

## PERMUTATIONS AND COMBINATIONS

Prof. SB DHAR

The study of Permutations and Combinations is concerned with determination of the number of different ways of arrangements and selections of objects out of a given number of objects, without actually listing them.

Example:

If there are two things A and B, they can be arranged as AB or BA i.e. at first place A comes and at second place B comes, or in other case B comes at first place and A comes at second place.

It shows that two possibilities exist for these arrangements.

If we are to select these A and B at a time, then we can have any one of these arrangements i.e. only one way of selection is there.

Arrangements are called **Permutations** and selections are called **Combinations**. The number of arrangements is always more than the number of combinations.

### FUNDAMENTAL THEOREM

If one operation can be performed in  $m$  ways, and another in  $n$  ways then the number of ways the two operations can be performed is  $(m \times n)$ .

### **Permutation**

The arrangement of different (distinct) objects in a particular order is called Permutation.

Examples:

- (a) If there are 3 different letters  $a, b, c$  and they are to be put in a row, then they can be put like:  $abc$ , or  $acb$ , or  $bac$ , or  $bca$ , or  $cab$ , or  $cba$  (i.e. 6 ways)
- (b) The number of permutations for  $n$  things

$$\text{taken } r \text{ at a time is } {}^n P_r = \frac{n!}{(n-r)!}$$

### MULTIPLICATION THEOREM

If an operation can be performed in  $m$  ways and another in  $n$  ways, then the two operations in succession can be performed in  $m \times n$  ways.

Example:

If a person can go by bus from Delhi to Gurgaon in three ways and from Gurgaon to Faridabad in two ways, then he can go to Faridabad via Gurgaon in  $3 \times 2$  ways.

### ADDITION THEOREM

If an operation can be performed in  $m$  ways and another independent operation can be performed in  $n$  ways, then either of the two can be performed in  $(m+n)$  ways.

Example:

If a person can go from Delhi to Ghaziabad in 2 ways by bus routes, 3 ways by metro routes and 1 way by rail route then he can reach Ghaziabad by either route in  $2+3+1$  ways.

Example:

Find the number of words, with or without meaning, that can be formed with the letters of the word CHALK.

*Solution:*

The word CHALK contains 5 letters, all different. Therefore, the number of words formed with or without meaning taking all together =  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Example:

Find the number of word, with or without meaning that can be formed with the letters of the word IITIAN.

*Solution:*

The word IITIAN contains not all different letters, but 3 I, 1A, 1N and 1T.

When the letters occur more than once in a word, we divide the factorial of the number of all letters by the

factorial of the numbers repeating the letters. Hence the number of words =  $\frac{6!}{3! \times 1! \times 1! \times 1!} = 120$

**Example:**

How many different words can be formed with the letters of the word SARITA such that the vowels are always together?

**Solution:**

The grouping of the letters will be S, R, T, (AAI). The letters AAI will be always together in the group in  $\frac{3!}{2! \times 1!} = 3$ . The four letters S,R,T,(AAI) will be arranged in  $4! = 24$ . So the total ways of arrangements =  $24 \times 3 = 72$ .

**Example:**

Find the number of permutations of the letters of the word REMAINS such that the vowels always occur in odd places.

**Solution:**

REMAINS has seven letters, out of which E, A, I (three) are vowels and R, M, N, S (four) are consonants.

(1) (2) (3) (4) (5) (6) (7)

Vowel are to take place at (1), (3), (5), (7).

And at the rest places the consonants will appear.

3 vowels can be put in 4 different places in  ${}^4P_3$  ways and rest 4 places can be filled by 4 different letters in  ${}^4P_4$  ways. Hence the total number of ways =  ${}^4P_3 \times {}^4P_4$

### Combination

The arrangements of different things without keeping a particular order in mind is called Combination.

Examples:

(a) For three letters  $a, b, c$ , the number of combinations for all together will be 1 (i.e. abc).

(b) Number of Combinations of  $n$  distinct things

$$\text{taken } r \text{ at a time} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

**Example:**

In how many ways can a committee of 1 man and 3 women be formed from a group of 6 men and 5 women?

