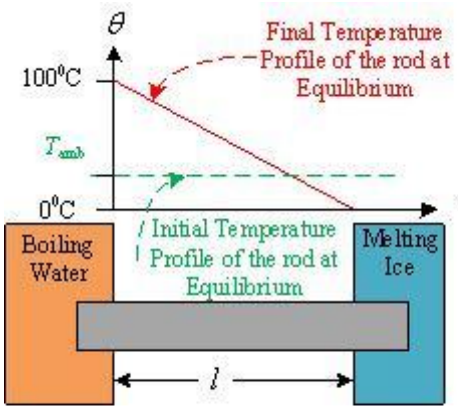
Heat – Transfer of Heat : Illustrations to Answers of Objective and Subjective Questions (Typical)

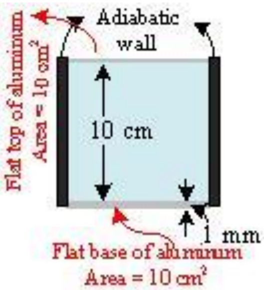
I-01	Rate of conduction of heat is $u=\frac{\Delta Q}{\Delta t}=\frac{kA\Delta\theta}{l}\Rightarrow k=\frac{ul}{A\Delta\theta}$, here k is the thermal conductivity which is characteristic to the material of the rod, while A and l are depending on geometry of the rod and $\Delta\theta$ is thermal operating condition which decide conduction of heat u from a specific material. **Hence answer is (d).**
I-02	Each of the given options is being analyzed. (a) Transfer of heat from one place to the other can take place through conduction, if there is any object connecting source of heat and object at other place which makes **option (a) to be correct**. (b) In presence of air in the room convection process will transfer heat and this makes **option (b) to be correct**. (c) When two objects are at a distance transfer of heat by radiation will take place as it is independent of medium. This makes **option (c) to be correct**. (d) Since all the three modes of heat transfer are equally valid hence answer is (d). **Thus answer is option (d).**
I-03	Given that an object at temperature T_1 is placed in chamber at temperature T_2 such that $T_2>T_1$. Therefore transfer of heat will be there from higher to lower temperature to lower temperature i.e. from room to the object. Since chamber is evacuated the mode of transfer of heat would be radiation. Accordingly as per Newton's Law of cooling $\Delta u=e\sigma A\left(T_2^4-T_1^4\right)$, here e is emissivity of surface of the object, σ is absorptive power of the body as per Kirchhoff's Law and A is surface area of the object which depends upon its geometry. Thus all the three factors are constants. Accordingly, $\Delta u\propto\left(T_2^4-T_1^4\right)$. **Thus answer is option (d).**
I-04	As per Stefan and Boltzmann's Law rate of radiation $\Delta u=e\sigma AT^4\Rightarrow\Delta u\propto T^4$, here e is emissivity of surface of the object, σ is absorptive power of the body as per Kirchhoff's Law and A is surface area of the object which depends upon its geometry. Since. These constants take care of geometry and properties of surface. Hence, the given statement $\Delta u\propto T^n\big\|_{n=4}$ applies to all bodies. **Thus option (b) is correct.**
I-05	As per As per Stefan and Boltzmann's Law rate of radiation is $\Delta u=e\sigma AT^4\Rightarrow\frac{\Delta u_2}{\Delta u_1}=\frac{T_2^4}{T_1^4}$. Accordingly the ratio is $=\left(\frac{T_2}{T_1}\right)^4=\left(\frac{273+20}{273+10}\right)^4=(1.04)^4=1.15\Rightarrow u_1:u_2::1:1.15$. It is to essential to remember that in this law temperature are on Thermodynamic scale**. Hence answer is option (a).** **N.B.:** Difference in values given in the options is discretely large and without actually calculating a mathematical judgment can be made that $(1.04)^4=1.15$, and accordingly question can be answered.
I-06	A metal rod with one end in the furnace shall experience transfer of heat by conduction as per $u=\frac{\Delta Q}{\Delta t}=\frac{kA\Delta\theta}{l}\Rightarrow k=\frac{ul}{A\Delta\theta}$, convection through ambient air and radiation at every point of the rod depending upon its temperature. Thus possible answers are (a) and (c). Now, between options (b) and (d), decrease of temperature cannot be stated with certainty as temperature of

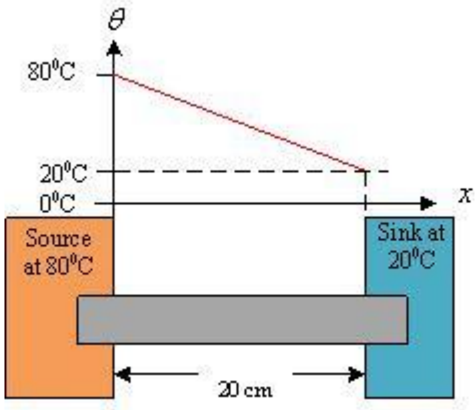
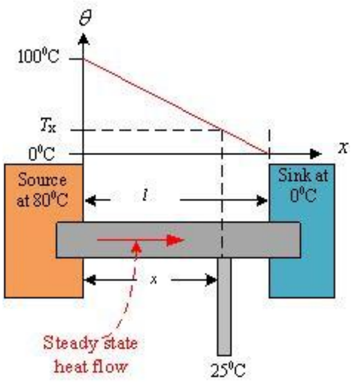
	the other end of the rod and other parameters of conduction are not defined. In case the length of the rod protruded outside furnace is at ambient temperature then option (b) is valid. But, if temperature of rod at other end is above ambient temperature then initially temperature of the rod would initially decrease and then again increase to attain temperature being maintained at the other end. This condition is satisfied in option (d). **Hence answer is (d).**
I-07	Newton's Law of cooling is about rate of cooling of an object due to radiation at a known ambient temperature when temperature of the heat radiating body T is above ambient temperature T_0 through a small value i.e. $T-T_0<$ and is expressed as $\frac{dT}{dt}=-bA\left(T-T_0\right)$. Accordingly each of the option is being analyzed. (a) The Wien's Displacement Law states maximum frequency of radiation by a and object at a given temperature and is expressed as $\lambda_m T=b$, thus it is not about rate of cooling and **hence option (a) is incorrect**. (b) Kirchhoff's Law is about ratio of emissive power to absorption power of any body is equal to emissive power of black body at that temperature and is expressed as $\frac{E(\text{body})_T}{a(\text{body})_T}=E(\text{blackbody})_T$. This is also not related to rate of cooling and **hence option (b) is incorrect**. (c) Stefan's Law states rate of emission of energy as $u=e\sigma AT^4$. Thus if the object A is at a temperature T kept in a ambient at a temperature T_0. Thus net rate of transfer $\Delta u_r=u_e-u_a=e\sigma AT^4-e\sigma AT_a^{\ 4}$. It leads to $\Delta u_r=e\sigma A\left(T^4-T_a^{\ 4}\right)=e\sigma A\left(\left(T_a+\Delta T\right)^4-T_a^{\ 4}\right)$. This resolves into $\Delta u_r=e\sigma AT_a^{\ 4}\left(\left(1+4\frac{\Delta T}{T}+\text{higher powers of }\frac{\Delta T}{T}\right)-1\right)$, since $\frac{\Delta T}{T}$, here $\Delta T=T-T_a$ is small accordingly it simplifies to $\Delta u_r=4e\sigma AT^4\frac{\Delta T}{T}=4e\sigma AT^3\Delta T=b_1A\Delta T$. Another source of dissipation of heat is convection and its of dissipation is approximated to $\Delta u_c=b_2A\Delta T$. Thus total loss of heat is $\Delta u=\Delta u_1+\Delta u_2=b_1A\Delta T+b_2A\Delta T=\left(b_1+b_2\right)A\Delta T...(1)$. Further, from heat capacity $\Delta Q=ms\Delta\theta\Rightarrow\Delta u=\frac{\Delta Q}{\Delta t}=ms\frac{\Delta\theta}{\Delta t}..(2)$. Combining equations (1 & 2) we get $ms\frac{\Delta\theta}{\Delta t}=\left(b_1+b_2\right)A\left(T-T_a\right)\Rightarrow\frac{d\theta}{dt}=\frac{\left(b_1+b_2\right)A}{ms}\left(T-T_a\right)=b\left(T-T_a\right)$, here $b=\frac{\left(b_1+b_2\right)A}{ms}$. Thus it is seen that this Stefan's Law is in close approximation to Newton's Law of cooling. **Hence option (c) is correct.** (d) Planck's Law states that energy contained in radiation is discrete and expressed as $E=nh\upsilon$,here υ is frequency of radiation, n is an integer and h is Planck's constant. Thus this law is far away from Newton's Law of cooling. Hence this option is incorrect. **Thus correct answer is option (c).**
I-08	In the given case rate of cooling i.e. fall of temperature can be determined from Newton's Law of Cooling (NLC) and accordingly, $\frac{dT}{dt}=-bA\left(T-T_0\right)$. In light of this each of the graph is being analyzed – • Graph(a) shows decrease in $\frac{dT}{dt}$ with fall in temperature and is approximated and hence **this graph is the answer.** • Graph (b) depicts constant rate of fall of temperature and is nut feasible as per NLC and **hence this is incorrect.** • Graph (c) shows slow rate of cooling at higher temperature and increase in rate of cooling with reduction

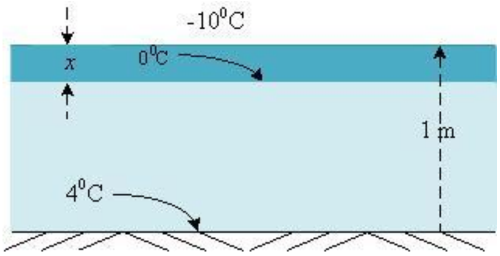
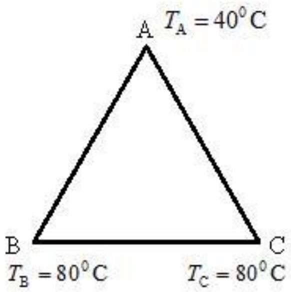
	in temperature. This is not in accordance with NLC. **Hence it is incorrect answer**. • Graph (d) shows no cooling at initial temperature and sudden cooling at infinite rate. This is not tenable with any of the laws of radiation. **Hence it is incorrect answer.** **Thus correct answer is graph (a).**
I-09	As per Newton's Law of Cooling $\frac{dT}{dt}=-bA\left(T-T_a\right)\Rightarrow\frac{dT}{T-T_a}=-bAdt$. On integration $\int\frac{dT}{T-T_a}=-\int bAdt\Rightarrow\ln\left(T-T_a\right)=-bAt+C$. This is a straight line with (-)ve slope and initial value at $t=0$ depending upon initial difference in temperature. **Thus, answer is option (a).**
I-10	As per Newton's law of cooling $\frac{\Delta\theta}{\Delta t}=b\left(T-T_a\right)$, here coefficient $b=\frac{\left(b_1+b_2\right)A}{ms}$ where $b_1=4e\sigma AT^3$ and b_2 is arbitrary constant regulated by convection process while constituents of ms are parameters of the body. Thus occurrence of T^3 in the coefficient would reduce rate of cooling with fall of temperature from 65^0C to 60^0 C to 60^0C to 55^0C, despite $\Delta T=65-60=60-55=5^0\text{C}=5\text{K}$. Hence, answer is option (c). **N.B.:** Qualitatively the above analysis is correct. But quantitatively analysis would require to take in Kelvin scale. As regards the factor ΔT it is same both in Celsius and Kelvin, and hence would not influence results either quantitatively or qualitatively.
I-11	The process of heat transfer is defined by equation $\Delta Q=\frac{kA\Delta\theta}{l}\Delta t$. At the start i.e. at time $t=0^-$ the rod is in equilibrium with ambient temperature. As soon as the two ends dipped at different temperature, hear transfer takes starts as a transient process, which disturbs initial equilibrium until a new equilibrium is established with temperature of the two ends are as defined. In this context each of the opition is being analyzed –  **Option (a):** From the above initial equilibrium will move into in-equilibrium, due to transient conditions, until new equilibrium is established **Thus this option is incorrect.** **Option (b):** With ends of the rod at discretely different temperatures a temperature gradient would be established along the length of the rod due to heat transfer. Hence, specific temperature cannot be assigned to the rod. **Thus this option is incorrect.** **Option (c):** Taking forward the analysis at option (b) even after attaining specific temperature cannot be assigned to the rod. **Thus this option is incorrect.** **Option (d):** Once steady state is reached, by its nomenclature its state would continue to be so unless conditions of the rod are changed. **Thus this option is correct.** **Hence answer is option (d).**
I-12	As per Kirchhoff's Law of radiation $\frac{E(\text{body})_T}{a(\text{body})_T}=E(\text{blackbody})_T$. Thus black body simultaneously absorbs and radiates heat and therefor what it does not is both reflection and refraction of radiation. **Hence answers is option (c) and (d).**
I-13	In summer temperature water of a calm river remains lower than that of land because of greater mass and low specific heat as compared to the land at the shore i.e. $m_1s_1=m_2s_2$ Therefore, for same radiation on on surface of sea and land temperature rose on land $\Delta\theta_{\text{land}}>\Delta\theta_{\text{sea}}$ and this will cause convection current in air with an upward drift. Thus a partial vacuum would be created on the surface of the land. In turn the denser air

