Physics - Not a Subject to Learn

PREAMBLE: *In an effort to help students to Grow with Concepts, and necessity of mathematics and physics in building thought process, this sub-column on Physics has become relevant. Making a beginning of this column in 9th e-Bulletin, it would be our endeavour to address one concept from first principle in each Bulletin.*

Physics is a phobia for most of the students in non-mathematics stream and sometimes it is seen that Physics is apparently tough to even a few students of mathematics. It would not be incorrect to say that mathematics is abstract without physics and physics is science fiction without mathematics. With this premise and that mathematics is a language of solving problems, and not a problem, ***phobia for physics is evident if it is treated as a subject to learn****. Growth of physics, from predator age, has taken place due to advent of men to observe the physical world around them. In this world everything is necessarily –*

- *Identifiable for presence,*
- *Quantifiable and measurable in magnitude,*
- *It is possible to correlate cause and effect in terms of underlying variables.*

Galileo made this journey experimental, reproducible and verifiable, with equations of motion, a shift in paradigm from observational or intuitional learning of physics. In such a situation it is ironical to say "I am learning physics"; rather ***it requires visualization of every concept of physics in real life and in surrounding****. It is no exaggeration to say that manifestation of laws of physics can be realized even in human behaviour. Physics become intuitive as one grows with study of laws of physics and their correlation with the observation, and application to analyse the observations in the surrounding. As journey of growing with concepts proceeds, such behavioural coincidence shall be brought out to establish relevance. It will be observed that all across the understanding of physics there is consistency of mathematical rationality, the very foundation of Physics*

In this regard a beginning is being made with pre-requisites for understanding physics; it has been in common practice without taking its formal cognizance. In this chapter an attempt is made to share understanding from first principle, based on conceptual problems encountered during mentoring. It may not go in line with sequential treatment of the subject in text books or reference books, as journey proceeds. A play list of videos relevant to this chapter, at its end, can be used by interested readers based on their convenience.

1.0. **PREREQUISITES FOR UNDERSTANDING PHYSICS:** Any physical quantity has a magnitude and could be significantly different for different physical entities viz. radius of electron is $2.8179403267 \times 10^{-15}$ m, diameter of hair (17 to 181 μm), length of book 20 cm, distance between home and school is 1.5 km, distance between Kashmir to Kanyakumari is 3,884 km, radius of the Earth 6.37×10^{3}km, mean distance between the Earth and the Sun is 1.5×10^{8} km, while in astronomical scale distances are in millions of light years; here light year is the distance travelled by light in one year; velocity of light is 2.99792458×10^{8}m/s. Analysis of these magnitudes breaks down into few more concepts: **a)** Nature of quantity, and **b)** Magnitude of quantity – *i) Scalar Quantity and ii) Vector Quantity*. Quantities that have only magnitude are called **Scalar Quantity** viz. mass, length, distance, work, power, energy etc. while those quantities having both magnitude and direction, as essential parameters, are called **Vector Quantity,** viz. Acceleration, Force, Surface Area, etc. All arithmetic operations (+, - , × and ÷ are applicable on Scalar quantities. Whereas, Vectors quantities follow + and – operations as usual but × operation is applicable in different manner with restriction on ÷ operation. Understanding these operations on vector quantities is a part of vector algebra elaborated in Chapter G-03 in Common Section of the Mentor's Manual.

2.0 Measurement: Understanding of physics started and grown with observations; sharper the observation better is the understanding. This is the secret of all discoveries in Physics, be it any scientist. These observations unless quantified lack scientific objectivity and thus remain subjective to each observer, an unscientific approach. Scientific technique of quantification of observation is known as **Measurement**.

2.1 Quantification: This open up questions about many considerations in physics, viz. Units, Rounding of numbers, Least Count of Instrument, Significant Digits, Error in measurement, Computation of Errors, Order of magnitude, and Dimensional analysis. Each of them is in conformance to the basic principles of mathematics, studied upto this point, and this makes mathematics inseparable from physics.

2.2 Units: *The moment any observation is quantified it is either multiple or a fraction of some reference quantity called* ***Unit****, and similar quantity can be represented in multiple units each of the is scalable w.r.t. other.* This scalability of units for a quantity Q can be represented as: $Q = nu$, here u is the **unit** *the fixed reference quantity*, and n is the number of times the unit occurs in the quantity called **magnitude** *which is a real number.* Thus representation of a quantity in different system of units can be made as: $A = n_1 u_1 = n_2 u_2 = n_3 u_3 \ldots.$, here suffix 1, 2, 3.., represent system of units.

2.2.1 System of Units: Every society growing into a country has its own convention of representing a quantity. The convention used as a fixed reference of different set of quantities is called a system of units. As the understanding of physics grew there were three basic quantities Mass, Length and Time and these could be used to explain all phenomenon that were observed in nature until Nineteenth Century. It is only with understanding of heat, electricity, light more basic quantities were identified each with a different basic unit. Growth in understanding of physics lead to use of these basic units to explain other physical quantities called *derived quantities each having its own unit called derived unit*, viz.

