(....conrd.) LET's LEARN OUR NUMBER SYSTEMS

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We shall discuss here the

(a) Binary Number System,
(b) Octal Number System, and
(c) Hexadecimal Number System

Earlier to this we studied the **Decimal Number System** containing the digits 0,1,2,3,4,5,6,7,8, and 9 (i.e 10 digits).

The base of the **Decimal Number System** is 10 (equal to the digits available for indicating a number).

BINARY NUMBER SYSTEM

It consists of only two digits 0 (ZERO) and 1 (ONE). The base of this number system is 2.

The modern binary system was devised by the mathematician **Gottfried Leibnitz** in 1679.

Binary Number System uses two types of electronic pulses: **Absence** of pulse shows **0** and the **Presence** of pulse shows **1**.

Each binary digit is called as **bit**. Left-most bit of a number is known as Most Significant Bit (MSB) and the Right-most bit is known as Least Significant Bit (LSB).

Decimal	**Binary**	**Read as**
0	0	Zero
1	1	One
2	10	One Zero
3	11	One One

Decimal	Binary	Read as
4	100	One Zero Zero
5	101	One Zero One
6	110	One One Zero
7	111	One One One
8	1000	One Zero Zero Zero
9	1001	One Zero Zero One
10	1010	One Zero One Zero

Explanation:

Take a number 193.

It is a decimal number. It is mathematically written to be identified as $(193)_{10}$.

It means $(193)_{10} = 1x10^2 + 9x10^1 + 3x10^0$

Take another number $(1879.345)_{10}$.

It means $(1879.345)_{10} = 1x\ 10^3 + 8x10^2 + 7x10^1 + 9x10^0 + 3x10^{-1} + 4x10^{-2} + 5x10^{-3}$

Similarly, in Binary

(a) $(10)_2 = 1x2^1 + 0x2^0$ (in decimal)
(b) $(010101)_2 = 0x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 0x2^1 + 1x2^0$ (in decimal).
(c) $(1101001.101)_2 = 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0 + 1x2^{-1} + 0x2^{-2} + 1x2^{-3}$ (in decimal).
(d) $(10)_2$ has 2 bits.
(e) $(010101)_2$ has 6 bits.

Positional Value

In Decimal Number System, the positions are: Ones, Tens, Hundreds, Thousands etc., from the RIGHT.

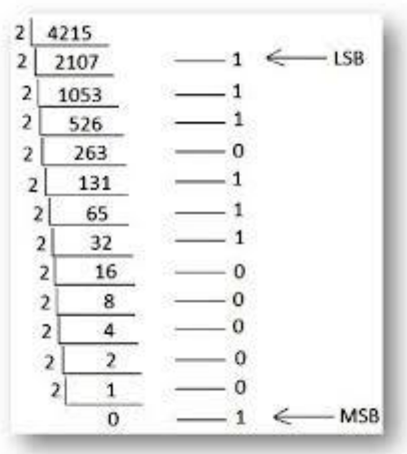
In Binary Number System, the positions are: Ones, Twos, Fours, Eights, etc.

As we move further Left, every number place gets 2 times bigger.

Note:

(a) A group of 4 bit is called **nibble** and group of 8 bit is called **byte**.
(b) Value of digit is determined by the position of digit in the number, where lowest value is for the Right-most position and each successive position to the Left has a higher place value.

Conversion from Decimal to Binary



$(4215)_{10} = (1000001110111)_2$, Or

$4x10^3 + 2x10^2 + 1x10^1 + 5x10^0 = 1x2^{12} + 0x2^{11} + 0x2^{10} + 0x2^9 + 0x2^8 + 0x2^7 + 1x2^6 + 1x2^5 + 1x2^4 + 0x2^3 + 1x2^2 + 1x2^1 + 1x2^0$

MSB (Most Significant Bit) is written in the extreme LEFT and the LSB (Least Significant Bit) is written in the extreme RIGHT (i.e. from Bottom to Top).

Operations

Addition

The RULES for addition are as under:

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0, carry 1 to be added to the LEFT column.

Example:

1 1 1 **Carried digits**

```
   0 1 1 0 1 0
 +1 0 1 1 0 0
  --------------
 1 0 0 0 1 1 0
  ---------------
```

Explanation:

$(011010)_2 = (26)_{10}$, and

$(101100)_2 = (44)_{10}$

$\Rightarrow 26+44=70 \Rightarrow (1000110)_2=(70)_{10}$

Multiplication

The RULES for multiplication are as under:

1 x 0 =0

0 x 1 =0

1 x 1 =1

It is done as in the Decimal Number System.

