# Illustrations of Answers to Practice Questions on Physics

## (Topic-Units, Dimensions and Measurements)

**Illustration 1:** Given formula is for measurement of approximate Surface Tension based on height of liquid in a capillary, where, density of liquid $[\rho] = \mathrm{ML^{-3}}$, Acceleration due to Gravity $[g] = \mathrm{LT^{-2}}$ , radius of the capillary $[r.] = \mathrm{L}$ and height of rise of liquid in the capillary $[h] = \mathrm{L}$. In dimensional analysis, numerical coefficients are dimensionless and, therefore, $S = (\mathrm{ML^{-3}})(\mathrm{LT^{-2}})(\mathrm{L})(\mathrm{L}) = \mathrm{ML^{-3+3}T^{-2}} = \mathrm{MT^{-2}}$. The final dimension is arrived at by applying Theory of Indices.

**Illustration 2:** . In question dimension of Temperature is used as K, instead of SI Units as θ, and hence used in solution. Given formula is measurement transfer of heat $[Q]$ through a solid whose cross-sectional area $[A] = \mathrm{L^2}$, length $[d] = \mathrm{L}$, across the length $(d)$ temperature difference is $[\theta] = [\theta_2 - \theta_1] = \mathrm{K}$ is and time for transfer of heat $Q$ is $[t] = \mathrm{T}$. Further, heat $Q$ is energy and hence it is dimensionally $[Q] = \mathrm{ML^2T^{-2}}$ . Therefore, dimensionally $[k] = \frac{[Q][d]}{[A][\theta_2-\theta_1][t]} = \frac{(\mathrm{ML^2T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{K})(\mathrm{T})} = \mathrm{MLT^{-3}K^{-1}}$.

**Illustration 3:** In given formula $Q = \int i.dt$, therefore $[Q] = [i][t] = \mathrm{IT}$; Moreover, Energy $E = VIt$, where $V$ is the potential difference, $I$ is current through $V$ and $t$ is time of flow of current. Therefore, dimensionally $[V] = \frac{[E]}{[I][t]} = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{IT}} = \mathrm{ML^2T^{-3}I^{-1}}$; dimension of current $(I)$ is I. Further, dimensionally $[Q] = [C][V] \rightarrow [C] = \frac{[Q]}{[V]} = \frac{\mathrm{IT}}{\mathrm{ML^2T^{-3}I^{-1}}} = \mathrm{M^{-1}L^{-2}T^4I^2}$.

**Illustration 4:** In SI $\underline{1\ N/m^2}$ factor for unit of Young's Modulus in MKS is $(\mathrm{N/m^2})$ into CGS $(\mathrm{dynes/cm^2})$. And unit $1\mathrm{N} = 1\ \mathrm{kg} - \mathrm{m/s^2}$ , likewise, $1\mathrm{dyne} = 1\mathrm{g} - \mathrm{m/s^2}$. Accordingly, converting individual units in SI into CGS $1\frac{\mathrm{N}}{\mathrm{m^2}} = \frac{1\ \mathrm{kg-m/s^2}}{\mathrm{m^2}} = \frac{((1000\mathrm{g})(100\mathrm{cm}))/\mathrm{s^2}}{(100\mathrm{cm})^2} = \frac{10^5\mathrm{dynes}}{10^4\ \mathrm{cm}^2} = 10\ \mathrm{dynes/cm^2}$.

**Illustration 5:** In dimensional analysis, all addends must have same dimension as that of the quantity being equated. $[y]$, where $y$ is distance has dimension L . Likewise, $[y] = \mathrm{L}$. Therefore, $[ax] = [a].\mathrm{L} = \mathrm{L}$, hence $[\mathrm{a}] = \frac{\mathrm{L}}{\mathrm{L}} = 1$, this is true when $[a]$ is dimensionless. Further, $[bt] = \mathrm{L} = [b].\mathrm{T} \rightarrow [b] = \frac{\mathrm{L}}{\mathrm{T}} = \mathrm{LT^{-1}}$ and $[ct^2] = \mathrm{L} = [c].\mathrm{T^2}$, this leads to $[c] = \frac{\mathrm{L}}{\mathrm{T^2}}$.

