Illustrations to Answers of Objective Questions (Typical) on Newton's Laws of Motion (Part-I)

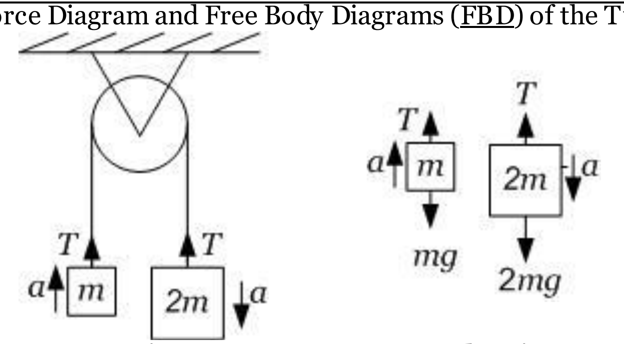
I-01

Let R is the radius of the earth and angular velocity of an object on the equator of the earth w.r.t. its Center, origin of the Frame of Reference (FOR) would be $\omega = \frac{2\pi\times1}{24\times60\times60} = 7.27\times10^{-5}$. Then Centrifugal force on a stationary train at the equator would be $F_{cs} = m\times R\times\omega^2 = m\frac{v_e{}^2}{R}$, here m is the mass of the train. Thus net force exerted on the rails would be $F_s = m\times g - F_{cs}$. The earth is rotating about its axis passing through the origin FOR in direction from East towards West. Let V be the speed of the train. Therefore, speed of train moving from West-East w.r.t FOR would be $V_{te} = V + V_e$ and the centrifugal force on the train would be $F_{te} = m\times\left(g - \frac{v_{te}{}^2}{R}\right)$.And for the train moving from East-West speed would be w.r.t. FOR would be $V_{tw} = V - V_e$ and the centrifugal force would be $F_{tw} = m\times\left(g - \frac{V_{tw}{}^2}{R}\right)$. In each of the cases R remains unchanged.

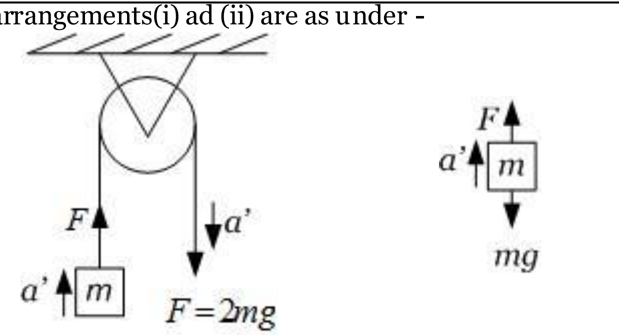
The subtrahend in F_{tw} is since larger than the subtrahend F_{te}, and hence, $F_{te} < F_{tw}$, and hence answer would be **option (b).**

I-02

Force Diagram and Free Body Diagrams (FBD) of the Two arrangements(i) ad (ii) are as under -



Force Diagram **Free Body Diagram**
Arrangement in Fig (i)

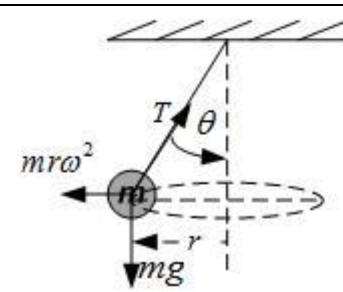


Force Diagram **Free Body Diagram**
Arrangement in Fig (ii)

Writing Force Eqn. for arrangement in Fig.(i) for mass $2m$ is $2mg - T = 2ma \rightarrow T = 2m(g-a)$ and for mass m would be $mg - T = -ma \rightarrow T = m(g+a)$ Using these Two Eqns. for T, $2m(g-a) = m(g+a) \rightarrow g = 3a \rightarrow a = g/3$.
While, for in Fig. (ii) for mass m is $mg - F = -ma' \rightarrow 2mg = m(g+a') \rightarrow a' = g$ i.e. $a' > a$. Thus answer is **Option (b).**

