COORDINATE GEOMETRY: STRAIGHT LINE

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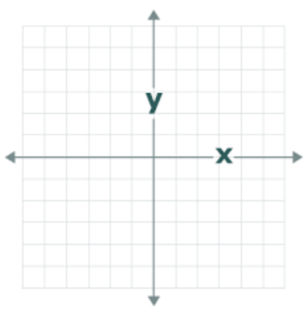
INTRODUCTION

Rene Descartes (1596-1650) was a French Mathematician who is credited as the father of Analytical Geometry, bridge between Algebra and Euclid Geometry. This Analytical Geometry is called in plane language, Coordinate Geometry.

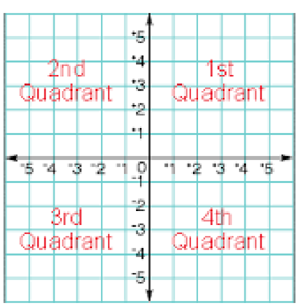
He used two mutually perpendicular lines to divide the plane into four equal parts. He named the horizontal line as **x-axis (abscissa)** and the vertical line as **y-axis (ordinate)**.

Example:1



Example:2

The parts of the plane divided by these lines in four equal parts were named as **1st Quadrant, 2nd Quadrant, 3rd Quadrant** and **4th Quadrant,** respectively in anticlockwise direction starting from the right side.



The intersection point of the **abscissa** and the **ordinate** was called **Origin**. This is the most important point on the plane because all the journeys to fix the points on the plane start from the origin.

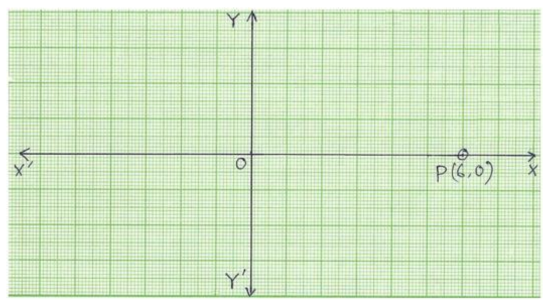
The right side on the x-axis from the origin was assumed to be the Positive side (+), the Left side on the x-axis, the Negative side (-), the up side on the y-axis as Positive (+) and the downward on the y-axis as Negative side (-).

To denote a point on the plane, two things are required: One the distance on x-axis with directional sign (+ or -) and the other one the distance along y-axis with directional sign. **This ordered pair is called coordinates of the point and represented as (x, y)**.

x is called the abscissa part and **y** is called the ordinate part of the point. This is an ordered pair because the first part is always a distance on x-axis and the second a distance on y-axis. If some part is not there, it is denoted by zero (0).

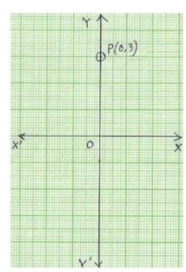
Example:3

The point **P (6, 0)** is on x-axis. It is on the right side of the origin. P covers +6 units distance on x-axis and 0 units distance on y-axis.



Example:4

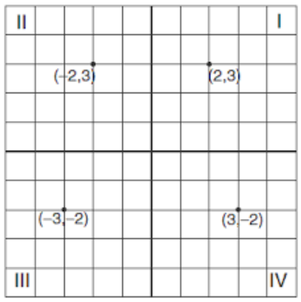
The point **P (0, 3)** is on y-axis. It is on the upper side of origin. The distance covered on x-axis is 0 and the distance on y-axis is +3 units above the origin.



OX is (+) side, OX' is (–) side. OY is (+) side and OY' is (-) side.

Example:5

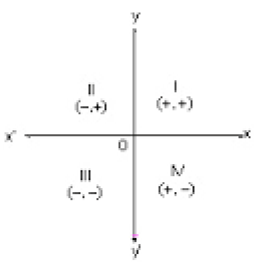
Points in different Quadrants with different signs show their positions.



Example:6

The signs of abscissa and the ordinates of the points in the corresponding Quadrants are shown below:

Ist Q (+, +); 2nd Q (-, +); 3rd Q(-, -); 4th Q(+, -)



SECTION FORMULAE

If the coordinates of the points A is (x_1, y_1) and B (x_2, y_2) then the coordinates of the point P dividing the line segment AB into the ratio m:n is given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

Here the point P lies inside the line segment AB. It is also called internal division.