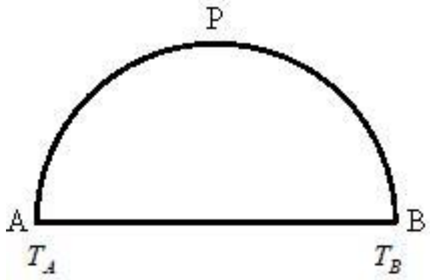
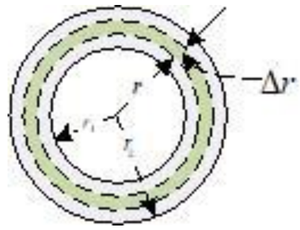
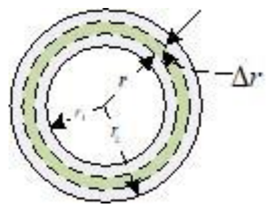
	on the surface of the sea would tend to fill partial vacuum on lands surface, like convection current. *Thus cause of mild wind on the shore of a calm river is convection current*. Hence, **answer is option (b).**
I-14	Surface of charcoal is like black body and therefore it will tend absorb Q_{a-c} all heat Q_i incident on it i.e, $Q_{a-c} \approx Q_i$. As against this shinning steel piece will reflect Q_{r-s} major part of the heat incident on it Q_i i.e $Q_{r-s} < Q_i$. In this context each option is being analyzed – **Option (a):** Therefore heat absorbed by steel rod $Q_{a-s} = Q_i - Q_{r-s}$. Thus $Q_{a-s} < Q_{a-c}$. This is in contradiction to the statement at (a). **Hence, option (a) is incorrect.** **Option (b):** *In light of analysis in option (a) leading to $Q_{a-s} < Q_{a-c}$ at equilibrium state steel piece would be cooler than charcoal. This inference is in contradiction to the option (b).* **Hence, option (b) is incorrect.** **Option (c):** When both the charcoal and the steel pieces are picked up by bare hands transfer of heat between the pieces and hand surface would be by conduction. Therefore, $\frac{\Delta Q}{\Delta t} = \frac{kA\Delta\theta}{l} \Rightarrow \frac{\Delta Q}{\Delta t} \propto k\Delta\theta$. Since, steel is good conductor of heat as compared to the charcoal such that coefficients of conductivity are $k_c > k_s$. *Therefore, rate of transfer of heat for steel would be more than that for charcoal. Since, sense of hotter or cooler on touch is caused by rate of transfer of heat. Accordingly, steel piece will be felt hotter than charcoal*. **Hence, option (c) is correct.** **Option (d):** Blackbody as compared to shinning surface has higher emissivity. *Therefore, when both pieces picked up from a hot state in the lawn and placed in cold chamber, the rate of loss of heat of charcoal piece is faster than steel rod*. **Therefore, option (d) is correct.** **Thus answer is option (c) and (d).**
I-15	Highest intensity of radiation vis-à-vis its wavelength as per Wien's Displacement Law is $\lambda_m T = b = \text{Const.}$ Therefore, $\lambda_{m-1}T_1 = \lambda_{m-2}T_2 \Rightarrow \frac{T_2}{T_1} = \frac{\lambda_{m-1}}{\lambda_{m-2}} = \left.\frac{\upsilon_{m-2}}{\upsilon_{m-1}}\right\|_{\upsilon=\frac{c}{\lambda}} \Rightarrow T \propto \upsilon$ .. **Option (a):** From the proportionality $T \propto \upsilon$ when temperature is doubled, frequency of maximum intensity of radiation would also double. **Thus option (a) is correct**. **Option (b):** Analysis at option (a) is reverse of the statement at option (b). **Hence option (b) is incorrect**. **Option (c):** As per Stefan and Boltzmann Law $u = e\sigma AT^4 \Rightarrow \frac{u_2}{u_1} = \left(\frac{T_2}{T_1}\right)^4 \Rightarrow \frac{u_2}{u_1} = \left.(2)^4\right\|_{T_2=2T_1} = 16$. This answer is supported by option (c) and **hence Option (c) is correct**. **Option (d):** In this option answer is given to be 8 which in contradiction to the answer already proved at (c). **Hence this option is incorrect.** **Thus correct answer is option (a) and (c).** **N.B.:** Stefan-Boltzmann Law specifies relationship between λ_m and T while in answer is asked on relationship between υ_m and T . This requires conversion of the equation with the relationship $\upsilon = \frac{c}{\lambda}$.
I-16	As per Stefan and Boltzmann's Law $u = e\sigma AT^4$ Since the sphere and hollow sphere are of same material and radii therefore $(e\sigma A)_{\text{solid\_sphere}} = (e\sigma A)_{\text{hollow\_sphere}}$. Accordingly, both will emit equal radiation in the beginning as they are heated to same temperature. **Thus option (a) is correct.** With logic discussed above rate of absorption of heat from surrounding will also be the sam**e. Thus option (b) is also correct.** Considering the heat capacity $\Delta Q = ms\Delta T \Rightarrow u = \frac{\Delta Q}{\Delta t} = \frac{ms\Delta T}{\Delta t} \Rightarrow \frac{\Delta T}{\Delta t} = \frac{u}{ms}$. Since material of solid sphere and hollow sphere are same their specific heat capacities $s_{\text{s-s}} = s_{\text{h-s}}$ but $m_{\text{s-s}} > m_{\text{h-s}}$. Accordingly,

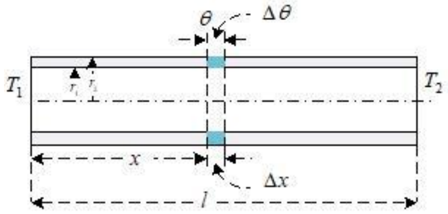
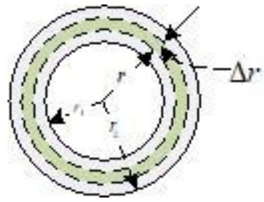
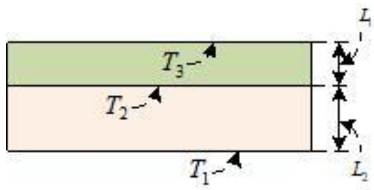
	$s_{\text{s-s}}\cdot m_{\text{s-s}} > s_{\text{h\_s}}\cdot m_{\text{h-s}} \Rightarrow (ms)_{\text{s-s}} \neq (ms)_{\text{h-s}}$. Therefore, using the above derivations $\frac{\left(\frac{dT}{dT}\right)_{\text{s-s}}}{\left(\frac{dT}{dT}\right)_{\text{h-s}}} = \frac{\frac{u}{(ms)_{\text{s-s}}}}{\frac{u}{(ms)_{\text{h-s}}}} = \frac{(ms)_{\text{h-s}}}{(ms)_{\text{s-s}}}$. And hence $\left(\frac{dT}{dT}\right)_{\text{s-s}} \neq \left(\frac{dT}{dT}\right)_{\text{h-s}}$. Thus ionitial rate of colling will not be same and contradicts option (c). **Thus option (c) is incorrect.** The two spheres at equal initial temperature have different rate of cooling and hence at every instant, after the initial, they will have different temperature. This is in contradiction to statement in option (d). **Thus option (d) is incorrect.** **Thus answer is option (a) and (b).**
I-17	With the given orientation of the slab $A=0.1\times0.1=0.01\,\mathrm{m}^2$ and $l=00.1\,\mathrm{m}$. Further, temperature difference between reservoirs is $\Delta T=90-10=80^0\,\mathrm{C}$. From conduction of heat $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}$. Using the given data $\frac{\Delta Q}{\Delta t}=\frac{0.8\times0.01\times80}{0.01}=64\,\mathrm{J.s^{-1}}$. Since answer asked is heat flowing in one minute $H=\frac{\Delta Q}{\Delta t}\times60=64\times60=3840\,\mathrm{J}$. **Hence answer is 3840 J.**
I-18	Given that $k=0.025\,\mathrm{J.s^{-1}.m^{-1}.^0C^{-1}}$, thickness of heat insulator $l=0.1\,\mathrm{m}$, area of the heat insulator is $A=0.80\,\mathrm{m}^2$. And temperature difference liquid-nitrogen and atmosphere is $\Delta T=300-80=220\,\mathrm{K}$ $=220^0\mathrm{C}$. Since atmosphere is at higher temperature and hence rate heat flowing into liquid nitrogen is $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}=\frac{0.025\times0.80\times220}{0.01}=440\,\mathrm{W}$. **Hence, answer is 440 W.** **N.B.:** Unit of heat energy is Joule, but rate of flow of heat is power i.e. $\mathrm{J.s^{-1}}$ which is equivalent to Watt
I-19	We know that $\frac{F-32}{9}=\frac{C}{5}\Rightarrow C=\frac{5}{9}(F-32)$, therefore $\Delta C=C_{\text{body}}-C_{\text{room}}=\frac{5}{9}\left(F_{\text{body}}-F_{\text{room}}\right)=\frac{5}{9}(97-47)=\frac{250}{9}$. Further, area of body covered with cloth $A=1.6$m2, and thickness of cloth $t=0.5\times10^{-2}\,\mathrm{m}$. Therefore rate of flow of heat from the body $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}=\frac{0.04\times1.6\times\frac{250}{9}}{0.5\times10^{-2}}=3.56\times10^2\,\mathrm{J.s^{-1}}$. **Hence answer is 356 J.s^{-1}**
I-20	Rate of evaporation of water is $m=100$ g per minute therefore amount of heat absorbed by water per-minute is $Q=m\times L=0.100\times2.26\times10^6=2.26\times10^5\,\mathrm{J.m^{-1}}$. It works out to $\frac{\Delta Q}{\Delta t}=\frac{2.26\times10^5}{60}=3.77\times10^3\,\mathrm{J.s^{-1}}$. Since the heat is transferred to the water through the bottom of the container having area $A=25\times10^{-4}\,\mathrm{m}^2$ and thickness $l=1\times10^{-2}\,\mathrm{m}$ and thermal conductivity $k=50\,\mathrm{W.m^{-1}.^0C^{-1}}$. From conduction of heat $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}$. Substituting the values, it leads to $3.77\times10^3=\frac{50\times\left(25\times10^{-4}\right)\times\Delta T}{1\times10^{-3}}\Rightarrow\Delta T=\frac{3.77\times10^4}{50\times25}=30.16$. Thus temperature of th bottom of the vessel above boiling point of water is $T=100+\Delta T=100+30.1-130.1^0\,\mathrm{C}$. Accordingly, based SGs in the given data **answer is T=130^{0}C**.
I-21	conduction of heat $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}$. Thus from given data $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}=\frac{46\times\left(0.04\times10^{-4}\right)\times(100-0)}{1}=1.84\times10^{-2}$ J.s-^{1}. Given the latent heat of fusion of ice, the rate of melting of ice would be