Area $=$ Length $(l \text{ in cm}) \times$ Breadth $(b \text{ in cm}) = lb \text{ cm}^2$,

Density $= \frac{\text{Mass } (m \text{ in gm})}{\text{Voulme } (v \text{ in cm}^3)} = \left(\frac{m}{v}\right) \text{ gm. cm}^{-3}$

Speed $= \frac{\text{Distance } (s \text{ in m})}{\text{Time } (t \text{ in sec})} = \left(\frac{s}{t}\right) \text{ m. sec}^{-1}$

Broadly, there were three system of units on predominant use viz. FPS (**F**oot, **P**ound and **S**econd), CGS (**C**entimetre, **G**ram and **S**econd) and MKS (**M**eter, **K**ilogram and **S**econd). In 1970 an International System of Units, called SI, was introduced . It has Seven basic units with their symbols and dimensions as shown in Table I. Each basic unit is independent and assigned an independent dimension which has found a great usefulness in deriving and/or verifying equations or relations of derived physical quantities. This shall be elaborated in para 5 *Dimensional Analysis* later in this chapter. There are Two basic dimensionless quantities viz angle and solid angle as shown in Table II, with their units defined. There are few quantities which are both dimensionless and unit-less viz. relative density, refractive index and many more that shall be encountered in the forward journey into physics.

Table I: Basic and Fundamental Dimensions System of International Units

Basic Unit	SI Unit Symbol	Measure	Dimension Symbol
meter	m	Length	L
kilogram	kg	Mass	M
second	s	Rime	T
ampere	A	Electric Current	I
kelvin	K	Temperature	θ
mole	mol	Amount of Substance	N
candela	cd	Luminosity	J

Table II: Dimensionless Basic Units

Basic Unit	SI Unit Symbol	Measure
Radian (α)	rad	Angle
Steradian (Ω)	sr	Solid Angle

SI Units have certain characteristics which make them widely acceptable viz. (a) Rational : One unit for one quantity viz Joule is for energy of any form viz. mechanical, heat, electrical. (b) Coherent: Every unit can be derived or expressed from basic or fundamental units and (c) Metric: It is based on decimal system which is internationally acceptable.

Units of same type are scalable and Conversion Factor (K_{ij}) for a unit from i^{th} system of units into j^{th} system of units can be expressed as . $K_{ij} = (k_a)^p (k_b)^q (k_c)^r$ …..Here, say a physical quantity X in i^{th} system has a unit is unit derived from constituent units a, b, c….. such that $1 \cdot a_i \equiv k_a \cdot a_j$, $1 \cdot b_i \equiv k_b \cdot b_j$, $1 \cdot c_i \equiv k_c \cdot c_j$ and expressed as $X_i = (a_i)^p (b_i)^q (c_i)^r$ and same quantity in j^{th} system has a unit $X_j = (a_j)^p (b_j)^q (c_j)^r$. This is applied to two example .

Example 1: Convert speed of 120 kmph ($\text{km} \cdot \text{h}^{-1}$) into meter-per-sec ($m.s^{-1}$). Then for distance component in the unit ratio is $1.\text{km} \equiv 1000.\text{m}$, hence $k_a = \frac{\text{km}}{\text{m}} = \frac{a_i}{a_j} = 1000$ and for time component of unit $1 \cdot \text{hr} \equiv 3600 \cdot \text{s}$. Hence $k_b = \frac{\text{hr}}{\text{s}} = \frac{b_i}{b_j} = 3600$. Therefore, $K_{ij} = (k_a)^1 (k_b)^{-1} = (1000)^1 (3600)^{-1} = \frac{5}{8}$. Thus $120 \text{ kmph} \equiv 120 \times \frac{5}{8} = 75 \text{ m} \cdot \text{s}^{-1}$.

Example 2: Convert density $1{,}500 \text{ kg} \cdot \text{m}^{-3}$ in MKS into CGS i.e. $\text{g} \cdot \text{cm}^{-3}$. Here, for mass the unit ratio is $1.\text{kg} \equiv 1000.\text{gm}$, hence $k_a = \frac{\text{kg}}{\text{g}} = \frac{a_i}{a_j} = 1000$ and like wise basic dimension of volume unit ratio is $1.\text{m} \equiv 100.cm$, hence $k_b = \frac{\text{m}}{\text{cm}} = \frac{b_i}{b_j} = 100$. Therefore, $K_{ij} = (k_a)^1 (k_b)^{-3} = (1000)^1 (100)^{-3} = 10^{-3}$. Accordingly, $1500 \text{ kg} \cdot \text{m}^{-3} \equiv 1500 \times 10^{-3} g \cdot \text{cm}^{-3} = 1.5\, g \cdot \text{cm}^{-3}$.

2.2.2 Rounding of Numbers: In calculations conversion of a fraction in metric form is often encountered with either terminating decimal places more than that of relevance or non-terminating decimal places. This is where the result is required to be rounded to a certain place of digit which can be either for decimal number or integers. Principle of decision on place value of a digit to be rounded is discussed while elaborating Significant Digits at para 1.2.5. Principle of rounding of a number is elaborated here with basic rules –

Rule 1: Typically if requirement of answer is Two places of digits, then a third, a lower digit is considered, for rounding lowest digit; in instant case it is Two.

Rule 2: If third digit is *greater than five* the second digit then Second digit is increased by 1 the result is reported or used, e.g. $4.38 \approx 4.4, 4.86 \approx 4.9$.

Rule 3: If third digit is *less than five* the second digit then Second digit is remains unchanged and the result is reported or used, e.g. $4.34 \approx 4.3, 4.82 \approx 4.8$.

