LET's LEARN OUR NUMBER SYSTEMS

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A **Number System** is a system of **Numeration**. It is a writing system for expressing numbers. In other words, it is a mathematical notation for representing numbers of a given set, using digits or symbols in a consistent manner.

It is a fact that everything around us is numbers. Numbers originated with life. We cannot think of life without numbers. Nature is full of numeric properties. Numbers speak. Mathematically, numbers make mathematics. Numbers were systematically studied firstly as abstractions by Greek philosophers **Pythagoras** and **Archimedes**. Let us learn about the different types of **NUMBERS**.

NUMBER AND NUMERAL

Number is defined as a ***mathematical object*** that is used to count.

The examples are: 1, 2, 3,...., and so forth.

A ***notational symbol*** that is used to denote a ***number*** is called a ***numeral***.

Numerals are used for labels, or for ordering.

NUMBER LINE

A number line is a line that represents all the numbers. It is a straight line. Natural Numbers, Whole Numbers, and Integers are represented as dots. The Real numbers are represented by continuous line.

We shall study here:

(a) Natural Numbers
(b) Whole Numbers
(c) Integers
(d) Rational Numbers
(e) Irrational Numbers
(f) Real Numbers
(g) Complex Numbers
(h) Roman Numbers, and
(i) Some especial numbers

NATURAL NUMBERS

The numbers that are used for counting or ordering the objects are called Natural Numbers. They are also called Cardinal numbers, or Positive Integers.

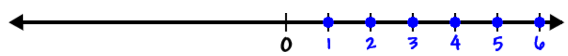
The primary method to represent a Natural Number was to put a mark for each object. ***The Egyptians*** developed system of numerals. ***The Babylonians*** developed place value system of the numbers.

Italian Mathematician ***Giuseppe Peano*** studied Natural Numbers and laid down some axioms:

(a) Each Natural Number has a successor.
(b) Every Natural Number has a predecessor except 1.
(c) Each Natural Number has unique predecessor.
(d) Set of Natural Numbers is represented by N.

N={1,2,3,4,....}

On Number Line, Natural Numbers are denoted by the dark dots on the right of 0 at equal distance. There is no Natural Number in the **LEFT** of 1.



There are infinite Natural Numbers. The least Natural Number is 1 and the largest Natural Number is not defined. The right hand side number on the Number Line is always greater than any number on its left hand side.

Natural numbers satisfy:

(a) Closure Property,
(b) Commutative Property,
(c) Associative Property, and
(d) Distributive Property for Multiplication distributed over Addition only.

Note:

(1) Natural Numbers are neither closed for Subtraction nor Division.
(2) There exists Multiplicative Identity only.
(3) There exists no Additive Identity in set of Natural Numbers.

PROPERTIES OF ADDITION

Closure Property

It states that if ***a*** and ***b*** are any two Natural Numbers, and their sum ***(a+b)*** is also a Natural Number, then it is said that the set of Natural Numbers is closed for addition for all Natural Numbers.

Example: 2 and 5 are Natural Numbers. The sum (2+5)= 7 is also a Natural Number. Hence the set of Natural Numbers is closed for Addition.

Commutative Property

If ***a*** and ***b*** are any two Natural Numbers and ***(a+b)=(b+a)***, then the set of Natural Numbers is called to obey Commutative Property of Addition for all Natural Numbers.

Example: 3+4= 4+3

Associative Property

If ***a, b,*** and ***c*** are any three Natural Numbers and ***a+(b+c)=(a+b)+c***, then we say that the set of Natural Numbers follows Associative Property of Addition for all its members.

Example: 3+(4+5)=(3+4)+5

Distributive Property

If ***a, b,*** and ***c*** are any Natural Numbers and ***a x (b+c)=axb + axc***, then it is said that ***a*** is distributed multiplicatively over the addition of ***b*** and ***c***.

Example: 4x(5+6)=4x5 + 4x6

Existence of Additive Identity

If there exists a number in the set of Natural Numbers such that its addition with any other Natural Number gives the Number itself, then the adding unique number is called Additive Identity. 0 is called the Additive Identity.