**Solution:**

1 man can be selected from 6 men in  ${}^6C_1$  ways and 3 women from 5 women in  ${}^5C_3$ . So the total number of ways =  ${}^6C_1 \times {}^5C_3$ .

**Example:**

There is a set of 5 black balls and 3 red balls. Selections of 5 balls are to be made such that at least 3 of them are black balls. Find the number of selections.

**Solution:**

There may be following possibilities:

(a) 3B, and 2R or

(b) 4B, and 1R or

(c) 5B, and 0R

Hence the total selections

$$= {}^5C_3 \times {}^3C_2 + {}^5C_4 \times {}^3C_1 + {}^5C_5 \times {}^3C_0$$

### Properties to be memorized

- (1) Factorial  $n$  is written as  $n!$
- (2)  $n! = n(n-1)(n-2)(n-3)\dots\dots 3.2.1$
- (3)  $0! = 1$  (assumption)
- (4)  $\frac{1}{(-n)!} = 0$
- (5) Factorial of proper fraction or negative integer is not defined.
- (6) If  $p$  is a prime number,  $n$  is a positive integer, then the greatest integer amongst  $1, 2, 3, \dots, (n-1), n$  which is divisible by  $p$  is  $\left[ \frac{n}{p} \right] p$ , where  $[.]$  represents the greatest integer less than or equal to  $n/p$ .
- (7) Greatest exponent of 3 in  $180!$ , dividing it completely is calculated by

$$\left[ \frac{180}{3} \right] + \left[ \frac{180}{3^2} \right] + \left[ \frac{180}{3^3} \right] + \left[ \frac{180}{3^4} \right] + \left[ \frac{180}{3^5} \right] + \dots$$

$$= 60 + 20 + 6 + 2 + 1 + 0 + \dots = 89.$$

(8) Exponent of 2 in  $20 \cdot 19 \cdot 18 \dots 12 \cdot 11 = \frac{20!}{10!} =$   
 Exponent of 2 in  $20!$  - Exponent of 2 in  $10! = 18 - 8 = 10$

(9) Number of zeros at the end of  $(60!) =$  Exponent of 10 in  $(60!) = \min \{E_2(60!), E_5(60!)\} = E_5(60!) = 14$

(10) One- one mapping of a finite set A onto itself is also called a permutation of A. Hence total number of permutations of n membered set is =  $n!$

(11)  ${}^n P_r$  represents the number of arrangements of r distinct things out of n distinct things.

(12)  ${}^n P_r = \frac{n!}{(n-r)!}$  where n is a natural number and  $0 \leq r \leq n$ .

(13)  ${}^n P_r$  is always a natural number.

(14)  $r \cdot {}^{n-1} P_{r-1}$  represents the number of permutations of n different things taken r at a time, when one particular thing is always included.

(15)  $\frac{r!}{(r-p)!} {}^{n-p} P_{r-p}$  represents the number of permutations of n different things taken r at a time, when p particular things are always included.

(16)  $p! \left[ r - (p-1) \right] {}^{n-p} P_{r-p}$  represents the number of permutations of n different things taken r at a

time, when p particular things are always together.

(17)  ${}^{n-p} P_r$  represents number of permutations of n different things, taken r at a time, when p particular things are not taken.

(18)  $p!(n-p+1)!$  represents the number of permutations of n different things taken all at a time, when p particular things always occur together.

(19)  $n! \left[ m!(n-m+1)! \right]$  represents the number of permutations of n different things, taken all at a time, when m particular things never occur together.

(20)  ${}^n C_r$  represents the number of selections of r distinct things out of n distinct things.

(21)  ${}^n C_r = \frac{n!}{r!(n-r)!}$  where n is natural number and  $0 \leq r \leq n$ .

(22)  ${}^n C_r + {}^n C_{r+1} = {}^{n+1} C_{r+1}$  (Pascal Law)

(23)  ${}^n C_r = {}^n C_{n-r}$  (Complementary Law)

(24) If  ${}^n C_r = {}^n C_s$  then either  $r = s$  or  $r + s = n$ .

