed above; here PE is due to shape and not the position . Therefore, *it must comply with the LCM and LCE of colliding objects*. In this experiment two balls of mass  $m_1$  and  $m_2$  moving with velocities  $\overline{v}_1$  and  $\overline{v}_2$  collide and are supposedly known. After the collision, both the masses without splitting, acquire velocities  $\overline{v}'_1$  and  $\overline{v}'_2$ , which are unknown. This is essential condition to determine four unknown variables. The above case is analysed below with a generic example of oblique collision in two dimensions, and it has been simplified by reducing variables for collision in one dimension. This simplification requires identifying *line of impact*, *a line joining centres of the two* 



*spherical balls* (the colliding objects), **point of collision** *P*, on line of impact where the two Ball touch each other. **Tangential line** is the line perpendicular to the line of impact at the point of collision. Direction of velocity vectors of

both the balls pre- and post-collision are shown in the figure. There is no transfer of momentum on the tangential line, while all the interaction during collision is along the line of impact.

Since, velocity is a vector it has two parameters and, therefore, the Two unknowns vectors  $\overline{v}'_1$  and  $\overline{v}'_2$  shall have Four parameters to be determined to find complete solution. Since, each vector can be resolved along two perpendicular directions and formulate four equations, a necessary condition for the solution.

*Mathematical analysis* of elastic collision to determine post-collision velocities, the unknown, follows. Since it is a dynamic interaction along line of impact, hence velocity components of colliding ball long this line pre- and post-collision shall remain unchanged;  $v_1 \sin \alpha = v'_1 \sin \theta$ , and  $v_2 \sin \beta = v'_2 \sin \varphi$ . While, as per LCM,  $m_1 v_1 \cos \alpha + m_2 v_2 \cos \beta = m_1 v'_1 \cos \theta + m_2 v'_2 \cos \varphi$ . Likewise, as per LCE total KE shall remain unchanged; thus  $\frac{1}{2}m_1 v_1^2 \cos^2 \alpha + \frac{1}{2}m_2 v_2^2 \cos^2 \beta = \frac{1}{2}m_1 v'_1^2 \cos^2 \theta + \frac{1}{2}m_2 v'_2^2 \cos^2 \varphi$ .

Likewise, as per equation of LCE,  $\frac{1}{2}m_1(v_1^2\cos^2\alpha - v_1'^2\cos^2\theta) = \frac{1}{2}m_2(v_2'^2\cos^2\varphi - v_2^2\cos^2\beta)$ . Second equation is  $m_1(v_1\cos\alpha - v_1'\cos\theta)(v_1\cos\alpha + v_1'\cos\theta) = m_2(v_2'\cos\varphi - v_2\cos\beta)(v_2'\cos\varphi + v_2\cos\beta)$ . Dividing, this transformed equation of LCE with that of LCM:  $(v_1\cos\alpha + v_1'\cos\theta) = (v_2'\cos\varphi + v_2\cos\beta)$ . This equation deduces to  $(v_1\cos\alpha - v_2\cos\beta) = (v_2'\cos\varphi - v_1'\cos\theta) = -(v_1'\cos\theta - v_2'\cos\varphi)$ . This equation is interpreted as **velocity of approach is equal to velocity of separation of colliding objects, and is a corollary of combined LCM and LCE (Corollary 1)**.

Thus from the two equations above, one from LCM and the other from corollary 1, it is a *solution of linear simultaneous equations*, needed to arrive at :

$$v_{1}' \cos \theta = \frac{(m_{1}-m_{2})v_{1} \cos \alpha + 2m_{2}v_{2} \cos \beta}{m_{1}+m_{2}} \quad [Multiply \ \mathbf{m_{2}} \ to \ equation \ from \ Cor.1 \ and \ subtract \ it \ from \ equation \ from \ LCM]$$
$$v_{2}' \cos \varphi = \frac{2m_{1}v_{1} \cos \alpha + (m_{1}-m_{2})v_{2} \cos \beta}{m_{1}+m_{2}} \quad [Multiply \ \mathbf{m_{1}} \ to \ equation \ from \ Cor.1 \ and \ subtract \ it \ from \ equation \ from \ LCM].$$

