MECHANICS-II: Newton's Laws of Motion

Some real life situations are being brought out to relate concepts Newton's Laws of Motion to happening around us. ***First situation*** is of glass fill of water is kept on the table, when we move close to it want to drink we lift it. The urge of thrust is so strong that we do not realize that some effort is being made to lift the glass.

In ***Second situation*** a box of instrument is kept on the floor, near teacher's table in a class. These instruments are to be used by Physics teacher to explain concepts of mechanics to the class. A teacher enters the class and calls Two of the students in the class. The one of the student is unenthusiastic about it unwillingly makes effort to lift the box and place it on teacher's table. The other student being enthusiastic to learn, happily joins hands to lift the box and place it on table. Expression of both the students clearly depict relative difference in their efforts to do the same work, together.

In ***Third Situation*** a man steals your bag on a railway platform and tries to run away. As soon as notice it, you will first race to reach the thief. A soon as you catch hold of the thief he tries to run faster. In an effort to be successful have to apply extra force so that the thief does not escape.

In ***Fourth Situation*** an object is released on a smooth inclined surface joining a smooth horizontal surface. The object continues to slide unless it is obstructed by another object. Interaction between the sliding object and obstruction involves forces.

Fifth Situation calls for visualization of ride Sky Wheel, one feels of weightlessness when the cradle in which we are sitting descends on the periphery of the sky wheel, while feel gaining wait when the wheel ascends.

More of such situations can be observed in day to day experiences to visualize as to how does mass, force and acceleration are coming into play. Discussion to follow start with Newton's Laws of Motion, a subject matter of ***classical mechanics or Newtonian Mechanics***, and is the basic concept behind these observations. It recognizes existence of an external force that can change state of rest or motion of an object. It is a set of Three Laws : Newton's First Law of Motion – also known as ***law on inertia***, Newton's Second Law of Motion – also known as ***law of acceleration***, Newton's Third Law of Motion – also known as *law of reaction or Cause and Effect*. Each of these law and associated mathematical concepts are elaborate below.

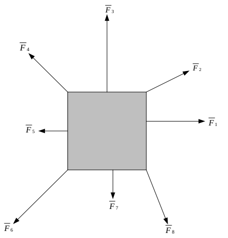
Newton's First Law of Motion (NFLM) : ***A body in an inertial frame of reference continues to be in a state of rest or motion with a constant velocity so long it is in a state of equilibrium***. Every body in this universe is experiencing force of one or the other kind from its surrounding. Such a situation cannot be called a state of No force or Zero Force. But, *when all the forces (a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is (a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is Zero and is like a state of No Force*.

Forces acting at any point on an object shown can be conceptually shown in a manner where end of a vector is beginning of another vector. In case of equilibrium, it is a closed polygon, having zero resultant of all the forces. While in case of in-equilibrium it is an open polygon, having resultant of all the forces, having a resultant i.e, an equivalent of all the forces represented by a vector joining the starting point of the first vector and end point of the last vector in the sequence of open polygon.

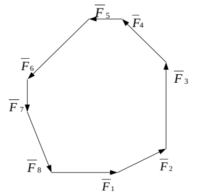
All the forces acting on the object, as shown in the figure below, can also be represented in star like formation. Mathematically, resultant of these forces is :

$$\bar{F}_{eq} = F_x\hat{\imath} + F_y\hat{\jmath} = \sum_{k=1}^{k=n}\left(F_{xk}\hat{\imath} + F_{yk}\hat{\jmath}\right);$$

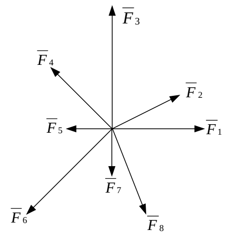
Here, F_{xk} is the *x*-component of k^{th} force, F_{yk} is the *y*-component of k^{th}. ***In case of equilibrium*** $F_{xk} = F_{xk} = 0$ or $\left|\boldsymbol{F_{eq}}\right| = \mathbf{0}$ **,** while ***in case of in-equilibrium*** either F_{xk} may be zero or F_{xk} may be zero, but both cannot be zero, i.e. $\left|\boldsymbol{F_{eq}}\right| \neq \mathbf{0}$.



Representation of Forces on a Object

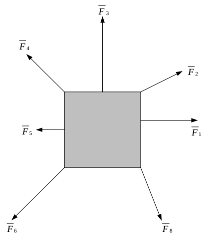


Closed Polygon of Vectors

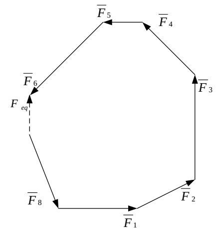


Forces on Cartesian plane

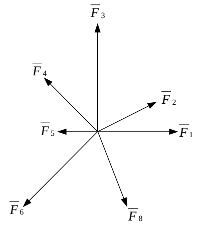
An object on a Horizontal Plane Acted upon by Eight Forces (Equilibrium Condition)