(25)  ${}^n C_r$  is greatest when  $r = \frac{n}{2}$ , if n is even;  $r = \frac{n+1}{2}$  or  $\frac{n-1}{2}$  if n is odd.

(26) Number of combinations of n distinct things taken r at a time, when k particular things always occur is  ${}^{n-k} C_{r-k} \cdot 0 \leq k \leq r$

(27) Number of combinations of  $n$  distinct things taken  $r$  at a time, when  $k$  particular things never occur is  ${}^{n-k}C_r$ ,  $1 \leq k \leq r$

(28) Number of total selections of one or more things from  $n$  distinct things is  ${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$ .

(29) Number of permutations of  $n$  things, taken all at a time, of which  $p$  are of one kind and  $q$  are of another kind =  $\frac{n!}{p!.q!}$ .

(30) Number of permutations of  $n$  things, taken all at a time, of which  $p_1$  are of one kind,  $p_2$  are of second kind,  $p_3$  are of third kind, and so on....  $p_r$  are of  $r^{\text{th}}$  kind, such that  $p_1 + p_2 + p_3 + \dots + p_r = n$  is  $\frac{n!}{p_1!.p_2!.p_3! \dots p_r!}$ .

(31) Number of permutations of  $n$  things, of which  $p$  are of one kind,  $q$  are of another kind and rest all are distinct is given by  $\frac{n!}{p!.q!}$ .

(32) If  $r$  things are to be arranged allowing repetition. Assume  $p_1, p_2, p_3, \dots, p_r$  to be integers such that the first thing occurs exactly  $p_1$  times, the second occurs exactly  $p_2$  times, etc then the total number of Permutations of these  $r$  things is given by  $\frac{(p_1 + p_2 + \dots + p_r)!}{p_1!.p_2! \dots p_r!}$ .

(33) Number of circular permutations of  $n$  distinct things is  $(n-1)!$ . When the things are distinct, the arrangements in clock-wise and anti-clockwise orders are taken into account.

Note : If  $n$  persons are arranged in a straight line, there are  $n!$  different ways in which this can be done. If they are placed round a circular

table, each arrangement will be repeated  $n$  times, so there are  $(n-1)!$  different arrangements. Or

We can regard any one person as 'head' and place  $(n-1)$  persons in  $(n-1)!$  different ways i.e. the number of ways of circular permutations of  $n$  different persons taken all at a time is  $(n-1)!$ .

(34) Number of circular permutations of  $n$  things like beads, flowers etc where distinction is not possible is  $\frac{(n-1)!}{2}$ . When the things are not distinct, the arrangements in clock wise and anti-clockwise orders are not taken in account, they are considered to be same.

(35) Number of circular permutations of  $n$  different things taken  $r$  at a time is  $\frac{{}^nP_r}{r}$  when clockwise is different from the anticlockwise arrangement.

(36) Number of circular permutations of  $n$  different things taken  $r$  at a time is  $\frac{{}^nP_r}{2r}$  when clockwise and anticlockwise are not different but considered same.

(37)  $n$  non-concurrent and non-parallel lines cut in  ${}^nC_2$  points.

(38) If  $n$  points are there, out of which no three are collinear, then the number of lines =  ${}^nC_2$ .

(39) If  $n$  points are there, out of which  $m$  are collinear, then the number of lines =  ${}^nC_2 - {}^mC_2 + 1$  as  $m$  points form one line only.

(40) In a polygon of  $n$  sides out of which no three points are collinear, the number of diagonals is  ${}^nC_2 - n$ .

