BEAUTY GENERATING CONCEPTS OF MATHEMATICS

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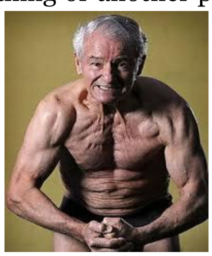
Beauty cannot be defined in words. It can be felt, smelt, heard or seen only. It is an absolute thing. Beauty may relate to anything like perception, race, culture or era. It is correctly said that beauty is in the eye of the beholder. In mathematics, beauty is a reality, and not abstract.

Mathematically, $beauty = simplicity + depth$

There are so many concepts that generate beauty in mathematics. Proof of a theorem is one of these. The beautiful proof is that which

(a) either uses minimum assumptions, or
(b) is based on new and original insights.

In day to day life, we find a thing or another person's body more attractive. It is due to the symmetry and proportionality. This is another concept that leads to beauty. Physical attraction depends on ratio. If a face is in proportion, there is more likely to notice it. Even scientists believe that proportional bodies are also sign of good health.



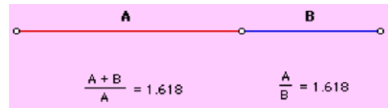
Leonardo da Vinci's drawings of the human body emphasised its proportion. The ratio of the distances *(foot to navel) : (navel to head)* is very considerable for beautiful look.

Leonardo da Vinci's "Vitruvian Man", showing the golden ratio in body dimensions

We find people to be attractive because of the proportions of the length of the nose, the position of the eyes and the length of the chin all conform to some particular ratio. This particular ratio is called Golden Ratio.

Let us have a look on mathematical facts that beautify the things:

(1) ***PHI (Φ): Golden Ratio:*** It is also called the divine proportion. It is calculated as the ratio between A and B in the following figure.

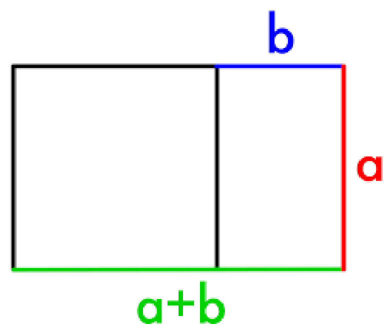


It is denoted by Φ, and pronunciated as *phi*.

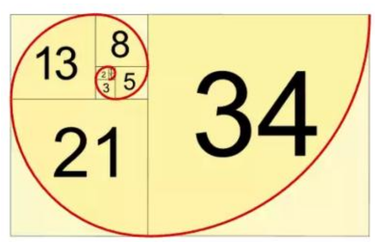
Golden Ratio is generated as below too:

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

(2) ***Golden Rectangle :*** The following figure depicts a Golden Rectangle.



(3) Golden Spiral

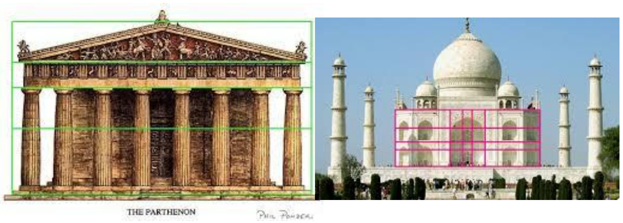


Proven fact: *Measure the length and width of your face. Divide the length by the width. If it comes nearer to 1.6, it is a beautiful person's face.*

It means beautiful face is about $1\frac{1}{2}$ *times longer than it is wide.*

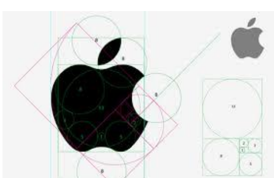
Many beautiful things in the world are with golden ratio. The Great Pyramid of Giza built in 2570BC exhibits the golden ratio.

The Parthenon, a temple of the Greek goddess Athena (447-432BC) or, the Tajmahal has golden ratio proportion.

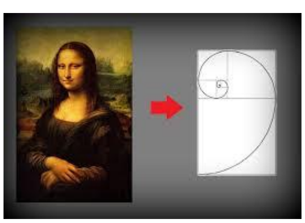


In ancient times architecture was a field of mathematics. Architects were simply mathematicians that someone would hire. Geometry is the guiding principle between the two areas. Mathematics, however, is indispensable to the understanding of structural concepts and calculations.

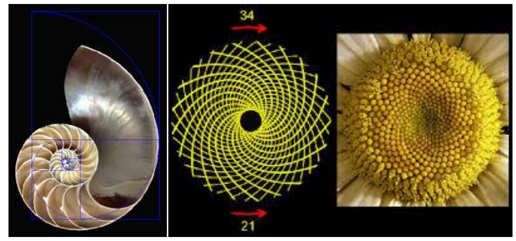
The logo of apple is a very good example of golden ratio.



Famous Mona Lisa portrait is the best examples of golden ratio.



The nature is full of examples of golden ratio concepts. The following figures are self-explanatory.



(4) ***Fibonacci Numbers:*** Each number of this sequence is *phi* times the last. Some of the first terms of this sequence are:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765.

