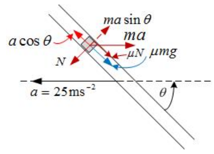
Code: Phy/NLM-II/S/001

Newton's Laws of Motion - II: Illustration of Subjective Questions (Typical)

I-01

Given that mass of the block is 1 kg. Translation acceleration of the horizontally placed disc is $a = 25\text{m.s}^{-2}$ towards left. Since the block is free to move in the groove, acceleration on the block along the groove shall be $a\cos\theta$ as is the figure. Therefore, frictional forces shall be acting against the acceleration on the block. There are Two frictional forces one on account of weight of the block $= \mu \times mg$. The second is due normal reaction $N = ma\cos\theta$ by the edge of groove, caused by pseudo-force (ma).

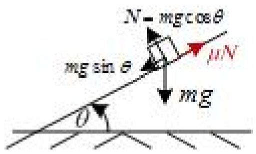


Thus second frictional force shall be $\mu ma\sin\theta$ Thus total Frictional force is $= \mu \times mg + \mu \times ma\sin\theta$. And this frictional force is equivalent to a retardation $= \mu \times (g + a \times \sin\theta)$

Accordingly, net acceleration shall be $a\cos\theta - \mu(g + a\sin\theta) \rightarrow 25 \times 0.8 - 0.4(10 + 25 \times 0.6) \rightarrow 20 - 0.4(10 + 15) \rightarrow 20 - 10 = 10\textbf{m.s}^{-2}$

I-02

Normal reaction for a plane inclined at 45° would be $\frac{mg}{\sqrt{2}}$ and hence frictional force for block A shall be $0.2\frac{mg}{\sqrt{2}}$ and for Block B shall be $0.3\frac{mg}{\sqrt{2}}$. Thus net force on Block A prompting it slide down would be $\frac{mg}{\sqrt{2}}(1-0.2) = 0.8\frac{mg}{\sqrt{2}}$and on block B shall be Block A prompting it slide down would be $\frac{mg}{\sqrt{2}}(1 - 0.3) = 0.7\frac{mg}{\sqrt{2}}$.



And as per NSLM acceleration experienced by Block A is $0.8\frac{g}{\sqrt{2}}$ and by Block B is $0.7\frac{g}{\sqrt{2}}$. Let distance covered by Block B when it is in line with block B is x m, then distance covered by block B shall be $x + \sqrt{2}$ m. Since both blocks start moving from rest at same instance and hence from Second Eqn. of Motion for block A, $x + \sqrt{2} = 0 \times t + \frac{1}{2}(0.8\frac{g}{\sqrt{2}}t^2)$, here t is time taken to cover distance $x + \sqrt{2}$. Likewise for block B, $x = 0 \times t + \frac{1}{2}(0.7\frac{g}{\sqrt{2}}t^2)$.

Accordingly, answers for Two parts are -

(a) Subtracting 1st Eqn. for block B from the Eqn. for block A and using given value of g it leads to $\sqrt{2} = \frac{1}{2}(0.1\frac{g}{\sqrt{2}}t^2) \rightarrow \sqrt{2} = \frac{1}{2}(\frac{1}{\sqrt{2}}t^2) \rightarrow t^2 = 4\ t = 2$ **sec**.

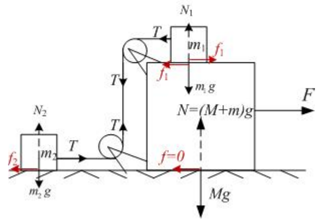
(b) Now to determine distance traveled by Two Blocks is

   (i) Block B: Use the equation for it with value of v and g, it leads to $x = \frac{1}{2}(0.7\frac{10}{\sqrt{2}}2^2) = 7\sqrt{2}\textbf{m}$.

   (ii) Block A: And distance covered by block A is $x + \sqrt{2} = 7\sqrt{2} + \sqrt{2} = 8\sqrt{2}$ **m**

