Mechanics-IV: FLUID MECHANICS

Understanding of nature of fluids dates to prehistoric period and is evident from evolution of boat, syringe (पिचकारी) , use of water for irrigation etc. It got formalized by Archimedes, Pascal and many more scientists. In study of mechanics till now we consider matter as a rigid body, which under application of external force has negligible relative displacement between various parts of the rigid body. Inside volume of solid even at atomic level their relative position remains unchanged with negligible space available within molecules. This is referred to as molecular bonding, which assigns it a property of retaining shape. But, fluids have large inter-molecular space where molecular bounding is feeble. As a result ***firstly*** *molecules perform random chaotic motion and velocity of molecules performing such motion is a dependent upon internal energy. This internal energy, in the form of kinetic energy, is manifestation of heat energy contained in the liquid and temperature of fluid is an indicator of the heat content. This can be experienced in a situation where a class is required to attend extra class, after school hour, and students are hungry. In another situation the same class, when schools starts, has first period free. It will be seen that activity level of the class in second situation is much higher as compared to the earlier situation; this difference is due to exhaustion of energy by students. This is only a representative analogy.* ***Secondly****, feeble molecular bonding in fluids provides them a property to flow from higher pressure to lower pressure.*

Fluids are of two types – a) Liquids and b) Gases. In gases inter-molecular spacing is sufficiently large as compared to the size of the molecule, on account of this kinetic energy of the molecules lets them fill available space and exert a force on the walls of the container. The physics of such behaviour of gases shall be analysed later while dwelling Kinetic Theory of Gases in Chapter V on Heat. Nevertheless, liquids, which have inter-molecular space lesser than gases, when attain a velocity greater than escape velocity evaporation occurs, otherwise it is a case of surface tension, which is included here, in elaboration as an integral part of chapter on Mechanics.

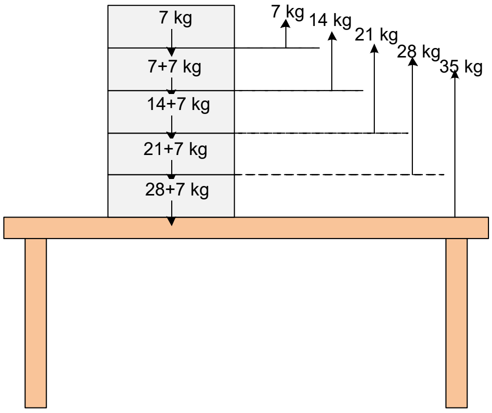
Fluid mechanics can also be classified in two distinct studies – ***a)*** *Fluid Statics and* ***b)*** *Fluid Dynamics, and elaboration of both of these are covered here.*

Fluid Statics: Archimedes law of buoyancy in 250 B.C. was a great revelation in understanding behaviour of a solid when immersed in liquid. Before, understanding buoyancy review of concept of density is essential. Visualize a school having two sections A and B of a class which are accommodated in rooms of same size with Ten benches each. For some reason there are 30 students in section A and 50 students. Obviously, occupancy of students on benches in section A is 3 students per bench while in class it is 5 students per bench. Thus, students in class B are more densely packed on benches as compared to class A.

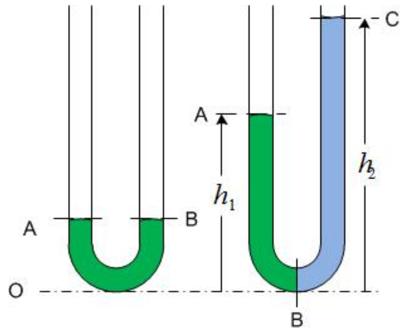
In physics, property of a matter pertaining to its mass and space (volume) occupied by it is called density and is mathematically represented as $\rho = \frac{M}{V}$; here, M is mass of the material, V is its volume and ρ is the density. Volume depends upon shape and is determined geometrically. Unit of density in MKS system is kg/m3 and its dimension is $[\rho] = ML^{-3}$.

In above example of Two sections A and B of a class occupancy i.e. space occupied by students is the available benches which is same in both the rooms. Thus density of students in section A is lesser than that in Section B, because sizes of and number of benches are same. This concept of density is extrapolated to compare of the density of different material and it is called ***Relative Density(RD).*** Accordingly, $RD = \frac{\rho}{\rho_w}$, where, ρ – is the density of material under consideration and ρ_w is the density of water at 4^0C which is 1 gm/cc in CGS system and in MKS system is 1000 kg/m3. Choice of water was made initially for its value in CGS system and continues to be used to define RD in MKS and IS.

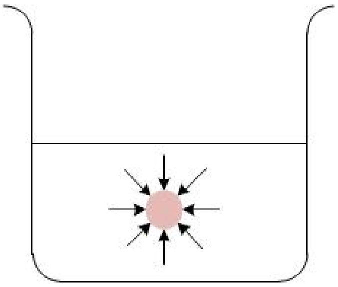
Taking forward learning of mechanics in previous section force exerted by a stack of Five boxes of books of Physics, each weighing 7 kg kept on the table, as shown in the figure, is $F = 7 \times 5 = 35\,\text{kg}$. The stack remains in place unless it is displaced. Force exerted by each box, progressively, on the lower box and reaction offered by the lower surface is shown in the figure.

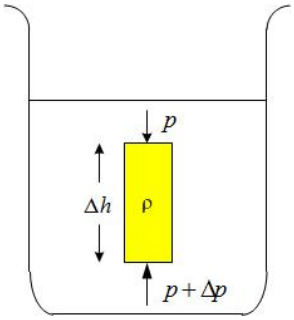


This analogy is extended to liquid. Taking a simple case of a U-tube in which liquid, in green colour, of density ρ_1 is poured slowly to fill a small part of its height. It will be seen that liquid in both arms of the U-tube rise to same heights equally. Once the liquid level has settled, the height of liquid in both arms above the level O is same at A and B,. Now another liquid of blue colour having density ρ_2, insoluble in earlier liquid of green colour, in Blue colour is slowly poured in one of the arms. Here density of liquids are $\rho_2 < \rho_1$. It will be seen that level of liquid of density ρ_1, in the arm, in which liquid of density ρ_2 is being poured, lowers down (level B descends) while in another arm its level rises (level A ascends) . Keep pouring liquid of density ρ_2, until layer separating the two liquids is at the midpoint of the U-tube, as shown in the figure. In such a state, it will be observed that height h_1 of liquid column of density ρ_1 and height h_2 of liquid column of density ρ_2 is such that $\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$. This observation lays foundation in understanding of fluid mechanics in general and fluid statistics in particular.