Example:

```
     1 1 1 0
  x      1 0
   ............
    0 0 0 0
  1 1 1 0 x
 ...............
 1 1 1 0 0
 ..............
```

Check:

$(1110)_2 = (14)_{10}$

$(10)_2 = (2)_{10}$

$14x2 = (28)_{10}$ and $(11100)_2 = (28)_{10}$

Subtraction

0 – 0 = 0

0 – 1 = 1, borrow 1

1 – 0 = 1

1 – 1 = 0

Example:

```
   1 1 0 0 1 1
   – 1 0 1 1 1
  ----------------
    1 1 1 0 0
 ....................
```

Check:

$(110011)_2=(51)_{10}$

$(10111)_2=(23)_{10}$

$(51)_{10}$- $(23)_{10}$= $(28)_{10}$=$(11100)_2$

Division

Division is done as in decimal.

```
      1 0 1
    _____________
1 0 1 ) 1 1 0 1 1
    – 1 0 1
      -----
       1 1 1
     –  1 0 1
        -----
         1 0
```

Check:

Divisor $(101)_2 = (5)_{10}$

Dividend $(11011)_2 = (27)_{10}$

Quotient $(101)_2 = (5)_{10}$

Remainder $(10)_2 = (2)_{10}$

Dividend = (Divisor)(Quotient)+(Remainder)

OCTAL NUMBER SYSTEM

It consists of 8 digits from 0 to 7 i.e., 0, 1, 2, 3, 4, 5, 6, and 7. It contains 8 digits, so the base of this number system is 8.

Octal numerals can be made from binary numerals by grouping consecutive binary digits into groups of three (starting from Right).

Example:

(a) Binary representation for $(75)_{10}$ (in decimal) is $(01001011)_2$

Hence, the octal of $(75)_{10}$ is

$(001)_2$ $(001)_2$ $(011)_2$ = $(113)_8$

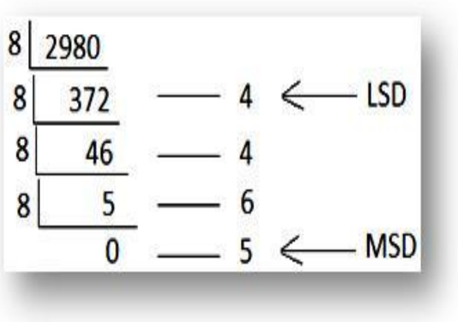
$= 1x8^2+1x8^1+3x8^0$

(b) The other examples are :
$(03105)_8$, and
$(4237.23)_8$

Note:

(a) It takes exactly three binary digits to represent an octal digit.
(b) Binary 000 is same as octal digit 0, binary 001 is same as octal 1, and so on.

Conversion from Decimal to Octal



Therefore,

$(2980)_{10} = (5644)_8$

HEXADECIMAL NUMBER SYSTEM

It consists of 16 types of digits from 0 to 9 and alphabets A, B, C, D, E, F. The base of number system is 16.

Digits from 10 to 15 are represented by A for 10, B for 11, C for 12, D for 13, E for 14, and, F for 15.

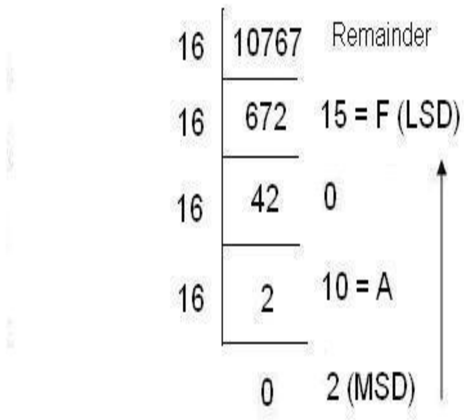
As numeric digits and alphabets are used to represent digits, this number system is also called as **Alphanumeric Number System**.

It is widely used in computer system.

Examples:

1. $(AF38)_{16}$
2. $(CE7.5B)_{16}$

Conversion from Decimal to Hexadecimal



i.e. $(10767)_{10} = (2A0F)_{16}$

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