Example: 0+ 5=5, 0+9=9. But 0 is not a member of the set of Natural Numbers, hence there ***exists no additive Identity*** in the set of Natural Numbers.

PROPERTIES OF MULTIPLICATION

Closure Property

It states that if ***a*** and ***b*** are any two Natural Numbers, and their product ***(axb)*** is also a Natural Number, then it is said that they are Closed for Multiplication.

For example: 2 and 5 are Natural Numbers. The product (2x5)= 10 is also a Natural Number.

Commutative Property

If ***a*** and ***b*** are two Natural Numbers and ***(axb)=(bxa)***, then they are called to obey Commutative Property for Multiplication.

Example: 3x4= 4x3

Associative Property

If ***a, b,*** and ***c*** are any three Natural Numbers and ***ax(bxc)=(axb)xc***, then we say that Natural Numbers follow Associative Property of Multiplication.

Example: 3x(4x5)=(3x4)x5

Distributive Property

If ***a, b,*** and ***c*** are Natural Numbers and ***ax(b+c)=axb+axc,*** then it is said that ***a*** is distributed multiplicatively over addition of ***b*** and ***c***.

Example: 4x(5+6)=4x5 + 4x6

Existence of Multiplicative Identity

If there exists a number in the set of Natural Numbers such that its product with any other Natural Number gives the Number itself, then the multiplier unique number is called Multiplicative Identity. 1 is called the Multiplicative Identity.

Example: 1x 5=5, 1x9=9. 1 is in the set of Natural Numbers. So, multiplicative Identity exists in the set of Natural Numbers.

IMPORTANCE OF NATURAL NUMBERS

(a) Natural numbers are the origin for making of other number sets.
(b) Natural numbers give existence to Negative Integers through Additive Inverse.
(c) Negative Integers are mathematically additive inverse of natural numbers set. They correspond one to one, i.e., Additive Inverse is unique or in other words, it can be said that there is one and only one additive inverse for each number. Additive inverse means the number that makes the sum of self and the other number equal to zero. Additive Inverse is always a negative number of the number.

For example:8 is a Natural Number. Negative of (8) is (-8). The sum of (8) and (-8)= (8)+(-8)=0.

Accordingly, (-8) is called the Additive Inverse of (8).

(d) Natural Numbers give way for the existence of the Rational Numbers through Multiplicative Inverse. There exists multiplicative inverse for each of the natural numbers. They correspond one to one, or mathematically, we may say that there exists one and only one multiplicative inverse for each number.

Multiplicative inverse means the number that makes the product of any number with it equals to 1.

For example: 3 is a natural Number. (1/3) is called multiplicative inverse of 3 as the product of (3) and (1/3)= 1.

Note:

(1) Some mathematicians have started assuming 0 as a Natural Number under ISO 31-11. ISO 31-11 was the part of International Standard ISO 31 that defined mathematical signs and symbols in 1992. It was superseded in 2009 by ISO 80000-2. Its definition included 0 as a Natural Number.

(2) Inclusion of 0 in the set of Natural Numbers gives rise to a new definition: "A Natural Number is either a Positive Integer (1,2,3,...) or a Non-negative Integer (0,1,2,3,...)".
First definition is used in Number Theory and the Second is used in Sets Theory and Computer Science.

(3) Computer scientists often start from zero for enumerating items like loop counters.

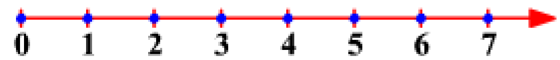
WHOLE NUMBERS

Set of Natural Numbers with Zero is called Whole Numbers. It is in general denoted by W.

W={0,1,2,3,....}

In Whole Numbers there is no fractional or decimal part and no negatives.

On a Number Line, Whole Numbers are denoted by the thick **BLUE** dots. There is no number in the LEFT of 0.



There is no end of Whole Numbers. The smallest Whole Number is 0 and there exists no greatest Whole Number.

All right hand side numbers are greater than any number on the LEFT hand side of it.

