

## Illustration of Answers Objective Questions – Kinematics

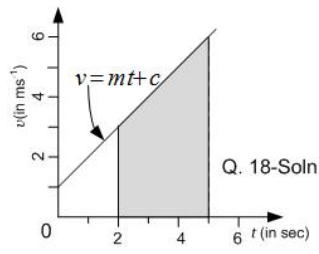
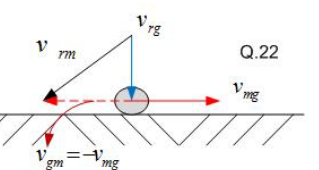
**Note: Students are advised to refer to Question while consulting the illustration of answer not matching to those listed. This shall avoid repetition of question.**

Q-1	<p>Distance travelled due north is <math>OP' = OP \cos 30 = 200 \frac{\sqrt{3}}{2} = 100\sqrt{3}</math>, <b>Answer is Option (b)</b></p>	
Q-2	<p>Given that <math>OA = 30\text{km}</math>, <math>AB = 40\text{km}</math>. Therefore, Distance travelled is <math>OA + AB = (30 + 40)\text{ km} = 70\text{ km}</math>. But displacement is <math>OB = \sqrt{(OA)^2 + (AB)^2} = \sqrt{30^2 + 40^2} = 50\text{ km}</math> and Angle <math>\alpha = \tan^{-1} \left( \frac{AB}{OA} \right) = \tan^{-1} \left( \frac{40}{30} \right) = \tan^{-1} \left( \frac{4}{3} \right)</math>. Since angle is measured in anti-clockwise direction at an angle <math>= \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{4}{3} \right) \right)</math>. <b>Answer is Option (a).</b></p>	
Q-3	<p>Total Displacement <math>\vec{d} = (3.5\hat{i} + 4\hat{j}) + (-4.5\hat{i}) + (-4.5\hat{j}) = (3.5 - 4.5)\hat{i} + (4 - 4.5)\hat{j} = -1\hat{i} - 0.5\hat{j}</math>. <b>Answer is Option (c).</b></p>	
Q-4	<p>From the given function <math>x = f(t)</math> Position, velocity and acceleration is determined successively -</p> <p>(i) Position of particle at <math>t = 5\text{sec}</math> is obtained by substituting value of constants and <math>t</math>. Therefore, <math>x = 2 \cdot 5 + 3 \cdot 5^2 - 1 \cdot 5^3 = 10 + 75 - 125 = -40\text{m}</math>.</p> <p>(ii) Velocity of the particle at <math>t = 5\text{sec}</math> is <math>v = \frac{d}{dt} (2 \cdot t + 3 \cdot t^2 - 1 \cdot t^3) \Big _{t=5} = 2 + 6 \cdot t - 3 \cdot t^2 \Big _{t=5}</math> <math>\rightarrow 2 + 6 \cdot 5 - 3 \cdot 5^2 = 2 + 30 - 75 = -43\text{ms}^{-1}</math></p> <p>(iii) Acceleration <math>t = 5\text{sec}</math> is <math>a = \frac{d}{dt} v \Big _{t=5} = \frac{d}{dt} (2 + 6 \cdot t - 3 \cdot t^2) \Big _{t=5} = (6 - 6 \cdot t) \Big _{t=5} = 6 - 6 \cdot 5</math> <math>= -24\text{ms}^{-2}</math></p> <p><b>Answer is matching with answer in option (d)</b></p>	
Q-5	<p>Since, <math>u</math> is upward and <math>g</math> is downward, hence, we have equation <math>h = ut - \frac{1}{2}gt^2</math>. Further, both <math>u</math> is integer, while <math>h</math> is in given in accuracy of First decimal and hence <math>g = 10</math>, taken to be of lowest accuracy operand i.e. integer with Two SDs. Accordingly, <math>25 = 30 \cdot t - \frac{1}{2} \cdot 10 \cdot t^2</math>. This is a equation of form <math>5 \cdot t^2 - 30 \cdot t + 25 = 0 \rightarrow t^2 - 6 \cdot t + 5 = 0 \rightarrow (t - 1)(t - 5) = 0</math>. Thus possible values of time are <math>= 1\text{ sec } t = 5\text{ sec}</math>. Since, time taken to reach maximum height according to equation <math>v = u - gt \rightarrow 0 = 30 - 10 \cdot t \rightarrow t = 3\text{ sec}</math> and accordingly it will taken another 3 sec to descend to ground, thus total time of flight is 6 sec. Therefore, time (<math>t</math>) for ball to be at height 25 m while descending shall be any where <math>3 &lt; t &lt; 6</math>, i.e. <math>t = 5\text{ sec}</math>. <b>Answer is Option (a)</b></p>	

<p>Q-6</p>	<p>(i) Position of particle at <math>t = 0</math>sec is obtained by substituting value of constants and <math>t = 0</math> in the given expression. Therefore, <math>x _{t=0} = 6 + 2 \cdot 5 + 3 \cdot 5^2 - 1 \cdot 5^3 = 6 + 10 + 75 - 125 = -34</math>m. And <math>x _{t=3} = 6 + 2 \cdot 3 + 3 \cdot 3^2 - 1 \cdot 3^3 = 6 + 6 + 27 - 27 = 12</math>m</p> <p>Therefore, average velocity of the particle is <math>\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x _{t=3} - x _{t=0}}{3-0} = \frac{12 - (-34)}{3} = \frac{46}{3} = 15.3 = 15 \text{ ms}^{-1}</math></p> <p>(ii) Velocity of particle at <math>t = 0</math>sec is <math>v _{t=0} = \frac{dx}{dt} \Big _{t=0} = (2 + 3 \cdot 2t - 1 \cdot 3t^2) \Big _{t=0} = 2 \text{ m} \cdot \text{s}^{-1}</math> and velocity of particle at <math>t = 3</math>sec is <math>v _{t=3} = (2 + 3 \cdot 2t - 1 \cdot 3t^2) \Big _{t=3} = (2 + 6 \cdot 3 - 3 \cdot 9) \text{ m} \cdot \text{s}^{-1}</math>. It comes to <math>-7 \text{ m} \cdot \text{s}^{-1}</math>.</p> <p>Therefore, average acceleration of the particle is <math>\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v _{t=3} - v _{t=0}}{3-0} = \frac{-7-2}{3} = \frac{-9}{3} = -3 \text{ ms}^{-2}</math></p> <p><b>Answer is Option (a)</b></p>
<p>Q-7</p>	<p>Each option shall have to be analyzed-</p> <p>(a) Is not possible since given that <math>v_z(t) = 0</math> and hence, at least one component <math>a_z(t) = \frac{d}{dt} v_z(t) = 0</math>. <b>False</b></p> <p>(b) Since, <math>a_z(t) = \frac{d}{dt} v_z(t) = 0</math>, but whether <math>a_x(t) = 0</math> or <math>a_y(t) = 0</math> would depend upon information of <math>v_x(t)</math> and <math>v_y(t)</math>, which may or may not be Zero. Hence, may have more than one component of acceleration Zero. <b>True</b></p> <p>(c) By definition and illustration at (a) above <math>a_z(t) = 0</math>. <b>True</b></p> <p>(d) This is possible by illustration at (b) above. <b>True</b></p> <p><b>Hence Answer is (b), (c) and (d)</b></p> <p><b>Note:</b> Language of the question needs to be checked to ensure expected answer is singular or plural, or any other nuance.</p>
<p>Q-8</p>	<p>In such a problem each case has to be analyzed as under-</p> <p>(a) When <math>v_x &gt; 0</math>, the object if at <math>x &gt; 0</math>. would be moving away from O, and if at <math>x &lt; 0</math> then towards O. Same is true for <math>v_y &gt; 0</math>. Thus direction of moving of object is dependent on its position which is not defined and hence cannot be definitely stated.</p> <p>(b) When <math>v_x &lt; 0</math>, the object if at <math>x &gt; 0</math>. would be moving towards from O, and if at <math>x &lt; 0</math> then away O. Same is true for <math>v_y &lt; 0</math>. Thus direction of moving of object is dependent on its position which is not defined and hence cannot be definitely stated.</p> <p>(c) It leads to Four cases</p> <ol style="list-style-type: none"> <li>(i) If <math>x &gt; 0</math> and <math>v_x &lt; 0</math>, then both <math>x \cdot v_x &lt; 0</math>, the object would be moving towards O along X axis. The same is true for <math>x &lt; 0</math> and <math>v_x &gt; 0</math>.</li> <li>(ii) If <math>y &gt; 0</math> and <math>v_y &lt; 0</math>, then both <math>y \cdot v_y &lt; 0</math>, the object would be moving towards O along Y axis. The same is true for <math>y &lt; 0</math> and <math>v_y &gt; 0</math>.</li> <li>(iii) If both <math>x \cdot v_x &lt; 0</math> and <math>y \cdot v_y &lt; 0</math>, then object is moving towards O along both X and Y axes</li> <li>(iv) If either of <math>x \cdot v_x &gt; 0</math> or <math>y \cdot v_y &gt; 0</math> then along object is moving along corresponding axis away from O, but definitely towards O on the other axis</li> </ol> <p>(d) It also leads to Four cases</p> <ol style="list-style-type: none"> <li>(i) If <math>x &gt; 0</math> and <math>v_x &gt; 0</math>, or If <math>x &lt; 0</math> and <math>v_x &lt; 0</math>, In either case <math>x \cdot v_x &gt; 0</math> and the object, along X axis, would be moving away from O</li> <li>(ii) If <math>y &gt; 0</math> and <math>v_y &gt; 0</math>, then both <math>y \cdot v_y &gt; 0</math>, the object, along Y axis, would be moving away from O.</li> <li>(iii) If both <math>x \cdot v_x &gt; 0</math> and <math>y \cdot v_y &gt; 0</math>, then object is moving away from O along both X and Y axes</li> <li>(v) If either of <math>x \cdot v_x &lt; 0</math> but <math>y \cdot v_y &gt; 0</math> then is moving along O only along X-axis and if <math>x \cdot v_x &gt; 0</math> but <math>y \cdot v_y &lt; 0</math> the object would be moving along O only Y-axis.</li> </ol> <p><b>The with certainty it is only case (c) when object would be moving towards O, i.e, Option (c).</b></p>