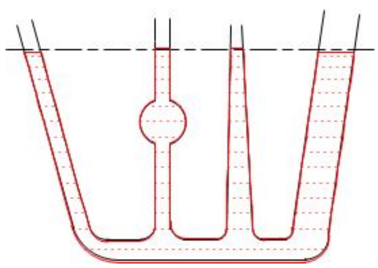


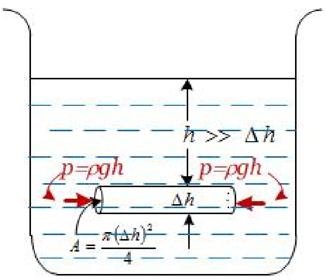
Static Fluid Pressure: Let there be a vessel filled with liquid. A small ball of volumetric density equal to that of water is immersed in liquid below the surface of liquid in the vessel. It will be observed that the ball remains in place. It implies that the fall is in equilibrium which is possible only if resultant force acting on the ball along any line drawn through it the ball is Zero. Now, let a thin cylindrical volume, of cross-sectional area A and thickness Δh, placed horizontally at a depth h from the surface in a liquid tank be considered. Let upper surface is under a downward pressure p and lower surface under a upward pressure $p + \Delta p$. The volume is since in equilibrium resultant vertical force on the volume must be ZERO. Accordingly, $pA + \rho g(A\Delta h) = (p + \Delta p)A$. Here, $\rho h(A\Delta h)$ is the weight of the volume. It leads to, $p + \rho h\Delta h = p + \Delta p$, or $\rho g\Delta h = \Delta p$, or $\frac{dp}{dh} = \rho g$. Therefore, net vertical pressure at a depth is $p = \int(\rho g)dh + C = \rho gh + C$. Here, C is integration constant such that at $h = 0$, i.e. on the surface of the liquid pressure is p_0, and, thus using these values in the instant integration we arrive at $C = p_0$. It implies that at any depth h below the surface of the liquid pressure is $p = p_0 + \rho gh$, i.e. as depth increases pressure increases. This is based on the assumption that density of the liquid is uniform, and is valid for incompressible liquids. This is, however, not valid for compressible fluid, typically gases.





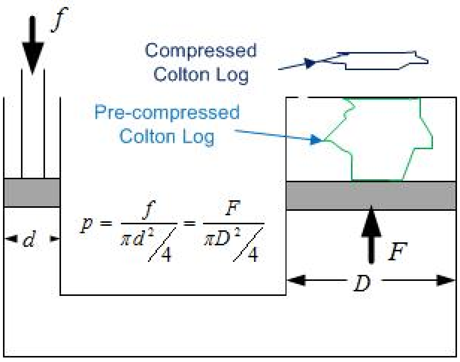
This principle of Fluid Pressure, helps to explain why despite different shapes of fluid column open to an environment having uniform pressure, height of fluid column is same. *It needs to be noted that this equality implies to equal Height of Fluid Column despite different length of Fluid Columns.*

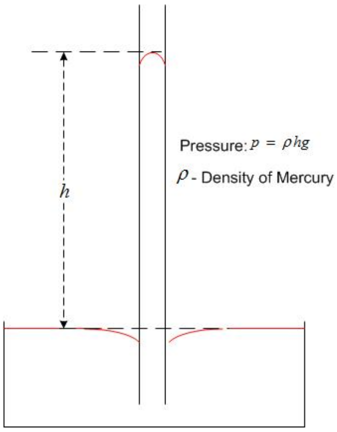




Next question that remains unanswered is "*what is the pressure on vertical-circular surface of the volume?*". It is experienced that liquids have a tendency to flow, therefore, under vertical pressure liquid starts flowing sideways to fill the available space, while volume of liquid is constant. This leads to conclusion that at any point horizontal pressure on the side walls is also same as the vertical pressure.

PASCAL's PRINCIPLE: *The mechanics of fluid pressure was studied by* ***Blaise Pascal*** and based on his experimental observation he formulated a principle in his treatise '***Equilibrium of Liquids***', which was found only after his death and published in 1663. The principle states that – "***A change in pressure at any point in an enclosed fluid at rest is transmitted undiminished to all points in the fluid***". It has found a wide application in ***Hydraulic Press or Levers*** as illustrated in the diagram. This principle has bearing on the principle of Static Fluid Pressure and demonstrates as to how with little effort large loads can be managed. A refinement to the Pascal's illustration stipulates that pressure at same height is constant. This principle is used to amplify fluid force on load in proportion to the surface area of fluid that supports the load and that at application of efforts i.e. $P_L = P_E$; $F = P_L A_L$ and $f = P_E A_E$; $F_L = \frac{A_L}{A_E} F_E$, here F corresponds to the load and f is the effort, while A_L is the surface area supporting load and A_E is the surface area of liquid on which effort is applied. At a first glance, it will be seen that though rationale of force violates Principle of Lever, discussed earlier in Mechanics, but a further analysis reveals that it is in conformance with the principle of Conservation of Energy, common to both. The volume (V_E) of liquid is displaced in effort column is moved into load column (V_L). Since liquid is incompressible, therefore, $V_L = V_E$, here $V_L = A_L x_L$ and $V_E = A_E x_E$, where x_E is the displacement of effort and x_L is the displacement of load, It leads to or $\frac{A_L}{A_E} = \frac{x_E}{x_L}$. Accordingly, using this relationship together with hydraulic force ratio, it leads to $F_L = \frac{x_E}{x_L} F_E$. Alternately, $F_L x_L = F_E x_E$. Both of these relationships are in conformance with principle of level and conservation of energy. Therefore, this hydraulic press is also called Fluid Lever.





Application of Pascal's Principle has been made in pressure gauge where one surface of the liquid is open to environment of which pressure is to be measured, while the other surface of the liquid is exposed to reference. In case of ***barometer***, which measures absolute pressure, reference is vacuum, while pressure gauge, ambient pressure is reference and measurand is pressure inside an enclosed surface. Diagrammatic representation of barometer is shown in figure.

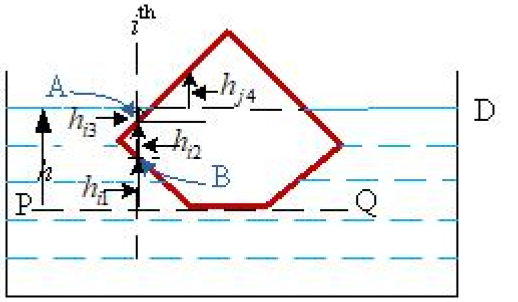
ARCHIMEDES PRINCIPLE: In Fluid Statics Principle of Buoyancy was promulgated by Archimedes 250 BC based experimental observation, but in different situations. He evolved a scientific way to solve problem posed upon him by the king to determine purity of Gold. This principle states that – "***upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and it acts in the upward direction at the centre of mass of the displaced fluid***"

This problem is analysed in two situations :**a)** Floating Objects, and **b)** Fully immersed Object. Each of the situations is being discussed separately.