Note: All Natural Numbers are Whole Numbers but all Whole Numbers are not Natural Numbers. Example: 0 is not a Natural Number but 0 is a Whole Number.

PROPERTIES OF ADDITION

The set of Whole Numbers follows:

(a) Closure Property
(b) Commutative Property
(c) Associative Property
(d) Distributive Property

Note: Additive Identity {0} exists in the set of Whole Numbers.

PROPERTIES OF MULTIPLICATION

The set of Whole Numbers follows:

(a) Closure Property
(b) Commutative Property
(c) Associative Property
(d) Distributive Property

NOTE:

(1) Multiplicative Identity {1} exists in the set of Whole Numbers.
(2) There exists no Additive Inverse in the set of Whole Numbers.
(3) There exists no Multiplicative Inverse in the set of Whole Numbers.
(4) Whole Numbers set is not closed for Subtraction and Division.

INTEGERS

Integer is defined as a Number that is not a fraction. It is a Whole Number. The name is derived from the Latin ***Integer*** which means "***whole***".

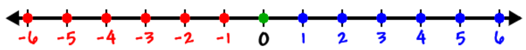
There are three types of Integers:

(a) The Negative Integers
(b) Zero Integer, and
(c) The Positive Integers

Integers are represented by ***I*** or ***Z***. ***Z*** is due to German word ***Zahlen*** meaning **numbers**.

Z= {….,-3,-2,-1, 0, 1, 2, 3, ….}

On Number Line, the Integers are described as the thick dots at equal distances.



There are infinite numbers of Integers. There exists neither any least Integer nor any greatest Integer.

Each number is greater than its left hand side numbers. Going Right is Ascending Order (i.e.,in Increasing Order) and Going Left is Descending order (i.e., in Decreasing Order).

Integers contain all Natural numbers, all Whole Numbers, and Negative of all Natural Numbers.

Set of Natural Numbers is also called Positive Integers and is denoted by I^+ or Z^+. They have been denoted by **BLUE** dots on the Number Line.

Zero is neither Positive nor Negative Integer. It is denoted by I^o or Z^o . It has been denoted by **GREEN** dot.

The ***Olmec*** and ***Maya*** civilizations used ***0*** as a separate number as early as 1^{st} Century BC, but the usage could not spread beyond Mesoamerica. The use of numeral **0** in modern times is the work of Indian Mathematician ***Brahmgupta*** in 628. Romans do not have any symbol for zero. They used word "***nulla***" meaning ***none***.

Negative Integers have been denoted by **RED** dots on the Number Line. It is also denoted by I^- or Z^-.

Positive Integers with Zero are also called Non-negative Integers. Negative Integers with Zero are called Non-Positive Integers.

PROPERTIES OF ADDITION

The set of Integers obey

(a) Closure Property for Addition, Subtraction and Multiplication
(b) Commutative Property for Addition and Multiplication
(c) Associative Property for Addition and Multiplication
(d) Distributive Property of Multiplication over Addition and Subtraction
(a) Additive Identity {0}, and Multiplicative Identity {1}, both exist in the set of Integers.
(b) Additive Inverse exists in the set of Integers as there is negative of all positive Integers and there is positive of all negative integers.
(a) Set of Integers is not closed for Division.
(b) The set of Integers has Additive Inverses but does not have Multiplicative Inverses.

Examples:

(1) 2 is an integer. (1/2) is its multiplicative inverse as (2)x(1/2)=1. But (1/2) is not a member of the set of Integers. Hence multiplicative Inverse does not exist.

RATIONAL NUMBERS

A Rational Number is a number in mathematics that represents a comparison of two numbers. The number that can be expressed as a ratio is called a Rational Number. In other words, the numbers that can be expressed in the form of p/q, where p and q are integers and especially q is not zero are called Rational Numbers.

Example: ½, 7/3, 0/1, , 1/1, etc.

Rational Numbers set is represented by Q. it was first denoted by ***Giuseppe Peano*** after ***quoziente***, an Italian word for "***quotient***".