Rule 4: If third digit is *equal to five* then it leads to Three cases-

(a) If *decimal form of calculation has only three digits* and *Second digit is an odd number* then it is increased by 1 and the result is reported or used, e.g. $4.55 \approx 4.6, 4.35 \approx 4.4$.

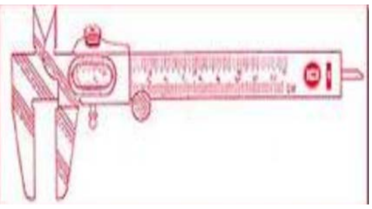
(b) If *decimal form of calculation has only three digits* and *Second digit is an even number* then it remains unchanged and the result is reported or used, e.g. $4.45 \approx 4.4$, $4.85 \approx 4.8$

   i. If *decimal form of calculation has more than three digits* and then *Second digit, even if it is an even ,* it is increased by 1 and the result is reported or used, e.g. $4.451 \approx 4.5$, $4.850001 \approx 4.9$. Rational of this sub-rule shall become clear during elaboration of Significant Digits in para 2.2.4.

2.2.2.1 Rule of exception: There is another situation where one wishes to distribute pencils to 34 deprived students, and has a discounted offer only on pencil box each containing 10 pencils. While extending generosity it is obvious for the person to ensure no child is let out. In this situation the practical choice is to purchase 4 pencil boxes, be it even at extra cost, and leave surplus 6 pencils either for use of students in future or distribute additional pencil to most deserving students. This is engineering approach of handling a problem and just not numbers.

2.2.3 Least Count of Instrument: It is the smallest quantity that is measurable with available instrument. The most accessible instrument is scale for

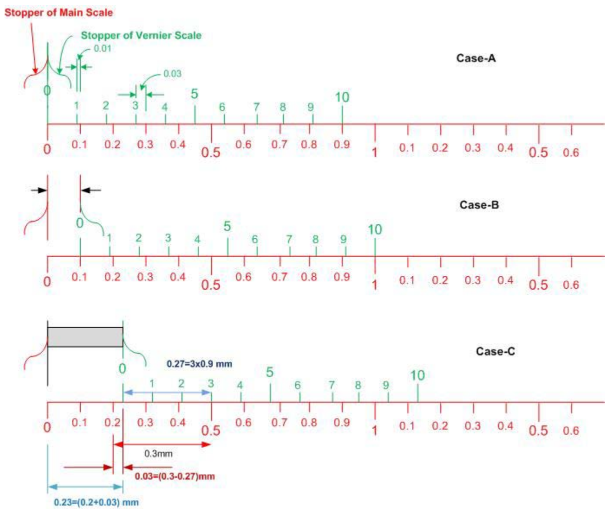
measurement of length. A typical centimetre scale is shown in the figure which can accurately measure length in steps of 0.1 cm or 1 mm. If an object has a length such that one end matches with Zero of scale and other anywhere between mark A or B. In this case as per rules of rounding discussed above, the measured length (l) shall be recorded as $1.75 \leq 1.8 \leq 1.85 \approx 1.8$ cm. This applies to equally any other measurable scale and, therefore, maximum error that can occur is equal to least count. Computation of error in an experiment involving multiple quantities being measured with each involving different least count is being elaborated in para 1.2.6 Estimation of Error. Subsequent development lead to evolution of Vernier Calliper. Here, typically a *Vernier scale* (VS) is devised such that its Ten equal parts (V) are equal to 9 parts (M) of *Main scale* (MS). Such $10\text{ V} = 9\text{ M} \rightarrow 1\text{M} - 1\text{V} = 0.1\text{M}$. Thus Least Count for a Centimetre Main Scale in this case is $0.1 \times 0.1\text{cm} = 0.01$ cm. This kind of proportioning of divisions of Vernier is scale achieved by simple geometrical method of drawing a fraction on a number line. Typical Vernier Calliper is shown in the figure. Principle of Vernier Scale is explained with Three cases in the figure below.



In case-A Zeros of MS and VS are matching this created a shift of every division of VS towards left of the corresponding division of MS and the shift is of the order of $n \times LC$. Typically for division 1 of VS it is 0.01 cm, for division 3 it is 0.03 cm and ultimately for division 10 of VS it is 0.1 cm and thus it matched with 0.9 cm mark on MS.

In case-B mark 10 of VS is matched with 10 of MS and the shifts of every mark of VS are similar but in reverse direction, such that division 10 of VS matched with 0.1 cm of MS.

In case-C while measuring a length Zero of VS is between 0.2 cm and 0.3 cm of MS, and this is where utility of VS comes in. It is seen that Division 3 Matches with 0.5 cm of MS, then in accordance with Case B discussed above shift of 0 of VS beyond 0.2 cm of main scale is $= (0.5 - 0.2) - 3 \times 0.09 = 0.3 - 0.27 = 0.03$ cm. Therefore, actual length being measured is $= 0.2 + 0.03 = 0.23$ cm.



There could be situation where none of the divisions of VS match with any mark on MS and this involves application of Rounding of numbers discussed at para 1.2.4 Significant Digits.

Principle of VS has been extended to measurement of higher precision in the form of *Screw Gauge* also called *Micrometer* shown in figure. These are available in different geometry to suit particular application. In this there are two scale One Circular scale and other axial scale such that *Circular scale moves axially one division on completion of one revolution and this distance is called **pitch** of instrument.* Longer perimeter of circular scale permits larger number of legible divisions and thus decreases of least count and thereby increase in the *precision* of the instrument discussed at para 1.2.5.