I-03

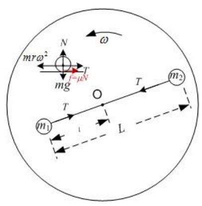
[**Note:** This case involves consideration of frictional force on mass m_1 and m_2 in the range Zero to their maximum values. And this needs to be decided first. This makes it an interesting problem] Accordingly, $f_1 \leq \mu \cdot m_1 \cdot g \rightarrow f_{1max} = (0.3) \cdot 20 \cdot 10 = 60$ N and $f_2 \leq \mu \cdot m_2 \cdot g \rightarrow = f_{2max} = (0.3) \cdot 5 \cdot 10 = 15$ N. Thus, $f_1 \neq f_2$, but given that $f_1 = 2 \cdot f_2$. This is possible only when mass m_1 is in state of rest w.r.t M. This is a case of static friction and under this condition $f_1 < f_{1max}$ and thus mass m_1 and mass M shall have same acceleration (a) under influence of force F. String connecting mass m_1 and m_2 are considered to be in-extensible, since nothing otherwise is defined in the problem. Therefore, mass m_2 shall also be experiencing same acceleration a as that of mass m_1 and M, and $f_2 = f_{2max} = 15$. Accordingly as per given condition $f_1 = 2 \cdot f_{2max} = 2 \cdot 15 = 30$N.



Given that $f_1 = 2 \cdot f_{2max} = 2 \cdot 15 = 30$N. Since, coefficient of friction of mass M w.r.t. ground is not defined it is taken to be Zero and hence frictional force on mass M is also $f = 0$. Such an assumption is important, and shall have to be taken unless universal quantity like g occurs.
Now, since motion of given masses is in horizontal direction, and hence, analyzing figure with all forces in action, horizontal forces on mass M shall form an equation $F - f_1 = M \cdot a$. Here, $f_1 \leq f_{1max}$. $\rightarrow F - 30 = 50a$
As regards mass m_1 equation of horizontal forces shall be $f_1 - T = m \cdot a \rightarrow 30 - T = 20 \cdot a \rightarrow T = 30 - 20a$. And for mass m_2 the equation shall be $T - f_2 = m_2 \cdot a \rightarrow f_2 = 15 = T - 5a \rightarrow T = 15 + 5a$.
Equating two values of T, it leads to $30 - 20a = 15 + 5a \rightarrow 25a = 15 \rightarrow a = \frac{15}{25} = \frac{3}{5}$ m.s^{-2}
It together with equation of mass M leads to $F = 30 + 50a = 30 + 50\frac{3}{5} = 30 + 30 = 60$N. And eqn. for mass m_2 leads to $T = 15 + 5\frac{3}{5} = 15 + 3 = 18$N.
Accordingly, desired Three variables are and the solution is $F = 60$N, $T = 18$N and $a = \frac{3}{5}$ m.s^{-2}

I-04

Geometrical setup of the problem is shown in the figure, where turn table is horizontal and rotating about its center at an angular speed ω. Hence, each mass would experience a centrifugal force $= mr\omega^2$. Accordingly force on mass m_1 would be $f_1 = m_1 l \omega^2$ and on mass m_2 would be $f_2 = m_2(L-l)\omega^2$. Now, that there is no friction on mass m_2. And coefficient of friction of mass m_1 is 0.5 hence maximum frictional force on it would be $f_{f1_{max}} = \mu N_1 = \mu m_1 g \rightarrow 0.5 \cdot 10 \cdot 10 = 50$ N.



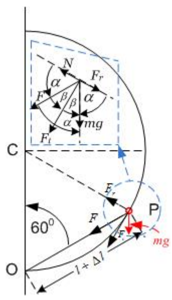
Since both masses are connected with a string of length $L = 0.3$ m, $l = 0.124$ m and hence a tension T in the string will act as centripetal force. Thus horizontal forces, which will affect equilibrium position of the masses, along diameter of the turn table, will have satisfy equation for mass m_1 as $f_1 = T + f_{f1} \rightarrow m_1(0.124)\omega^2 = T + f_{f1} \rightarrow 10(0.124)10^2 = T + f_{f1} \rightarrow 124 = T + f_{f1}$. Likewise, for mass m_2 the equation shall be $f_2 = T + f_{f2} \rightarrow m_1(L-l)\omega^2 = T \rightarrow 5(0.3-0.124)10^2 = T \rightarrow T = (0.176)500 = 88$ N. Substituting value of T in equation for mass m_1, it leads to $124 = 88 + f_{f1} \rightarrow f_{f1} = 124 - 88 = 36$ N. **This is part (a) of the solution.**

At ω_max when mass m_1 tends to slip $f_{f1} \rightarrow f_{f1_{max}} = 50$ N and $m_1 \cdot l \cdot \omega^2 = m_2(L-l)\omega^2 + f_{f1_{max}} \rightarrow 10 \cdot (0.124) \cdot \omega^2 = 5 \cdot (0.3 - 0.124) \cdot \omega^2 + 50 \rightarrow (1.24 - (0.5)(0.176))\omega^2 = 50 \rightarrow 0.36\omega^2 = 50 \rightarrow \omega = \sqrt{\frac{50}{0.36}} = 11.75$ rad.s^{-1}. **This is part (b) of the solution.**

