MECHANICS-II: Dynamics of Particles (Newton's Laws of Motion)

Some real life situations are being brought out to relate concepts Newton's Laws of Motion to happening around us. ***First situation*** is of glass fill of water is kept on the table, when we move close to it want to drink we lift it. The urge of thrust is so strong that we do not realize that some effort is being made to lift the glass.

In ***Second situation*** a box of instrument is kept on the floor, near teacher's table in a class. These instruments are to be used by Physics teacherto explain concepts of mechanics to the class. A teacher enters the class and calls Two of the students in the class. The one of the student is unenthusiastic about it unwillingly makes effort to lift the box and place it on teacher's table. The other student being enthusiastic to learn, happily joins hands to lift the box and place it on table. Expressions of both the students clearly depict relative difference in their efforts to do the same work, together.

In ***Third Situation*** a man steals your bag on a railway platform and tries to run away. As soon as notice it, you will first race to reach the thief. A soon as you catch hold of the thief he tries to run faster. In an effort to be successful to escape the thief applies extra force to run away.

In ***Fourth Situation*** an object is released on a smooth inclined surface joining a smooth horizontal surface. The object continues to slide unless it is obstructed by another object. Interaction between the sliding object and obstruction involves forces.

Fifth Situation calls for visualization of ride Sky Wheel, one feels of weightlessness when the cradle in which we are sitting descends on the periphery of the sky wheel, while feel gaining wait when the wheel ascends.

More of such situations can be observed in day-to-day experiences to visualize as to how does mass, force and acceleration come into play unconsciously. ***Galileo*** made a conscious effort to analyze motion of a free fall and on an inclined plane in his book '***Two New Sciences***' in the form of a dialogue. He first introduced concept of impressed and impelling force to elaborate phenomenon of equilibrium, acceleration and resistance in motion. In the book he further elaborated uniform motion through Four axioms and Six theorems. It continued in explaining acceleration through Twenty Two Theorems involving Thirty Eight Propositions, followed by projectile motion. Later, ***Newton*** in his book '***Principa - The Mathematical Principes of Natural Philosophy***' while propounding Laws of Motion acknowledged that "*....* ***By First two laws and first two corollaries Galileo discovered that descent of body observed the duplicate ratio of time, and that the motion of projectile was in curve of a parabola; experience agreeing with both, unless so far as these motions are a little retarded by resistance of air***. ....". It is just a coincidence that year of death on Galileo and birth of Newton is same year 1642. At this point it worth recalling Albert Einstein who had said that "*A hundred times a day I remind myself that my inner and outer life depend on the labours of other men, living and dead, and that I must exert myself in order to give in the measure as I have received and am still receiving*". The form and clarity with which these laws are now presented is just a perpetuation of the spirit.

Newton through eight definitions in his book laid premise to formalize Galileo's observations with his own and the doctrine action-and-reaction to promulgate Laws of Motion which formed foundation of classical mechanics or Newtonian Mechanics, and these laws are known after his name as Newton's Laws of Motion.

Position, displacement and acceleration are specific to a point, called reference, if motion is along a line. But, force is either in direction of displacement or against it. But, consideration of a motion either in a plane or in space enhances sense of direction from a point to coordinates called directional vectors, called Frame of Reference (FOR). There are views that force is independent of FOR, but, in accelerating frames experience of reaction changes creating a paradox. This shall be discussed with examples as the subject matter proceeds.

Discussions to follow start with Newton's Laws of Motion, the basic concept behind these observations. It recognizes existence of an external force and its impact on change of state of rest or motion of an object. These laws are in a set of Three Laws : Newton's First Law of Motion – also known as ***law on inertia***, Newton's Second Law of Motion – also known as ***law of acceleration***, Newton's Third Law of Motion – also known as ***law of reaction or Cause and Effect***. Each of these law and associated mathematical concepts are elaborate below.

Newton's First Law of Motion (NFLM) : ***A body in an inertial frame of reference continues to be in a state of rest or motion with a constant velocity so long it is in a <u>state of equilibrium</u>***. Every body in this universe is experiencing force of one or the other kind from its surrounding. Such a situation cannot be called a state of No force or Zero Force. But, *when all the forces (a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is (a vector quantity) are represented by sides of a polygon, it is a case of equilibrium i.e. resultant of all forces acting on the body is Zero and is like a state of No Force*. Force is directly not perceivable, at least it was not perceivable at times of Galileo and Newton, but acceleration was. A body in state of rest or motion has an identical kinematic quantity to be Zero and that is acceleration. Thus, conversely it is deduced that a body in equilibrium has Zero net force and in turn has Zero acceleration.

An object observed from Two frames of references in uniform motion with respect to each other they are equivalent in classical mechanics and are called Galilean Invariant Frames.

Force has magnitude and direction, making it a vector quantity. Vectors can be represented graphically by a line having length scaled in proportion to its magnitude and direction from a reference same as that of the force.

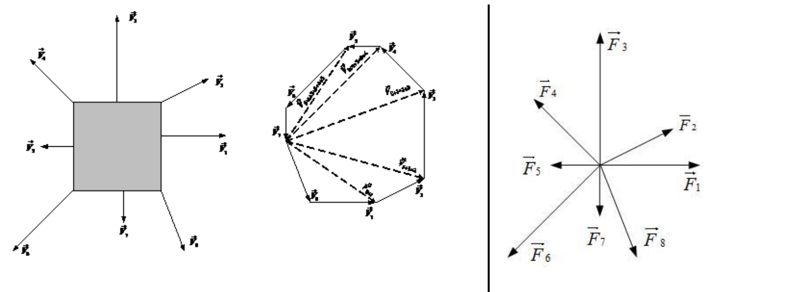
When more than one force is acting on an object they can be graphically represented with vector lines emanating from a point. Net force on the object is determined by taking any one force with the other by principle of *Parallelogram of Forces* to determine its resultant. This process is continued by taking the resultant with one of the remaining forces. If such successive resultants of last-but-one-force are equal and opposite to last of the forces, then these forces are in equilibrium and shall have Zero net force. Alternately, using principle of **Triangle of Vectors** to construct a *Polygon of Vectors* such that head of last vector converges on the tail of the first vector representing an equilibrium condition. In case such convergence does not occur, it would be a case of in-equilibrium and it would lead to a set of lines and cannot be called a shape known as polygon.

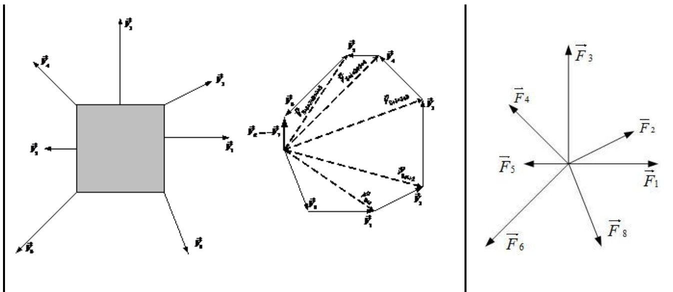
Mathematically, resultant, or equivalent of forces acting on the object is -

$$\vec{F}_{eq} = F_x\hat{\imath} + F_y\hat{\jmath} = \sum_{k=1}^{k=n}\left(F_{xk}\,\hat{\imath} + F_{yk}\,\hat{\jmath}\right);$$

Here, F_{xk} is the x-component of k^{th} force, F_{yk} is the y-component of k^{th}. ***In case of equilibrium*** $F_{xk} = F_{xk} = 0$ or $\left|\boldsymbol{F_{eq}}\right| = \mathbf{0}$ **,** while ***in case of in-equilibrium*** one of the F_{xk} and F_{xk} may be zero, but both cannot be zero, i.e. $\left|\boldsymbol{F_{eq}}\right| \neq \mathbf{0}$.