Representation of Forces on a Object



Open Polygon of Vectors



Forces on Cartesian plane

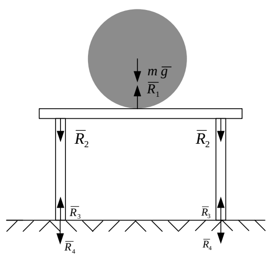
An object on a Horizontal Plane Acted upon by Seven Forces (In-equilibrium Condition)

Newton's Second Law Of Motion (NSLM): The conditions of in-equilibrium in the above illustration gives rise to proposition of Newton's Second Law of Motion, to provoke a thought as to what would happen. Newton propounded the effect of in-equilibrium of forces acting on an object as Second Law of Motion which states that : ***in an inertial frame of reference net force acting on an object causes an acceleration of the object in the direction of the force, such that Force is equal to product to the mass of the object and the acceleration***. Mathematically, it is expressed as: $\bar{F} = m\bar{a}$. This has led to evolution of a new term ***Momentum*** ($\bar{p} = m\bar{v}$) and rate of change of momentum: $\bar{F} = m\bar{a} = m\frac{d\bar{v}}{dt} = \frac{d}{dt}(m\bar{v}) = \frac{d}{dt}(\bar{p})$. This definition of rate of change of momentum becomes very useful in analysing propulsion of rockets which eject mass during their motion and will be analysed as we proceed in study of dynamics.

Newton's Third Law of Motion (NTLM): In fourth situation, visualized in the beginning of this, the sliding object exerts a force (Cause) on the obstructing object. But, what happen to the sliding object (Effect) as a consequence. This was analysed by Newton and propended as **Third Law** which states that : ***when an object exerts a force (action or cause) on another object, the second object simultaneously exerts an equal-and-opposite-force (reaction or effect) on the first object***.

Newton's Third Law of motion has its manifestations in Two forms, first is in Inertial Frame of Reference (IFOR) w.r.t which object is in state of rest. And second is Non-inertial Frame of Reference (NFOR) where frame of reference is itself accelerating.

Newton's Third Law in IFOR:



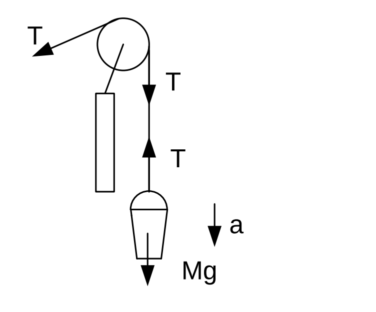
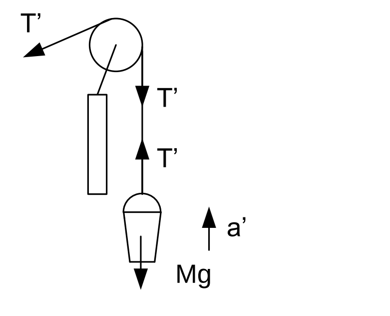
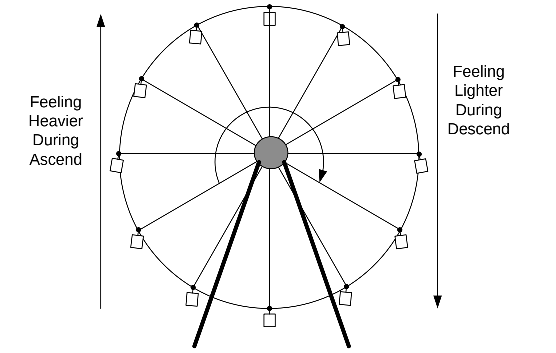
A Ball of Mass *m* is experiencing a gravitational acceleration $\bar{g}$. But, the acceleration is stopped by the table top on which it is kept . As a result, as per NSLM, the table top exerts a force ***R₁*** on the ball to stop the acceleration of the ball under gravity and thus ball stays at rest on the table top. It is a case of equilibrium in IFRM. Accordingly, the ball is in a state of equilibrium such that $m\bar{g} = \bar{R}_1$.

It is also observed that, despite the ball on the rectangular Table Top, placed at its centre, which is exerting an upward force $\bar{R}_1$, the table top remain at a state of rest on the earth's surface, the IFRM. As per NFLM this is possible only when $\bar{R}_1 = 2\bar{R}_2$; here, $\bar{R}_2$ is the force exerted on pair of table legs, one behind the other.

Despite $2\bar{R}_2$ force on the table legs, their motion is stopped by the earth's surface, an IFRM, on which the table is kept. This can only happen in a state of equilibrium i.e. $2\bar{R}_2 = 2\bar{R}_3$. Here, $\bar{R}_3$ is force exerted by the earth's surface on each pair of legs. Thus reaction (effect) of the table top, and the earth's surface in IFRM is equal to action (cause) i.e. weight of the ball i.e. $m\bar{g} = 2\bar{R}_2 = 2\bar{R}_3$.

Here, for a moment discussions on $2\bar{R}_4$ shown in the figure are put on hold till discussions on NTLM in NFOR, to follow, are completed. But, it can be realized from the impression of legs of the table, that it leaves, on bare ground surface. These impressions are visible when table is removed, and is indicative of force causing depression on the surface in the form of the impressions.