2.2.4 Significant Digits (SD): Elaboration of SD in metric system is being made. It is the most practical aspect in measurement taking two types of metre scale. **Case 1**: It has graduation markings of cm and millimetre all along. **Case 2:** It has markings at an interval of 10 cm viz, 0, 10, 20…100 over the entire length. In addition it has centimetre marking 0, 1, 2..10 on first 10 cm length of the scale. Now, for a length measured to be 7.3 cm there is no sense in writing 07.3 cm. The leading zero is insignificant. Nevertheless, recording of this measurement as 7 cm is incorrect with First scale which has a capability to measure accurately upto 1 mm, while recording 7.0 cm with Second scale is incorrect since it cannot record upto decimal places of a 1 cm. Accordingly, following rules have been formulated for rationalizing SDs. Each rule is supported with an example where SDs are underlined.

Rule 1: In case of integers -

- **i.** All leading zeros, i.e. zeros to the leftmost non zero digits are insignificant, viz. 00<u>37</u>.
- **ii.** All trailing zeros, i.e. zeros to the rightmost non-zero digit are insignificant, viz. <u>24</u>000.
- **iii.** All zeros sandwiched between significant digits are significant, viz. <u>4147</u>, <u>400167</u>, <u>4010039</u> have 4, 6 & 7 SDs respectively.

Rule 2: In case of decimal number (metric system) -

- i. If number is < 1 then -
  - a. All zeros between decimal and leftmost non-zero digit are insignificant, viz. 0.00<u>56</u>, i.e. there are 2 SDs.
  - b. All trailing zeros after last are non-zero digit are significant, viz. 0.00<u>560</u>, i.e. there are 3 SDs.
- ii. If number is ≥ 1 then –
  - a. All zeros leading leftmost non-zero digit are insignificant, but all trailing zeros are significant, viz. 000<u>20.0</u> has 3 SDs respectively (this is in conjunction with Rule 2.i.b above).

2.2.4.1 Least and Most Significant Digits: No instrument can give exact quantity to be measured. This is an implementation aspect of the Rules of SDs. Accordingly, the *leftmost significant digit is called **Most Significant Digit (MSD)** or most certain digit***,** while, *rightmost significant or most certain digit is called **Least Significant Digit (LSD),** or most doubtful or uncertain digit. Eventually, **place value of LSD is the least count of measurement or maximum permissible error**.* Let us compare five typical length with measuring tools shown in the **Table-III** at the end of the chapter, using measurements articulated for a practical view. None of the instrument can give an exact quantity to be measured and accordingly error in measurement is defined. *Estimation of Error* an important role in journey of physics and is elaborated in Para3.

2.2.4.2 Arithmetical Operation and SDs: There is a famous phrase–"*strength of a chain is that of its weakest link*". Likewise, reliability of a calculation is of the order of minimum SDs among the operands. Arithmetic operations may involve measurement taken with different instruments having different least count and thus require rationalization of the results such that they are reliable. In light of this application of principles of SD in arithmetic operations have been simplified into a set of rules as under, and its applications are shown in **Table-IV.** This principle is in use in electronic machines and can be verified on MS Excel, which is readily available, with a care that **Round Function** is used.

Rule 1: Sequence of arithmetic operations are as per BODMAS RULE.

Rule 2: *Addition and Subtraction* (Column 2 & 3 in Table IV) -

a. Convert all operands into Decimal form by padding trailing Zeros to match the operand with largest number of decimal places, among addends or subtrahends.
b. Perform Addition and Subtraction operation as usual.
c. Report result in minimum number of decimal places among operands.

Rule 4: Division and Multiplication(Column 2 & 3 in Table IV):

a. Convert all quantities in scientific notation, retaining LSDs (Column 2 & 3 in Table IV).
b. Perform Multiplication and Division as usual.
c. Report result in least precise operand, having smallest SDs be it addition/subtraction, multiple/division or compound arithmetic operation at every stage of BODMAS rule.

Note: 1. It is pertinent to highlight that among SDs, the LSD is unreliable being a result of rounding, while rest of the digits in SDs are reliable digits (RDs). This leads to a relationship $\mathrm{SDs} = \mathrm{RDs} + 1$.
2. During arithmetic operations if unit of some quantity is required to be converted the number of SDs remain unchanged.
3. Constants are treated to have infinite SDs. Therefore, calculations involving a constant, rules of SDs of variable are applied e.g. $d = 2r$, SD for r shall apply to d.
4. In calculations involving multiple steps one extra uncertain digit is retained and only while reporting final result rules of SD are applied.
5. This error analysis is simplified using differential calculus.

2.2.5 Accuracy and Precision: These are two different parameters. Taking an example to understand difference between accuracy and precision related to an example of assessment of human crowd by three set of persons viz. P, Q and R. Actual strength of crowd is 1000 persons. The Set P makes an assessment of 1100, 1200, 1300, 1400 persons in the crowd, *averaging out to 1250*, while set Q assesses 900, 1000, 1100, 1200 persons, *averaging out to 1050*. The third set R makes an assessment 950, 1000, 1050, *averaging out to 1000*. Assessment made by set P and Q are equally precise in hundreds but set Q is more accurate since it is more close to the actual size of the crowd. But, the set R is more precise than P and Q since assessment is in a deviation is in lower steps which are 50, as compared to that of P and Q having a larger step size i.e. 100. But, R is most accurate among the three sets, being closest to actual. Thus, observation set R is both accurate and precise. But, this example is of an assessment of crowd by just an observation and no measurement is involved. Hence, the example is good for understanding the concept but, does not approach in physics.