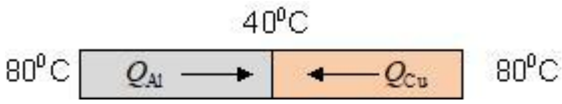
	$\frac{\Delta Q}{\Delta t}=mL \Rightarrow 1.84\times10^{-2}=m\times\left(3.36\times10^{5}\right)\Rightarrow m=\frac{1.84}{3.36}\times10^{-7}=5.5\times10^{-8}$ kg = $5.5\times10^{-8}\times10^{3}=5.5\times10^{-5}$ g.s^{-1}. **Hence answer is** 5.5×10^{-5} **g.**
I-22	Icebox is dipped in water at 20^{0}C whose parameters are given, Therefore, transfer of heat by conduction from water to ice. Accordingly, $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}$ …(1) and this is responsible for melting of ice such that $\frac{\Delta Q}{\Delta t}=mL$ …(2) Here, m rate of melting of ice per second. Combining equations (1 & 2) and using the given data $\frac{0.06\times\left(2400\times10^{-4}\right)\times\left(20-0\right)}{2.0\times10^{-3}}=m\times\left(3.4\times10^{5}\right)\Rightarrow m=\frac{0.12\times2.4}{6.8}\times10^{-5}=4.2\times10^{-4}$. Kg.s^{-1}. This is equivalent to $m=\left(4.2\times10^{-4}\right)\times3600=1.5$ kg.Hr^{-1}. **Hence, answer is 1.5 kg.h^{-1}** **N.B.:** Since, this quantity of melting of ice is too small to be measurable from the scale of the given data. Therefore answer needs to be represented in measurable unit keeping into consideration significant digits. Accordingly, though in question nothing is stated about unit of time while asking rate of melting of ice, it converted from Kg.s^{-1} into kg.Hr^{-1}.
I-23	Water seeps out of pitcher through pours of its walls and evaporates absorbing heat from the water contained in the pitcher. In the process temperature of water starts falling down until it becomes constant. Thus heat equation can be formed such that $m\times L=\frac{-\Delta Q}{\Delta t}=-\frac{kA\Delta T}{l}\Rightarrow\Delta T=-\frac{m\times L\times l}{k\times A}$. Using the given data we get $\Delta T=-\frac{\left(0.1\times10^{-3}\right)\times\left(2.27\times10^{6}\right)\times\left(1.0\times10^{-3}\right)}{0.80\times\left(200\times10^{-4}\right)}=-14^{0}$C. Accordingly, Temperature equation is $\Delta T=T_w-T_a\Rightarrow T_w=T_a+\Delta T\Rightarrow 42-14=28^{0}$ C. **Hence answer is 28^{0}C** N.B.: Since heat is being lost by water contained inside pitcher and hence ΔQ is taken to be (-)ve.
I-24	Transfer of heat through steel frame from one to other end is through conduction and accordingly $\frac{\Delta Q}{\Delta t}=\frac{kA\Delta T}{l}$. Using the given data $\frac{\Delta Q}{\Delta t}=\frac{45\times\left(0.20\times10^{-4}\right)\times\left(40-20\right)}{0.60}=0.03\text{J.s}^{-1}=0.03\text{W}$. **Thus answer is 0.03 W.**
I-25	 The system is of the cylindrical vessel having adiabatic walls and flat bottom of aluminum is shown in the figure. **Assume** that - **(1)** transfer of heat from water to the ambient is taking place through its flat bottom having outside temperature to be 20^{0}C, **(2)** no heat transfer is taking from top surface of water through convection to the ambient, and **(3)** temperature of water is approximately constant at 50^{0}C. Thus rate of transfer of heat through flat bottom and identical top by conduction is $\frac{\Delta Q}{\Delta t}=\frac{k\times2A\times\Delta T}{l}$, using the given data $\frac{\Delta Q}{\Delta t}=\frac{200\times2\times\left(10\times10^{-4}\right)\times\left(50-20\right)}{1\times10^{-3}}=12000$ J.s^{-1}. Further, for fall of temperature by 1^{0}C the amount of heat lost by water is $\Delta Q=ms\Delta T=\left(A\times h\times\sigma\right)\times s\times\left(50-49\right)\Rightarrow\Delta Q=\left(0.10\times\left(10\times10^{-4}\right)\times1000\right)\times4200\times1=4.2\times10^{2}$, using the given data. Accordingly, time taken for fall of temperature by 1^{0}C is $t=\frac{\Delta Q}{\frac{\Delta Q}{\Delta t}}=\frac{4.2\times10^{2}}{12000}=0.035$ s. **N.B.:** Based on assumptions answer would change, therefore, whenever any assumption is made it must be

	specified. Such questions can be expected as a part of subjective tests.
I-26	Since there mo heat loss through radiation and hence only mode of flow of heat is conduction which is constant at any cross-section of the rod and, therefore, $\frac{\Delta Q}{\Delta t}=\frac{k\times 2A\times\Delta T}{l}$. Accordingly, $\Delta T\propto l\Rightarrow\frac{\Delta T_{20}}{\Delta T_9}=\frac{20}{9}\Rightarrow\Delta T_9=\frac{9}{20}\times\Delta T_{20}=\frac{9}{20}\times(80-20)=27^0$. Thus, $\Delta T=27=T_0-T_9\Rightarrow T_9=80-27=53^0$ C. **This part (a) of the answer.** Going back to equation of conduction of heat, using given data, $\frac{\Delta Q}{\Delta t}=\frac{385\times\left(0.20\times10^{-4}\right)\times(80-20)}{0.20}=2.31$J.s^{-1}. **This is part (b) of the answer**. **Thus answer (a) 53^{0}C and (b) 2.31 J.s^{-1}** 
I-27	If another rod with one end at 250C touches the rod having heat transfer at point where temperature is 250C there will not be any heat flow from the another rod. Given system as shown in figure is having constant heat transfer from source to sink at the equilibrium condition. Applying the form of equation of a line for which coordinates of two points on it are known. In the instant these two points are (0,100) and (100,0). Accordingly, equation of line is $\theta_x-\theta_1=\frac{T_2-T_1}{l}(x-x_1)\Rightarrow 25-100=\frac{0-100}{100}(x-0)\Rightarrow x=75$. This is the distance from the hot end. Therefore distance from the cold end is $l-x=100-75=25$cm. **Hence answer is 25 cm.** 
I-28	Given that the box is cubical having volume 216 cm^3, Therefore sides of the cube are $V=L^3=216\Rightarrow L=(216)^{\frac{1}{3}}=6$cm and therefore surface area of cube is $A=6L^2=6\times6^2=216$ cm^2. It is required to find thermal conductivity (k) of the wooden cubical box of thickness $l=0.1$cm with $\frac{dQ}{dt}=100$ W heater inside and temp difference across walls $\Delta T=5^0$ C. Applying equation of heat for conduction $\frac{dQ}{dt}=\frac{kA\Delta T}{l}\Rightarrow k=\frac{\frac{dQ}{dt}\times l}{A\times\Delta T}=\frac{100\times\left(0.1\times10^{-2}\right)}{\left(216\times10^{-4}\right)\times5}=9.2\times10^{-1}$ W.m^{-1}.^{0}C^{-1}. **Thus answer is 0.92 W.m^{-1}.^{0}C^{-1}.**
I-29	In the given problem when the block of mass M is released from state of rest it attains a constant speed of falling $\frac{dh}{dt}=10$ cm.s^{-1} and at that point steady state temperature of water is 1^{0}C while temperature outside the water container is 0^{0}C. Thus from the given data $\Delta T=1^0$ C and $\frac{dW}{dt}=Mg\frac{dh}{dt}=M\times10\times\left(10\times10^{-2}\right)=M$ J.s^{-1}. This is case of thermo-mechanical equilibrium where $\frac{dW}{dt}=\frac{dQ}{dt}$. For conduction of heat $\frac{dQ}{dt}=\frac{kA\Delta T}{l}=\frac{0.50\times0.05\times1}{2.0\times10^{-3}}=12.5$J.s^{-1}. Using the equilibrium equation with values arrived at above $M=12.5$kg. **Hence answer is 12.5 Kg.**

I-30	In this problem rate of transfer of heat by conduction $\frac{dQ}{dt}=\frac{kA\Delta T}{l}$ through ice layer of thickness $l=0.10\,\text{m}$ has to be equal to rate of solidification of ice where $\frac{dQ}{dt}=L\frac{dM}{dt}=\frac{L(\sigma A dl)}{dt}$. Equating the two, with the given data we get, $\frac{1.7\times1\times(0-(-10))}{10\times10^{-2}}=\left(\frac{1\times dl\times1000}{dt}\right)\times(3.36\times10^{5})\Rightarrow\frac{dl}{dt}=\frac{1.7\times10^{2}}{3.36\times10^{8}}=0.5\mu\,\text{m.s}^{-1}$. **Therefore answer of part (a) is** 5.0×10^{-7} **m.s^{-1}.** Combining the two equation of $\frac{dQ}{dt}$ we get $\frac{kA\Delta T}{l}=\frac{L(\sigma A dl)}{dt}\Rightarrow dt=\frac{L\sigma}{k\Delta T}ldl$. To determine time taken to form a layer of ice of 10 cm thickness on integration we get $\int_0^t dt=\frac{L\sigma}{k\Delta T}\int_0^{0.1} ldl\Rightarrow t=\frac{L\sigma}{k\Delta T}\left[\frac{l^2}{2}\right]_0^{0.1}=\frac{(3.36\times10^{5})\times1000}{1.7\times10\times2}\times(1\times10^{-2})=1\times10^{5}\,\text{s}=\frac{1\times10^{5}}{60\times60}=27.5$ Hrs. **Therefore answer of part (b) is 27.5 Hrs.**
I-31	In a state of steady-state of thickness of the ice (x) formed, will always remain as top layer since $\sigma_{\text{ice}}<\sigma_{\text{water}}$, and rate of heat transfer by conduction would be $\frac{dQ_{\text{ice}}}{dt}=\frac{dQ_{\text{water}}}{dt}$. In the lake while area would remain same for both water and ice, but thickness of ice and water layers would be $l_{\text{ice}}=x$ and $l_{\text{iwater}}=1-x$, respectively. Since, during conduction $\frac{dQ}{dt}=\frac{kA\Delta T}{l}$. Accordingly, in equilibrium the equation would be $\frac{0.5\times A\times(4-0)}{1.0-x}=\frac{1.7\times A\times(0-(-10))}{x}\Rightarrow2.0\times x=17(1-x)\Rightarrow x=\frac{17}{19}=0.89\text{m}$. **Hence answer is 89 cm.**  **N.B.:** Here data on density of water and latent heat of fusion of ice are redundant and should not be confused.
I-32	The rate of flow of heat by conduction in the instant case is $\frac{dQ}{dt}=\frac{KA\Delta T}{l}$. Accordingly heat flow in each section, using the given data would be as under  **Section AB :** $\frac{dQ}{dt}=\frac{K_{AB}A(T_A-T_B)}{dt}=\frac{50\times1\times10^{-4}\times(40-80)}{0.20}=-1\text{W}$. (-)ve sign shows that heat flow in the section is from B to A. **Section AC :** $\frac{dQ}{dt}=\frac{K_{AC}A(T_A-T_C)}{dt}=\frac{400\times1\times10^{-4}\times(40-80)}{0.20}=-8\text{W}$. (-)ve sign shows that heat flow in the section is from C to A **Section BC :** $\frac{dQ}{dt}=\frac{K_{BC}A(T_A-T_B)}{dt}=\frac{200\times1\times10^{-4}\times(80-80)}{0.20}=0\text{W}$. Since both the ends of the rod are at same temperature leading to $\Delta T=0$. **Thus, answer is 1W, 8W and Zero.**

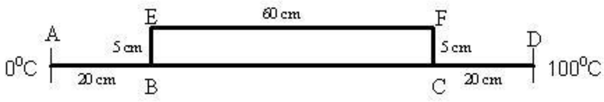
I-33	The rate of flow of heat by conduction in the instant case is $\frac{dQ}{dt}=\frac{KA\Delta T}{l}$. In the given case there are Two path of heat conduction- (1) straight path AB where $l_{AB}=2r$ and (ii) semicircular path APB where $l_{APB}=\pi r$. Accordingly, - $\frac{dQ_{AB}}{dt}=\frac{KA(T_A-T_B)}{2r}$ and $\frac{dQ_{APB}}{dt}=\frac{KA(T_A-T_B)}{\pi r}$. The ratio required to be determined is  $\frac{Q_{APB}}{Q_{AB}}=\frac{\frac{dQ_{APB}}{dt}}{\frac{dQ_{AB}}{dt}}=\frac{\frac{KA(T_A-T_B)}{\pi r}}{\frac{KA(T_A-T_B)}{2r}}=\frac{2}{\pi}\Rightarrow Q_{APB}:Q_{AB}::2:\pi$. **Hence answer is 2:π.**
I-35	In the instant case heat transfer is through conduction where $\frac{dQ}{dt}=\frac{kA\Delta T}{l}$. Let us take an element of tube of length l having radius r and thickness Δr. Heat transfer through $\frac{dQ_r}{dt}=\frac{k(2\pi rl)\Delta\theta}{dr}=K\Rightarrow\Delta\theta=\frac{K}{2k\pi l}\cdot\frac{dr}{r}$. The rate of heat transfer in steady state remains constant all through every infinitesimal element, without temp buildup along thickness of the tube. Hence,  $\int_{\theta_1}^{\theta_2}d\theta=\frac{K}{2k\pi l}\cdot\int_{r_1}^{r_2}\frac{dr}{r}\Rightarrow\theta_2-\theta_1=\frac{K}{2k\pi l}[\ln r]_{r_1}^{r_2}\Rightarrow K=(\theta_2-\theta_1)\cdot\frac{2k\pi l}{\ln\frac{r_2}{r_1}}$. Using the given data the rate of heat flow $K=(30-120)\cdot\frac{2\times0.15\times3.14\times0.50}{\ln\frac{1.2}{1}}=-232.4$ J.s^{-1}. Here (-)ve sign is indicative of the fact that during radial flow of heat temperature gradient is (-)ve along (+)ve direction of r, therefore absolute value of heat flow is 232 J.s-1, considering SGs. **Hence answer is 232 J.s^{-1}.**
I-36	In the instant case heat transfer is through conduction where $\frac{dQ}{dt}=\frac{kA\Delta T}{l}$. Let us take an element of washer, along its width, having radius r and thickness Δr. Heat transfer through $\frac{dQ}{dt}=\frac{k(2\pi rd)\Delta\theta}{dr}=K$. The rate of heat transfer in steady state remains constant all through every infinitesimal element, without temp buildup along thickness of the tube. Hence,  $\int_{\theta_1}^{\theta_2}d\theta=\frac{K}{2k\pi d}\cdot\int_{r_1}^{r_2}\frac{dr}{r}\Rightarrow\theta_2-\theta_1=\frac{K}{2k\pi d}[\ln r]_{r_1}^{r_2}\Rightarrow K=(\theta_2-\theta_1)\cdot\frac{2k\pi d}{\ln\frac{r_2}{r_1}}$. Since $\theta_1>\theta_2$ hence, $\frac{dQ}{dt}=-(\theta_1-\theta_2)\cdot\frac{2k\pi d}{\ln\frac{r_2}{r_1}}$ Here (-)ve sign is indicative of the fact that during radial flow of heat temperature gradient is (-)ve along (+)ve direction of r, therefore absolute value of heat flow is $\frac{2\pi kd(\theta_1-\theta_2)}{\ln\left(\frac{r_2}{r_1}\right)}$. **Hence answer is** $\frac{2\pi kd(\theta_1-\theta_2)}{\ln\left(\frac{r_2}{r_1}\right)}$