Newton's Third Law in NFOR: It is an excellent example of out of box visualization of scientific principles in surrounding. Those living in rural background must have experienced that when a bucket is released in a well, its weight for a moment apparently decreases. On the contrary, when bucket is pulled out of well it requires more force than that required to hold it stationary in hair. Similar experience one gets when during a sky-wheel ride.

 $T + Ma = Mg$	 $T' - Ma' = Mg$	
Experience of a Bucket Being Dropped in a Well	Experience of a Bucket Being Dropped in a Well	Experience on a Sky-wheel

When bucket is released in the well, it is descending down with an acceleration $\boldsymbol{a}$ w.r.t. earth, the IFRM. Thus, the bucket NFRM. Now, if bucket, which is experiencing $\boldsymbol{g}$ has to be transformed into IFRM, it shall have to be subjected to a retardation $\boldsymbol{a}$ w.r.t. to itself. Thus $\boldsymbol{Ma}$, is virtual force shall have to be assumed. This will lead to $T = Mg - Ma = M(g - a)$. This virtual force is called ***pseudo force is against the direction of acceleration of NFRM,*** which in the instant case is bucket, and hence bucket is experienced to be lighter.

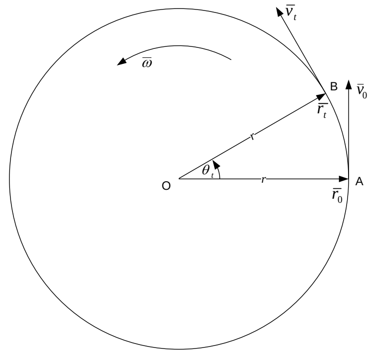
Likewise, when bucket is pulled out of well with an acceleration ***a'*** w.r.t earth the IFRM, the pseudo force would add to gravitational pull and thus $T' = Mg + Ma' = M(fg + a')$. This is conformance with experience of additional force, or effort, to pull out the bucket, as compared to that of holding it in place in the well.

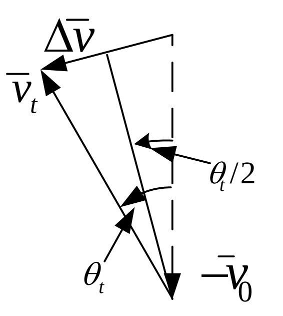
This stipulation of Pseudo Force is transformation of Non-inertial Frame into state of virtual equilibrium like that of IFRM, where problem is transformed into application of NFLM.

This is now appropriate stage to examine what happens to effect $2\bar{R}_4$ shown in the figure having a ball kept on the table. Earth surface was considered to be IFRM. The $2\bar{R}_4$ effect of $2\bar{R}_3$ should be causing acceleration earth. Accordingly, this problem also should have been analysed on the lines of NTLM in NFRM But, $\boldsymbol{M_e}$ us very large as compared to $\boldsymbol{M}$ ($\boldsymbol{M_e}$,>>$\boldsymbol{M}$) such that acceleration of earth ($\boldsymbol{a_e}$) as per NSLM would be $a_e = \frac{Mg}{M_e} \rightarrow 0$. This is the reason that despites earth being n constant acceleration, due to rotatory and revolving motion, which again has much smaller angular speed that the physical objects being observed, is taken as IFRM. Circular and rotatory motion shall be discussed little later.

Next comes sky wheel where we find that cradles are radial when they are at top of the wheel or on the bottom of the wheel. Otherwise, they remain suspended with a tilt outwards. Understanding, the cause of this observation requires concepts of circular motion.

Uniform Circular motion: In this a particles is taken to be *revolving around a fixed point with a constant radius and with a constant angular speed* $\bar{\omega} = \frac{d\bar{\theta}}{dt}$, *a vector quantity*, here $\bar{\theta}$ is the angular displacement on a plane a vector quantity. Likewise, there exists angular acceleration $\bar{\alpha} = \frac{d\bar{\omega}}{dt}$, and its will be used while working with rotational motion. Since angles are measured in clockwise direction and hence $+\bar{\omega}$ is upwards, coming out of a surface, and $-\bar{\omega}$ is inwards, entering a surface. This circular motion is analysed below.