**Illustration 6:** Heat is energy and therefore, $[H] = ML^2T^{-2}$ . Therefore, dimensional equation comes out to be $[H] = [I]^a[R]^b[t]^c \rightarrow \mathrm{ML^2T^{-2}} = \mathrm{I^a}[R]^{\mathrm{b}}\mathrm{T^c}$. As per Ohms Law $[R] = \frac{[V]}{[I]}$; moreover electrical energy $E = VIt$, where $V$ is the potential difference, $I$ is current through $V$ and $t$ is time of flow of current. Therefore, dimensionally $[V] = \frac{[E]}{[I][t]} = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{IT}} = \mathrm{ML^2T^{-3}I^{-1}}$, and $[R] = \frac{\mathrm{ML^2T^{-3}I^{-1}}}{\mathrm{I}} = \mathrm{ML^2T^{-3}I^{-2}}$. Accordingly, as per dimensional identity, $\mathrm{ML^2T^{-2}} = \mathrm{I^a(ML^2T^{-3}I^{-2})^bT^c} = \mathrm{M^bL^{2b}T^cI^{a-2b}}$. Since, each dimension is independent and, therefore, corresponding indices on both sides must be equal. Thus, for dimension M, $\boldsymbol{b} = \mathbf{1}$; for dimension of L, $2b = 2$ and it corroborates values of $b$ arrived earlier; for dimension of T, $-3b + c = -2 \rightarrow c = -2 + 3 \rightarrow \boldsymbol{c} = \mathbf{1}$; and for dimension of I, $0 = a - 2b = a - 2 \rightarrow \boldsymbol{a} = \mathbf{2}$.

**Illustration 7:** We know that $[\sin\theta] = y$ is also dimensionless, so also $\theta = \sin^{-1} y$ is also dimensionless, being ratio of length of arc and radius of arc. Therefore, $\left[\frac{x}{a} - 1\right] = \frac{L}{L}$ is dimensionless and hence $\left[\frac{x}{a}\right] = \frac{L}{L}$ is also dimensionless or $[a] = [x] = L$ . Now taking L.H.S. $\left[\frac{dx}{\sqrt{ax - x^2}}\right] = \frac{[dx]}{[\sqrt{ax - x^2}]} = \frac{L}{\sqrt{L^2}} = \frac{L}{L}$ is also dimensionless. Both sides of given equation ave dimensional equality where $[a] = L$

**Illustration 8:** Linear Momentum $[\bar{p}] = [\bar{F} \times \bar{V}] = (MLT^{-2}).(LT^{-1}) = ML^2T^{-3} \neq ML^{-1}T^{-2}$ , it is not the answer.

Work done by a force $[w] = [\bar{F}.\bar{D}] = (MLT^{-2}) \cdot (L) = ML^2T^{-2} \neq ML^{-1}T^{-2}$, it is not the answer.

Energy per unit volume $[e] = \frac{[E]}{[V]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2} = ML^{-1}T^{-2}$, it is one of the correct choices

Pressure $[p] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} = ML^{-1}T^{-2}$, it is one of the correct choices.

***Note:*** *In dimensional analysis vector and scalar quantities are treated equally.*

**Illustration 9:** Dimensional analysis is based on representation of physical quantities in terms of base quantities; hence answer (a) is true.

Answer (b) is true, as it is the fundamental premise on which dimensional analysis is built.

Answer (c) corollary of (b) and hence true.

Answer (d) is true since all quantities expressed as ratio of identical quantities are zero, e.g. strain, angle etc.

**Illustration 10:** 1 century = 100 years = 100 × 365 days = 100 × 365 × 24 × 60 × 60 minutes. Therefore duration of journey 90 minutes $= \frac{90}{100 \times 365 \times 24 \times 60 \times 60}$ centuries

Or, 90 $minutes$ = 2.85388E − 08 centuries = 2853.88E − 05 milli − centuries. In these calculations least number of significant digits is Two hence answer shall be written as 28E − 05 milli − centuries.

***Logic:*** *while rounding a result of a calculations involving multiplications answer is expressed in number of least significant digits, which is Two in this case. Therefore, result of calculation in Third digit from MSD shall have to be rounded to Second digit from MSD. The Third digit is 5, but if digit left to the one being rounded is even, in the instant case this digit is 8, an even number, and hence it shall remain unchanged in final answer.*

**Illustration 11:** $[\boldsymbol{g}] = \frac{[\boldsymbol{T^2}]}{[\boldsymbol{l}]}$, and accuracy of is $\frac{\Delta l}{l} \times 100 = \frac{0.1}{20} \times 100 = 0.5\%$ and $\frac{\Delta t}{t} \times 100 = \frac{1}{90} \times 100 = 1.1\%$. Therefore, $\frac{\Delta g}{g}\% = 2\frac{\Delta t}{t}\% + \frac{\Delta l}{l}\% = (2 \times 1.1)\% + 0.5\% = 2.7\% = 3\%$

***Logic:*** *Error is compared with respect to base value and can be (+)ve or (-)ve. Since, possibility of maximum error is considered and hence all errors are taken to be positive. Likewise,* $[\boldsymbol{l}]$ *is in the denominator mathematically using method of partial derivative* $\frac{\Delta g}{g} = 2\frac{\Delta t}{t} - \frac{\Delta l}{l}$ . *But, considering uncertainty in* ± *errpr all errors are taken to be (+)ve as worst case condition. Therefore, calculation of accuracy in this case shall be* $\frac{\Delta g}{g}\% = 2\frac{\Delta t}{t}\% + \frac{\Delta l}{l}\%$.

