Mechanics – Part I: Kinematics

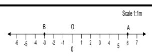
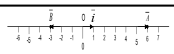
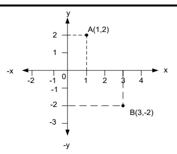
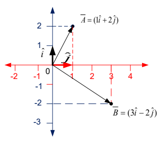
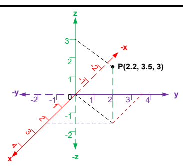
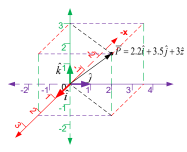
Human eye perceives that object first which is moving w.r.t its surroundings. This becomes starting point of understanding kinematics.Kinematics is a study of motion of a particle or a body, without getting into cause of motion, i.e. the force or torque. When cause of motion, i.e. force or torque, is an integral part of the study it is called dynamics. Galileo, was the first experimental physicist, who formulated basic equations of motion called 'kinematics'.

Study of motion involves identifying ***position of an object***. Motion It can be changing along a straight line and can be simplified into one *One dimensional motion*. The moment motion is on a curved path it is a matter of consideration whether it is simpler consider the path on a plane i.e. *Two dimensional motion*. Likewise, if path of motion cannot be aligned on a plane then it is spatial motion or *Three dimensional motion*. Thus based on extent of space involved motion can be classified as linear motion parabolic motion, circular motion, elliptical motion, spiral motion, oscillatory motion and so on. Each of these motion will be analyzed as the journey proceeds.

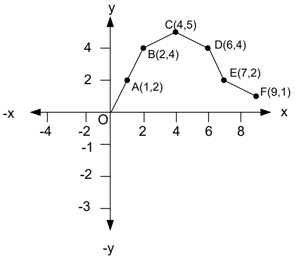
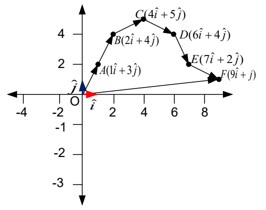
In kinematics every object is assumed as a point object, while in real geometry every object has a geometry and thus it violates the assumption. Despite, relevance of kinematics cannot be ruled out. Thus, practical aspect of geometry has been incorporated by definition two terms (a) *rectilinear motion* which pertains to a point, and (b) *translational motion* it pertains to an object. In translational motion there is no relative motion between different parts or particles of the object. This helps to idealize any physical object, which satisfies conditions of translational motion, into a point object. Cases, where condition of translational motion is not valid are analyzed as rotational motion and explained with **D' Alembert's Principle** later in Part-III.

Easiest comparative visualization of translational motion is an object with a definite base moving on a surface, and rotational motion is an ball rolling on a surface, every particle of the ball is having a relative motion with respect to the Rotational motion is a subject matter of dynamics to be illustrated in successive chapters.

Understanding change in position is beginning of exploring kinematics. Position is expressed with the help of coordinates or vectors (1D, 2D or 3D) which is a correlation of an object with a reference systems. Position of a point in 1D is like use of a number line, while representations in 2D and 3D is done using Cartesian coordinate system. These vectors can also represented in polar coordinates, which is separately dealt with in Chapter III Foundation Mathematics, and shall be elaborated in Physics wherever necessary.

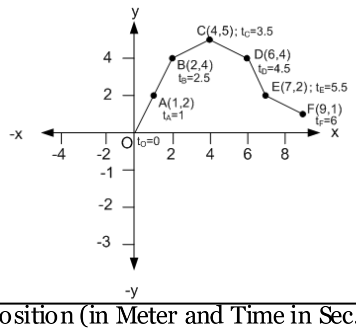
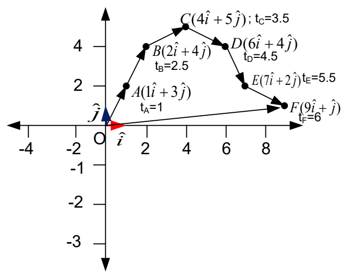
 Scale: 1 Part is equal to 1 Meter O: is the reference point A: is right of O by 6 Meter B: is left of O by 3 Meter	 $\hat{\imath}$: Unit Vector $\bar{A}$: Vector represented by OA $(= 6\hat{\imath})$ $\bar{B}$: Vector represented by OB $(= -3\hat{\imath})$	 Point A & B on x-y plane as: Point A: (1,2) where, x=1, 5=2 Point B: (3,-2) Where, x=3, y=-2	 Vector $\bar{A}$ and $\bar{B}$ on $\hat{\imath}-\hat{\jmath}$ plane Position Vector of point A is $\vec{A} = \hat{\imath} + 2\hat{\jmath}$, and is represented by line OA. Likewise position vector of point B is $\vec{B} = 3\hat{\imath} - 2\hat{\jmath}$ is represented by OB.	 Point P in x-y-z space is defined as (2.2, 3.5, 3) where, its resolution on three axes are x=2.2, y=3.5 and z=3	 Point P in $\hat{\imath}-\hat{\jmath}-\hat{k}$ space is represented by lone OP. Position vector of point P is $\overline{OP} = 2.3\hat{\imath} + 3.5\hat{\jmath} + 3\hat{k}$. Its resolution along three orthogonal directions is vectors $2.3\hat{\imath}$, $3.5\hat{\jmath}$ and $3\hat{k}$
Coordinates	Vectors	Coordinates	Vectors	Coordinates	Vectors
1D Representation		2D Representation		3D Representation	
Concepts of coordinates have been correlated together; may please zoom, as required, for legibility					

Next study is of ***change in position***. This change in position is gradual and always follows a path. Length of the path is called ***distance*** traced (***d***) and is scalar quantity. This distance may be different for different path. But, relative change in position is called ***displacement*** ($\vec{r} = r\hat{r} = \overrightarrow{OQ} - \overrightarrow{OP}$) of a point from position P to Q**.** It is a vector having a magnitude $\boldsymbol{r}$ and direction i.e. angle $\boldsymbol{\theta}$ in radians represented by unit vector $\hat{\boldsymbol{r}} = \boldsymbol{r}e^{j\theta}$. Mathematically, displacement is Final Position Vector MINUS initial position vector.