I-03

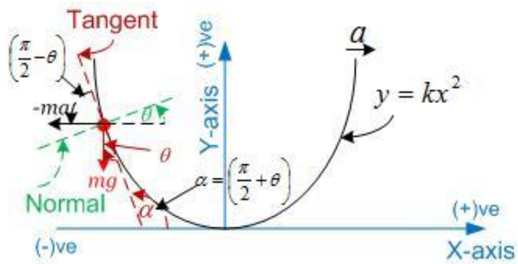
Let the ball is on a horizontal circular path of radius r and string makes angle θ with vertical line of suspension, and string is under tension T. By geometry of circular pendulum $sin\theta = \frac{r}{L}$. Then in condition of uniform circular motion, while vertical component T will cancel out since $T\cdot cos\theta = mg$. And horizontal components of T would bed $\cdot sin\theta = mr\omega^2 \rightarrow mL\,sin\theta\,\omega^2 \rightarrow T = mL\omega^2 \rightarrow mL\cdot sin\theta\,\omega^2 \rightarrow T = mL\omega^2$. Thus with limiting values of T, and that of m and L, limiting value of angular velocity would be $\omega = \left(\frac{T_{max}}{ML}\right) = \sqrt{\frac{324}{0.5\times0.5}} = \sqrt{324\times4} = \sqrt{81\times16}\} = 36$. Thus answer is **Option (b).**



I-04

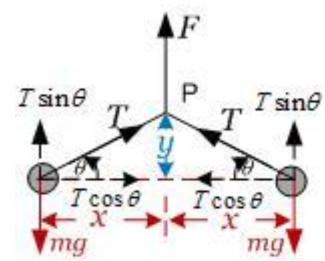
Under gravity the block would tend to slip down with gravitational pull parallel to the plane $= mg\cdot sin\theta - f$. Here, f is the force of friction and θ is angle on inclination of the plane. This motion can be retarded with a force P as given. When the block is just prevented from sliding down $P = P_1$, the block would tend to slide down and hence force

<table>
<tr><td></td><td>friction $f = mg \cdot sin\theta - P_1$.
It shall be in the upward direction i.e. opposite to the direction tending to be in motion and shall be parallel to P i.e. (+ve). As, P is increased frictional force $f \to 0$. Thereafter, it will have tendency to slide up, thereby reversing direction of frictional force against direction of motion till $P_2 = mg \cdot sin\theta + f$. Thus, f will become (-)ve. Answer is Option (a).</td></tr>
<tr><td>I-05</td><td>

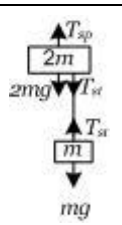


Wire frame in shape of parabola is given to be $y = \mathrm{k}x^2$. The frame exerts a force ma on the bead which in turn as per Newton's Third Law of Motion (NTLM) exerts a reaction $-ma$ as shown in the figure. For the bead to stay in any position, without sliding, it is essential that components gravitational force (mg) and reaction $-ma$ should be zero. It leads to $\mathrm{mg} \cdot \cos\theta + (-\mathrm{ma}) \cos\left(\frac{\pi}{2} - \theta\right) = mg \cdot \cos\theta - ma \cdot \sin\theta = 0 \Rightarrow \tan\theta = \frac{\mathrm{g}}{\mathrm{a}}$ ..(a)

