COMMON ERRORS IN MATHEMATICS

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Students solving mathematical problems, usually commit 3 types of errors:

(a) Concept error (*i.e.* inability in identifying the proper fact),
(b) Misused data error (*i.e.* copying the wrong data), and
(d) Calculation error (*i.e.* miscalculation)

The reason of error is:

1. The missing link of the inter-relations between the various facts. Mathematics being the subject of Cumulative study, does not allow random approach. Or,
2. The lack of practice of concepts with pen on notebook. Practice of mathematical facts is more important than their knowledge because Mathematics is NOT a Spectator Sport.

Let us discuss Some Examples!

1. The Error relating to division by ZERO

Everyone knows that $\frac{0}{4}=0$ or, $0\div 4=0$. This is correct.

But $\frac{4}{0}=0$ or, $4\div 0=0$ is NOT correct.(why?)

Because division by 0 is not defined.

Remember: Never divide anything by zero.

Importance of ignorance of the above fact is shown below:

(1) Assume , A=B
(2) Multiply both sides by B, $AB=B^2$
(3) Subtract A^2 from both sides, $AB - A^2=B^2 - A^2$
(4) Factorize both sides, A(B-A)=(B-A)(B+A)
(5) Cancel out (B-A) from both sides, A=B+A
(6) Use fact of (1)i.e. A=B, A=A+A
(7) Simplify, A=2A
(8) Divide both sides by A, 1=2

Is it not a funny result? Certainly, it is a miracle, but why? Because we did a mistake at step (4). Both sides were divided by (B-A) which was actually equal to 0. ***Remember: We cannot divide any number by 0.***

2. The Error relating to Parentheses.

The symbol ( ) is called Parentheses. Parentheses play an important role in finding the correct solution. If we ignore them, then we shall reach wrong result.

Ex: Square 5x

Correct	**Incorrect**
$(5x)^2=(5)^2(x)^2=25x^2$ (why?)	$5x^2$

Because 5 has not been squared. Every quantity within the parentheses should be squared.

Remember: Do not ignore parentheses.

Ex: Square -3

Correct	**Incorrect**
$(-3)^2=(-3)(-3)=9$	$-3^2 = -(3)(3) = -9$(why?)
	Because – (negative) is not squared.

Remember: Do not ignore use of parentheses when negative sign appears in the expression.

Ex: Subtract 6x-2 from x^2+2x-7

Correct	**Incorrect**
x^2+2x-7- (6x-2)	x^2 +2x-7- 6x – 2
= x^2+2x-7-6x+2 = x^2-4x-5	= x^2 - 4x – 9 (why?)

Because 6x-2 is not put in parentheses to distribute the negative sign over it. Only 6x is being multiplied by negative and -2 is ignored.

Remember: Do not forget to distribute the sign before the parentheses over its all terms.

3. The Error relating to the cancelation of factors

Ex: Solve: $2x^2=x$

Correct	**Incorrect**
$2x^2=x\Rightarrow 2x^2-x=0$	$2x^2=x$
$\Rightarrow x(2x-1)=0$	$2x=1$
	(On cancelling x from both sides)

$\Rightarrow x=0$, or, $x=\frac{1}{2}$ (Two values) $x=\frac{1}{2}$ (Only one value) (why?)

Because on cancelling x from both sides, one value of x is eliminated.

Remember: Never cancel any variable from both sides. By doing so, root is eliminated.

4. **The Error relating to the square root**

There is a misconception that $\sqrt{9}$ is ± 3.

The value ± 3 is absolutely wrong. Square root is always POSITIVE or ZERO of Real quantities.

The correct answer is: $\sqrt{9}=3$. To find -3, proceed as: $-\sqrt{9}=-\left(\sqrt{9}\right)=-(3)=-3$.

The misconception of $\pm$ sign comes from the solution of the equation $x^2 = 9$.

On solving this, we get x = ±3. It appears that square roots of both sides are taken.

This is not correct. No square root has been taken.

The correct procedure is:

$x^2=9\Rightarrow x^2-9=0\Rightarrow (x-3)(x+3)=0\Rightarrow$

$x=3$ Or $x=-3\Rightarrow x=\pm 3$.

It is a joint result. It is **NOT** Positive **AND** Negative. It is Positive **OR** Negative.

Remember: Square root is always Positive or Zero and Never negative of Real quantities.

5. **The Error relating to the ambiguity in writing fractions**

Often we use "**/** " symbol for writing fractions. Symbol **/** means ÷(division).

Some write the fraction $\frac{a+b}{c+d}$ as $a+b/c+d$ which is **wrong** because it is $a+\frac{b}{c}+d$. This is NOT equal to the original fraction. The correct way of writing this fraction with **/** symbol is $(a+b)/(c+d)$.

Remember: Do not ignore parentheses. They help in removing doubt.

6. **The Error relating to Trigonometry**

Ex: Degree vs Radian

Is $\sin 30^0$ = sin30?

$\sin 30^0=\frac{1}{2}$ and sin30 ≠ $\sin 30^0$ because in **sin30**, **30** is in ***radians*** and in $\sin 30^0$, 30 is in ***degrees***. $1 radian \cong 57.29^0$; $30 radians \cong 1718.87^0$. So, $\sin 30 \cong -0.98$.

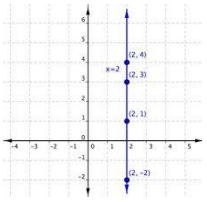
In $30^{\mathbf{0}}$, **0** does not indicate power of 30. **0** indicates a symbol for degree. Symbol for radian is c. 30^c means 30 radians but generally it is not written.

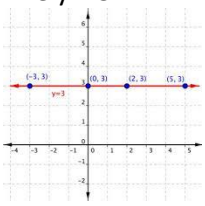
Remember: Do not forget to distinguish between the degrees and the radians.

7. **The Error relating to Coordinate Geometry**

Ex: Draw Graphs of x = 2, and y = 3

Line x = 2 Line y = 3





x = 2 is a line parallel to y-axis and NOT parallel to x-axis. x = 2 is a vertical line.

y = 3 is a line parallel to x-axis and NOT parallel to y-axis. y = 3 is a horizontal line.

Remember:

x=a is a line parallel to y-axis on which y changes and x is always a.

y=b is a line parallel to x-axis on which x changes and y is always b.

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