	Linear distance between points: $OA = \sqrt{5}$ m;: $AB = \sqrt{5}$ m; $BC = \sqrt{5}$ m; $CD = \sqrt{5}$ m; $DE = \sqrt{5}$ m; $EF = \sqrt{5}$m Route Distance: OF: $= OA + AB + BC + CD + DE + EF = 6\sqrt{5}$ m		**Displacements (in Polar form):** $\overrightarrow{OA} = \sqrt{5}e^{j\,63.5^0}m$;$\overrightarrow{OB} = \sqrt{5}e^{j\,63.5^0}$m; $\overrightarrow{BC} = \sqrt{5}e^{j\,26.5^0}$m;$\overrightarrow{CD} = \sqrt{5}e^{-j\,26.5^0}$m; $\overrightarrow{DE} = \sqrt{5}e^{-j\,63.5^0}$m;$\overrightarrow{EF} = \sqrt{5}e^{j\,-26.5^0}$;m $\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF}$ $= \sqrt{82}e^{j\,7.33^0}$m In Cartesian form it would be: $\overrightarrow{OF} = (1\hat{\imath} + 3\hat{\jmath}) + \big((2\hat{\imath} + 4\hat{\jmath}) - (1\hat{\imath} + 3\hat{\jmath})\big)$ $+\big((4\hat{\imath} + 5\hat{\jmath}) - (2\hat{\imath} + 4\hat{\jmath})\big)$ $+\big((6\hat{\imath} + 4\hat{\jmath}) - (4\hat{\imath} + 5\hat{\jmath})\big)$ $+\big((7\hat{\imath} + 2\hat{\jmath}) - (6\hat{\imath} + 4\hat{\jmath})\big)$ $+\big((9\hat{\imath} + 1\hat{\jmath}) - (7\hat{\imath} + 2\hat{\jmath})\big)$ $= 9\hat{\imath} + \hat{\jmath} =$ (Position Vector of Final Position) MINUS (Position Vector of Final Position). ***Conclusion:*** *Path of displacement does not influence net displacement.*
Representation of Distance (Scalar)		Representation of Displacement (Vector)	

The third is ***rate of change of position***. This rate of change position is what gets perceived; higher the rate of change, faster is the perception of the change. Having recognised the change in position in separate forms viz. scalar and vector the rate of change of position also has two forms ***speed*** (scalar) and ***velocity*** (vector). The diagram above is being used with additional information in respect of time-stamping in respect of each position to determine speed ($\boldsymbol{s}$) and velocity ($\vec{v}$) for the same trajectory For a better understanding. Mathematically it is represented as under:

$s = \frac{\Delta d}{\Delta t}$; since, distance ($d$) is scalar and hence Δd is also scalar, moreover t is scalar. Accordingly, s is also scalar. So long Δt is finite s so calculated is ***average speed*** and expressed as s_{av}. While, ***average velocity*** is $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$, where $\Delta\vec{\boldsymbol{r}}$ is displacement during time interval Δt and ***instantaneous velocity*** at time $\boldsymbol{t}$ is $\vec{v}_t = \frac{\Delta\vec{r}}{\Delta t}\Big|_{\Delta t \to 0} = \frac{d\vec{r}}{dt}$. There is another convention to represent average velocity, it is $\bar{v}$. But, as $\Delta t \to 0$, magnitude of $\Delta d = |\Delta\vec{\boldsymbol{r}}|$, therefore, *magnitude of instantaneous speed and magnitude of instantaneous velocity have always same magnitude, except that the earlier is scalar while the latter is a vector.*Graphical representation of average velocities is shown below. It needs to be noted in the diagram below that length of the line segments OA, AB, etc. are only representative of displacements and neither the speed nor velocity, they are used with time stampings of terminal points to calculate speed and velocity. In general expression is $\vec{v}_t$ written as $\vec{v}$.

	Speed between points: $s_{OA} = \sqrt{5}$ m /sec; $s_{AB} = \frac{\sqrt{5}}{\frac{3}{2}} = \frac{2\sqrt{5}}{3}$ m /sec; $s_{BC} = \sqrt{5}$ m /sec; $s_{CD} = \sqrt{5}$ m /sec; $s_{DE} = \sqrt{5}$ m /sec; $s_{EF} = \frac{\sqrt{5}}{\frac{1}{2}} = 2\sqrt{5}$ m /sec; and $s_{OF} = \frac{\sqrt{5}(1 + 0.\dot{6} + 1 + 1 + 1 + 2)}{6}$ $= 1.1\sqrt{5}$ m /sec		Velocity between points: $\bar{v}_{OA} = \sqrt{5}e^{j\,63.5^0}$ m /sec; $\bar{v}_{AB} = \frac{2\sqrt{5}}{3}e^{j\,63.5^0}$ m /sec; $\bar{v}_{BC} = \sqrt{5}e^{j\,26.5^0}$ m /sec; $\bar{v}_{CD} = \sqrt{5}e^{-j\,26.5^0}$ m /sec; $\bar{v}_{DE} = \sqrt{5}e^{-j\,63.5^0}$ m /sec; $\bar{v}_{EF} = \sqrt{5}e^{j\,-26.5^0}$ m /sec; and $\bar{v}_{OF} = \frac{\sqrt{82}}{6}e^{j\,7.33^0}$ m /sec;
Position (in Meter and Time in Sec.)	Speed Between Positions	Position (in Meter) and Time (in Sec.)	Velocity Between Positions
Rate of Change of Distance - Speed (Scalar)		Rate of Change of Displacement - Vector (Scalar)	