Q-9	<p>Each option is being analyzed separately -</p> <p>(a) Since, <math>v_z(t) = \frac{d}{dt} r_z(t) \rightarrow \int v_z(t) dt = r_z(t) + C \rightarrow 0 = r_z(t) + C</math> or <math>r_z(t) = C_z</math> which may be zero if <math>C_z = 0</math> or may not be zero if <math>C_z \neq 0</math>. Thus, it is per sure that at least one component <math>r_z(t)</math> and may or may not be identically zero. Since, there is uncertainty in this answer, hence it is <b>True</b>.</p> <p>(b) Since, from (a) above, <math>r_z(t)</math> may be identically zero only when <math>C_z = 0</math>. Likewise value of components <math>r_x(t) = 0</math> and <math>r_y(t) = 0</math> would depend upon information of <math>v_x(t)</math> and <math>v_y(t)</math>, which may or may not be Zero. Hence, may have more than one component of acceleration Zero. <b>True</b></p> <p>(c) By definition and illustration at (a) above uncertainty for <math>r_z(t)</math> to be zero depends upon value of <math>C_z</math> and hence it is <b>False</b></p> <p>(d) This is not possible by illustration of uncertainty at (c) above. <b>False</b></p> <p><b>Answer is Option (a) and (b)</b></p>
Q-10	<p>Average speed <math>\bar{s} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{30+30}{3+3} = 10 \text{ ms}^{-1}</math>, since distance is scalar it is +ve and added.</p> <p>Average velocity <math>\bar{v} = \frac{\bar{y}_f - \bar{y}_i}{t_f - t_i} = \frac{0-0}{6-0} = 0 \text{ ms}^{-1}</math>. Since ball starts from ground and returns to ground hence <math>\bar{y}_f = \bar{y}_i = 0</math>, while <math>t_i = 0</math> sec and <math>t_f = 3 + 3 = 6</math> sec.</p> <p><b>Thus answer is Option(b)</b></p>
Q-11	<p>In this problem taking each option and verify which of them do not satisfy given condition</p> <p>(a) <math>\frac{dx}{dt} &lt; 0</math>, if point is at <math>x &gt; 0</math>, then it is moving towards O, but as soon as it crosses O it will start moving away from O, Hence this is not valid</p> <p>(b) <math>\frac{dx}{dt} &gt; 0</math>, if point is at <math>x &lt; 0</math>, then it is moving towards O, but as soon as it crosses O it will start moving away from O, Hence this is not valid</p> <p>(c) <math>\frac{dx^2}{dt} &lt; 0</math>, in this case irrespective of <math>0 &lt; x &lt; 0, x^2 &gt; 0</math> and <math>\frac{dx^2}{dt} = 2x \cdot \frac{dx}{dt}</math>. It leads to Two cases –</p> <p>a. If <math>x &gt; 0</math> and point is moving towards O as given, then <math>\frac{dx}{dt} &lt; 0</math> and hence <math>x \cdot \frac{dx}{dt} &lt; 0</math></p> <p>b. If <math>x &lt; 0</math> and point is moving towards O as given, then <math>\frac{dx}{dt} &gt; 0</math> and hence <math>x \cdot \frac{dx}{dt} &lt; 0</math></p> <p><b>Thus in either situation i.e. at <math>0 &lt; x &lt; 0</math> the particle is moving towards O.</b></p> <p>(d) <math>\frac{dx^2}{dt} &gt; 0</math>, in this case irrespective of <math>0 &lt; x &lt; 0, x^2 &gt; 0</math> and <math>\frac{dx^2}{dt} = 2x \cdot \frac{dx}{dt}</math>. It leads to Two cases –</p> <p>a. If <math>x &gt; 0</math> and point is moving towards O as given, then <math>\frac{dx}{dt} &lt; 0</math> and hence <math>x \cdot \frac{dx}{dt} &lt; 0</math></p> <p>b. If <math>x &lt; 0</math> and point is moving towards O as given, then <math>\frac{dx}{dt} &gt; 0</math> and hence <math>x \cdot \frac{dx}{dt} &lt; 0</math></p> <p>Since, in either situation i.e. at <math>0 &lt; x &lt; 0</math> when the particle is moving towards O, <math>\frac{dx^2}{dt} = 2x \frac{dx}{dt} \neq 0</math>. Hence this case is not valid.</p> <p><b>Answer is Option (c).</b></p>
Q-12	<p>Problem states only <math>x(t) &gt; 0</math> and nothing more and hence we have to consider <math>x = kt^n</math> with various possibilities as under -</p> <p>a. <math>n = 0</math>, then <math>\frac{dx}{dt} = 0</math>, this is not valid since given that <math>\frac{dx}{dt} &gt; 0</math> and <math>x &gt; 0</math></p> <p>b. <math>1 &gt; n &gt; 0</math>, then <math>\frac{dx}{dt} = nkt^{n-1}</math>, and <math>\frac{x}{t} = \frac{kt^n}{t} = kt^{n-1}</math>s, it leads to <math>\frac{dx}{dt} &lt; \frac{x}{t}</math>.</p> <p>c. <math>n = 1</math> then <math>\frac{dx}{dt} = k</math>, and <math>\frac{x}{t} = \frac{kt}{t} = k</math>, it leads to <math>\frac{dx}{dt} = \frac{x}{t}</math>.</p> <p>d. <math>n &gt; 1</math> then <math>\frac{dx}{dt} = nkt^{n-1}</math>, and <math>\frac{x}{t} = \frac{kt^n}{t} = kt^{n-1}</math>, it leads to <math>\frac{dx}{dt} &gt; \frac{x}{t}</math>.</p> <p><b>Thus possible cases satisfying given conditions are <math>\frac{dx}{dt} (&gt; \text{OR} = \text{OR} &lt;) \frac{x}{t}</math>, i.e. Option (d)</b></p>