Rational Numbers are of two types:

(a) Terminating decimal expression. Example ½ =0.5, and
(b) Non-terminating or recurring decimal expression. Example: 1/3=0.333333

Note:

(1) Sum, difference, product and division of two non-zero rational numbers is always a rational number.
(2) Every Natural Number is a Rational Number.
(3) Every Whole Number is a Rational Number.
(4) Every Integer is a Rational Number.
(5) There are infinite Rational Numbers.

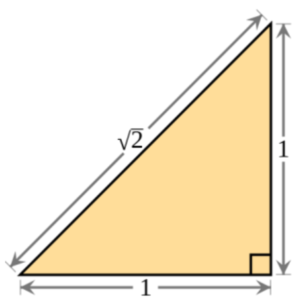
(6) There are infinite Rational Numbers between any two Rational Numbers.

PROPERTIES

(a) Rational Numbers set is closed for Addition, Subtraction, Multiplication and Division by a non-zero number as division by zero is NOT defined.
(b) Rational Numbers set obeys commutative, and associative properties for addition, and Multiplication.
(c) It does not obey Commutative, and Associative Properties for Subtraction and Division.
(d) It obeys the Distributive Property of Multiplication over Addition and Subtraction both.
(e) There exists Additive Identity and Multiplicative Identity in the set of Rational Numbers.
(f) There exists Additive and Multiplicative Inverses.
(g) Set of Rational Numbers cannot be expressed in Roster form i.e., we cannot make the list of all rational numbers as there are infinite rational numbers between any two numbers.
(h) Set of Rational numbers can be expressed on Number line as a non-stop continuous line.

IRRATIONAL NUMBERS

The numbers that cannot be expressed in the form of p/q are called Irrational Numbers. The numbers that are not Rational are called Irrational Numbers. The numbers that are neither terminating nor recurring are called Irrational Numbers.



√2 is an irrational number.

Set of Irrational Numbers is denoted by $\boldsymbol{Q^c}$.

Example: √2, √3,... are Irrational Numbers but √4 is not an Irrational Number as it terminates to 2.

Irrational Numbers can also be expressed on Number Line though they do not have a fixed numerical value. This is done by using **Pythagoras Theorem** that says that the square of the hypotenuse in the right angle triangle is equal to the sum of the squares of the other two sides.

The sum, difference, product and division of two Irrational Numbers is NOT always an Irrational Number.

Examples:

(a) 2+ √3, and 2- √3 are two different Irrational numbers. The sum of these two numbers is 2, which is not an Irrational number.
(b) For the irrational numbers 2+ √3, and 2- √3, the product is 1 which is not an irrational number.
(c) For the irrational numbers √3-2, and √3+2, the difference is also not an irrational number as it is -4, a rational number.
(d) If 2√3, and 5√3 are two irrational numbers then their quotient is 2/5, a rational number and not an irrational number.

Note:

(1) **There are some especial Irrational numbers: e, π, golden ratio (1+√5)/2, $\log_2 3$, e^π, π^e, etc**
(2) Greek Mathematician Pythagoras believed that all numbers were rational.
(3) **Hippasus**, a student of Pythagoras proved using Geometry that square root of 2 cannot be written as a fraction and so it was not a rational number. The followers of Pythagoras could not accept it and Hippasus was drowned at sea as a punishment from the gods.

SURDS

When it is not possible to remove the radical sign (√) from the number, the number is called a surd. Surd is, infact, another name for Irrational Number.

Examples:

(a) √2 (square root of 2) cannot be simplified further, hence it is a surd.

(b) $(8)^{1/3}$ (cube root of 8) can be simplified as 2. Hence it is not a surd.

Note:

(1) Around 820 AD ***al-Khwarismi*** (a Persian mathematician from whom we got name

"**Algorithm**" called Irrational Numbers "**Inaudible**" which was later translated to the Latin ***surdus*** (meaning **deaf** or **mute**).

(2) Surds are simplified by rationalizing the denominator of the expression.

CONJUGATE OF SURDS

If x and y are two surds and their sum is a rational number, then both surds are called conjugate of each other.