Next comes the ***rate of change of velocity***, and this is called ***acceleration***. We do not talk of rate of change of speed. *Reason for this will become more clear with journey into dynamics involving Newton's Law of Motion, a little later.* Like, average velocity and instantaneous velocity, acceleration is also analysed as average acceleration $\left(\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}\right)$. Here, $\vec{v}_2$ and $\vec{v}_1$ are velocities at instances t_2 and t_1 respectively. While, instantaneous acceleration $\vec{a}_t = \frac{d\vec{v}}{dt} = \frac{\Delta\vec{v}}{\Delta t}\Big|_{\Delta t \to 0}$. In general expression is $\vec{a}_t$ written as $\vec{a}$.

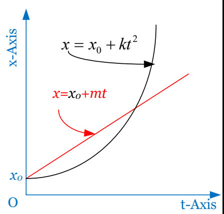
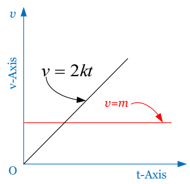
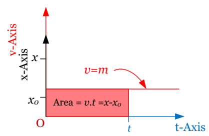
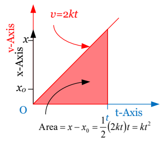
GALILEO's EQUATION OF MOTION (GEM): Galileo had studied motion of bodies and suggested three equation correlating displacement, velocity and acceleration, all of the three quantities being vectors. Velocity in a plane or in space can be expressed as Two or Three orthogonal vectors and analysed independently. Likewise, acceleration can also be analysed independently on the corresponding orthogonal unit vectors.

Motion In a Straight Line: In kinematics acceleration is considered to be constant. It is based on the fact that here cause of acceleration and/or its change is not the consideration, this is presently postponed until study of dynamics starts. *If acceleration, being a vector, is constant it constant it means it is constant in magnitude and direction both.* This restriction is first applied to motion in a straight line, where vector shall have either (+)ve value or (-)ve value depending upon reference direction. *Accordingly, expressions of variation in velocity and distance with time can be conveniently expressed in scalar form* as under :

$v = u + at$ ... (1) ; s= $ut + \frac{1}{2}at^2$ ... (2); and $v^2 = u^2 + 2as$ ... (3)

Note: *In most of the texts displacement, in above equations, is denoted as **s**. Accordingly, the same convention is being followed for convenience of readers.* ***In this text these three GEM shall be referred to as GFEM, GSEM and GTEM, respectively, as an abbreviations for Galileo's First, Second and Third Equation of Motion.***

With understanding of calculus and definitions of velocity $\left(v = \frac{dx}{dt}\right)$ and acceleration $\left(a = \frac{dv}{dt}\right)$ velocity shall be changing as long as $a \neq 0$, with exception that whatever be the value $a = C$. Thus relationships between, $x, t, v,$ and a can be visualized with $x - t$ and $v - t$ graphs for initial displacement at $t = 0$ from reference point O is x_0 and initial velocity is also 0, shall be as under.

<table>
<tr>
<td rowspan="2">
</td>
<td rowspan="2">
</td>
<td rowspan="2">$v = \frac{d}{dt}(x_0 + mt)$
$= m$
$v = \frac{d}{dt}(x_0 + kt^2)$
$= 2kt$</td>
<td>
</td>
<td>
</td>
<td rowspan="2">$x = \int vdt = mt + C$
$x|_{t=0} = m \cdot 0 + C = x_0$
$x - x_0 = vt$
$x = \int 2kt \cdot dt = kt^2 + C$
$x|_{t=0} = k \cdot 0 + C = x_0$
$x - x_0 = kt^2$</td>
</tr>
<tr>
<td>$v = m$</td>
<td>$v = 2kt$</td>
</tr>
<tr>
<td>Typical $x - t$ Graph</td>
<td colspan="2">$v - t$ Graph Derived from $x - t$ Graph</td>
<td colspan="3">$x - t$ Graph Reconstructed from $x - t$ Graph</td>
</tr>
</table>

Generalized form of equations listed above can also be obtained mathematically from basic definition of velocity and acceleration. Since, $a = \frac{dv}{dt}$; or, $adt = dv$, therefore, $\int_0^t adt = \int_u^v dv$;$a \int_0^t dt = \int_u^v dv$; $a[t]_0^t = [v]_u^v$; $a[t - 0] = [v - u]$. It is simplified into $\boldsymbol{v = u + at}$**; this is equation (1).**

Now with next level of integration of Eqn (1) w.r.t. time displacement in time t would be $s = \int_0^t vdt = \int_0^t (u + at)dt$. It leads to $s = \left[ut + \frac{1}{2}at^2\right]_0^t$; or $\boldsymbol{s = ut + \frac{1}{2}at^2}$**, this is equation (2).**

Third equation is since without variable t this can be evolved by replacing $t = \frac{\boldsymbol{v-u}}{\boldsymbol{a}}$ from eqn. (1) in eqn. (2). It leads to $s = u\left(\frac{\boldsymbol{v-u}}{\boldsymbol{a}}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 = \left(\frac{\boldsymbol{v-u}}{\boldsymbol{a}}\right)\left(u + \frac{\boldsymbol{v-u}}{\boldsymbol{2}}\right) = \left(\frac{v-u}{a}\right)\left(\frac{v+u}{2}\right) = \frac{\boldsymbol{v^2-u^2}}{\boldsymbol{2a}}$. It simplifies into $\boldsymbol{v^2 = u^2 + 2as}$**, this is equation (3).** Alternatively, distance covered in time (t) for a motion under uniform velocity is $s = \frac{(v+u)}{2}t = \frac{(v+u)}{2}\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a}$, it is of the sa.me form as Eqn. (3).