Q-13	<p>In each case eventually it leads to variation of <math>x</math> w.r.t. <math>t</math>. and accordingly abscissa is identified with <math>t</math> and ordinate is identified with <math>x</math>. Thus, each case is being analyzed separately as under -</p> <p>Case (i): Constant speed <math>s = \frac{dx}{dt}</math>, since object is moving <math>x</math> cannot be constant with varying time in <math>x - t</math> graph. Since, no other information is given about the path of moving particle, it shall be constant velocity along (+)ve X-axis [<b>Graph A</b>], (-)ve X-axis [<b>Graph C</b>]. Thus graph A and C match with this case.</p> <p>Case (ii): Given that <math>a = \frac{d^2x}{dt^2} = 3t \rightarrow \frac{dx}{dt} = \frac{3}{2}t^2 + C_1 \rightarrow x = \frac{3}{6}t^3 + C_1t + C_2</math>. This is cubic equation and for all (+)ve values of <math>t</math> displacement will continue to rise till infinity. Thus best representative <b>graph is D</b>.</p> <p>Case (iii): This case is similar to displacement particle thrown vertically upward, with acceleration due to gravitation (<math>g</math>) acting in (-)ve direction with respect to initial velocity represented by equation <math>x = ut - \frac{1}{2}gt^2</math>. It will lead to a parabolic curve and is best represented by <b>graph E</b>.</p> <p>Case (iv): Given graph E represents <math>x - t</math> curve, then for <math>v - t</math> curve <math>\frac{d}{dt}x = \frac{d}{dt}\left(ut - \frac{1}{2}gt^2\right)</math>. It leads to <math>v = u - gt</math>, this is equation of a line with point of inflection at <math>t = \frac{u}{g}</math>. The <b>graph C</b>, best represents this case.</p> <p><b>In Answer Matching of graph is consolidated.</b></p>
Q-14	<p>Acceleration <math>a = \frac{dv_x}{dt} = C \neq 0</math>, given that <math>C</math> is constant. Since object is moving in <math>x</math> direction for <math>t &gt; 0</math>. It leads to two cases -</p> <p>Case 1: If <math>C &gt; 0</math>, then <math>v_x(t) &gt; 0 _{t&gt;0}</math> hence <math>v_x \cdot \frac{dv_x}{dt} &gt; 0</math> i.e. <b>+ve</b>.</p> <p>Case 2: If <math>C &lt; 0</math>, then <math>v_x(t) = 0 + C \cdot t = C \cdot t &lt; 0 _{t&gt;0}</math>. Since, object is moving in <math>x</math> direction hence necessarily <math>v_x(t) &gt; 0</math>, therefore <math>\frac{dv_x}{dt} \not\leq 0</math> or <math>C \not\leq 0</math>. Thus this case is not valid with given criteria.</p> <p><b>Answer is Option (c).</b></p>
Q-15	<p>This is a case of free fall under gravity. In this case total distance traversed during fall in <math>t = n</math> sec is <math>s_n = 0 \cdot n + \frac{1}{2}g \cdot n^2 = \frac{g}{2} \cdot n^2</math>, and total distance traversed during fall in <math>(n + 1)</math> sec is <math>s_{n+1} = 0 \cdot (n + 1) + \frac{1}{2}g \cdot (n + 1)^2 = \frac{g}{2} \cdot (n + 1)^2</math>. Thus, distance traversed in last <math>(n + 1)^{th}</math> sec is <math>\Delta s_{n+1} = s_{n+1} - s_n = \frac{g}{2}((n + 1)^2 - n^2) = \frac{g}{2}(2n + 1)</math>. Thus for <math>n = 1, \Delta s_{n+1} = \frac{g}{2} \cdot 3</math> and <math>s_n = \frac{g}{2} \cdot 1</math>, thus <math>\Delta s_{n+1} &gt; s_n</math>. For <math>n = 2, \Delta s_{n+1} = \frac{g}{2} \cdot 5</math> and <math>s_n = \frac{g}{2} \cdot 4</math>, thus <math>\Delta s_{n+1} &gt; s_n</math>. For <math>n = 3, \Delta s_{n+1} = \frac{g}{2} \cdot 7</math> and <math>s_n = \frac{g}{2} \cdot 9</math>, thus <math>\Delta s_{n+1} &lt; s_n</math>. Thus point of inflection of the relationship occurs between 2 sec and 3 sec.</p> <p><b>Hence, answer shall be Option (b)</b></p>
Q-16	<p>Given the sign convention that (+)ve is upward and hence, all the variable pointing upward shall be (+)ve (<math>x, v</math> and <math>a</math>) <math>&gt; 0</math>. Likewise, variables pointing downward shall be (-)ve (<math>x, v</math> and <math>a</math>) <math>&lt; 0</math>. In the instant problem magnitude of <math>a_y</math> is in question w.r.t. sign convention which is (-)ve irrespective of direction of travel be it upward or downward. Thus On the way up <math>a_y &lt; 0</math>, On the way down <math>a_y &lt; 0</math>.</p> <p><b>Answer is option (d)</b></p>
Q-17	<p>This is a case of absolute acceleration and this is due gravity. Since, the ball is projected upward, i.e. in (=)ve direction, the acceleration due to gravity shall be downward i.e. <math>(-)9.8ms^{-2}</math>.</p> <p><b>Hence, answer is Option (d)</b></p>

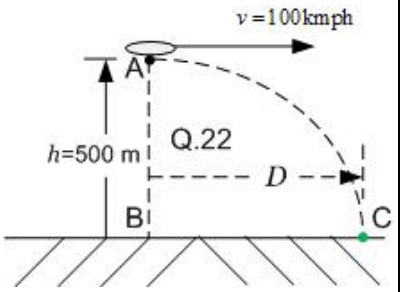
<p>Q-18</p>	<p>Motion of a particle in straight line can be expressed on <math>v - t</math> graph as <math>v = mt + c</math>. In this case <math>c = 1</math>. The intercept on <math>v</math>-axis. Slope of line <math>m = \frac{v_5 - v_2}{5 - 2} = \frac{6 - 3}{5 - 2} = \frac{3}{3} = 1</math>. Distance (<math>s</math>) traversed by the particle is <math>= \int_2^5 v dt = \int_2^5 (t + 1) dt = \left[ \frac{t^2}{2} + t \right]_2^5 = \left[ \left( \frac{5^2}{2} + 5 \right) - \left( \frac{2^2}{2} + 2 \right) \right] = \left[ \left( \frac{5^2}{2} + 5 \right) - \left( \frac{2^2}{2} + 2 \right) \right]</math> ., and is shown in gray colour in the graph. This resolves into <math>s = \left[ \frac{1}{2} (25 - 4) + 3 \right] = [10.5 + 3] = 13.5 \approx 14</math> m. Here, principle of rounding of digits is applied while reporting final result. <b>Hence, answer is Option (b).</b></p>	
<p>Q-19</p>	<p>Equation of motion gives <math>v _t = u + at</math>, therefore, <math>v _3 = 5 + 0.75 \times 3 = 7.25</math> and at a later time <math>v _5 = 5 + 0.75 \times 5 = 8.75</math>. Therefore, distance travelled can be found by formula <math>v^2 = u^2 + 2as \rightarrow s = \frac{v^2 - u^2}{2a} = \frac{(8.75)^2 - (7.25)^2}{2 \times 0.75} = \frac{(8.75 - 7.25)(8.75 + 7.25)}{1.5} = \frac{1.5 \times 16}{1.5} = 16</math> m. <b>Hence, answer is Option (a).</b></p>	
<p>Q-20</p>	<p>Given that <math>u = 15 \text{ ms}^{-1}</math>, and at maximum height final velocity is <math>v = 0 \text{ ms}^{-1}</math>. Therefore, <math>h = \frac{v^2 - u^2}{2(-g)} = \frac{0 - 15^2}{2(-10)}</math>. Since, given data is for Two SDs, and hence <math>g = 10 \text{ ms}^{-2}</math>, i.e. with Two SDs. Accordingly, <math>h = \frac{225}{20} = 112.5 \approx 110</math> m. <b>Hence, answer is Option (c).</b></p>	
<p>Q-21</p>	<p>Initial velocity of nut-bolt in vertical direction is <math>u_v = 2 \sin 45^\circ = \sqrt{2}</math>, and in horizontal direction is <math>u_h = 2 \cos 45^\circ = \frac{2}{\sqrt{2}} = \sqrt{2}</math>. The object is experiencing an acceleration <math>g = -10 \text{ ms}^{-2}</math>. This value of <math>g</math> is taken based on SDs of the given data with a (-)ve sign. The object reaches ground at a height <math>-10\text{m}</math>, the (-)ve sign is assigned to depth below the point of projection. Therefore, to determine distance of fall from stand there are several ways. Here it is decided to determine vertical velocity of the nut-bolt when it touches ground, and it would be <math>v_v^2 = u_v^2 + 2(-g)(-h) = 2 + 2 \times 10 \times 1.5 = 32 \rightarrow v_v = \pm 4\sqrt{2}</math> m. Since, while touching ground vertical velocity is downward and hence it will be <math>v_v = -4\sqrt{2}</math>. Thus time taken by the object to touch ground will be <math>v_v = u_v + (-g)t \rightarrow -4\sqrt{2} = \sqrt{2} - 10 \times t</math>. It leads to <math>-5\sqrt{2} = -10 \times t \rightarrow t = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}</math> sec. Since, travel of object in horizontal is free of acceleration i.e. <math>a_h = 0 \text{ ms}^{-2}</math>, therefore, total horizontal distance travelled is <math>s = u_h \times t = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1\text{m}</math>. Hence answer is <b>option (b)</b>. <b>Note:</b> It is not essential to spend time in remembering formulae for various cases. Any problem can be solved by starting with basic equations. <u>Normally such problems involve quantities causing minimum calculations.</u> With practice, all associated problems become intuitive and it becomes possible to handle any twist in problem.</p>	
<p>Q-22</p>	<p>In this velocities of man and rain drops are given with respect to ground. And speed of rain drops w.r.t. man is to be determined, i.e. its magnitude. Therefore, <math>\vec{v}_{rm} = \vec{v}_{rg} + \vec{v}_{gm} = \vec{v}_{rg} - \vec{v}_{mg}</math>. Thus <math>v_{rm} =  \vec{v}_{rm}  = \sqrt{v_{rm}^2 + v_{mg}^2 + 2v_{rm} \cdot v_{mg} \cdot \cos \theta}</math>, here <math>\theta = 90^\circ</math> i.e. angle between direction of rain drop w.r.t ground and direction of man w.r.t. ground. This leads to <math>v_{rm} = \sqrt{v_{rm}^2 + v_{mg}^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5</math>. Since <math>\sqrt{25}</math> is a surd, therefore, its value</p>	

is (+)ve. **Answer is Option (c).**

Q-23 Slope of  $v - t$  graph, which are line segments in zones  $0 \leq t \leq 2$ ,  $4 \leq t \leq 6$ , is  $m = \frac{15}{2} \text{ms}^{-2}$  and in zones  $2 \leq t \leq 4$ ,  $6 \leq t \leq 8$ , is  $m = -\frac{15}{2} \text{ms}^{-2}$ . This slope  $m = \frac{dv}{dt}$ , and is nothing but acceleration of the ball along the curve. Therefore, equation of line segments shall be  $v - v_1 = m(t - t_1) \rightarrow v = m(t - t_1) + v_1$  with corresponding values if the variables. Distance covered by ball during  $0 \leq t \leq 4$  is  $s = \int_0^4 v dt = \int_0^2 v dt + \int_2^4 v dt$ . This is equal to area under of  $v - t$  graph i.e. of triangle  $0 \leq t \leq 4$  sec  $= \frac{1}{2} \times 4 \times 15 = 30\text{m}$ . And change of acceleration at  $t = 6$  sec is  $= a_{6+} - a_{6-} = m_{6+} - m_{6-} = \left(-\frac{15}{2}\right) - \frac{15}{2} = -15 \text{ms}^{-2}$ .