Graphical representation of a typical system of Seven forces is shown both in star and polygon formation, for both the kind of cases equilibrium and in-equilibrium.





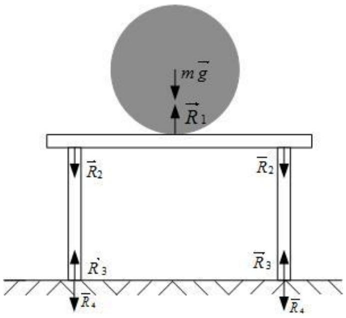
Representation of Forces on a Object	Closed Polygon of Vectors	Forces on Cartesian plane	Representation of Forces on a Object	Open Polygon of Vectors	Forces on Cartesian plane
An object on a Horizontal Plane Acted upon by Eight Forces (Equilibrium Condition)			An object on a Horizontal Plane Acted upon by Seven Forces (In-equilibrium Condition)		

Newton's Second Law Of Motion (NSLM): The conditions of in-equilibrium in the above illustration gives rise violation of NFLM and, therefore, it would tend to change state of motion. When net force is Zero, acceleration is Zero as per NFLM, and, therefore, there must be some relationship between force and acceleration, a natural consequence. Quantification of this relationship is core contention of Newton's Second Law of Motion (NSLM). Newton, through experimental observations propounded that the effect of in-equilibrium of forces acting on an object NSLM which states that : ***in an inertial frame of reference net force acting on an object causes an acceleration of the object in the direction of the net force (resultant), such that Force is equal to product of the mass of the object and the acceleration***. Mathematically, it is expressed as: $\bar{F} = m\bar{a}$. This has led to evolution of a new term ***Momentum*** ($\bar{p} = m\bar{v}$) and rate of change of momentum: $\bar{F} = m\bar{a} = m\frac{d\bar{v}}{dt} = \frac{d}{dt}(m\bar{v}) = \frac{d}{dt}(\bar{p})$. This definition of rate of change of momentum becomes very useful in *analysing propulsion of rockets* which eject mass during their motion or any other situation where there is change of both mass and velocity, due to parting of components or any other reason. This will be analysed as we proceed in study of dynamics.

Mathematical evidence of NSLM starts with NFLM and is extended by correlating experimental observations. Taking $F = f(m, x, v, a)$, as per NFLM, a body at rest ($x -$ position remains constant) or uniform motion ($v -$ velocity remains constant) then automatically $\left(a = \frac{dv}{dt} = 0\right)$ is in equilibrium and hence net Force $F = 0$. This reduces the relation, using NFLM, to $F = f(m, a) = f\left(m, \frac{dv}{dt}\right) \rightarrow F = f\left(\frac{d(mv)}{dt}\right)$. Experimentally observed that if mass (m) is kept constant then $F \propto a$, and if acceleration (a) is kept constant then $F \propto m$. The earlier relationship of Force can be verified by observing acceleration of an object on a smooth surface with different angles of inclination using Kinematic Equations. The latter can be verified by suspending a mass from one end of a light spring and setting it in horizontal circular motion by holding other end. Extension in length of spring is proportional to the force $(F \propto \Delta l)$, here $\Delta l = l - l_0$; l_0- is normal length of spring and l - is length of string at any instant of circular motion. Using analysis of circular motion, to be discussed little later, acceleration experienced by the suspended mass shall be derived as $a = l\omega^2$. It can be verified that force on any mass in uniform circular where angular velocity (ω) is constant radial acceleration (a) is also constant. Thus it comes to $F \propto m$. Combining these Two experimental observations it is concluded that $F = ma = m\left(\frac{dv}{dt}\right)$. It can be further combined into $F = \frac{d(mv)}{dt} = \frac{d}{dt}p$, here $p(= mv)$, is called momentum. Since, m is a scalar quantity and v is vector and hence p is also a vector being scalar multiple of vector v.

Galileo had used term ***momento*** interchangeably for momentum while elaborating Force and motion of a body. But, Newton in the book categorically defined "the quantity of motion is measure of same arising from the velocity and quantity of matter conjunctly" (Definition II and I). This has definition has sustained as Momentum and is referred in explanation of NSLM.

Newton's Third Law of Motion (NTLM): In fourth situation, visualized in the beginning, the sliding object exerts a force (Cause) on the obstructing object. But, what happen to the sliding object (Effect) as a consequence. This was analysed by Newton and propounded as **Third Law** which states that: ***when an object exerts a force (action or cause) on another object, and if it remains in equilibrium, the second object simultaneously exerts an equal-and-opposite-force (reaction or effect) on the first object.***



Mathematical verification of NTLM- If Body of mass M is placed on a table and it remains in a state of rest. The body exerts a force on the table called weight caused by acceleration due to gravity $\vec{F} = m\vec{g}$. But, the body is since in a state of rest there must be another force, let it be called ***reaction***, causing the condition of equilibrium such that $\vec{F} + \vec{R} = 0 \rightarrow \vec{F} = -\vec{R}$. It implies that Reaction caused by a force is equal and opposite to it and is consistent with NTLM. Similarly reaction on a body of a person is pushed back or forth on application of sudden acceleration or breaking. The effect is momentary since muscles takes over the control soonafter and person repositions himself to a comfortable posture.

NTLM has its manifestations in Two forms, first is in Inertial Frame of Reference (IFOR) w.r.t. which object is in state of rest. And second is Non-inertial Frame of Reference (NFOR) where there is relative acceleration of object w.r.t. FOR.

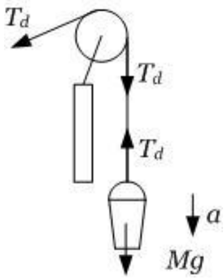
Newton's Third Law in IFOR: Above example of a Ball of Mass m being extended to analyze reactions wherever they are occurring is experiencing a gravitational acceleration $\vec{g}$. But, the acceleration is stopped by the table top on which it is kept . As a result, as per NTLM, the table top exerts a reaction $\vec{R}_1$ on the ball to stop the acceleration of the ball under gravity and thus ball stays at rest on the table top. It is a case of equilibrium in IFRM. Accordingly, the ball is in a state of equilibrium such that $m\vec{g} + \vec{R}_1 = 0$ or, $m\vec{g} = -\vec{R}_1$. Ultimately, net force exerted on earth through legs of the table is so small that acceleration caused by it to the earth, as per NSLM, is infinitesimally small and perceived to be ZERO. This makes earth to be IFRM and is explained mathematically as under-

In this example the ball is kept on the centre to idealize reactions by four legs of the table to be symmetrically equal. Unequal reactions offered by legs of table, when the ball is kept away from its centre can be understood once principle of Moment of Force is clear; this is covered in next Section on Mechanics. Thus taking $\vec{R}_1 + 4\vec{R}_2 = 0$, for table in rest $\vec{R}_1 = -4\vec{R}_2$ where $\vec{R}_2$ is the reaction offered by each of the Four legs. In the Figure only Two fronts legs are shown, and rear Two legs are masked by the Two front-legs, and hence are invisible.