- (41) Number of triangles formed with  $n$  points out of which no three are collinear is  ${}^n C_3$ .
- (42) Number of triangles formed with  $n$  points out of which  $m$  points are collinear is  ${}^n C_3 - {}^m C_3$ .
- (43) Number of triangles formed with  $n$  points when none of the side is common to the side of the polygon is  ${}^n C_3 - {}^n C_1 - {}^n C_1 \cdot {}^{n-4} C_1$   
 Explanation: (Total number of triangles from all  $n$  vertices taking 3 at a time) – (total number of triangles whose two sides are the consecutive sides of the polygon) – (total number of triangles whose only one side is the side of the polygon). Or, when no side of the triangle is the side of the polygon is formed by taking alternate vertices of the polygon.
- (44) Number of parallelograms with two sets of lines one containing  $m$  lines and the other containing  $n$  lines is  ${}^m C_2 \times {}^n C_2$ .
- (45) Number of squares in two system of parallel lines one containing  $m$  lines and another containing  $n$  lines, perpendicular to each other is  $\sum_{r=1}^{m-1} (m-r)(n-r); (m < n)$  Note: In chess board  $m = n = 9$ .
- (46)  $(n!+1)$  is not divisible by any number between 2 and  $n$ .
- (47) Number of hand-shakes by  $n$  persons =  ${}^n C_2$
- (48) Number of ways of selecting  $r$  things out of  $n$  identical things is 1.
- (49) Number of ways of selecting zero or more i.e. at least one thing from a group of  $n$  identical things is  $(n+1)$ .
- (50) Number of ways of selecting of some or all out of  $(p+q+r)$  things out of which  $p$  are alike,  $q$  are of second kind and rest are of third kind is  $[(p+1)(q+1)(r+1)]-1$ .
- (51) Number of ways of selecting one or more things from  $p$  identical things of one kind,  $q$  identical things of second kind,  $r$  identical things of third kind and  $n$  different things is  $(p+1)(q+1)(r+1)2^n - 1$
- (52) The number of ways answering one or more of  $n$  questions =  $2^n - 1$
- (53) The number of ways of answering one or more of  $n$  questions when each question has an alternative =  $3^n - 1$ .
- (54) The number of ways of answering all of  $n$  questions when each question has an alternative =  $2^n$ .
- (55) If  $p_1, p_2, p_3, \dots, p_k$  are distinct prime numbers and  $n_1, n_2, n_3, \dots, n_k$  are positive integers, then the number of divisors of  $A = p_1^{n_1} \cdot p_2^{n_2} \cdot p_3^{n_3} \dots p_k^{n_k}$  is  $(n_1 + 1)(n_2 + 1)(n_3 + 1) \dots (n_k + 1)$ .
- (56) Total number of proper divisors =  $(n_1 + 1)(n_2 + 1)(n_3 + 1) \dots (n_k + 1) - 2$ .
- (57) Sum of all divisors =  $(p_1^0 + p_1^1 + p_1^2 + \dots + p_1^{n_1}) (p_2^0 + p_2^1 + p_2^2 + \dots + p_2^{n_2}) \dots (p_k^0 + p_k^1 + p_k^2 + \dots + p_k^{n_k})$ .
- (58) Number of ways in which  $(m+n)$  distinct items can be divided into two unequal groups containing  $m$  and  $n$  items =  ${}^{m+n} C_m$

(59) Number of ways in which  $(m+n+p)$  things are divided into unequal groups containing  $m, n, p$  things is  ${}^{m+n+p}C_m \times {}^{n+p}C_n$ .

(60) Number of ways to distribute  $(m+n+p)$  things among 3 persons in groups containing  $m, n, p$  items is = (No. of ways to divide)  $\times$  (No. of groups) =  ${}^{m+n+p}C_m \times {}^{n+p}C_n \times 3!$ .

(61) Number of ways in which  $mn$  different things divided equally into  $m$  groups, each containing  $n$  things and the order of the groups is not important is  $\left(\frac{(mn)!}{(n!)^m}\right) \frac{1}{m!}$ .

(62) Number of ways in which  $mn$  different things divided equally into  $m$  groups, each containing  $n$  things and the order of the groups is important is  $\left(\frac{(mn)!}{(n!)^m} \times \frac{1}{m!}\right) m!$ .

(63) Number of ways of dividing  $n$  identical things among  $r$  persons, each one of whom, can receive 0, 1, 2, or more things is  ${}^{n+r-1}C_{r-1}$ . or,  
Number of ways of dividing  $n$  identical things into  $r$  groups, if blank groups are allowed is  ${}^{n+r-1}C_{r-1}$ .