Buoyancy of Floating Objects: An object in a steady state of flotation is in translational and rotational equilibrium. Translational equilibrium implies $\vec{F}_g + \vec{F}_b = 0 \rightarrow \vec{F}_g = -\vec{F}_b$, i.e. the two forces are equal in magnitude and opposite in direction. Since, $\vec{F}_g$ is vertical downward and passes through its COM, therefore, $\vec{F}_b$ must also pass through COM and in vertically upward direction.

Further, rotational equilibrium implies torques in clockwise and anti clockwise directions are equal. As per D'Almbert's Principle rotation of an object due to torque caused by force out of COM. Since, both the forces acting on a floating object are along vertical line passing through COM, and therefore their torques $\vec{\tau} = \vec{r} \times \vec{F} = 0\big|_{r=0}$. Accordingly the body would continue to float without turning.

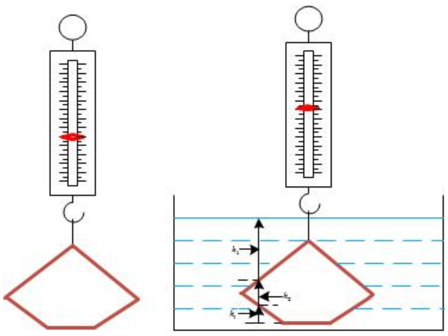
For elaboration a small horizontal elemental area Δa_i, along i^{th} vertical line, fluid force along upper surface at A would be



$\widehat{f}_A = p_A \Delta a_{iA} \cos\alpha(-\hat{n}) = -\rho g h_{i3} \Delta a_i \hat{n}$ i.e. in downward direction, here $\hat{v}$ is unit vector in upward direction. It is to be noted on slanting surfaces at A and B, $\Delta a_{iA} \cos\alpha = \Delta a_{iB} \cos\beta = \Delta a_i$, i.e. projection of inclined surfaces of the solid along the horizontal is are same. Here, α and β are angles of upper and lower surfaces of solids at A and B, respectively. Likewise, force on the bottom surface at B, along the line would be $\widehat{f}_B = p_B \Delta a_{iB} \cos\beta(\hat{n}) = \rho g (h - h_{i1}) \Delta a_i \hat{n}$. Thus net force on the Δa_i of the liquid bearing the load will be $\vec{f}_i = \vec{f}_{iB} - \vec{f}_{iA} = \rho g (h - h_{i1}) \Delta a_i \hat{v} - \rho g h_{i3} \Delta a_i \hat{v} = \rho g (h - (h_{i1} + h_{i3})) \Delta a_i \vec{\hat{v}}$. It leads to $\vec{f}_i = \rho g h_{i2} \Delta a_i \hat{v}$. Accordingly, net force along i^{th} line on the object would be $\vec{f}_i = (p_i \Delta a_i)\hat{v}$ i.e. it magnitude in upward direction is $f_i = p_i \Delta a_i$ Thus total upward force is $F_b = \sum f_i = \sum (p_i \Delta a_i) = \rho_l g \sum (h_{i2} \Delta a_i) = \rho_l g \sum \Delta v_i = \rho_l V_d g$; here $\sum (h_{i2} \Delta a_i) = \sum \Delta v_i = V_d$ is the volume of liquid displaced, and force F_b *is called* ***Buoyancy Force.***

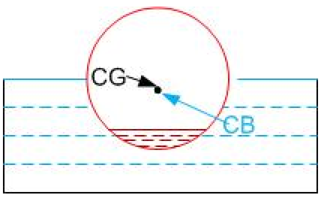
As regards weight of the object acting on the bottom surface of the liquid is $F_g = Mg = V\rho_s g$ and would act vertically along the line of gravity, since g is vertical. An object in a steady state of flotation has **(i)** F_g and F_g *in equilibrium i.e.* $V_d \rho_l = V\rho_s$, **(ii)** *the line of buoyancy force coincides with the vertical line of centre of gravity (CG), which keep the body to keep it floating without any kind of rotational effect i.e. torque of produced by distributed mass of the solid about its COM is equal and opposite to the torque produced by displaced liquid about the COM of the liquid called* ***Centre of Buoyancy (CB)****; it is in accordance with D'Almbert's Principle* **(iii)** *as long as (ii) is valid, position of COM of solid and that of displaced liquid is immaterial* **(iv)** if object is partially immersed in liquid then $V_d < V$ it implies that $\rho_s < \rho_l$, and **(v)** object shall float in fully immersed condition only if $\rho_s = \rho_l$; such an object shall stay floating at any depth, and this is what artificially managed in submarines.

The principle of buoyancy knowingly, or unknowingly with mere experience since immemorial times, has found application in increasing load bearing capacity of boats which are made of wood lighter than water; so also in design of ships which are made of steel much denser metal than water, yet carry very heavy loads.

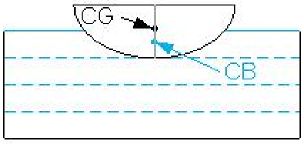


Buoyancy of Immersed Objects: In case of fully immersed object, the above analysis shall apply with $h_{j4} = 0$, and h_{i3}will depend upon shape and depth of submergence of object in liquid. Another deviation in the analysis is that weight of the object is greater than weight of liquid displaced i.e. $\sum h_{i2} A_i \rho_s g > \sum h_{i2} A_i \rho_l g$. This leads to reduction of weight of a denser object immersed in lighter liquid. This principle causes reduction in effort to take out an object immersed in deep water viz. well, river, lake and sea.

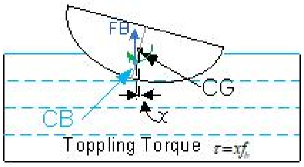
Stability During Float: Extending the Archimedes principle, together with principle of ***Centre of Gravity (CG)*** and moments. Mass distribution of floating object is different and may be non-homogenous, while mass distribution of homogenous displaced liquid is based on volume immersed. This could lead to three situations.



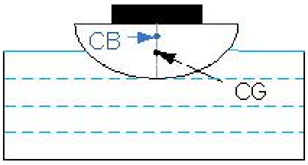
Situation a) In this CG and CB coincide. If the object *is slightly tilted* and left to negotiate with liquid, it would continue to be in that position. As per D' Alembert's Principles since centre of gravity acts as fulcrum, since CG and CB are coinciding there would be no torque caused by buoyancy and state of ***stable equilibrium*** would continue to be there.

