# Mechanics of Rigid Bodies : Illustrations of Answers to Objective Questions (Typical)

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| I-01 | Determination of position vector centre of mass $(\vec{R}_{cm})$ is based on $\vec{R}_{cm} = \frac{1}{\sum_i m_i}\sum_i m_i\vec{r}_i = \frac{1}{M}\sum_i m_i\vec{r}_i$, here acceleration of frame has no implication, and hence (A) is correct. As regards acceleration in a inertial frame as per NSLM $\vec{a}_{cm} = \frac{\vec{F}}{M}$, but when mass $M$ is in an non-inertial frame having an acceleration $\vec{a}_f$ force experienced by it would be $\vec{F} = (\vec{a}_{cm} - \vec{a}_f)M$ and hence (B) is wrong; this can be better explained with experiencing weight in a lift accelerating during ascend and during descend. Accordingly, out of the given **options (c) is the answer.** |
| I-02 | Statement (A) implies that $\vec{P} = \sum_i m_i\vec{v}_i =$Constant. On integrationg it leads to - $\int \vec{P}dt = \sum_i \int m_i\vec{v}_i dt$, which is mathematically $\int \vec{P}dt = \sum_i \left( m_i \int \vec{v}_i dt - \int \left( \frac{d}{dt} m_i \times \int \vec{v}_i dt \right) dt \right)$. By conservation of mass $\frac{d}{dt} m_i = 0$ and hence second term will disappear and it would lead to $\int \vec{P}dt = \sum_i m_i(\vec{r}_i + \vec{C}_i) = \sum_i m_i\vec{r}_i + \sum_i m_i\vec{C}_i = M\vec{R} + \vec{C}$. In this with (A) LHS is linearly increasing with time and so also in RHS for each of the constituent mass First terms would continue to increase linearly i.e. vector $\vec{R}$ of COM is not fixed. But, the second term is fixed bias created by initial condition. This implies that centre of mass is not fixed ***i.e. while A implies B does not.***<br><br>Statement (B) implies that $\vec{R} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \rightarrow M\vec{R} = \sum_i m_i\vec{r}_i$. Differentiating it, $\frac{d}{dt}M\vec{R} = \sum_i \left( m_i \frac{d}{dt}\vec{r}_i + \vec{r}_i \frac{d}{dt} m_i \right)$. Since by conservation of mass second term of RHS is Zero and both the terms of derivative ob LHS are constant, it leads to $0 = \sum_i m_i\vec{v}_i = \sum_i \vec{p}_i$. This is possible if either $\vec{v}_i$ for all constituent masses are Zero or resultant of all $\vec{p}_i$ is constant at Zero. Thus, ***(B) implies (A).***<br>***Thus conclusions of analysis of both the premises lead to answer (d).*** |
| I-03 | Statement (A) implies that $\vec{P} = \sum_i m_i\vec{v}_i = 0$ i.e. either $\vec{v}_i$ for all constituent masses are Zero or resultant of all $\vec{p}_i$ is constant at Zero. In case resultant of all velocities is $\sum_i \vec{v}_i = 0$ then kinetic energy cannot be zero since it involves square of the magnitudes of velocity (scalar quantity). Thus, while ***(A) implies (B) does not***. |

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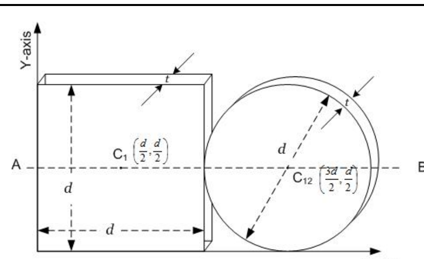
| | As regards statement (B) it implies that $KE = \sum_i \left(\frac{1}{2} m_i v_i^2\right) = \frac{1}{2}\sum_i m_i v_i^2 = 0$ is possible when velocity of all particles is zero. Thus, while ***(B) implies (A) does not***.<br>***Thus conclusions of analysis of both the premises lead to answer (d).*** |
| Q-04 | Q-04, HCV-I,Obj-SCQ,pp.157 |
| | Consider the following Two statements –<br>(A) Linear momentum of system of a particle is independent of the frame of reference.<br>(B) Kinetic energy of system of a particle is independent of frame of reference<br>Then,<br>(a) Both A and B are true<br>(b) A is true but B is false<br>(c) A is false but B is true<br>(d) Both A and B are false |
| I-04 | Statement (A) implies that $\vec{P}_R = \sum_i m_i \vec{v}_{i\_R}$ and $\vec{v}_{i\_R} = \vec{v}_{i\_S} + \vec{v}_{R,S}$ here $\vec{P}_R$ is linear momentum in FOR R; $\vec{v}_{i\_R}$ is velocity of particle in FOR R; $\vec{v}_{i\_S}$ is velocity of particle in FOR S and $\vec{v}_{R,S}$ is velocity of FOR R w.r.t. S. Thus addend $\vec{v}_{R,S}$ will create a dependence of linear momentum on FOR. Hence, **premise A is false**.<br>Likewise, in statement B, $KE_p = \frac{1}{2}\sum_i m_i v_{i\_R}^2$ and component $\vec{v}_{R,S}$ in KE makes it dependent on FOR, and hence **premise B is false**.<br>***Thus conclusions of analysis of both the premises lead to answer (d).*** |
| I-05 | Since, $\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$, given that $\lvert\vec{r}_i\rvert = R$, but distribution of mass of particles in the system is not given.<br>Hence, if all $\vec{r}$ are coincident then by principles of vector addition $\frac{\left\lvert\sum_i \vec{r}_i\right\rvert}{n} = R$, here number of vectors is $n$. And if any of the vector is divergent then $\frac{\left\lvert\sum_i \vec{r}_i\right\rvert}{n} < R$. Combining both the cases **answer is (b)** |
| I-06 | The masses are arranged as shown in the figure. With material density $\rho$ mass of square plate shall be $M_1 = \rho(d^2 t)$ with its COM at $C_1\left(\frac{d}{2}, \frac{d}{2}\right)$ and for circular plate shall be $M_2 = \rho\left(\frac{\pi d^2}{4} t\right)$ with its COM at $C_1\left(\frac{3d}{2}, \frac{d}{2}\right)$. Hence, coordinates of combined COM shall be<br><br><br>$\overline{Y} = \frac{d}{2}$, since each of objects is symmetrical about line AB passing through $C_1$ and $C_2$ and parallel to X- |

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| | Axis. Whereas, along X-axis is, $\overline{X}=\frac{M_1\left(\frac{d}{2}\right)+M_2\left(\frac{3d}{2}\right)}{M_1+M_2}=\frac{\rho d^2t\left(\frac{d}{2}+\frac{\pi}{4}\cdot\frac{3d}{2}\right)}{\rho d^2t\left(1+\frac{\pi}{4}\right)}=\frac{\frac{d}{2}\left(\frac{3\pi+4}{4}\right)}{\frac{\pi+4}{4}}=\frac{d}{2}\left(\frac{3\pi+4}{\pi+4}\right)=\frac{d}{2}\cdot\frac{13.42}{7.14}<d$. Thus, COM of compound mass it shall be inside square plate. Hence, **answer shall be (b)** |
| I-07 | Let at time $t=0$, two identical particles A and B of mass $m$ are at position vectors $\vec{r}_{a\_0}$ and $\vec{r}_{b\_0}$ along X-axis for simplification; particles are taken to be at rest at the initial instance both. Therefore centre of mass shall be initially at $\vec{R}_0=\frac{m\vec{r}_{a\_0}+m\vec{r}_{b\_0}}{2m}=\frac{1}{2}\left(\vec{r}_{a\_0}+\vec{r}_{b\_0}\right)$. Particle A remains at rest i.e. $\vec{r}_{a\_0}=\vec{r}_{a\_t}$, while the particle B has an acceleration $\vec{a}$. Therefore, position vector of particle at any instant $t$ would be $\vec{r}_{b\_t}=\vec{r}_{b\_0}+\left(0\times t+\frac{1}{2}\vec{a}t^2\right)$. Therefore COM of the two masses shall be at $\vec{R}_t=\frac{m\vec{r}_{a\_t}+m\vec{r}_{b\_t}}{2m}=\frac{1}{2}\left(\vec{r}_{a\_0}+\vec{r}_{b\_0}+\frac{1}{2}\vec{a}t^2\right)=\vec{R}_0+\frac{1}{2}\left(\frac{1}{2}\vec{a}\right)t^2$. Thus, as per Second equation of kinematics, acceleration of COM is $\frac{1}{2}\vec{a}$, and hence **answer (b).** |
| I-08 | Internal forces in a system are always in pairs i.e. equal and opposite. In this question two physical quantities linar momentum $\left(\vec{P}=\sum_i m_iv_i\right)$ and kinetic energy $\left(KE=\frac{1}{2}\sum_i m_iv_i^2\right)$ are compared in different context. Taking derivative of linear momentum $\frac{d}{dt}\vec{P}=\sum_i m_i\frac{d}{dt}v_i=\sum_i m_ia_i\rightarrow\Delta\vec{P}=\vec{F}_{ext}\Delta t$. Thus in accordance with NSLM they are is the external forces which changes linear momentum. But, taking derivative of kinetic energy $\frac{d}{dt}KE=\frac{1}{2}\sum_i m_i\frac{d}{dt}\left(v_i^2\right)=\frac{1}{2}\sum_i m_i(2v_i)a_i$. It leads to $\Delta KE=\left(\sum_i m_iv_ia_i\right)\Delta t$, thus change in kinetic energy in a time $\Delta t$ is due acceleration and its consequent effect on velocity of constituent masses. These are used to analyse given options -<br>• From the above analysis **option (a) is incorrect** internal forces are stated to change linear momentum.<br>• Option (b) is correct in respect of linear momentum. As regards KE, internal forces when affect $a_i$ of constituent particles, change in $v_i$ affects the kinetic energy as well and is in accordance with basic definition. Hence, ***both the cases in option (b) are correct***.<br>• From the logic invalidating option (a) ***the option (c) is incorrect***.<br>• Since internal forces change kinetic energy of the system as in option (b) and hence taking both P and KE invariant in ***option (d) is incorrect.***<br>Accordingly ***answer is option (b)*** |
| I-09 | Let $m$ and $v$ are mass and velocity of the bullet and $M$ and $V=0$ are mass and velocity of block. After bullets gets embedded then as per principle of Conservation of Monentum |