For both bodies to be at equilibrium without frictional force coming into play necessary requirement is $T = m_1 \cdot l \cdot \omega^2 = m_2 \cdot (L - l) \cdot \omega^2 \to m_1 \cdot l = m_2 \cdot (L - l) \to (m_1 + m_2)l = m_2 \cdot L \to l = \frac{m_2}{m_1 + m_2} L \to$ $l = \frac{5}{15} 0.3 = 0.1$ m, this distance of mass m_1 from O. Accordingly, distance of mass m_2 from O is $L - l = 0.3 - 0.1 = 0.2$ m. **This forms part (c) of the solution**.

I-05

Given the geometry of the system where lengths CE=OP and hence $\angle CPO = 60° \to \angle OCP$ and hence $\triangle OCP$ is equilateral therefore length OP $= R$. Therefore, for holding ring at spring of natural length $\frac{3}{4}R$ is stretched by a length $R - \frac{3}{4}R = \frac{1}{4}R$ and hence it would exert a force $F = \frac{1}{4}R \cdot \frac{mg}{R} \to \frac{mg}{4}$.
Free body diagram of the ring is shown in the figure. In this angles $\alpha = 60°$ and $\beta = 30°$ between vectors are shown in inset, and are determined geometrically.
It forms part (a) of the solution

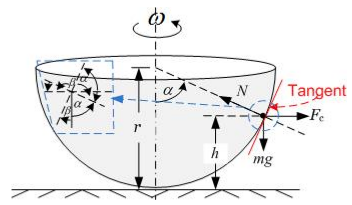


Tangential force $F_t = F \cdot \cos 30° + mg \cos 30° = (\frac{mg}{4} + mg) \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}mg}{8}$ N. Thus acceleration of ring, along the tangent at point as per NSLM, calculates to $a_t = \frac{\frac{5\sqrt{3}mg}{8}}{m} = \frac{5\sqrt{3}g}{8}$ m $\cdot$ s^{-2} tend to accelerate the ring P, along it.
And, normal force is derived from forces acting on the ring are F and weight mg. Accordingly, $F_r = mg \cos 60° - F \cdot \cos 60° \to F_r = (mg - F) \cos 60° \to (\frac{mg}{4} - mg)\frac{1}{2} = -\frac{3mg}{8}$ N. Therefore, $N = -F_r$, it implies that $|N| = |F_r| = \frac{3mg}{8}$ N

Thus Tangential acceleration $a_t = \frac{5\sqrt{3}g}{8}$ m $\cdot$ s^{-2} and normal reaction $N = \frac{3mg}{8}$ N, form part (b) of the solution .

I-06

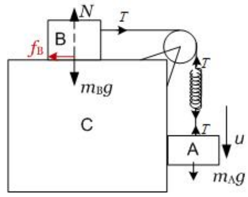
On the particle three forces are acting, one is gravitational, second is centrifugal force due to rotation about its vertical axis, and third is reaction offered by inner surface of the bowl. Since, inner surface of bowl is frictionless, there will not be frictional force. For the ball to remain stationary w.r.t. bowl net tangential force must be zero. All forces are sown in the figure and geometrically $\cos\alpha = \frac{r-h}{r} \to \sin\alpha = \sqrt{\frac{r^2 - (r-h)^2}{r^2}} = \frac{\sqrt{h(2r-h)}}{r}$ and $\beta = \frac{\pi}{2} - \alpha$.



Thus equation of forces for equilibrium on tangential line at point P would be $F_c \cos\alpha = mg \cos\beta$, here $F_c = m(\sqrt{r^2 - (r-h)^2})\omega^2$.
Accordingly, substituting values $m(\sqrt{h(2r-h)})\omega^2 \frac{(r-h)}{r} = mg\frac{\sqrt{h(2r-h)}}{r} \to \sqrt{h(2r-h)})\omega^2(r-h) = g(\sqrt{h(2r-h)}) \to \omega^2(r-h) = g \to h = r - \frac{g}{\omega^2}$ m .**This is answer for part (a) of the question**
From the above $\omega^2 = \frac{g}{r-h} \to \omega = \sqrt{\frac{g}{r-h}}$. Since $0 < h \leq r$, as increases in the range $h \to r$ the $\omega \to \infty$, therefore minimum ω limit for $h \to 0$ the value is $\omega_{min} = \sqrt{\frac{g}{r}}$. **This answer for part (b) of the problem.**