**Illustration 12:** One Joule $= \left(1\text{kg} \times \frac{1\text{m}}{\text{sec}^2}\right) \times 1\text{m}$. Since, 1 calorie $= 4.2\,\text{J} = 4.2 \times \frac{1}{\text{x}} \times \left(\frac{1}{y}\right)^2 \times \left(\frac{1}{y}\right)^{-2}$ U. Therefore, 1 calorie $= 4.2x^{-1}y^{-2}z^2\text{U}$

**Illustration 13:** Energy through inductance is $E = \frac{1}{2}LI^2$ and hence dimensionally $[E] = [L][I]^2$**.** Likewise, **e**nergy through inductance is $E = \frac{1}{2}CV^2$ and hence dimensionally $[E] = [C][V]^2$. Equating R.H.S of two equations of dimension of energy $[L][I]^2 = [C][V]^2 \rightarrow \left[\frac{L}{C}\right] = \left[\frac{V}{I}\right]^2$. As per Ohm's Law dimensionally $\left[\frac{V}{I}\right] = [R] \rightarrow \left[\frac{L}{C}\right] = [R]^2$.

**Illustration 14:** As per Faraday's Law $V = \frac{d\emptyset}{dt}$. Energy gained by an electron in travelling through a potential difference $V$, is $[E] = [e][V]$. Moreover, energy of a photon is $[E] = [h][\nu]$**.** Thus equating RHS of these dimensional equations of energy $[h][\nu] = [e][V] \rightarrow \left[\frac{h}{e}\right] = \left[\frac{V}{\nu}\right]$. This is the dimension of magnetic flux.

**Illustration 15:** Energy per unit volume in dielectric is $\left[\frac{U}{V}\right] = [\varepsilon_0 E^2] = \frac{\text{ML}^2\text{T}^{-2}}{\text{L}^3} = \text{ML}^{-1}\text{T}^{-2}$ . Further, $[\mu_0\varepsilon_0] = \left[\frac{1}{c^2}\right] = \frac{1}{(\text{LT}^{-1})^2} = \frac{1}{\text{L}^2\text{T}^{-2}}$ , here $\varepsilon_0$ is permittivity of free space and $c$ is the velocity of light. Thus $\left[\frac{E^2}{\mu_0}\right] = \left[\frac{\varepsilon_0 E^2}{\mu_0\varepsilon_0}\right] = \frac{\text{ML}^{-1}\text{T}^{-2}}{\frac{1}{\text{L}^2\text{T}^{-2}}} = (\text{ML}^{-1}\text{T}^{-2}).(\text{L}^2\text{T}^{-2}) = \text{MLT}^{-4}$

**Illustration 16:** Since $\bar{l} = \bar{r} \times \bar{p}$ hence $[l] = [r][p] = [r][mv] = (\text{L})(\text{MLT}^{-1}) = \text{ML}^2\text{T}^{-1}$. Magnetic field is since coupled the current flowing in a loop hence $m = Ia$, where $I$ is the current through a loop having cross-sectional area $a$. Therefore, $[\mu] = [Ia] = \text{I}.\text{L}^2$. Hence , $\left[\frac{L}{\mu}\right] = \frac{\text{ML}^2\text{T}^{-1}}{\text{L}^3\text{I}} = \text{MT}^{-1}\text{I}^{-1}$

**Illustration 17:** Force between Two charges $F = \frac{q_1q_2}{4\pi\epsilon_0 r^2} \rightarrow \text{MLT}^{-2} = \left[\frac{q_1q_2}{4\pi\epsilon_0 r^2}\right] = \left[\frac{q^2}{\epsilon_0}\right].\frac{1}{L^2} \rightarrow \left[\frac{q^2}{\epsilon_0}\right] = \text{ML}^3\text{T}^{-2}$. Further, $[hc] = \left[\frac{E}{\frac{1}{T}}\right](LT^{-1}) = (ML^2T^{-2})(T)(LT^{-1}) = ML^3T^{-2}$. Therefore, $\left[\frac{e^2}{\varepsilon_o hc}\right] = \left[\frac{q^2}{\epsilon_0}\right]\left[\frac{1}{hc}\right] = \frac{\text{ML}^3\text{T}^{-2}}{ML^3T^{-2}}$=1, or it is dimensionless.