In these equations acceleration ***a*** *in direction of initial velocity* ***u*** *is treated as* ***+ve****, and will tend to give velocity at any subsequent moment, where* ***t*** *is* ***+ve****, time. While, acceleration when in direction opposite to that of* ***u*** *is taken as* ***−ve*** *and called* ***retardation or deceleration****.* Further, based on initial velocity at any instant (t_1) velocity at any subsequent instant (t_2), the time is taken as $t = t_2 - t_1$. Likewise, in any attempt to determine velocity at any previous instant (t_1) based on known velocity at any point of time (t_2), the time of previous instant will be $t_1 = t_2 - t$, velocity at time t_2 will be *u and velocity at* time t_1 will become v, and rest of the equations shall remain unchanged.

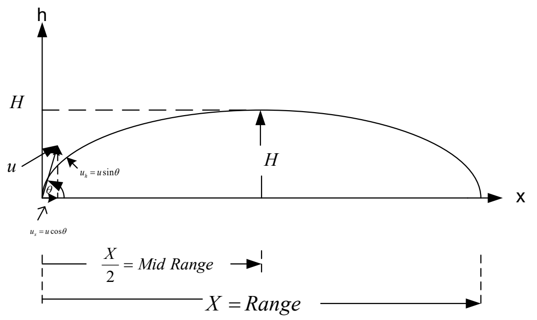
Motion Under Gravity: *In case of motion under gravity, the equations of motion get slightly modified in respect of notations for acceleration (a) and displacement (h), and instead acceleration under gravity (g) and depth (h) are used. Under gravity fall, (+) ve depth is measured downward while acceleration is also downwards i.e. toward earth the equations are of the form:*

$v = u + gt$ ... (4) ; $h = ut + \frac{1}{2}gt^2$ ... (5); and $v^2 = u^2 + 2gh$ ... (6)

But, in case of an upward throw initial velocity until it is rising is upward as also the height, but acceleration (g) is downward, hence being vector it value is $(-g)$. Thus it requires care to adopt proper sign convention since v, u, g and h are vectors.

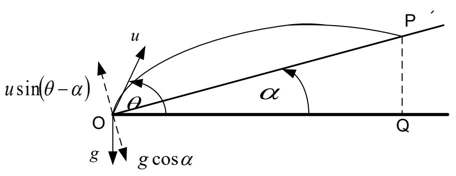
Thus,, maximum height $\boldsymbol{h}$ attainted by of a particle projected vertically with a velocity $\boldsymbol{u}$ upwards shall be when the velocity $\boldsymbol{v}$ will be Zero ($\boldsymbol{v}$=0) and this happen under retardation $-\boldsymbol{g}$. Accordingly, as per Eqn. (6) $0 = u^2 - 2gh$, or $h = \frac{u^2}{2g}$ and time to reach the height $\boldsymbol{h}$ would be, from Eqn. (4) would be $0 = u - gt$; or $t = \frac{u}{g}$, Further, total time taken by the particle to reach the ground be $T = 2t = \frac{2u}{g}$. This, value of T can be verified using Eqn. (5) us taking total displacement $h = 0$, when object return to the same height from which it was projected.

Motion in Plane: Representation of plane is best done using Cartesian coordinates. In this position, velocity, acceleration, and displacement is done using 2D coordinates with direction vectors $\hat{\imath}$ and $\hat{\jmath}$ along x-y axes respectively. Analysis of motion of a particle under gravity when projected at an angle with horizontal, called angle of projection is called ***Projectile Motion***. This can be simplified by resolving velocity in its Two components horizontal and vertical components. In projectile motion acceleration is only due to gravity along $(\vec{a}_y = -\hat{\jmath}g)$ and hence its horizontal component of acceleration is $\vec{a}_x = -\hat{\imath}g\cos 90^0 = -\hat{\imath}g \cdot 0 = 0$. Hence, horizontal component of velocity $\boldsymbol{v_x}$ does not change till it touches ground. Nevertheless, vertical component of velocity $\boldsymbol{v_h}$ remains under influence of $\boldsymbol{g}$. Different text books have come up with different theorems, articles and/or problems. Nevertheless, simplest and fundamental approach is to resolve projected velocity in horizontal and vertical direction and analyze them independently with convergence at a point as per statement of the problem or premise. Basic mathematical formulations remains confined to three equations of motion to vertical and horizontal components of motion.



a. Maximum height (H) that is attained by the projectile with an initial velocity $\boldsymbol{u}$ and angle of inclination θ is: $H = \frac{u^2 \sin^2\theta}{2g}$
b. Time of Flight is the time taken by the projectile, projected from ground, to reach back: $T = \frac{2u\sin\theta}{g}$
c. Distance from the point of projection, where the projectile shall touch the ground, Range: $X = u\cos\theta T = u\cos\theta \frac{2u\sin\theta}{g} = \frac{u^2(2\sin\theta\cos\theta)}{g}$ $= \frac{u^2\sin 2\theta}{g}$
d. Angle of projection (θ) for projectile, having initial velocity $\boldsymbol{u}$ to attain maximum height, using ***principle of Maxima-Minima***, is $\frac{dX}{d\theta} = \frac{u^2}{2g}\cos 2\theta = 0$; since u and $\theta \neq 0$, necessary condition is $\cos 2\theta = 0$; or $2\theta = \frac{\pi}{2}$; or $\theta = \frac{\pi}{4}$ or 45^0.
Alternately, in a simple maximum span X will be when $\sin 2\theta = 1$, the maximum possible value, it implies $2\theta = \frac{\pi}{2}$; or $\theta = \frac{\pi}{4}$ or 45^0.