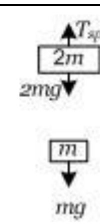
Slope of wire at the point is $\frac{dy}{dx} = \frac{d}{dx} \cdot kx^2 = 2kx = \tan(\pi + \theta) = -\cot\theta \to \tan\theta = -\frac{1}{2kx}$ ... (b). Equating values of $\tan\theta$ from Eqns. (a) and (b) $\frac{g}{a} = -\frac{1}{2kx}$. It leads to $x = \frac{a}{2kg}$. Hence answer is Option (b).</td></tr>
<tr><td>I-06</td><td>Slope of $\vec{p}$ is $\tan\theta_p = -\frac{\sin \mathrm{kt}}{\cos \mathrm{kt}}$. Further, Force $\vec{F} = \frac{d}{dt}\vec{p} = kA(-\sin kt\ \hat{\imath} - \cos(kt)\hat{\jmath}) \to \tan\theta_F = \frac{\cos \mathrm{kt}}{\sin kt}$. Since $\tan\theta_p \cdot \tan\theta_F = \left(= -\frac{\sin \mathrm{kt}}{\cos \mathrm{kt}}\right)\left(\frac{\cos \mathrm{kt}}{\sin kt}\right) = -1$. This is the condition for the Two vectors being orthogonal it $\left|\theta_p - \theta_F\right| = 90^0$.
Hence answer is Option (d)</td></tr>
<tr><td>I-07</td><td>Qn. No 41, on Kinematics in set Code:Phy/Kinx/O/001 is apparently similar to this. There in illustration to the answer used geometrical identity by correlating x and y with given a; this approach worked since correlation of velocities is associated with displacements, a geometrical entity, but it would be incorrect to apply it here.



In the instant case, it is related to correlation of forces and hence, it has to be analyzed with force equations. Further, the Two equal masses m placed symmetrically, it leads to $F = 2mg = 2T\sin\theta \to T = \frac{F}{2\sin\theta}$. Moreover, along the line joining the Two masses $T\cos\theta = m\frac{dx}{dt} \to T = \frac{m}{\cos\theta} \cdot \frac{dx}{dt}$. Equating the Two values of T, it leads to $\frac{F}{2\sin\theta} = \frac{m}{\cos\theta} \cdot \frac{dx}{dt} \to \frac{dx}{dt} = \frac{F}{2m} \cdot \cot\theta = \frac{F}{2m} \cdot \frac{x}{y}$. Expressing y geometrically $y = \sqrt{a^2 - x^2}$, it would be $\frac{dx}{dt} = \frac{F}{2m} \cdot \cot\theta = \frac{F}{2m} \cdot \frac{x}{\sqrt{a^2 - x^2}}$. Thus answer is Option (b).
N.B.: Such fine nuances in principles, despite problems appearing similar are often encountered. Therefore, it is extremely essential to analyze the question before one proceeds with wrong concepts and consequently ends up with wring answer.</td></tr>
<tr><td>I-08</td><td>



FBD of the Two masses suspended from spring, in steady-state is shown in the figure on the left. Accordingly in steady state force equation for B would $T_{st} = mg$ and for A would be $T_{st} = T_{st} + 2mg \to T_{st} = mg + 2mg \to T_{st} = 3mg$.
But, as soon as the string is cut the FBD of the Two masses shall become independent for motion and the FBD in new case is as shown in the figure on the right. Accordingly, force equation for A would be $T_{sp} - 2mg = 2ma \to 3mg - 2mg = 2ma \to mg = 2ma \to a = \frac{g}{2}$. Thus acceleration of mass A is $\frac{g}{2}$. Whereas, for mass B, T_{st} would disappear as soon as the string is cut. Hence, it will make a free fall under gravity and hence its acceleration would be g.