Examples:

(a) $4+\sqrt{5}$ is a conjugate of $4-\sqrt{5}$.

(b) $-4+\sqrt{5}$ and $-4-\sqrt{5}$ are conjugates of each other.

FINDING SQUARE ROOTS OF A SURD $\sqrt{2+\sqrt{3}}$

Steps:

(a) Assume $\sqrt{2+\sqrt{3}}=\sqrt{a}+\sqrt{b}$ where ***a***, and ***b*** are positive rational numbers.

(b) Square both sides and equate rational part to rational part and the irrational part to irrational part.

$2+\sqrt{3}=a+b+2\sqrt{ab}$

$a+b=2$ ...(i)

$2\sqrt{ab}=\sqrt{3}$ ..(ii)

(c) Find out ***(a-b)*** from the formula $(a-b)^2=(a+b)^2-4ab$.

$a-b=1$ (iii)

(d) Solve equations (i) and (iii)

$a=\frac{3}{2}, b=\frac{1}{2}$

(e) The required square root is $\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}$.

Note:

Assume $\sqrt{a}-\sqrt{b}$ for the square root of $\sqrt{2-\sqrt{3}}$.

REAL NUMBERS

The collection of all Rational Numbers and Irrational Numbers together is termed as the set of Real Number. It is represented by R. Real numbers can be represented on Number Line.

Real numbers set contains infinite numbers. All Natural Numbers, 0, Negatives of all Natural Numbers, all Rational Numbers, and all Irrational Numbers are Real Numbers.

The sum, difference, product and quotient of two Real Numbers is always a Real Number. The sum, difference, product or quotient of a Rational and an Irrational Number is always an Irrational Number.

PROPERTIES

(a) Set of Real Numbers is closed for Addition, Subtraction, Multiplication and Division by a non-zero number as division by zero is NOT defined.

(b) Set of Real Numbers obeys commutative, and associative properties for addition, and Multiplication.

(c) Set of Real Numbers does not obey Commutative, and Associative Properties for Subtraction and Division.

(d) Set of Real Numbers obeys the Distributive Property of Multiplication over Addition and Subtraction both.

(e) Set of Real Numbers contains Additive Identity {0}, and Multiplicative Identity {1}.

(f) Set of Real Numbers contains Additive and Multiplicative Inverses.

(g) Set of Real Numbers cannot be expressed in Roster form i.e., we cannot make the list of all Real Numbers.

(h) Set of Real Numbers can be expressed on Number Line as a non-stop continuous line.

(i) Set of Real Numbers are also represented in Interval forms as (2,4), [2,4], (2,4], [2,4) etc.

(j) Real numbers are uncountable.

Note: The adjective Real was introduced in 17^{th} Century by French Mathematician ***Rene Descartese***, by distinguishing Real and Imaginary roots of polynomials.

KNOW MORE ABOUT NUMBERS

(1) The usage or study of numbers is called arithmetic.

(2) 0,1,2,3,4,5,6,7,8,9 are called **digits**.

(3) 10, 11, 54,.., etc are called **numbers**.

(4) A number divisible by 2 is called an **even number**.

(5) A number not divisible by 2 is called an **odd number**.

(6) Numbers greater than 1 and **not prime** are called **composite numbers**.

(7) A number greater than 1 and having exactly two factors are called **prime numbers**.

(8) Two numbers are called **co primes** if their HCF is 1.

(9) Prime numbers that differ by 2 are called **twin primes**.

(3,5),(5,7),(11,13),(17,19),(29,31),

(41,43),(59,61),(59,61),(71,73)

(10) The actual value of the number is called **face value**. The product of the number with the value of its **place** (Ones, Tens, Hundreds ...) is called **place value** of the number**.**

DIVISIBILITY RULES

Divisibility by 2: A number is divisible by 2 if its unit's digit is any one of 0,2,4,6, and 8.

Divisibility by 3: A number is divisible by 3 if the total of its digits is divisible by 3.

Divisibility by 4: A number is divisible by 4 if the number formed by the last 2-digits is divisible by 4.