**Answer is Option (a)**  
**Note:** Here integration can be avoided by replacing it with area of corresponding triangles represented by the graph. While applying mathematics in physics, using appropriate mathematical formulation helps to gain speed, which is crucial in examinations.

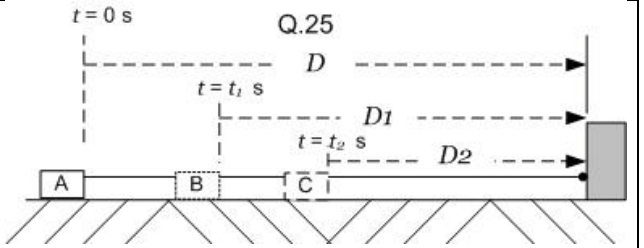
Q-24 Velocity of plane in SI is  $v = \frac{150 \times 1000}{60 \times 60} = 41.7 \text{ms}^{-1}$ . Shadow of plane on the ground is at B, vertically below the plane. Time taken by packet to reach ground  $h = u \cdot t + \frac{1}{2}gt^2 = \frac{g}{2}t^2$ , since at the time of drop vertical velocity of packet is Zero, while acceleration due to gravity  $g = 5\text{ms}^{-2}$  velocity. In this case both  $h$  and  $g$  vertically downward are taken to be (+)ve. Therefore,  $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 125}{10}} = 5$  sec, Therefore, distance of fall of packet shall be  $D = vt = 41.7 \times 5$  m. It works out to  $D = 208.5 \text{m} \approx 210 \text{m}$ . This is in conformance with the principles of SDs. **Answer is Option (c)**



Q-25 Part (i) – Velocity of car w.r.t. ground is  $a$  kmph. And velocity of insect while flying from wall towards car is  $(-b)$  kmph, and while flying from car towards wall is  $b$  kmph. Since insect continues to fly, without stop at  $b$  kmph, till car take a sharp turn at the wall. Hence, time of fly of insect ( $T$ ) is equal to time taken by car to cover a distance to reach the wall. Thus,  $T = \frac{D}{a}$  hrs, and distance covered by fly is  $= b \times \frac{D}{a} \text{ km} = D \left(\frac{b}{a}\right) \text{ km}$ .

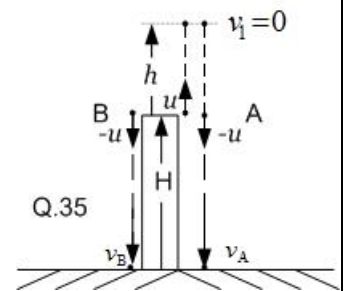
Part (ii) – When both are flying towards each other in opposite direction, and they meet after time  $t'$  then  $a \times t_1 + b \times t_1 = D \rightarrow t_1 = \frac{D}{a+b}$  hrs and car reaches position B at a distance from the wall  $D_1 = D - a \times t_1 \rightarrow D - a \left(\frac{D}{a+b}\right) = \frac{bD}{a+b}$ . Then, insect starts flying back towards wall with a speed  $b$  kmph and takes time  $t_2 = \left(\frac{bD}{a+b}\right) \times \frac{1}{b} = \left(\frac{D}{a+b}\right)$  hrs, and car reaches at position C having travelled another distance  $a \times t_2$  km. Thus car at position C is away from the wall  $D_2 = D_1 - a \times \left(\frac{D}{a+b}\right) = \frac{bD}{a+b} - \frac{aD}{a+b} = \left(\frac{b-a}{b+a}\right) D$ .

It is seen that at start of first trip distance between car and wall is  $D$  km, at start of 2<sup>nd</sup> trip it is  $\left(\frac{b-a}{b+a}\right) D$  and it keep on reducing by a factor  $\left(\frac{b-a}{b+a}\right)$  in every successive trip till it reduces to ZERO. It becomes a convergent geometric series  $Dr^n$  such that  $Dr^n \rightarrow 0 |_{r < 1 \text{ and } n \rightarrow \infty}$ . In this case  $b > a$  both are of (+)ve magnitude and hence  $r = \frac{b-a}{b+a} < 1$ . Thus the only necessary condition for distance between car and insect reducing to ZERO is  $n \rightarrow \infty$ , i.e. **it will take infinite trips. Thus answer is Option (d)**

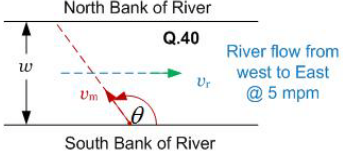
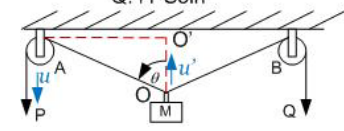


<p>Q-26</p>	<p>Slope of line <math>m = \tan(-\theta) = -\tan \theta</math>. Hence equation of line OP is <math>y = -\tan \theta \cdot x</math>. Travel of particle along X-axis shall be <math>x = ut</math>, since there is no acceleration along it. But travel of particle along Y-axis shall be <math>y = 0 + \frac{1}{2}(-g) \cdot t^2</math>. Therefore, trajectory of particle in X-Y plane can be defined by eliminating <math>t</math> in equations of <math>x</math> and <math>y</math> as <math>y = -\frac{g}{2} \cdot \left(\frac{x}{u}\right)^2</math>. Thus coordinates of A can be determined by point of intersection of the trajectory and the line, using their equations respectively. Thus, <math>y = -\tan \theta \cdot x = -\frac{g}{2} \cdot \left(\frac{x}{u}\right)^2 \rightarrow x = \frac{2 \tan \theta u^2}{g}</math>. Therefore, corresponding value of <math>y = -\tan \theta \left(\frac{2 \tan \theta u^2}{g}\right) = -\frac{2 \tan^2 \theta}{g} \cdot u^2</math>. Thus length of span over inclined plane <math>l = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{\left(\frac{2u^2 \tan \theta}{g}\right)^2 + \left(-\frac{2 \tan^2 \theta}{g} \cdot u^2\right)^2} = \frac{2u^2 \tan \theta}{g} \sqrt{1 + \tan^2 \theta} = \frac{2u^2 \tan \theta}{g} \sec \theta</math>. It resolves into, <math>l = \frac{2u^2}{g} \tan \theta \cdot \sec \theta = \frac{2u^2}{g} \sin \theta</math>. Answer is <b>Option (c)</b>.</p> <p><b>Note: Solution of problem becomes simple using Coordinate Geometry, i.e. equation of line, curve and point of intersection,</b></p>	
<p>Q-27</p>	<p>Let object start a free fall at the instance bullet is fired at it horizontally. Since initial horizontal velocity of bullet and object <math>u = 0 \text{ ms}^{-1}</math>, both are experiencing an acceleration due to gravity. Let in time <math>t</math> the bullet hits the object, during which free fall of the object is <math>h = 0 \cdot t + \frac{1}{2}g \cdot t^2 \rightarrow t = \sqrt{\frac{2h}{g}}</math>. The Bullet will also descend through same height <math>h</math>. For the bullet to be able to hit the object, necessary condition is <math>= \frac{d}{u}</math>, since the bullet is not experiencing any acceleration in horizontal direction. Thus equating <math>t = \frac{d}{u} = \sqrt{\frac{2h}{g}} \rightarrow d = u \cdot \sqrt{\frac{2h}{g}}</math>, a necessary condition for the ball to hit the bullet. <b>Answer is Option (d)</b></p>	
<p>Q-28</p>	<p>This is a problem of type pursuit where particle A is pursuing B and the sequence continues till the converge. Speed of approach of particles A and B separated at a distance <math>a</math> is sum of components of speed of respective particles along the line of separation at any instant. Initially it is <math>v_a =  (v + v \cos 90^\circ)  = v</math>. Accordingly, time of convergence <math>t = \frac{a}{v}</math> sec. <b>Answer is Option (d)</b></p>	
<p>Q-29</p>	<p>Speed of car before change of speed at the instance <math>t</math> is <math>v_{t-} = 50\hat{j}</math> and speed after turn is <math>v_{t+} = -50\hat{i}</math>. Therefore, net change of speed at the instance is <math>\Delta v = v_{t+} - (v_{t-}) = -50\hat{i} - 50\hat{j} = 50 \times \sqrt{2} \angle 225^\circ</math>. It calculates to 70 kmph and graphically in direction south-west. <b>Answer is Option (b)</b></p>	
<p>Q-30</p>	<p>Rate of change of distance from initial position during <math>0 \leq t \leq t_1</math> is <math>\frac{dx}{dt} = \frac{\Delta x}{\Delta t} = \frac{a}{t_1}</math> is constant as graph is a straight line. And during <math>t &gt; t_1</math> the graph is parallel to <math>t</math>-axis and the equation of graph for that portion is <math>x = a</math>, i.e. the particle stops. Hence, <b>Answer is Option (d)</b></p>	
<p>Q-31</p>	<p>Let position of the particle at time <math>t_0 = 0</math> sec be A <math>(x_A, y_A)</math>. Initial velocity <math>u</math> is parallel to X-axis, and acceleration (let it be <math>-a</math>) acting parallel to X-axis, but it is retarding i.e. in opposite direction indicated by (-)ve sign. Therefore, it is only X-coordinates of point would change with time. Since, acceleration is acting against velocity, therefore, for any position of particle B at <math>t &gt; 0</math> the X-coordinates <math>x_B &lt; x_A</math>.</p>	