This projectile motion leads to a wide range of problems available in text books. Two typical cases are a) a range on an inclined plane and, b)projection on an inclined plane, are elaborated below. It is to highlight that each of the case requires appropriate resolution of velocity and acceleration, to reach the solution.



Component of Velocity of The Particle Perpendicular to the Plan: $u\sin(\theta - \alpha)$

Retardation Due to Gravity Perpendicular to the Plane: $g\cos\alpha$

Therefore, Time of Flight T for the particle to strike the inclined plane: $\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$

During T particle will continue to travel horizontally through a distance OQ with a velocity: $u\cos\theta$

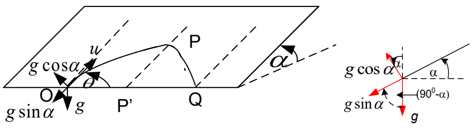
Therefore, Particle will strike the plane at point P, such that: $OP = \frac{OQ}{\cos\alpha} = \frac{(u\cos\theta)T}{\cos\alpha}$

$$= \frac{u\cos\theta}{\cos\alpha} \cdot \frac{2u\sin(\theta-\alpha)}{g\cos\alpha} = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

(a): Motion of a Particle Projected with Velocity $\boldsymbol{u}$ at an Angle θ Striking a Plane Inclined to Horizontal at an Angle α

The above case can also be transformed by eliminating variable t in motion of vertical and horizontal motion which will give equation of parabolic trajectory of projectile. Point of intersection of the trajectory and equation of line of shadow of the trajectory on the plane will be the point where the particle shall hot on the plane. ***This involves use of coordinate geometry.***

Another case is not much different, except that motion of article is on smooth plane inclined at an angle α with the horizontal.



Here, dotted lines represent lines of greatest slope, and is of great relevance.

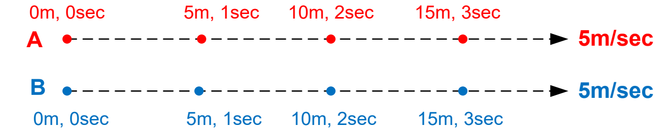
Acceleration component perpendicular to the plane ($g\cos\alpha$) has no role in the motion of particle on the smooth plane.

Component of acceleration along the plane $g\sin\alpha$ and the line of greatest slope and the component of initial velocity along the line will decide the time of descend $T\left(=\frac{2u\sin\theta}{g\sin\alpha}\right)$ of the particle to the horizontal surface.

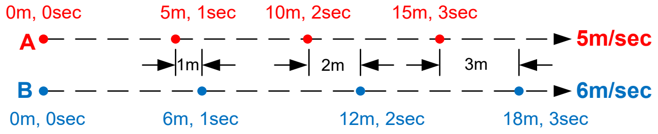
Having determined $\boldsymbol{T}$, height on the inclined plane (PP') and the span (OQ) can be determined as usual.

(b) Motion of a Particle Projected at an Angle θ with the Intersecting Edge of a Plane Inclined at an Angle α with the Horizontal

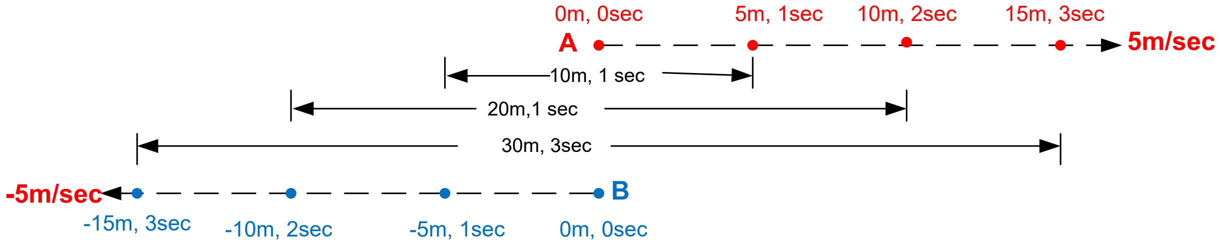
Relative Velocity: In this world existence of everything is relative i.e. difference between two quantities to be compared. An an example A is elder (difference of age), taller than B (difference of height) heavier by weight etc., and the antonym of the adjective applies to when B is compared to A on same parameters. Likewise, perceiving movement of an object w.r.t. its surrounding was the starting point of Kinematics, and is being analysed as ***relative velocity*** a vector quantity. Initially it is being done for Two objects having same starting point.



Case 1:Objects A and B moving in same direction, each at Velocity 5 m/sec



Case 2: Objects A and B moving in same direction at velocities 5m/sec and 6 m/sec, respectively



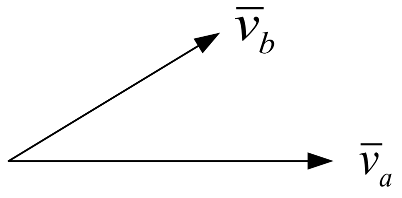
Case 3:Objects A and B moving in Opposite direction with velocities 5 m/sec and -5m/sec, respectively

In the above figure, velocities are expressed as scalar since they are along, parallel lines, virtually lines. Accordingly, displacement of the two objects , along the line, relative to each other is tabulated below.