Let's look at the ratio of each number in this sequence to the one before it:

$\frac{1}{1}=1$, $\frac{2}{1}=2$, $\frac{3}{2}=1.5$,

$\frac{5}{3}=1.666...$ $\frac{8}{5}=1.6$ $\frac{13}{8}=1.625$

$\frac{21}{13}=1.61538...$ $\frac{34}{21}=1.61905..$

$\frac{55}{34}=1.61764..$ $\frac{89}{55}=1.61861..$

If we keep going, we reach Golden Ratio i.e., 1.618 033 988 7....

We can say that the Fibonacci sequence is encoded in the number $\frac{1}{89}$.

(5) Prime Numbers: The number that is divisible by only 1 and itself is called Prime Number. They are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, ...

The largest known prime number (as of December 2005) has 9,152,052 digits. According to Euclid (~300BC), there are infinitely many prime numbers.

(6) Twin primes: Twin primes are the primes that differ by 2. Viz. 2, *3, 5, 7,* 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, ... Some twin primes are: (3,5), (5,7), (11,13), (17,19),...

(7) Prime Triplets: The set of three prime numbers of the form (p, p + 2, p + 6) or (p, p + 4, p + 6) with the exception of (2,3,5) and (3,5,7) are called prime triplets. There are infinitely many triples of primes.

(8) Have a fun with the following mathematical calculation: Give all letters from A to Z a running number.

A(1) B(2) C(3) D(4) E(5) F(6)
G(7) H(8) I(9) J(10) K(11) L(12)
M(13) N(14) O(15) P(16) Q(17)
R(18) S(19) T(20) U(21) V(22)
W(23) X(24) Y(25) Z(26)

then

H-A-R-D-W-O-R- K
8+1+18+4+23+15+18+11 = 98%

and

K-N-O-W-L-E-D-G-E
11+14+15+23+12+5+4+7+5 = 96%

But

A-T-T-I-T-U-D-E
1+20+20+9+20+21+4+5 = **100%**

Can we conclude mathematically, while ***Hard Work*** and ***Knowledge*** will get you close, ***Attitude*** will get you there?

It is simply a fun, for some it may be true.

(9) Patterns of numbers

(a)

7^2	49
67^2	4489
667^2	444889
6667^2	44448889
66667^2	4444488889
666667^2	444444888889
6666667^2	44444448888889

(b)

$03 \times 37 = 111$ & $1+1+1 = 03$

$06 \times 37 = 222$ & $2+2+2 = 06$

$09 \times 37 = 333$ & $3+3+3 = 09$

$12 \times 37 = 444$ & $4+4+4 = 12$

$15 \times 37 = 555$ & $5+5+5 = 15$

$18 \times 37 = 666$ & $6+6+6 = 18$

$21 \times 37 = 777$ & $7+7+7 = 21$

$24 \times 37 = 888$ & $8+8+8 = 24$

$27 \times 37 = 999$ & $9+9+9 = 27$

(c)

$987\,654\,321 \times 09 = 08\,888\,888\,889$

$987\,654\,321 \times 18 = 17\,777\,777\,778$

$987\,654\,321 \times 27 = 26\,666\,666\,667$

$987\,654\,321 \times 36 = 35\,555\,555\,556$

$987\,654\,321 \times 45 = 44\,444\,444\,445$

$987\,654\,321 \times 54 = 53\,333\,333\,334$

$987\,654\,321 \times 63 = 62\,222\,222\,223$

$987\,654\,321 \times 72 = 71\,111\,111\,112$

$987\,654\,321 \times 81 = 80\,000\,000\,001$

(d)

$1 \times 8 + 1 = 9$

$12 \times 8 + 2 = 98$

$123 \times 8 + 3 = 987$

$1234 \times 8 + 4 = 9876$

$12345 \times 8 + 5 = 98765$

$123456 \times 8 + 6 = 987654$

$1234567 \times 8 + 7 = 9876543$

$12345678 \times 8 + 8 = 98765432$

$123456789 \times 8 + 9 = 987654321$

(e)

$1 \times 9 + 2 = 11$

$12 \times 9 + 3 = 111$

$123 \times 9 + 4 = 1111$

$1234 \times 9 + 5 = 11111$

$12345 \times 9 + 6 = 111111$

$123456 \times 9 + 7 = 1111111$

$1234567 \times 9 + 8 = 11111111$

$12345678 \times 9 + 9 = 111111111$

$123456789 \times 9 + 10 = 1111111111$

(f)

$9 \times 9 + 7 = 88$

$98 \times 9 + 6 = 888$

$987 \times 9 + 5 = 8888$

$9876 \times 9 + 4 = 88888$

$98765 \times 9 + 3 = 888888$

$987654 \times 9 + 2 = 8888888$

$9876543 \times 9 + 1 = 88888888$

$98765432 \times 9 + 0 = 888888888$

(g)