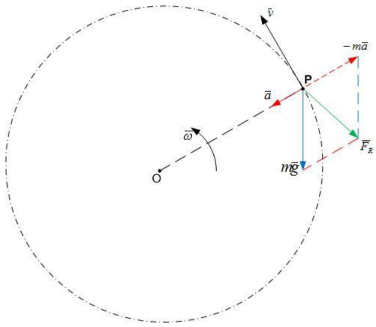




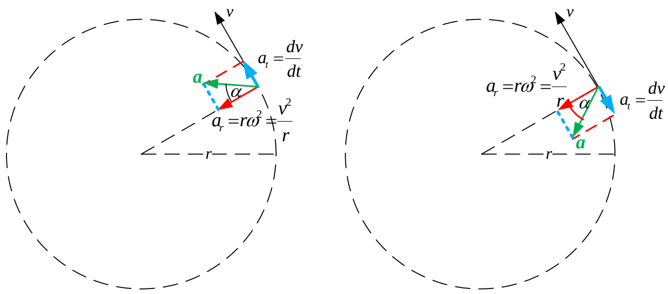
Linear velocity at any instant is $\bar{v} = \bar{\omega} \times \bar{r}$, a vector cross product.. Since, axis of rotation ($\hat{\omega}$) and radius of rotation ($\hat{r}$) are perpendicular to each other hence. Accordingly, $v = |\bar{v}| = |\bar{\omega}||\bar{r}|$, $v = r\omega$; *since r and ω are constant, basic premise of circular motion, hence v is also constant*. Let, θ_t be the displacement of radial vector during time Δt when velocity vector of the particle changes from $\bar{v}_0$ to $\bar{v}_t$. Accordingly, $\Delta\bar{v} = \bar{v}_t - \bar{v}_0$. Since, v is constant in the vector diagram for $\Delta\bar{v}$, both v_t and v_0 forming an isosceles triangle are also constant at an angle $\theta_t = \omega\Delta t$ and geometrically length of the third side $\Delta v = r\theta_t = 2v\sin\frac{\theta_t}{2}$.

As such, acceleration of the article performing circular motion shall be: $a = \frac{\Delta v}{\Delta t}\Big]_{\Delta t \to 0} = \frac{2v \sin\frac{\theta_t}{2}}{\Delta t}$. Substituting v and Δt from the above: $a = \frac{2r\,\omega \sin\frac{\theta_t}{2}}{\frac{\theta_t}{\omega}} = r\omega^2 \frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}$. Further, geometrically $\frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}\Big|_{\Delta\theta_t \to 0} = 1$ and hence, $a = r\omega^2$. A close observation of the vector diagram reveals that, $\Delta\theta_t \to 0$, $\Delta\bar{v}$ tends to become perpendicular to instantaneous velocity of the particle, performing circular motion, i.e. radially inwards called ***centripetal acceleration a vector $\bar{a}$***. This centripetal acceleration is keeping the particle perform uniform circular motion, else the particle will run away. This experience can be obtained with water soaked in a wet handkerchief, when it is rotated holding its one end.

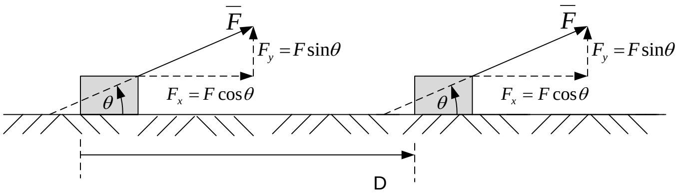
Review of NTLM in NFOR : This, is the point to discuss why cradles of sky wheel get automatically tilted outwards and observe as to how NTLM in NFOR automatically comes into play. The cradle, being fixed on to rim at point P of the sky-wheel is performing circular motion. Accordingly, it will experience a constant acceleration $\bar{a}$**,** and thus the cradle becomes a NRFM. When, the forces on the cradle are transformed to IFRM, an observer on the ground, an IFRM, this centripetal acceleration is considered as causing a pseudo force as shown in the diagram below. Resultant of the pseudo force and gravitational pull, by IFRM, is outwards and it causes tilting of cradle outwards, depending upon its magnitude and direction, except on the highest and lowest points where pseudo and gravitational force are collinear. If accidently cradle gets unhinged to the rim it would run away outwards, with instantaneous velocity at the time of release and would perform projectile motion under gravity, with no role of centripetal acceleration. There are numerous situations encountered in daily life, involving circular motion on IFRM and NFRM. Thus pseudo force is called ***Centrifugal Force***.



There is another situation when a particle is experiencing **non-uniform circular motion,** it can happen when velocity of particle performing circular motion has its velocity ***v*** is either accelerating or retarding in which case the trace of particle would be growing spiral or collapsing spiral, respectively. In this case net ***tangential acceleration*** of the particle shall be not focussed towards centre of the spiral, unlike uniform circular motion. The net acceleration shall be drifted forward from the centre of the spiral, i.e. in the direction of velocity in case of tangential acceleration; and backward, i.e. against the velocity, in case of tangential retardation. Mathematically, this is elaborated as $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2}$ and angle of drift is $\alpha = \tan^{-1}\frac{dv/dt}{r\omega^2}$, and supported with necessary illustration diagram. A similar effect of drift in gravitational pull is experienced when one moves from equator towards poles. But, it is not due to non-uniform circular motion, rather it is due to pseudo force on an object which remains un-displaced due to predominant gravitational force by the earth, while centrifugal force caused by rotation of the earth which is radial to the axis of rotation but not the gravitational force which is along the radial joining the object and centre of the earth. Thus as one questions the observations, more of integration of different concepts is involved, and such problems and articles would be found in references cited below.



Work, Power and Energy: A person lifting an object placed on floor and placing on a raised platform does a work. But, displacing an object displaced on a smooth horizontal surface is not. This requires to understanding **definition of work** (***W***) in physics according to it ***work is the product of Force and displacement caused by it in the direction of the force.*** Mathematically, it is expressed as DOT product of force and displacement, both of which are vectors, while work is scalar. $W = \bar{F} \cdot \bar{D} = FD\cos\theta$, where W is work, $\bar{F}$ is force acting on an object, $\bar{D}$ is the displacement of the object under influence of force $\bar{F}$ and θ is the angle between vectors $\bar{F}$ and $\bar{D}$. Thus, alternatively, work is product of displacement and force in the direction of displacement, a mathematical equivalent. The SI unit of Force in Newton, Distance us Meter and accordingly unit of Work is Newton Meter and also called Joule, which is more widely used in Heat. Dimensionally, $[W] = [F][D] = [MLT^{-2}][L]$. In the expression of work both θ and $\cos\theta$ are dimensionless, and accordingly dimension of work reduces to $[W] = [ML^2T^{-2}]$.