	<p>What is asked is to compare magnitude displacement in first 10 Sec (<math>x_A</math>) and next 10 sec (<math>x_B</math>). Here, value of X-coordinate at first 10<sup>th</sup> sec i.e. <math>t_1 = t_0 + \Delta t = 0 + 10 = 10</math> secs (<math>x_1</math>) is <math>x_1 = x_0 - (ut_1 - at_1^2) = x_0 - 10u + 100a = 100a - 10u</math>. And value of X-coordinate in next <math>\Delta t = 10</math> secs i.e. (<math>t_2 = t_1 + \Delta t = 10 + 10 = 20</math>) (<math>x_2</math>) is <math>x_2 = x_0 - (ut_2 - at_2^2)</math>. This resolves to <math>x_3 = 0 - 20u + 400a = 400a - 20u</math>. Thus, magnitude of displacement in <math> x_A  =  x_2 - x_1  =  100a - 10u - x_1 </math> and <math> x_B  =  x_3 - x_1  =  400a - 20u - x_1 </math></p> <p>Thus magnitude of displacement, with respect to <math>x_1</math> (the reference point) would depend upon value of <math>u</math> and <math>a</math>, which are not known, and it would not be possible to compare <math> x_A </math> and <math> x_B </math>. It implies information is incomplete to answer question. <b>Answer is Option (d)</b></p>
Q-32	<p>Let the time of two travels be <math>t</math> and hence distance travelled at velocity <math>v_1</math> will be <math>x_1 = v_1 t</math>, and distance travelled at velocity <math>v_2</math> will be <math>x_2 = v_2 t</math>. Accordingly, average velocity <math>\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_1 + x_2}{t + t} = \frac{v_1 t + v_2 t}{2t} = \frac{v_1 + v_2}{2}</math>. <b>Hence, answer is Option (a).</b></p>
Q-33	<p>Let the distance of two travels be <math>x</math> and hence time of travel with velocity <math>v_1</math> will be <math>t_1 = \frac{x}{v_1}</math>, and another time of travel with at velocity <math>v_2</math> will be <math>t_2 = \frac{x}{v_2}</math>. Accordingly, average velocity <math>\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x + x}{t_1 + t_2} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}}</math>. This can be transformed into <math>\frac{2}{\bar{v}} = \frac{1}{v_1} + \frac{1}{v_2}</math>. <b>Hence, answer is Option (c).</b></p>
Q-34	<p>The moment stone is released from elevator, its velocity (<math>v</math>) is same as that of the escalator at the time of release. But, after that it makes a free fall with initial velocity (<math>v</math>), but the only acceleration acting on it that due to gravity i.e. <math>g</math> downward. <b>Hence answer is Option (d)</b></p>
Q-35	<p>The ball A and B are thrown with velocity (+)<math>u</math> m/sec and (-)<math>u</math> m/sec, upward and downward, respectively. All upward and downward directions are (+ve) and (-)ve respectively. The ball A rises to height <math>h = \frac{u^2}{2g}</math>, and when it descends to the point of throw, same as that of ball B, it acquires velocity (-)<math>u</math> m/sec. Now the velocity of both the balls at height <math>-h</math> from the initial point, when it reaches the ground, would be <math>v^2 = u^2 + 2(-g)(-H) = u^2 + 2gH</math>. Therefore velocity of ball at ground would be, <math>u = \sqrt{v^2 + 2gH}</math>. Since, velocity of ball (<math>= -u</math>), depth of travel (<math>= -H</math>) and acceleration (<math>= -g</math>) are all in same direction for both balls and hence <math>v_A = v_B</math>. <b>Answer is Option(c).</b></p>
Q-36	<p>In projectile motion velocity <math>\vec{v} = v_x \hat{i} + v_y \hat{j}</math> is always tangential to its trajectory. Therefore, direction of velocity of projectile, an angle (<math>\alpha</math>) with horizontal, is such that <math>\tan \alpha = \frac{v_y}{v_x}</math>. Therefore, acceleration (<math>g</math>) to be perpendicular to <math>\vec{v}</math>, necessary condition is <math>\tan \alpha = \frac{v_y}{v_x} = 0 \rightarrow v_y = 0</math>.</p> <p>And acceleration is due to gravity (<math>\vec{a} = -g\hat{j}</math>) which is always directed vertically downward towards. At highest point in the trajectory of the projectile <math>v_y = 0</math>, while, <math>v_x = V \cos \theta</math> remains constant. At all other point on the trajectory <math>v_x \neq 0</math>. Thus, it is only the highest point of the projectile where its velocity <math>\vec{v} = v_x \hat{i}</math> while the acceleration <math>\vec{a} = -g\hat{j}</math> where <math>\vec{v}</math> and <math>\vec{a}</math> are mutually perpendicular.</p> <p><b>Hence answer is Option (c).</b></p>
Q-37	<p>Since both the bullets are fired horizontally, simultaneously and from same place, their initial vertical velocities are ZERO, and they have to traverse same height to hit the ground. Therefore, the time would be from <math>h = 0 \cdot t + \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}</math>, would be same. <b>Hence, answer is Option (c).</b></p>





Q-38	<p>Let <math>u</math> is the velocity of the projectile fired at an angle <math>\theta = 15^\circ</math>, has a range 50 m. Then vertical velocity is <math>u_y = u \sin \theta</math>. Therefore, time of flight would be <math>-u \sin \theta = u \sin \theta - gt \rightarrow t = \frac{2u \sin \theta}{g}</math>. Accordingly, range of projectile would be <math>R = u_x \cdot t = u \cos \theta \cdot \frac{2u \sin \theta}{g} = \frac{u^2 \sin 2\theta}{g} = \left  \frac{u^2}{g} \cdot \frac{1}{2} \right _{2\theta=30^\circ} = 50 \rightarrow u^2 = 100g</math>. Now keeping <math>u</math> to be same when it is fired at an angle <math>\theta' = 45^\circ</math> thus, new range would be <math>R' = \left  \frac{u^2 \sin 2\theta'}{g} \right _{\theta'=45^\circ} = \frac{u^2}{g} = \frac{1}{g} \cdot 100g = 100 \text{ m}</math>. <b>Answer is Option (d).</b></p>	
Q-39	<p>It has been seen in the Illustration that range of a projectile is <math>R = \frac{u^2 \sin 2\theta}{g}</math>. Accordingly, for projectile A and B, their ranges would be <math>R_A = \frac{u_A^2 \sin 2\theta}{g}</math> and <math>R_B = \frac{u_B^2 \sin 2\theta}{g}</math>. Despite angle of projection known, the other variable in range is <math>u_A</math> and <math>u_B</math>, which are unknown. Hence, ranges of Two projectiles cannot be compared. <b>Answer is Option (d).</b></p>	
Q-40	<p>The stipulations of the problem are shown in the figure. To cross the river at a given speed <math>v_m</math> its component across the river (<math>v_m \sin \theta</math>) is decisive and time taken by it would be <math>t = \frac{w}{v_m \sin \theta}</math>. For minimum possible time either <math>\frac{dt}{d\theta} = 0</math>, or component of velocity across the river should be maximum i.e. <math>\frac{d}{d\theta}(v_m \sin \theta) = 0</math>. The later case is mathematically simple since <math>v_m</math> is a given constant. Therefore, for <math>\frac{d}{d\theta} \sin \theta = \cos \theta = 0</math>, the condition is the <math>\theta = \pm \frac{\pi}{2}</math>. <math>\theta \neq -\frac{\pi}{2}</math>, since it amount to traversing against North bank on land. Hence, <math>\theta = \frac{\pi}{2}</math>, i.e. northward. <b>Answer is Option (a).</b></p>	
Q-41	<p>Given that end P is moving with a velocity <math>u = -\frac{dy}{dt}</math>, here <math>y</math> is vertical descend of point P. Let at any point of time length of string AO be <math>l</math> and makes an angle <math>\theta</math> with the vertical and <math>\frac{dy}{dt} = \frac{dl}{dt} = -u</math>. Now in triangle AOO', <math>l^2 = (AO')^2 + (OO')^2</math>. Since, pulleys are fixed and hence <math>AO'</math> is constant, on differentiating the Pythagorean identity <math>2l \frac{dl}{dt} = 2(OO') \frac{d}{dt}(OO') = 2l(-u) = 2l \cos \theta \cdot u'</math>. It simplifies into <math>u' = -\frac{u}{\cos \theta}</math>. Since <math>u</math> is downwards, hence (-)ve sign signifies Point O moving upwards. Since Point O is fixed to mass rigidly and its velocity is that of mass M.</p> <p><b>Hence, answer is Option (b)</b></p>	