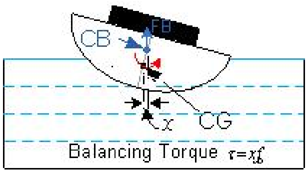


Situation b) In this CG of Object is above CB. When the object is tilted in any direction, say clockwise, the CB would also shift clockwise and F_b would compound turning torque, on the object about CG, and this is called ***Toppling Torque.*** This is a cascading effect in same direction until the object topples and sinks. Such a situation is called ***dynamic in-equilibrium***.



Situation c) In this CG of object is below CB. When the object is the displaced in any direction, say clockwise, the though CB would shift clockwise, F_b would exert a torque about CG in anticlockwise direction. Thus it would tend to neutralize the tilting. Therefore, this torque is called ***Balancing Torque***. Thus the floating object starts oscillating about its mean position, until it oscillating energy is dissipated and finally the object settles down to rest i.e. pre-tilting position called mean position. This is called ***dynamic equilibrium***.

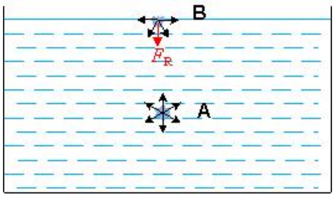


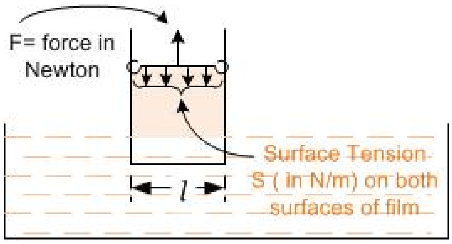


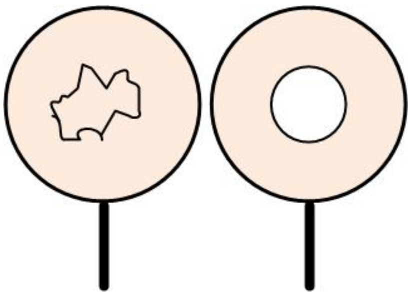
Shift in CB depends more upon shape of the floating objects, which are generally symmetrical about vertical axis in its static equilibrium. But, shift in CB requires to be observed closely, which generally conforms to the above analysis.

The Archimedes Principle seems to produce nearly correct results for objects having high mass to surface area ration. But, when this ratio is small causing floating conditions, like that of a metal sheet, it fails. This is a boundary condition where Surface Tension comes into play; illustration of this phenomenon is made in the next paragraph.

SURFACE TENSION: It must have been observed that a thin metal sheet placed horizontally on the surface of water keeps floating, and when it is tilted slightly it submerges into water. This floatation cannot be explained with principle of buoyancy. It requires understanding and analysis of interaction of molecules inside liquid. A molecule inside liquid is surrounded by large number of molecules each of which experiences gravitational force from surrounding molecules and experiences a state of static equilibrium as shown for molecule A. This molecule under consideration when moved near the surface, it creates a state of in-equilibrium resulting into a net downward pull F_R as shown for molecule B. Existence of this force can be realized with creation of a liquid film in a rectangular frame which is adjustable in one direction. It requires a U-shaped frame with a sliding arm to make a closed surface. The frame is dipped in soap solution and when taken out in horizontal position the sliding arm is closed to opposite fixed side of the frame. But, when the frame is held vertical with a small force the sliding arm can be pulled off. Magnitude of the force (F) on the adjustable arm is $F = 2Sl$, here S is the surface tension per unit length and l is the width of the film. Here, multiplier 2 is due to two surfaces of the film formed by the frame, and each surface exerts a force on the adjustable arm.







In another experiment a film is formed in a closed frame. On this film a light loop of thread, wetted in the same solution, is carefully placed on it. The loop retains a shape as it is left on the film. Now the film inside the loop is punctured with a fine needle, it is observed that the loop rearranges itself in shape of a circle. As per geometry circle is a shape whose area is largest for a perimeter. Further, assuming that the film has a surface energy, largest area being taken out by the circular loop, created by the puncture, remnant film shall have least surface energy, and is in accordance with the ***principle of least potential*** which states that ***every body tries to occupy position of least potential energy.*** The concept of ***surface energy*** is another evolution in the journey.

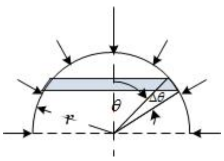
Surface Energy: Molecules inside liquid and those at surface experience molecular interaction in a state of equilibrium. Equilibrium of molecules inside liquid is due to balanced radial forces within the sphere of influence, as shown above. Radius of the sphere of influence is about 10-15 molecular diameter. While, molecules near the surface due to hemispherical distribution of molecules creates an inward pull. Though, liquids have a tendency to remain in shape of the vessel, its molecules experience random chaotic motion (*this shall be a subject matter during illustrations on Heat, as Brownian motion*) and undergo elastic collisions due energy, in the form of heat, acquired by them. These molecules as they tend to approach surface of liquid loose velocity due to retardation caused by inwards pull created by imbalance of molecular forces, as shown in the figure above. This retardation is similar to that on a particle projected upwards on the earth's surface. In the process these molecules gain potential energy following laws of conservation of energy. Few molecules, having remnant kinetic energy may leave the surface of liquid, but would return back to the liquid surface due to molecular pull. Some of such molecules coming out of surface of liquid might get swept away by either wind or velocity being sufficient to lead them to escape zone of influence of the liquid. This velocity can e compared with escape velocity illustrated in gravitational field.

Surface energy of a film is illustrated in the example of the film created by a moving of a U-shaped frame, shown above. Total force tends to shrink the film. If the moving arm is slowly moved through a distance x, to increase the size of the film, the amount of work done on the film (Potential energy of the film: $U = W = S2lx = SA$, here, A is the total surface area, taking both the faces of the film.

Pressure Inside a Bubble & Drop: The concept of *Surface Energy* is useful in analysing pressure inside bubble and drop of liquid.

Pressure inside Drop

A horizontal hemisphere of a small drop, as shown in the figure, is being considered. Let a small area be taken from a circular strip, parallel to the cross-section of the hemisphere, on the surface of the drop. The area of the element shall be $\Delta A = (2\pi r\sin\theta)(rd\theta)$. The ambient pressure on it would be radially inward and uniform.



The force along the cross-section of the hemisphere shall be cancelled out to elements of the strip which are diametrically opposite.