Despite $4\vec{R}_2$ force on the table legs, their motion is stopped by the earth's surface, an IFRM, on which the table is kept. This can only happen in a state of equilibrium i.e. $4\vec{R}_2 = 4\vec{R}_3$. In turn $4\vec{R}_3$, force exerted by the earth's surface on each pair of legs, produces a counter force as per NTLM $= 4\vec{R}_4$. Thus, weight of the ball is transmitted on the earth $mg = 4R_4$. As per NSLM it should produce an acceleration in the earth such that $mg = M_e a_e$, here, $M_e = 5.98 \times 10^{24}$ kg. Accordingly, $a_e = \frac{m}{M_e} g$. Since, $m \ll M_e$ it leads to $\frac{m}{M_e} \to 0$ or $a_e \to 0$. This is the reason earth remains IFOR

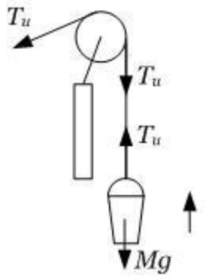
Here, for a moment discussions on $4\bar{R}_4$, shown in the figure, are put on hold till discussions on NTLM in NFOR are completed. But, existence of $4\bar{R}_4$ can be realized from the impression of legs of the table, that it leaves, on bare ground surface. These impressions are visible when table is removed, and is indicative of compression caused by the force on the surface on which it is resting.

Newton's Third Law in NFOR: It is an excellent example of out of box visualization of scientific principles in surrounding. Those living in rural background must have experienced that when a bucket is released in a well, its weight for a moment apparently decreases. On the contrary, when bucket is pulled out of well it requires more force than that required to hold it stationary in air. Similar experience one gets when during a sky-wheel ride.



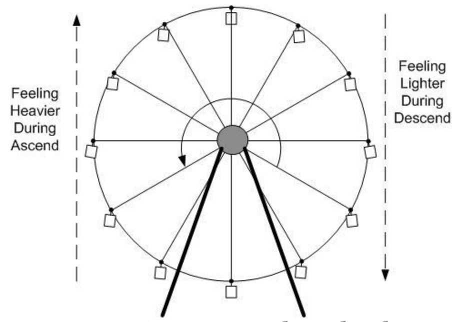
$T_d + Ma = Mg$

Experience of a Bucket Being Dropped In a Well



$T_u - Ma = Mg$

Experience of a Bucket Being Pulled Out of a Well



Experience on a Sky-wheel

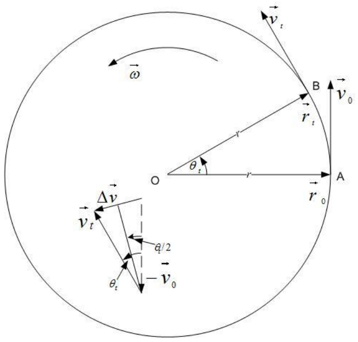
The bucket of mass m in a state of rest which is experiencing ***g*** w.r.t. IFRM making it weighs $W = mg$. Thus as per NTLM force exerted in form of tension is rope is $T = W = mg$. When bucket is released in the well to with acceleration ***a*** w.r.t. earth, the relative vertical acceleration of the bucket w.r.t. the IFOR is $a_d = g - a$. And, tension in the rope as per NTLM shall be $T_d = m(g - a)$. Thus the bucket, while moving down in the well, is experienced to be lighter than its weight when held at rest. Likewise, for the bucket pulled up, relative vertical acceleration w.r.t. IFOR shall be $a_u = g + a$ and, therefore, force required to pull out the bucket, as per NTLM, shall be $T_u = m(g + a)$. Thus the bucket, while pulling it out of the well is experienced to be heavier, and can be measured

This difference in force arising out of relative acceleration is also explained by visualizing a virtual force called **Pseudo Force**. Conceptually, *Pseudo Force* on the accelerating object in NFRM, is taken to be $F_P = m(-a) = -ma$, i.e. product of mass of accelerating object and negative of its acceleration .w.r.t. IFR It is thus a way to normalize an NFOR into IFOR under consideration in the problem. This is achieved by considering acceleration of NFRM relative to IFRM. Thus the concept of Pseudo Force produces same results as that obtained with the consideration of relative acceleration, elaborated above, and is experimentally verifiable. Accordingly, *stipulation of Pseudo Force of an accelerating object is transformation of its state of un-equilibrium, causing acceleration into virtual equilibrium with its own reference.*

This is now appropriate stage to examine what happens to effect $4\vec{R}_4$ shown in the figure having a ball kept on the table. Earth surface was considered to be IFRM. The $4\vec{R}_2$ effect of $4\vec{R}_3$ should be causing acceleration earth. Accordingly, this problem also should have been analysed on the lines of NTLM in NFRM But, $\boldsymbol{M_e}$ us very large as compared to ***M*** (***M***$\boldsymbol{M_e}$,>>***M***) such that acceleration of earth ($\boldsymbol{a_e}$) as per NSLM would be $a_e = \frac{Mg}{M_e} \to 0$. This is the reason that despites earth being in constant acceleration, due to rotatory and revolving motion, which again has much smaller angular speed, as compared to that of the physical objects being observed, is taken as IFRM. Circular and rotatory motion of earth shall be discussed little later.

Next is the sky wheel where we find that cradles are radial when they are at top of the wheel or on the bottom of the wheel. Otherwise, they remain suspended with a tilt outwards. Understanding, the cause of this observation requires concepts of circular motion.

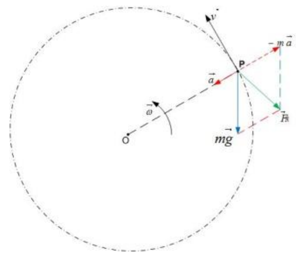
Uniform Circular motion: In this a particle is taken to be *revolving around a fixed point with a constant radius and with a constant angular speed* $\vec{\omega} = \frac{d\vec{\theta}}{dt}$*, a vector quantity,* here $\vec{\theta}$ is the angular displacement on a plane, a vector quantity. Likewise, there exists angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$, and its will be used while working with rotational motion. Since angles are measured in clockwise direction and hence $+\vec{\omega}$ is upwards, coming out of a surface, and $-\vec{\omega}$ is inwards, entering a surface. This circular motion is analysed below.



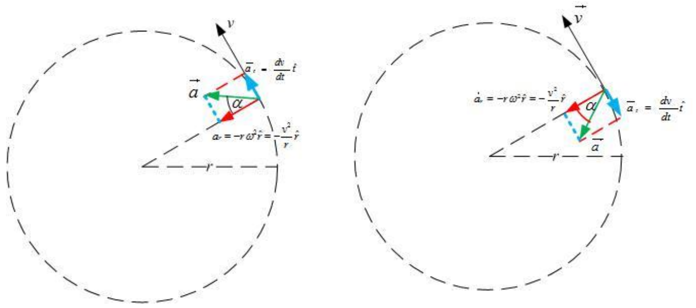
Linear velocity at any instant is $\vec{v} = \vec{\omega} \times \vec{r}$, ***a vector product, also called dot product of vectors***. Here, unit vector along axis of rotation $(\hat{\omega})$ and radius of rotation $(\hat{r})$ are perpendicular to each other. Accordingly, $v = |\vec{v}| = |\vec{\omega}||\vec{r}| \rightarrow v = r\omega$. *Since r and ω are constant, basic premise of circular motion, hence v is also constant*. Let, θ_t be the displacement of radial vector during time Δt when velocity vector of the particle changes from $\vec{v}_0$ to $\vec{v}_t$. Accordingly, $\Delta\vec{v} = \vec{v}_t - \vec{v}_0$. Velocity, v being constant, in the vector diagram, both $\vec{v}_t$ and $\vec{v}_0$ are also constant, at an angle $\theta_t = \omega\Delta t$, and geometrically length of the third side $\Delta v = r\theta_t|_{\theta_t \rightarrow 0} = 2v\sin\frac{\theta_t}{2}\Big|_{\theta_t \rightarrow 0}$.