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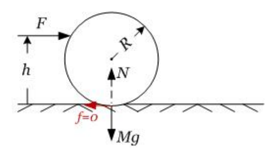
| | $mv + M \times 0 = (M+m)v_f \rightarrow v_f = \frac{mv}{M+m}$, here $v_f$ is the final velocity. With this analysis of each option is –<br>• Moment of block post embedding of bullet changes from Zero to $M\left(\frac{mv}{M+m}\right)$, ***hence option (a) is incorrect***.<br>• Kinetic energy of the block post embedding changes from Zero to $\frac{1}{2}M\left(\frac{mv}{M+m}\right)^2$, ***hence option (b) is incorrect***.<br>• Since block is placed on a horizontal surface hence its height $(h)$above ground remains unchanged post embedding and hence its gravitational, potential energy $(= Mgh)$, $M$ and $g$ remaining constant, it remains unchanged, ***hence option (c) is correct***.<br>• Embedding of bullet in block is causing a plastic deformation and which shall internal energy of the block and hence its temperature and ***hence option (d) is incorrect***<br>The above analysis leads to ***as answer option (b)*** |
| I-10 | Since, horizontal surface is smooth and $\mu = 0 \rightarrow f = \mu(Mg) = 0$. Inabsence of friction, there wuld be only lipping of sphere, without angular rotation. Therefore, force $F$ will only cause translational motion of sphere and as per NSLM, centre of sphere would accelerate horizontally at $a = \frac{F}{M}$, in which there is no dependence on height $(h)$)of application of $F$. This rules out option (a), (b and (c), and goes in favour of otion (d). Hence, ***answer is (d).***<br><br> |
| I-11 | Let an object of mass $M$ falling vertically has a velocity of fall $U$ breaks into two unequal masses $m_1$ and $m_2$ at height $h$ above ground. Only vertical gravitational acceleration causing a force $Mg$ on the object, and $m_1 g$ and $m_2 g$ on split masses is acting. Since, there is only vertical force acting on the split masses COM would move downward along $\vec{g}$ . As regards horizontal displacement there is no force in this direction acting either on the object or split masses, therefore, as per NFLM there woild be no horizontal displacement of COM. Therefore, **answer is (c)** |
| I-12 | Let an object of mass $M$ is at rest and hence its momentum $P = 0$. Since, no external force is defined causing breaking of the object in two equal masses $m_1$ and $m_2$ and start travelling with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively. Then, as per NFLM $P = m_1\vec{v}_1 + m_2\vec{v}_2 = 0 \rightarrow \frac{m_1}{m_2} = (-)\frac{\vec{v}_2}{\vec{v}_1}$. From this equation following inferences can be drawn –<br>• Since $m_1$ and $m_2$ are unequal scalar quantities and hence, by ratio-proportion, magnitudes $v_1$ and $v_2$ of vectors $\vec{v}_1$ ***and*** $\vec{v}_2$ ***shall also be unequal***.<br>• In ration proportion LHS is scalar and therefore for equality of proportion $\frac{\vec{v}_2}{\vec{v}_1}$ shall have to be scalar. This is possible only when $\vec{v}_1$ ***and*** $\vec{v}_2$ ***are colinear.*** |

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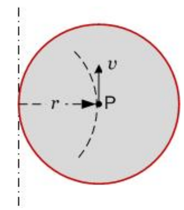
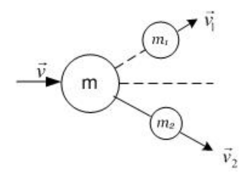
| | • In the derived equation (-)ve sign on RHS indicates that direction of $\vec{v}_1$ vector is opposite to the $\vec{v}_2$.<br>The above inferences satisfy option (d), and ***answer is (d).*** |
| I-13 | COM of a ring is at its centre and hence shall be at point P, located at distance $r$ from axis of rotation as shown in the figure. The ring is filled with light disc and hence its mass is considered negligible. Another particle pass is placed at centre of the disc. Thus mass concentrated at point P is $2m$. When the system is rotated in a circular parth of radius $r$ at velocity velocity $v$, trajectory of the mass $2m$ shall pass through P. This is a case of circuar motion and hence an external force equal to centripetal force $2m\left(\frac{v^2}{r}\right)$ shall have to be applied on the sysem to maintain the rotation. Accordingly, ***answer is (c).***   |
| I-14 | Momentum of nucleus before emission is $\vec{P} = m\vec{v}$, and emiision of α-particle momentii of α-particle and the remainibg nuleus shall be $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$, resectively. As per principle of conservation of momentum (PCM) $\vec{P} = \vec{p}_1 + \vec{p}_2 \rightarrow m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 \rightarrow \vec{v} = \frac{1}{m}(m_1\vec{v}_1 + m_2\vec{v}_2)$. Since, $m_1 \neq m_2$ and so also $\vec{v}_1 \neq \vec{v}_2$ only condition thuat satisfies PCM is the equation is $\vec{v}_1$ is parallel to $m_1\vec{v}_1 + m_2\vec{v}_2$. Hence, ***answer is (d).***   |
| I-15 | Vector of COM is $\vec{R} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$. For COM to be at origin $\vec{R} = 0 = R$. This proposition is being analysed for each of the given options to verify if it satisfies $\vec{R} = 0$<br>• Since numerator is scalar multiles of vector $\vec{r}_i$ and therefore requirement is $\sum_i m_i\vec{r}_i = 0$ and not the count but as stipluated ***at (a) , hence this option is not correct.***<br>• Again going by definition of $\vec{R}$ it is not the equal masses on either side of the origin, but $\sum_i m_i\vec{r}_i = 0$, and therefore, ***option (b) is not correct***.<br>• Going by analysis of option (a) and (b), this option too does not lead to $\sum_i m_i\vec{r}_i = 0$, and ***option (c) is not correct.***<br>• Option (d) takes all particles on X-axis and it is similar to otion (a). Hence, ***option (d) is also not correct***.<br>Thus none of the options are correct.<br>*[N.B.: In MCQ questions, it might happen options are so twisted that none of them apply to the given condition, and may lead to none of the options.]* |
| I-16 | Vector of COM is $\vec{R} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$. For COM to be at origin $\vec{R} = 0 = R$. This proposition is being analysed for each of the given options to verify if it satisfies $\vec{R} = 0$ . In light of this each of the option needs to be verified- |

- When coordinates of all particles are (+)ve, they for $\vec{R}$ shall be (+) and COM shall not be at origin, hence ***option (a) is not correct***.
- When coordinates of all particles are (-)ve, they for $\vec{R}$ shall be (-) and COM shall not be at origin, hence ***option (b) is not correct***.
- Inlight (a & b) above vector of COM can be written as

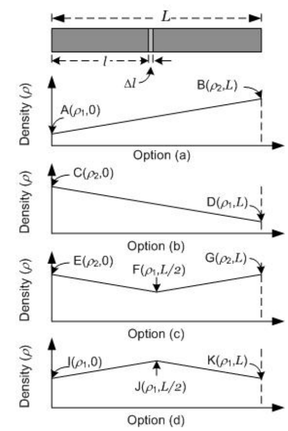
$$\vec{R}=\frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}=\frac{\sum_{i=1}^{j} m_i\vec{r}_i+\sum_{i=j+1}^{k} m_i\vec{r}_i+\sum_{i=k+1}^{l} m_i\vec{r}_i+\sum_{i=l+1}^{m} m_i\vec{r}_i}{\sum_{i=1}^{k} m_i}$$ where, $\sum_{i=1}^{j} m_i\vec{r}_i$, $\sum_{i=j+1}^{k} m_i\vec{r}_i$, $\sum_{i=k+1}^{l} m_i\vec{r}_i$ and $\sum_{i=l+1}^{m} m_i\vec{r}_i$

correspond to particles in 1st, 2nd, 3rd and 4th quadrant.
  - Non-negative coordinates mean positive coordinates, but preceeding it with **<u>may be</u>** implies it is not necessary. This makes ***option (c) correct***.
  - Option (c) when put in other words with some (+) and others (-)ve and that makes ***option (d) correct***

Accordingly, ***answer is option (c) and (d).***

I-17

*N.B.: This question can be easily answered by symmetry of the mass distribution, which plays an important role in analysis. Nevertheless, to train the thought process the each of four options are being analysed.*