Q-42	<p>Each option is being analyzed since it is of type multiple choice.</p> <p>(i) Since tip of the minute hand of a clock reaches back to the same position after one hour. Hence, its displacement <math>\Delta s = 0</math> in one hour. <b>Option (a) is correct.</b></p> <p>(ii) Distance covered is <math>\Delta x = 2\pi l</math>, here <math>l</math> is the length of minute hand hence <math>\Delta x = 2\pi l \neq 0</math></p> <p>(iii) Speed of the tip <math>\frac{\Delta x}{60 \times 60} \neq 0</math>, since numerator <math>\Delta x \neq 0</math></p> <p>(iv) Average velocity of tip <math>\bar{v} = \frac{\Delta s}{60 \times 60} = 0</math>, since <math>\Delta s = 0</math> <b>Option (d) is correct.</b></p> <p><b>Answer- Options (a) and (d) are correct</b></p>
Q-43	<p>Each option is being analyzed since it is of type multiple choice.</p> <p>(i) <math>u_0 = \frac{dx}{dt} = \frac{d}{dt} u(t-2) + a(t-2)^2 = u + 2a(t-2) = u - 2a _{t=0} \neq u</math>. Hence this option is <b>incorrect</b></p> <p>(ii) Acceleration of the particle is <math>\frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d}{dt} (u + 2a(t-2)) = 2a \neq a</math>. Hence this option is <b>incorrect</b></p> <p>(iii) During verification of option (b) at (ii) above it is found that <math>\frac{d}{dt} \left( \frac{dx}{dt} \right) = 2a</math>. <b>This option is correct</b></p> <p>(iv) <math>x = u(t-2) + a(t-2)^2 = u(2-2) + a(2-2)^2 _{t=2} = 0</math> I.e, particle is at origin, <b>Option (d) is correct.</b></p> <p><b>Answer-Options (c) and (d) are correct</b></p>
Q-44	<p>Each option is being analyzed since it is of type multiple choice.</p> <p>(i) Shortest distance between two points A and B is the length of the line joining them. If a particle is travelling only in one direction along the line then average speed <math>= \frac{\Delta x}{\Delta t}</math> and so also average velocity <math> \vec{v}  = \frac{\Delta x}{\Delta t}</math>. The moment particle travels either to-or-fro or deviates from straight line before it reaches from A to B, average speed <math>&gt;  \vec{v} </math>. <b>The Option (a) is correct</b></p> <p>(ii) At highest point in trajectory of projectile, <math> \vec{v} _{0+} =  \vec{v} _{0-}</math> this makes <math>\frac{d}{dt}  \vec{v}  = 0</math>, but acceleration <math>a = g = \left  \frac{d\vec{v}}{dt} \right  \neq 0</math>. <b>Thus Option (b) is also correct</b></p> <p>(iii) Given that <math>\bar{v} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = 0</math>, it does not warrantee that <math>\vec{v}_t \neq 0 _{t_1 &lt; t &lt; t_2}</math>. <b>Hence Option (c) is also correct.</b></p> <p>(iv) This case is different from (c) since motion is restricted on a straight line, which is not the case in case (c). In this case it is given that <math>\bar{v} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = 0</math> and that infinite acceleration are not allowed</p>

i.e.  $\Delta t \neq 0$  it means as  $\Delta t \rightarrow 0$ ,  $\Delta x \rightarrow 0$ , this leads to an indeterminate quantity which is not possible and hence Option (d) is incorrect. Statement “infinite acceleration are not allowed” makes this case different from case (c).

**Answers is Options (a), (b) and (c).**

Q-45 Each option is being analyzed since it is of type multiple choice.

- (i) Instantaneous speed is  $= \frac{\Delta x}{\Delta t} \Big|_{\Delta x \rightarrow 0}$  while instantaneous velocity is  $= \frac{\Delta \vec{x}}{\Delta t} \Big|_{\Delta x \rightarrow 0}$ . Thus in limits  $\Delta x = \Delta \vec{x}$ , hence it is not possible for instantaneous velocity and speed. Moreover, unless question states average speed and velocity it is to be treated as instantaneous velocity and speed. **Thus this option is incorrect.**
- (ii) In case of circular motion velocity at every instant is changing due to direction of displacement, but speed (distance covered) remains constant. **Hence Option (b) is correct**
- (iii) For  $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \neq 0$ , it is essential that  $\Delta \vec{v} \neq 0$ . **Hence Option (c) is not correct.**
- (iv) Since (b) is possible and hence  $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \neq 0$  while speed is constant. Example is circular motion.

**Hence Option (d) is correct**

**Answer is Options (b) and (d)**

Q-46 Each option is being analyzed since it is of type multiple choice.

- (i) We have  $v = u + at$ , and sign of  $a$  and  $v$  are different then, object will initially slow down and but after velocity  $v$  becomes zero then it starts increasing. Here, use of is in the question is to be noted in question “.... object is slowing down....”, it talk of present perfect tense, which is possible. **This option is correct.**
- (ii) We have  $s = ut + \frac{1}{2}at^2$ . If  $u$  is (+)ve and  $a$  is (-)ve, at that instant magnitude of velocity would tend to reduce. So also is true for  $u$  is (-)ve and  $a$  is (+)ve. **Hence this option is correct at that instance.**
- (iii) In motion of a particle, point of inflection, where velocity passes through zero while changing direction. This happen since acceleration is non zero. Therefore,  $a \neq 0$ . **Hence, this option is**

	<p><b>incorrect.</b></p> <p>(iv) If velocity is zero during <math>t_1 \leq t \leq t_2</math>, hence during this interval <math>\Delta v = 0</math>, therefore acceleration <math>a = \frac{\Delta v}{\Delta t} = 0</math>, numerator being zero. <b>Hence, this option is correct.</b></p> <p><b>Answer is options (a), (b), and (d)</b></p>
Q-47	<p>Each option is being analyzed since it is of type multiple choices.</p> <p>(i) For acceleration at <math>t = 0</math>, it is essential that <math>\Delta v = v_{0+} - v_0 = 0</math>. Given that <math>v_0 = 0</math>, but there is no information on value of <math>v_{0+}</math>, this option cannot be assertively stated to be correct. <b>Thus this option is incorrect.</b></p> <p>(ii) Considering analysis at (i) above, <math>a_{t=0}</math> may be zero if <math>v_{0+} = 0</math>. <b>Thus this option is correct.</b></p> <p>(iii) <b>If acceleration is Zero during <math>0 \leq t \leq 10</math></b>, then as per <math>v = 0 + 0 \cdot t = 0</math> during the interval, thus there would be no displacement hence speed shall also remain zero. <b>Thus this option is correct.</b></p> <p>(iv) This condition is corollary of case analyzed at (iii) above hence true. <b>Thus this option is correct.</b></p> <p><b>Answer is options (b), (c) and (d)</b></p>
Q-48	<p>Each option is being analyzed since it is of type multiple choices.</p> <p>(i) Unless specifically indicated velocity and speed are treated as instantaneous. Hence this statement is correct in limits <math>\Delta t \rightarrow 0</math>. <b>This option is correct.</b></p> <p>(ii) Average velocity is <math>\frac{\Delta \vec{x}}{\Delta t}</math> over an interval <math>\Delta t \neq 0</math>. It can be equal to average speed if-and-only-if despite different terminal points terminal <math> \Delta \vec{x} </math> is equal to distance. This since cannot be guaranteed based on given information. <b>Hence this option is incorrect</b></p> <p>(iii) If speed of a particle is zero, hence it is not moving, hence there cannot be any displacement and velocity has to be zero. <b>Hence this option is incorrect</b></p> <p>(iv) If speed of particle is never zero, hence it will definitely cover a distance during a given interval. Hence average speed during the interval cannot be zero, since distance is a scalar. <b>Hence this option is incorrect</b></p> <p><b>Answer – Only option (a)</b></p>
Q-49	<p>Each option is being analyzed since it is of type multiple choices.</p> <p>(i) The graph is a straight line and its equation would work out to <math>y - y_1 = \left(\frac{0-10}{10-0}\right)(x - x_1)</math>. Taking <math>(x_1, y_1) \equiv (0,10)</math>, the equation becomes <math>y = -1t + 0 + 10 = -1t + 10</math>, here slope of the line <math>\frac{dy}{dt} = m = -1</math> and this same as acceleration. Thus <math>a = -1</math> is a constant. <b>This option is correct</b></p> <p>(ii) Since slope is uni-directional and the velocity is changing from (+) ve to (-)ve at <math>t=10</math> sec, this is</p>

the point when particle is turning around. **Hence this option is incorrect.**

(iii) Since displacement is  $s = \int_{t_1}^{t_2} y dt = \int_{t_1}^{t_2} (-1t + 10) dt = \left[ -\frac{t^2}{2} + 10t \right]_{t_1}^{t_2}$  This cannot be equal to zero either (a)  $t_1 = t_2$  Or (b) graph is upto 20 sec only this is due to (-) slope with initial velocity (+ve). Since the graph is over a range  $t = 0$  to  $t = 30$ . **Hence, this option is incorrect**