If the point of division P is outside the line segment AB i.e. P is dividing externally, then n becomes negative and the coordinates of point P becomes as below:

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right).$$

DISTANCE BETWEEN TWO POINTS

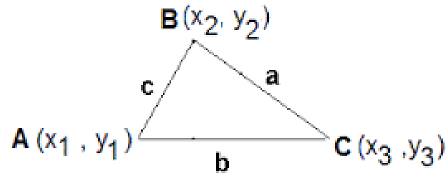
Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The distance is always positive (+).

AREA OF A TRIANGLE

Shoelace formula Or Gauss formula



The area of the triangle ABC

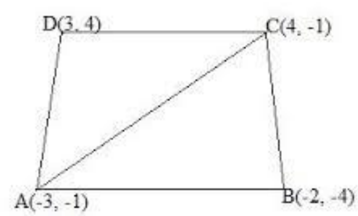
$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{bmatrix}$$

$$= \frac{1}{2}\left[x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3\right]$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

AREA OF A QUADRILATERAL



Area of a quadrilateral with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, $D(x_4, y_4)$ in continuation is given by

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

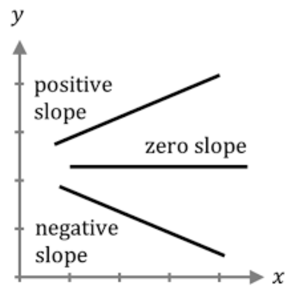
$$= \frac{1}{2}\left[x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4\right]$$

SLOPE OF A LINE

The slope of a line is defined as the ratio of the rise and the run of the point on a plane.

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

It is also called **gradient**. It is a real number. Its value can be anything, i.e. positive, zero or negative.

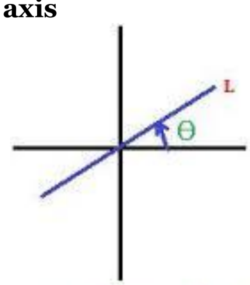


The slope is always calculated with respect to the positive direction of x-axis.

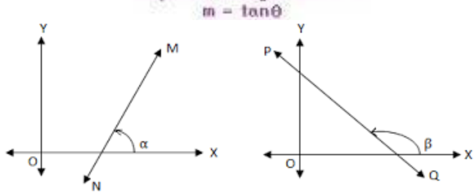
If the line is inclined with the positive side of x-axis in the angle θ, the value of the slope of the line is given by $\tan\theta$. In general, it is denoted by m.

The inclination and the slope are different. The inclination is the angle made by the line and the slope is the value of tangent of that angle made by the line with the positive side of the x-axis in the anticlockwise direction.

(a) Angle with axis



slope of straight line L is m = tanθ



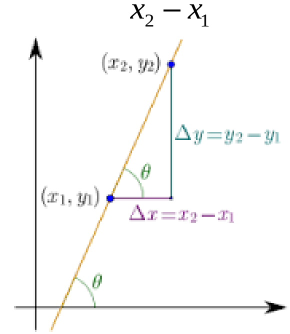
When the angle made by the line with the positive direction of x-axis in anticlockwise direction is an acute angle like α, the slope **tanα** is **positive**. When the angle made is obtuse as β, then the slope **tanβ** is **negative**.

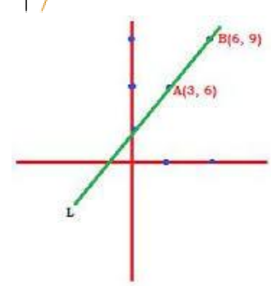
Examples:

1. If a line makes an angle of 60° with the positive side of x-axis in the anticlockwise direction, then the slope of this line will be tan 60°.
2. If a line makes an angle of 60° with the negative side of x-axis in clockwise direction, then the slope of the line will be tan (180°-60°) because the line is making 120° with the positive side of x-axis.
3. If a line makes an angle of 60° with the negative side of x-axis in anticlockwise direction, then the slope of the line will be tan (180°+60°).
4. If a line is making angle of 60° with the positive side of y-axis in anticlockwise direction, then the slope of the line will be tan (90°+60°).
5. If a line is making an angle of 60° with the positive side of y-axis in the clockwise direction, then the slope will be tan (90°-60°).
6. If a line makes an angle of 60° with the negative side of y-axis in the anticlockwise direction, then the slope of the line will be tan (180°+90°+60°).
7. If a line is making an angle of 60° with the negative side of y-axis in clockwise direction, then the slope of the line will be tan (180°+90°-60°).