In another example involving measurement an approach in physics, a man of 72 kg is weighed on three different machines with three sets of observations: machine A weighs 70, 70.5, 70.3 kg, averaging to 70.3 kg; machine B weighs 72.9, 73, 73.1 Kg averaging to 73.0 kg; while machine C weighs 72, 72.1 72.2 kg, averaging to 72.1 Kg. All the three machines are equally precise in measuring weight upto first decimal place i.e. 70.3, 73.9 and 72.1 Kg respectively . But, machine C measures weight closest to the actual weight is more accurate.

This infers that ***precision*** *is about close conformance of measurement on repetitive measurement*; while ***accuracy*** *is about close conformance of measurement to the actual value*.

3.0 Error Estimation: Any observation, howsoever accurate it may be, is bound to be a subject of: **a)** instrumental error, **b)** observational error, **c)** manual error, **d)** time dependent variation, **e)** environmental error etc. All these errors are real life propositions, yet uncertain and if they are ignored would lead to unreliability of studies which may contradict the basic premise of growth of concepts. While, there are other techniques to handle different types errors, here emphasis is only on error caused by scale of instrument.

3.1 True Value, Absolute Error & Percentage Error: In handling errors, an important prerequisite in journey of physics, defining, these three terms is essential. Let any physical quantity A is to be measured, repetitive observations are carried out to arrive at average value of the quantity $a_m = \frac{\sum_1^n a_i}{n}$, here $a_1, a_2, a_3 \dots a_n$ form a set of n observation, which would confirm the premise of error a reality. This a_m is treated as ***true value*** for all practical purposes. There error in i^th measurement is $\Delta a_i = a_i - a_m$, it may be (+) ve or (-)ve depending upon $a_i > a_m$ or $a_i < a_m$. Since, $(\pm)$ve errors are distributed around a_m, accordingly $a_m = a_i - \Delta a_i$ and is independent of error. Therefore, ***absolute error*** is defined as $\Delta a = \frac{\sum_1^n |\Delta a_i|}{n}$ and it needs to be carefully noted that in this expression absolute value i.e. unsigned values of error are taken. But, ***relative error*** is defined as $\delta a = \frac{\Delta a}{a_m} \times 100 = \pm x\%$, where $x = \frac{\Delta a}{a_m} \times 100$ a numerical value. Here, important observations that need to be made are – **a)** notation for percentage error is different from absolute error, **b)** it is calculated as relative to true value, **c)** it is expressed in the form of $\delta a = \pm x\%$. Relative error is important when magnitude of true value differs widely viz. diameter of marble (2 cm) hair and diameter of a football (22 cm) measured with same scale having *least count of 1mm.*

3.2 Algebra of Error Estimation: It is seen that during mathematical operation error are additive and hence it is also called propagation of errors.

3.2.1 Error in Addition and Subtraction: Let error in sum two similar quantities is to be determined given that they are $a = x \pm \delta a\%$ and $b = y \pm \delta b\%$, here and x and y are measured values from two different sources, while $\delta a\%$ and $\delta b\%$ are percentage errors in the two measurement, based on available information. Then error in the required sum is $a+b=(x\pm\Delta a)+(y\pm\Delta b)$ or $a+b=(x+y)\pm(\Delta a+\Delta b)\rightarrow\delta(a+b)\%=\pm\left(\frac{\Delta a+\Delta b}{x+y}\right)\times 100$. Likewise, $a-b=(x\pm\Delta a)-(y\pm\Delta b)$, which implies that $a-b=(x-y)\pm(\Delta a+\Delta b)$. But, the order relative error is $\delta(a-b)\%=\pm\left(\frac{\Delta a+\Delta b}{a-b}\right)\times 100$. This inference might look contradictory, but is based on the logic of uncertainty of error. It is to be noted that absolute errors of respective operands are added and then relative error is calculated as percentage of sum or difference of measured values and then prefixed with $\pm$sign. The basic logic behind it is the uncertainty of error. In a particular case it may be compensating among associated operands or may be summative and thus maximum possible error is arrived at. Accordingly, a generic formula for error during summation is$(\sum a_i)\%=\sum a_i\,\%$ and during difference of quantities $\delta(a_i-a_j)\%=\frac{(\Delta a_i+\Delta a_j)}{a_i-a_j}\times 100$, *where individual relative error is first converted into absolute error.*

3.2.2 Error in Multiplication and Division: Taking a case $c=a\times b$, given that $a=x\pm\Delta a$ and $b=y\pm\Delta b$. Therefore, the product is $c=z\pm\Delta c=(x\pm\Delta a)(y\pm\Delta b)=xy+x\cdot\Delta b+y\cdot\Delta a+\Delta a\cdot\Delta b$. In its final form It is the absolute error is approximated to $\Delta c\approx x\cdot\Delta b+y\cdot\Delta a$. Here, the term $\Delta a\cdot\Delta b$ is ignored being insignificant, since it is product of two small quantities; errors which are always small. Thus, relative error $\delta c=\frac{\Delta c}{c}=\frac{x\cdot\Delta b}{xy}+\frac{y\cdot\Delta a}{xy}$. It makes it convenient to represent relative error as $\delta c=\frac{\Delta b}{y}+\frac{\Delta a}{x}$ which can also be written as $\delta c\%=\delta a\%+\delta b\%$ …… i.e. summation of percentage of all the multiplicands.