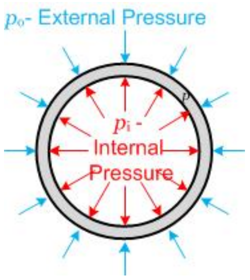
The component of pressure perpendicular to the cross-section shall be= $p(2\pi r^2\sin\theta\, d\theta)\cos\theta = \pi r^2 p\sin 2\theta d\theta$. Here, p is the pressure inside the drop.

Thus overall atmospheric force on the cross-section shall be $F_A = \int_0^{\pi/2}\pi r^2 p\sin 2\theta d\theta = \pi r^2 p\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \pi r^2(p - p_o)$, here, p_o is the ambient pressure outside the drop.

While, edge of the hemisphere would exert a force on account of Surface Tension caused by surface of the bubble $F_s = 2\pi r S$. Equating the two forces $\pi r^2\left(p - p_0\right) = 2\pi r S$, It leads to $p = \frac{2S}{r} + p_0$.

Pressure inside Bubble

Bubble is like a Twin drop, but it has a small difference from the adjoining example such that inside and outside bubble there is gas at different pressures and these are separated by a thin film of liquid.



Accordingly, it requires consideration and analysis of the two surfaces. For outer surface is balanced by difference $(p - p_o)$, where p_o is the ambient pressure and p is the outwards pressure exerted by liquid film. While, the inner surface is balanced by difference $(p_i - p)$ where p_i is the outward pressure exerted by gas trapped inside the bubble, while p is same as considered for outer surface. Thus net force on the surface of the bubble is $F_S = [(p_i - p) - (p - p_o)]\pi r^2$.

The diametric-cross-sectional edge will experience force of Two surfaces (inner and outer) $F_S = 2\times(2\pi r)S = 4\pi r S$, here, r is the radius of bubble having a liquid film of negligible thickness.

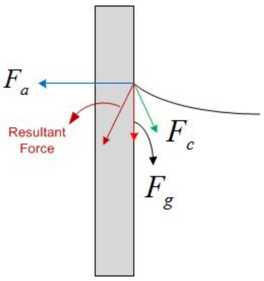
Thus, in a state of equilibrium of the bubble $F_G = F_S$, thus $[(p_i - p) - (p - p_o)]\pi r^2 = 4\pi r S;\ p_i - p_o = \frac{4S}{r} \rightarrow p_i = \frac{4S}{r} + p_o$.

Capillary Action: This is an effect of surface tension which causes extraction of water and nutrients from soil by plants, a beginning of life. Understanding capillary actions requires deeper understanding of inter molecular forces. Force of attraction between homogenous molecules (belonging to same substance) is called ***force of cohesion***(f_c) and similar force between heterogeneous molecules (belonging to different substances) is called ***force of adhesion*** (f_a). The capillary actions is a result of interaction between: **a)** difference between forces of cohesion and adhesion, and **b)** gravitational force on molecules of liquid in vicinity of the capillary.

Top surface of water in a capillary is called **meniscus**. Shape of meniscus is concave when $f_a > f_c$, observed in water and most of the liquids. But when $f_a < f_c$ the meniscus becomes convex, as it happen in mercury. Both the cases are discussed separately. Another, important property of the liquid is its fluidity, i.e. it would start flowing in the direction of force, unless equilibrium exits. According to this property, *net force on a liquid, in equilibrium must be perpendicular to the surface.* This property need not be confused to be in contradiction to the surface tension illustrated earlier; rather it is an extension of surface tension and is illustrated below.

Shape of Liquid Surface
$(f_a < f_c)$

The resultant force on molecules in contact with the container is downwards towards the solid container. Accordingly, depending on the shapes of solid following cases arise in respect of the surface of the liquid:



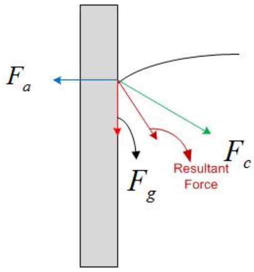
Case a- *A sheet or the plane of container:* A curved surface rising on the **walls of solid surface**

Case b- *A rod or a cylinder in:* A circular curved surface rising on the walls of solid surface

Case c- *A Capillary* (*tube of small diameter*)*:* A concave curved surface tending to be hemisphere and rising above the surface of liquid outside capillary

Shape of Liquid Surface
$(f_a > f_c)$

The resultant force on molecules in contact with the container is downwards inside the liquid in the container. depending on the shapes of solid following cases arise in respect of the surface of the liquid:



Case a- *A sheet or the plane of container:* A curved surface tending to dip towards the walls of solid surface

Case b- *A rod or a cylinder in:* A circular curved tending to dip towards the walls of solid surface

Case c- *A Capillary (tube of small diameter:* A convex curved surface tending to be inverted hemisphere, and dipping below the surface of the liquid outside capillary

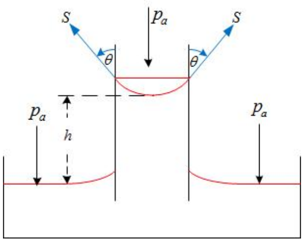
The above analysis requires consideration of forces, shown in the figure, acting on the molecules of liquid in vicinity of the solid surface which could be either of container, separator or a capillary. These would determine direction of the resultant force, which has to be perpendicular to the surface of liquid in a state of equilibrium. Direction of the resultant force shall decide shape of surface of liquid. The angle between two lines at the point of contact of liquid and solid: **a)** perpendicular to the surface of liquid, and **b)** tangent to the surface of solid is called ***Angle of Contact***.

Height of Liquid Inside Capillary: This capillary prompts to analyze height of height of liquid column in a capillary. It depends upon: **a)** density of liquid in capillary, **b)** pressure above surface of liquid, and **c)** radius of capillary.

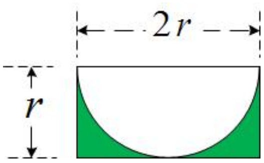
Variables involved in this analysis are : p_0- atmospheric pressure above base liquid level, p_i – pressure above liquid in capillary then, h – height of the liquid in capillary above base liquid, ρ is the density of liquid, and r – radius of capillary. The force created by rise or fall of liquid in capillary $F_g = \pi r^2 h\rho g$; this force is in the direction opposite to rise/fall of liquid. Force created by surface tension $F_S = 2\pi r S\cos\theta$. These two forces are in equilibrium in steady-state of the liquid column in a capillary. In the illustration below open capillary is taken which leads to $p_i = p_0$.