As such, magnitude of acceleration of the article performing circular motion shall be: $a = \frac{\Delta v}{\Delta t}\Big|_{\Delta t \rightarrow 0} = \frac{2v\sin\frac{\theta_t}{2}}{\Delta t}$. Substituting v and Δt from the above: $a = \frac{2r\omega\sin\frac{\theta_t}{2}}{\frac{\theta_t}{\omega}} = r\omega^2\frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}$. Further, geometrically $\frac{\sin\frac{\theta_t}{2}}{\frac{\theta_t}{2}}\Bigg|_{\Delta\theta_t \rightarrow 0} = 1$ and hence, $a = r\omega^2$. A close observation of the vector diagram reveals that, as $\Delta\theta_t \rightarrow 0$, so also $\Delta\bar{v}$ tends to become perpendicular to instantaneous velocity of the particle, performing circular motion, i.e. radially inwards called ***centripetal acceleration a vector*** $\bar{\boldsymbol{a}}$. This centripetal acceleration is the reason behind keeping the particle to perform uniform circular motion, else the particle will run away from the orbit. This experience can be obtained with water soaked in a wet handkerchief, when it is rotated holding its one end, starts sprinkling away..

Review of NTLM in NFOR : A common observation in respect of cradles of sky wheel, why do they get automatically tilted outwards? Analysis of this observation explains, how does NTLM in NFOR automatically comes into play? The cradle, being fixed on to rim at point P of the sky-wheel is performing circular motion. Accordingly, it will experience a constant acceleration $\vec{\boldsymbol{a}}$**,** and thus the cradle becomes a NRFM. When, the forces on the cradle are transformed to IFRM, an observer on the ground, an IFRM, *reaction to this centripetal force caused by centripetal acceleration, appears in the form of a force equal and opposite to the centripetal force; this force is called* ***centrifugal force*** and is nothing other than as a pseudo force as shown in the diagram. Resultant of the Centrifugal Force and gravitational pull, by IFRM, is outwards and it causes tilting of cradle outwards. Angle of tilt depends upon its magnitude and direction, but the highest and lowest points are exceptions. At these points the two forces are collinear. If accidently cradle gets unhinged to the rim, the centripetal force and in turn centrifugal force would cease to exit. In effect the cradle would run away, with instantaneous tangential velocity at the time of release. But, under action of gravity it would perform projectile motion. There are numerous situations, encountered in daily life, involving circular motion on IFRM and NFRM. Thus pseudo force is called ***Centrifugal Force***.

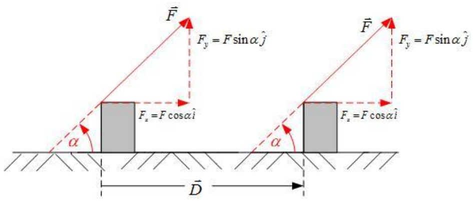


There is another situation when a particle is experiencing **non-uniform circular motion.** It can happen when velocity of particle performing circular motion with velocity $\boldsymbol{v}$ is either accelerating or retarding. In this case the trace of particle would be either growing spiral or a collapsing spiral, depending upon whether particle is under acceleration or retardation, respectively. In this case net ***acceleration*** of the particle shall be not be radially inward, unlike uniform circular motion. The net acceleration shall be drifted forward from the centre of the spiral, i.e. in the direction of



velocity in case of tangential acceleration; and backward, i.e. against the velocity, in case of tangential retardation. Mathematically, this is elaborated as $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2}$ and angle of drift is $\alpha = \tan^{-1}\frac{dv/_{dt}}{r\omega^2}$, and supported with necessary illustration diagram. A similar effect of drift in gravitational pull is experienced when one moves from equator towards poles. It is a special case of combined effect of centrifugal and gravitational acceleration and shall be elaborated in next Section.

Work, Power and Energy: Common understanding of work and that in physics is not the same. A person lifting an object placed on floor and placing on a raised platform does a work. But, displacing an object on a smooth horizontal surface is not. This requires to understand **definition of work** ($\boldsymbol{W}$) in physics; according to it ***work is the product of Force and displacement caused by it in the direction of the force.*** Mathematically, it is expressed as ***DOT product*** of force and displacement, both of which are vectors. Mathematically Dot product of vectors is a scalar, $W = \vec{F} \cdot \vec{D} = FD\,cos\theta$, where W is work done, a scalar quantity, by a force $\vec{F}$ applied on an object, $\vec{D}$ is the displacement of the object under influence of force $\vec{F}$ and θ is the angle between vectors $\vec{F}$ and $\vec{D}$. Thus, alternatively, work is product of displacement and force in the direction of displacement, a mathematical equivalent. The SI unit of Force in Newton, Distance is Meter and accordingly unit of Work is Newton Meter and also called Joule Dimensionally, $[W] = [F][D] = (MLT^{-2}) \cdot (L)$. In the expression of work both θ and $\cos\theta$ are dimensionless, and accordingly dimension of work reduces to $[W] = ML^2T^{-2}$.



It is to be remembered that work is scalar quantity. If displacement has a component in the direction of force work (+)ve, or the component of force in the direction of displacement, and is done by agent of force on the object, and if the component of displacement is against direction of force work is (-)ve and is done by reaction, as per NTLM, f the force offered by the object. It is in accordance with NTLM. This discussion shall be extended little later into concept of *conservation of energy*.

Power (P) is the rate of doing work. It can be compared with two vehicles climbing on an inclined road starting from same point. Time taken vehicle A to reach destination is 2 Hours, while time taken by vehicle B to reach destination is 3 hours. Then in common parlance it is said that vehicle A has more power than the B. Accordingly, **Work** is mathematically expressed as $P = \frac{W}{T}$, and thus unit is Joules per Second or Joule/Sec. and dimensionally it reduces to $[P] = \frac{ML^2T^{-2}}{T} = ML^2T^{-3}$.

In another example, vehicle A after full-tank filling makes 5 trips between two places, but other vehicle B after full-tank filling makes 7 trips. Then in common parlance ***energy***, total work done, of vehicle B is greater than that of A, and in this power has no consideration. Mathematically energy ($\boldsymbol{E}$) is expressed as $E = P \times T$, and accordingly unit of energy is [(Joules/Sec) X (Sec) = (Joule)]. *Energy in classical mechanics is considered to be conservative.* The ***Law of Conservation of Energy (*LCE*)*** *states that energy can neither be created nor destroyed; it can be transformed from one form to the other.* In mechanics, energy is considered to be of two forms; one is ***Potential Energy (PE)***, *due to position of an object* and the other is ***Kinetic Energy (KE)***, *due to velocity of an object*. Correlation between PE and KE shall be discussed a little later, after elaboration of concept of PE.