**Illustration 18:** As per Wein's Law $\lambda_m t = b$ where, $\lambda_m$ is wavelength of radiation having maximum intensity, $t$ is temperature and $b$ is Wein's Constant. Therefore, dimensionally $[b] = [\lambda_m t]$ . Also $v = \lambda f$, or $[\lambda] = \left[\frac{v}{f}\right] \rightarrow \frac{\text{LT}^{-1}}{\text{T}^{-1}} = \text{L}$. Further, as per Stephenan-Boltzmann Law heat energy radiated by a black body $= \sigma a t^4$ , here, $Q$ is heat energy dissipated per unit time, $a$ is the surface area of the black body, $t$ is temperature of the black body and $\sigma$ is Stephen's Constant. Thus dimensionally $[\sigma] = \frac{\frac{[Q]}{T}}{[a][t^4]}$Therefore, $[\sigma b^4] = [\sigma][b]^4$. Substituting dimensions of constituent quantities $[\sigma b^4] = \frac{\frac{[Q]}{T}}{[a][t^4]} \cdot [\lambda_m t]^4 = \frac{\frac{[Q]}{T}[\lambda_m]^4}{[a]}$. Since, $Q$ is heat energy $[Q] = \text{ML}^3\text{T}^{-2}$ . It leads to $[\sigma b^4] = \frac{(\text{ML}^2\text{T}^{-2})}{\text{T}} \cdot \left(\frac{1}{\text{L}^2}\right) \cdot \text{L}^4 = \text{ML}^4\text{T}^{-3}$.

**Illustration 19:** Dimensionally $[CV] = \left[\frac{\varepsilon_0 A}{d}\right][V] = [\varepsilon_0]\left[\frac{A}{d}\right][V] = [\varepsilon_0][V]\left(\frac{L^2}{L}\right) = [\varepsilon_0][V](L)$, likewise, $[\rho\varepsilon_o] = \left[\frac{RA}{l}\right][\varepsilon_o] = [R][\varepsilon_o]\left(\frac{L^2}{L}\right) = [R][\varepsilon_o](L)$. Therefore, $\left[\frac{CV}{\rho\varepsilon_o}\right] = \frac{[\varepsilon_0][V](L)}{[R][\varepsilon_o](L)} = \left[\frac{V}{R}\right] = [I] = \text{I}$

**Illustration 20:** We know that $E = h\nu = (\hbar)(2\pi\nu)$. Thus, $Ec = (\hbar c)(2\pi\nu) \rightarrow [\hbar c] = \left[\frac{Ec}{\nu}\right] = \frac{(\mathrm{ML^2T^{-2}})(\mathrm{LT^{-1}})}{\mathrm{T^{-1}}}$. It leads to $[\hbar c] = \mathrm{ML^3T^{-2}}$.

**Illustration 21**: Young's Modulus of of elasticity $Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{Fl}{A\Delta l} = \frac{(1\times10)\times2}{\frac{\pi}{4}\times.(4\times10^{-4})^2(84\times10^{-4})} = 1.989\times10^{11}$ N/m². Applying logic of significant digits $Y = 2.0\times10^{11}$, since minimum number of SGs in given data is One. Now error in calculation $\Delta Y = \left(2\times\frac{\Delta d}{d} + \frac{\Delta(\Delta l)}{l}\right)Y = \left(2\times\frac{0.01}{0.4} + \frac{0.05}{0.8}\right)\times 2.0\times10^{11}$. Thus error is $Y = (5\times10^{-2} + 6.25\times10^{-2})\times 2.0\times10^{11} = 1.125\times10^{-2}\times2.0\times10^{11} = 2.25\times10^{9} = 2\times10^{9}$. Thus with the logic of maximum number of significant digits in addition $Y + \Delta Y = (2.0 + 0.02)\times10^{11}$ N/m²

**Illustration 22**: When more than one value are available then best approximation is to be made using Two values of the resistance box, which are arranged in ascending or descending order. Since, number $(n)$ of values is even then the values to be chosen are $\left(\frac{n}{2}-1\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ and their average is taken. In the event of $n$ being odd then $\left(\frac{n}{2}+1\right)^{th}$ is best approximation.

**Illustration 23**: Diameter of wire is measured to be 2.49 mm . This is a case of multiplication of quantities with different SDs and hence answer will have SDs equal to quantity with minimum SDs. Accordingly, required surface area is $= \pi dl = 3.14\times2.49\times10^{-1}\times\mathbf{6.5} = 50.829 = 51$ cm²

**Illustration 24**: (a) $[Torque] = [Force][Distance]$ and $[Work] = [Force][Distance]$ nboth are dimensionally identical

(b) $[Momentum] = [Mass][Velocity] = (\mathrm{M})(\mathrm{LT^{-1}}) = \mathrm{MLT^{-1}}$ and

$[Planck' Constant] = [Energy][Wavelength] = \mathrm{ML^2T^{-2}}$ both are dimensionally different

(c) $[Stress] = \left[\frac{Force}{Area}\right] = \frac{MLT^{-2}}{L^2} = ML^{-3}T^{-2}$ and $[Young's\ Moulus] = \left[\frac{Stress}{Strain}\right] = [Stress]$, since strain is dimensionless and both are dimensionally identical.

(d) We now that $\varepsilon_o\mu_o = \frac{1}{c^2}$, here $c$ is speed of light, therefore dimensionally both $\varepsilon_o\mu_o$ and $\frac{1}{\mathrm{Speed}^2}$ are identical.