I-37	This is the problem of conduction of heat through a tube involving basic equation as per Fourier's Law is $\frac{dQ}{dt}=-kA\frac{d\theta}{dl}$, and is being appled to different geometrical configurations given in the problem.  **Part (a)**: It is a case of axial Conduction. As shown in the figure an element of pipe of thickness Δx is taken a distance x from the end whose cross-section is at temperature T_1. Let cross-section of pipe having area $A=\pi\left(r_2^2-r_1^2\right)$. Thus heat flow through the cross-sectional element of tube of thickness Δx is, $\frac{dQ}{dt}=-k\pi\left(r_2^2-r_1^2\right)\frac{d\theta}{dx}=K$ this is constant since in steady state there is no absorption of heat and therefore constant temperature of each cross-section. $Kdx=-k\pi\left(r_2^2-r_1^2\right)d\theta \Rightarrow \int_0^l Kdx=-\int_{T_1}^{T_2} k\pi\left(r_2^2-r_1^2\right)d\theta \Rightarrow Kl=-k\pi\left(r_2^2-r_1^2\right)\left(T_2-T_1\right)=k\pi\left(r_2^2-r_1^2\right)\left(T_1-T_2\right)\cdot$ This resolves into $K=\frac{dQ}{dt}=\frac{k\pi\left(r_2^2-r_1^2\right)\left(T_1-T_2\right)}{l}$. **Hence answer of part (a) is** $\frac{k\pi\left(R_2^2-R_1^2\right)\left(T_2-T_1\right)}{l}$ **Part (b)** : This is a case of radial conduction. As per Fourier's Law heat transfer by conduction through a small element, as shown in the figure, is $\frac{dQ}{dt}=-\frac{k\left(2\pi rd\right)\Delta\theta}{dr}=K$, having radius r and outer radius $r+\Delta r$ maintained at temperatures T_1 and T_2 respectively. The rate of heat transfer in steady state remains constant all through every infinitesimal element, without temp buildup along thickness of the tube. Hence, $\int_{T_1}^{T_2} d\theta=-\frac{K}{2k\pi d}\cdot\int_{r_1}^{r_2}\frac{dr}{r} \Rightarrow T_2-T_1=-\frac{K}{2k\pi d}\left[\ln r\right]_{r_1}^{r_2} \Rightarrow T_1-T_2=\frac{K}{2k\pi d}\cdot\ln\frac{r_2}{r_1}$. It leads to rate of heat $K=\frac{dQ}{dt}=\left(T_1-T_2\right)\cdot\frac{2k\pi d}{\ln\frac{r_2}{r_1}}$ **Hence answer is** $\frac{2\pi kd\left(T_1-T_2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$ 
I-38	The two slabs are as shown in the figure of thickness L_1 and L_2 having equal area A and having temperatures at their outer surfaces as T_1 and T_3 respectively. The common surface of the Two slabs is at temperature T_2 such that $T_1>T_2>T_3$. Therefore, $\frac{dQ_1}{dt}=\frac{K_1A\left(T_1-T_2\right)}{L_1}$ and $\frac{dQ_2}{dt}=\frac{K_2A\left(T_2-T_3\right)}{L_2}$. Let the thermal conductivity of the composite slab be K, then rate of heat transfer through the system would be $\frac{dQ}{dt}=\frac{KA\left(T_1-T_3\right)}{L_1+L_2}$. Since the system is at equilibrium and therefore rate heat entering the system at temperature T_1 is same at every layer of system including that at temperature T_3 and therefore, $\frac{dQ}{dt}=\frac{dQ_1}{dt}=\frac{dQ_2}{dt}$. Accordingly, equating $\frac{dQ}{dt}$ and $\frac{dQ_1}{dt}$ we get $\frac{K\left(T_1-T_3\right)}{L_1+L_2}=\frac{K_1\left(T_1-T_2\right)}{L_1}$. It leads to $\frac{K\left(T_1-T_3\right)}{L_1+L_2}=\frac{K_1\left(T_1-T_2\right)}{L_1} \Rightarrow K\frac{L_1}{K_1\left(L_1+L_2\right)}\left(T_1-T_3\right)=T_1-T_2$ ...(1). Likewise, equating $\frac{dQ}{dt}$ and 

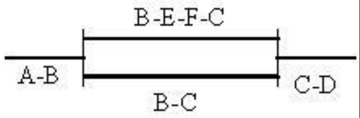
	$\frac{dQ_2}{dt}$ we get $\frac{K(T_1-T_3)}{L_1+L_2}=\frac{K_2(T_2-T_3)}{L_2}\Rightarrow K\frac{L_2}{K_2(L_1+L_2)}(T_1-T_3)=T_2-T_3...(2)$. Adding (1) and (2) $K\frac{L_1}{K_1(L_1+L_2)}(T_1-T_3)+K\frac{L_2}{K_2(L_1+L_2)}(T_1-T_3)=T_1-T_3\Rightarrow K\left(\frac{L_1}{K_1(L_1+L_2)}+\frac{L_2}{K_2(L_1+L_2)}\right)=1$. It solves into $K\left(\frac{K_2L_1+K_1L_2}{K_1K_2(L_1+L_2)}\right)=1\Rightarrow K=\frac{K_1K_2(L_1+L_2)}{K_2L_1+K_1L_2}$. **Thus answer is** $\frac{K_1K_2(L_1+L_2)}{L_1K_2+L_2K_1}$.
I-39	This is the case of series combination of two rods of copper $\left(K_1=390\,\text{W.m}^{-1}.{}^0\text{C}^{-1}\right)$ and steel $\left(K_2=46\,\text{W.m}^{-1}.{}^0\text{C}^{-1}\right)$ having equal length L and area A and hence thermal conductivity of the series combination would be $K=\frac{K_1K_2(L+L)}{LK_2+LK_1}\Rightarrow K=\frac{390\times46\times2}{390+46}=82$ W.m^{-1}.^{0}C^{-1}. Thus with the given temperature difference across the combination rate of heat transfer would be $\frac{dQ_{\text{comb}}}{dt}=\frac{82A(100-0)}{2L}=4100\frac{A}{L}$. Let temperature at the junction of the combination be θ therefore rate of heat transfer through copper rod would be $\frac{dQ_{\text{Cu}}}{dt}=\frac{390A(\theta-0)}{L}=390\frac{A\theta}{L}$. In state of equilibrium rate of heat transfer through every section of the combination would be same accordingly $\frac{dQ_{\text{comb}}}{dt}=\frac{dQ_{\text{Cu}}}{dt}\Rightarrow 4100\frac{A}{L}=390\frac{A\theta}{L}\Rightarrow\theta=\frac{4100}{390}=10.5\,{}^0$C. **Hence answer is 10.5^{0}C.**
I-40	Rate of heat transfer by conduction is $\frac{dQ}{dt}=\frac{KA\Delta\theta}{l}$. Since temperature difference across both the copper and aluminum rods connected in parallel are $\Delta\theta=60-20=40^0$C, therefore $\frac{dQ}{dt}=\frac{dQ_{\text{Cu}}}{dt}+\frac{dQ_{\text{Al}}}{dt}$. On substituting the given data $\frac{dQ}{dt}=\frac{390\times(1\times10^{-4})\times40}{1}+\frac{200\times(1\times10^{-4})\times40}{1}=2.36\,\text{J}$. **Hence answer is 2.36 J.**
I-41	Rate of heat transfer through conduction is $\frac{dQ}{dt}=\frac{KA\Delta\theta}{l}$.  Accordingly, in aluminum section towards the cold junction $\frac{dQ_{\text{Al}}}{dt}=\frac{K_{\text{Al}}A\Delta\theta}{l}=\frac{200\times(0.20\times10^{-4})\times(80-40)}{0.20}=0.80$ and in copper section towards the cold joint is $\frac{dQ_{\text{Cu}}}{dt}=\frac{K_{\text{Cu}}A\Delta\theta}{l}=\frac{400\times(0.20\times10^{-4})\times(80-40)}{0.20}=1.60$. Thus total heat taken out of the cold joint is $\frac{dQ}{dt}=\frac{dQ_{\text{Al}}}{dt}+\frac{dQ_{\text{Al}}}{dt}=0.8+1.6=2.4\,\text{J.s}^{-1}$. Therefore, net rate of heat taken out of the cold joint in one minute is $=2.4\times60=144\,\text{J.m}^{-1}$. Hence answer is 144 J

I-42

Let temperature at points B and C are T_B and T_C respectively. The cross-sectional area of each part of the frame be A and thermal conductivity of the material be K. Heat transfer through conduction is $\frac{dQ}{dt}=\frac{KA\Delta\theta}{l}$.



The system can be visualized as shown in the figure and accordingly heat flowing in section AB is $\frac{dQ_{AB}}{dt}=\frac{KA(T_B-0)}{0.20}=\frac{KAT_B}{0.20}$; in section B-C it is $\frac{dQ_{BC}}{dt}=\frac{KA(T_C-T_B)}{0.60}$; in section B-E-F-C it is $\frac{dQ_{BEFC}}{dt}=\frac{KA(T_C-T_B)}{0.05+0.60+0.05}=\frac{KA(T_C-T_B)}{0.70}$ and in section C-D it is $\frac{dQ_{CD}}{dt}=\frac{KA(100-T_C)}{0.20}$. When system is in equilibrium



$\frac{dQ_{AB}}{dt}=\frac{dQ_{CD}}{dt}=170$ J.s^{-1}. Accordingly, in section A-B, $\frac{KAT_B}{0.20}=170\Rightarrow T_B=\frac{34}{KA}$ and in section C-D it is $\frac{KA(100-T_C)}{0.20}=170\Rightarrow 100-T_C=\frac{34}{KA}\Rightarrow T_C=100-\frac{34}{KA}$. Therefore, heat flowing in section B-C is $\frac{dQ_{BC}}{dt}=\frac{KA(T_C-T_B)}{0.60}$; substituting the intermediate temperatures derived above

$$\frac{dQ_{BC}}{dt}=\frac{KA\left(\left(100-\frac{34}{KA}\right)-\frac{34}{KA}\right)}{0.60}=\frac{100KA-68}{0.60}$$ and

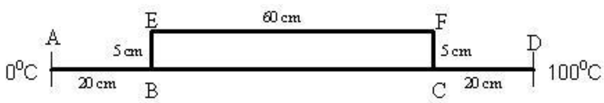
$$\frac{dQ_{BEFC}}{dt}=\frac{KA\left(\left(100-\frac{34}{KA}\right)-\frac{34}{KA}\right)}{0.70}=\frac{100KA-68}{0.70}.$$ It leads to Since $$\frac{\frac{dQ_{BC}}{dt}}{\frac{dQ_{BEFC}}{dt}}=\frac{\frac{100KA-68}{0.60}}{\frac{100KA-68}{0.70}}\Rightarrow\frac{\frac{dQ_{BC}}{dt}}{\frac{dQ_{BEFC}}{dt}}=\frac{0.7}{0.6}.$$

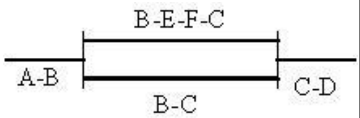
Applying componendo, $$\frac{\frac{dQ_{AB}}{dt}}{\frac{dQ_{BEFC}}{dt}}=\frac{1.3}{0.6}\Rightarrow\frac{dQ_{BEFC}}{dt}=130\times\frac{0.6}{0.7}=60$$ J. Thus answer is 60 J.