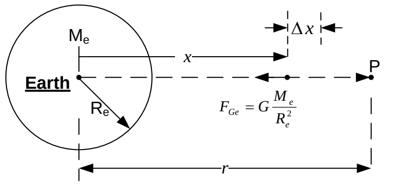


While **power (P) is rate of doing work**. It can be compared with two vehicles climbing on an inclined road starting from same point. Time taken vehicle A to reach destination is 2 Hours, while time taken by vehicle B to reach destination is 3 hours. Then in common parlance it is said that vehicle A has more power than the B. Accordingly, **Work** is mathematically expressed as $P = \frac{W}{T}$, and thus unit is Joules per Second or Joule/Sec. and dimensionally it reduces to $[P] = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$.f

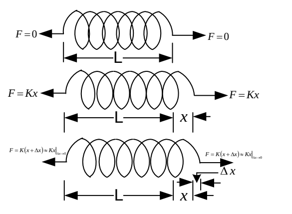
In another, situation vehicle A after one full tank filling in it makes 5 trips, but the vehicle B after full tank filling makes 7 trips than in common parlance ***energy*** of vehicle B is greater than that of A, and in this power has no consideration. Mathematically energy (***E***) is expressed as $E = P \times T$, and accordingly unit of energy is [(Joules/Sec) X (Sec) = (Joule)]. Energy in classical mechanics is considered to be conservative. The ***Law of Conservation of Energy (*****LCE*****)****states that energy can neither be created nor destroyed, it can be transformed from one form to the other*. In mechanics energy is considered to be two form; one is ***Potential Energy****, due to position of an object* and the other is ***Kinetic energy****, due to velocity of an object*. Here, discussion is limited to these two forms.

This is the time to review concept of work when $0^0 \le \theta < 90^0$ and $90^0 < \theta \le 180^0$. In the earlier case $0 < \cos\theta \le 1$, force $\bar{F}$ has a component in the direction of $\bar{D}$, and mathematically net work done is +ve. While in latter case $-1 \le \cos\theta < 0$ and force $\bar{F}$ has a component against the direction of $\bar{D}$, and mathematically net work done is -ve. Thus +ve work done by external agent exerting the force stores energy in the object. Here, it is to be noted that when an external force is so exerted that displacement is slow without causing any acceleration. Accordingly, as per NSLM, it is a case NTLM in IFOR, and thus there will be an equal and opposite force of reaction. Therefore, in terms of reaction or restraining force, work done by external force causing change of position is stored as energy in the object in IFRM; this is called potential energy. Two typical examples of potential energy are being elaborated here under:

Potential in Gravitational Field: Work done in moving a unit mass from Earth's surface, against the gravitational pull, up to a point P at a radial distance r : $PE = \sum \Delta w = \int_{R_e}^{r} \left(G\frac{M_e}{x^2}(-\hat{x}) \right) \cdot d\bar{x} = GM_e \left[\frac{1}{x}\right]_{R_e}^{r} = GM_e \left[\frac{1}{r} - \frac{1}{R_e}\right] = \frac{GM_e}{r}\Big|_{taking, \frac{GM_e}{R_e}=0}$. Here, gravitational force, is in direction $-\hat{x}$; while displacement Δx is also in direction $\hat{x}$ and shall be discussed, a little later in this series. In present context, till **Law of Gravitation** is discussed, in the formulation of PE acceleration due to gravity (***g***) as per GEM and NSLM is used instead and thus formulation of PE becomes: $PE = \sum \Delta w = \int_0^h m\bar{g} \cdot d\bar{h} = [mgh]_0^h = mgh$. Further, in above derivation, it is assumed that Potential at Earth's surface is Zero, instead at infinity, -ve sign is not used with $d\bar{h}$. Hence the PE at point P for a unit mass, calculated above, is called as relative **Potential** w.r.t. Earth's surface. The moment mass of the object being moved is considered, other than unity, it becomes **Potential Energy**.



Potential in Gravitational Field: Potential Energy: When spring is stretched/ compressed by length x it requires a force in the direction of push/pull= kx. An incremental pull/push over an infinitesimal length Δx would call upon external work, stored in the form of energy in the system: $\Delta W = -k\bar{x} \cdot \Delta\bar{x}$ external work, stored in the form of energy in the system: $\Delta W = -k\bar{x} \cdot \Delta\bar{x}$. Hence, $W = PE = \int_{x=0}^{l} -k\bar{x} \cdot d\bar{x} = -\frac{1}{2}k[x^2]_0^l = -\frac{1}{2}kl^2$. This is the absolute Potential Energy of the spring when stretched by length *l*. When *l=0,* its PE=0. It is to be noted that in the above derivation of **Potential Energy** of spring, primary parameter is spring constant k which control restraining force. This restraining force is being overcome by external force to cause displacement without acceleration, and there is no role of mass in it. This is equally valid when spring is compressed by an external force. Thus, in case of spring its elongation or compression change in PE remains uninfluenced, while in case of Gravitational field of change in PE of an object while descending reverses to that while ascending.