Divisibility by 5: A number is divisible by 5 if its unit's digit is either 0 or 5.

Divisibility by 6: A number is divisible by 6 if it is divisible by both 2 and 3.

Divisibility by 7: A number is divisible by 7 if the difference of the double of the last digit and the number formed by rest of the digits is divisible by 7.

Example: 679.

Double of 9=18; Difference of 67 and 18= 49,

49 is divisible by 7.

Hence 679 is divisible by 7

Divisibility by 8: A number is divisible by 8 if the number formed by last three digits is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of all its digits is divisible by 9.

Divisibility by 10: A number is divisible by 10 if its unit digit is 0.

Divisibility by 11: A number is divisible by 11 if the difference of the total of digits at odd places and the total of digits at even places is divisible by 11.

Divisibility by 12: A number is divisible by 12 if it is divisible by 3 and 4 both.

Divisibility by 14: A number is divisible by 14 if it is divisible by 2 and 7 both.

Divisibility by 16: A number is divisible by 16 if the number formed by last 4 digits is divisible by 16.

COMPLEX NUMBERS

The numbers in the form of $x+iy$ where x and y are real numbers and $i=\sqrt{-1}$ are called complex numbers. Complex Numbers may be purely real or purely imaginary. The set of complex numbers is represented by C.

Set of complex Numbers is closed for Addition, Subtraction, Multiplication and Division by a non-zero number. Set of complex Numbers follows Commutative, Associative, and Distributive Properties. Set of Complex Numbers contains Additive Identity, Additive Inverses, Multiplicative Identity and Multiplicative Inverses.

Complex Numbers are represented on the ***Argand Plane***. This plane is also called ***Gaussian Plane*** or ***Complex Plane***. In this plane, the ordinate or y-axis of Descartes Plane becomes imaginary. When a single letter is used to denote a Complex Number, it is sometimes called an ***affix***.

Complex Number $x+iy$, where x and y are real numbers, are represented in the following forms:

(a) Cartesian Form : $z = x+iy$
(b) Polar Form: $z = r(\cos\theta + i\sin\theta)$
(c) ***Eulerian*** Form: $z = re^{i\theta}$ where r is the modulus and θ is the principal argument.
(d) Ordered Pair Form: $z = (x, y)$ where x is real part, and y is imaginary part.

Note:

(1) x is called the real part of z.

(2) y is called the imaginary part of z

(3) $x - iy$ is called the conjugate of $x + iy$

(4) $|x + iy|$ is called the modulus or magnitude of complex number $x + iy$ and its value is $\sqrt{x^2 + y^2}$

(5) $\tan^{-1}\frac{y}{x}$ is called the argument or amplitude of the complex number $x + iy$

(6) 0 is a complex number that is purely real and purely imaginary.

ROMAN NUMERALS

Roman Numerals originated in ancient Rome. Roman Numerals use ***seven symbols*** (I, V, X, L, C, D, M) for representing numbers.

I for 1	II for 2	III for 3
V for 5	X for 10	L for 50
C for 100	D for 500	M for 1000

Rules for using Roman Numerals:

(a) Any numeral can be repeated maximum up to 3 times.

(b) Repetition means addition. Example: III means 1+1+1=3, X+X=20

(c) Only I, X, C and M can be repeated. Examples: CC=200, MMM=3000

(d) V, L, and D cannot be repeated. Examples: VV≠ 10, it is not allowed; LL≠100, it is not allowed; DDD≠1500, it is not allowed.

(e) When a numeral of lower value is written to the right of a numeral of higher value, the value of all the numerals are added. Example: DCLVIII=500+100+50+5+1+1+1=658

(f) When a numeral of lower value is written to the left of a numeral of higher value, then the value of lower numeral is subtracted from the value of higher numeral. Example: XL=50-10=40, CLIX=100+50-1+10=159

(g) V is never written to the left of X.

(h) If a horizontal line is drawn over the numerals then their value becomes 1000 times. Example: XV=15, $\overline{XV} = 15000$

Note:

(1) Romans did not use any symbol for 0 (zero).