**Illustration 25**: Number of significant digits are arrived at by using rules for SDs rules for. In 23000 SDs are 2 sincre trailing zero are neglected b'coz there is no decimal, in 23.000 all zeros trailing decimal are included and hence SDs are 5, and in 02030.0 leading zeros to the leftmost digit are excluded, while zero sandwiched between digits and trailing decimal point are included and accordingly SDs are 5

**Illustration 26**: Formula for a parallel plate capacitor is $C = \frac{\varepsilon_o\varepsilon_r A}{d}$, here unit of capacitance $(C)$ is Farad, and unit of area $(A)$ is meter-squre, unit of distance $(d)$ between parallel plates is meter, and relative permittivity of the dielectric $(\varepsilon_r)$ being a ratio is unit less and therefore $\varepsilon_o = \frac{Cd}{\varepsilon_r A}$ and thus it shall have unit $\frac{\text{Farad}-\text{meter}}{\text{meter}-\text{square}} = \frac{\text{Farad}}{\text{meter}}$

**Illustration 27**: Law of gravitation stipulates $F = G\frac{m_1m_2}{r^2} \rightarrow G = \frac{Fr^2}{m_1m_2}$ and hence unit of $G$ is N-m²/kg²

**Illustration 28**: In expression $1-e^{-\alpha t}$, the subtrahend $e^{-\alpha t}$ must also be dimensionless which implies $[\alpha t]=1 \rightarrow [\alpha]=\mathrm{T}^{-1}$. Further, $[x(t)]=\mathrm{L}=\left[\frac{v_0}{\alpha}\right]=\frac{[v_0]}{\mathrm{T}^{-1}} \rightarrow [v_0]=\mathrm{LT}^{-1}$

**Illustration 29**: From the given equation $D=-\frac{n(x_2-x_1)}{(n_2-n_1)} \rightarrow [D]=\left[\frac{n(x_2-x_1)}{(n_2-n_1)}\right]=\frac{[n][x_2-x_1]}{[n_2-n_1]}$. Now by definition of variables, dimensions of $[n]=\frac{1}{\mathrm{L}^3\mathrm{T}}=\mathrm{L}^{-3}\mathrm{T}^{-1}$, $[x_2-x_1]=\mathrm{L}$ and $[n_2-n_1]==\frac{1}{\mathrm{L}^3}=\mathrm{L}^{-3}$. Accordingly, $[D]=\frac{(\mathrm{L}^{-3}\mathrm{T}^{-1})\cdot\mathrm{L}}{\mathrm{L}^{-3}}=\mathrm{LT}^{-1}$

**Illustration 30**: . In question dimension of current is taken as A instead of I as per SI Units, hence retained. From Ampere's Law, $F=Bil \rightarrow [F]=[B]\cdot[i]\cdot[l] \rightarrow\rightarrow [B]=\frac{\mathrm{MLT}^{-2}}{\mathrm{AL}}=\mathrm{MT}^{-2}\mathrm{A}^{-1}$. Therefore, $[P]=\frac{(\mathrm{MT}^{-2}\mathrm{A}^{-1})^2\cdot\mathrm{L}^2}{\mathrm{M}}=\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{A}^{-2}$.

**Illustration 31**: Let $m \propto c^p G^q h^r \rightarrow m=Kc^p G^q h^r$, therefore, $[m]=[Kc^p G^q h^r]$here $K$ is dimensionless proportionality constant. Dimensionally, $[c]=\mathrm{LT}^{-1}$, $[G]=\frac{[Fr^2]}{[m^2]}=\frac{(\mathrm{MLT}^{-2})\cdot\mathrm{L}^2}{\mathrm{M}^2}=\mathrm{ML}^3\mathrm{T}^{-2}$, and $[h]=\frac{[E]}{[v]}=\frac{ML^2T^{-2}}{T^{-1}}=ML^2T^{-1}$, then, $\mathrm{M}=(\mathrm{LT}^{-1})^p(\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^2)^q(ML^2T^{-1})^r=\mathrm{M}^{-q+r}\mathrm{L}^{p+3q+2r}\mathrm{T}^{-p+2q-r}$. Therefore, as per theory of indices, $-q+r=1, p+3q+2r=0, -p-2q-r=0$. Solving these three simultaneous equations, we get, $p=\frac{1}{2}$, $q=-\frac{1}{2}$ and $r=\frac{1}{2}$. Therefore, $m \propto c^{\frac{1}{2}}G^{-\frac{1}{2}}h^{\frac{1}{2}}$

**Illustration 32**: Relative Density (RD) $=\frac{\text{Weight of body in air}}{\text{Weight loss in water}}$. During addition or subtraction of quantities error is added for worst case conditions. Thus $RD=\frac{(5.00\pm 0.5)}{(1.00\pm 0.1)}=\frac{\left(5.00\pm\frac{0.05}{5.00}\sim\times 100\right)}{\left(1.00\pm\frac{0.1}{4.00}\times 100\right)}$. This resolves into $RD=\frac{\left(5.00\pm\frac{0.05}{5.00}\sim\times 100\right)}{\left(1.00\pm\frac{0.1}{4.00}\times 100\right)}=5.0\pm\frac{0.05}{5.00}\times 100\pm 5\times\frac{0.10}{1.00}\times 100=5.0\pm(1+10)\%=5.0\pm 11\%$. Here, $\left(\pm\frac{0.05}{5.00}\sim\times 100\right)\left(\pm\frac{0.1}{4.00}\times 100\right)$is ignored being too small.