From above $g = \omega^2(r-h)$. Given are r and ω, therefore, $\Delta g = \omega^2 \Delta h$. Given that $\Delta h = 10^{-4}$. Taking $g = 9.8$ m $\cdot$s^{-2} would depend upon $\Delta g = {\omega_{min}}^2 \Delta h \rightarrow \sqrt{\frac{g}{r}}^2 \Delta h = \sqrt{\frac{9.8}{0.1}}^2 10^{-4} = \frac{9.8}{0.1} \times 10^{-4} = 9.8 \times 10^{-3}$ m$\cdot$s^{-2}. **This is answer for part (c) of the question**.

I-07



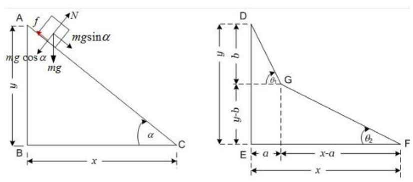
Given that both the masses A and B are moving with uniform velocity, and none of the masses are experiencing any force as per NFLM. This can happen when weight of the mass A has stretched the spring fully; let extension in spring is Δl. Thus $K\Delta l = m_A g \rightarrow \Delta l = \frac{m_A g}{1980} = \frac{2 \times 9.8}{1960} = 0.01$ m. Accordingly, force on both ends of the spring and tension in the string shall be $T = K\Delta l = 2 \times 9.8 = 19.6$ N.

With the given condition of uniform velocity, horizontal force on mass B must also be in equilibrium and accordingly $f_B = T = 19.6 \rightarrow \mu N_B = 19.6 \rightarrow 0.2 \times m_B \times g = 19.6 \rightarrow 0.2 \times m_B \times 9.8 = 19.6 \rightarrow m_B = \frac{19.6}{1.96} = 10$ kg. **This is part (a) of the answer**.

Answer to part (b) of the question is energy in the spring $E = \frac{1}{2}K\Delta x^2 = \frac{1}{2}1960 \times (0.01)^2 = 0.098$ J.

It is to be noted that mass A is suspended vertically and its component along perpendicular to the surface shall make an angle 90° and hence normal reaction on the surface shall be Zero [$\cos 90° = 0$. Thus frictional force between vertical surface of block C and block A shall be Zero

I-08

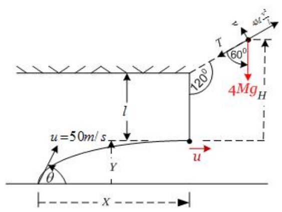


Free body diagram for Fig (a) are shown and will b used to arrive at generic equation of velocity V_C. The same equation shall be used in parts for Fig (b) for Two section, with different inclinations, to derive velocity V_F. Block is free to slide along inclined plane and acceleration shall be $a_{AC} = g(\cos\alpha - \mu\sin\alpha)$.

Thus according to kinematics ${V_C}^2 = 2a_{AC} y \csc\alpha = 2g(\cos\alpha - \mu\sin\alpha)y\csc\alpha = 2g(y\cot\alpha - y\mu) \rightarrow {V_C}^2 = 2g(x - \mu y)$. This equation of mechanics has been reduced geometrical relationship, and shall be used to solve for ${V_F}^2$ in Fig. (b)

Now in Fig. (b) there are Two slope DG and GF with inclinations θ_1 and θ_2 respectively. And, generic equation of Fig. (a) ${V_G}^2 = 2g(a - \mu b)$ and ${V_F}^2 = {V_G}^2 + 2g((x-a) + \mu(y-b))$. It leads to ${V_F}^2 = 2g(a - \mu b) + 2g((x-a) + \mu(y-b)) = 2g(x - \mu y)$. Thus $V_C = V_F = \sqrt{2g(x - \mu y)}$. **This is answer to the problem.**

I-9



Solution to this problem involves kinematics, conservation of momentum, conservation of energy and centrifugal action. Let u is the velocity of the combined mass $4M = M + 3M$ post collision, and v is the velocity of the combined mass before centrifugal force $= 4M\frac{v^2}{l}$ and tension $T = 4Mg\cos 60°$ in the string are in equilibrium, a necessary condition for string to remain stretched until it turn through an angle 120°. As bob tends to swing beyond 120° its potential energy will increase and in turn velocity would decrease. This would reduce centrifugal force which keep string stretched. Moreover, with increase of the swing angle, angle between the string and g would reduce, which will cause increase in component of $4Mg$ centripetal force. This imbalance creates condition of collapse of the string.