- **Option (a):** gradient of density $k_a=\frac{\rho_2-\rho_1}{L}$. Therefore, centre of mass shall be at $\overline{L}=\frac{\int_0^L l(\rho_0+k_a l)dl}{\int_0^L(\rho_0+k_a l)dl}=\frac{\int_0^L(\rho_0 l+k_a l^2)dl}{\left[\rho_0 l+\frac{k_a l^2}{2}\right]_0^L}=\frac{\left[\rho_0\frac{l^2}{2}+k_a\frac{l^3}{3}\right]_0^L}{(2\rho_o+k_a L)\frac{L}{2}}=\frac{(3\rho_0+2k_a L)\frac{L^2}{6}}{(2\rho_o+k_a L)\frac{L}{2}}$. It leads to $\overline{L}=\left(\frac{3\rho_0+2k_a L}{2\rho_o+k_a L}\right)\frac{L}{3}\neq\frac{L}{2}$. Thus, COM shall lie between such that $0<\overline{L}<\frac{L}{2}$, i.e. COM is not at the centre, hence, ***option (a) is correct***.
- **Option (b):** gradient of density $k_b=\frac{\rho_1-\rho_2}{L}=(-)k_a$. This is mirror image of option (a) and hence COM shall be $\frac{L}{2}<\overline{L}<\frac{L}{2}$, hence, ***option (b) is correct.***
- **Option (c):** gradient of density in section $0\le l\le\frac{L}{2}$ is $k_c=\frac{\rho_1-\rho_2}{\frac{L}{2}}=(-)2\frac{\rho_2-\rho_1}{L}=(-)2k_a$, and in section $\frac{L}{2}\le l\le L$, gradient is $k_c'=\frac{\rho_2-\rho_1}{\frac{L}{2}}=2\frac{\rho_2-\rho_1}{L}=2k_a$. Accordingly,

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| | $\overline{L}=\frac{\int_0^{L/2} l(\rho_0-2k_a l)dl+\int_{L/2}^{L} l(\rho_0+2k_a l)dl}{\int_0^{L/2}(\rho_0-2k_a l)dl+\int_{L/2}^{L}(\rho_0+2k_a l)dl}$. It leads to<br>$\overline{L}=\frac{\rho_0\int_0^L ldl-2k_a\left(\int_0^{L/2} l^2dl-\int_{L/2}^{L} l^2dl\right)}{\rho_0\int_0^L dl-2k_a\left(\int_0^{L/2} ldl-\int_{L/2}^{L} ldl\right)}=\frac{\rho_0\frac{L^2}{2}-\frac{2k_a}{3}\left(\frac{L^3}{8}-\left(L^3-\frac{L^3}{8}\right)\right)}{\rho_0 L-\frac{2k_a}{2}\left(\frac{L^2}{4}-\left(L^2-\frac{L^2}{4}\right)\right)}=\frac{1}{2}\frac{\left(\rho_0+k_a\frac{L}{2}\right)L}{\left(\rho_0+k_a\frac{L}{2}\right)}=\frac{L}{2}$, i.e. COM is at $\frac{L}{2}$, hence, ***option (c) is incorrect.***<br>• **Option (d):** Mathematical formulation in this case shall be same as at option (c) except that in section $0\le l\le\frac{L}{2}$ is $k_d=\frac{\rho_2-\rho_1}{\frac{L}{2}}=2\frac{\rho_2-\rho_1}{L}=2k_a$, and in section $\frac{L}{2}\le l\le L$, $k_d'=\frac{\rho_1-\rho_2}{\frac{L}{2}}=(-)2\frac{\rho_2-\rho_1}{L}=(-)2k_a$. This is left for reader to determine $\overline{L}$ on the lines of option (c) and it will be seen that $\overline{L}=\frac{L}{2}$, i.e. COM is at the centre of the rod and ***hence option (d) is incorrect***.<br>***Answer is options (a) and (b).***<br>***N.B.:*** *Here a twist has been given in question by stating COM is not in the centre and needs to be carefully noted* |
| I-18 | NFLM stipulates that a body in uniform motion or state of rest continues to be so unless resultant of external forces $R=\sum_i \vec{F}_{ext\_i}=0$ and as per NSLM $\sum_i \vec{F}_{ext\_i}=m\vec{a}\rightarrow\sum_i \vec{F}_{ext\_i}\rightarrow\vec{a}=0\Big\|_{m\neq 0}$. Thus analysis of given options –<br>• It is not necessary as per NFLM for body to be in a state of rest, hence ***option (a) is not correct***.<br>• As per NSLM, in given condition $\vec{a}=0$, the sustem will not accelerate, hence ***option (b) is correct***.<br>• If body is in state of motion then as per NFLM, it will move is conditional and hence ***option (c) is correct***<br>• As per NSLM in given condition $\vec{a}=0$ and system will not accelerate and ***option (d) is not correct***.<br>***Thus answer is option (b) and (c).*** |
| I-19 | Given that $\vec{F}_{ext}\neq 0=Ma$, here $M$ is mass the system and hence as per NSLM acceleration of the COM is . $a\neq 0$, one of the variable whose value is part of the answer; this automatically rules out option (a) and (c) where $a=0$ . Now in respect of second variable $v_0$ at any instant $t$ its value as per First equation of |

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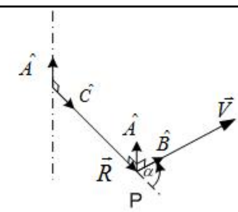
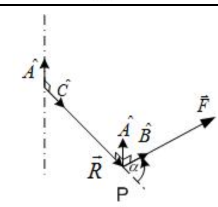
| | kinematics $v_0 = u + at$, which depends upon the only variable $u$, the initial velocity, which is unknown. If , $u = v_0 - at$ then only $v_0 = 0$ ***is possible as in option (bd)*** with $a \neq 0$, and if $u \neq v_0 - at$ then $v_0 \neq 0$ ***is possible as in option (d)*** . Thus ***answer is (b) and (d).*** |
| I-20 | Let us consider the two balls a system of masses having a mass $M = 2m$, where $m$ is mass of each ball. One system is thrown up, the only external force acting on it is gravitational force $F_{ext} = Mg$ and hence acceleration of COM of $M$ shall be $a = \frac{F_{ext}}{M} = \frac{Mg}{M} = g$. Thus, in firnal form of acceleration it rules out Option (a) since it is independent of direction of two balls; Option (b) since it is independent of the masses of two balls; Option (c) since it depends upon speed of two balls. The only option left is (d) where given that $a = g$, the same has been established. Hence ***answer is (d).*** |
| I-21 | Let a block of mass $M$ moving with velocity $\vec{V}$ breaks will have momentum $\vec{P} = M\vec{V}$ into two parts of masses $m_1$ and $m_2$ and move with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively and there momentum shall be $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$. Since, - consequence of breaking is question for two parameters momentum and kinetic energy, each of this shall be analysed to answer the question –<br>• The block is moving in air and breaks, and no external force is mentionedand hence as per NFLM, momentum shall be conserved, and hence $\vec{P} = \vec{p}_1 + \vec{p}_2$, therefore ***option (a) is correct***<br>• *As a corollary* ***option (c) is incorrect***<br>• Breaking of particle is result of internal forces, and some part of kinetic energy of initial mass $KE = \frac{1}{2}MV^2$, and of the two split masses is $KE_1 = \frac{1}{2}m_1v_1^2$ and $KE_2 = \frac{1}{2}m_2v_2^2$. As per principle of conservation of energy $KE = KE_1 + KE_2 + \Delta KE$, here , $KE = KE_1 + KE_2 + \Delta KE$ As regards $\Delta KE$ it is not equal to ZERO, since it is energy spent in splitting of the mass into two, and hence $KE \neq (KE_1 + KE_2)$ . Thus, while total energy is conserved kinetic energy is not hence ***option (b) is incorrect***. As a consequence total kinetic energy would change, i.e. ***option (d) is correct***.<br>Thus ***answer is option (b) and (d).*** |
| I-22 | Unit vector along the axis of rotation $(\vec{A})$ is perpendicuar to the plane of vectors, velocity of particle $(\vec{B})$ and distance of particle from the axis of rotation $(\vec{C})$ . Since, $\vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos 90^0 = 0$, angle $\theta = 90^0$ as shown in the figure. Hence, ***answer is (c).***  |
| I-23 | Unit vector along the axis of rotation $(\vec{A})$ is perpendicuar to the plane of vectors, resultant force on the particle $(\vec{B})$ and distance of particle from the axis of rotation $(\vec{C})$ . Since, $\vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos 90^0 = 0$, angle $\theta = 90^0$ as shown in the figure. Hence, ***answer is (c).***  |