Accordingly, accelerations of A and B are $\frac{g}{2}$, g . Hence answer is Option (b).</td></tr>
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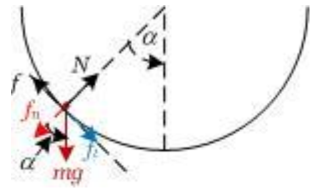
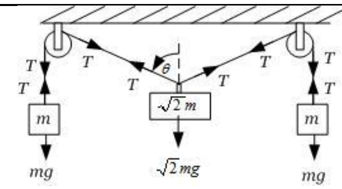
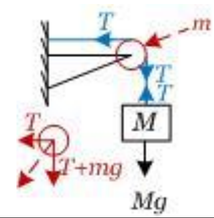
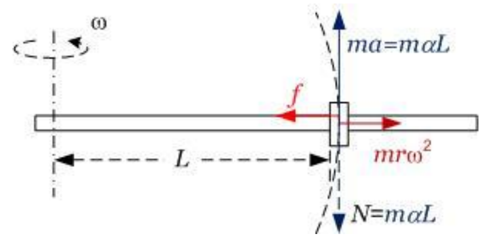
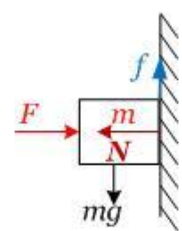
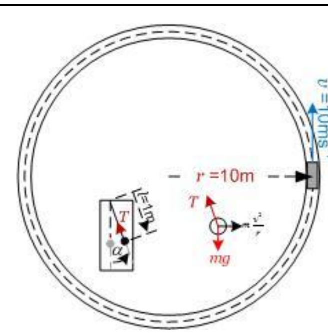
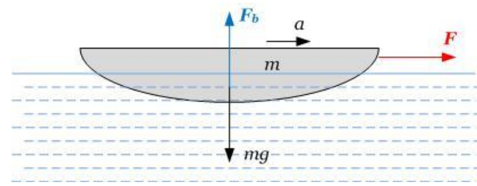
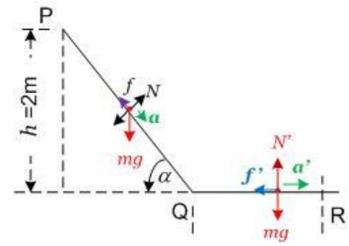
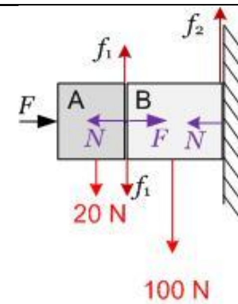
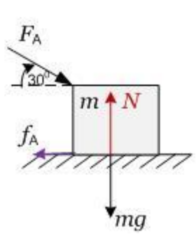
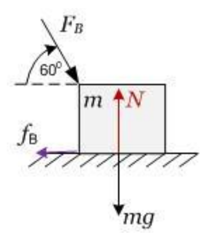
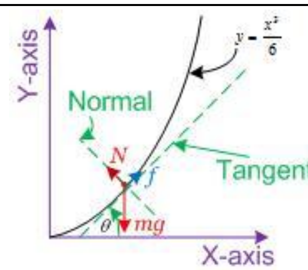
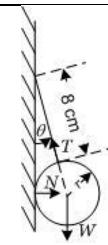
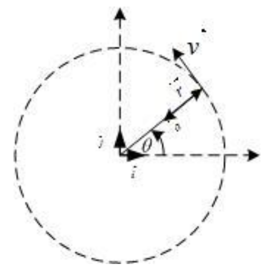
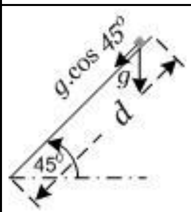
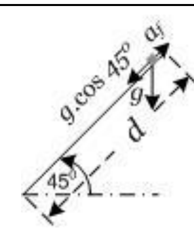
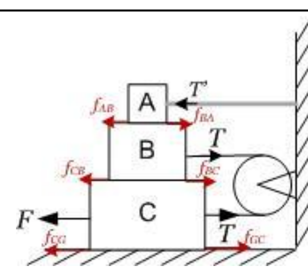
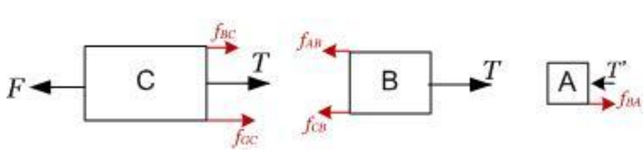
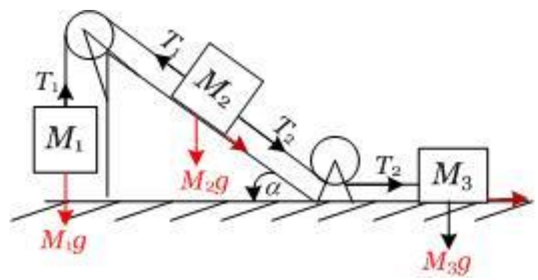
I-09	At the maximum height that insect can crawl shall be a state of equilibrium and hence as per force equations $\mathrm{N} = \mathrm{f_n} = \mathrm{mg}\cos\alpha$, hence frictional force that shall prevent the slipping down of the insect shall be $\mathrm{f} = \mu\mathrm{mg}\cos\alpha$. Whereas, the force that shall cause slipping is tangential component of ravitational pull $\mathrm{f_t} = \mathrm{mg}\cos\left(\frac{\pi}{2} - \alpha\right) = \mathrm{mg}\sin\alpha$. At the state of equilibrium $\mathrm{f} = \mathrm{f_t} \rightarrow \mu\mathrm{mg}\cos\alpha = \mathrm{mg}\sin\alpha \rightarrow \cot\alpha = \frac{1}{\mu}$. With the given value of μ, it would lead to $\cot\alpha = 3 \rightarrow \cot^{-1}3$. Hence, answer is **Option (a).** 
I-10	Since system is symmetrical about vertical line passing through mass $\sqrt{2}m$ and the Two strings joint. Therefore, horizontally it will be in equilibrium. And for vertical equilibrium necessary condition is $2T\cos\theta = \sqrt{2}mg$. Further, for equilibrium of masses m, the condition is $T = mg$. Combining these equations, $2mg\cos\theta = \sqrt{2}mg \rightarrow \cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{2}} \rightarrow \theta = 45^0$. Thus answer is **Option (c).** 
I-11	In the state of equilibrium $T = Mg$. And at the point of clamp where pulley is supported, vertical force shall be $F_v = T + mg = Mg + mg = (M+m)g$. Further horizontal pull shall be $F_h = Mg$. Hence, vreaction offered by clamp on the pully shall be vector addition of perpendicular forces F_h and F_v i.e. $\vec{F} = \vec{F_h} + \vec{F_v}$. Thus magnitude of net force offered by clamp on pully shall be $F = \lvert\vec{F}\rvert = \sqrt{{F_h}^2 + {F_h}^2} = \sqrt{(M+m)^2 + M^2}\cdot g$. Thus answer is **Option (d).** 