(iv) Average speed of particle  $0 \leq t \leq 10$  is  $= \frac{0-10}{10-0} = -1$  and likewise Average speed of particle  $10 \leq t \leq 210$  is  $= \frac{-10-0}{20-10} = -1$ . Both are same. **Hence this option is correct.**

**Options (a) and (d) are correct.**

Q-50 Each option is being analyzed since it is of type multiple choices. For convenience of analysis Eight points A, B, C, D, E, F, G and H have been marked in red.

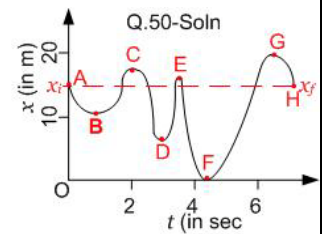
(i) Points B, C, D, E, F and G are **only six points** where x-t curve has points of inflection where slope is where slope is zero and hence particles comes to rest. **This Option is correct.**

(ii) Maximum speed is at that point when  $\frac{dx}{dt}$  is large, and by inspection of graph it is in section between E and F. Graph at  $t = 6$  sec is outside the identified zone of maximum speed. **Hence this option is incorrect.**

(iii) Since slope in sections of the graph AB, ED and EF are (-)ve, **hence this option is incorrect.**

(iv) Since position ( $x_f$ ) of the particle at the end of the graph (Point H) is equal to initial value ( $x_i$ ) at the starting point A,  $\bar{v} = \frac{x_f - x_i}{t - 0} = \frac{0}{t} = 0$ . i.e. it is not negative. **Hence this option is incorrect.**

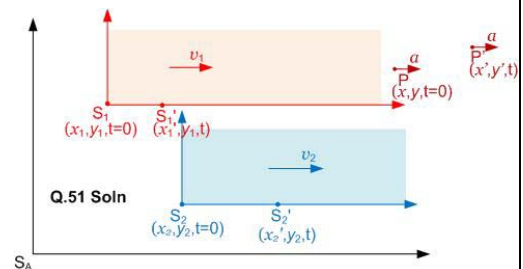
**Answer Option (a) is correct.**



Q-51 Each option is being analyzed since it is of type multiple choices.

(i) It is to be noted that only magnitudes of acceleration from Two frames of reference are given, while acceleration is a vector. Moreover, their initial velocities are not known. Hence, it cannot be said with certainty that the Two frames at rest with respect to each other. **Hence, this option is incorrect.**

(ii) For this case a figure is conceptualized where frames  $S_1$  and  $S_2$  are shown w.r.t to an absolute frame  $S_A$ . Frame  $S_1$  with its origin has coordinates  $(x_1, y_1, t = 0)$  w.r.t.  $S_A$  is shown. Likewise, frame  $S_2$  with its origin has coordinates  $(x_2, y_2, t = 0)$  w.r.t.  $S_A$  is shown. For simplification, both the frames are taken to be moving parallel to X-Axis of  $S_A$ , with velocities  $v_1$  and  $v_2$  respectively and are not accelerating as stipulated for this case.. Therefore, new position of the origins of two frames after a time  $t$  would be  $x_1' = x_1 + v_1 \cdot t$  and  $x_2' = x_2 + v_1 \cdot t$ , respectively. Particle P ( $x, y$ ) at  $t=0$  is having an acceleration  $a$  parallel to X-axis w.r.t.  $S_1$  and  $S_2$ , and reaches at position P' after a time  $t$ . Therefore, for new position w.r.t.  $S_A$ ,  $x' = x + at^2$ , here another simplification is initial velocity of particle P is Zero. Therefore displacement of particle w.r.t.  $S_1$  and  $S_2$  would be  $\Delta x_1 = x' - x_1'$  and  $\Delta x_2 = x' - x_2'$ , respectively. Substituting values,  $\Delta x_1 = (x + at^2) - (x_1 + v_1 \cdot t)$  and  $\Delta x_2 = (x + at^2) - (x_2 + v_1 \cdot t)$ , respectively. Thus acceleration of particle w.r.t.  $S_1$  and  $S_2$ , where  $x, x_1, x_2, v_1$  and  $v_2$  are constants, it would be –



- I. Acceleration w.r.t.  $S_1$  is  $a_{p1} = \frac{d^2}{dt^2} \Delta x_1 = \frac{d}{dt} \left( \frac{d}{dt} ((x + at^2) - (x_1 + v_1 \cdot t)) \right) = \frac{d}{dt} (2at - v_1) = 2a$ ,
- II. Acceleration w.r.t.  $S_2$  is  $a_{p2} = \frac{d^2}{dt^2} \Delta x_2 = \frac{d}{dt} \left( \frac{d}{dt} ((x + at^2) - (x_2 + v_1 \cdot t)) \right) = \frac{d}{dt} (2at - v_2) = 2a$

Therefore, acceleration of frame  $S_1$  with respect to particle would be  $a_{1p} = -a_{p1} = -2a$ . Thus acceleration of frame  $S_1$  w.r.t.  $S_2$  would be  $a_{12p} = a_{1p} + a_{p2} = -2a + 2a = 0$

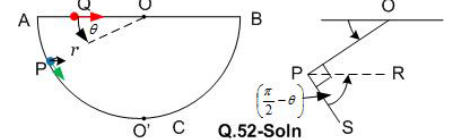
For particle to be experiencing an acceleration, in the instant case, with respect to  $S_1$  and  $S_2$  it is not necessary for the frames to be accelerated w.r.t. each other, while they may be moving with a constant velocity. In such a situation acceleration of particle would be same w.r.t. both the frames, as much as neither of the frame is accelerated w.r.t. each other. **Hence, this option is correct.**

(iii)  $|\vec{a}|$  of  $S_2$  with respect to  $S_1$   $|\vec{a}| = |\vec{a}_{s2} - \vec{a}_{s1}| = \sqrt{a_{s2}^2 + a_{s1}^2 + 2a_{s1} \cdot a_{s2} \cdot \cos \theta}$  would depend upon angle  $\theta$  between them which can  $0 \leq \theta \leq 180^\circ$ . Given that  $a_{s2} = a_{s1} = 4 \text{ ms}^{-2}$ ,  $a = 0$  when  $\theta = 180^\circ$  and  $a = 2 \times 4 \text{ ms}^{-2} = 8 \text{ ms}^{-2}$  and therefore  $0 \leq a \leq 8 \text{ ms}^{-2}$ . Since  $\theta$  is not known, hence  $a = 0$  or  $a = 8 \text{ ms}^{-2}$  can not stated with certainty. **Hence, this option is incorrect**

(iv)  $|\vec{a}|$  of  $S_2$  with respect to  $S_1$   $|\vec{a}| = |\vec{a}_{s2} - \vec{a}_{s1}| = \sqrt{a_{s2}^2 + a_{s1}^2 + 2a_{s1} \cdot a_{s2} \cdot \cos \theta}$  would depend upon angle  $\theta$  between them which can  $0 \leq \theta \leq 180^\circ$ . Given that  $a_{s2} = a_{s1} = 4 \text{ ms}^{-2}$ ,  $a = 0$  when  $\theta = 180^\circ$  and  $a = 2 \times 4 \text{ ms}^{-2} = 8 \text{ ms}^{-2}$  and therefore  $0 \leq a \leq 8 \text{ ms}^{-2}$ . **This option is correct.**

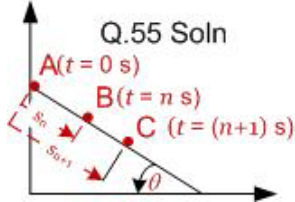
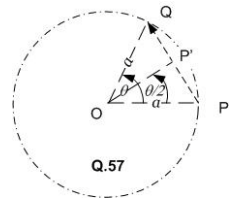
**Answer is options (b) and (d)**