**Illustration 33**: Total weigh (= 5.3 + 0.200 + 0.375 +0.075 =5.950 = 6.0). In this least SDs are Two and so shall be answer. First LSD to the right of decimal shall decide LSD in the sum and all other digits right to it will be dropped. Since, next digit right to it is 5 , the LSD shall be rounded retaining SDs equal to Two..

**Illustration 34**: Since $V=I\times R=3.56\times 15.479=55.10524=55.1$. Since minimum SDs are Three, and taking Four SDs and rounding it Three SDs would be the answer.

**Illustration 35**: Principally $\Delta x=\frac{X}{100}\left(\frac{2}{3}\times\%P+2\times\%Q+1\times\%R+\frac{5}{2}\times S\right)$. It is seen that maximum contribution in error is caused by S

**Illustration 36:** Fundamental units are 1. Length-metre (m), 2. Mass- kilogram (kg), 3. Time- second (s), 4. Electric current- Ampere (A), 5. Thermodynamic Temperature – Kelvin (K), 6. Amount of substance – mole (mol), and 7. Luminous Intensity – Candela (cd).

**Illustration 37:** $E=hv \rightarrow Joule=h\left(\frac{1}{s}\right) \rightarrow h=Joule-s$.

**Illustration 38:** As per Ohm's Law in an inductive element $V=IX$ whereX is reactance and has unit Ohm, same as that for resistance.

**Illustration 39:** It is known that energy in an inductor is $E=\frac{1}{2}I^2L$ and hence dimensionally $[E]=\mathrm{I}^2[L]$, and power absorbed in a resistor $P=I^2R$, since Power by definition is rate of doing work and hence $[P]=\frac{[E]}{T}=\mathrm{I}^2[R]\rightarrow[E]=\mathrm{TI}^2[R]$. This equating dimensions of $[E]=\mathrm{I}^2[L]=\mathrm{TI}^2[R]$. Or, $\left[\frac{R}{L}\right]=\frac{\mathrm{I}^2}{\mathrm{TI}^2}=\mathrm{T}^{-1}$.

**Illustration 40:** It is known that unit of energy and work are same and so also their dimensions Hence, $[E]=[F][D]=(\mathrm{MLT}^{-2})\cdot\mathrm{L}=\mathrm{ML}^2\mathrm{T}^{-2}$.

**Illustration 41:** Electric potential is expressed as $V=I\cdot R\rightarrow[V]=\mathrm{I}[\mathrm{R}]$, and power consumed by a resistor is $P=\frac{dE}{dt}=I^2R\rightarrow ML^2T^{-3}=\mathrm{A}^2[R]\rightarrow[R]=\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{I}^{-2}$. Substituting $[R]$ in dimensional equation of $[V]$, we get $[V]=\mathrm{I}\cdot(\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{I}^{-2})=\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{I}^{-1}$. Since in answer $Q$ has been instead of $A$ where $A=QT^{-1}$ i.e. rate of flow of charge and hence moderated answer shall be $[V]=\mathrm{I}\cdot(\mathrm{ML}^2\mathrm{T}^{-3}\mathrm{I}^{-2})=\mathrm{ML}^2\mathrm{T}^{-3}(\mathrm{QT}^{-1})^{-1}=\mathrm{ML}^2\mathrm{T}^{-2}\mathrm{Q}^{-1}$. Here, $\mathrm{I}=\mathrm{QT}^{-1}$ as required in question despite not as per SI Units.

**Illustration 42:** As per Kinetic Theory of Gases $PV=NkT$, here $P$ is pressure of gas has dimension $(\mathrm{ML}^{-1}\mathrm{T}^{-2})$, $V$ is volume of gas $\mathrm{L}^3$, $N$ is number of gas molecules is dimensionless, $k$ is Boltzmann Constant, its dimension has to be determined and $T$ is temperature of gas has dimension $\theta$ as per System of International Units and is retained. Thus dimensionally the equation is $(\mathrm{ML}^{-1}\mathrm{T}^{-2})\cdot\mathrm{L}^3=[k]\cdot\theta\rightarrow[k]=\mathrm{ML}^2\mathrm{T}^{-2}\theta^{-1}$,