N.B.: This is an example of simplification using compnendo of algebra in simplifying the calculations

I-43

Let temperatures at point B and C of the system be are T_B and T_C respectively. While, the cross-sectional area of each section is same the lengths of each section of heat conduction are shown in the figure. And conductivity of section BEFC is $K_1=780$ J.s^{-1}.m^{-1}.^{0}C^{-1} while that of section BC is $K_2=390$ J.s^{-1}.m^{-1}.^{0}C^{-1} Therefore rate of conduction of heat in section BCDEF is



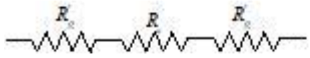
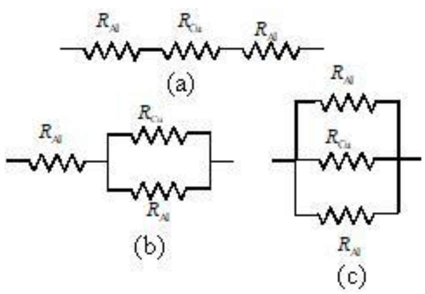


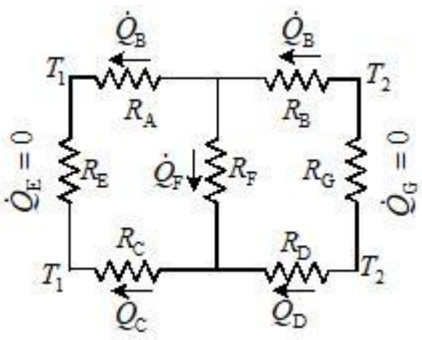
$$\frac{\frac{dQ_1}{dt}}{\frac{dQ_2}{dt}}=\frac{\frac{K_1A(T_C-T_B)}{0.05+0.60+0.05}}{\frac{K_2A(T_C-T_B)}{0.60}}\Rightarrow\frac{\frac{dQ_1}{dt}}{\frac{dQ_2}{dt}}=\frac{0.60K_1}{0.70K_2}=\frac{0.60\times780}{0.70\times390}=\frac{12}{7}\Rightarrow\frac{dQ_1}{dt}:\frac{dQ_2}{dt}::12:7$$

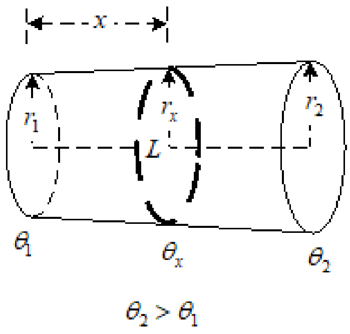
. **Hence answer is 12:7.**

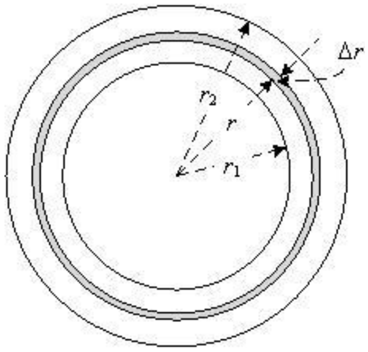
I-44

Heat transfer through conduction is $\frac{dQ}{dt}=\frac{KA\Delta\theta}{l}=\frac{\Delta\theta}{R}$, here $R=\frac{l}{KA}$ is thermal resistance of the heat

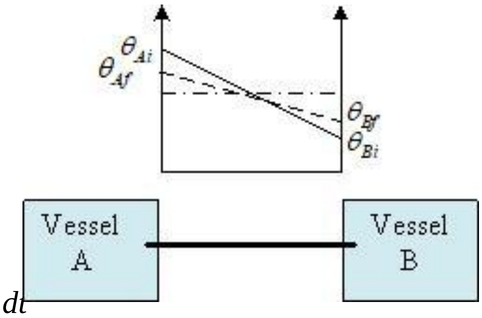
	conduction medium. The problem is solved part-wise. **Part (a):** In case of heat conduction through window $R_g = \dfrac{2\times10^{-3}}{1\times(1\times2)} = 1\times10^{-3}$ hence $\dfrac{dQ_g}{dt} = \dfrac{(40-32)}{1\times10^{-3}} = 8000$ J.s^{-1}. **This is answer of Part (a)** **Part (b):** In this case air film of 1mm thickness is sandwiched between two glass panes each of thickness 1 mm it forms a series combinations of resistances with effective resistance such that $R_g^{'} = \dfrac{1\times10^{-3}}{1\times(1\times2)} = 5\times10^{-4}$ and $R_a = \dfrac{1\times10^{-3}}{0.025\times(1\times2)} = 2\times10^{-2}$. Thus in this  case effective thermal resistance $R = R_g^{'} + R_a + R_g^{'} = 2\times5\times10^{-4} + 2\times10^{-2} = 21\times10^{-3}$. Hence, net rate of conduction of heat is conductivity would be $\dfrac{dQ_g}{dt} = \dfrac{(40-32)}{21\times10^{-3}} = 381$ J.s^{-1}. **This is answer of Part (b).** **Hence answers are (a) 8000 J.s^{-1} (b) 381 J.s^{-1}**
I-45	Heat transfer through conduction is $\dfrac{dQ}{dt} = \dfrac{KA\Delta\theta}{l} = \dfrac{\Delta\theta}{R}$, here $R = \dfrac{l}{KA}$ is thermal resistance of the heat conduction medium. Let thermal resistance of part A is R_A and resistance of part B is R_A. Therefore, thermal current through the combination is $\dfrac{dQ}{dt} = \dfrac{\theta_1 - \theta_2}{R_A + R_B}$. And at junction of A-B whose temperature is $\theta_{\text{J}} = 70\,^0$C. Therefore, $\dfrac{dQ}{dt} = \dfrac{\theta_1 - \theta_{\text{J}}}{R_A} = \dfrac{\theta_{\text{J}} - \theta_2}{R_B}$. Accordingly, $\dfrac{R_A}{R_B} = \dfrac{100-70}{70-0} = \dfrac{30}{70}$. When rods are interchanged net resistance between the two heat source and sink would remain unchanged but junction temperature would take a new value $\theta_{\text{J}}^{'}$ such that $\dfrac{dQ}{dt} = \dfrac{\theta_1 - \theta_{\text{J}}^{'}}{R_B} = \dfrac{\theta_{\text{J}}^{'} - \theta_2}{R_A} \Rightarrow \theta_{\text{J}}^{'}R_B - \theta_2 R_B = \theta_1 R_A - \theta_{\text{J}}^{'}R_A \Rightarrow \theta_{\text{J}}^{'}(R_A + R_A) = \theta_1 R_A + \theta_2 R_B$. Using the given data and the derived values $\theta_{\text{J}}^{'}(R_A + R_B) = 100R_A + 0 \Rightarrow \theta_{\text{J}}^{'} = 100\dfrac{R_A}{R_A + R_B}\,^0$C. Using invertendo-componendo the junction temperature in new condition is $\theta_{\text{J}}^{'} = 100\times\dfrac{30}{100} = 30\,^0$C. **Hence answer is 30^{0}C** .
I-46	Heat transfer through conduction is $\dfrac{dQ}{dt} = \dfrac{KA\Delta\theta}{l} = \dfrac{\Delta\theta}{R}$, here $R = \dfrac{l}{KA}$ is thermal resistance of the heat conduction medium. With the given data electrical analog of the heat conduction systems is shown in the figure for all the three cases respectively. Therefore, equivalent thermal resistance in the arrangement (a) would be $R_{\text{a}} = R_{\text{Al}} + R_{\text{Cu}} + R_{\text{Al}} = 2R_{\text{Al}} + R_{\text{Cu}}$. Accordingly temperature  difference across two ends of the arrangement would be $\Delta\theta = \dfrac{dQ_a}{dt}R_a = 40\times\dfrac{l}{A}\left(\dfrac{2}{200} + \dfrac{1}{400}\right) = \dfrac{l}{2A}$. Now in arrangement (b) is a series parallel combination whose equivalent resistance is

	$R_b = R_{\mathrm{Al}} + \frac{R_{\mathrm{Al}}R_{\mathrm{Cu}}}{R_{\mathrm{Al}}+R_{\mathrm{Cu}}} = \frac{l}{200A} + \frac{\frac{l}{200A}\times\frac{l}{400A}}{\frac{l}{200A}+\frac{l}{400A}} = \frac{l}{150A}$. Therefore, rate of heat transfer maintaining temperature difference across extreme ends would be $\frac{dQ_b}{dt} = \frac{\Delta\theta}{R_b} = \frac{\frac{l}{2A}}{\frac{l}{150A}} = 75$ W **is the answer for arrangement (b).** Next in arrangement (c) $\frac{1}{R_c} = \frac{1}{R_{\mathrm{Al}}} + \frac{1}{R_{\mathrm{Cu}}} + \frac{1}{R_{\mathrm{Al}}} = \frac{R_{\mathrm{Al}}+2R_{\mathrm{Cu}}}{R_{\mathrm{Al}}R_{\mathrm{Cu}}} = \frac{\frac{l}{200A}+\frac{2l}{400A}}{\frac{l}{200A}\times\frac{l}{400A}} = 800\frac{A}{l}$. Therefore rate of heat transfer maintaining temperature difference across extreme ends would be $\frac{dQ_c}{dt} = \frac{\Delta\theta}{R_c} = \frac{l}{2A}\times\frac{900A}{l} = 400$ W **is the answer for arrangement (c).** **Thus answer is 75 W and 400 W.** **N.B.:** This is a case of analysis of a problem of thermal conduction using its electrical analog
I-47	Heat transfer through conduction is $\frac{dQ}{dt} = \frac{KA\Delta\theta}{l} = \frac{\Delta\theta}{R}$, here $R = \frac{l}{KA}$ is thermal resistance of the heat conduction medium. It is apparent from the figure that point B is mid of length CD and hence net lengths of section F-C-B and E-D-B $l_{\mathrm{FCB}} = l_{\mathrm{EDB}} = \frac{3}{2}l$. Accordingly, resistances $R_{\mathrm{FCB}} = R_{\mathrm{EDB}} = \frac{3l}{2KA}$ while $R_{\mathrm{AB}} = \frac{l}{KA}$. Now let temperature of the junction B is T_{B} and there is no loss of heat to the atmosphere and hence net rate of heat transfer at the junction, as per principle conservation of energy is Zero, Accordingly, the equation would be $\frac{dQ_{\mathrm{FB}}}{dt} + \frac{dQ_{\mathrm{AB}}}{dt} + \frac{dQ_{\mathrm{EB}}}{dt} = 0 \Rightarrow \frac{2KA(T_3 - T_B)}{3l} + \frac{KA(T_1 - T_B)}{l} + \frac{2KA(T_2 - T_B)}{3l} = 0 \Rightarrow T_1 + \frac{2}{3}(T_2 + T_3) = \left(\frac{4}{3}+1\right)T_B$. It solves into $T_{\mathrm{B}} = \frac{3T_1 + 2(T_2 + T_3)}{7}$. **Hence answer is** $\frac{3T_1 + 2(T_2 + T_3)}{7}$
I-48	Heat transfer through conduction is $\frac{dQ}{dt} = \frac{KA\Delta\theta}{l} = \frac{\Delta\theta}{R}$, here $R = \frac{l}{KA}$ is thermal resistance of the heat conduction medium. With the given data there would be no flow of heat through elements E and G as their temperature are kept constant at T_1 and T_2 respectively, such that $T_2 > T_1$. Let temperature of junction of rods A,B and F be T_{ABF} and likewise temperature of junction of rods C,D and F be T_{CDF}. From the rate of heat flows assumed in the given arrangement $\dot{Q}_{\mathrm{B}} = \dot{Q}_{\mathrm{A}} + \dot{Q}_{\mathrm{F}}$ and $\dot{Q}_{\mathrm{C}} = \dot{Q}_{\mathrm{D}} + \dot{Q}_{\mathrm{F}}$. With the given thermal conductivities of different rods, their resistances would be 

	$R_A = R_A = \frac{l}{AK_o}$; and $R_F = \frac{l}{4AK_o}$. Accordingly, $\dot{Q}_B = (T_2 - T_{ABF}) \times \frac{2AK_0}{l}$; $\dot{Q}_A = (T_{ABF} - T_1)\times\frac{AK_0}{l}$; $\dot{Q}_D = (T_2 - T_{CDF})\times\frac{2AK_0}{l}$; $\dot{Q}_C = (T_{CDF}-T_1)\times\frac{AK_0}{l}$ and $\dot{Q}_F = (T_{ABF}-T_{CDF})\times\frac{4AK_0}{l}$. From these values heat transfer equations are rewritten as under – $\cdot(T_2 - T_{ABF})\times\frac{2AK_0}{l} = (T_{ABF}-T_1)\times\frac{1AK_0}{l} + (T_{ABF}-T_{CDF})\times\frac{4AK_0}{l} \Rightarrow 2(T_2-T_{ABF}) = (T_{ABF}-T_1)+4(T_{ABF}-T_{CDF})$ It leads to $7T_{ABF} - 4T_{CDF} = 2T_2 + T_1$...(1) $(T_2 - T_{DCF})\times\frac{2AK_0}{l} + (T_{ABF}-T_{CDF})\times\frac{4AK_0}{l} = (T_{CDF}-T_1)\times\frac{1AK_0}{l} \Rightarrow 2(T_2-T_{DCF})+4(T_{ABF}-T_{CDF}) = (T_{CDF}-T_1)\cdot$ It leads to $7T_{CDF} - 4T_{ABF} = 2T_2 + T_1$...(2). Since RHS of equation (1) and (2) are equal and hence equating LHS we get $7T_{ABF} - 4T_{CDF} = 7T_{CDF} - 4T_{ABF} \Rightarrow T_{ABF} = T_{CDF}$. **Part (a):** Since both the ends of rod F are same temperature the rod has uniform temperature. Accordingly, taking any of the two equation, say Eqn (1), we get temperature of rod F as $T_{ABF} = T_{CDF} = T_F = \frac{T_1 + 2T_2}{3}$, **hence proved.** **Part (b):** Since there is no flow of heat in rod B and hence rate of flow heat from source is sum of the rate of flow of heat in rod B and D $Q = \frac{(T_2 - T_{\mathrm{ABF}})}{R_B} + \frac{(T_2 - T_{\mathrm{CDF}})}{R_B} = \frac{(T_2 - T_{\mathrm{ABF}})}{R_B} + \frac{(T_2 - T_{\mathrm{ABF}})}{R_B}$. Using the values derived above $Q = \left(T_2 - \frac{T_1+2T_2}{3}\right)\left(2\times\frac{2AK_0}{l}\right) = \frac{4K_0A(T_2-T_1)}{3l}$. **Hence, answer of part (b) is** $\frac{4K_0A(T_2-T_1)}{3l}$
I-49	 As per Fourier's Law of heat conduction $\frac{dQ}{dt} = -kA\frac{d\theta}{dl} = K$. In the given conical rod a small element of thickness Δx and radius r is taken at a distance x from the end having radius r_x. In steady state equation for rate of conduction of heat through every infinitesimal section of thickness Δx is constant K can be written as $K\int_0^L \frac{dx}{r_x^2} = -k\int_{\theta_1}^{\theta_2} \pi d\theta$. From geometry $r_x = r_1 + (r_2 - r_1)\frac{x}{L}$ and $dr_x = \left(\frac{r_2-r_1}{L}\right)dx \Rightarrow dx = \left(\frac{L}{r_2-r_1}\right)dr_x$ Accordingly substituting in the integration $\frac{KL}{r_2-r_1}\int_{r_1}^{r_2}\frac{dr_x}{r_x^2} = -k\int_{\theta_1}^{\theta_2}\pi d\theta$. Solving the integral equation $\frac{KL}{r_2-r_1}\left[-\frac{1}{r_x}\right]_{r_1}^{r_2} = -k\pi[\theta]_{\theta_1}^{\theta_2} \Rightarrow \frac{KL}{r_2-r_1}\left[\frac{1}{r_2}-\frac{1}{r_1}\right] = k\pi(\theta_2-\theta_1)$. This solves into $K\frac{L}{r_1r_2} = -k\pi(\theta_2-\theta_1) \Rightarrow K = -\frac{k\pi r_1 r_2(\theta_2-\theta_1)}{L}$. Here, flow of heat is taken to be from end of radius r_1 to