This is the time to review concept of work when $0^0 \leq \theta < 90^0$ and $90^0 < \theta \leq 180^0$. In the earlier case $0 < \cos\theta \leq 1$, force $\vec{F}$ has a component in the direction of $\vec{D}$, and mathematically net work done is +ve. While in latter case $-1 \leq \cos\theta < 0$ and force $\vec{F}$ has a component against the direction of $\vec{D}$ and mathematically net work done is -ve. Thus +ve work done by external agent exerting the force stores energy in the object. Here, it is to be noted that when an external force is so exerted that rate of displacement is slow down it is a case of retardation, and when rate of displacement increases it is a case of acceleration. Accordingly, as per NSLM, in conjunction with NTLM in IFOR, while agent of force does (+)ve work on the object, while the object stores the work i.e. (-)ve work. Thus there would be a balance of work. Accordingly, *work done by reaction or restraining force is stored as energy in the object in IFRM; this is called **potential energy***. Two typical examples of potential energy are being elaborated here under:

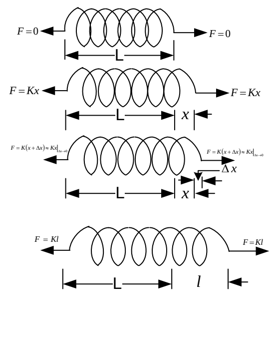
Potential in Gravitational Field: Taking an integrated view, involves ***Law of Gravitation***, to be covered in next section on Mechanics. But, taking a preliminary view force exerted by earth on an object, as per the law, $F_e = G\frac{Mm}{(R_e+h)^2} = G\frac{Mm}{{R_e}^2\left(1+\frac{h}{R_e}\right)^2} \approx G\frac{Mm}{{R_e}^2}$, here G is gravitational constant, M is the mass of the earth, m is the mass of the object under consideration, R_e is radius of the earth and h is the height of object above earth's surface, such that $h \ll R_e$ such that $\frac{h}{R_e} \to 0$. This expression has been approximated to

$F_e = \left(\frac{GM}{{R_e}^2}\right)m = gm$, here $g = \frac{GM}{{R_e}^2}$. Now when an object of mass is lifted above earth surface, through a height h, without acceleration, $W = \sum \Delta w = -\int_0^h m\vec{g} \cdot d\vec{h} = -[mgh]_0^h = -mgh$. This work is stored in the object by virtue of its position w.r.t. earth at height h it is called ***Potential Energy (PE)*** $= -W = -(-mgh) = mgh$. If mass of the object under consideration is unity then $PE = mgh = mP$, here $P = gh$ is called potential of the object, retraining force (mg) is against direction of displacement.

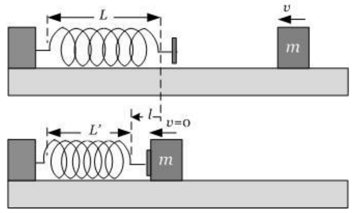
Mathematical correlation between magnitude of PE and KE is : $\Delta PE = \Delta w = F\Delta x \rightarrow dw = d(KE) = \left(m\frac{dv}{dt}\right)dx = m\left(\frac{dx}{dt}\right)dv$. It leads to $d(KE) = m(vdv)$. It leads to $\int d(KE) = m\int vdv \rightarrow KE = \frac{1}{2}mv^2$. This relationship can also be derived, using kinematics and NLM taking example of an object of mass m lifted through a height h, without acceleration, against gravity. In doing so an external force $F = -mg$ is applied on the object and work done in changing its position over a height is $PE = -W = mgh$. Here, g is acceleration due to gravity. Now, if the object is dropped $(u = 0)$ from height h above the ground then velocity of the object, when it reaches ground as per kinematics, will be $v^2 = 2gh$. As per conservation of energy $PE = KE \rightarrow mgh = Cv^2$. As per Third equation of Kinematics $0 = u^2 + 2(-g)h \rightarrow u^2 = 2gh$. In the instant case $u \rightarrow v$, accordingly $mgh = C(2gh) \rightarrow C = \frac{1}{2}m$. Thus $KE = \frac{1}{2}mv^2$, is the ***equivalence of the two fom of energy.***

Kinetic Energy of an object can also be derived from NSLM and definition of work from the perspective of principle of Conservation of Energy (COE). If an object of mass m Kg is subjected to an external force F N such that it experiences a free body displacement s m in the direction of F. In that case work done is $W = Fs$. This displacement is caused by an acceleration $a = \frac{F}{m}$ in the direction of force. If initial velocity of the body in the direction of force is u m/s then after traversing the distance s, velocity of the body as per TEM would be $v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$. Thus external work done on the body is $W = (ma) \times \left(\frac{v^2 - u^2}{2a}\right) = \frac{1}{2}m\left(v^2 - u^2\right) = \frac{1}{2}mv^2\Big|_{u^2=0}$. Considering that initially the body was at rest i.e. $u = 0$, as per COE the work done on the object is converted in kinetic energy such that $W = KE = \frac{1}{2}mv^2$. Accordingly, taking $u \neq 0$, $W = \frac{1}{2}m\left(v^2 - u^2\right) = \Delta KE$.

Potential Energy in Spring: When spring is stretched or compressed by length x it requires an external force in the direction of stretch/compression $= kx$, and corresponding equal and opposite reaction called *restraining force* as per NTLM is produced by the spring. Here, k is called *spring constant* taken to be uniform during stretch/compression. This linear nature of force is compared spring PE is compared with gravitational PE. An incremental stretch/compression by an infinitesimal length Δx would call upon external work on the spring, which in turn is stored in the form of PE in the spring: $\Delta W = -k\bar{x} \cdot \Delta\bar{x}$. Total PE stored in spring when compressed/stretched by length l is $PE = -W = -\int_{x=0}^{l} -k\bar{x} \cdot d\bar{x} = \frac{1}{2}k[x^2]_0^l = \frac{1}{2}kl^2$. If a mass m is attached to a light spring, and the spring is stretched/compressed by a length l is released, the mass will gain kinetic energy, when released by the spring as per *principle of COE* would be $KE = \frac{1}{2}mv^2 = \frac{1}{2}kl^2$. Accordingly, the mass at this position of the spring will have $v = \sqrt{\frac{k}{m}}l$. This can

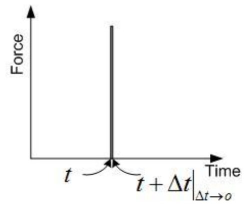


be best verified with a light spring placed on a smooth table, with one end fixed, and another end free. A mass m with a velocity v in line with length of spring towards its free end, impinges on it, as shown in the figure. The mass has kinetic energy $= \frac{1}{2}mv^2$ at the time it impinges on the spring. When the mass comes to rest it kinetic energy becomes Zero, but the spring gets compressed by a length l, thereby the spring acquires a potential energy $= \frac{1}{2}kl^2$. It is demonstrates equivalence of kinetic energy of mass when spring is in its natural length, uncompressed, with the potential energy

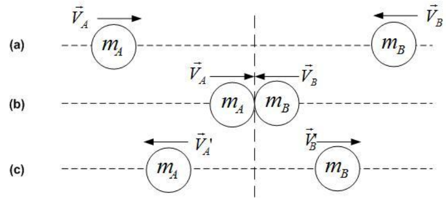


of the compressed spring when mass comes to rest. i.e. it validates principle Fof conservation of energy.