(64) Number of ways of dividing  $n$  identical things among  $r$  persons, each one of whom, can receive at least one thing is  ${}^{n-1}C_{r-1}$ . or,  
Number of ways of dividing  $n$  identical things into  $r$  groups, if blank groups are not allowed is  ${}^{n-1}C_{r-1}$ .

(65) Number of ways in which  $n$  identical things can be divided into  $r$  groups so that no group contains less than  $m$  things and more than  $k$  ( $m < k$ ) is :

Coefficient of  $x^n$  in the expansion of  $(x^m + x^{m+1} + \dots + x^k)^r$ .

(66) If  $n$  distinct things are arranged in a row, then the number of ways in which they can be de-arranged so that none of them occupies its original place is  $n! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots + (-1)^n \frac{1}{n!} \right\}$ .

(67) If  $r$  things occupy the original places and none of the remaining  $(n-r)$  things occupies its original places, then the number of such ways is  ${}^nC_r \cdot (n-r)! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^{n-r} \frac{1}{(n-r)!} \right\}$ .

(68) Number of bijections from  $A$  to  $A$ , containing  $n$  elements, such that  $f(x) \neq x$ , for all  $x \in A$  is,  $n! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^n \frac{1}{n!} \right\}$ .

(69) If  $x_1 + x_2 + x_3 + x_4 + \dots + x_r = n$  where  $x_1, x_2, x_3, x_4, \dots, x_r$  and  $n$  are non-negative integers, then total number of non-negative integral solutions = coefficient of  $x^n$  in  $(x^0 + x^1 + \dots + x^n)^{-r} = (1-x)^{-r} = {}^{n+r-1}C_{r-1}$ .

(70) To solve  $x_1 + x_2 + x_3 + \dots + x_m \leq n$  introduce a dummy variable  $x_{m+1} \geq 0$  such that  $x_1 + x_2 + x_3 + \dots + x_{m+1} = n$  and the solution shall be =  ${}^{n+m+1-1}C_{m+1-1}$ .

(71) Number of ways of getting a total of  $r$  when 2 dice are thrown is = coefficient of  $x^r$  in the expansion of  $(x^1 + x^2 + x^3 + \dots + x^6)^2$  when the dice are numbered from 1 to 6. Or,  $(r-1)$  if  $2 \leq r \leq 7$  and  $(13-r)$  if  $8 \leq r \leq 12$ .

(72) Number of ways of getting a total of  $r$  when three dice are thrown is = coefficient of  $x^r$  in the

expansion of  $(x^1+x^2+x^3+\dots+x^6)^3$  when the dice are numbered from 1 to 6 Or  $\frac{(r-1)(r-2)}{2}$  when  $3 \leq r \leq 8$  and  $\frac{(19-r)(20-r)}{2}$  when  $13 \leq r \leq 18$  and 25 if  $r = 9$  or 12 and 27 if  $r = 10$  or 11.

(73) Number of ways of getting a total of  $r$  in throw of  $n$  dice if they are  $m$ -faced = coefficient of  $x^r$  in the expansion of  $(x^1+x^2+x^3+\dots+x^m)^n$ .

(74) Maximum number of points of intersection of  $n$  circles =  ${}^n P_2$

(75) The number of ways in which  $N$  can be resolved into two factors which are co-prime to each other is  $2^{n-1}$  where  $n$  is the number of different factors in  $N$ .

**Some solved examples:**

1. There are 27 boys and 14 girls in a class. One boy and one girl are to be selected to represent the class for a function. In how many ways can the teacher make this selection?

*Solution:*

The teacher will have to perform two operations for this selection:

- (a) Selecting a boy from the 27 boys, and
- (b) Selecting a girl from the 14 girls.

One boy out of 27 boys can be selected in  ${}^{27}C_1$  ways and one girl out of 14 girls can be selected in  ${}^{14}C_1$  ways. This will be done one by one hence by the fundamental principle of counting, the required number of ways is  ${}^{27}C_1 \times {}^{14}C_1$ .