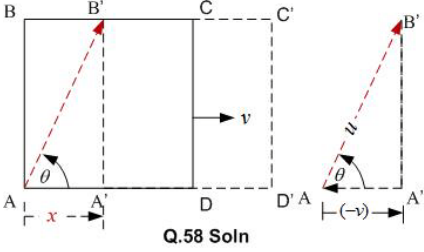
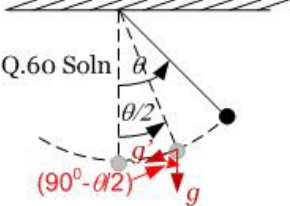
Q-52 **It is to be noted carefully that both P and Q pass off point A at  $t = 0$ , with same horizontal velocity  $v$** , over frictionless bowl and string respectively. As Q slides its velocity increases upto point O' right



below center of the bowl O, and then while traversing upward its velocity decreases. The increase in velocity of P, while sliding down increases its horizontal velocity  $v$  and on reaching back to B at height of A, it comes back to  $v$ . Thus all along traversing semicircular arc ACB its horizontal velocity  $v_h > v$  and hence time taken by P to reach B would be  $t_p < t_q$ . **Answer is Option (a)**

Q-53 Velocity of ball dropped from ground from a height  $h$ , will have initial velocity  $u = 0$  and hence velocity at any height would be  $v = \sqrt{2gh}$  downward (-ve) But, after bounce if it goes to height  $\frac{h}{2}$  then its initial velocity at bounce shall be  $v' = \sqrt{2g \frac{h}{2}} = \sqrt{gh}$ , upward (+ve). Thus, considering sign conventions possible options

	<p>are (a) and (c). Since, <math>v - h</math> graph is not a straight line therefore Option (c) is ruled out. Further, <math>v - h</math> relation is quadratic, with higher (-)ve <math>v</math> for higher <math>h</math> and for bounce since velocity is lower and hence lower height but (+)ve. These, stipulations are there in option (a). <b>Answer is Option (a)</b></p>
<p>Q-54</p>	<p>Given that <math>u = 0</math> and <math>a = -\frac{10}{11}t + 10</math>. Since, acceleration is +ve in given time zone maximum velocity the area under the curve i.e. <math>v_{max} = \int_{t=0}^{t=11} a \cdot dt</math> since <math>a = \frac{dv}{dt}</math>. Thus <math>v_{max} = \int_{t=0}^{t=11} \left(-\frac{10}{11}t + 10\right) \cdot dt</math>. On resolves into <math>v_{max} = \left[-\frac{10}{11} \cdot \frac{t^2}{2} + 10t\right]_0^{11} = -5 \times 11 + 10 \times 11 = 55 \text{ ms}^{-1}</math>. <b>Answer is Option (b)</b></p>
<p>Q-55</p>	<p>Acceleration due to gravity <math>g \text{ ms}^{-2}</math> is vertically downward. Since block starts, from rest, sliding down, hence acceleration along the plane would be <math>g \sin \theta</math>. Hence, <math>x_{n-1} = 0 \cdot n + \frac{1}{2} \cdot g \sin \theta \cdot (n-1)^2 = \frac{g \sin \theta}{2} (n-1)^2</math>; <math>x_n = \frac{g \sin \theta}{2} n^2</math>, and <math>x_{n+1} = \frac{g \sin \theta}{2} (n+1)^2</math>. Hence, <math>s_n = x_n - x_{n-1} = \frac{g \sin \theta}{2} (n^2 - (n-1)^2) = \frac{g \sin \theta}{2} \cdot (2n-1)</math>. And <math>s_{n+1} = x_{n+1} - x_n = \frac{g \sin \theta}{2} ((n+1)^2 - n^2) = \frac{g \sin \theta}{2} \cdot (2n+1)</math>. Thus <math>\frac{s_n}{s_{n+1}} = \frac{\frac{g \sin \theta}{2} \cdot (2n-1)}{\frac{g \sin \theta}{2} \cdot (2n+1)} = \frac{(2n-1)}{(2n+1)}</math>. <b>Thus answer is option (c)</b></p> 
<p>Q-56</p>	<p>Equation of graph on <math>v-x</math> plane is <math>v = \frac{dv}{dx} \cdot x + v_0 = \left(\frac{-v_0}{x_0}\right) \cdot x + v_0</math>. Further, manipulating <math>\frac{dv}{dx} = \frac{dv/dt}{dx/dt} = \frac{a}{v} \rightarrow v = -\left(\frac{x_0}{v_0}\right) \cdot a</math>. Thus, equation of the given graph transforms into <math>\left(\frac{x_0}{v_0}\right) \cdot a = \left(\frac{-v_0}{x_0}\right) \cdot x + v_0</math>. It is simplified into <math>a = \left(\frac{-v_0}{x_0}\right)^2 x - v_0 \left(\frac{x_0}{v_0}\right) = \left(\frac{x_0}{v_0}\right)^2 x - v_0 \left(\frac{x_0}{v_0}\right)</math>. This equation has (+)ve slope and (-)ve intercept on on ordinate. <b>Thus answer is option (a)</b></p>
<p>Q-57</p>	<p>Given that a particle (P) is moving in circular path of radius <math>a</math> with an angular speed <math>\omega</math>. Therefore angle traversed in time <math>t</math> would be <math>\theta = \omega t</math>, and reach a point Q. Therefore, magnitude of displacement in time <math>t</math> would length of chord PQ. Since OPQ is an isosceles triangle with <math>OP = OQ = a</math>, hence <math>PQ = 2PP' = 2(a \sin \frac{\theta}{2}) = 2a \sin \frac{\omega t}{2}</math>. Hence <b>answer is Option (b)</b></p> 

<p>Q-58</p>	<p>Let the square frame be ABCD at time <math>t = 0</math> and is moving with a velocity <math>v</math>. After a time the frames would be displaced to position A'B'C'D' such that displacement of vertex shall be <math>x = vt</math>. The particle after projected from point A with a speed <math>u</math> at an angle <math>\alpha</math> so as to hit the vertex B. Relative velocity of particle w.r.t. frame should be such that its horizontal component should lead to cancellation of relative velocity of frame, i.e. <math>u \cos \theta - v = 0 \rightarrow \cos \theta = \frac{v}{u} \rightarrow \theta = \cos^{-1} \left( \frac{v}{u} \right)</math>. <b>Answer is Option (b).</b></p>	 <p style="text-align: center;">Q.58 Soln</p>
<p>Q-59</p>	<p>Let stone A be projected with velocity <math>v_a \angle \theta_a</math> and stone B be projected with velocity <math>v_b \angle \theta_b</math>. Therefore, vertically upward velocities of the two stones shall be <math>v_a \sin \theta_a</math> and <math>v_b \sin \theta_b</math> respectively and their time of flights shall be <math>T_a = \frac{2v_a \sin \theta_a}{g}</math> and <math>T_b = \frac{2v_b \sin \theta_b}{g}</math>. Let, <math>T_a &gt; T_b</math> then during <math>0 \leq t \leq T_b</math> the relative velocity of stone A w.r.t shall be <math>\vec{v}_{ab} = \vec{v}_a - \vec{v}_b</math> and during this period relative acceleration would be zero because both are under acceleration <math>\vec{g}</math> vertically downward. Therefore, during <math>0 \leq t \leq T_b</math> relative position of stone A w.r.t. B shall be varying linearly with time. But, at <math>T_b</math> stone B becomes stationary, while, stone A will continue to perform projectile motion, with it ne velocity and height at that instant. Since projectile motion is parabolic.</p> <p><b>Hence, Answer is Option (b)</b></p>	
<p>Q-60</p>	<p>Given that bob is released from rest at an angle <math>\theta = 60^\circ</math>. At this position it experiencing an acceleration <math>g = 10 \text{ m}_S^{-2}</math> vertically downward, which remains constant in magnitude and direction. But, since pendulum is fixed to ceiling and hence its tangential acceleration at given angle <math>\theta = 30^\circ</math> would be <math>= g \cos(90^\circ - \theta/2) = g \cos 60^\circ = \frac{g}{2} = 5 \text{ m}_S^{-2}</math>.</p> <p><b>Thus answer is Option (b).</b></p>	 <p style="text-align: center;">Q.60 Soln</p>



Q-61

Since, vertex A is fixed and C is moving toward right with a velocity  $v \text{ ms}^{-1}$  and point O is mid point of AC and hence rate of displacement of point O, by principle of proportionality, would be  $\frac{d}{dt}(AO) = \frac{v}{2} \text{ ms}^{-1}$ . When rhombus is in shape of square in triangle AOB which is a right angle isosceles triangle ABO, side AB and vertex A are fixed and  $BO = AO$ . Further, also  $AB^2 = AO^2 + BO^2$ . Differentiating the equation w.r.t.  $t$  it would be  $0 = 2AO \frac{d}{dt}(AO) + 2BO \frac{d}{dt}(AO) \rightarrow \frac{d}{dt}(AO) = -\frac{d}{dt}(AO)$ . Hence, if velocity of A is  $(+v)$  along X-axis, velocity of B is  $(-)$ ve i.e. along Y axis towards X-axis  $\frac{v}{2} \text{ ms}^{-1}$ .

**Thus answer is Option (c)**