(b) Two points given

When the value of the angle made by the line with axes is not known, and the coordinates of the two points on the line are known, then the slope is calculated as $m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$





Slope of a straight line

Slope of line L is given by :

$$m_{AB} = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-6}{6-3} = 1$$

It means the line makes an angle of 45^o with the positive side of x-axis in the anticlockwise direction, or the slope is 1.

(c) Equation of a line given

When the equation of a line is given, the slope is calculated by the following way:

Let the equation of the line be $ax+by+c=0$.

The slope is given by the value of $-\frac{coefficient\ of\ x\ with\ proper\ sign}{coefficient\ of\ y\ with\ proper\ sign} = -\frac{a}{b}$

Note:

(a) When the lines are parallel to each other, their slopes are equal.
(b) When the lines are perpendicular to each other, the product of their slopes is always equal to -1.
(c) When a line is parallel to x-axis, the slope is 0.
(d) When a line is perpendicular to x-axis, the slope is said to be undefined, but for calculation purposes, it is taken to be $\frac{1}{0}$ and not ∞.
(e) The three points are called to be collinear when the slopes of each of the twos are equal, i.e. A, B, C are collinear iff $m_{AB} = m_{BC} = m_{AC}$.
(f) The three points are called to be **collinear** when the area of $\Delta ABC=0$.
(g) The three points are called to be

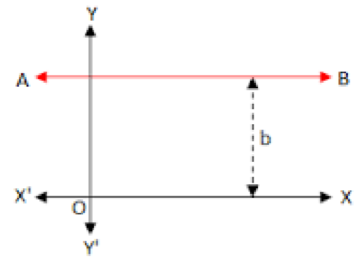
Collinear when either of the points divide the line segments made by the other twos externally or internally. For example, A, B, C are collinear if B divides the line segment AC, or A divided line segment BC, of C divides line segment AB.

EQUATION OF A LINE

Equation of a line is unique. The two conditions are required to determine the equation of a line. These two conditions may be

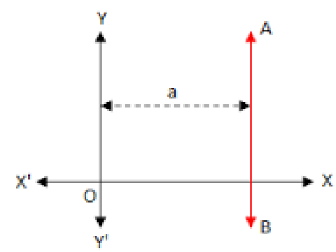
(a) Line is parallel to either of the axes.
(b) Line is passing through a point and its inclination or slope is known.
(c) Line is passing through two points.
(d) Line makes intercepts with axes.
(e) The length of perpendicular to the line from the origin and its inclination with positive side of x-axis in the anticlockwise direction is known.

(a) Equation of a line Parallel to x-axis



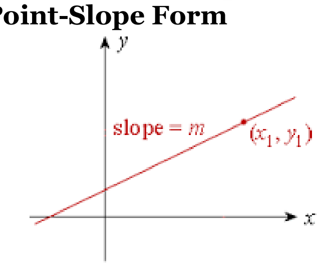
When the line parallel to x-axis is at a distance b from x-axis, the equation of such line is given by $y = b$. If b is in the positive direction, it is $y = +b$ and if it is in the negative direction, it is given by $y = -b$.

(f) Equation of a line Parallel to y-axis



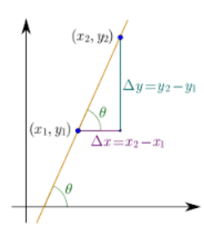
When the line is parallel to y-axis and it is at a distance a from it, the equation of this line is given by $x = a$. If this line is on the right side of y-axis, its equation is given by $x = +a$ and if it is on the Left side of y-axis, then it is given by $x = -a$.

(g) Equation of a line when a point on the line is given and the slope of the line is known Point-Slope Form



Let a point on the line be (x, y).
The slope of the line segment joining (x_1, y_1) and (x, y) will be $m = \frac{y-y_1}{x-x_1}$
Or, $(y - y_1) = m (x - x_1)$

Two-Point Form



If the line passes through the two points (x_1, y_1) and (x_2, y_2).