As per law of conservation of energy in classical mechanics, an object from a height shall while losing PE should get KE equivalent to the loss of PE and in turn velocity. But, a question may arise how to relate velocity component of KE to the PE. Here, TGEM comes into play. In simple case a ball of mass m, at a height has $PE = mgh$. And for a fall through height h, gain in velocity from an initial velocity $u = 0$, as per TGEM, $v^2 = 2gh$. Accordingly, ***equivalence of the two form of energy*** $\boldsymbol{KE = PE} = m\left(\frac{v^2}{2}\right) = \frac{1}{2}mv^2$.

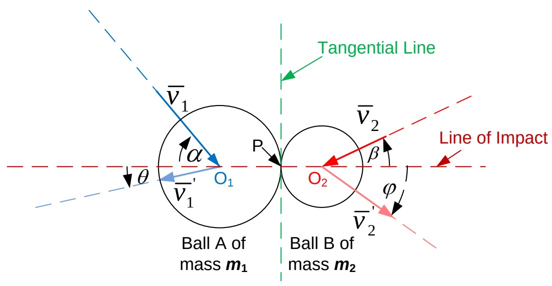
Momentum-Impulse-Collision: The concept of momentum was introduced during discussions on NSLM, which is being extended into ***Law of Conservation of Momentum*** **(LCM*)***. Together with this, concept Law of Conservation of energy will be helpful to conceptualize Impact or impulse and Collision.

An experiment is suggested, where a tightly closed box, with air inside, has a centrally placed lid. The box, is placed in sunlight and gets heated developing an internal force causing self-opening of the box lid; there is no external force. It is seen that the lid, having mass ***m***, moves with a high velocity, and the box, having mass ***M***, is pushed in a direction opposite to the lid with a velocity relatively much smaller than that of the lid. This follows laws of momentum according to which external force $\bar{F}=0=\frac{d}{dt}\bar{p}=\frac{d}{dt}\left((M+m)\bar{v}_0\right)$, and $\bar{P}$=0. Initial velocity $\bar{v}_0$ of the combined mass $M+m\neq 0$ and hence $\bar{v}_0=0$. When the lid open let the lid has velocity $\bar{v}_m$ and the container has velocity $\bar{v}_M$. Therefore, in absence of external force, $\bar{P}$ would continue to be Zero i.e. $\bar{P}=0=M\bar{v}_M+m\bar{v}_m$, or $\bar{v}_M=-\frac{m}{M}\bar{v}_m$. This mathematical analysis is in conformance with the example experiment cited above. This experiment can be tried and if need be some stove can be used in open, to heat the box, with a care to avoid any accident.

Nevertheless, in presence of an external force or no force , as per NSLM, $\bar{F}=\frac{d}{dt}\bar{p}$ and hence $\bar{J}=\int_{\bar{p}_i}^{\bar{p}_f}d\bar{p}=\int_{t_i}^{t_f}\bar{F}dt$; or $\bar{J}=\bar{p}_f-\bar{p}_i$. *This simplifies the analysis to initial and final momentum or in turn velocities* of each of the interacting components of the object on which force is applied.

Elastic Collision: It is pertinent to understand that in the case of elastic collisions on a horizontal plane essential condition for collision are – (a) Both the initial velocity vector should be on same plane and anti-parallel, and (b) Pre-collision positions of the object such that trace of displacements has a point of convergence called collision.

During impact there would be elastic deformation, but during deformation it would store energy, and release the stored energy during restoration of shape prior to collision. This is similar to the conversion of $PE\leftrightarrow KE$ in case of spring discussed above; here PE is due to shape and not the position . Therefore, *it must comply with the LCM and LCE of colliding objects*. In this experiment two balls of mass $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ moving with velocities $\bar{\boldsymbol{v}}_1$ and $\bar{\boldsymbol{v}}_2$ collide and are supposedly known. After the collision, both the masses without splitting, acquire velocities $\bar{\boldsymbol{v}}'_1$ and $\bar{\boldsymbol{v}}'_2$, which are unknown. This is essential condition to determine four unknown variables. The above case is analysed below with a generic example of oblique collision in two dimensions, and it has been simplified by reducing variables for collision in one dimension. This simplification requires identifying ***line of impact***, *a line joining centres of the two spherical balls* (the colliding objects), ***point of collision P***, on line of impact where the two Ball touch each other. **Tangential line** is the line perpendicular to the line of impact at the point of collision. Direction of velocity vectors of both the balls pre- and post-collision are shown in the figure. There is no transfer of momentum on the tangential line, while all the interaction during collision is along the line of impact.