Work Energy Theorem (WET): It is extrapolation of NSLM together with kinematics. In the two examples discussed above are ideal cases where motion is lossless i.e. it involves forces called are ***conservative forces (CF)***. In case of free fall it is assumed that air or medium does not offer any resistance or friction. Likewise, in stretch/compression of spring the system is considered to be frictionless i.e. without loss of energy. It leads to, a mathematical interpretation that $\Delta PE + \Delta KE = 0 \rightarrow \Delta PE = -\Delta KE$, i.e. *in any displacement loss/gain of potential energy is gain/loss of kinetic energy*. This statement needs to be read carefully that loss of potential energy is associated with gain of kinetic energy and vice-versa. But, cases encountered in day to day life involve some kind of friction or loss of energy $\Delta PE + \Delta KE = \Delta W$, here ΔW is work done by other forces which are called ***non-conservative forces (NCF)***. Elaboration of friction in next section would make the concept of NCF more explicit. Energy associated with NCF, in mechanical processes gets converted in heat energy, and thus validity of WET is maintained.



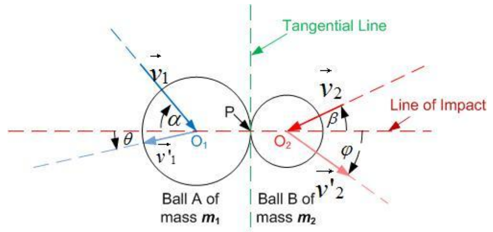
Momentum-Impulse-Collision: The concept of momentum was introduced during discussions on NSLM, which is being extended into ***Law of Conservation of Momentum (*LCM*)***. This involves understanding of impulse and instantaneous impulse, which are being elaborated. As per NSLM $\vec{F} = \frac{d}{dt}\vec{P} \rightarrow \int d\vec{P} = \int \vec{F}\,dt$. It leads to $I = \int_{t_1}^{t_2} \vec{F}\,dt = \int d\vec{P} \rightarrow \vec{I} = P_{t=t_2-t_2} = \Delta P$. This the *integral form of NSLM and here **I** is called **Impulse***. Thus *impulse of a force is change of linear momentum*. Accordingly, if mass of an object, remains constant, during application of a force for a duration t_1 to t_2 then impulse on the mass is $\vec{I} = m(\vec{v}_2 - \vec{v}_1)$, where $\vec{v}_2$ is the velocity of mass at instance t_2 and $\vec{v}_1$ is the velocity of mass at instance t_1. Impulse is a vector quantity and shall have corresponding component along $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ direction vectors. Accordingly, *impulsive force will effect into changes in object in respect of its both: a) direction and b) speed*. Consider another situation, where a very large force acts on a body for a very short interval called instance. This can be visualized in that kind of sports where a ball is hit by bat, racket, hockey, foot or hand for that matter even boxing. Accordingly, $\vec{I}_{inst} = \int_t^{t+\Delta t} \vec{F}\,dt = m(\vec{v}_{t+\Delta t} - \vec{v}_t) = m\Delta\vec{v} = \Delta\vec{P}$. If, $\Delta t \rightarrow 0$ and F is so large that other finite forces viz. gravity, friction etc. can be considered insignificant then $\vec{I}_{inst} = \int_t^{t+\Delta t} \vec{F}\,dt = \Delta\vec{P}$ is a finite quantity called ***Instantaneous Impulse***, and is shown in figure.



Extending this concept of instantaneous impulse where two balls, moving in opposite directions along a line are considered to be a system with no external force, collide as shown in the figure in three stages – **a)** pre collision, **b)** at the instance of collision and **c)** post collision. The two balls at the instance of collision, come in contact to separate from each other is infinitely small time, and each of the ball experiences an interactive linear impulse such that for ball A impulse is: $\vec{I}_A = \int \vec{F}_{A,B}\,dt = \Delta\vec{P}_A$, and for ball B: $\vec{I}_B = \int \vec{F}_{B,A}\,dt = \Delta\vec{P}_B$. As per NTLM it leads to $\vec{F}_{A,B} = -\vec{F}_{B,A}$. Here, $\vec{F}_{A,B}$ is force on ball A by Ball B, and $\vec{F}_{BA}$ is force on ball B by Ball A. These are conservative forces applicable to elastic bodies which do not undergo any loss of energy during collision. Therefore, $\Delta\vec{P}_A + \Delta\vec{P}_B = \int \vec{F}_{A,B}\,dt + \int(-\vec{F}_{A,B})dt = \int \vec{F}_{AB}\,dt - \int \vec{F}_{AB}\,dt = 0$. This mathematical expression can also be represented as $m_A(\vec{V}_A' - \vec{V}_A) + m_B(\vec{V}_B' - \vec{V}_B) = 0 \rightarrow m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}_A{}' + m_B\vec{V}_B{}'$. This is called ***principle Conservation of Linear Momentum*** (CLM) and is *expressed in terms of linear momentum of elastic objects, which in absence of any external force, remains conserved during collision, it remains constant and before and after collision*. Together with this, concept *Law of Conservation of Energy* will be helpful to conceptualize phenomenon of Collision. This leads to ***Impact Momentum Principle*** (***IMP***) according to which for a body of mass m it is mathematically, $\vec{P}_i + \vec{I} = \vec{P}_f \rightarrow m\vec{v}_i + \vec{I} = m\vec{v}_f$, here $\vec{P}_i$ is initial momentum of the body, $\vec{P}_f$ is the final momentum of the body after an impulse $\vec{I}$ impinges on it.

Another experiment which demonstrates CLM with NTLM is being elaborated. Take a tightly closed box, with air inside, has a centrally placed lid. The box, with its lid on its vertical side, is placed on a horizontal smooth surface $(\vec{v}_0 = 0)$, in sunlight to heat up and develop an internal pressure causing self-opening of the lid; there is no external force. It is seen that after some time the lid, having mass ***m,*** moves with a high velocity, and the box, having a heavier mass ***M>>m***, is pushed in a direction opposite to the lid with a velocity relatively much smaller than that of the lid. This follows LCM and accordingly $\vec{F} = 0 = \frac{d}{dt}\vec{P} = \frac{d}{dt}\big((M+m)\vec{v}_0\big)$, and $\vec{P} = Const$. Since, the combined mass $M + m \neq 0$ and $\vec{v}_0 = 0$, hence $\vec{P} = 0$. When the lid opens, it has a velocity $\vec{v}_m$ and the container has velocity $\vec{v}_M$. In absence of external force, $\vec{P}$ would continue to be Zero as per CLM, and. $\vec{P} = 0 = M\vec{v}_M + m\vec{v}_m$, or $\vec{v}_M = -\frac{m}{M}\vec{v}_m$. This experiment can be tried and if need be some stove can be used in open, to heat the box, with a care to avoid any accident.

Collision: *Newton had experimentally determined that when two objects collide, their speeds after the collision depend on the material from which they are made, .and is independent of mass* and evolved a new term called ***coefficient of restitution*** (***COR***) or resilience (e) and mathematically expressed as $e=(-)\frac{velocity\ of\ seperation}{velocity\ of\ approach}=(-)\frac{relative\ speed\ after\ collision}{relative\ speed\ before\ collision}=(-)\frac{v_{1f}-v_{2f}}{v_{1i}-v_{2i}}$. Here, v_{1i} and v_{2i} are velocities of objects 1 and 2, respectively, before collision. Likewise, v_{1f} and v_{2f} are velocities of objects 1 and 2, respectively after collision. When it comes to relative velocity it makes no difference whether it is taken w.r.t. object 1 or 2, but reference has to be same for before and after collision. Further, (-) sign accounts for separation to signify it as as reverse of the approach. Accordingly, collisions are classified in Three categories: **a)** Elastic collision where $e=1$, **b)** Plastic collision where $e=0$, and **c)** Non-elastic collision when $0<e<1$. This phenomenon of collision is being analysed in a more generic manner when lines of motion of two objects are intersecting each other which is called as *Oblique Collision*, and is shown in the figure, But, when the colliding objects are travelling along same line it is called *Direct Collision* and in common parlance it is *Head-on Collision*.