Then its slope is $m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

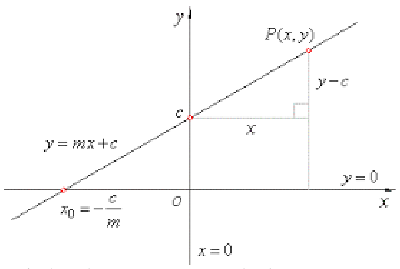
If a general point on the line be (x, y), then the slope of line segment joining (x, y) and (x_1, y_1) will also be $m = \tan\theta = \dfrac{y - y_1}{x - x_1}$

Hence $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Or, $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(h) Slope-Intercept Form

Case I: When the intercept with y-axis is positive



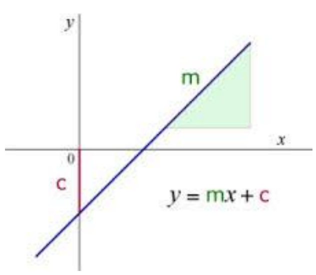
If the slope of the line is **m** and the intercept with y-axis is **c**, then $m = \tan\theta = \dfrac{y - c}{x}$

Or, ***y-c=mx*** or ***y=mx+c***

Note:

The intercept with x-axis in this case can be calculated by putting ***y* = *o*** in the equation, i.e., $x_0 = -\dfrac{c}{m}$. Here c and m will be with their proper signs.

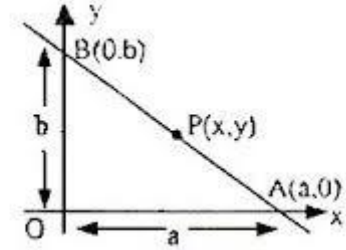
Case II: When the intercept with y-axis is negative



$m = \tan\theta = \dfrac{y + c}{x}$

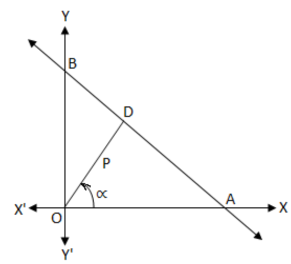
Or, ***y=mx-c***, where ***c*** is with proper sign i.e. below origin.

(i) Intercept Form



The line AB is passing through the two points A (a, o) and B (o, b). Using two points form of the equation of the line, it is $\dfrac{x}{a} + \dfrac{y}{b} = 1$. Here, intercepts ***a*** and ***b*** are with their proper signs.

(j) Normal Form



The equation of the line AB is in the form of p (the perpendicular distance from the origin to the line) and angle α made by the perpendicular with the positive side of x-axis in the anticlockwise direction is given by :

$$x\cos\alpha + y\sin\alpha = p.$$

Note: ***p*** is always taken to be positive as the distance is conventionally considered positive.

This equation is derived using two-point form or intercepts form.

OA is hypotenuse of the right angled ΔODA.

From ΔODA, ∠ODA=90°.

$$\cos\alpha = \frac{p}{OA} \Rightarrow OA = \frac{p}{\cos\alpha}$$

OB is the hypotenuse of the Right angled ΔODB.

From ΔODB, ∠ODB=90° and ∠OBD=α,

$$\sin\alpha = \frac{p}{OB} \Rightarrow OB = \frac{p}{\sin\alpha}$$

Using intercepts form, the equation of AB is given by

$$\frac{x}{OA}+\frac{y}{OB}=1 \Rightarrow \frac{x}{p/\cos\alpha}+\frac{y}{p/\sin\alpha}=1$$

Or, $\boldsymbol{x\cos\alpha + y\sin\alpha = p}$

GENERAL EQUATION OF A LINE

All the forms of the equation of a line are in One degree in x and y. We may say that One degree equation in x and y will represent always a line. This is the reason, one degree equation is also called linear equation.

The general equation of One degree is written as $\boldsymbol{Ax+By+C=0}$, where A, B, C are real numbers.

Conversion of $Ax+By+C=0$ in

(a) $\boldsymbol{y=mx+c}$ form: $y=-\frac{A}{B}x-\frac{C}{B}$.