These two velocity components of colliding balls along the line of impact together with velocity equivalence along tangential line, brought out in the beginning of this mathematical analysis, is enough to determine the vectors  $\overline{v}'_1$  and  $\overline{v}'_2$ 

Determination of *impact of collision* on each of the colliding balls uses LCM represented as:  $m_1(v_1 \cos \alpha - v'_1 \cos \theta) = m_2(v'_2 \cos \varphi - v_2 \cos \beta)$ . Here, impact on collision ball of mass A is  $J_1 = m_1(v_1 \cos \alpha - v'_1 \cos \theta)$ , and impact on ball B is  $J_2 = m_2(v_2 \cos \beta - v'_2 \cos \varphi)$ , and as per LCM  $J_1 = -J_2$ , this is in conformance with NTLM (*Corollary 2*).

**Non-elastic Collision:** it is different from elastic collision in respect of restoration of shape of colliding objects after collision; in non-elastic collision original shape or position is not restored. This calls for introduction of a new term *e* **Coefficient of Restitution**. This can be compared with a mass *m* dropped from a height H on a horizontal surface bounces back to a height *h*. In such a case  $e = \sqrt{\frac{h}{H}}$  and as per LCE  $e \le 1$ . From TGEM,  $h = \frac{v^2}{2g}$  and  $H = \frac{u^2}{2g}$ . Accordingly,  $e = \frac{v}{u}$ , here *u* is the velocity of the mass at the time of impacting on the horizontal surface and *v* is the velocity of the mass leaving the surface after impact. Looking it from the case of elastic collusion, *u* is the velocity of approach to the horizontal surface where impact is taking place and *v* is the velocity of separation from the surface. Taking energy considerations, kinetic energy pre-collision  $KE_1 = \frac{1}{2}mu^2$  and post-collision  $KE_2 = \frac{1}{2}mv^2$ ; or  $\frac{v}{u} = \sqrt{\frac{KE_2}{KE_1}} = e$ , alternately,  $KE_2 = e^2$ .  $KE_1$ . Extending the analogy of non-elastic collision, in context of LCM in IFRM where horizontal surface of mass *M* remains static having Zero velocity,  $p_i = kp_f$ . Here,  $p_i = mu$  and  $p_f = mv$ . Accordingly  $\frac{p_f}{p_i} = \frac{1}{f} = \frac{v}{u} = e$ ; or  $mv = e \cdot mu$ , or  $p_f = e \cdot p_i$  in other words in non-elastic collision post-collision momentum is equal to coefficient of restitution multiplied to pre-collision momentum.

This analogy can be applied to oblique collision where equations of velocity equivalence along tangential line remain unchanged while equations of LCM and LCE would be:  $e(m_1v_1\cos\alpha + m_2v_2\cos\beta) = m_1v'_1\cos\theta + m_2v'_2\cos\varphi$ ; as per LCM- for non-elastic collision and it leads impact equation as :  $m_1(ev_1\cos\alpha - v'_1\cos\theta) = m_2(v'_2\cos\varphi - ev_2\cos\beta)$ . Further, as per LCE the energy balance is:  $e^2(\frac{1}{2}m_1v_1^2\cos^2\alpha + \frac{1}{2}m_2v_2^2\cos^2\beta) = \frac{1}{2}m_1v'_1^2\cos^2\theta + \frac{1}{2}m_2v'_2^2\cos^2\varphi$ . Alternatively,  $m_1(e^2v_1^2\cos^2\alpha - {v'_1}^2\cos^2\theta) = m_2({v'_2}^2\cos^2\varphi - e^2v_2^2\cos^2\beta)$ , or  $m_1(ev_1\cos\alpha - v'_1\cos\theta)(ev_1\cos\alpha + v'_1\cos\theta) = m_2(v'_2\cos^2\theta) = m_2(v'_2\cos^2\theta)$   $m_2(v'_2 \cos \varphi - ev_2 \cos \beta)(v'_2 \cos \varphi + ev_2 \cos \beta)$ . Combining thus equations with impact equation:  $(ev_1 \cos \alpha + v'_1 \cos \theta) = (v'_2 \cos \varphi + ev_2 \cos \beta)$ , it leads to equation of velocity of approach and separation as:  $e(v_1 \cos \alpha - v_2 \cos \beta) = -(v'_1 \cos \theta - v'_2 \cos \varphi)$ , transformation of Corollary 1 to non-elastic collision. This equation together with LCM for the case leads to:

$$v_1' \cos \theta = \frac{(m_1 - em_2)v_1 \cos \alpha + (1 + e)m_2v_2 \cos \beta}{m_1 + m_2} \quad [Multiply \ \mathbf{m_2} \ to \ equation \ from \ Cor.1 \ and \ subtract \ it \ from \ equation \ from \ LCM]$$
$$v_2' \cos \varphi = \frac{(1 + e)m_1v_1 \cos \alpha + (m_1 - em_2)v_2 \cos \beta}{m_1 + m_2} \quad [Multiply \ \mathbf{m_1} \ to \ equation \ from \ Cor.1 \ and \ subtract \ it \ from \ equation \ from \ LCM].$$

This is the most general equation for collision, and gets easily transformed to specific case in isolation or in multiple combination of the following options:

- **a.** Elastic collision by choosing e = 1, **b.** One dimensional Collision by choosing  $\alpha = 0$  and  $\beta = 0$
- **c.** Collision with static object by choosing  $v_2 = 0$ , **d.** Collision of two bodies of uniform masses by choosing  $m_1 = m_2$

**Rocket Propulsion:** This is another interesting case of change in momentum in IFRM, where rocket, without any external force, is self-propelled to move against gravity by ejection of mass at  $\mathbf{r}$  kg/sec in opposite direction, a consequence of NTLM. Let initial mass of the rocket be  $(\mathbf{m}_i)$  and it is moving at a velocity  $(\mathbf{v}_i)$ , thus initial momentum be  $p_i = m_i \cdot v_i$ , a product of two variables dependent upon time. Since there is no external force and hence :  $\frac{d}{dt}p_i = 0 = \frac{d}{dt}(mv) = m\frac{d}{dt}v + v\frac{d}{dt}m$ . Accordingly, at any point of time ( $\mathbf{t}$ ) during propulsion  $p_i = m_iv_i = p = mv = (m_i - rt)v_t$ . Thus, velocity of the rocket at any time ( $\mathbf{t}$ ):  $v_t = \frac{m_iv_i}{(m_i - rt)}$ . This reduces velocity of rocket v as a function of time, where  $m_i, v_i$  and r are constants. Therefore,  $\int_0^v dv_t = \int_0^t \frac{m_iv_i}{(m_i - rt)} dt$ ;  $v = \frac{mv}{r} ln\left(\frac{m_i}{m_i - rt}\right)$ . Taking ejection of mass w.r.t. rocket being propelled at velocity  $\mathbf{u}$ , as per LCM in NFOR: 0 = ru - mv, or  $\frac{mv}{r} = u$ . Substituting in the above equation , velocity of rocket, starting from rest, at any time later is  $v = u ln\left(\frac{m_i}{m_i - rt}\right)$ . It is a case of simple algebraic manipulations of equations, formulated from concepts of physics, to determine end result.

**Summary:** Analysis of varieties of problems, representing different situation involve concepts of Newton's Laws of Motion, Circular Motion, Work-power-energy, and conservation of energy and momentum. Many such situations can also be observed in real life and taking problems from the cited as Reference would help to build an insight in the phenomenon occurring around. A deeper journey into the problem solving would make integration and application of concepts intuitive. This is absolutely true for any real life situation which requires multi-disciplinary knowledge in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person.

Varieties of problems, representing different situation that are generally encountered in real life are covered in the books cited as Reference. Best clue to solve problems in physics is to visualize the problem statement in surrounding, and then apply the known concepts to the problem. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory.