I-12	The tangential acceleration of the bead place at a distance L from axis of its rotation, when the rod is set in an angular acceleration α, is $a = L\alpha$. Thus a normal recation offered by rod is $N = mL\alpha$, therefore frictional force between the rod and the bead is $f = \mu mL\alpha$. And the centrifugal force would be $f_c = mL\omega^2$. At the instant when the bead is about to slip $f = f_c \rightarrow \mu mL\alpha = mL\omega^2\rvert_{r=L} \rightarrow \omega = \rightarrow \sqrt{\mu\alpha}$. From a state of rest time taken by the rod to acuquire angular velocity ω, as per Galelio's Fist Equation would be $\omega = \alpha t \rightarrow t = \frac{\omega}{\alpha} = \frac{\sqrt{\mu\alpha}}{\alpha} \rightarrow \sqrt{\frac{\mu}{\alpha}}$. Thus answer is **Option (a).** 
I-13	Normal force N is reaction by the surface of wall and hence $F = N$, accordingly, frictioonal force shall be $f = \mu N = 0.5 \times 5 = 2.5N$. But, threshhold value static friction (maximum) is equal to force tending to displace it. In he instant case posible displacement is vertical and hence $f_{max} = mg = 0.1 \times 9.8$ N, it is equal to fricttional force acting on the block. Hence,answer is **Option (b)**. ***N.B.:*** *Since value of g is not given, in competitve tests generally it is given, and therefore chossing it* $10\ ms^{-1}$, *would lead to* $f_{max} = 1 \approx 0.98$, *and would not change the answer.* 
I-14	The car, a non-inertail frame, is under ceptrepetal force buy virtue of it circular motion and according bob would experience a radial inward centrepetal force $f_{cp} = -m\frac{v^2}{r}$. Since, the bob remain suspended in equilibrum there would be corresponding radially outward centrifugal force $f_{cf} = m\frac{v^2}{r}$. The bod, which is suspended by strin is also vertical equilibrium, is acted upon Two forces tension in the string T and gravitatonal force mg. Resolving Tension T along radial for radial equilibrium $f_{cp} = -m\frac{v^2}{r} = -T\sin\alpha$, likewise along certical direction $mg = T\cos\alpha$. Combining Two equation $\tan\alpha = \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{rg} = \frac{10^2}{10\times10} = 1 = \tan 45^0$. Hence answer is **Option (c).** ***N.B.: Zoom-in the figure as required*** 