(b) Intercepts form: $\frac{x}{(-C/A)}+\frac{y}{(-C/B)}=1$

(c) Normal form:

$$\frac{-Ax}{\sqrt{A^2+B^2}}+\frac{-By}{\sqrt{A^2+B^2}}=\frac{C}{\sqrt{A^2+B^2}}$$

Or, $x\cos\alpha + y\sin\alpha = p$ where

$$\cos\alpha=\frac{-A}{\sqrt{A^2+B^2}}, \sin\alpha=\frac{-B}{\sqrt{A^2+B^2}},$$

and $p=\frac{C}{\sqrt{A^2+B^2}}$.

Note: Keep $\boldsymbol{p}$ always positive and then evaluate value of angle α, otherwise the equation conversion will not be correct.

DISTANCE OF A POINT FROM A LINE

Distance of a point (x_1, y_1) from a line $\boldsymbol{Ax+By+C=0}$ means perpendicular distance of a point from the line. It is given by

$$d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

Note: distance should always be taken positive.

Distance between two parallel lines

Let the equation of the lines be $\boldsymbol{y=mx+c_1}$ and $\boldsymbol{y=mx+c_2}$.

$$d=\left|\frac{c_1-c_2}{\sqrt{1+m^2}}\right|$$

LOCUS AND EQUATION OF LOCUS

Locus is defined as the path traversed by the point.

The following rules are followed to find the equation of the locus of the point.

(a) Assume the point to be (h, k) whose locus is to be determined.
(b) Fulfill the conditions given for the movement of the point.
(c) Simplify the equation generated in step b.
(d) Replace ***h*** by ***x*** and ***k*** by ***y***.
(e) The equation in x and y is the required equation of the locus of the point.

Example 7:

Find equation of the locus of the centroid of the triangle whose vertices are (a *cost*, a *sint*), (b *sint*, $-b$ *cost*) and (1, 0); t being a parameter.

Solution:

Let the centroid be (h, k), then

$$h=\frac{a\cos t+b\sin t+1}{3}, \text{ and}$$

$$k=\frac{a\sin t-b\cos t+0}{3}$$

Or, $a\cos t+b\sin t=3h-1$, and $a\sin t-b\cos t=3k$

On solving these equations, we get

$$\sin t=\frac{3bh+3ak-b}{\sqrt{a^2+b^2}}, \text{ and } \cos t=\frac{3ah-3bk-a}{\sqrt{a^2+b^2}}$$

Using the identity,

$\sin^2 t+\cos^2 t=1$, we get

$$(1-3h)^2+9k^2=(a^2+b^2)$$

The required locus is found by replacing h by x and k by y as $(1-3x)^2+9y^2=(a^2+b^2)$

INTERSECTION OF TWO GIVEN LINES

Point of Intersection of two lines is calculated by solving the equation of the lines for x and y.

The coordinates of point is given by (x, y).

Let the lines be $\boldsymbol{a_1x+b_1y+c_1=0}$ and $\boldsymbol{a_2x+b_2y+c_2=0}$.

These lines will be

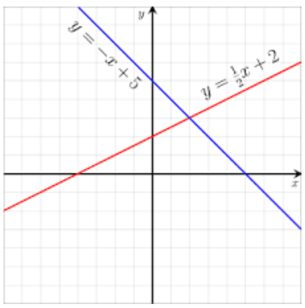
(a) Parallel and distinct iff $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

(b) Coincident iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) Intersecting iff $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Example:8

Find the point of intersection of the lines $y = \frac{1}{2}x + 2$ and $y = -x + 5$.



Obviously, x=2 and y=3. So the point of intersection is (2, 3).

Example:9

If the lines x+3y-9=0, 4x+by-2=0, and 2x-y-4=0 are concurrent, then find the equation of the line that passes through the point (b,0) and concurrent with the given lines.

Solution:

Given that the three lines

x+3y=9 ...(i)

2x-y=4 ..(ii)

4x+by=2 .(iii)

are meeting at one point. To find out this meeting point, let us solve equations (i) and (ii).

x=3, y=2. So, the point is (3, 2).

Line (iii) also passes through (3, 2) hence b=-5.

Now equation of the line that passes through (b, 0) or (-5, 0) and point of concurrency (3, 2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Or, $y - 0 = \frac{2-0}{3-(-5)}(x-(-5)) = \frac{2}{8}(x+5)$

Or, $4y - x = 5$

IMAGE OF A POINT

Image of a point means the plane mirror image.