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 $m_1 + m_2$ 

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- 4. Sears & Zemansky; University Physics with Modern Physics.
- 5. I.E. Irodov; Problems in General Physics



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**GROWING WITH CONCEPTS - Chemistry** 

(....Contd.) ORGANIC CHEMISTRY : BASIC PRINCIPLES AND TECNIQUES Kumud Bala

#### Nomenclature of unsaturated hydrocarbons:

Open chain hydrocarbons which contain carbon-carbon double or triple bonds in the molecules are called unsaturated hydrocarbons. These are further classified into two types (i) Alkene (ii) Alkyne

**ALKENES:** Unsaturated aliphatic hydrocarbons containing a carbon-carbon double bond (>c=c<) are called alkene. They are also called olefins (Greek-olefiant =oil forming)

General formula=  $C_nH_{2n}$  where  $n = 2, 3, 4, \dots$ .

Primary suffix = ene

Common names: Alkane - ane + ylene = Alkylene

IUPAC name : Alkane - ane + ene = Alkene

The positions of the double bonds are indicated by the Greek letters  $\alpha,\beta,\gamma,\ldots$ ...etc. in the common system while Arabic numericals, i.e., 1,2,3,4, etc. are used in the IUPAC system. For example

n	Formula	Common	IUPAC
		name	name
	$CH_2=CH_2$	Ethylene	Ethene
2			
	$CH_3CH=CH_2$	Propylene	Propene
3			
	$CH_3CH_2CH=CH_2$	α-Butylene	But-1-ene
4		-	
	$CH_3CH=CHCH_3$	β-Butylene	But-2-ene
4			
	$CH_3CH_2CH_2CH=CH_2$	α-	Pent-1-ene
5		Pentylene	
	CH <sub>3</sub> CH <sub>2</sub> CH=CHCH <sub>3</sub>	β-Pentylene	Pent-2-
5			ene
	$CH_3CH_2CH_2CH_2CH=CH_2$	α-Hexylene	Hex-1-ene
6		-	
	$CH_3CH_2CH_2CH=CHCH_2$	β-Hexylene	Hex-2-ene
6		•	
	CH <sub>3</sub> CH <sub>2</sub> CH=CHCH <sub>2</sub> CH <sub>3</sub>	γ-Hexylene	Hex-3-ene
6		· •	

**ALKYNEs:** Unsaturated aliphatic hydrocarbons containing a carbon-carbon triple bond (-C=C-) are called alkyne. In the common system , they are called acetylenes after the name of the first member of this family i.e., acetylene General formula :  $C_nH_{2n-2}$  where n = 2,3,4,... etc.

Primary suffix : yne

Common name : Acetylene and its alkyl derivatives

IUPAC name: Alkane – ane +yne =alkyne The position of the triple bond on the parent chain is designated by lowest possible Arabic numerals. For example:

n	Formula	Common	IUPAC
		name	name
	CH=CH	Acetylene	Ethyne
2			
	CH <sub>3</sub> -C≡CH	Methyl	Propyne
3		acetylene	
		or allylene	
	CH <sub>3</sub> CH <sub>2</sub> -C≡CH	Ethyl	But -1-
4	5	acetylene	yne
	$CH_3-C\equiv C-CH_3$	Dimethyl	But-2-
4		acetylene	yne
	CH <sub>3</sub> CH <sub>2</sub> CH <sub>2</sub> -C≡CH	n-proyl	Pent -1-
5		acetylene	yne
	$CH_3CH_2-C=C-CH_3$	Ethyl	Pent -2-
5		methyl	yne
		acetylene	
	CH <sub>3</sub> CH <sub>2</sub> CH <sub>2</sub> CH <sub>2</sub> -C≡CH	n-butyl	Hex-1-
6		acetylene	yne
	$CH_3CH_2CH_2-C\equiv C-CH_3$	Methyl –	Hex -2-
6		n-propyl	yne
		acetylene	

# Rule for IUPAC nomenclature of unsaturated hydrocarbons (Alkene and Alkyne)

While naming compounds containing multiple (double and triple) bonds, the following addition rules are followed:

1. The parent chain must contain the multiple bonds regardless of the fact whether it also denotes the longest continuous chain of carbon atoms or not. For example, in structure (1), the parent chain consists of five carbon atoms as indicated. Through the longest chain contain six carbon atoms it is not the parent chain since it does not include the double bond.