For simplification of the problem the object are taken to be spherical balls. The line perpendicular to the surface of the colliding balls at the point of contact is called Line of Impact (LOI) its direction vector is $\hat{n}$. If the collision is considered in 2-D, then line perpendicular to LOI is called Tangential line with direction vector $\hat{t}$, but if collision is being considered in 3-D, then it becomes a tangential plain which is normal to $\hat{n}$. The problem is simplified by considering 2-D collision. Momentum of the balls before collision, is resolved as $\vec{P}_{1i}=P_{1i_n}\hat{n}+P_{1i_t}\hat{t}$ and $\vec{P}_{2i}=P_{2f_n}\hat{n}+P_{2i_t}\hat{t}$. Likewise, post collision momentums are $\vec{P}_{1fi}=P_{1f_n}\hat{n}+P_{1f_t}\hat{t}$ and $\vec{P}_{2f}=P_{2f_n}\hat{n}+P_{2f_t}\hat{t}$. This will simplify vector analysis to scalar analysis along $\hat{n}$ and $\hat{t}$.

At this point another concept of ***Smooth Collision*** is introduced, acceding to which at the instance of collision, impact is only along $\hat{n}$ and no tangential force exists. As this journey proceeds, in the next section existence of impact forms a potential case for occurrence phenomenon of ***friction***. But, in smooth surface there is no friction and hence it is called *smooth collision*.

Applying this concept of smooth collision in the instant case, since there is no tangential force, $P_{1i_t}=P_{1f_t}\rightarrow m_1V_{1i_t}=m_1V_{1f_t}$ and $P_{2i_t}=P_{2f_t}\rightarrow m_2V_{2i_t}=m_2V_{2f_t}$. Since, mass of two balls remains intact, i.e. unchanged, the tangential velocities of the Two balls shall also remain unchanged i.e. $V_{1i_t}=V_{1f_t}$ and $V_{2i_t}=V_{2f_t}$. *This concept reduces Oblique collision to Direct collision and its analysis gets simplified.*

During collision, along the line of impact, ball B exerts an impact $I_{AB}\hat{n}$ on ball A and likewise on ball B exerts an impact $I_{BA}\hat{n}$. As per NTLM, $I_{AB}\hat{n}=-I_{BA}\hat{n}\rightarrow I_{AB}=-I_{BA}$. Thus, according to IMP, for ball A it would be $m_1V_{1i_n}+I_{AB}=m_1V_{1f_n}$ and accordingly for ball B, $m_2\left(-V_{2i_n}\right)+I_{BA}=m_2V_{2f_n}\rightarrow -m_2V_{2i_n}-I_{AB}=m_2V_{2f_n}$. It is to be noted that V_{2i_n} is taken as (-)ve since it in direction opposite to $\hat{n}$, while V_{1f_n} and V_{2f_n} being unknown are taken to be (+)ve, with its sign left to be part of solution. Adding these two equation, $m_1V_{1i_n}-m_2V_{2i_n}=m_1V_{1f_n}+m_2V_{2f_n}$ ..Eqn (1) This is in conformance with the CLM. This is a stage where only One equation is there with V_{1f_n} and V_{2f_n} unknown, while rest of the terms in equation are unknown. This is where Newton's experimentally determined ***empirical constant e*** called **coefficient of restitution,** it is characteristically specific to colliding masses, and provides a relief with another equation involving V_{1f_n} and V_{2f_n} . Accordingly, $e=-\frac{V_{1f_n}-V_{2f_n}}{V_{1i_n}-(-V_{2i_n})}\rightarrow e\left(V_{1i_n}+V_{2i_n}\right)=V_{2f_n}-V_{1f_n}$..Eqn (2) This leads to solution of Two simultaneous equations to determine Two variables V_{1f_t} and V_{2f_t}. Accordingly, by taking [Eqn(1)] PLUS $[(m_1)\times\ \text{Eqn}(2)\ ]$ leads to:

$$\left(m_1V_{1i_n}-m_2V_{2i_n}\right)+(m_1)e\left(V_{1i_n}+V_{2i_n}\right)=\left(m_1V_{1f_n}+m_2V_{2f_n}\right)+(m_1)\left(V_{2f_n}-V_{1f_n}\right)$$

$$=>\left(m_1V_{1i_n}-m_2V_{2i_n}\right)+(m_1)e\left(V_{1i_n}+V_{2i_n}\right)=(m_1+m_2)V_{2f_n}\rightarrow \boldsymbol{V_{2f_n}}=\frac{\boldsymbol{(1+e)m_1V_{1i_n}+(em_1-m_2)V_{2i_n}}}{\boldsymbol{m_1+m_2}}$$

Likewise, by [Eqn(1)] MINUS $[(m_2)\times\ \text{Eqn}(2)\ ]$ leads to:

$$\left(m_1V_{1i_n}-m_2V_{2i_n}\right)-(m_2)e\left(V_{1i_n}+V_{2i_n}\right)=\left(m_1V_{1f_n}+m_2V_{2f_n}\right)-(m_2)\left(V_{2f_n}-V_{1f_n}\right)$$

$$=> (m_1V_{1i_n} - m_2V_{2i_n}) - (m_2)e(V_{1i_n} + V_{2i_n}) = (m_1+m_2)V_{1f_n} \rightarrow \boldsymbol{V_{1f_n}} = \frac{\boldsymbol{V_{1i_n}(m_1 - em_2) - (1+e)m_2V_{2i_n}}}{\boldsymbol{m_1+m_2}}$$

V_{1f_n} and V_{2f_n} together with V_{1f_t} and V_{2f_t} would give the post collision velocity vectors. Using this generic solution multiple cases can be solved, some of the typical cases are as under –

a) **e=1**; $V_{1f_n} = \frac{V_{1i_n}(m_1-m_2)-2m_2V_{2i_n}}{m_1+m_2}$ **and** $V_{2f_n} = \frac{2m_1V_{1i_n}+(m_1-m_2)V_{2i_n}}{m_1+m_2}$. This is a case of elastic collision and should conform LCE and accordingly $\frac{1}{2}m_1{V_{1i_n}}^2 + \frac{1}{2}m_2{V_{2i_n}}^2 = \frac{1}{2}m_1{V_{1f_n}}^2 + \frac{1}{2}m_2{V_{2f_n}}^2 \rightarrow m_1({V_{1i_n}}^2 - {V_{1f_n}}^2) = m_2({V_{2i_n}}^2 - {V_{2f_n}}^2)$. It leads to $m_1(V_{1i_n}+V_{1f_n})(V_{1i_n}-V_{1f_n}) = m_2(V_{2i_n}+V_{2f_n})(V_{2i_n}-V_{2f_n})$. Eqn. (1) above, as per LCM, leads to $m_1(V_{1i_n}-V_{1f_n}) = m_2(V_{2i_n}+V_{2f_n})$. Taking this as divisor to the equation as per LCE we get : $\frac{m_1(V_{1i_n}+V_{1f_n})(V_{1i_n}-V_{1f_n})}{m_1(V_{1i_n}-V_{1f_n})} = \frac{m_2(V_{2i_n}+V_{2f_n})(V_{2i_n}-V_{2f_n})}{m_2(V_{2i_n}+V_{2f_n})}$. It leads to $(V_{1i_n}+V_{1f_n}) = (V_{2i_n}+V_{2f_n}) \rightarrow V_{1i_n} - V_{2i_n} = -(V_{1f_n} - V_{2f_n})$. This corroborates that purely elastic collision, where there is no loss of energy, and is conservative in nature. This can be applied to all collisions where $e = 1$.