For example, we know that the plane mirror image is always at the same distance from the object at which it is at a distance from the mirror.

We shall study here the three cases of the images:

(a) Image of a point about x-axis

Let a point be (2, 3). Obviously, this point is at a distance 2 from y-axis and at a distance 3 units from x-axis. If we want its image about x-axis, it means x-axis should work as a plane mirror, i.e., the point in First Quadrant will image in IV Quadrant. It means only the ordinate part of the coordinates will change to negative. Hence the image of (2, 3) in x-axis will be (2, -3).

Similarly, the image of (2, -3) will be (2, 3); image of (-2, 3) will be (-2, -3); image of (-2, -3) will be (-2, 3).

(b) Image of a point about y-axis

Image about y-axis means, y-axis will work as plane mirror. Hence the x coordinate of the point will change to negative.
For example:
Image of (2, 3) about y-axis will be (-2, 3); image of (-2, -3) will be (2, -3).

(c) Image of a point about a line

Consider a point (x_1, y_1) and the line be ***ax+by+c=0***. Here the line will work as a plane mirror. Let the mirror image of the point be (x', y'). The line segment joining (x_1, y_1) and (x', y') be perpendicular to the given line ***ax+by+c=0***.

The image coordinates can be calculated by the following formula:

$$\frac{x'-x_1}{a} = \frac{y'-y_1}{b} = -\frac{2(ax_1 + by_1 + c)}{a^2 + b^2}.$$

Note: *a, b, c, x_1, y_1* will be taken with proper sign.

IMAGE OF A LINE ABOUT X-AXIS

Let the line be ax+by+c=0. Its image about x-axis will be ax-by+c=0. Only y will change from positive to negative or vice-versa.

Image of a line about y-axis

Let the line be ax+by+c=0. Its image about y-axis will be –ax+by+c=0. Only x will change from positive to negative or vice-versa.

IMAGE OF A LINE ABOUT A LINE

Let the line be ax+by+c=0 and the line about which the image is to be found is Ax+By+C=0. If the reflected line is a'x+b'y+c'=0, then the line Ax+By+C=0 will be the internal bisector of the angle made by the lines ax+by+c=0 and a'x+b'y+c'=0.

This can be theoretically, very easily be calculated if we could find the two distinct points on the given line. By this we shall find out their images in two points and then a line passing through those two points.

Example:10

Find the distance from the origin to the image of (1, 1) with respect to the line x+y+5=0.

Solution:

First find the image of the point (1, 1) w.r.t the line x+y+5=0 using the formula

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\frac{2(ax_1+by_1+c)}{a^2+b^2}$$

Here (x, y) is the image of (x_1, y_1), or

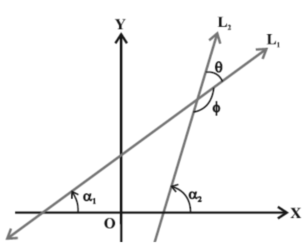
$x_1=1$, $y_1=1$, a=1,b=1,c=5

Hence $\frac{x-1}{1}=\frac{y-1}{1}=-\frac{2(1+1+5)}{1+1}=-7$

Or, x=-6, y=-6

Now the distance between (0, 0) and (-6, -6) is given by $6\sqrt{2}$ using distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

ANGLE BETWEEN TWO LINES



Consider the two lines L_1 and L_2.

L_1 makes α_1 angle with x-axis and L_2 makes α_2 with the positive side of x-axis.

Let slopes be $m_1=\tan\alpha_1$ and $m_2=\tan\alpha_2$.

The angle between L_1 and L_2 is either ϕ or θ as shown in the figure.

Obviously,

$$\alpha_1+\theta=\alpha_2 \Rightarrow \theta=\alpha_2-\alpha_1;\ \theta=180^0-\phi$$

Or, $\tan\theta=\tan(\alpha_2-\alpha_1)=\frac{\tan\alpha_2-\tan\alpha_1}{1+\tan\alpha_2\tan\alpha_1}$

$$=\frac{m_2-m_1}{1+m_2m_1} \qquad \text{...(i)}$$

And, $\tan\phi=\tan(180^0-\theta)=-\tan\theta=\frac{m_2-m_1}{1+m_2m_1}$

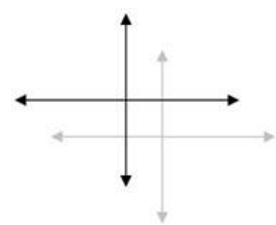
$$\Rightarrow \tan\theta=-\frac{m_2-m_1}{1+m_2m_1} \qquad \text{...(ii)}$$

On combining results (i) and (ii),

$$\tan\theta=\pm\frac{m_2-m_1}{1+m_2m_1}$$

SHIFTING OF ORIGIN OR CHANGE OF ORIGIN

Shifting of origin means the whole coordinate system is moved to some other point keeping the axes intact.