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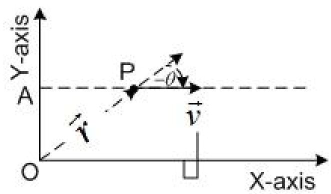
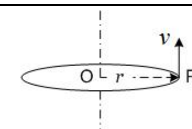
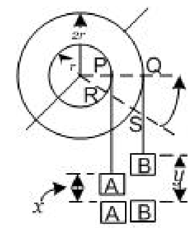
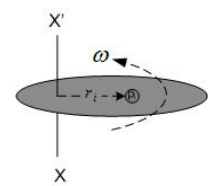
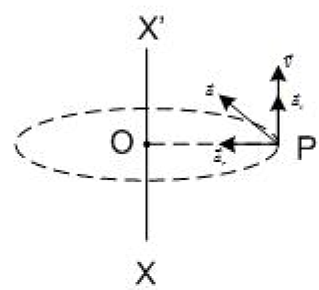
| I-24 | The system is shown in the figure. Angular momentum of the particle is $\vec{L} = \vec{r} \times \vec{v} = rv\sin(-\theta)\hat{n} = v(r\sin\theta)(-\hat{n})$. In this factor $v$ is constant, despite changing $r$ and $\theta$ another factor $r\sin\theta$ is constant and $(-\hat{n})$is unit normal vector in opposite direction. Accordingly, angular moment is product of three constants and hence it is also constant. Hence, **answer is (b)**   |
| I-25 | In the given system as shown in the figure, $r$ known, dependence of $\omega$ is required to be ascertained which turn out to be $\omega \propto v$ . Hence, options (a), (b) and (c) are ruled out, and the only ***correct option is (d).***   |
| I-26 | Given quantities are redfined in figure. System has two coaxially fixed pulleys, and hence if smaller puller of radius $r$ rotates through an angle $\theta$ about its axis, another pulley of radius $2r$ shall allso rotate throgh same angle. Since strings do not slip, hence in rotation of the system through an angle $\theta$ string attached to masses A and B shall be released by length $x = r\theta$ and $y = (2r)\theta = 2r\theta$. Thus it is evident that $y = 2x$, hence ***answer is (c).***   |
| I-27 | A body as shown in the figure is rotating about axis X-X' with a uniform angular velocity $\omega$. Then resultant force experienced by particle $P_i$, to keep uniformy rotating with $\omega$, shall $\vec{F}_i = -m_i r_i \omega^2 \hat{r}$, i.e. centripetal force and shall be in the direction vector $(-)\hat{r}_i$. This dirction vector $(-)\hat{r}_i$ is perpendicular to the vertical axis of rotation, i.e. horizontal and intersection axis X-X' ***inconformance with option (c).*** This ***rules out option (a),;*** since it is not skewed about the axis **option (b) is also ruled out;** ***option (d) is ruled out*** since ***option (c) is correct.*** *Hence,* ***answer is (c).***   |
| I-28 | It rotation of a body is non uniform it implies $\frac{d\omega}{dt} = \alpha \neq 0$ and therefore at an instant when particle has parameters angular velocity $\omega$, tangential velocity $v$, radial distance from axis $r$, will have radial acceleration $\vec{a}_r = (-)\frac{v^2}{r}\hat{r}$ and tangential acceleration $\vec{a}_t = \frac{dv}{dr}\hat{t}$, accordingly, resultant acceleration and in turn force will be in the horizontal plane of vectors $\hat{r}$ and $\hat{t}$ but, skewed in the direction of velocity if $\frac{dv}{dt} > 0$ and skewed backward if $\frac{dv}{dt} < 0$. This proves ***options (a), (c) and (d)*** **are incorrect** and ***option (b) is the answer***.   |
| I-29 | Position vector $\vec{r}$ is w.r.t origin and $\vec{\Gamma} = \vec{r} \times \vec{F} = rF\sin\theta\,\hat{n}$ it means $\vec{\Gamma}$ is perpedicular to the plane containing vectors $\vec{r}$ and $\vec{F}$ and in-turn vectors $\vec{r}$ and $\vec{F}$. Hence, $\vec{r} \cdot \vec{\Gamma} = 0$ and $\vec{F} \cdot \vec{\Gamma} = 0$ which makes both the conditiona in option (a) correct. In ***rest of the option (b) and (c) and (d)*** either of the coditions is stated nont equal to to zero and they ***are incorrect***. Hence, ***answer is (a).*** |

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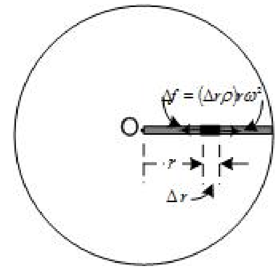
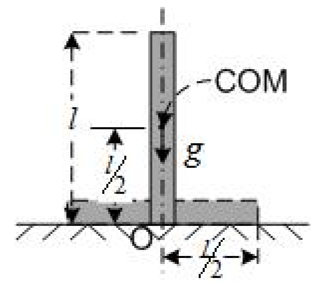
| I-30 | Let, uniform rod of length $l$ and mass $m$ has mass density per unit length $\rho = \frac{m}{l}$.   Since, table is mooth hence entire centipetal force shall get transferred on clamp, when the rod rotates through a angular velocity $\omega$ and this force shall be $F = \int_0^l (dr\rho) r\omega^2 = \rho\omega^2 \int_0^l rdr = \rho\omega^2 \left[\frac{r^2}{2}\right]_0^l = \frac{1}{2}\rho\omega^2 l^2 = \frac{1}{2}(\rho l)\omega^2 l = \frac{1}{2}m\omega^2 l$. This conforms with ***option (d) and it is correct.*** Rest of the ***options (a), (b) and (c) are incorrect***. Hence ***answer is (d)*** |
| I-31 | As shown in the figure, when a uniform rod is kept vertically, its CG is passing through its base. Since rod is on a smooth surface there will be no frictiobal surface. When it is rotated slightly , and allowed to make its free fall, there is no external force except gravitational force, and hence, COM sglich is ahall move along $g$ until it reaches O. And for this to happen lower end of the rod will get displaced from ts initial position O by $\frac{l}{2}$, and this can happen in any direction around point O. This inference is contained only in option (c).   |
| I-32 | Moment of inertia of a circular plate is $I = M\frac{R^2}{2}$, is a standard value. Now for disc P, $M_p = (4\pi r^2 t)\rho = 4\pi r^2 t$ and for disc Q $M_q = \left(4\pi(4r)^2 \frac{t}{4}\right)\rho = 16\pi r^2 t$. Now with known formulation of I, $I_p = M_p\frac{r^2}{2} = 4\pi r^2 t\frac{r^2}{2} = 2\pi r^4 t$, and $I_q = M_q\frac{(4r)^2}{2} = 16\pi r^2 t\frac{16r^2}{2} = 108\pi r^4$. Thus, it is evident from analysis $I_p < I_q$. Hence, ***answer (c).***<br>***N.B.:*** *There are standard values e.g. vlaue of I for a circular disc. If derivations of such stnadrad results are carried out, independently, it becomes easy to remember, Ths task of memorizing standard values is reduced drastically. It is, therefore, advised to carry out derivation of each formula and standard results.* |
| I-33 | It is known that $\vec{\Gamma} = I\vec{\alpha}$, here $\vec{\Gamma}$ is torque, $I$ is moment of inertia and angluar acceeration is $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$. Now, mass of disc A, $M_A = (4\pi r^2 t)\rho = 4\pi r^2 t$ and for disc B is $M_B = \left(4\pi(4r)^2 \frac{t}{4}\right)\rho = 16\pi r^2 t$. Now with known formulation of I, $I_A = 4\pi r^2 t\frac{r^2}{2} = 2\pi r^4 t$, and $I_B = 16\pi r^2 t\frac{16r^2}{2} = 108\pi r^4$. Since same torque is applied on both the discs hence $\vec{\Gamma} = I_A\frac{d}{dt}\vec{\omega}_A = I_B\frac{d}{dt}\vec{\omega}_B \rightarrow \frac{I_A}{I_B} = \frac{\left\lvert\frac{d}{dt}\vec{\omega}_B\right\rvert}{\left\lvert\frac{d}{dt}\vec{\omega}_A\right\rvert} = \frac{r_B\frac{d}{dt}v_B}{r_A\frac{d}{dt}v_A}$. From the given data |

of radii and derived values of MI, the conclusion can be written as $\frac{I_A}{I_B} = \frac{2\pi r^4 t}{108\pi r^4 t} = \frac{4r_A \frac{d}{dt} v_B}{r_A \frac{d}{dt} v_B}$.

Accordingly, $\frac{d}{dt} v_A = 216 \frac{d}{dt} v_B$. Since linear acceleration of a particle on rim A is greater than that of a particle on rim B, velocity of particles at any instant shall be $v_A > v_B$. Hence, ***answer is (a)***

**I-34**

Since tube is not filled with water, in state of rest on a horizontal plane, water will spread across length uniformply. Let mass of liquid is $m$, and length of the tube is $l$ having mass $M$ hence MI of liquid $I_l = \frac{ml^2}{12}$ and MI of tube $I_t = \frac{Ml^2}{12}$. But when the tube is set in rotation about its perpendicular bisector, the mass shall distribute uniformly, to fill both ends of the tube, across its cross-section. Accordingly, new value of moment of inertia of water shall be $I_l' = 2\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{4}$. In $I_l$ and $I_l'$ numerator is same but denominator in $I_l'$ and hence $I_l' > I_l$ is small i.e. on rotation moment of inertia of water increases, and hence **answer is (a)** .