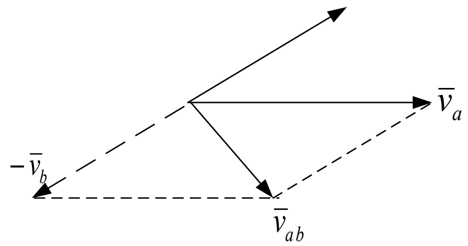
Time Stamp (Sec)	Case 1; Both A and B moving in Same Direction, with same Velocities				Case 2; Both A and B moving in Same Direction, with Velocities 5m/sec and 6 m/sec, respectively				Case 3; Both A and B moving in Opposite Same Direction, with Velocities 5m/sec and -5 m/sec, respectively			
	Displacement of Particles (m)		Relative Velocity of Particles (m/sec)		Displacement of Particles (m)		Relative Velocity of Particles (m/sec)		Displacement of Particles (m)		Relative Velocity of Particles (m/sec)	
	A w.r.t. B	**B w.r.t. A**	**A w.r.t. B**	**B w.r.t. A**	**A w.r.t. B**	**B w.r.t. A**	**A w.r.t. B**	**B w.r.t. A**	**A w.r.t. B**	**B w.r.t. A**	**A w.r.t. B**	**B w.r.t. A**
1 sec	**0**	**0**	**0**	**0**	**1**	**-1**	**1**	**-1**	**10**	**-10**	**10**	**-10**
2 sec	**0**	**0**	**0**	**0**	**2**	**-2**	**1**	**-1**	**20**	**-20**	**10**	**-10**
3 sec	**0**	**0**	**0**	**0**	**3**	**-3**	**1**	**-1**	**30**	**-30**	**10**	**-10**

Having established quantitative feel of the relative velocity which everyone encounters while moving on road where he observes vehicles passing by either in the same direction or oppositedirection.

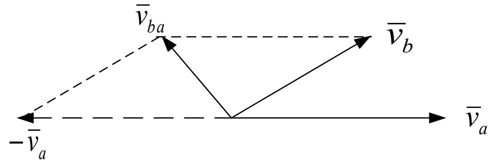
This concept of relative velocity is now being extended in vector form*; in** this the person experiencing relative velocity is called ***Observer, and whose relative velocity is being observed is called the ***Object***. It is graphically illustrated below, taking different direction of velocities of Observer and Object, where Observer is stated in a ***frame of reference***(FOR) from which Object is being observed.



Velocities of A - Observer, and B Object, with FOR Ground.

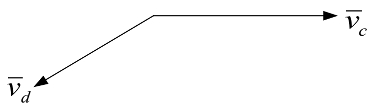


$\bar{v}_{AB}$: Relative Velocity of A w.r.t. B

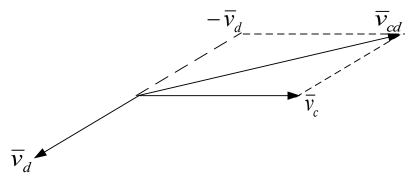


$\bar{v}_{BA}$: Relative Velocity of B w.r.t. A

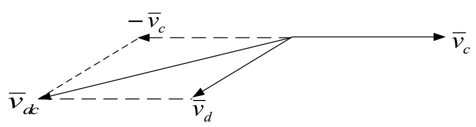
Relative velocities with $\bar{v}_{c=}\bar{v}_a$ and $\bar{v}_{d=}-\bar{v}_b$ follow similar treatment for $\bar{v}_c$ and $\bar{v}_d$, and is shown below:



Velocities of C- Observer, and D-Object, with FOR Ground

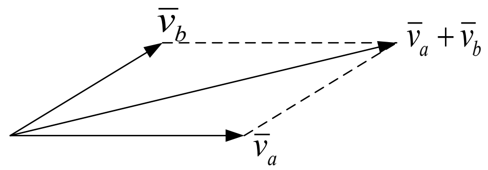


$\bar{v}_{CD}$: Relative Velocity of C w.r.t. D

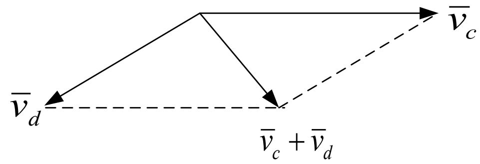


$\bar{v}_{DC}$: Relative Velocity of D w.r.t. C

There is another situation, when FOR of Observer is moving with reference to a stationary FOR and Object is moving w.r.t FOR of Observer. This situation is similar to a man moving inside a moving train *or* a bird flying inside a train.*In such a situation, velocity of the object with stationary FOR is sum of the two velocities of Observer with stationary FOR and velocity of Object with Observer as FOR,* as shown below.

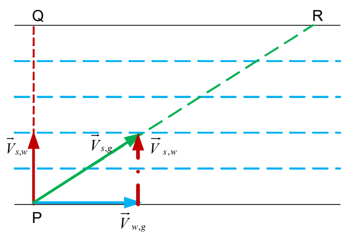


Velocity of Object B w.r.t Ground i.e. FOR of Observer A



Velocity of Object D w.r.t Ground i.e. FOR of Observer C

Some examples are used to illustrate the concept in real life. Taking a simple example with an assumption that Delhi Agra is a straight road with Mathura coming in between. Distance between Delhi to Mathura is 200 km, and moving forward another 100 km is Agra. Hence, distance between Agra to Delhi is 300 km. This is illustrated with a real life examples involving vectors, where sum of known velocities is required to be determined and where does the relative velocity. In case of a swimmer, in the figure, starts from point P and tries to swim across in direction PQ with a velocity $\tilde{V}_{s,w}$ w.r.t. water, taking it to be still, like that in a lake or a swimming pool, and he would reach Q. Now, the person swims in a river orienting himself across the , with same velocity, in a river with flowing at a velocity $\tilde{V}_{w,g}$ w.r.t. ground. In this case velocities of swimmer and water are with a common FOR, i.e. ground, and hence velocities would get added, accordingly, the man trying to swim right across the river reaches point R instead of point Q. An anecdote to represent

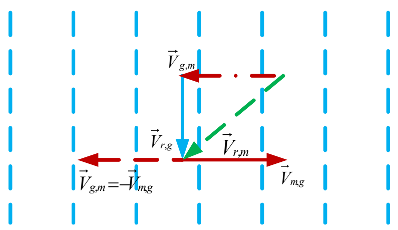


$$\vec{V}_{s,w} + \vec{V}_{w,g} = \vec{V}_{s,g}$$

cascading of vectors is given across is helpful to handle problems when velocities are given with some other frame of reference. Thus the swimmer out of experience intuitively chooses direction of swim to reach point Q, which a student of physics can do it in one try . A simple example distance between Delhi to Mathura (D_{SM}) is 200 km, and moving forward from Mathura to Agra ($D_{M,A}$) another 100 km is Agra. Hence, distance between Delhi to Agra (D_{DA}) is 300 km (=$D_{S,M}$ + $_{M,A}$).