	the end r_2 where temperatures are θ_1 and θ_2 such that $\theta_2 > \theta_1$. Since flow of heat during conduction is from higher to lower temperature hence there is (-)ve sign in the arrived value. **Thus answer is** $\frac{k\pi r_1 r_2\left(\theta_2-\theta_1\right)}{L}$.
I-50	As per Fourier's Law of heat conduction $\frac{dQ}{dt}=-KA\frac{d\theta}{L}$. Rate of rise of temperature $\frac{d\theta}{dt}=\frac{60-0}{10\times 60}=0.10$ ^{0}C.s^{-1}. With the given data $\frac{dQ}{dt}=-\frac{200\times\left(1\times 10^{-4}\right)}{0.20}\Delta\theta \Rightarrow Q_{\text{Total}}=\sum_{1}^{600}\Delta Q_n=0.10\sum_{1}^{600}\Delta\theta_n$. Now at $t=0\rightarrow\Delta\theta=0$ and one second later $t=1\rightarrow\Delta\theta=0.10\,^0$C. Therefore, based on mean value theorem $\theta_1=\frac{0+0.10}{2}=0.05\,^0$C. With the $\frac{d\theta}{dt}=0.10$ $\theta_2=0.05+0.10=0.15\,^0$C. The RHS forms an arithmetic progression whose sum is $\sum_{1}^{600}\Delta\theta_n=\frac{600}{2}\left[\theta_1+\left(600-1\right)\Delta\theta\right]=300\left(0.05+599\times 0.10\right)=17985$. Using this value in the equation for total heat transfer in 10 minutes $Q_{\text{Total}}=\sum_{1}^{600}\Delta Q_n=0.10\times 17985=1798$. Since in the given data SDs are Two and **hence answer is 1800**
I-51	 As per Fourier's Law of heat conduction $\frac{dQ}{dt}=-kA\frac{d\theta}{dr}$. Taking an elemental in between the two sphere such that $r_1\le r\le r_2$ of thickness Δr whose area is $A=4\pi r^2$. Temperature of sphere of radius $r_1=5\times 10^{-2}$ m is 50^{0}C and $r_1=20\times 10^{-2}$ m is 10^{0}C a. In steady state heat conduction $\frac{dQ}{dt}=K=100$ J.s^{-1}. Let thermal conductivity of the medium filled in between the Two sphere is k. Hence, the equation of heat conduction is rewritten as $\int_{r_1}^{r_2}\frac{K}{k\left(4\pi r^2\right)}dr=-\int_{\theta_1}^{\theta_2}d\theta \Rightarrow \frac{K}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2}=-\left[\theta\right]_{\theta_1}^{\theta_2}$. This solves into $\frac{K}{4\pi k}\left[-\frac{1}{r}\right]_{r_1}^{r_2}=-\left(\theta_2-\theta_1\right)\Rightarrow \frac{K}{4\pi k}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\left(\theta_2-\theta_1\right)\Rightarrow k=\frac{K}{4\pi}\left(\frac{r_1-r_2}{r_1 r_2}\right)\cdot\frac{1}{\theta_2-\theta_1}$. On substituting the given data $k=\frac{100}{4\pi}\left(\frac{5\times 10^{-2}-20\times 10^{-2}}{\left(5\times 10^{-2}\right)\times\left(20\times 10^{-2}\right)}\right)\cdot\frac{1}{10-50}=\frac{100}{4\pi}\times\frac{15}{1}\times\frac{1}{40}=2.98$ W.m^{-1}.^{0}C^{-1}. **Thus as per SDs the answer is 3.0 W.m^{-1}.^{0}C^{-1}.**
I-52	Let temperature of water in the two vessels A and B initially at temperatures θ_1 and θ_2 such that $\theta_1 > \theta_2$. Rate of heat conduction is $\frac{dQ}{dt}=\frac{KA\Delta\theta}{L}$ …(1) As heat is conducted from water in vessel A to B, temperature of water in vessel A falls $\frac{dQ}{dt}=ms\frac{d\theta}{dt}$ and likewise temperature of water in vessel B rises with an equal amount since mass of water in both the vessels is same as much as heat transfer is also same. Therefore, combining the two equations for vessel A, during an infinitesimal time, equation (1) would be - $\frac{dQ}{dt}=\frac{ms\left(\theta_A-\theta_A^{'}\right)}{dt}=\frac{KA\left(\theta_A^{'}-\theta_B^{'}\right)}{L}\Rightarrow \theta_A^{'}=\theta_A-\frac{KA}{msL}\left(\theta_A^{'}-\theta_B^{'}\right)dt$.

Likewise for vessel B equation (2) would be–



$$\frac{dQ}{dt}=\frac{ms\left(\theta_B^{'}-\theta_B\right)}{dt}=\frac{KA\left(\theta_A^{'}-\theta_B^{'}\right)}{L}\Rightarrow\theta_B^{'}=\theta_B+\frac{KA}{msL}\left(\theta_A^{'}-\theta_B^{'}\right)dt$$

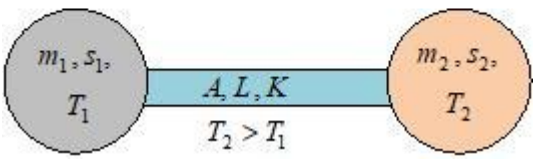
Subtracting Eqn (2) from Eqn (1) we get $\theta_A^{'}-\theta_B^{'}=\theta_A-\theta_B-\frac{KA}{msL}\left(\theta_A^{'}-\theta_B^{'}\right)dt-\frac{KA}{msL}\left(\theta_A^{'}-\theta_B^{'}\right)dt$. It simplifies into $\theta_A^{'}-\theta_B^{'}=\theta_A-\theta_B-\frac{2KA}{msL}\left(\theta_A^{'}-\theta_B^{'}\right)dt\Rightarrow\Delta\theta'=\theta_A-\theta_B-\frac{2KA}{msL}\Delta\theta'dt$. Differentiating this equation w.r.t. t we get $\frac{d\Delta\theta'}{dt}=-\frac{2KA}{msL}\Delta\theta'\Rightarrow\frac{d\Delta\theta'}{\Delta\theta'}=-\frac{2KA}{msL}dt$. Here, $\Delta\theta'$ which is driving the forward process of heat transfer while $\left(\theta_A-\theta_B\right)$ becomes pre-transfer constant and hence $\frac{d\left(\theta_A-\theta_B\right)}{dt}=0$. Accordingly integrating the equation in its last form $\int_{\theta_{1i}-\theta_{2i}}^{\frac{\theta_{1i}-\theta_{2i}}{2}}\frac{d\Delta\theta'}{\Delta\theta'}=-\frac{2KA}{msL}\int_0^t dt\Rightarrow\left[\ln\Delta\theta'\right]_{\theta_{1i}-\theta_{2i}}^{\frac{\theta_{1i}-\theta_{2i}}{2}}=-\frac{2KA}{msL}t$. It further solves into $\frac{2KA}{msL}t=-\left[\ln\left(\frac{\theta_{1i}-\theta_{2i}}{2}\right)-\ln\left(\theta_{1i}-\theta_{2i}\right)\right]\Rightarrow t=\frac{msL}{2KA}\left[\ln\left(\theta_{1i}-\theta_{2i}\right)-\ln\left(\frac{\theta_{1i}-\theta_{2i}}{2}\right)\right]$. On its final form

$t=\frac{msL}{2KA}\ln 2$. **Hence answer is** $\frac{msL}{2KA}\ln 2$ **s.**

N.B.: In this case choice between $\Delta\theta$ and $\Delta\theta'$ as discussed above is important as it changes the characteristic of the equation

I-53

Let temperature of mass m_1 and m_2 are initially at temperatures T_1 and T_2 such that $T_2>T_1$. Rate of heat conduction is $\frac{dQ}{dt}=\frac{KA\Delta T}{L}$ …(1) As heat is conducted from mass m_2 to m_1 and, temperature of mass m_2 falls $\frac{dQ}{dt}=m_2s_2\frac{dT}{dt}$ and likewise temperature of mass m_1 rises.



Therefore, combining the two equations for mass m_2, during an infinitesimal time it would be -

$$\frac{dQ}{dt}=\frac{m_2s_2\left(T_2-T_2^{'}\right)}{dt}=\frac{KA\left(T_2^{'}-T_1^{'}\right)}{L}\Rightarrow T_2^{'}=T_2-\frac{KA}{m_2s_2L}\left(T_2^{'}-T_1^{'}\right)dt \quad \ldots (1)$$

Likewise for mass m_1, it would be–

$$\frac{dQ}{dt}=\frac{m_1s_1\left(T_1^{'}-T_1\right)}{dt}=\frac{KA\left(T_2^{'}-T_1^{'}\right)}{L}\Rightarrow T_1^{'}=T_1+\frac{KA}{m_1s_1L}\left(T_2^{'}-T_1^{'}\right)dt \quad \ldots (2)$$

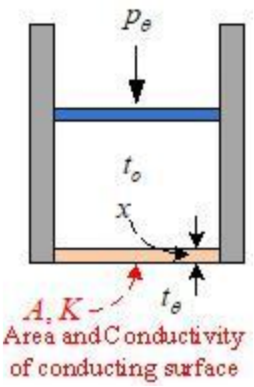
Subtracting Eqn (2) from Eqn (1) we get $T_2^{'}-T_1^{'}=T_2-T_1-\frac{KA}{m_2s_2L}\left(T_2^{'}-T_1^{'}\right)dt-\frac{KA}{m_1s_1L}\left(T_2^{'}-T_1^{'}\right)dt$. It leads to $T_2^{'}-T_1^{'}=T_2-T_1-\frac{KA}{L}\left(\frac{1}{m_2s_2}+\frac{1}{m_1s_1}\right)\left(T_2^{'}-T_1^{'}\right)dt\Rightarrow\Delta T'=T_2-T_1-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1m_2s_1s_2}\right)\Delta T'dt$, here temperature difference between the two masses at any instant t is $\Delta T'=T_2^{'}-T_1^{'}$.

Differentiating this equation w.r.t. t we get $\frac{d\Delta T'}{dt}=-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1m_2s_1s_2}\right)\Delta T'\Rightarrow\frac{d\Delta T'}{dt}=-\lambda\Delta T'$, here $\lambda=\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1m_2s_1s_2}\right)$ On separating the variables $\frac{d\Delta T'}{\Delta T'}=-\lambda dt$. Here , $\Delta T'$which is driving the forward process of heat transfer while $\Delta T=(T_2-T_1)$becomes pre-transfer constant and hence $\frac{d(T_2-T_1)}{dt}=0$. Accordingly, on integration of the equation in its final form we get $\int_{T_2-T_1}^{T_{2t}-T_{1t}}\frac{d\Delta T'}{\Delta T'}=-\lambda\int_0^t dt\Rightarrow[\ln\Delta T']_{T_2-T_1}^{T_{2t}-T_{1t}}=-\lambda t\Rightarrow\ln\frac{T_{2t}-T_{1t}}{T_2-T_1}=-\lambda t\Rightarrow T_{2t}-T_{1t}=(T_2-T_1)e^{-\lambda t}$. **Hence answer is** $(T_2-T_1)e^{-\lambda t}$ **°C, here** $\lambda=\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1m_2s_1s_2}\right)$

N.B.: In this case choice between ΔT and $\Delta T'$as discussed above is important as it changes the characteristic of the equation

I-54

Rate of heat transfer through conducting surface and , using given data is , at any instant when temperature of gas in cylinder is T, then $\frac{dQ}{dt}=\frac{KA\Delta\theta}{l}\Rightarrow\frac{dQ}{dt}=\frac{KA(T_\theta-T)}{x}\ldots(1)$



Since gas is monatomic and hence, $U=n\frac{3}{2}RT$, therefore $\frac{dU}{dt}=n\left(\frac{3}{2}R\right)\frac{dT}{dt}\Rightarrow\frac{dU}{dt}=nC_V\left.\frac{dT}{dt}\right|_{C_V=\frac{3}{2}R}$. Since pressure in the gas is balanced by atmospheric pressure p_θ, therefore heat transfer from conducting base shall utilized in constant pressure process therefore $\frac{dQ}{dr}=nC_p\frac{dT}{dt}\ldots(2)$

Further $C_p=C_V+R=\frac{3}{2}R+R=\frac{5}{2}R$.