I-15	Since resistance is negligible hence $F = Ma \rightarrow 5 \times 10^4 = 3 \times 10^7 \times a \rightarrow a = \frac{5}{3} \times 10^{-3}$. Therefore, speed of the ship after being pulled over a distance of 3m from a state of rest can be obtained from Galeilio's Second Eqn of Motion $v^2 = u^2 + 2as \rightarrow v^2 = 0 + 2 \times \frac{5}{3} \times 10^{-3} \times 3 = 10^{-2}$. It leads to $v = 10^{-1}$ ms-1= 0.1 ms-1. Hence answer is **Option (c).** 
I-16	Solution to this problem is simple based on energy equivalence and potential energy of mass at P $(= mgh)$ is utilized in overcoming friction and particle comes to rest. Further, given that energy lost in track PQ $= \frac{1}{2}mgh$. Thus, force analysis on this portion of the track, as shown in figure, is redundant. On track QR retardation force is $f' = \mu N' = \mu mg$. Hence energy lost in track QR $= \frac{1}{2}mgh = f'x = (\mu mg)x$. It leads to $\frac{1}{2}h = \mu x \rightarrow \mu x = \frac{1}{2} \times 1 = 1$. This is achieved nearest with values of μ and x at (b), hence answer is **Option (b).** 
I-17	Block A ahall experience forces F and N in horizonatl direction such that $F = -N$, while in vertical direction 20N due to gravity balanced by $20 = -f_1 = -\mu_1 F$, where f_1 is tending to avoid slip of the block. As regards Block B horizontal forces are same as that on Block A, but equilibrium of vertixal forces is $100 + f_1 = f_2 \rightarrow f_2 = 100 + 20 = 120$ N, here $f_2 = -\mu_2 N$, but information on μ_2 is redundant. Thus answer is **Option (b)** ***N.B.:*** *Questions may at times contain redundant information and they must be identified correctly and thus solution going complex, can be avoided.* 
I-18	During pushing an object its velocity is consdered to be negligible and hence frictional force would be at its threshhold value i.e. $f = \mu N$. Hence, when force is applied then $f_A = F_A \cos 30^0 = \mu(mg + F_A \sin 30^0) \rightarrow \frac{\sqrt{3}}{2}F_A = \frac{\sqrt{3}}{5}\left(mg + \frac{1}{2}F_A\right) \rightarrow \frac{1}{2}\left(1 - \frac{1}{5}\right)F_A = \frac{1}{5}mg \rightarrow \frac{2}{5}F_A = \frac{1}{5}mg \rightarrow F_A = \frac{1}{2}mg$. Likewise, for $f_B = F_B \cos 60^0 = \mu(mg + F_B \sin 60^0) \rightarrow \frac{1}{2}F_B = \frac{\sqrt{3}}{5}\left(mg + \frac{\sqrt{3}}{2}F_B\right) \rightarrow \frac{1}{2}\left(1 - \frac{3}{5}\right)F_B = \frac{\sqrt{3}}{5}mg \rightarrow \frac{1}{5}F_B = \frac{\sqrt{3}}{5}mg \rightarrow F_B = \sqrt{3}mg$. Therefore, $\frac{F_A}{F_B} = \frac{\frac{1}{2}mg}{\sqrt{3}mg} = \frac{1}{2\sqrt{3}}$. Answer is **Option (d)**  
I-19	Let, position of the mass m where it can be placed without slipping be (x, y), where maximum height of climb shall be y. At this point tangential component of gravitational force is equal to frictional force such that $mg\cos(90^0 - \theta) = \mu N \rightarrow mg\sin\theta = \mu mg\cos\theta \rightarrow \mu = \tan\theta$. Further from coordinate geometry slope of tangent at a point on a curve is $m = \frac{d}{dx}y = \frac{d}{dx}\left(\frac{x^3}{6}\right) = \frac{x^2}{2} = \tan\theta$. Equating the Two values of $\tan\theta$ it leads to $\mu = \tan\theta \rightarrow 0.5 = \frac{x^2}{2} \rightarrow x = 1$. Using this value of x, maximum height of the mass, without slipping, shall be $y = \frac{1^3}{6} = \frac{1}{6}$m. Hence answer is **Option (c).** 
I-20	With the given data $\Delta\vec{v} = (6\hat{j}) - (6\hat{i} - 2\hat{j}) = -6\hat{i} + 8\hat{j}$. Therefore force acted upon the block $\vec{F} = m \cdot a = m \cdot \frac{\Delta\vec{v}}{\Delta t}$. It is given that $\Delta t = 10$ s, therefore, $\vec{F} = 5 \cdot \frac{-6\hat{i}+8\hat{j}}{10} = -3\hat{i} + 4\hat{j}$.nce answer is **Option (b).**