Likewise, taking $p=\frac{q}{r}$, given that $q=x\pm\Delta q$ and $r=y\pm\Delta r$. Then $p\pm\Delta p=\frac{x\pm\Delta q}{y\pm\Delta r}=\frac{(x\pm\Delta q)(y\mp\Delta r)}{y^2-(\Delta r)^2}=\left.\frac{(x\pm\Delta q)(y\mp\Delta r)}{y^2}\right|_{(\Delta r)^2\to 0}$. It leads to $p\pm\Delta p=\frac{xy\pm x\Delta r+y\Delta q}{y^2}=\frac{x}{y}\pm\frac{\Delta q}{x}\left(\frac{x}{y}\right)\mp\frac{\Delta r}{y}\left(\frac{x}{y}\right)$. In case of maximum possible error it comes to $\frac{\Delta p}{p}=\pm\left(\frac{\Delta q}{x}+\frac{\Delta r}{y}\right)\rightarrow\delta p\%=\delta q\%+\delta r\%$.

Likewise, for $p=q^n$, then extrapolating the logic of error in multiplication it comes to $\delta p\%=n(\delta q\%)$. *Here, individual absolute error is first converted into relative error.*

3.2.3: Illustrative Examples: This is a point complete logic of quantification, involving significant digits, their mathematical operations, computation of error and its estimation of overall error, all consolidated for application in an example, given below.

Example: A typical pendulum is used to determine value of acceleration due to gravity g, where $T=2\pi\sqrt{\frac{7(R-r)}{5g}}$, here, T is time period of pendulum and R and r are radii.

Data:

a. Time Period $\{0.52, 0.56, 0.57, 0.54, 0.59\}$ is in a set of observations with least count of stopwatch used in observations is 0.01 sec.

b. Radii $R=60\pm 1$ mm and $r=10\pm 1$ mm

Solution: $\boldsymbol{T}=\frac{\mathbf{0.52+0.56+0.57+0.54+0.59}}{\mathbf{5}}=0.556\approx 0.56$, 1st level rounding of mean (actual) time period to LSD of data.

Or, $\Delta T=\frac{|0.56-0.52|+|0.56-0.56|+|0.56-0.57|+|0.56-0.54|+|0.56-0.59|}{5}$. It leads to $\Delta T=\frac{0.04+0+0.01+0.02+0.03}{5}=\frac{0.1}{5}=0.02$, there $\delta T\%=\frac{\Delta T}{T}\%$, which comes to $\delta T\%=\frac{0.02}{0.56}\times 100=3.57\%$

For the Two radii $\delta R\%=\frac{1}{60}\times 100=1.67\%$ and $\delta R\%=\frac{1}{10}\times 100=10\%$, but in given formula it comes as $(R-r)$ and therefore, $\Delta(R-r)=\Delta R+\Delta r$ therefore $\delta(R-r)=\frac{\Delta R+\Delta r}{R-r}=\frac{1+1}{50}$. Thus $\delta(R-r)\%=\frac{2}{50}\times 100=4\%$.

Since, error is to be estimated in calculated value of g the equation is transformed for the purpose $g=\frac{28\pi^2}{5}\cdot\frac{(\boldsymbol{R-r})}{\boldsymbol{T^2}}$, and relative error $\delta g\%=\delta(R-r)\%+2\times\delta T\%=4.00\%+2\times 3.57\%=4.00\%+7.14\%$. It works out to $\delta g\%=11\%$, conforming to arithmetic SDs in the given data.

4.0 Scientific Notation: In this any number is expressed as $\boldsymbol{a}$[Decimal].$\boldsymbol{b}$x10^n; here, $1\leq\boldsymbol{a}\leq 9$, $0\leq\boldsymbol{b}\leq 9999$ ….[any number of digits based on precision of observations] , and $-999999\ldots\leq\boldsymbol{n}\leq 9999$..[any number of digits based on order of magnitude brought out at para 1.3.1]. Scientific notation has advantage of retaining degree of precision viz. exact count of <u>17</u>00 persons on the principle of Significant Digits (SDs), also called Significant Figures (SigFigs), is 2 resulting into an approximation to hundreds, while in scientific notation it is <u>1.7</u>x10^3 retains precision in count of person.

4.1 Order of Magnitude: Various magnitudes stated above are in different order but representation of radius of electron, radius of the earth, mean distance between sun to earth and velocity of light are expressed in scientific notations as $\boldsymbol{a}.\boldsymbol{b}\times 10^n$. Here '$\boldsymbol{a}$' is a digit such $1\leq\boldsymbol{a}\leq 9$, '$\boldsymbol{b}$' is number, having any number of digits depending on accuracy of measurement, and '$\boldsymbol{n}$' is an integer and is called **order of magnitude**. If $1\leq a<5$ then order of magnitude is n, and if $5\leq a\leq 9$ then order of magnitude is $n+1$. Nomenclature for order of magnitude as per SI Units is listed in **Table-5.** In actual use symbol of unit of quantity to be expressed symbol is suffixed to both SI symbol and prefix viz. mm, cm, mg, kg etc.