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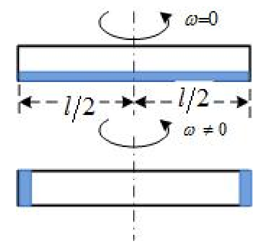
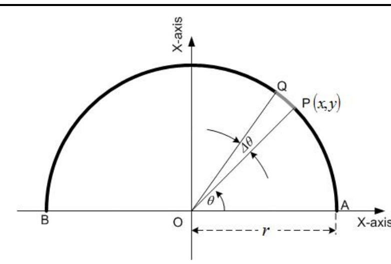



**I-35**

This can be answered by radial symmetry about centre O. But, an anlytical answer can be evolved by taking a small section of wire PQ of length $r\Delta\theta$, between two radial displaced by an angle $\Delta\theta$, having a mass $\Delta m = \left(\frac{M}{\pi r}\right) r\Delta\theta = \frac{M}{\pi}\Delta\theta$, here $\frac{M}{\pi r}$ is mass per unit length. It is to be noted that wire is only along arc. There is no wire along diameter, if t were it would change the answer. Taking, $\Delta\theta \to 0$, coordinates of element PQ, shall be $x = r\cos\theta$ and $y = r\sin\theta$. And $I_{xx} = \frac{M}{\pi}\int_0^{\pi}(r\sin\theta)^2 d\theta = \frac{Mr^2}{\pi}\int_0^{\pi}\sin^2\theta\, d\theta$ and likewise, $I_{xx} = \frac{Mr^2}{\pi}\int_0^{\pi}\cos^2\theta\, d\theta$. Further, by *Perpendicular Axis Theorem* required moment of inertia shall be $I_O = I_{XX} + I_{YY} = \frac{Mr^2}{\pi}\left(\int_0^{\pi}\sin^2\theta\, d\theta + \int_0^{\pi}\cos^2\theta\, d\theta\right) = \frac{Mr^2}{\pi}\int_0^{\pi}(\sin^2\theta + \cos^2)d\theta = \frac{Mr^2}{\pi}\int_0^{\pi} d\theta$. It leads to a line

$I_O = \frac{Mr^2}{\pi}\cdot\pi = Mr^2$. Hence, ***answer is (a).***




**I-36**

Moment of inertia is $I = Mk^2$, here $M$ is the mass and $k$ is radius of gyration, which depends upon geometry of mass fistribution and not mass density. Two bodies are identical shall have same $k$ and volume $V$, and hence $I_1 = M_1k^2$ and $I_2 = M_2k^2$, but and depend upon density $\rho$ and in instant case

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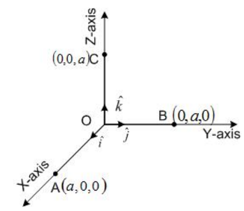
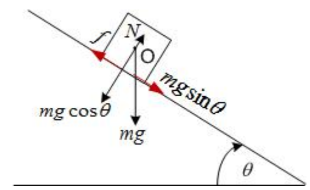
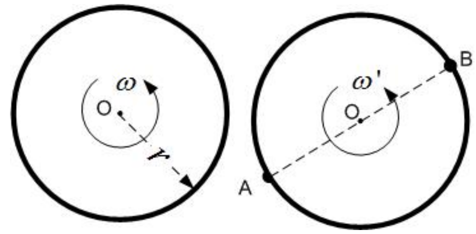
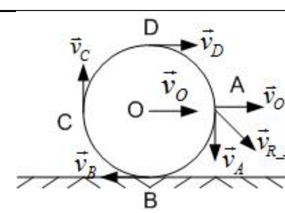
| | density of first body of aluminum is lower than that of iron i.e. $\rho_1 < \rho_2$ .Therefore, $\frac{I_1}{I_2} = \frac{V\rho_1 k^2}{V\rho_1 k^2} = \frac{\rho_1}{\rho_2}$, and hence, with the knowledge of densities of aluminium and iron, $I_1 < I_2$ hence ***answer is (a).*** |
| I-37 | Let mass of three particles, as shown in figure, and identified as A, B, and C be $m_1$, $m_2$ and $m_3$ . Then position vector of centre of mass shall be $\vec{R} = \frac{m_1 a\hat{i} + m_2 a\hat{j} + m_3 a\hat{k}}{m_1 + m_2 + m_3}$.  <br>Given that $\vec{R} = 0$, is not possible since all the masses are along (+)ve $\hat{i}$, $\hat{j}$ and $\hat{k}$. Hence, determining $I_{zz} = I_{xx} + I_{yy}$ by Perpendicuar Axis Theorem is not possible as principle of COM and Moment of Inertia can not be incosistent. Hence, answer can not be deduced with given information. Hence, ***answer is (d)*** |
| I-38 | The information given is utilized to construct figure. Normal force, assuming cubical block of side $a$ has uniform density, so as to have its COM at O, its centre. Since Normal force shall pass through O,hence its torque at O shall be Zero.  <br>***Hence, answer is (a)*** |
| I-39 | Moment of inertia of circular ring is $I = Mr^2$. When it is revolving with an angular velocity $\omega$ , around its ventre O, its angular momentum would be $\Gamma = I\omega = Mr^2\omega$. While it is revolving, two particles A and B, each of mass $m$, are attached to the ring at two diamatrically opposite points new moment of inertia would become $I' = (M + 2m)r^2$, let its angular velocity change to $\omega'$ and accordingly its angular momentum becomes  <br>$\Gamma' = I'\omega' = (M + 2m)r^2\omega'$. By principle of conservation of angular momentum it would lead to $\Gamma = \Gamma' \rightarrow Mr^2\omega = (M + 2m)r^2\omega' \rightarrow \omega' = \frac{M\omega}{M + 2m}$. This conslusion is consistent with option (b). Hence ***answer is (b)***<br>***N.B.:*** *The nature of attachment is without any external force, hence instead of principle of energy conservation, principle of conservation of momemtum has been applied in this case.* |
| I-40 | Persons, sitting on rotating stool initially with arms stretched when fold his arms, no external force is applied. Folding of arms would change moment of inertia of the system $I$, and accordingly its angular velocity $\omega$ would also change, but its angular momentum would remain $\Gamma = I\omega = I'\omega'$. Hence, ***answer is option (c).*** |
| I-41 | When a wheel is rolling with velocity $v_0$ , without slipping as there is no mention of slipping, its centre is moving with velocity $v_0$ in the direction of rolling with same velocity , as much as each particle on the periphery also with a velocity $v_0$ in direction tangential to the point on circular periphary , as hown in the figure.  <br>Points A and C are at the level of centre of the wheel O. Therefore, taking point A, since $v_A = v_0$ and angle between $\vec{v}_A$ and $\vec{v}_0$ is $90^0$ its speed would $\sqrt{2}v_0$ be along diagonal of the square, |

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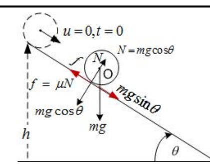
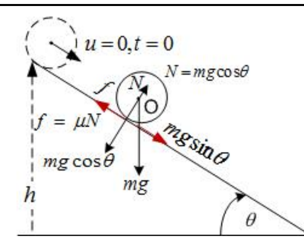
| | and same can be verified for particle C.. Thus ***answer is (c).*** |
| I-42 | Let M is the mass of wheel and is set to revolve at angular speed $\omega$ and translation velocity of COM $v$. Let time taken to cover both angular and linear displacements simultaneously be $t$. Accordingly, $d = ut + \frac{1}{2}at^2$ and $\theta = \omega t + \frac{1}{2}\alpha t^2$. In motion of a cylindrical or spherical object on a surface four cases can occur **(i)** $\omega = 0, v \neq 0$, pure sliding and can occor on a smooth surface; **(ii)** $v = r\omega$, will have pure rolling. Since wheel is pushed forward, and hence at point of contact with base surface will be backward; **(iii)** $v > r\omega$, in this case, particle of wheel in contact with the base surface has forward motion and hence frictional force shall be backward; (***iv***) $v < r\omega$, it is a case when particle of wheel in contact with base surface is travelling backward and hence frictional force would act forward. In the instant case $\theta = \omega t = 2\pi$ and hence peripheral distance covered by particle in time $t$ would be $p = r\theta = r(\omega t) = 2\pi r = 125.6$ cm, while linear ditance covered by COM $d = vt = 60$ cm. Since duration is same, it implies $v < r\omega$ it belongs to case (iv). Hence, from above analysis, frictional force on wheel would act forward. ***Thus, answer is (a)*** |
| I-43 | Point to be noted is scotter is moving on a frictionless surface. It implies that there is no role of friction in motion of scooter. Motion of scooter on road due power transmitted by engine to the wheel is inly due to friction. In absence of friction, on a fricionless surface, increase in petrol supply would only increase angular velocity of the wheel, and thus input power would be stored in rotational kinetic energy of wheel, without increase in linear velocity of scooter. ***Hence, answer is (d).*** |
| I-44 | <br>Moment of inertia of a the given objects of mass M are $I_{SS} = \frac{2}{5}MR^2$, $I_{SS} = \frac{2}{3}MR^2$, and $I_D = \frac{2}{3}MR^2$. Since, incline surface is stated to be smooth, there would be pure slipping and hence time taken by each of the object, to reach the botton, would be governed by second equations of kinematics and is independent of their MIs, and would be same. ***Hence, answer is (d).*** |
| I-45 | <br>Moment of inertia of a the given objects of mass M are $I_{SS} = \frac{2}{5}MR^2$, $I_{SS} = \frac{2}{3}MR^2$, and $I_D = \frac{2}{3}MR^2$. Since, surface is frictiona and hence each of he object would experiencea frictional force drag $f = \mu Mg\cos\theta$ and a forward push $F = Mg\sin\theta$. Since, pur rolling occurs, therefore, linear acceleration $a = \frac{F-f}{M} = \frac{Mg\sin\theta - \mu Mg\cos\theta}{M} = g\sin\theta - \mu g\cos\theta$. And , angular acceleration $\alpha = \frac{fR}{I} = \frac{\mu Mg\cos\theta}{I}$. . Since, $I$ is different for the three objects and hence $\alpha$ would also be different, despite each having same frictional force $f$. But, since $a$ is independent of $I$, as per second equation of kinematics, time to reach bottom would be same. ***Hence, answer is (d).***<br>***N.B.:*** *For pure rolling necessary condition is $a = R\alpha$, and $I$ is dependent on , despite objects having same mass and radius, and hence their angular velocities $\omega$ on reaching the ground would be different. It can be verified that since vertical fall for all objects, on the incline, is same for all objects, as per Law of conservation of energy $\Delta PE = \Delta KE_L + \Delta KE_R$, here, $\Delta KE_L$ is change in change in kinetic energy of* |