b) **e=1 and $\boldsymbol{m_1 = m_2}$:** $V_{1f_n} = -V_{2i_n}$ **and** $V_{2f_n} = V_{1i_n}$ i.e. velocities of colliding objects, along line of impact are interchanged

c) **e=0**: $V_{1f_n} = \frac{m_1V_{1i_n}-m_2V_{2i_n}}{m_1+m_2}$ and $V_{2f_n} = \frac{m_1V_{1i_n}-m_2V_{2i_n}}{m_1+m_2}$, it leads to $V_{1f_n} = V_{2f_n}$, i.e. after collision both balls plastically stick to each other and move with same velocity.

d) $\boldsymbol{m_2 = \infty, V_{2i_n} = 0}$ **and e=1:** $V_{1f_n} = \frac{V_{1i_n}(m_1-m_2)}{m_1+m_2} = \frac{m_1-m_2}{m_1+m_2}V_{1i_n} = \left.\frac{\frac{m_1}{m_2}-1}{\frac{m_1}{m_2}+1}V_{1i_n}\right|_{\frac{m_1}{m_2}\rightarrow 0} = -V_{1i_n}$, i.e. the colliding ball rebounds with same velocity.

e) $\boldsymbol{V_{2i_n}}$ **is +ve, i.e travelling in direction of $\boldsymbol{V_{1i_n}}$:** $V_{1f_n} = \frac{V_{1i_n}(m_1-em_2)+(1+e)m_2V_{2i_n}}{m_1+m_2}$ and $V_{2f_n} = \frac{(1+e)m_1V_{1i_n}-(em_1-m_2)V_{2i_n}}{m_1+m_2}$

f) **e=1, $\boldsymbol{V_{2i_n} = 0}$ and both $\boldsymbol{m_1 = m_2}$ and $\boldsymbol{m_2}$ are finite:** As per conservation of energy accordingly $\frac{1}{2}m_1{V_{1i_n}}^2 = \frac{1}{2}m_1{V_{1f_n}}^2 + \frac{1}{2}m_1{V_{2f_n}}^2 \rightarrow {V_{1i_n}}^2 = {V_{1f_n}}^2 + {V_{2f_n}}^2$. Thus, V_{1i_n}, V_{1f_n} and V_{2f_n} shall constitute orthogonal triplet. Actual vectors V_{1i}, V_{1f} and V_{2f}, shall depend upon V_{1i_t} and V_{2i_t}.

Thus, any specific case can be solved with this generic solution using equation derived from CLM and COR, constituting laws of collision. It is pertinent to understand that in the case of elastic collisions on a horizontal plane essential condition for collision are – **(a)** Both the initial velocity vector should be on same plane and not parallel, and **(b)** Pre-collision positions of the objects are such that trace of displacements has a point of convergence called collision.

Rocket Propulsion: This is another interesting case of change in momentum in IFRM, where rocket, without any external force, is self-propelled to move against gravity by ejection of mass at $\boldsymbol{r}$ kg/sec in opposite direction, a consequence of NTLM. Let initial mass of the rocket be $(\boldsymbol{m_i})$and it is moving at a velocity $(\boldsymbol{v_i})$, thus initial momentum be $p_i = m_i \cdot v_i$, a product of two variables dependent upon time. Since there is no external force and hence: $\frac{d}{dt}p_i = 0 = \frac{d}{dt}(mv) = m\frac{d}{dt}v + v\frac{d}{dt}m$. Accordingly, at any point of time ($\boldsymbol{t}$) during propulsion $p_i = m_iv_i = p = mv = (m_i - rt)v_t$. Thus, velocity of the rocket at any time ($\boldsymbol{t}$): $v_t = \frac{m_iv_i}{(m_i-rt)}$. This reduces velocity of rocket v as a function of time, where m_i,$v_i$and r are constants. Therefore, $\int_0^v dv_t = \int_0^t \frac{m_iv_i}{(m_i-rt)}dt$; $v = \frac{mv}{r}ln\left(\frac{m_i}{m_i-rt}\right)$. Taking ejection of mass w.r.t. rocket being propelled at velocity $\boldsymbol{u}$, as per LCM in NFOR: $0 = ru - mv$, or $\frac{mv}{r} = u$. Substituting it in the above equation, velocity of rocket, starting from rest, at any time later is $v = u\ ln\left(\frac{m_i}{m_i-rt}\right)$. It is a case of simple algebraic manipulations of equations, formulated from concepts of physics, to determine end result.

Summary: *Analysis of varieties of problems, representing different situation involve concepts of Newton's Laws of Motion, Circular Motion, Work-power-energy, and Conservation of Energy and Momentum. Many such situations can also be observed in real life and sailing into the problems in References cited below, would help to build an insight in the phenomenon occurring around. A deeper journey into the problem-solving would make integration and application of the concepts intuitive. This is absolutely true for any real life situation which requires multi-disciplinary knowledge in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory; an overall blend of personality of a person.*

References:

1. *NCERT; PHYSICS, Text Book for Class XI (Part I and II), and Exemplar Problems.*
2. *S.L.Loney; The Elements of Statistics and Dynamics: Part 1 – Statics and Part 2 – Dynamics.*
3. *H.C. Verma; Concepts of Physics, (Vil 1 & 2).*
3. *Resnick, Halliday, Resnick and Krane; Physics (Vol I and II).*
4. *Sears &Zemansky; University Physics with Modern Physics.*
5. *I.E. Irodov; Problems in General Physics*

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Source: Swayam Prabha, MHRD Project, IIT-PAL, CH_22, Physics by Prof. Sanjeev Sanghi, IIT Delhi; Language – English

1. LAWS OF MOTION: NEWTON'S FIRST LAW OF MOTION (CH_22) - https://www.youtube.com/watch?v=sW7f_IuvT6E
2. LAWS OF MOTION: NEWTON'S SECOND AND THIRD LAWS OF MOTION (CH_22 - https://www.youtube.com/watch?v=BZDfM6SDaFQ
3. FORCES ON BODIES: PROCEDURE TO SOLVE PROBLEMS (CH_22) - https://www.youtube.com/watch?v=gIahHPH3018
4. FORCES ON BODIES: PROBLEMS INVOLVING MULTIPLE BODIES; UNIFORM CIRCULAR MOTION; BODY IN CONTACTWITH FLUID - https://www.youtube.com/watch?v=bwiof-yWa3M
5. PROBLEM SOLVING NEWTON'S SECOND LAW (CH_22) - https://www.youtube.com/watch?v=mamhKWrA8Ag
6. WORK AND ENERGY: BASIC CONCEPTS RELATED TO CONSTANT AND VARIABLE FORCES; KINETIC ENERGY (CH_22) - https://www.youtube.com/watch?v=0kzvyEtW9QA
7. WORK ENERGY THEOREM AND CONCEPT OF POTENTIAL ENERGY (CH_22) - https://www.youtube.com/watch?v=EcTQbUJcLJA
8. WORK ENERGY AND IMPULSE MOMENTUM PRINCIPLES; CONSERVATION OF MOMENTUM (CH_22) - https://www.youtube.com/watch?v=skJQGH8wp0Q
9. IMPACT AND COLLISION (CH_22) - https://www.youtube.com/watch?v=dqZ7O-JWsso

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