Since, velocity is a vector it has two parameters and, therefore, the Two unknowns vectors $\bar{\boldsymbol{v}}'_1$ and $\bar{\boldsymbol{v}}'_2$ shall have Four parameters to be determined to find complete solution. Since, each vector can be resolved along two perpendicular directions and formulate four equations, a necessary condition for the solution.

Mathematical analysis of elastic collision to determine post-collision velocities, the unknown, follows. Since it is a dynamic interaction along line of impact, hence velocity components of colliding ball long this line pre- and post-collision shall remain unchanged; $\mathrm{v}_1\sin\alpha=\mathrm{v}'_1\sin\theta$, and $\mathrm{v}_2\sin\beta=\mathrm{v}'_2\sin\varphi$. While, as per LCM, $m_1v_1\cos\alpha+m_2v_2\cos\beta=m_1v'_1\cos\theta+m_2v'_2\cos\varphi$. Likewise, as per LCE total KE shall remain unchanged; thus $\frac{1}{2}m_1v_1^2\cos^2\alpha+\frac{1}{2}m_2v_2^2\cos^2\beta=\frac{1}{2}m_1{v'_1}^2\cos^2\theta+\frac{1}{2}m_2{v'_2}^2\cos^2\varphi$.

Likewise, as per equation of LCE, $\frac{1}{2}m_1\left(v_1^2\cos^2\alpha-{v'_1}^2\cos^2\theta\right)=\frac{1}{2}m_2\left({v'_2}^2\cos^2\varphi-v_2^2\cos^2\beta\right)$. Second equation is $m_1(v_1\cos\alpha-v'_1\cos\theta)(v_1\cos\alpha+v'_1\cos\theta)=m_2(v'_2\cos\varphi-v_2\cos\beta)(v'_2\cos\varphi+v_2\cos\beta)$. Dividing, this transformed equation of LCE with that of LCM: $(v_1\cos\alpha+v'_1\cos\theta)=(v'_2\cos\varphi+v_2\cos\beta)$. This equation deduces to $(v_1\cos\alpha-v_2\cos\beta)=(v'_2\cos\varphi-v'_1\cos\theta)=-(v'_1\cos\theta-v'_2\cos\varphi)$. This equation is interpreted as ***velocity of approach is equal to velocity of separation of colliding objects, and is a corollary of combined LCM and LCE (Corollary 1).***

Thus from the two equations above, one from LCM and the other from corollary 1, it is a *solution of linear simultaneous equations*, needed to arrive at :

$v_1' \cos\theta = \frac{(m_1-m_2)v_1\cos\alpha+2m_2v_2\cos\beta}{m_1+m_2}$ [*Multiply* $\boldsymbol{m_2}$ *to equation from Cor.1 and subtract it from equation from LCM*]

$v_2' \cos\varphi = \frac{2m_1v_1\cos\alpha+(m_1-m_2)v_2\cos\beta}{m_1+m_2}$ [*Multiply* $\boldsymbol{m_1}$ *to equation from Cor.1 and subtract it from equation from LCM*].

These two velocity components of colliding balls along the line of impact together with velocity equivalence along tangential line, brought out in the beginning of this mathematical analysis, is enough to determine the vectors $\overline{\boldsymbol{v}}_1'$ and $\overline{\boldsymbol{v}}_2'$

Determination of ***impact of collision*** on each of the colliding balls uses LCM represented as: $m_1(v_1\cos\alpha - v_1'\cos\theta) = m_2(v_2'\cos\varphi - v_2\cos\beta)$. Here, impact on collision ball of mass A is $J_1 = m_1(v_1\cos\alpha - v_1'\cos\theta)$, and impact on ball B is $J_2 = m_2(v_2\cos\beta - v_2'\cos\varphi)$, and as per LCM $J_1 = -J_2$, this is in conformance with NTLM ***(Corollary 2).***

Non-elastic Collision: it is different from elastic collision in respect of restoration of shape of colliding objects after collision; in non-elastic collision original shape or position is not restored. This calls for introduction of a new term ***e Coefficient of Restitution***. This can be compared with a mass ***m*** dropped from a height H on a horizontal surface bounces back to a height ***h***. In such a case $e = \sqrt{\frac{h}{H}}$ and as per LCE $e \leq 1$. From TGEM, $h = \frac{v^2}{2g}$ and $H = \frac{u^2}{2g}$. Accordingly, $e = \frac{v}{u}$, here $\boldsymbol{u}$ is the velocity of the mass at the time of impacting on the horizontal surface and $\boldsymbol{v}$ is the velocity of the mass leaving the surface after impact. Looking it from the case of elastic collusion, $\boldsymbol{u}$ is the velocity of approach to the horizontal surface where impact is taking place and $\boldsymbol{v}$ is the velocity of separation from the surface. Taking energy considerations, kinetic energy pre-collision $KE_1 = \frac{1}{2}mu^2$ and post-collision $KE_2 = \frac{1}{2}mv^2$; or $\frac{v}{u} = \sqrt{\frac{KE_2}{KE_1}} = e$, alternately, $KE_2 = e^2.KE_1$. Extending the analogy of non-elastic collision, in context of LCM in IFRM where horizontal surface of mass ***M*** remains static having Zero velocity, $p_i = kp_f$. Here, $p_i = mu$ and $p_f = mv$. Accordingly $\frac{p_f}{p_i} = \frac{1}{f} = \frac{v}{u} = e$; or $mv = e \cdot mu$, or $\boldsymbol{p_f} = \boldsymbol{e} \cdot \boldsymbol{p_i}$ in other words **in non-elastic collision post-collision momentum is equal to coefficient of restitution multiplied to pre-collision momentum.**