Some problems need shifting of origin to be solved comfortably. For this we follow the following rules:

(a) Let the new origin be (h, k)
(b) Let the new coordinates of the point be (x', y')
(c) Let the original coordinates of the point be (x, y)
(d) The inter-related formula is x'=x-h and y'=y-k.
(e) The new coordinates w.r.t the new origin (h, k) of the old point (x, y) will be (x-h, y-k)

Example:11

Find the new coordinates of the point (3, 5) when the origin is shifted to (1, -3).

Here, x=3, y=5, h=1, k=-3

Let the new coordinates of the point be (x', y')

x'=x-h, y'=y-k

Hence, x'=3-1=2 and y'=5-(-3)=8 i.e., (2, 8)

Example:12

Write the new equation of the straight line 4x+5y-7=0 when the origin is shifted to (4, 6).

Here, we have to write the equation in the form of x' and y'.

So, x=x'+h and y=y'+k

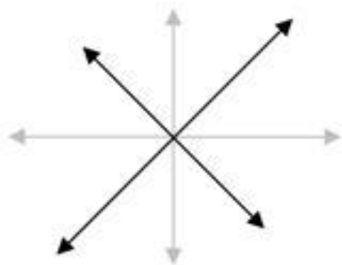
Or, x=x'+4 and y=y'+6

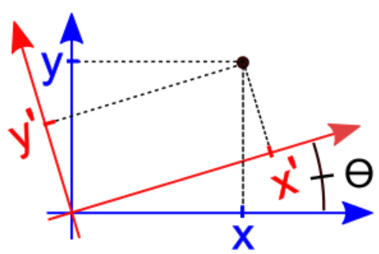
Hence 4x+5y-7=0 becomes 4(x'+4)+5(y'+6)-7=0⇒ 4x'+5y'+39=0

ROTATION OF AXES

Rotation of axes means the origin is at its point and the axes are rotated through some angle clockwise or anticlockwise.

Position of a point when the axes are rotated





Let the axes be rotated through an angle θ in anticlockwise direction. Let the original axes be blue lined and the new be red lined.

Let the old coordinates be (x, y) and the new coordinates be (x', y').

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

And

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

Example:13

If the coordinate axes are rotated through 45^0 in anticlockwise direction then write the new coordinates of (2, 6).

Solution13:

Given: x=2, y=6, $\theta=45^0$

$$x' = x\cos\theta + y\sin\theta = 2\cos 45^0 + 6\sin 45^0$$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2} + 3\sqrt{2}, \text{ and}$$

$$y' = -x\sin\theta + y\cos\theta = -2\sin 45^0 + 6\cos 45^0$$

$$= -2\left(\frac{1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2} + 3\sqrt{2}$$

i.e., the new coordinates will be $\left(\sqrt{2} + 3\sqrt{2}, -\sqrt{2} + 3\sqrt{2}\right)$.

Example:14

If the coordinate axes are rotated through an angle 30^0 about the origin, then transform the equation: $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$.

Solution14:

$x = x'\cos\theta - y'\sin\theta$ and $y = x'\sin\theta + y'\cos\theta$ where $\theta=30^0$

$x = x'\cos\theta - y'\sin\theta = x'\cos 30^0 - y'\sin 30^0$ or

$$x = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' - y'}{2}$$

$y = x'\sin\theta + y'\cos\theta = x'\sin 30^0 + y'\cos 30^0$ or

$$y = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' + \sqrt{3}y'}{2}$$

Replacing x and y in the given equation $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

We get, $\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 - 4\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right)$

$+\sqrt{3}\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 0$ Or, $\sqrt{3}y'^2 - x'y' = 0$

The shortest distance between two points is a straight line.

- **Archimedes**

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