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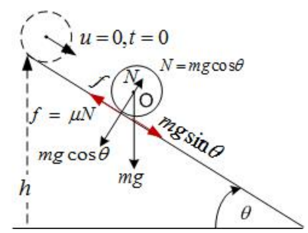
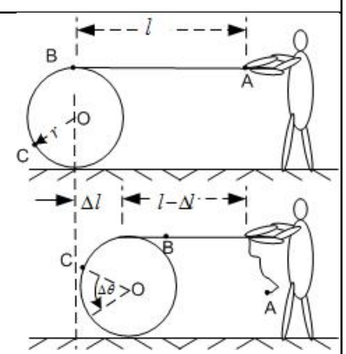
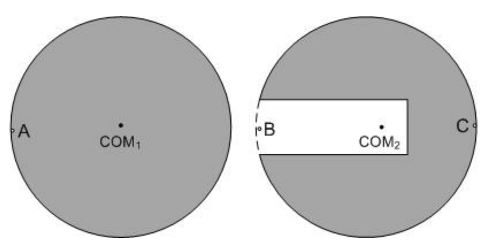
| | *linear motion, and* $\Delta KE_R$ *is change in rotational kinetic energy is same for all the three objects..* |
| I-46 | <br><br>Moment of inertia of a the given objects of mass M are $I_{SS} = \frac{2}{5}MR^2$, $I_{SS} = \frac{2}{3}MR^2$, and $I_D = \frac{2}{3}MR^2$. Since, surface is frictiona and hence each of he object would experiencea frictional force drag $f = \mu Mg\cos\theta$ and a forward push $F = Mg\sin\theta$. Since, pur rolling occurs, therefore, linear acceleration $a = \frac{F-f}{M} = \frac{Mg\sin\theta - \mu Mg\cos\theta}{M} = g\sin\theta - \mu g\cos\theta$. And , angular acceleration $\alpha = \frac{fR}{I} = \frac{\mu Mg\cos\theta}{I}$. .<br>Since, $I$ is different for the three objects and hence $\alpha$ would also be different, despite each having same frictional force $f$. But, since $a$ is independent of $I$, as per third equation of kinematics, linear velocity of each object on reaching the bottom would be same, so also their kinetic energy would be same.<br>It given that there is no pure rolling i.e. $a \neq R\alpha$, and $I$ is dependent on , despite objects having same mass and radius, and hence their angular velocities $\omega$ on reaching the ground would be different. But, as per Law of Conservation of Energy $\Delta PE = \Delta KE_L + \Delta KE_R$, here, $\Delta KE_L$ is change in change in linear kinetic energy which is same for all objects, as derived above, and hence arithmatically $\Delta KE_R$, ***change in rotational kinetic energy would also be same for all the three objects***. This can be verified, by applying equations oof rotational kinematics. ***Hence, answer is (d).*** |
| I-47 | <br><br>The given problem has been split in, for illustration, in two figures. In initial case man is holding end A of string wrapped over cylinder and particle B of the string is last point of contact of string with cylinder of radius $r$. Let another particle C of the string is at a linear length $l$ from particle B.<br>While pulling string cylinder undergoes pure rolling and therefore if point B moves forward by a distance $\Delta l$ the point rotates through an angle $\Delta\theta = \frac{\Delta l}{r}$ .<br>Thus eventually the man pulls the cylinder towards him by a distance $\Delta l$, the rolling causes unwrapoing of string of length by another length $\Delta l$ due to rotation of cylinder an angle $\Delta\theta$ . Accordingly to pull cylinder towards him through a length $\Delta l$ the string that passes through his hands is $2\Delta l$.<br>Thus for the man to keep string pulling the string, till cylinder reaches him, cylinder has to move towards man through a distance $l$ , hence is the length of the string that passes through hands of the man is $2l$. ***Hence, answer is (b).*** |
| I-48 | <br><br>It requires to be distinguish between rotation and revolution. An object when turn around an axis within its geometry called nternal axis it is rotation, and when axis lies outside its geometry then it is rotation. These definitios are applied to two shapes, One has $COM_1$ and is symmetrical about X-axis and Y-axis. In this Axis of rotation may pass through centre of mass or anywhere within the shape e.g point A. Thus axis of rottaion may pass through COM, but not compulsoary. ***This makes option (b) correct and makes makes requirement in option (a) as incorrect.***<br>In another shape is symmetrical about X-axis by un-symmetrical about Y- axis, due to a notch cut out of it. In this, through COM2 is outside its body but is within the outer periphary, so also point B, while point |

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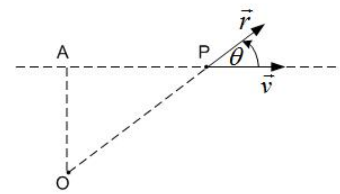
| | C is within the body. Thus axis or rotation is within outer periphary but not necessarily through the body. ***This makes option (d) correct but makes makes requirement in option (c) as incorrect. Thus option (b) and (d) ae correct.*** |
| I-49 | In inertial frame, equations (A) and (B) are valid. But, in non-inertial frame equation (A), angular momentum in inertial frame $L$ is based on instantaneous value of $\omega$ and hence it would be valid. But, as regards equation (B), $\frac{dL}{dt} = \Gamma = I\alpha$ where, $\alpha$ is angular acceleration of object. But, in non-inertial frame, it implies that i.e. $\alpha = \alpha_i = \alpha_{ni} + \alpha_{ni,i} \rightarrow \alpha_{ni} = \alpha_i - \alpha_{ni,i}$. Thus torque is dependent on $\alpha_i - \alpha_{ni,i}$ and not merely $\alpha_i$. These two conditions are specified only in option (b). ***Hence, answer is (b).*** |
| I-50 | The problem is explianed in figure where paticle P has a mass $m$ with a position vector $\vec{r} = r\hat{r}$ and moving along a stright line with a velocity $\vec{v} = v\hat{v}$. Accordingly, angluar momentum is<br>$\vec{L} = \vec{r} \times \vec{v} = (r\hat{r}) \times (v\hat{v}) = rv(\hat{r} \times \hat{v}) = rv(\sin(-\theta)\hat{n}) = (r\sin\theta)v(-\hat{n}) = pv(-\hat{n})$.<br><br><br>Here, $p$ is the perpendicular distance AO of the line from O and is fixed by the given geometry. Further, velocity of the particle is given to be constant (uniform) and $(-\hat{n})$ is unit normal vector toward the plane of vector $\vec{r}$ and . Thus all the constituent quantities of $\vec{L}$ so also it is. Analysing each of the given option leas to –<br>(a) For $\vec{L}$ to be zero either $r = 0$, or $v = 0$ or $\theta = 0$, none of these are given hence cannot be zero, and ***option (a) is incorrect***.<br>***(b)*** If point O is on straight line then $p = 0$ which being a coefficient will make $\vec{L} = 0$. Hence, ***option (b) is correct.***<br>***(c)*** For any point away from the line collinear to the velocity $p \neq 0$, and hence by corollary of (b) $\vec{L} \neq 0$, hence ***option (c) is correct.***<br>(d) $\vec{L}$ remains constant at any given point be it on the line or outside it, irrespective of position P of particle , hence ***option (d) is correct.***<br>***Accordingly, answers are (b), (c) and (d).*** |
| I-51 | In case of linear motion $\vec{F}\Delta t = \Delta\vec{P} \rightarrow \vec{F}\Delta t = m\Delta\vec{v}$ while in case of angular motion, $\Gamma\Delta t = \Delta\vec{L} \rightarrow (\vec{r} \times \vec{F})\Delta t = I\Delta\vec{\omega}$. With this analytical statement each option is being analysed .<br>***(a)*** Since, given body is rigid and hence both its mass and shaperemains unchanged. Given that $\vec{F} = 0$, therefore $\Delta\vec{L} = 0$ hence angular momentum would remain constant. Hence, ***option (a) is correct.***<br>(b) Since, mass of rigid body remains constant, hence with the given $\vec{F} = 0$ linear momentum would remain constant. Hence, ***option (b) is correct***.<br>(c) As per NSLM ,for $\vec{F} = 0$, $\Delta V = 0$ and $\Delta\omega = 0$, therefore kinetic energy of linear motion and rotational motion would also remain constant. Hence, ***option (c) is correct***.<br>(d) Rigid body by its definition maintains its geometry and mass density, and hence its moment of inertia remains constant. Hence, ***option (d) is correct***.<br>***Answer is (a), (b), (c) and (d).*** |