Accordingly combining eqn (1) and (2) alongwith value of $C_p=\frac{5}{2}R$ the equation is rewritten as $\frac{dQ}{dr}=nC_p\frac{dT}{dt}\Rightarrow\frac{KA(T_\theta-T)}{x}=\frac{5}{2}nR\frac{dT}{dt}\Rightarrow\frac{dT}{(T_\theta-T)}=\frac{2KA}{5nRx}dt$. On integrating the equation we get $\int_{T_\theta-T_0}^{T_\theta-T}\frac{dT}{(T_\theta-T)}=\frac{2KA}{5nRx}\int_0^t dt\Rightarrow[\ln(T_\theta-T)]_{T_\theta-T_0}^{T_\theta-T}=-\frac{2KA}{5nRx}t\Longrightarrow\ln\frac{(T_\theta-T)}{(T_\theta-T_0)}=-\frac{2KA}{5nRx}t\Rightarrow\frac{(T_\theta-T)}{(T_\theta-T_0)}=e^{-\frac{2KA}{5nRx}t}$.

This leads to $T=T_\theta-(T_\theta-T_0)e^{-\frac{2KA}{5nRx}t}$. Further as per Ideal Gas Equation $pV=nRT\Rightarrow p\Delta V=nR\Delta T$. Thus in the given case combining IGL and temperature of gas enclosed in an isobaric process is $p_\theta A\Delta l=nR(T-T_0)\Rightarrow\Delta l=\frac{nR}{p_\theta A}\left(\left(T_\theta-(T_\theta-T_0)e^{-\frac{2KA}{5nRx}t}\right)-T_0\right)\Rightarrow\Delta l=\frac{nR}{p_\theta A}(T_\theta-T_0)\left(1-e^{-\frac{2KA}{5nRx}t}\right)$.

Thus answer is $\frac{nR}{P_aA}(T_\theta-T_0)\left(1-e^{-\frac{2kAt}{5Rnx}}\right)$

	N.B.: This is again a case of integration of concepts.
I-55	As per Stefan's Law of heat radiation $\frac{dQ}{dt}=A\sigma T^4$. Accordingly using the given data $\frac{dQ}{dt}=1.6\times\left(6.0\times10^{-8}\right)\times\left(273+37\right)^4=887$ J.s^{-1}. **Hence answer is 887 J** **N.B.:** In Stefan's Law temperature is taken in Kelvin and hence necessary conversion is done.
I-56	As per Stefan's Law of heat radiation $\frac{dQ}{dt}=Ae\sigma T^4$. Accordingly using the given data $\frac{dQ}{dt}=\left(12\times10^{-4}\right)\times0.80\times\left(6.0\times10^{-8}\right)\times\left(273+20\right)^4=0.42$ J.s^{-1}. **Hence answer is 0.42 J** **N.B.:** In Stefan's Law temperature is taken in Kelvin and hence necessary conversion is done.
I-57	Surface area of the aluminum sphere is $S_{Al}=4\pi R^2$ and copper sphere is $S_{Cu}=4\pi\left(2R\right)^2=16\pi R^2$. As per Stefan's Law of heat radiation $\frac{dQ}{dt}=Ae\sigma T^4$. Since, both the sphere are heated to same temperature and hence $\frac{dQ_{Al}}{dt}=\left(4\pi R^2\right)e\sigma T^4$ and $\frac{dQ_{Cu}}{dt}=\left(16\pi R^2\right)e\sigma T^4$. **Part (a):** The required ratio is $\frac{\frac{dQ_{Al}}{dt}}{\frac{dQ_{Cul}}{dt}}=\frac{\left(4\pi R^2\right)e\sigma T^4}{\left(16\pi R^2\right)e\sigma T^4}=\frac{1}{4}$. **Hence answer is 1:4** **Part (b):** Rate of fall of temperature is $\frac{dQ}{dt}=ms\frac{d\theta}{dt}\Rightarrow\frac{d\theta}{dt}=\frac{1}{ms}\times\frac{dQ}{dt}$. Therefore, the required ratio is $\frac{\frac{d\theta_{Al}}{dt}}{\frac{d\theta_{Cu}}{dt}}=\frac{\frac{1}{m_{Al}s_{Al}}}{\frac{1}{m_{Cu}s_{Cu}}}\times\frac{\frac{dQ_{Al}}{dt}}{\frac{dQ_{Cul}}{dt}}=\frac{1}{\frac{4}{3}\pi R^3\rho_{Al}s_{Al}}\times\frac{\frac{4}{3}\pi\left(2R\right)^3\rho_{Cu}s_{Cu}}{1}\times\frac{1}{4}=8\times\frac{3.4}{1}\times\frac{390}{900}\times\frac{1}{4}=2.9$. **Hence ratio of rate of cooling is 2.9:1.**
I-58	As per Stefan's law $\frac{dQ}{dt}=Ae\sigma T^4$. Power of the bulb is rate of heat radiation and accordingly with the given data $100=\left(2\pi rl\right)\times0.8\times\left(6.0\times10^{-8}\right)T^4\Rightarrow T=\sqrt[4]{\frac{100}{\left(2\times\pi\times\left(4\times10^{-5}\right)\times1.0\right)\times4.8\times10^{-8}}}$. This solves to $T=\sqrt[4]{8.3\times10^{12}}=1.7\times10^3=1700$ K. **Hence answer is 1700 K.**
I-59	As per Stefan's heat absorption by the ball is $\frac{dQ}{dt}=A\sigma T^4$, and using the given data ppp $\frac{dQ}{dt}=\left(20\times10^{-4}\right)\times\left(6\times10^{-8}\right)\times\left(273+57\right)^4=1.2\times10^{-10}\times\left(3.30\times10^2\right)^4=1.4$ J.s^{-1}. **Thus answer of part (a) is 1.4 J.** When temperature of the ball is 200^0C in the same environment then net rate of heat flow

	$\frac{dQ}{dt}=\frac{dQ_{\text{ems}}}{dt}-\frac{dQ_{\text{abs}}}{dt}=A\sigma\left(T_{ball}^4-T_{amb}^4\right)$; assume $e=1$ and $a=1$. Using the results From part (a) $\frac{dQ}{dt}=1.2\times10^{-10}\left(\left(5.73\times10^2\right)^4-\left(3.3\times10^2\right)^4\right)=1.2\times\left(\left(4.73\right)^4-\left(3.3\right)^4\right)\times10^{-2}=4.58$ J.s^{-1}. Since ball is at higher temperature and therefore heat radiation is from the ball. **N.B.:** This is a case of superimposition of emissivity and absorptivity. Assume absorptivity $(a=1)$ and emissivity $(\text{e}=1)$ since these values are not given.
I-60	In the problem radius of spherical tungsten piece is 1 cm and hence its surface area $A=4\pi r^2=4\times3.14\times\left(1\times10^{-2}\right)^2=12.56\times10^{-4}$. As per Stefan's law $\frac{dQ_{\text{ems}}}{dt}=Ae\sigma T^4$. Thus with the given data $\frac{dQ_{\text{ems}}}{dt}=\left(12.56\times10^{-4}\right)\times0.30\times\left(6.0\times10^{-8}\right)\left(10^3\right)^4=22.6$ W. At the same time haet absorbed by the spherical piece as per Stefan's law is $\frac{dQ_{\text{abs}}}{dt}=A\sigma T^4=\left(12.56\times10^{-4}\right)\times\left(6.0\times10^{-8}\right)\left(3.0\times10^2\right)^4=0.02$ p. Therefore, heat required to maintain temperature of the ball at 1000 K is $\frac{dQ}{dt}=\frac{dQ_{\text{ems}}}{dt}-\frac{dQ_{\text{abs}}}{dt}\Rightarrow\frac{dQ}{dt}=22.6-0.02=22.58$ W. This as per SD is 22 W. **Hence answer is 22 W.**
I-61	The block is stated to be emitting radiation like a black body it implies it emissivity $e=1$ and absorptivity $a=1$. Since edge of the cubical block is l = 5.0 cm and hence its surface area is $A=6l^2=6\times5.0^2=150$ cm^2. Since block is heated to 227^{0}C therefore as per Stefan's Law $\frac{dQ_e}{dt}=A\sigma T_b^4$. Since Stefan's constant is $\sigma=6.0\times10^{-8}$ W.m^{-1}.^{0}K^{-1} and temperature of the block in Kelvin is $T_b=273+227=500$ K. this is responsible for heat emission by the block. However black being black body, it will absrb heat from ambient at temperature $T_a=273+27=300$ K. Thus net rate of heat loss by the body is $\frac{dQ}{dt}=\frac{dQ_e}{dt}-\frac{dQ_a}{dt}$. With the given data it leads to $\frac{dQ}{dt}=A\sigma\left(T_b^4-T_a^4\right)=\left(150\times10^{-4}\right)\times\left(6\times10^{-8}\right)\times\left(500^4-300^4\right)=49$ W…(1) Heat contained in the block is $Q=msT\Rightarrow\frac{dQ}{dt}=ms\frac{dT}{dt}$. Using the given data $\frac{dQ}{dt}=1.0\times400\frac{dT}{dt}=400\frac{dT}{dt}$…(2). Combining the equations (1) and (2), $4.0\times10^2\frac{dT}{dt}=49\Rightarrow\frac{dT}{dt}=\frac{49}{4.0\times10^2}=0.12$ ^{0}C.s^{-1}. **Hence answer is 0.12^{0}C.s^{-1}**
I-62	As per Stefan's Law net rate of heat emission of a body having emissivity e is $\frac{dQ}{dt}=eA\sigma\left(T_b^4-T_a^4\right)$. Here, $\sigma=6.0\times10^{-8}$ W.m^{-1}.^{0}K^{-1} is Stefan's constant , $T_b=500$ K is temperature of heat emitting body and $T_a=300$ K is ambient temperature. With the given data, for the bare copper surface of the sphere $\frac{dQ_{\text{Cu}}}{dt}=eA\sigma\left(500^4-300^4\right)\Rightarrow210=544eA\sigma$ ... (1). When surface of sphere is blackened $e=1$ the net rate of heat emission is $\frac{dQ_{\text{Black}}}{dt}=1\times A\sigma\left(500^4-300^4\right)\Rightarrow700=544A\sigma$…(2).