$$H_{3}^{5}C-CH_{2}-CH_{2}-CH_{2}-CH_{2}-CH_{2}$$
  
 $H_{2}C-CH_{3}$ 

[parent chain contains five

carbon atoms (correct)]

$$\begin{array}{c} {}_{\text{H}_3\text{C}\text{-}\text{CH}_2\text{-}\text{CH}_2\text{-}\text{CH}_2\text{-}\text{CH}_2}\\ {}^{\text{I}}_{\text{H}_2\text{C}\text{-}\text{-}\text{CH}_3}\end{array}$$

 $H_2 \cup H_3$  [parent chain contains six carbon atoms without double bond (wrong)]

2. If both double and triple bonds are present, the numbering of the parent chain should always be done from that end which is nearer to the double or the triple bond i.e., the lowest set of locants rule for the multiple bonds must be followed. For example:

$$H_3^1 C \longrightarrow CH \longrightarrow CH \longrightarrow CH \longrightarrow CH_2 \longrightarrow CH_3$$
  
Set of locants = (2, 5)  
(correct)

$$H_{3}^{8}C \xrightarrow{7}CH \xrightarrow{6}CH \xrightarrow{5}CH_{2} \xrightarrow{4}C \xrightarrow{3}C \xrightarrow{2}CH_{2} \xrightarrow{1}CH_{3}$$
  
Set of locants = (3, 6)  
(Incorrect)

3. If the multiple bonds are at similar position, there is a choice in numbering. The double bond is always given preference over the triple bond, for example

$$H_{1}^{5} = \underbrace{\overset{4}{\underline{C}}}_{2}^{4} - \underbrace{\overset{3}{\underline{C}}}_{3}^{4} H_{2}^{2} - \underbrace{\overset{2}{\underline{C}}}_{4}^{2} H = \underbrace{\overset{1}{\underline{C}}}_{5}^{4} H_{2}^{4} - \underbrace{\overset{constrained}{\underline{C}}}_{5}^{4} + \underbrace{\overset{constrained}{\underline{C}}}_{5}^{4}$$

4. If the organic compound contains only one double or the triple bond, its locant or the positional number is always placed before its suffix, for example.

$$H_{3}C \longrightarrow CH_{2} \longrightarrow CH \longrightarrow CH_{2} \longrightarrow Hex + 3 - ene \longrightarrow Hex - 3 - ene$$

If, however, both double and triple bonds are present, their locants are written before their respective suffixes, the terminal 'e' from the suffix 'ene' is dropped while writing the complete name of the organic compound. It may be emphasized here that the organic compound is named as derivative of alkyne rather than alkene, for example

$$H_{3}^{5}C - CH = CH = CH - CE CH$$

pent - 3 - en(e) + 1 - yne -----> pent-3-en-1-yne

5. If more than one double bond and triple bond are present, the prefixes like di, tri ,etc. are used before the primary suffix for the bond. For example, if 2 double bonds are present the word 'diene' is used. For 2 triple bonds, the word 'diyne' is used.

$$H_{2}^{1}C = C_{H_{3}}^{2} H_{C}^{4}H_{2}^{4} 2-\text{methylbuta-1,3-diene}$$

$$C_{H_{3}}^{C}H_{3}^{4} = C_{H_{3}}^{5}$$

$$H_{C}^{1}C = C_{H_{3}}^{2}CH_{C}^{4} = C_{H_{3}}^{5} 3-\text{methylpenta-1,4-diyne}$$

In some cases all the double or triple bonds present in the molecule cannot be included in the longest chain. In such cases the following prefixes are used for double and triple bonded groups.