Table 5 : Unit Multipliers as per SI Units

Sub-multiplier				Multiplier		
SI Symbol	**Value**	**Prefix**		**SI Symbol**	**Value**	**Prefix**
d	10^{-1}	deci		da	10^1	deca
c	10^{-2}	centi		h	10^2	hecto
m	10^{-3}	milli		k	10^3	kilo
µ	10^{-6}	micro		M	10^6	mega
n	10^{-9}	nano		G	10^9	giga
p	10^{-12}	pico		T	10^{12}	tera
f	10^{-15}	femto		P	10^{15}	peta
a	10^{-18}	atto		E	10^{18}	exa
z	10^{-21}	zepto		Z	10^{21}	zetta
y	10^{-24}	yocto		Y	10^{24}	yotta

5.0 Dimensional Analysis : It is defining dimension of a quantity in terms of its basic or fundamental quantities. As per System if International Units (SI) basic dimensions with their dimensions have been listed in Table-I at Para 2.2.1. These basic dimensions are used to obtain compound or derived dimension with some typical quantities, viz. Area =[Length]x[Breadth]=L^2]; since both length, breadth and height, being of same nature, have dimension L while $[Volume] = L^3$,

$[Velocity] = \frac{[Displacement]}{[Time]} = LT^{-1}$, $[Speed] = \frac{[Distance]}{[Time]} = LT^{-1}$

$[Acceleration] = \frac{[Change\ in\ Velocity]}{[Duration\ of\ Change\ , in\ time]} = LT^{-2}$

$[Force]$ =, $[Mass] \times [Acceleration] = MLT^{-2}$

$[Work]$= $[Force] \times [Displacement\ in\ the\ Direction\ of\ Force]$= ML^2T^{-2}

$[Power] = \frac{[Work\ Done]}{[Duration\ of\ Work,\ in\ Ttime]} = ML^2T^{-3}$,

$[Energy] = [Power]x[Duration\ of\ Use\ of\ Power] = ML^2T^{-2}$.

This dimensional analysis is used to define every physical quantity in terms of basic dimensions and shall be an integral part of forward journey in physics. This dimensional analysis is extremely useful in correlating different types of variables involved in a physical phenomenon. This shall dealt with appropriately in the forward journey into Physics. Dimensional analysis also finds application in deriving relationship of a physical quantity involving more than multiple physical quantities. So also it is useful in verifying conformity of an equation involving multiple addend/subtrahend. It has been formalized with following rules –

Rule 1: Quantities having same dimension can only be added or subtracted.
Rule 2: Quantities having different dimensions only can be multiplied or divided.
Rule 3: Dimensionless constants viz. Numbers, Pi, refractive index, relative density etc. And dimensionless variables viz. Trigonometric ratio have no role in dimensional analysis.
Rule 4 - Buckingham Pi Theorem: It states that "If there are n variables in a problem and there variables contain m primary dimensions (for example M, L T), the equation relating all the variables will have (n-m) dimensionless groups."

5.1 Application of Dimensional Analysis: It is being illustrated with a set of typical examples.

5.1.1 Consistency of Equation: Given that $S = ut + \frac{1}{2}at^2$. The equation to be consistent it is necessary that $[S] = [ut] + [at^2]$. This is First Galileo's equation of Motion where, displacement $[S] = L$, initial velocity $[u] = LT^{-1}$, acceleration $[a] = LT^{-2}$, and time $[t] = T$. Therefore, $L = (LT^{-1}) \cdot T + (LT^{-2}) \cdot T^2 \rightarrow L = L + L$, i.e. all terms are dimensionally consistent.

5.1.2 Determination of Dimension: Given that $U = \frac{A\sqrt{x}}{x^2+B}$, here U is potential energy whose dimension is ML^2T^{-2}, which is function of its distance travelled $[x] = L$, while A and B are dimensional constants. It is to determine $[AB]$. As per dimensional consistency $[x^2 + B] = [x^2] = [B]$, but $[x^2] = [x]^2 = L^2$, hence $[B] = L^2$. Further, $[\sqrt{x}] = [x]^{\frac{1}{2}} = L^{\frac{1}{2}}$. Using these dimensions $[U] = \frac{[A\sqrt{x}]}{[x^2+B]} \rightarrow ML^2T^{-2} = \frac{[A]\cdot L^{\frac{1}{2}}}{L^2} = [A] \cdot L^{-\frac{3}{2}}$. It leads to $[A] = (ML^2T^{-2}) \cdot L^{\frac{3}{2}} = ML^{\frac{7}{2}}T^{-2}$. Therefore, $[AB] = (L^2) \cdot (ML^{\frac{7}{2}}T^{-2}) = ML^{\frac{11}{2}}T^{-2}$.

5.2 Dimensional Exceptions: It may be noted that dimensions of velocity and speed are same. Likewise, different quantities viz. work and torque also same dimensions. As study of physics proceeds different physical quantities shall be encountered and at that point dimensional analysis of such new quantities shall carried out as an integral part of conceptual study.