Shape of Liquid Surface
$(f_a < f_c)$

Balancing fluid pressure and surface tension in liquid column: $\pi r^2 h\rho g = 2\pi r S$. Accordingly height of liquid column $h = \frac{2S\cos\theta}{\rho g r}$.

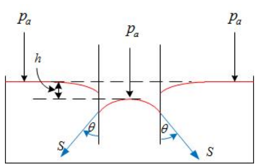


Shape of meniscus, which tends to be hemispherical as $\theta \to 0$, creates an error in height of liquid column. A correction Δh is accounted due weight of liquid in meniscus and it is: $\Delta h = \frac{\left((\pi r^2)r - \frac{4/3\pi r^3}{2}\right)\rho g}{\pi r^2 \rho g} = +\frac{r}{3}$

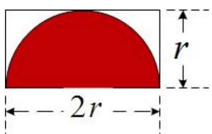


Shape of Liquid Surface
$(f_a > f_c)$

Balancing fluid pressure, due to liquid outside capillary, and surface tension in liquid column of in capillary, it leads to: $\pi r^2 h\rho g = 2\pi r S$. Accordingly height of liquid column $h = \frac{2S\cos\theta}{\rho g r}$.



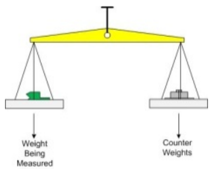
Shape of meniscus, which tends to be hemispherical, as $\theta \to 0$, creates an error in height of liquid column. A correction Δh is accounted due weight of liquid in meniscus and it is: $\Delta h = \frac{\left((\pi r^2)r - \frac{4/3\pi r^3}{2}\right)\rho g}{\pi r^2 \rho g} = \frac{r}{3}$. Nevertheless, in case of barometer, when mercury column is above the cup, it is the height of the column upto to the top of the cup is measured. Accordingly, the correction $\Delta h = -\frac{r}{3}$, and is left for verification by the mentor.



Situation that arises when length or depth of the tube is less than h, wherein, both the cases, liquid neither spills over the capillary nor liquid loosing contact with the capillary. In such cases $\theta \to \frac{\pi}{2}$, i.e. liquid surface inside tends to lose its curvature. This can be verified through experiment.

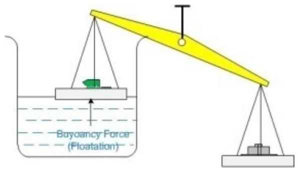
A simple experiment which *combines principles of gravity, buoyancy and surface tension covered in separate parts of mechanics are* brought out here under-

A-Gravitational Balance



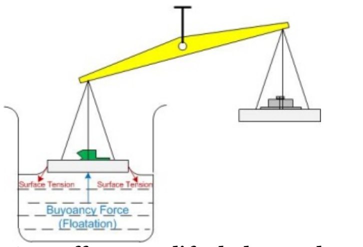
Equal weight placed on two separate pans of a **Beam Balance**. It is a result equilibrium of moments of forces on the beam to keep it horizontal. Thus equal weights are measured equal.

B-Effect of Floatation(Buoyancy)



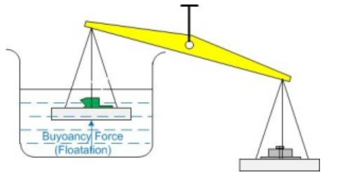
One of pan held above a liquid, with partial immersion, gives rise to **force buoyancy**. As a result, the held above the liquid *pan apparently becomes becomes lighter*.
This is effect of disbalance caused by force of buoyancy..

C-Effect of Surface Tension



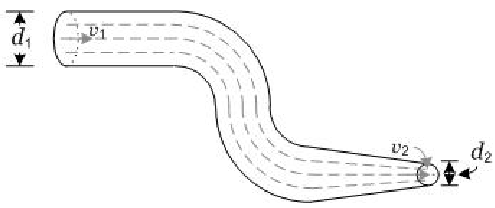
An effort to lift balance beam causes a downward force due to **surface tension** on the pan placed above the liquid surface..
As a result, the *pan in contact with liquid surface suddently appears to be heavier* and beam tilts towards the pan on liquid surface.

D-Effect of Submergence (Buoyancy)



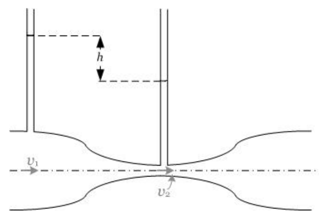
When one of the pan is allowed to immerse in liquid, the **buoyant force** is in direction opposite to gravitational force as in case (B). In this case effect of buoyant force is pronounced since full volume of pan with weights is immersed.*This apparanetly reduces the measure of weight*.

Fluid Dynamics: Flow of liquid is characterized in two forms: **a)** ***Streamline Flow*** and **b)** ***Turbulent Flow***. In *Streamline flow* of liquid is linear i.e. *flow is along a line viz. lines of field which do not cross each other*. Flow of liquid through a pipe of different cross-section and changing shapes these lines of flow bend, compress or rarefied depending on cross-section of the pipe, without intersecting each other, as shown in the figure. This is based on following consideration : **a)** Volume of liquid flow remains constant because of it incompressibility, **b)** Velocity of flow of liquid due to (a) shall increase/decrease with decrease/increase in cross-sectional are of pipe, and **c)** this continuity in stream of liquid flow shall conform with the law of conservation of energy. Accordingly, analysis is being made to arrive at a general equation for stream line flow.

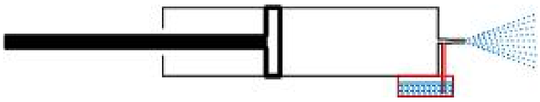


Let V is the volume of liquid entering the pipe shown in the figure. Therefore velocity of water at the entrance of the pipe shall be $v_1 = \frac{V}{\pi d_1^2/_4}$, and velocity of liquid leaving the pipe shall be $v_2 = \frac{V}{\pi d_2^2/_4}$. Thus, as per observation at (a and b) above, ratio of velocities of liquid at the entrance of the pipe and its exit shall be, $v_1 : v_2 :: d_2^2 : d_1^2$.