2. If all permutations of the letters of the word AGAIN are arranged in the order as in a dictionary. What is the 49<sup>th</sup> word?

*Solution:*

The dictionary starts with A, so let us Put A at the first place

A x1, x2, x3, x4. Four letters A, G, I, N will be arranged in 4! Ways i.e. 24 ways.

Next put G at the first place

G, y1, y2, y3, y4. Four letters A, A, I, N will be arranged in  $\frac{4!}{2! \times 1! \times 1!} = 12$  ways.

Next step start with I at first place as

I, z1, z2, z3, z4. Four letters A, A, G, N can be arranged in  $\frac{4!}{2! \times 1! \times 1!} = 12$  ways.

The total of these arrangements is  $24+12+12=48$ .

So the next arrangement will be the required word. It will start with N as first letter.

N, w1, w2, w3, w4. Where at second, third, fourth and fifth places will be filled by A, A, G, I as NAAGI.

3. Evaluate the number of permutations of  $n$  different things taken  $r$  at a time such that two specific things occur together.

*Solution:*

Two things are always together. It means, from the group of  $n$  things, 2 things are grouped as One. It means that 2 specific things are arranged in  $r$  places in  $(r-1)$  ways and those 2 things will be arranged in 2! ways i.e.  $2! \times (r-1)$  ways.

Now  $(n-2)$  things will be arranged in  $(r-2)$  places in  ${}^{n-2}P_{r-2}$  ways.

So, the total number of ways is  $2! \times (r-1) \times {}^{n-2}P_{r-2}$ .

4. Find the number of signals that can be sent by 6 flags of different colours taking one or more at a time.

*Solution:*

Total number of ways of making signals = use of (1 flag + 2 flags + 3 flags + 4 flags + 5 flags + 6 flags)

$$\text{i.e. } {}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$$

5 A man has 7 relatives, 4 of them are ladies and 3 gentlemen; his wife has 7 relatives and 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives?

*Solution:*

The possibilities may be

(a) 3 ladies from husband side and 3 gentlemen from wife side, i.e.  ${}^4C_3 \cdot {}^4C_3$

(b) 3 gentlemen from husband side and 3 ladies from wife side i.e.  ${}^3C_3 \cdot {}^3C_3$ .

(c) 2 ladies and 1 gentleman from husband side and 1 lady and 2 gentlemen from wife side i.e.  $({}^4C_2 \cdot {}^3C_1) \cdot ({}^3C_1 \cdot {}^4C_2)$

(d) 1 lady and 2 gentlemen from husband side and 2 ladies and 1 gentleman from wife side i.e.  $({}^4C_1 \cdot {}^3C_2) \cdot ({}^3C_2 \cdot {}^4C_1)$

The total ways is the sum of all the numbers i.e. (a)+(b)+(c)+(d).

6 Two finite sets have m and n elements, the total number of subsets of the first set is 56 more than the total number of subsets of the second. Find the values of m and n.

*Solution:*

$$2^m = 56 + 2^n, m=6, n=3$$

7 Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many different ways can we place the balls so that no box remains empty?

*Solution:*

Obviously each box will contain at least one ball as no box is to be empty.

Two cases are possible.

*Case I*

1<sup>st</sup> box 1 ball; 2<sup>nd</sup> box 1 ball and 3<sup>rd</sup> box 3 balls.

This is possible in  ${}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 \cdot ({}^3C_1)$  as in 1<sup>st</sup> box it will be any 1 ball out of 5 balls; in 2<sup>nd</sup> box out of 4 balls, any 1 will be put and in 3<sup>rd</sup> box any 3 balls from the remaining 3 balls will be put in.

Also note the box that contains 3 balls can be chosen in  ${}^3C_1$  ways.

*Case II*

1<sup>st</sup> box 1 ball; 2<sup>nd</sup> box 2 balls and 3<sup>rd</sup> box 2 balls.