**Illustration 43:** Dimensionally each of the given quantity is being analysed

(a) As per Newton's Law of Gravitation $F=G\frac{Mm}{r^2}\rightarrow[F]=\mathrm{MLT}^{-2}=[G]\mathrm{M}^2\mathrm{L}^{-2}\rightarrow[G]=\frac{\mathrm{MLT}^{-2}}{\mathrm{M}^2\mathrm{L}^{-2}}$ or $[G]=\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}$ ; it is not dimensionless

(b) As per Planck-Einstein relation $E=h\nu\rightarrow\mathrm{ML}^2\mathrm{T}^{-2}=[h]\mathrm{T}^{-1}\rightarrow[h]=\mathrm{ML}^2\mathrm{T}^{-1}$; it is not dimensionless

(c) Power of a lens is $P=\frac{1}{f}$, here $P$ is power of a Lens having dimension $[P]$ and $f$ is focal length having dimension $[L]$ and hence $[P]=\frac{1}{\mathrm{L}}=\mathrm{L}^{-1}$; it is not dimensionless

(d) Hence answer is (d)

**Illustration 44:** Dimensionally each of the given quantity is being analysed

(a) Dimensionally $[Force]=\mathrm{MLT}^{-2}$ has dimension, whereas Strain $[Strain]=\frac{\Delta l}{\mathrm{L}}=\mathrm{L}^0$ is dimensionless

(b) Dimensionally $[Force]=\mathrm{MLT}^{-2}$, and $[Stress]=\frac{\mathrm{MLT}^{-2}}{\mathrm{L}^2}=\mathrm{ML}^{-1}\mathrm{T}^{-2}$; both have different dimensions

(c) Angular velocity $[\omega]=\frac{[\text{Angle}]}{\mathrm{T}}=\mathrm{T}^{-1}$ , here angle being ration of lengths of arc to radius of arc is dimensionless. Further, Frequency $[f]=\frac{[\text{No of Cycles}]}{\mathrm{T}}=\mathrm{T}^{-1}$, here also [No of Cycles] is dimensionless. Hence, $[\omega]$ and $[f]$ have same dimensions.

(d) Dimensionally $[Energy]=[Work]=\mathrm{ML}^2\mathrm{T}^{-2}$, while Strain $[Strain]=\frac{\Delta l}{\mathrm{L}}=\mathrm{L}^0$ is dimensionless.

**Hence, (c) is the answer.**

**Illustration 45:** Dimensionally pressure (P) is Force Per unit area and hence $[P] = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}} = \mathrm{ML^{-1}T^{-2}}$ . Analyzing each of the given quantity-

(a) Dimensionally $[Force\ per\ unit\ Volume] = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^3}}$ has dimension different than that of $[P]$,

(b) Dimensionally Energy per unit volume is $\left[\frac{Energy}{Volume}\right] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{L^3}} = \mathrm{ML^{-1}T^{-2}}$, it is identical to that of pressure

(c) Dimensionally $[Force] = \mathrm{MLT^{-2}}$ and is different from that of pressure.

(d) Dimensionally Energy is equal to work i.e. $[Energy] = \mathrm{ML^2T^{-2}}$, it is different from that of pressure

**Hence, (b) is the answer**

**Illustration 46:** Dimensionally pressure

(a) $[Torque] = [Force] \cdot [Perp. Dist. From\ Fulcrum] = (\mathrm{MLT^{-2}}) \cdot \mathrm{L} = \mathrm{ML^2T^{-2}}$ , While, $[Worke] = [Force] \cdot [Disp. Along\ Force] = (\mathrm{MLT^{-2}}) \cdot \mathrm{L} = \mathrm{ML^2T^{-2}s}$ , both are different both are same

(b) $[Stress] = \frac{[Force]}{[Area]} = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}} = \mathrm{ML^{-1}T^{-2}}$ and $[Energy] = [Force] \cdot [Disp.] = (\mathrm{MLT^{-2}}) \cdot \mathrm{L} = \mathrm{ML^2T^{-2}s}$ , both are different,

(c) $[Force] = \mathrm{MLT^{-2}}$ and $[Stress] = \frac{[Force]}{[Area]} = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}} = \mathrm{ML^{-1}T^{-2}}$, both are different

(d) Dimensionally $[Force] = \mathrm{MLT^{-2}}$ and $[Worke] = [Force] \cdot [Disp. Along\ Force] = (\mathrm{MLT^{-2}}) \cdot \mathrm{L} = \mathrm{ML^2T^{-2}}$, both different from that of pressure.