At this point applying Law of Conservation led to a new revelation. Initially, axis of the pipe is considered to be horizontal along the length, irrespective of its cross-section. Height of the axis above ground is taken to be h, it leads to constant potential during the flow. Nevertheless, there is an imbalance in total energy of the liquid at entrance $\left(E_1 = V\rho gh + \frac{1}{2}(V\rho)v_1^2\right)$ and energy at exit $\left(E_1 = V\rho gh + \frac{1}{2}(V\rho)v_2^2\right)$. Since, $d_1 > d_1$; $E_2 - E_1 = \frac{1}{2}(V\rho)v_2^2 - \frac{1}{2}(V\rho)v_1^2 = \frac{1}{2}(V\rho)(v_2^2 - v_1^2) > 0$, there must be some other form of energy in the flowing water. This inequality led Daniel Bernoulli to investigate the flow of incompressible liquids and he propounded in 1738 relation between velocity and potential energy in the form of pressure and is known after his name as ***Bernoulli's Equation*** such that:$E_2 - E_1 = \frac{1}{2}(V\rho)(v_2^2 - v_1^2) = V\Delta p$. In a generalized form removing assumption of uniform height along the flow of liquid, as per energy conservation the energy equation shall be: $\left(\left(V\rho gh_2 + \frac{1}{2}(V\rho)v_2^2 + Vp_2\right) - \left(V\rho gh_1 + \frac{1}{2}(V\rho)v_1^2 + Vp_1\right)\right) = 0$. It leads to: $gh_1 + \frac{1}{2}v_1^2 + \frac{p_1}{\rho} = gh_2 + \frac{1}{2}v_2^2 + \frac{p_2}{\rho}$.

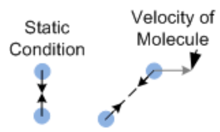


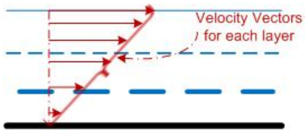
Thus, in streamline flow of liquid, under isothermal conditions, has energy three forms: **a)** ***Potential Energy*** arising due to height of liquid under gravitational field, **b)** ***Kinetic Energy***, due to velocity of flow, and **c) *Pressure Energy*,** caused by the pressure with which liquid is flowing. This principle, also called as *Bernoulli's Principle* is excellently demonstrated through a ***Ventury Tube***, a small reduction in diameter of pipe through which liquid is flowing. This is principle finds used in Ventury Meter to determine velocity of flow of liquid through a cross-section of a pipe, as shown in the figure above. Accordingly, equation of pressure difference, as per principle of Barometer, is $p_1 - p_2 = \rho g h$. Using this equation together with Bernoulli's Equation it leads to $\frac{\rho g h}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$, Accordingly, $(v_2^2 - v_1^2) = 2gh$, is used for determining velocity in a pipe at any cross-section. An excellent application of this Bernoulli's Principle is spray-machine which is used for spraying insecticide on farms, spray painting etc. It is a modified form a simple air-syringe. At it's outlet where velocity of air is high which causes low pressure condition. This low pressure at the outlet causes suction of liquid from a tank connected through a narrow tube. Liquid supplied through the tube get atomized in the high speed air being expelled by the syringe and thus sprayed. This atomization process is splitting of liquid in fine droplets which absorb kinetic energy of air in the form surface energy of droplets, as per principle of *Surface Tension*.

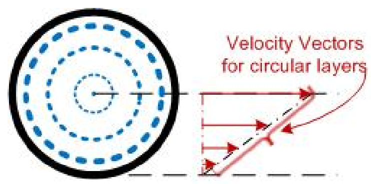


Manipulation of wings of birds is intuitive to them, but it is purely an application Bernoulli's principle which is used by engineers and scientists in design of wings of air craft, and improving aerodynamic performance of high speed vehicles.

Viscosity: Property of liquid to flow is influenced by inter-molecular forces within the volumes of liquid and gets manifested as force of friction. It involves both the forces viz. *Force of Adhesion* on the particles of liquid in contact with the static boundary and *Force Cohesion* within the volume of liquid. In static condition, when water is stationary molecules of liquid stay in equilibrium and experience a force of attraction. But, when a molecule is set in motion the force of attraction continues depleting as per laws of gravitation as they move away from sphere of influence of each other. In turn force exerted by the moving molecule tries to pull along with it the adjoining molecules which is having lower relative velocity. At the same time the molecules with lesser velocity try to retard the molecule moving at relatively higher velocity. This effect is called ***Viscosity*** and is different from *Surface Tension*. An understanding of Viscosity requires to understand behaviour of liquid during flow. Effect of viscosity is a gradient in velocity of flow, as shown in the figure. The solid surface over which liquid is flowing viz. river bed or canal is since static layer of liquid layer touching experiences : a) maximum intra-molecular force i.e. force of cohesion, b) a pressure due to head of liquid above it, and c) PLUS atmospheric pressure. But, the for the molecules above it decreases – i) in inverse proportion of square of the distance from the static surface, and ii) direct proportion of liquid head. As a result of these reducing forces on the molecules of liquid, velocity of liquid increases as they approach surface of liquid. This gradient exists in layers, and conceptual representation of the laminar is shown in the figure. This gradient is also true for adjoining walls of the liquid and is well depicted for circular tube in the figure. It is pertinent to observe three exceptions in case of laminar flow of liquid: **a)** Bernoulli considered uniform velocity of liquid across the direction of flow, an ideal situation, which does not exist due to viscosity, **b)** In wide stream velocity gradient is uniform along the width of stream, It is subject to following conditions - ***i)*** the flow under consideration is sufficiently away from the banks, and ***ii)*** bed of flowing liquid is parallel to the horizontal surface of the flowing liquid, and **c)** velocity of liquid flowing in pipe is radially uniform along the cross-section of the pipe, like annular rings having velocity gradient such that velocity is minimum in contact with pipe and maximum at its centre.





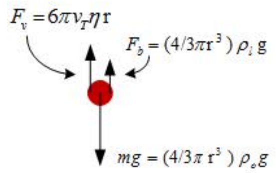


Taking this conceptual representation of viscosity as elaborated, next is analytical aspect and it is seen that ***Viscous Force*** in an open stream is proportional to – **a)** is proportional to the surface area and its nature is similar to shear force; it is attributed by the fact that number of interacting molecules are proportional to surface area, and **b)** gradient of velocity from the surface of flowing liquid. Accordingly, Viscous Force $F_v \propto A$and $F_v \propto \frac{dv}{dx}$; consolidating these two relationships, $F_v = \eta A \frac{dv}{dx}$; here, η is proportionality constant and dimensionally it is, $[\eta] = \frac{[F_v]}{[A]\left[\frac{dv}{dx}\right]} = \frac{(MLT^{-2})}{(L^2)\left(\frac{LT^{-1}}{L}\right)} = ML^{-1}T^{-1}$. Likewise, unit of η is N-s/m², which in CGS is Poise = 0.1 N-s/m². *Viscous Force is like Frictional force and its comes into play only when there is relative motion between adjoin molecules, be it of the same liquid or a container or different object moving inside the liquid.*