6.0 Conclusions: *Mechanics, being realizable in immediate surroundings, is the first step in study of Physics. Moreover, measurements form premise for all scientific discoveries. Mechanics would find its applications in other branches of physics viz. heat, electricity, magnetism, sound and optics. But, these basics, more than often are ignored, in pursuit of further journey into science. A good amount of practice is required to make the quantification of measurement intuitive, which is, otherwise, prone to error.* ***Such errors, despite correct applications of laws of physics and calculations may lead to error in answer or results and thus retard growth of an individual with concepts.***

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Table III: Measurement vis-à-vis Significant Digits (SDs)

Intrument Used	Range of Length	Minimum Graduation (Division)	Least Cout	Measurement					
				Measurement	Scientific Representat	GPE	MSD	LSD	SDs
Micrometre	0 to 5 cm	Main Scale : 1 mm Circular Scale: 1mm = 100 Div	0.01 mm	0.07 mm	7×10^{-2} mm	0.01 mm	0	7	2
				0.73 mm	7.3×10^{-2} mm		7	3	2
				2.04 mm	2.04×10^{-2} mm		2	4	3
				3.15 mm	3.15 mm		3	5	3
				4.60 cm	4.60 cm		4	0	3
				4.6″ cm	4.6 cm		4	6	2
Vernier Calliper	0 to 10 cm	Main Scale : 1 mm Sliding Scale: 9mm = 100 Div	0.1 mm	0.3 mm	3×10^{-1} mm	0.1 mm	3	3	1
				1.6 mm	1.6 mm		1	6	2
				5.2 mm	5.2 mm		5	2	2
				2.39 cm	2.39 cm		2	9	3
				9.05 cm	9.05 cm		9	5	3
				9.1 cm″	9.1 cm	0.05 mm	9	1	2
				9 cm″	9 cm	0.5 cm	9	9	1
Measuring Rod	0 to 1 m	10 cm	10 cm	25 cm*	2.5×10^{1} cm	10 cm	2	5	2
				60 cm	6.0×10^{1} cm		6	6	1
				60.0 cm*	6.00×10^{1} cm		6	0	3
Measuring Tape	0 to 10 m	0.5 cm	0.5 cm	5 mm	5 mm	0.5 cm	5	5	1
				3.7 cm*	3.7 cm		3	7	2
				5.0 cm	5.0 cm		5	0	2
				20.6 cm*	2.06×10^{1} cm		2	6	3
				7.34 m″	7.34 m		7	4	3
				7.30 m″	7.30 m		7	0	3
				7.3 m″	7.3 m		7	3	2
				5 m″	5 m		5	5	1
Surveying Chain	0 to 50 m	10 cm	10 cm	40.0 cm*	4.00×10^{1} cm	10 cm	4	0	3
				40 cm	4.0×10^{1} cm		4	0	2
				1.70 m	1.70 m		1	0	3
				1.7 m″	1.7 m		1	7	2
				33.80 m	3.380×10^{1} cm		3	0	4
				33.8 m″	3.38×10^{1} cm		3	8	3
				28 m″	2.8×10^{1} cm		2	8	2

Legend: GPE- Greatest Possible Error, MSD - Most Significant Digit, LSD - Least Significang Digit,
″ - Full accuracy of instrumn not used, * - Incorrect Measurement

Table IV: Basic Arithmatic Operations and Significant Digits

Particulars	Basic Arithmatic Operations			
	Additions	**Subtraction**	**Multiplication**	**Division**
(1)	**(2)**	**(3)**	**(4)**	**(5)**
Given data set	**2.34617**, 21.14, 9.790003, 7.562	52.309, -1.42987	(i) **5.1349** X(8.79345x10^{-4}) (ii) (5.6379x10^{-3})x(**79.35** x10^{3})	{**5.1349** X(8.79345x10^{-4})}÷ { (5.6379x10^{-3})x(**79.35** x10^{3})}
Actual Operation	2.346172 +21.140000 **+0.790003** +7.562000 Sum = **31.83**8173	52.**309**00 -1.42987 50.**879**13	Product: i) = 4.515386405x10-3 ii) = 4.4737X10-4	Division: =(4.5154x10^{-3})÷(4.474X10^{-4}) '=1.009249x10-1
Answer to be Reported	Sum is **31.84**	Difference is **50.879**	Product: i) = 4.5154x10-3 ii) = 4.474X10-4	Division: =**1.009** x10^{-1}
Analysis of Reported Answer	LSD of a number with largest uncetainity decides position of LSD in final value. Accordingly, 0.790003 with LSD at Sixth Decimal place is decisive. Least decimal places among operands are Two in Second operand, after that any value is uncertain. Hence certain result shall have Two decimal places with Second place least significant. Accordingly, Third decimal place in sum shall be rounded to **Second Decimal Place** and result reported.	LSD of a number with largest uncetainity decides position of LSD in final value. Accordingly, -1.42987 with LSD at Fifth Decimal place is decisive. Least decimal places among operands are Three in First operand, after that any value is uncertain. Hence certain result shall have Three decimal places with Third place being least significant. Accordingly, Fourth decimal place in difference shall be rounded to **Third Decimal Place** and result reported.	i) Minimum SDs in First case are in First operand and it is Five. Accordingly, Sifth digit in product shall be rounded to **Fifth place**. ii) Minimum SDs inSecond case are in Second operand and it is Four. Accordingly,in result Fifth digit in product shall be rounded to **Fourth place**.	Multiplication in numerator shall resilt into **Five SDs** and that in Denominator shall result into **Four SDs**. Thus in final form of division minimum of SDs among numerator and Denominators shall decide SDs, which in instant case is **Four**.

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