Take another case when a person travels from Delhi to Agra, a 300 km distance ($D_{D,A}$), and is required to come backwards from Agra to Mathura, 100km ($D_{A,M}$ =-$D_{M,A}$). Then displacement of the man from Delhi is 200 km ($D_{D,A}$ =$D_{D,A}$+ $D_{A,M}$) and arithmetically it equates $(200 = 300 - 100)$.

This is illustrated with an example of direction of raindrops perceived by a person on the ground. When the person is still and wind is not blowing, the rain drop appear to be falling vertically. But, as the person starts moving the rain drops appear to be approaching him from forward direction, i.e. the direction of motion. The experience is same when he changes direction $0 \le \theta < 2\pi$. Here, both velocities, of man $\tilde{V}_{m,g}$ and rain drop $\tilde{V}_{r,g}$ have a common FOR i.e. ground. But, the rain drops are seen by the man, i.e. change of FOR. This is the case where relative velocity shall have to be determined, and is illustrated in the figure. An anecdote of cascading of vectors in this case also is given across is simplest to remember and handle problems with simple addition of vectors.

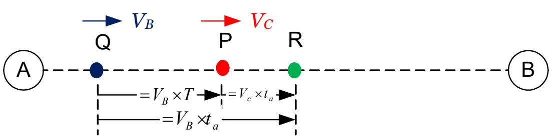


$$\vec{V}_{r,g} + \vec{V}_{g,m} = \vec{V}_{r,m}$$

Third situation could be when it is raining with a velocity $\bar{V}_{r,g}$, a windstorm arrives with a velocity $\bar{V}_{ws,g}$. In this velocity raindrop with respect to ground $(\bar{V'}_{r,g} = \bar{V}_{r,g} + \bar{V}_{ws,g})$ shall be determined, by addition of vectors, as done with the swimmer. With the velocity of rain drops so determined$(\bar{V'}_{r,g})$ apparent velocity of the rain drop to the man moving on the ground shall be $\bar{V}_{r,m} = (\bar{V'}_{r,g} - \bar{V}_{m,g}) = (\bar{V'}_{r,g} + \bar{V}_{g,m})$. In this case necessarily rain drops may not appear to becoming from front, as shown in previous example, the experience would depend upon direction of windstorm.

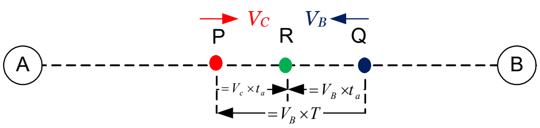
There is another interesting example which can be solved by classical method and using concept of relative motion and is illustrated through example as under.

Example: *Two towns A and B are connected by a regular bus service with a bus leaving in either direction at an interval every T minutes at a constant speed. A man cycling with a speed of* 20 *kmph in direction from A to B notices that a bus goes past him every* 18 *minutes on direction of his motion, and every* 6 *minutes on opposite direction. What is the interval T of the bus service and with what speed do buses on the road?*



Each of the cases is diagrammatically represented separately to illustrate mathematical equations relating associated quantities. When bus (Q) is moving from A to B in the direction of the cyclist (P). Let at any instant $t = 0$, a bus passes by the cyclist. Therefore, at this instant another following bus shall be behind the Bus and cyclist by a distance $= V_B \times T$, as per given interval, as per given periodicity of buses. The following bus shall pass by the cyclist after time $t_a = 12$ minutes, when he reaches at R and shall cover a distance $= V_c \times t_a$ and the bus shall cover a distance $= V_B \times t_a$. Hence, geometrically $V_B \times t_a = V_B \times T + V_c \times t_a \rightarrow (V_B - V_c)t_a = V_B \times T \rightarrow V_{B,C} \times t_a = V_B \times T$. It is to be noted that here, speeds are given which are scalar and are represented accordingly and not in vector notations. Despite, generic equation developed using classical mathematics in this case is also satisfying the conditions of relative velocity of Bus w.r.t. Cyclist.

Taking another case, when bus is moving in a direction opposite to that of the cyclist. In this case at an instant $t = 0$ the following bus is ahead of the cyclist at a distance $= V_B \times T$. The next bus that shall pass by the cyclist after a time $t_b = 6$ minutes, the cyclist shall move in direction of the bys by a distance $= V_c \times t_b$ and so also the bus shall move a distance $= V_B \times t_a$ toward cyclist to pass by him. Geometrically, $V_B \times T = V_B \times t_a + V_c \times t_a \rightarrow (V_B - (-V_c))t_a = V_B \times$



$T \to V_{B,C}' \times t_a = V_B \times T$. This reduces to simple algebraic problem of solution of Two simultaneous equations involving Two variables V_B and T, while V_c, t_a and t_b are given in the problem.