CH <sub>2</sub> =	$CH_3CH=$
Methylene	Ethylidene
CH <sub>2</sub> =CH-	HC≡C-
Vinyl or ethenyl	Ethynyl

For example:



#### Assignment:

 The IUPAC name of the compound: CH<sub>3</sub>C≡CCH(CH<sub>3</sub>)<sub>2</sub> is

 [A] 4,4-Dimethyl-2-butyne
 [B] 4-Methyl-2-pentyne
 [C] Methylisopropyl acetylene
 [D] 2-methyl-4- pentyl

2. The IUPAC name of the compound:  $(CH_3)_3C-CH=CH_2$  is

[A] 2,2-Dimethylbut-2-ene

[B] 2,2-Dimethylpent-3-ene

[C] 3,3-Dimethylbut-1-ene

[D] Hex-1-ene

3 The IUPAC name of the compound:  $CH_2$ =CH-CH<sub>2</sub>-CH=CH<sub>2</sub>

[A] Penta-1,4-diene

[B] Penta-1,5-diene

[C] 3-Ethylpenta-1,3-diene

[D] Pent-2-en-4-ene

4 Which one is correct structural formula of the following : Pent-1-en-4-yne

 $[A] CH = C-CH_2-CH = CH_2$ 

 $[B] CH_3-CH=CH-C=CH$ 

 $[C] CH_2 = CH - CH = CH - CH = CH_2$ 

 $[D] CH_2 = CH - CH_2 - CH = CH_2$ 

5 Which of the following is an vinyl group

**Answer** 1. [B] 2. [C] 3. [A] 4. [A] 5. [C]



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Together Each Achieves More (TEAM)

## QUIZDOM – Nov'16

#### TOPIC: INDIAN FREEDOM MOVEMENT (Part IV)

#### Phanindra Ivatury, Quiz Host

- 1. What nationalist political party was formed by Subhash Chandra Bose after his resignation from the Presidency of the Indian National Congress?
- 2. Subhash Chandra Bose was supposed to have married in 1937 while in Berlin, Germany. What is the name of his supposed wife?

#### Information: Their daughter's name is Anita Bose

- 3. Subhash Chandra Bose had allegedly died in a plane crash on 18 August 1945 while enroute to Tokyo. In which city did the supposed plane crash occur?
- 4. What rank did Subhash Chandra Bose secure when he passed the Indian Civil Services Examination in Britain in 1920-21?

*Information:* Subhash Bose who went to England on his father's insistence to study there, declined to join Indian Civil Service as that would mean serving under the British)

5. What resolution was passed with Nehru as the Congress President in the Lahore Session of Congress in December 1929?

**Information:** A call for complete independence. On the banks of river Ravi in Lahore, It is in Pakistan

6. What is the name of the book written by Nehru during his imprisonment between in 1942-1946 at the Ahmednagar fort for his participation in the Quit India Movement?

*Information:* The book is widely regarded as an Indian Classic The book was made into a TV series called "Bharat Ek Khoj" by noted Director Shyam Benegal in 1988,

- 7. Nehru made a historic speech as the first Prime Minister of Independent India at the midnight of 14 August 1947. With what title is this speech popularly referred to as?
- 8. from Solapur, Maharashtra was sent to China in 1938 by Jawaharlal Nehru as a part of the 5 doctor Indian Medical Mission deputed to treat patients of the Second Sino-Japanese War?

**Information:** The Medical Mission was sent to China by Nehru on the request of Chinese Communist General Zhu De. Dr.Kotnis treated hundreds of wounded soldiers of war in China from 1938 till his untimely death in 1942 due to epilepsy. A movie - Dr.Kotnis ki Amar Kahani was made by V Shantaram in 1946 where he himself potrayed the role of Dr.Kotnis

- 9. During one of his trips to Europe in the 1930s Jawaharlal Nehru turned down the offer made by a Italian dictator who wanted to meet him, thereby endorsing the spirit of freedom and democracy all over the world. Who was this dictator?
- 10. What is the Hindi name of the weekly publication started by Ambedkar in 1920 which when literally translated into English would mean "Leader of the silent"?
- 11. What is the name of the Pact signed by Ambedkar and Mahatma Gandhi on 24 September 1932 on the subject of providing separate electorates for untouchables?

**Information:** Signed at the Yerawada Jail, Pune where Gandhiji was imprisoned. Ambedkar was forced to drop his demand for a separate electorate. Instead a certain number of seats were reserved for the depressed classes.