Qualitatively, effect of viscosity in tubular flow is identical to that of a stream, but quantitatively is different. Experimentally it is observed that rate of flow of liquid $\left(\frac{V}{t}\right)$ in a tube is a function of – **a)** Pressure gradient along the tube i.e. $\frac{p}{l}$; here, l – is the length of the tube and p - is pressure difference in liquid flowing in the tube along the length, **b)** Viscosity of the liquid (η), and **c)** Radius of the tube (r). *Dimensional analysis* has been excellently used to determine relationship of these parameters on rate of flow of liquid. Accordingly, $\left[\frac{V}{t}\right] = k\left[\frac{p}{l}\right][\eta][r]$, here, k is taken to be dimensionless constant. Accordingly, $\frac{L^3}{T} = \left(\frac{ML^{-1}T^{-1}}{L}\right)^a (ML^{-1}T^{-1})^b (L)^c$. It leads to, $L^3T^{-1} = (ML^{-2}T^{-2})^a (ML^{-1}T^{-1})^b (L)^c = M^{a+b}L^{-2a-b+c}T^{-2a-b}$. Equating exponents of each of the Three fundamental dimension: **i)** for exponents of M, $0 = a + b$; **ii)** for exponents of L, $3 = -2a - b + c$, and iii) for exponents of T, $-1 = -2a - b$. Solution of these three simultaneous equations is, $a = 1, b = -1,$ and $c = 4$. Experimentally, value of dimension less constant (k) was determined to be $\frac{\pi}{8}$, seperately by *Jean Léonard Marie Poiseuille in 1838 and Heinrich Ludwig Hagen* and published later. It is also known as *Hagen–Poiseuille equation*, also known as ***Poiseuille Formula***: $\frac{V}{t} = \frac{\pi p r^4}{8\eta l}$. In the analysis brought above, it was assumed that the flow isothermal and temperatures are conducive to laminar flow.

Stoke's Law: In 1851, George Gabriel Stokes determined viscous force exerted on motion of a spherical object, having smooth surface, in a homogeneous liquid at a velocity having laminar flow. He propounded that viscous force is given by:$F = kv^a\eta^b r^c$; here, k is dimensionless constant, while, F - is the frictional force caused by liquid medium in which object is moving, v - is the flow velocity relative to the object, η - is the coefficient of viscosity, and r is the radius of the spherical object.

Again, using dimensional analysis the relationship of viscous force is arrived at $MLT^{-2} = (LT^{-1})^a (ML^{-1}T^{-1})^b L^c = M^b L^{a-b+c} T^{-a-b}$. Equating dimensional exponents, three simultaneous equations are: i) $1 = b$, ii) $1 = a - b + c$, and iii) $-2 = -a - b$. Accordingly, $a = 1, b = 1,$ and $c = 1$. Thus, the relationship comes out to be $F = kv\eta r$. Experimentally, Stoke determined $k = 6\pi$, thus the quation got formalized as $F = 6\pi v\eta r$, and is known as ***Stoke's Law***. In SI units, F is Newton, η in Poise, r in meters, and v in m/s. It is interesting to observe that apart from other parameters, for a spherical object, the only variable is velocity v. Further, if the object is moving under gravity, it experiences a constant acceleration and buoyant force. Thus, taking a free body diagram of the object it will initially accelerate and finally attain a constant velocity called *Terminal Velocity* (v_T) such that there is equilibrium of forces, represented by equation: $\left(\frac{4}{3}\pi r^3\right)\rho_o g = \left(\frac{4}{3}\pi r^3\right)\rho_l g + 6\pi v_T \eta r$. Accordingly, $\left(\frac{4}{3}\pi r^3\right)(\rho_o - \rho_l)g = 6\pi v_T \eta r$, and thus, $v_T = \frac{2r^2(\rho_o - \rho_l)g}{9\eta}$. This concept explains velocity of rain drops, never exceeds its limiting value. This is used to determine viscosity of a liquid.



In the event of either of – **a)** temperature increasing, molecules of liquid acquire sufficient velocity to trespass sphere of influence of molecules in immediate vicinity, **b)** there is obstruction in way of laminar flow, **c)** velocity of flow is large and sudden change of pressure, flow of liquid loses laminar flow. Such a flow is called **Turbulent Flow** where path of liquid cannot be predicted. This is like students leaving extra class after the school hours, unlike students leaving assembly ground to enter into their class rooms in rows of respective classes; the latter is streamline like. While, all other parameters, for a particular situation, are constant velocity is something which can affect laminar flow and it is defined as **Reynolds Number** (Re). Basically, it is representative of ratio of inertial force responsible for flow of liquid and viscous force. This concept was introduced by Stoke, but itwas Osborne Reynolds who popularized use of the concept in 1883. It was named by Arnold Sommerfeld in 1908 as Reynolds number $Re = \frac{\rho v D}{\eta}$. It is seen that – **a)** when $Re < 2000$, liquid flow is steady, **b)** for $2000 < Re < 3000$ flow is unstable i.e. it may suddenly change from steady to turbulent or vice-versa, and **c)** for $Re > 2000$ it is turbulent.

Summary: *It is observed that activities brought out in text books, specially in chapters pertaining to mechanics are affordable. Skipping these activities or just glancing pages containing them are detrimental to the process of building visualization of physics and in turn nature. They must be performed by every student; in case it is not possible then students must be motivated and mentored to perform them in small groups. It will help to create -* ***(a)*** *a strong foundation of physics and thus understanding of nature, and* ***(b)*** *take forward leaps in exploring physics.*

Illustration of Fluid Mechanics in this chapter has been done by drawing examples from real life experience. This is bound to help readers to evolve visualization of concepts and correlate phenomenon occurring around them. Analysis of varieties of problems, representing different situation involve concepts of physics. A deeper journey into the problem solving would make integration and application of concepts intuitive. This is the demand of real life situations which requires multi-disciplinary knowledge and a skill to appreciate multifacets of a problem and evolving an appropriate solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory. It is a process of building an overall personality through academics.

References:

1. *NCERT; PHYSICS, Text Book for Class XI (Part I and II), and Exemplar Problems.*
2. *S.L.Loney;The Elements of Statistics and Dynamics: Part 1 – Statics and Part 2 – Dynamics.*
3. *H.C. Verma; Concepts of Physics, (Vil 1 & 2).*
3. *Resnick, Halliday, Resnick andKrane; Physics (Vol I and II).*
4. *Sears &Zemansky; University Physics with Modern Physics.*
5. *I.E. Irodov; Problems in General Physics*

—00—