**I-52**

As per prallel axis theorem moment of inertia of a body is $I_{xx} = I + Md^2$, here $I = \frac{\sum m_i r_i^2}{\sum m_i}$ is moment of inertia of a body about an axis passing through its COM let us call it principal axis (PA) and it depends upon choice of axes, $M$ is mass of the body, $I_{xx}$ is Momememt of inertia aout any axis X-X parallel to the axis passing through COM and $d$ is distance of the line. Given that MI of same body about Two axis, A and B, out of which A is passing through COM and hence its distance with principal axis $d = 0 \rightarrow I_A = I$ while for axis B $I_B = I + Md^2$ such that $d \neq 0$ since it is not passing through PI and $d^2 > 0$ irrespective of $d$ is (+)ve or (-)ve . Accordingly, analysis of options goes as under –

(a) Taking an example of rectangular body its MI taken along an axis passing through COM and parallel to length is smaller than MI taken on an axis parallel to width. This is unlike circlular plate, ring or a sphere. Since, in the problem neither shape of body nor its oientation is given and hence it can relation between $I_A$ and $I_B$ can not be established and hence ***this option is incorrect.***

(b) From analysis at (a) if axis through COM is parallel to length and another axis is chosen anti parallel to the axis, but geometrically so placed that $I_A < I_B$ and henceit makes certainty of ***this option incorrect.***

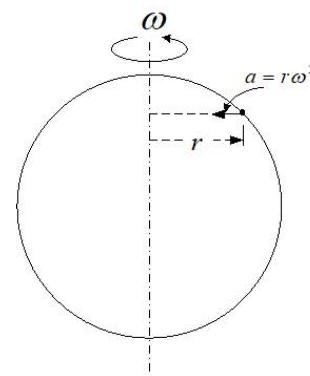
(c) Analysis above makes ***this option as correct.***

(d) In case axes are not parallel, but geometrical positioning is not known therefore, from above analysis this option can not be ascertained and hence ***this option is also incorrect.***

***Correct answer is option (c).***

**I-53**

Given that a sphere is rotating about a diameter without mentioning that diameter is about a fixed axis or not. Accordingly, the diameter is taken as fixed axis. Thus analysis of options goes as under –




(a) Every particle on the sphere, a rigid body would experience a radial acceleration $a = r\omega^2$ as shown in the figure. And there is no linear acceleration since sphere is rotating abot its diameter. Hence ***this option is incorrect.***

(b) Particles on the diameter always remain at diameter with $r = 0 \rightarrow a = 0$ from the analysis at (a) above, hence ***this option is correct***.

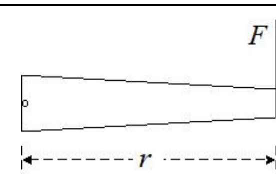
(c) Sphere is symmetrical about diameter and sphere is a rigid body and irrespective position of the particle on surface i.e. distance from the diameter, angular speed at any instant shall remain same and hence proposition ***option of different angular speeds is incorrect***.

(d) Linear speed is of a particly on surface is $|\vec{v}| = |\vec{r} \times \vec{\omega}| = r\omega \sin\theta$, here $\theta$ depends upon position of the article and hence despite $r$ and $\omega$ remaining constant linear speed $|\vec{v}|$ cannot be constant and hence **option (d) is incorrect.**

***Answer is option (b)***

**I-54**

It is known that $\vec{\Gamma} = \vec{r} \times \vec{F} = rF \sin\theta \,\vec{n} = rF\,\vec{n}$ , problem states that $\theta = 90^0$,. While, $\vec{\Gamma} = I\vec{\alpha} \rightarrow \vec{\alpha} = \frac{rF\,\vec{n}}{I}$. Thus, as per analysis-




(a) $\vec{F}$ is taken to be constant since no variation in it is defined. Likewise, $r = l$ i.e. length of the rod but its $I$ is dependent on pivot i.e. axis of rotation and hence would influence $\vec{\alpha}$ and ***hence this answer is incorrect***.

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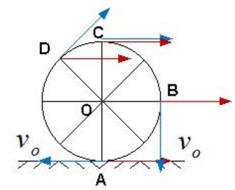
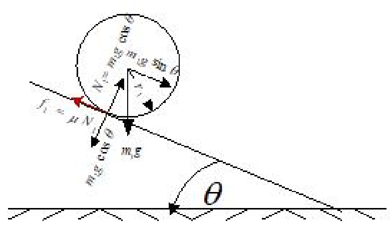
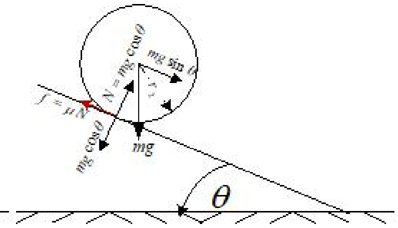
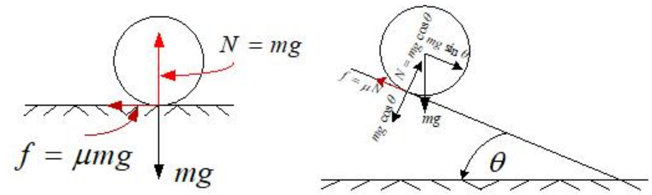
| | (b) Since parameters given in this option depend upon $\alpha$, which itself at (a) is incorrect and hence dependence of these parameters on position of pivot is incorrect. Thus, ***this option is incorrect***.<br>(c) Since, $\vec{p} = I\vec{\omega}$ and therefore by logic of (a & b) above and $\omega = \omega_0 + \alpha t$, ***this option is incorrect***.<br>(d) From the equation of torque, $\vec{\Gamma} = rF\,\vec{n}$ in given case both $r$ and $F$ are constant irrespective of the end chosen to be pivot and hence ***this option is correct.***<br>***Answer is option (d).*** |
| I-55 | Magnitude of velocity of all particles at the periphary of the wheel having a linear velocity $v_o$ is also $v_o$ but, in a direction of tangent at the point. Each particle shall have a linear velocity with linear velolicty $v_o$ parallel to the roadand a tangential velocity $v_o$ as shown for the points identified in the question. Accordingly, each of the option is being analyzed is supported with a figure.<br><br><br>(a) Linear and tangential velocity vectors of point A are equal and opposite and hence resultant velocity shall be Zero. Hence, ***this is correct option.***<br>(b) As shown in the figure resultant of linear and tangential velocities for point B and D shall be $v_0 < 2v_0$ and for point C shall be $2v_0$. Thus none of the points has velocity equal to $v_0$, hence this ***option is incorrect.***<br>(c) Resultant velocity of point C is $2v_0$ as already analyzed at (b) above, and hence ***this answer is correct.***<br>(d) Magnitude of velocity of point B is greater than $v_0$ as already analyzed at (b) above, and hence ***this answer is correct.***<br>***Thus answer is (a), (c) and (d).*** |
| I-56 | Taking the two spheres of masses $m_1$ and $m_2$ radii $r_1$ and $r_2$ respectively. Their MI shall be $I_1 = \frac{2}{5}m_1r_1^2$ and $I_2 = \frac{2}{5}m_2r_2^2$ respectively.<br><br><br><br><br>Torques and angular acceleration shall be experienced by the the two sphere shall be<br>$\Gamma_1 = f_1 r_1 = (\mu m_1 g\cos\theta)\, r_1 = I_1\alpha_1 \to \alpha_1 = \frac{(\mu m_1 g\cos\theta)\, r_1}{\frac{2}{5}m_1r_1^2} = \frac{5}{2}\frac{\mu g\cos\theta}{r_1}$ and $\Gamma_2 = (\mu m_2 g\cos\theta)r_2 = I_2\alpha_2$<br>which bu analogy works out to $\alpha_1 == \frac{5}{2}\frac{\mu g\cos\theta}{r_2}$. Since spheres are not slipping and hence<br>$v = r\omega \to a = r\alpha$. Accordingly, $a_1 = r_1\left(\frac{5}{2}\cdot\frac{\mu g\cos\theta}{r_1}\right) = \frac{5}{2}\mu g\cos\theta$ and likewise, $a_2 = \frac{5}{2}\mu g\cos\theta$ .Since linear accelerations of both the sphere are same and both roll down same slope from same height anf hence both would reach bottom of the incline together. Hence, among the given options only (c) is correct.<br>***Answer is (c).*** |
| I-57 | In the problem, moment of inertia of a hollow sphere is $I_{HS} = \frac{1}{2}MR^2$ and that of a solid sphere |

$I_{SS} = \frac{2}{5}MR^2$ for two sphere having same mass and radii. Both the sphere would experience similar frictional force $f$ that causes rolling, and hence angular acceleration of the spheres can be determined with $\Gamma = fR = I\alpha = M\,k^2R^2\alpha \rightarrow \alpha = \frac{f}{k^2MR}$. Here, $k^2$ is 0.4 for solid sphere and 0.5 for hollow sphere.
Thus, angular angular acceleration of solid sphere would be higher than that of hollow sphere since- **a)** it is nversly proprtional to $k^2$ , and **b)** $k^2$ is larger for hollow sphere. Since both the sphere are rolling where $v = R\omega$ and hence angular and linear speed attained by solid sphere while rolling down same inclined rough plane would also be higher. Using this analysis each of the option is being examined –

(a) Since angular acceleration of hollow sphere is less and than solid sphere, and hence time taken by it reach ground shall be more by second equation of kinematics. ***Hence, this option is incorrect***.
(b) Since, angular acceleration of solid sphere is higher, and speed attaied by it on reaching the bottom would also be more. Hence , ***this option is correct***.
(c) Kinetic energy of the sphere is $KE = KE_R + KE_L = Mgh$, where $KE_R$ is kinetic energy of rotational motion and $KE_L$ is kinetic energy of linear motion and $Mgh$ is potential energy utilized during descend through height $h$ on the rough inclined plane. Since, $Mgh$ is constant and same for both the given spheres. Hence, ***this option is incorrect***.
(d) From the above analysis, since linear velocities of both the sphere are not same and hence lineear momentum $p = Mv$ would also be different, hence ***this answer is incorrect***.