$1 \times 1 = 1$

$11 \times 11 = 121$

$111 \times 111 = 12321$

$1111 \times 1111 = 1234321$

$11111 \times 11111 = 123454321$

$111111 \times 111111 = 12345654321$

$1111111 \times 1111111 = 1234567654321$

$11111111 \times 11111111 = 123456787654321$

$111111111 \times 111111111 = 12345678987654321$

(h) Without 8

$12345679 \times 09 = 111111111$

$12345679 \times 18 = 222222222$

$12345679 \times 27 = 333333333$

$12345679 \times 36 = 444444444$

$12345679 \times 45 = 555555555$

$12345679 \times 54 = 666666666$

$12345679 \times 63 = 777777777$

$12345679 \times 72 = 888888888$

$12345679 \times 81 = 999999999$

(i) Inputs of 9

$9 \times 9 = 81$

$99 \times 99 = 9801$

$999 \times 999 = 998001$

$9999 \times 9999 = 99980001$

$99999 \times 99999 = 9999800001$

$999999 \times 999999 = 999998000001$

$9999999 \times 9999999 = 99999980000001$

$99999999 \times 99999999 = 9999999800000001$

$999999999 \times 999999999 = 999999998000000001$

(j) Input of 6

$6 \times 7 = 42$

$66 \times 67 = 4422$

$666 \times 667 = 444222$

$6666 \times 6667 = 44442222$

$66666 \times 66667 = 4444422222$

$666666 \times 666667 = 444444222222$

$6666666 \times 6666667 = 44444442222222$

$66666666 \times 66666667 = 4444444422222222$

$666666666 \times 666666667 = 444444444222222222$

(10) 0.999.... is equal to 1

X = 0.999...

10X =9.999...

10X-X =9.999... -0.999...

9X =9

X =1

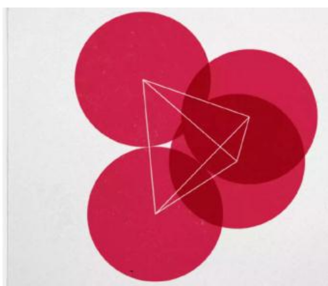
(11) The 4-colour theorem

The 4-Color Theorem was first discovered in 1852 by a man named ***Francis Guthrie***, who at the time was trying to color in a map of all the counties of England (this was before the internet was invented, there wasn't a lot to do).

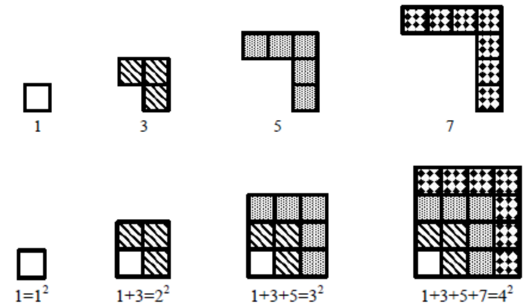
He discovered something interesting—he only needed a maximum of four colors to ensure that no counties that shared a border were colored the same. Guthrie wondered whether or not this was true of any map, and the question became a mathematical curiosity that went unsolved for years.

In 1976 (over a century later), this problem was finally solved by ***Kenneth Appel*** and ***Wolfgang Haken.***

(12) Four touching spheres form a tetrahedron



(13) ***Paul Erdös*** once said, "I know numbers are beautiful. If they aren't beautiful, nothing is." Mathematics is the study of patterns, structure and regularity.



(14) The derivative of an exponential is an exponential. $\dfrac{d}{dx}e^x = e^x$

(15) **Euler's Identity**

It links 5 fundamental mathematical constants with three basic arithmetic operations each occurring once. $1+e^{i\pi}=0$.

(16) **The Basel problem**, set as a challenge by ***Jakob Bernoulli*** in 1689 and triumphantly solved by ***Euler*** in 1735. $\dfrac{\pi^2}{6}=\sum_{n=1}^{\infty}\dfrac{1}{n^2}$

(17) **Equation expressing** the connection between π and odd numbers $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{9}-....$

(18) **Sum of an infinite geometric series** $\sum_{k=0}^{\infty}ar^k=\dfrac{a}{1-r}, |r|<1$

Nature is a good mathematician. All its presentations are well calculated. If nature is God, then God is certainly a good mathematician. God has used beautiful mathematics in creating this world. None will hesitate in accepting the fact that without mathematics, there is nothing we can do. Everything around us is mathematics and everything around us are numbers. It is up to us how we search, study, admire after its use. The mathematical concepts are fun and it will be good for learners and mentors if they are communicated and learnt that way.

Dr S.B. Dhar, is **Editor of this Quartrerly e-Bulletin**. He is an eminent mentor, analyst and connoisseur of Mathematics from IIT for preparing aspirants of Competitive Examinations for Services & Admissions to different streams of study at Undergraduate and Graduate levels using formal methods of teaching shared with technological aids to keep learning at par with escalating standards of scholars and learners. He has authored numerous books – Handbook of Mathematics for IIT JEE, A Textbook on Engineering Mathematics, Reasoning Ability, Lateral Wisdom, Progress in Mathematics (series for Beginner to Class VIII), Target PSA (series for class VI to class XII) and many more.
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