This analogy can be applied to oblique collision where equations of velocity equivalence along tangential line remain unchanged while equations of LCM and LCE would be: $e(m_1v_1\cos\alpha + m_2v_2\cos\beta) = m_1v_1'\cos\theta + m_2v_2'\cos\varphi$; as per LCM- for non-elastic collision and it leads impact equation as : $m_1(ev_1\cos\alpha - v_1'\cos\theta) = m_2(v_2'\cos\varphi - ev_2\cos\beta)$. Further, as per LCE the energy balance is: $e^2\left(\frac{1}{2}m_1v_1^2\cos^2\alpha + \frac{1}{2}m_2v_2^2\cos^2\beta\right) = \frac{1}{2}m_1{v_1'}^2\cos^2\theta + \frac{1}{2}m_2{v_2'}^2\cos^2\varphi$. Alternatively, $m_1\left(e^2v_1^2\cos^2\alpha - {v_1'}^2\cos^2\theta\right) = m_2\left({v_2'}^2\cos^2\varphi - e^2v_2^2\cos^2\beta\right)$, or $m_1(ev_1\cos\alpha - v_1'\cos\theta)(ev_1\cos\alpha + v_1'\cos\theta) = m_2(v_2'\cos\varphi - ev_2\cos\beta)(v_2'\cos\varphi + ev_2\cos\beta)$. Combining thus equations with impact equation: $(ev_1\cos\alpha + v_1'\cos\theta) = (v_2'\cos\varphi + ev_2\cos\beta)$, it leads to equation of velocity of approach and separation as: $e(v_1\cos\alpha - v_2\cos\beta) = -(v_1'\cos\theta - v_2'\cos\varphi)$, transformation of Corollary 1 to non-elastic collision. This equation together with LCM for the case leads to:

$v_1'\cos\theta = \frac{(m_1-\cdot em_2)v_1\cos\alpha+(1+e)m_2v_2\cos\beta}{m_1+m_2}$ [*Multiply* $\boldsymbol{m_2}$ *to equation from Cor.1 and subtract it from equation from LCM*]

$v_2'\cos\varphi = \frac{(1+e)m_1v_1\cos\alpha+(m_1-em_2)v_2\cos\beta}{m_1+m_2}$ [*Multiply* $\boldsymbol{m_1}$ *to equation from Cor.1 and subtract it from equation from LCM*].

This is the most general equation for collision, and gets easily transformed to specific case in isolation or in multiple combination of the following options:

a. Elastic collision by choosing $e = 1$, **b.** One dimensional Collision by choosing $\alpha = 0$ and $\beta = 0$
c. Collision with static object by choosing $v_2 = 0$, **d.** Collision of two bodies of uniform masses by choosing $m_1 = m_2$

Rocket Propulsion: This is another interesting case of change in momentum in IFRM, where rocket, without any external force, is self-propelled to move against gravity by ejection of mass at ***r*** kg/sec in opposite direction, a consequence of NTLM. Let initial mass of the rocket be $(\boldsymbol{m_i})$ and it is moving at a velocity $(\boldsymbol{v_i})$, thus initial momentum be $p_i = m_i \cdot v_i$, a product of two variables dependent upon time. Since there is no external force and hence : $\frac{d}{dt}p_i = 0 = \frac{d}{dt}(mv) = m\frac{d}{dt}v + v\frac{d}{dt}m$. Accordingly, at any point of time (***t***) during propulsion $p_i = m_i v_i = p = mv = (m_i - rt)v_t$. Thus, velocity of the rocket at any time (***t***): $v_t = \frac{m_i v_i}{(m_i - rt)}$. This reduces velocity of rocket v as a function of time, where m_i, v_i and r are constants. Therefore, $\int_0^v dv_t = \int_0^t \frac{m_i v_i}{(m_i - rt)} dt$; $v = \frac{mv}{r} ln\left(\frac{m_i}{m_i - rt}\right)$. Taking ejection of mass w.r.t. rocket being propelled at velocity $\boldsymbol{u}$, as per LCM in NFOR: $0 = ru - mv$, or $\frac{mv}{r} = u$. Substituting in the above equation , velocity of rocket, starting from rest, at any time later is $v = u\ ln\left(\frac{m_i}{m_i - rt}\right)$. It is a case of simple algebraic manipulations of equations, formulated from concepts of physics, to determine end result.

Summary: *Analysis of varieties of problems, representing different situation involve concepts of Newton's Laws of Motion, Circular Motion, Work-power-energy, and conservation of energy and momentum. Many such situations can also be observed in real life and taking problems from the cited as Reference would help to build an insight in the phenomenon occurring around. A deeper journey into the problem solving would make integration and application of concepts intuitive. This is absolutely true for any real life situation which requires multi-disciplinary knowledge in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person.*

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