This leads to $4M\frac{v^2}{l} = 4Mg\cos 60° \rightarrow v^2 = \frac{gl}{2} = \frac{50}{3}$.

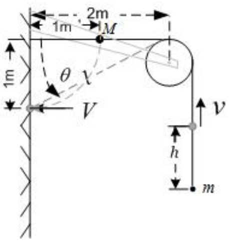
Taking principle of conservation of momentum $MV\cos\theta = 4Mu \rightarrow u = \frac{50}{4}\cos\theta = \frac{25}{2}\cos\theta$.

Taking energy balance equation post collision until limiting condition for stretched string $\frac{1}{2}4Mu^2 = \frac{1}{2}4Mv^2 + 4Mgl(1+\cos 60°) \rightarrow u^2 = v^2 + gl\frac{3}{2} \rightarrow u^2 = v^2 + 10\frac{10}{3}\frac{3}{2} \rightarrow (\frac{25}{2}\cos\theta)^2 = \frac{50}{3} + 100 \rightarrow = \frac{350}{3} \rightarrow \cos^2\theta = \frac{1400}{1875} = \frac{56}{75} \rightarrow \cos\theta = \sqrt{\frac{56}{75}} = 0.86 \rightarrow \theta = 30°$. **This part (a) of the answer.**

Time taken by bullet to the bullet of mass M to reach its highest point, at which it strikes hanging bob of mass $3M$mass, is $t = \frac{V^{\cdot}\sin\theta}{g}$; this is from GFEM equation of motion. During this time, bullet horizontally covers,with uniform velocity, a distance $X = (V\cos\theta)(\frac{V^{\cdot}\sin\theta}{g}) = \frac{V^2\sin 2\theta}{2g} = \frac{50^2 \times \sqrt{3}}{2 \times 2 \times 10} = 108.25$ m. Likewise, for height attained equation shall be $0 = V^2\sin^2\theta - 2gY \rightarrow Y = \frac{V^2\sin^2\theta}{2g} = \frac{50 \times 50 \times (\frac{1}{2})^2}{2 \times 10} = 31.25$ m. **These values of X and Y are part (b) of the answer.**

I-10

This solution involves principle of conservation of energy. Let the mass M will move down along a circular path, as shown in figure and shall have a velocity V when it touches the wall. The string shall remain stretched during movement of masses. At this instant the end of inclined portion of the string attached to the mass shall have a component of V along its inclined length $= V\cos\theta$. By geometry $\cos\theta = \frac{2}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}}$. When mass M reaches wall length of string between wall and pulley is $1+\sqrt{5}$ m as against 2 m in initial condition, eventually it increases by $(1+\sqrt{5}) - 2 = \sqrt{5} - 1$ m.



Since string is in-extensible, hence mass M will descend through a height, 1 m, and mass m will rise through a height $(\sqrt{5}-1)$ m, and shall have a velocity $v\frac{2}{\sqrt{5}}$. This makes information complete for evolving energy balance equation.

Accordingly, $(PE_{Mi} + KE(Mi)) + (PE_{mi} + KE(mi)) = (PE_{Mf} + KE(Mf)) + (PE_{mf} + KE(mf)) \rightarrow \Delta(PE_M + KE(M)) + (PE_m + KE(m))$.

Thus, $\Delta PE_M = -Mg \cdot 1 = -2 \times 9.8 = -19.6$ J; $\Delta PE_m = mg \cdot (\sqrt{5}-1) = 0.5 \times 9.8 \times 1.24 = 6.08$ J;

Thus, $\Delta KE_M = \frac{1}{2}Mv^2 = v^2$ J; $\Delta KE_m = \frac{1}{2}m(v\cos\theta)^2 = \frac{1}{4}v^2(\frac{2}{\sqrt{5}})^2 = \frac{v^2}{5} = 0.2v^2$ J;

Thus, $\Delta PE_M + \Delta PE_m + \Delta KE_M + \Delta KE_m = 0 \rightarrow \Delta KE_M + \Delta KE_m = -(\Delta PE_M + \Delta PE_m) \rightarrow v^2 + 0.2v^2 = 19.6 - 6.08 \rightarrow v = \sqrt{\frac{13.52}{1.2}} = 3.36 \approx 3.4$ m·s^{-1}

[**Note:** Variation in LSD of answer depends upon rounding of digits and LSDs od the quantities that appear in calculation. It is a practice to take result of each intermediate calculation to SDs one more than LSD. And final result is rounded to appropriate LSD.]