- 12. The new congress led Government in 1947 invited Ambedkar into the Union Cabinet. What portfolio did Ambedkar hold?
- 13. Dr. Sharada Kabir is the original name of Ambedkar's second wife whom he married on 15 April 1947. What name did she adopt later or how is she popularly known as?

**Information:** Ambedkar met her when he went for diabetic treatment in Mumbai after the completion of drafting of Indian constitution. He felt the need of a companion who was also a good cook and from the medical field. Dr.Savita took care of him for the rest of his life.



Quiz Host is a Post-Graduate in Public Personnel Management and Winner of Kulapati K.M.Munshi Medal in Public Relations, has quizzing as a hobby. He has so far hosted over 200 Quizzing events on various platforms all over the globe. He currently works with the Comptroller & Auditor General of India, New Delhi having worked on intra/inter-national assignments. E-mail ID: phanindraivaturi@yahoo.com

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## Answers to QUIZDOM : Oct'16

(1) THE BOYCOTT MOVEMENT CONSISTED OF BOYCOTT OF FOREIGN GOODS AND ALSO SOCIAL BOYCOTT OF AN INDIAN WHO USED FOREIGN GOODS WHILE THE SWADESHI MOVEMENT CONSISTED OF THE USAGE OF GOODS PRODUCED BY ONESELF OR IN INDIA.; (2) CHAMPARAN AGITATION; (3) CHAURI CHAURA; (4) FROM SABARMATI OR AHMEDABAD TO DANDI BETWEEN 12 MARCH TO 6 APRIL (24 DAYS) 388 KMS/240 MILES. ; (5) INDIAN SAILORS OF THE ROYAL INDIAN NAVY ON BOARD SHIP AND STORE ESTABLISHMENTS OF BOMBAY HARBOUR. ALSO KNOWN POPULARLY AS THE ROYAL INDIAN NAVY MUTINY.; (6) SURENDRANATH BANERJEE (1848-1925); (7) DEPRESSED CLASSES; (8) SUBHASH CHANDRA BOSE ; (9) LOK NAYAK; (10) CHITTARANJAN DAS; (11) RABINDRANATH TAGORE; (12) THE INTERNATIONAL DAY OF NON VOILENCE; (13) GANDHI-IRWIN PACT ; (14) 1915 (June 5, 1915 to be precise); (15) GOPALA KRISHNA GOKHALE (Bapu himself had admitted this in many of his speeches); (16) KARAMACHAND GANDHI AND PUTLI BAI GANDHI; (17) 1938 & 1939

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#### S R Ε L Ν E Μ Ε Т L Ν F Ν Т Υ L Е Т D Α Ε C Т С 0 S Α Ν Ν L 0 Q U Α D R Α Ν Т S T L Ν Т С G Ε Ν 0 Т Α Ν Y S н U L L V Ε т Ν С 0 R 0 Α Ρ Ε R Μ U Т Α Т Т 0 Ν S Ε G 0 Т Μ L G Α R I Н Ε

### Answer To Crossword Puzzle : Trigonometry & Basic Algebra

## **CROSSWORD PUZZLE Nov'16 : Mathematical Beauty**

### Prof. S.B. Dhar



#### Across:

- 8 A repeated decorative design
- 10 Name of a Theorem giving minimum number of colors to color maps
- 12 Field of mathematics for designing
- 13 Pairs of primes that differ by 2
- 14 Name of Identity connecting five fundamental constants

#### Down:

- 1 A ratio whose approximate value is 1.618033.
- 2 The property of being exactly similar facing each other
- 3 Tallest building in the world
- 4 Derivative of itself
- 5 The condition of being easy to understand
- 6 Relates with the theorem
- 7 Name of mathematician who developed a sequence resulting golden ratio
- 8 Leonardo da Vinci's art drawings uses this art
- 9 Ability to become aware of something through senses
- 11 A problem challenged by Jakob Bernoulli and solved by Euler

## (Answer to this Crossword Puzzle shall be provided in Supplementary Bulletin No 1-3 Dt. 1<sup>st</sup> Dec'16)

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Every end, is a pause for a review, before re-continuing of a journey far beyond ...