I-21	Force diagram of the problem is shown in figure where Vertical component of tension in the string would be balaned by weight i.e. $T\cdot\cos\theta = W$ and $T = W\cdot\sec\theta$. With the given geometry, $\sec\theta = \frac{(8+5)}{\sqrt{(8+5)^2-5^2}} = \frac{13}{12}$. Hence, answer is **Option (d).** 
I-22	Position vector of particle in polar coordinates is $R\angle\theta$, and since options are given in cartesian coordinates and hence $\vec{R} = R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}$, therefore velocity vector shall be $\vec{V} = \frac{d}{dt}\vec{R} = \frac{d}{dt}(R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath})$. It leads to $\vec{V} = R\left(-\sin\theta\frac{d\theta}{dt}\hat{\imath} + \cos\theta\frac{d\theta}{dt}\hat{\jmath}\right)$. Since, $V = R\frac{d\theta}{dt} = R\omega$ hence, $\vec{V} = R\omega(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})$. And accelertaion is $\vec{a} = \frac{d}{dt}\vec{V} = \frac{d}{dt}\left(R\omega(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})\right)$. It further comes to $\vec{a} = R\omega\left(\frac{d}{dt}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})\right) = R\omega\left(-\cos\theta\frac{d\theta}{dt}\hat{\imath} - \sin\theta\frac{d\theta}{dt}\hat{\jmath}\right)$. It calculates to $\vec{a} = R\omega(-\omega\cdot\cos\theta\,\hat{\imath} - \omega\cdot\sin\theta\,\hat{\jmath}) = -R\omega^2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) = -R\left(\frac{V}{R}\right)^2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) =$ $-\frac{V^2}{R}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$. Thus answer is **Option (c).** 
I-23	 A smooth block when slides down a distance d on a 45^0 incline, therefore effective acceleration shall be $a = g\cos 45^0 = \frac{g}{\sqrt{2}}$. Thus effecive time of slide over a distance d is as per Galelio's First Equation of Motion shall be $d = 0 + \frac{1}{2}at^2 = \frac{g}{2\sqrt{2}}t^2 \to t = \sqrt{\left(2\sqrt{2}\frac{d}{g}\right)}$. Now when it slides on a rough incline, having coefficent of friction as μ, frictional retardation shall be $a_f = \mu\left(\frac{g}{\sqrt{2}}\right) = \frac{\mu g}{\sqrt{2}}$. Thus, effective acceleration shall be  $a' = a - a_f = \frac{g}{\sqrt{2}} - \frac{\mu g}{\sqrt{2}} = \frac{g}{\sqrt{2}}(1-\mu)$. Accordingly time taken on rough surfaceshall beTherefore, time taken to cover a distance d is as per Galelio's Second Equation shall be $d = \frac{1}{2}a't'^2 \to t' = \sqrt{\frac{2d}{a'}} = \sqrt{\frac{2d}{\left(\frac{g}{\sqrt{2}}(1-\mu)\right)}} \to