A clear understanding of these typical cases should be helpful in solving problems involving relation motion. In reality it is difficult to find a frame which purely stationary, since each may be so in a limited sense, but is in motion other object or frame. In such a situation relative velocity of two unconnected objects can be determined based on relative velocities of objects known to be connecting the target objects. All that it needs is to arrange the relative velocity vectors in a cascaded order and take their sum viz. $\vec{v}_{a,e} = \vec{v}_{a,b} + \vec{v}_{b,c} + \vec{v}_{c,d} + \vec{v}_{d,e}$.

Frame of Reference referred to above are those which are not accelerating i.e. either they are stationary or moving with a constant velocity and are called ***Inertial Frame of Reference (IFOR)***. This specific identity of FOR discussed above is considered essential, since soonafter this, study of Dynamics will require introductionand use of ***Non-inertial Frame of Reference (NIFOR).***

Conclusion: Varieties of problems, representing different situation that are generally encountered in real life are covered in the books cited as Reference. Best clue to solve problems in physics is to visualize the problem statement in surrounding, and then apply the known concepts to the problem. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving,and making formulations more intuitive rather than a burden on memory.

References:

1. *NCERT; PHYSICS, Text Book for Class XI (Part I and II), and Exemplar Problems.*
2. *S.L.Loney, The Elements of Statistics and Dynamics: Part 2 – Dynamics.*
3. *H.C. Verma; Concepts of Physics, (Vil 1 & 2).*
3. *Resnick, Halliday, Resnick and Krane; Physics (Vol I and II).*
4. *Sears & Zemansky; University Physics with Modern Physics.*
5. *I.E. Irodov; Problems in General Physics*

—00—

Play list of Videos on Contents of This Chapter

1. **Swayam Prabha, MHRD Project, IIT-PAL, CH_22 Physics, by Prof. Sanjeev Sanghi, IIT Delhi, Language-English:**
   - **a.** AN INTRODUCTION TO KINEMATICS (CH_22) - https://www.youtube.com/watch?v=Z63Ch4W6PMM ,
   - **b.** MOTION IN A STRAIGHT LINE (CH_22 - https://www.youtube.com/watch?v=bq03lFQfNOw&t=50s ,
   - **c.** Motion In a Straight Line (CH_22) and Relative Velocity - https://www.youtube.com/embed/O7kTOkri4AE
   - **d.** INTRODUCTION TO VECTORS (CH_22) - https://www.youtube.com/watch?v=rYCipFGjzXI
   - **e.** Planar Motion (CH_22) - https://www.youtube.com/embed/drtdGg89Psw
   - **f.** Problems in Kinematics (CH_22) - https://www.youtube.com/embed/_JNGmr6onlg

2. **Pradeep Kshetrapal Channel, by Praddep Kshetrapal, Language – Bilingual English-Hindi:**
   - **a.** XI_10.Kinematics- Introduction - https://www.youtube.com/watch?v=XYz02ckbKDE
   - **b.** XI-3.01.Motion in One dimension (2014) - https://www.youtube.com/watch?v=mmSGbicQmzs
   - **c.** XI-3-01 Rest and motion (2016) - https://www.youtube.com/watch?v=mYfIkMBkcV8
   - **d.** XI-3.02.Motion in One dimension (2014) - https://www.youtube.com/watch?v=Rcu5ei5UysI
   - **e.** XI-3-02 Distance and velocity part-1(2016) l - https://www.youtube.com/watch?v=0U03wVx7MXs
   - **f.** XI-3.03.Average Speed (2014) - https://www.youtube.com/watch?v=i5HxUeJamrU
   - **g.** XI-3.4.Relative motion (2014) - https://www.youtube.com/watch?v=qfvSHivH86w

h. XI-3-04 Relative Motion (2016) - https://www.youtube.com/watch?v=qfvSHivH86w
i. XI-3-05 Numerical on Relative motion (2016) - https://www.youtube.com/watch?v=mWoQX-EdDOI
j. XI-3.05.Accelerated motion (2014) - https://www.youtube.com/watch?v=evW750fjXD8
k. XI-3-06 Accelerated motion intro. (2016) - https://www.youtube.com/watch?v=blutD5F1zwI
l. XI-3-07 Motion under gravity (2016) - https://www.youtube.com/watch?v=NhfjjFQVZDM
m. XI-3.06. Graph in 1-D Motion (2014) - https://www.youtube.com/watch?v=XkZeXiPysxw
n. XI-3.07.V-T Graph (2014) - https://www.youtube.com/watch?v=kgKild17VZ8
o. XI-3.08.Numericals based on graphs (2014) - https://www.youtube.com/watch?v=9FjT5EdIXMI
p. XI-3-08 Calculus in physics (2016) - https://www.youtube.com/watch?v=3hRaStIzRQM
q. XI-3-09 Differentiation part-1 (2016) - https://www.youtube.com/watch?v=H55q-fKTw4c
r. XI-3-10 Integration (2016) - https://www.youtube.com/watch?v=W6ATW1iesXE
s. XI-Kinematics-HCV, Objective level-1 (2016) - https://www.youtube.com/watch?v=TXSs6cFupAI
t. XI-Kinematics-HCV, Objective level-2 (2016) - https://www.youtube.com/watch?v=NDXnjcKMqvk
u. XI-3-9 Kinematics 1-D Numericals (2015) - https://www.youtube.com/watch?v=ZGXynm6xshM
v. XI-3-10 Kinematics numerical SCV (2015) - https://www.youtube.com/watch?v=3JEyBnxGamY
w. XI-3.13.Motion in one dimension Numericals(2014) - https://www.youtube.com/watch?v=EShPT6I9yf8
x. XI-3.14.One dimension Numericals2 (2014) - https://www.youtube.com/watch?v=iz6Cidsr468
y. XI-3.15.One dimension motion Numerical-3(2014) - https://www.youtube.com/watch?v=TG2hxWyNVNA

—00—