***Answer, is option (b)***

I-58




Rolling of a sphere on a plane requires a frictional force and in absence of this sphere cannot roll. Necessary requirement of frictional force $f = \mu N = \mu Mg\sin\theta$, here $\theta$ is the angle between normal to the surface and direction of $\theta$ Since no mention of any external force is made it is considered to be zero. Thus, anlysing the problem for each option –

(a) In this case $\theta = 0 \rightarrow \sin\theta = 0 \rightarrow f = 0$ and hence no rolling would take place. But, since there is no external force, it would also not slide on smooth horizontal surface, it would remain in state of rest. Thus question of rolling ruled out, which makes ***this option is an incorrect answer***.
(b) In case of smooth inclined plane though $\theta \neq 0$, yet $f = 0$ since $\mu = 0$ and hence necessary condition for rolling is not satisfied. This makes ***this option is correct answer***.
(c) On a rough plane $\mu \neq 0$ that makes $f \neq 0$ which satisfies condition of rolling. Thus in the event of any external force sphere would tend to roll. This makes ***this option is incorrect answer***.
(d) On inclined rough plane, $\theta \neq 0$, $\mu \neq 0$ ad hence $f \neq 0$, which satisfies condition of rolling. Further, there is an external force due to gravity $F = mg\sin\theta$ which would set the sphere to roll. Hence, ***this option is incorrect answer*** .

***Correct answer is option (b).***

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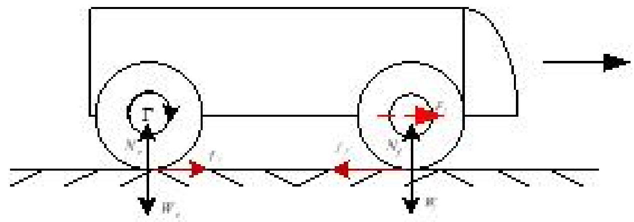
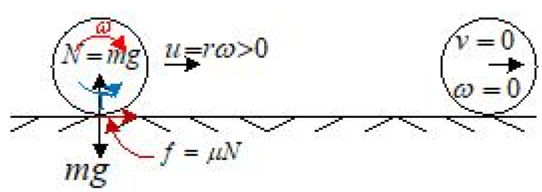
| I-59 | Given is rear-wheel driven car in which for motion of car in forward direction torque $\Gamma$ in clockwise direction is supplied to the wheel by engine as shown in the figure. Since frictional force on rear wheels $f_r$ acts opposite to the direction of motion, and frictional force at the point of contact of rear wheels with the road must act in anti-clockwise direction as shown in the figure.<br><br>As regards, front wheels rotate due to push $F_f$ created by linear motion of car, in forward direction, due to rotation of rear wheels. Therefore, it would experience frictional force $f_f$ in backward direction, setting front wheels to also rotate in clockwise direction, in unison with rear wheels. Accordingly, each of the given option is being examined –<br>(a) ***This option is correct*** and is discussed in analysis above.<br>(b) Direction of friction is analyzed to be in backward direction and hence ***this answer is correct.***<br>(c) Rotation of rear wheel is cause of action the frictional force $f_r$ on rear wheel, has to be more than or equal to effect frictional force $f_f$ on the front wheel. Considering some assorted losses during motion of car, magnitude of frictional force of rea wheel shall be greater that frctional force of front wheel, which has an overall effect of motion of car.Accordingly, ***this option is correct***.<br>(d) It is analyzed at (c) above $f_R > \|-f_F\| \rightarrow f_R + f_F > 0$ and hence net frictional force on car shall be in forward direction, ***which makes thos option incorrect***.<br>***Answer is (a), (b) and (c).*** |
| I-60 | In pure rolling on an inclined plane $v = r\omega$, $a = \frac{mg\sin\theta - f}{m} \rightarrow f = m(g\sin\theta - a)$ ,<br>$fr = I\alpha = \left(\frac{2}{5}mr^2\right)\left(\frac{a}{r}\right) \rightarrow f = \frac{2}{5}ma$ while frictional force is $f = \mu mg\cos\theta$. Condition for pure rolling is $\mu > \frac{2}{7}g\tan\theta$ it leads to $f > \left(\frac{2}{7}g\tan\theta\right)(mg\cos\theta) \rightarrow f = \frac{2}{7}mg^2\sin\theta$. But, it is given that $\mu = \frac{1}{7}g\tan\theta$ this would make actual frictional force $f_{actual} = \frac{1}{7}mg^2\sin\theta \rightarrow f_{actual} < f$ would be less than that required for pure rolling. Therefore, extra push created by downward gravitational force would cause translational motion together with the rotation of sphere about its centre. Thus, ***answer is option (c).*** |
| I-61 | In case of a sphere rolled on a horizontal surface if it slows down it means it is apparently experiencing a linear retardation. But, actually it is experiencing angular retardation. Since, during rolling $v = r\omega$ and, therefore, when $\omega = 0 \rightarrow v = 0$.Accordingly each of the option is being analyzed –<br><br>(a) When sphere is rolling forward, particle of sphere in contact with horizontal surface is tending to move backward in clockwse direction. Eventually, frictional force $f = \mu N = \mu mg$ would act in forward direction. Thus, torque $\Gamma = fr = I\alpha$ would act in anti-clockwise direction to create an angular retardation. As result of the retardation during rolling $(v = r\omega)$ linear velocity would decrease. Further, as per first equation of kinematics applied to angular motion after certain time $t$ angular |

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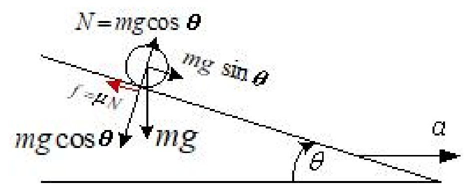
| | velocity of sphere would become ZERO to make it come to rest, i.e. $0 = \omega_0 - \alpha t$. This is ***consistent with option (a) and is correct.***<br>(b) From the analysis at (a) above since angular velocity is decreasing and hence ***option (b) is incorrect.***<br>(c) Since with decrease of $\omega$ linear velocity $v$ is also decereasing and hence linear momentum $p = mv$ would also decrease until sphere comes to rest. Accordingly, ***option (c) is incorrect***.<br>(d) Since $\omega$ is retarding as sphere rolls, as analyzed at (a) above, angular momentum $L = I\omega$ would also decrease. Thus this ***option (d) is also correct***.<br>***Answer is option (a) and (d)*** |
| I-62 | Force digram of the problem is shown in the figure, where car is having an acceleration n horizontal road $a = g \tan\theta$. Component of this acceleration along the inclined plane is $a' = a\cos\theta = (g\tan\theta)\cos\theta = g\sin\theta$. Since ball is on inclined plane it will tend to roll down and hence frictional force with given parameters would be $f = \mu N = \mu mg\cos\theta$. Condition for force $f'$ that will allow the sphere to roll down the plane with an acceleration $g\sin\theta$ would be $mg\sin\theta - f' = m(a\sin\theta) = ma\sin\theta \rightarrow f' = 0$. Since, this leads to that sphere tending to roll down would remain in its position; virtually the the sphere in a non-inertial frame with an acceleration down the plane $g\sin\theta$ causing a pseudo force to create an equilibrium. Accordingly analysis of options is as under -<br><br>(a) If sphere is set on pure rolling, the torque created by $\Gamma = fr$ created by friction about COM of sphere, would continue to roll the sphere at the same posotion on the inclined plane. Thus, this ***option (a) is correct***.<br>*(b)* Equlibrium of forces along the plane will not let the sphere slip down as per NFLM. Hence, ***option (b) is incorrect***.<br>(c) Equilibrium of forces along the plane would not allow increase of velocity as per NSLM. Hence ***option (c) is incorrect***.<br>(d) By logic at (c), ***option (d) is also incorrect***.<br>***Thus answer is (a).***<br>***N.B.:*** *This phenmenon of sphere rolling at sae position can be visualized from the fact that gravitational force along the slope, and psuedo force on the sphere caused by acceleration of car are equal and opposite. Thus, the roll of the sphereof the inclined plane is nullified by the displacement of the inclined plane due to accelerated car. Virtually position of the sphere remains unchanged. Nevertheless, rolling of sphere about its centre continues due to torque created by the frictional force.* |