This will be done in  ${}^5C_1 \cdot {}^4C_2 \cdot {}^2C_2 \cdot ({}^3C_2)$  ways as any 1 ball can be put in the 1<sup>st</sup> box; any 2 balls in the 2<sup>nd</sup> box and any 2 balls in the 3<sup>rd</sup> box from the remaining 2 balls.

Note the boxes that contain 2 balls can be chosen in  ${}^3C_2$  ways.

The required number = sum of all the 2 cases.

8 A train going from Mumbai to Delhi at nine intermediate stations. Six persons enter the train during the journey with six different tickets of the same class. How many different sets of tickets may they have had?

*Solution:*

Assume Mumbai,  $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9$ , Delhi as all stations.

Find all possible different tickets available during the journey, i.e. at intermediate stations.

Obviously at  $s_1$ , 9 different tickets will be available; at  $s_2$ , 8 different tickets will be available and so on.

Total number of different tickets =  $9+8+\dots+1=45$



Six different tickets will be from these 45.

Hence the required number of ways =  ${}^{45}C_6$

9 Find the remainder obtained when  $1! + 2! + 3! + \dots + 200!$  is divided by 14.

*Solution:*

$n!$  is divisible by 14 if it is divisible by 2 and 7, and it is possible only when  $n \geq 7$ .

i.e.  $7! + 8! + 9! + \dots + 200!$  is divisible by 14.

But  $1! + 2! + 3! + 4! + 5! + 6! = 873 = 14 \times 62 + 5$

i.e. the whole expression is when divided by 14 will leave remainder 5.

10 Find the number of integral solutions for the system of equations  $x_1 + x_2 + x_3 + x_4 + x_5 = 20$  and  $x_1 + x_2 = 15$  where  $x_i \geq 0$  for all  $i = 1, 2, 3, 4, 5$ .

*Solution:*

From the given equations

$x_1 + x_2 = 15 \Rightarrow$  the number of solutions =  ${}^{15+2-1}C_{2-1}$ , and

$x_3 + x_4 + x_5 = 5 \Rightarrow$  the number of solutions =  ${}^{5+3-1}C_{3-1}$

Hence the total number of solutions = product of the above twos.

11 One million people can be identified by 3 letters initials, then find the number of letters in the alphabet.



Dr S.B. Dhar, is **Editor of this Quarterly e-Bulletin**. He is an eminent mentor, analyst and connoisseur of Mathematics from IIT for preparing aspirants of Competitive Examinations for Services & Admissions to different streams of study at Undergraduate and Graduate levels using formal methods of teaching shared with technological aids to keep learning at par with escalating standards of scholars and learners. He has authored numerous books – Handbook of Mathematics for IIT JEE, A Textbook on Engineering Mathematics, Reasoning Ability, Lateral Wisdom, Progress in Mathematics (series for Beginner to Class VIII), Target PSA (series for class VI to class XII) and many more.

**e-Mail ID:** maths.iitk@gmail.com

*Solution:*

Given  $n \times n \times n = 10^6$

$\Rightarrow n = 10^2 \Rightarrow n = 100$ .

12 Find the number of negative terms of the sequence  $\{x_n\} = \frac{{}^{n+4}P_4}{(n+2)!} - \frac{143}{4n!}$ .

*Solution:*

Simplify  $\{x_n\} = (4n^2 + 28n - 95) / 4n!$

For negative terms,  $4n^2 + 28n - 95 < 0$  and this is possible for  $n = 1$  and  $2$  only hence only 2 negative terms are there.

13 Find the greatest number of points in which  $m$  straight lines and  $n$  circles can intersect.

*Solution:*

(i) 2 straight lines intersect in at most one point  $\Rightarrow m$  lines intersect in  ${}^mC_2 \times 1$  points.

(ii) 2 circles intersect in at most two points  $\Rightarrow n$  circles intersect in  ${}^nC_2 \times 2$  points.

(iii) a line intersects a circle in at most 2 points  $\Rightarrow m$  lines and  $n$  circles intersect in  $(mn) \times 2$  points.

Hence the maximum number of points of intersection of  $m$  lines and  $n$  circles = sum of all above.