(2) 4000 or more than 4000 cannot be written in Roman Numerals without using bar on the numerals because none of the numerals can be repeated more than 3 times.

WRITING NUMERALS IN WORDS

There are two ways of writing and reading numbers in mathematics:

(a) Indian Numbering System or Indian Place Value Chart

(b) International Place Value Chart

Indian Place Value Chart

Periods Consisting Of Two Places

SANKH	TEN SANKH, SANKH
PADMA	TEN PADMA, PADMA
NEEL	TEN NEEL, NEEL
KHARAB	TEN KHARAB, KHARAB
ARAB	TEN ARAB, ARAB
CRORE	TEN CRORE, CRORE
LAKH	TEN LAKH, LAKH
THOUSAND	TENTHOUSAND, THOUSAND

Period Consisting Of Three Places

ONE HUNDRED, TEN, ONE

Separators in Indian System

CRORE	LAKH	THOUSAND	ONE
00,	00,	00,	000
12,	34,	56,	789

Correct Way of Writing Numbers in Words

TWELVE CRORE THIRTY FOUR LAKH FIFTY SIX THOUSAND SEVEN HUNDRED EIGHTY NINE

Wrong Way of Writing Number:

TWELVE CRORES THIRTY FOUR LAKHS FIFTY SIX THOUSANDS SEVEN HUNDRED EIGHTY NINE

TWELVE CRORE, THIRTY FOUR LAKH FIFTY SIX THOUSAND, SEVEN HUNDRED EIGHTY NINE

TWELVE CRORE, THIRTY FOUR LAKH, FIFTY SIX THOUSAND, AND SEVEN HUNDRED EIGHTY NINE

Note: No COMMA between the periods, no PLURALS of the period, and nowhere AND is used in writing and reading the big numbers in words.

International Place Value Chart

All the Periods Consist Of Three Places as

(a) **TRILLIONS** consists of Hundred Trillions, Ten Trillions, Trillions
(b) **BILLIONS** consists of Hundred Billions, Ten Billions, Billions
(c) **MILLIONS** consists of Hundred Millions, Ten Millions, Millions
(d) **THOUSANDS** consists of Hundred Thousands, Ten Thousands, Thousands
(e) ONES consists of Hundreds, Tens, Ones

Separators in International System

MILLIONS	THOUSANDS	ONES
000,	000,	000
123,	456,	789

Correct Way Of Writing Numbers In Words

ONE HUNDRED TWENTY THREE MILLION FOUR HUNDRED FIFTY SIX THOUSAND SEVEN HUNDRED EIGHTY NINE.

Note: No COMMA between the periods, no PLURALS of the period, and nowhere AND is used in writing and reading the big numbers in words.

SOME SPECIAL NUMBERS

Perfect numbers

A perfect number is a Positive Integer whose twice is equal to the sum of its all divisors.

(a) 6 is a Perfect Number. Its divisors are: 1, 2, 3, and 6.

$2x6=1+2+3+6$

(b) The next Perfect Numbers are: 28, 496, ...
(c) The general formula for the n^{th} Perfect Number is $(2^n-1)2^{n-1}$ where n=2, 3,...

Narcissistic numbers

It is equal to the sum of the cubes of its digits.

$153=1^3 + 5^3 + 3^3$

$370=3^3 + 7^3 + 0^3$

$371=3^3 + 7^3 + 1^3$

$407=4^3 + 0^3 + 7^3$

Perfect digital invariant

$1634=1^4 + 6^4 + 3^4 + 4^4$

Taxi cab number

$1729=1^3 + 12^3 = 10^3 + 9^3 = 7 \times 13 \times 19 = 19 \times 91$.

Mathematics makes life. Everyone knows mathematics. Even dogs know counting. If someone thinks it is not true then he should put three dog biscuits in his pocket for three dogs, and then give one each to only two of them. Watch what happens...ha ha ha...

Numbers are very interesting. We shall continue with some new number systems like Binary, Octal, Hexadecimal, etc and their inter-relationships in the next e-bulletin….

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