t' =$. It is given that $\frac{t'}{t} = n$. It leads to $n^2 = \frac{2\sqrt{2}\frac{d}{g}\frac{1}{(1-\mu)}}{2\sqrt{2}\frac{d}{g}} = \frac{1}{1-\mu}$. This can be further solved usin proprties of ratio-proportion and accordingly, $\frac{1-\mu}{1} = \frac{1}{n^2} \to -\mu = \frac{1-n^2}{n^2} \to \mu = 1 - \frac{1}{n^2}$. Since, it a case of sliding and hence $\mu \to \mu_k$, accordingly it would be $\mu_k = 1 - \frac{1}{n^2}$. Accordingly, answer shall be **option (a).** ***N.B.:*** *This is an excellent example of patience in handling variables, with proper care, works so well that most of them cancel out, and resulting expression is easy to reduce in its simplext form.*
I-24	Force diagram of the system is shown in the figure where $f_{AB} = f_{BA} = \mu M_A g = 0.25\times 3\times 10 = 7.5$ N. Likewise, $f_{BC} = f_{CB} = 0.25\times(4+3)\times 10 = 17.5$ N and $f_{BG} = 0.25\times(8+4+3)\times 10 = 37.5$ N. Frictional forces acting on each of the masses is shown. Tension T along the string shall be unifrm   FBD are also shown in the figure and analysis is strated from force F external to the system and each mass is analyzed independently. Thus, for equilibrium of mass C, $F = T + f_{GC} + f_{BC} = T + 37.5 + 17.5 = (T + 55)$ N. Likewise for mass B, force equation is $T = f_{AB} + f_{cB} = 17.5 + 7.5 = 25$ N. And for mass A $T' = f_{BA} = 7.5$ N, this equation is redundant since mass A is not moving. Using, eralier Two equation $F = 55 + 25 = 80$ N. Hence, answer **is option (c).**

I-25	The force diagram of the given system is explicit and hence going forward FBD has been skipped. Given that mass M_1 is moving downwards with uniform velocity it implies $T_1 = M_1 g$. Since all mases are connected with inestensible strings and hence M_2 and M_3 shall also be moving with uniform velocity and thus shall force equation balanced along the line of motion. Accordingly, $T_1 = T_2 + M_2 g \cos(90^2 - \alpha) + f_1 \rightarrow M_1 g = T_2 + M_2 g \sin\alpha + \mu M_2 g \cos\alpha \rightarrow M_1 g = T_2 + M_2 g(\sin\alpha + \mu \cos\alpha)$. Likewise for mass M_2, force equilibrium is $T_2 = \mu M_3 g$. It leads to $M_1 g = \mu M_3 g + M_2 g(\sin\alpha + \mu \cos\alpha) \rightarrow M_1 = \mu M_3 + M_2(\sin\alpha + \mu \cos\alpha) \rightarrow M_1 = 0.25 \times 4 + 4(0.6 + 0.25 \times 0.8) = 1 + 4 \times 0.8 = 4.2$ kg. And, tension in horizontal portion of the string is $T_2 = 0.25 \times 4 \times 10 = 10$ kg. Hence, answer is **Option (d).** 

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