**Hence, (a) is the answer**

**Illustration 47:** For dimensional equality of given Equation $[x] = [Ay] = [B \tan Cz]$, here $\tan Cz$ is dimension less and so also $Cz$. Therefore, $[x] = [B]$ and $C = z^{-1}$ ,. Likewise, $[x] = [B] = [Ay]$, hence $[y] = \left[\frac{B}{A}\right]$. Thus, $y$ is not having dimension of $x$. **Hence, (d) is the answer**

**Illustration 48:** Taking dimensional analysis of each quantity -

(a) Dimension $[Resistivity] = \left[\frac{RA}{l}\right] = [R] \cdot \mathrm{L}$ . And power of resistance $P = I^2R \rightarrow \mathrm{ML^2T^{-3}} = \mathrm{I^2[R]} \rightarrow \mathrm{[R]} = \mathrm{ML^2T^{-3}A^{-2}}$. Fundamental dimension for Current is (A) which in converted to $\mathrm{I} = \mathrm{QT^{-1}}$. Therefore, $\mathrm{I^{-2}} = \mathrm{Q^{-2}T^2}$ and thus substituting it $\mathrm{[R]} = (\mathrm{ML^2T^{-3}}) \cdot (\mathrm{Q^{-2}T^2}) = \mathrm{ML^2T^{-1}Q^{-2}}$. Thus , $[Resistivity] = \left[\frac{RA}{l}\right] = (\mathrm{ML^2T^{-1}Q^{-2}}) \cdot \mathrm{L} = \mathrm{ML^3T^{-1}Q^{-2}}$. This is the given dimension, as required in question though not as per SI Unit.

(b) Dimensionally $[Conductivity] = \frac{1}{[Resistivity]} = \frac{1}{\mathrm{ML^3T^{-1}Q^{-2}}} = \mathrm{M^{-1}L^{-3}T^1Q^2}$. This is not the given dimension.

(c) Dimension of Resistance is $\mathrm{[R]} = \mathrm{ML^2T^{-1}Q^{-2}}$ , it is not the given dimension.

(d) Since One of the above i.e. at (a) is matching with the given dimension and hence this choice is not applicable.

**Hence, (a) is the answer**

**Illustration 49:** Dimensional equality requirement has to be tested in the given option for Two unequal quantities

(a) Let $[A] = \mathrm{M^aL^bT^c}$ and $[B] = \mathrm{M^dL^eT^f}$ such that $a \neq d,\ b \neq e$ and $e \neq f$ as given. In that case, $\left[\frac{A}{B}\right] = \frac{\mathrm{M^aL^bT^c}}{\mathrm{M^dL^eT^f}} = \mathrm{M^{a-d}L^{b-e}T^{c-f}}$. In this dimensional expression $a-d,\ b-e$ and $e-f$ exist as per given condition and hence the mathematical operation is meaningful.,

(b) Dimensionally unequal quantities, already given, can not be added
(c) Dimensionally unequal quantities, already given, cannot be subtracted.
(d) Since option (a) is true hence this Option shall nt be applicable.

**Hence, (a) is the answer**

**Illustration 50:** As per Coulomb's Law $[F] = \left[\frac{Q^2}{\varepsilon_0 L^2}\right] \to [\varepsilon_0] = \frac{[Q^2]}{(\mathrm{MLT^{-2}}).\mathrm{L^2}} = \frac{[Q^2]}{(\mathrm{MLT^{-2}}).\mathrm{L^2}} = \frac{\mathrm{I^2T^2}}{\mathrm{ML^3T^{-2}}} = \mathrm{M^{-1}L^{-3}T^4I^2}$. Thus $[\varepsilon_0]$ matches with Option (b)

Now, from Maxwell's equation $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$. Therefore, $c^2 = \frac{1}{\mu_0\varepsilon_0} \to [\mu_0] = \frac{1}{[c^2][\varepsilon_0]} = \frac{1}{(\mathrm{LT^{-1}})^2\cdot(\mathrm{M^{-1}L^{-3}T^4I^2})} = \frac{1}{\mathrm{L^2T^{-2}}\cdot(\mathrm{M^{-1}L^{-3}T^4I^2})} = \frac{1}{\mathrm{M^{-1}L^{-1}T^2I^2}} = \mathrm{MLT^{-2}I^{-2}}$ . Thus $[\mu_0]$ matches with Option (c)

**Illustration 51:** In Question, dimension of current is used as A and not I, as per SI Units, and hence retained in illustration. Dimensionally energy stored in an inductor carrying current is $[E] = [L]\cdot\mathrm{A^2} = \mathrm{ML^2T^{-2}}$, it leads to $[L] = \mathrm{ML^2T^{-2}A^{-2}}$. Likewise, power of a resistor is $[P] = [R]\cdot\mathrm{A^2} = \mathrm{ML^2T^{-3}} \to [R] = \frac{\mathrm{ML^2T^{-3}}}{\mathrm{A^2}} = \mathrm{ML^2T^{-3}A^{-2}}$.

Further, from charge on a capacitor is dimensionally $[Q] = [CV] = \mathrm{AT}$.

Combining all these in given expression $\left[\frac{L}{RCV}\right] = \frac{[L]}{[R]\cdot[CV]} = \frac{\mathrm{ML^2T^{-2}A^{-2}}}{(\mathrm{ML^2T^{-3}A^{-2}}).\mathrm{AT}} = \mathrm{A^{-1}}$; this matches with option (c).

**Hence, (c) is the answers**

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