	Combining the two equations $\frac{210}{700}=\frac{544eA\sigma}{544A\sigma}\Rightarrow e=\frac{210}{700}=0.3$. **Hence answer is 0.3**
I-63	As per heat emitted by a black body is $\frac{dQ}{dt}=A\sigma T^4$. **Part (a):** Therefore heat emitted by spherical ball A is $\frac{dQ_A}{dt}=\left(20\times10^{-4}\right)\times\left(6.0\times10^{-8}\right)\times300^4=0.97$ J. **Thus answer of part (a) is 0.79 J** **Part (b):** Heat emitted by inner surface of hollow spherical shell B is $\frac{dQ_B}{dt}=\left(80\times10^{-4}\right)\times\left(6.0\times10^{-8}\right)\times300^4=3.9$ J. **Thus answer of part (a) is 2.9 J** **Part (c):** Heat emitted by outer surface of ball A and inner surface of shell B are unequal as derived at (a) and (b) above. Thus $\frac{d\Delta Q}{dt}=\frac{dQ_B}{dt}-\frac{dQ_A}{dt}$ But, the temperatures of both A and B are at 300 K, This is possible only when excess heat emitted by shell B is returned. Accordingly, $\frac{d\Delta Q}{dt}=3.9-0.97=2.93$ J. Thus considering SDs **the answer of part (c) is 2.9 J.** **Thus answer of each part is (a) 0.97 J (b) 3.0 J (c) 2.9 J**
I-64	As per Stefan's Law in the given case $\frac{dQ}{dt}=Ae\sigma\left(T_2^4-T_1^4\right)=A\sigma\left(T_2^4-T_1^4\right)$, since for blackened surface $e=1$ and temperatures are in Kelvin and hence $T_2=273+27=300$ K and $T_1=273+17=290$ K. Accordingly as per given data $\frac{dQ}{dt}=A\sigma T^4=\left(1\times10^{-4}\right)\times\left(6.0\times10^{-6}\right)\times\left(300^4-290^4\right)=62$ W. As regards transfer of heat through rod with its sides outside chamber thermally insulated $\frac{dQ}{dt}=\frac{KA\Delta T}{L}=\frac{K\times\left(1.0\times10^{-4}\right)\times(17-0)}{50\times10^{-2}}=3.4\times10^{-3}\times K$ W. In state of steady state both the rate of heat transfer are equal and hence $3.4\times10^{-3}\times K=62\Rightarrow K=\frac{62}{3.4}\times10^3=1.8\times10^3$ $W.m^{-1}.^0C^{-1}$. **Hence answer is** 1.8×10^2 $\mathbf{W.m^{-1}.^0C^{-1}}$**.**
I-65	As per Stefan's Law in the given case rate of heat radiation is $\frac{dQ_R}{dt}=Ae\sigma T_2^4=A\sigma T_2^4$, since end radiating heat acts like a black body and hence $e=1$ and temperatures are in Kelvin and given that $T_2=800$ and $T_1=300$ K. And rate of heat conduction through rod is $\frac{dQ_C}{dt}=\frac{KA\Delta T}{L}=\frac{KA\left(T_2-T_1\right)}{L}$. In steady state $\frac{dQ_R}{dt}=\frac{dQ_C}{dt}\Rightarrow A\sigma T_2^4=\frac{KA\left(T_2-T_1\right)}{L}\Rightarrow K=\frac{L\sigma T_2^4}{T_2-T_1}$. Using the given data $K=\frac{0.2\times\left(6.0\times10^{-6}\right)\times800^4}{800-300}=98$ $W.m^{-1}.K^{-1}$. **Hence answer as per SDs is 980** $\mathbf{W.m^{-1}.K^{-1}}$**.**
I-66	As per Newton's Law of Cooling $\frac{dQ}{dt}=-hA\left(T_t-T_a\right)$, here h is heat transfer coefficient of the surface, A A is the cooling surface area, T_t is temperature of cooling body at any instant and T_a is ambient temperature. Amount of heat lost by 100 ml of water on cooling from 40^0C to 35^0C in $t_W=5$ minutes is

	$\Delta Q_W = ms\Delta T = V\rho_W S_W \Delta T$. Density of water $\rho_W = 1000\,\text{kg.m}^{-3}$ and hence with given data $\Delta Q_W = \left(100\times10^{-6}\right)\times1000\times4200\times\left(40-35\right) = 2.1\times10^{3}$ cal. Let t is the time taken by the given K-oil in cooling under similar conditions then $\Delta Q_K = \left(100\times10^{-6}\right)\times800\times2100\times\left(40-35\right) = 8.4\times10^{2}$ Since cooling surface is identical and hence cooling rates shall also be the same accordingly $\frac{\Delta Q_W}{t_W} = \frac{\Delta Q_K}{t_K} \Rightarrow t_K = \frac{\Delta Q_K}{\Delta Q_W} t_W$. Therefore the given data and derived values $t_K = \frac{840}{2100}\times5 = 2$ min, **Hence answer is 2 min.**
I-67	As per Newton's Law of Cooling $\frac{dQ}{dt} = -hA\left(T_t - T_a\right)$, here h is heat transfer coefficient of the surface, A A is the cooling surface area, T_t is temperature of cooling body at any instant and T_a is ambient temperature. Heat lost by body in cooling from temperature T_1 to T_2 under ambient temperature is $\Delta Q_{1-2} = ms\Delta T = ms\left(T_1 - T_2\right) = ms(50-45) = 5\,\text{ms}$ and cooling further from temperature T_2 to T_3 under ambient temperature is $\Delta Q_{2-3} = ms\Delta T = ms\left(T_2 - T_3\right) = ms\left(45-40\right) = 5ms$. Therefore, with the given data rate of cooling in first case is $\left(\frac{dQ}{dt}\right)_1 = \frac{5ms}{5} = ms$ and in second case is $\left(\frac{dQ}{dt}\right)_2 = \frac{5ms}{8} = \frac{5}{8}ms$. But, as per for applying Newton's Law of Cooling we shall take average temperature which in first case is $T_{1_avg} = \frac{50+45}{2} = 47.5\,^0$C, while in case 2 it is $T_{1_avg} = \frac{45+40}{2} = 42.5\,^0$C. Accordingly, as per Newton's Law of cooling in first case $\left(\frac{dQ}{dt}\right)_{1-\text{NLC}} = -hA\left(47.5 - T_a\right)$ and in second case $\left(\frac{dQ}{dt}\right)_{2-\text{NLC}} = -hA\left(42.5 - T_a\right)$. Taking ratios of rate of cooling $\frac{\frac{dQ_1}{dt}}{\frac{dQ_2}{dt}} = \frac{\left(\frac{dQ}{dt}\right)_{1-\text{NLC}}}{\left(\frac{dQ}{dt}\right)_{2-\text{NLC}}} \Rightarrow \frac{ms}{\frac{5}{8}ms} = \frac{-hA\left(47.5 - T_a\right)}{-hA\left(42.5 - T_a\right)}$. Solving the equation we get $8\left(42.5 - T_a\right) = 5\left(47.5 - T_a\right) \Rightarrow \left(8-5\right)T_a = 8\times42.5 - 5\times47.5 = 34\,^0$C. **Hence answer is 34^{0}C.**
I-68	As per Newton's Law of cooling $\frac{dQ}{dt} = k(T - T_a)$, here constant k is same in both the cases with different contents in calorimeter. Further temperature fall in both the cases is from 50^{0}C to 45^{0}C which leads to same average temperature $T = \frac{50+45}{2} = 47.5\,^0$C of the cooling object, as well as ambient temperature is also same. Another concept involved in the problem is Water equivalent of calorimeter, this is derived from amount of heat gained or lost by a substance of mass m kg, s cal.kg^{-1}.0C^{-1} specific heat and ^{0}C change of temperature. Since specific heat of water in CGS is 1 cal.g^{-1}.0C^{-1}, while all other substances have their specific heat different from it and there for $S = ms$ is called Heat capacity of the object and is also terms as water equivalent of the object. Accordingly for calorimeter it is designated as S_C. In instant case copper being good conductor of heat and hence both water and copper would be at same temperature and in transfer of heat total heat capacity in case 1 shall be $S_1 = S_C + S_{W1}$ and in case 2 it shall be $S_2 = S_C + S_{W2}$

	Now taking both the cases separately – **Case 1:** $S_{W1}=\left(50\times10^{-3}\right)\times\left(1\times10^{3}\right)=50$ and hence rate of loss of heat from the given data will calculate to $\frac{dQ_1}{dt}=\frac{\left(S_C+S_{W1}\right)\left(T_i-T_f\right)}{t_1}=\frac{\left(S_C+50\right)\times(50-45)}{10\times60}=\frac{1}{120}\left(S_C+50\right)$ **Case 2:** $S_{W2}=\left(100\times10^{-3}\right)\times\left(1\times10^{3}\right)=100$ and hence rate of loss of heat from the given data will calculate to $\frac{dQ_2}{dt}=\frac{\left(S_C+S_{W2}\right)\left(T_i-T_f\right)}{t_2}=\frac{\left(S_C+100\right)\times(50-45)}{18\times60}=\frac{1}{12\times18}\left(S_C+10\right)$ As discussed with NLC, since rate of cooling is same at and hence equating rate of colling in both the cases $\frac{dQ_1}{dt}=\frac{dQ_2}{dt}\Rightarrow=\frac{1}{120}\left(S_C+50\right)=\frac{1}{12\times18}\left(S_C+100\right)\Rightarrow18S_C+900=10S_C+1000\Rightarrow8S_C=100$. It solves into water equivalent of calorimeter $S_C=12.5$ g. **Hence answer is 12.5 g.**
I-69	Given that a metal ball of mass $m=1$kg is heated by means of a heater of 20W in a room at 20^0C, the temperature of the ball, rises to 50^0C. This question is split in Four parts – **Part (a):** At steady state when temp of ball is 50^0C the rate of supply of heat $\frac{dQ_S}{dt}=20$ W is equal to rate of loss of heat $\frac{dQ_L}{dt}$ and hence $\frac{dQ_L}{dt}=\frac{dQ_S}{dt}=20$ W. **Hence answer of part (a) is 20 W.** **Part (b):** Rate of loss of heat when ball is at 30^0C, as per Newton's Law of cooling, $\frac{dQ_T}{dt}=k\left(T-T_a\right)$ accordingly at rate of cooling at 50^0C would be $\frac{dQ_{50}}{dt}=k\left(50-20\right)=30k$ and at 30^0C it would be $\frac{dQ_{30}}{dt}=k\left(30-10\right)=10k$. Since, cooling object is same and hence would be same in both the cases, moreover, rate of loss of heat at 50^0C is $\frac{dQ_{50}}{dt}=k\left(T-T_a\right)=20$ Watts, as determined in part (a). Hence, therefore, $\frac{\frac{dQ_{30}}{dt}}{\frac{dQ_{50}}{dt}}=\frac{10k}{30k}\Rightarrow\frac{dQ_{30}}{dt}=\frac{1}{3}\times\frac{dQ_{50}}{dt}=\frac{20}{3}$ W. **Hence answer of part (b) is $\frac{20}{3}$ W.** **Part (c):** When Ball is at 20^0C , temperature difference with ambient $\Delta T=0$, and as per NLC all the heat is absorbed by the body and there is no loss of heat i.e. there is no loss of heat or $\frac{dQ_{20}}{dt}=0$ W and atpart (b) it has been derived that $\frac{dQ_{30}}{dt}=\frac{20}{3}$W. It is given that temperature of the ball rises uniformly, as against NLC, and heat input is constant at 20 W. Hence average heat loss would be $\frac{dQ_{avg}}{dt}=\frac{\frac{dQ_{20}}{dt}+\frac{dQ_{30}}{dt}}{2}=\frac{0+\frac{20}{3}}{2}=\frac{10}{3}$ W. Therefore amount of heat loss in 5 minutes to raise temperature of ball from 20^0C to 30^0C is $Q_L=\frac{dQ_{avg}}{dt}\times t=\frac{10}{3}\times5\times60=1000$ J. Hence answer of Part (c) is 1000 J. **Part (d):** As per energy balance equation $Q_S=Q_A+Q_L$**.** The ball is being heated by means of heater of power $P=20$ and hence in 5 minutes heat supplied **is** $Q_S=W\times t=20\times5\times60=6000$ J and hence heat

	absorbed by the ball is $Q_A = Q_S - Q_L = 6000 - 1000 = 5000$ J. Further heat absorbed by a body for rise of temperature ΔT **is** $Q_A = ms\Delta T$ **,** with given data and value of Q_A derived above **,** $5000 = 1 \times s \times (30-20) \Rightarrow s = \frac{5000}{10} = 500$ J.kg^{-1}.^{0}C^{-1}**. Thus answer of part (d) i.e. specific of the ball is 500** J.kg^{-1}.K^{-1}. **Thus part wise answers are (a) 20 W (b)** $\frac{20}{3}$ **W (c) 1000 J (d) 500 J.kg^{-1}.K^{-1}**
I-70	Rate of rise of temperature of metal block of heat capacity $S = 80$J.^{0}C^{-1}, heated by heater of power say P W is $\frac{dT_{20}}{dt} = 2\,^0$C.s^{-1}. Therefore, $\Delta Q = S\Delta T \Rightarrow \frac{dQ_{20}}{dt} = S\frac{dT_{20}}{dt}$ accordingly, $p = \frac{dQ}{dt} = 80 \times 2 = 160$ W. **Thus answer of part (a) is 160 W.** When heater is switched off at 30^{0}C, the rate of fall of temperature is $\frac{dT_{30}}{dt} = -0.2$ J.^{0}C^{-1}; here (-)ve sign is attributed to rate of fall of temperature. At this instance there is no supply of heat and hence fall of temperature is due to radiation $\frac{dQ_{30}}{dt} = -S\frac{dT_{30}}{dt} = -80 \times (-0.2) = 16$W. **Thus is part (b) of the answer is 16 W.** As per Newton's Law of Cooling rate of loss of heat $\frac{dQ}{dt} = k(T_t - T_a)$, here k is heat transfer coefficient of the surface T_t is temperature of cooling body at any instant and T_a is ambient temperature. In part (b) it is derived that $\frac{dQ_{30}}{dt} = 16 = k(30-20) \Rightarrow k = \frac{16}{10} = 1.6$. Therefore, $\frac{dQ_{25}}{dt} = 1.6(25-20) = 8$ W. **Thus answer of part (c) is 8W.** It is assumed that $\frac{dQ_{25}}{dt} = 8$W is the average rate of heat radiation, and, therefore, heat balance equation for the metal block to attain 30^{0}C is $P \times t = \frac{dQ_{25}}{dt} \times t + S\Delta T \Rightarrow \left(P - \frac{dQ_{25}}{dt}\right)t = 80 \times (30-20) = 800$. Using value of $P = 160$W and $\frac{dQ_{25}}{dt} = 8$ W determined at part (a) and (c) of the problem above $t = \frac{800}{160-8} = 5.3$ s. **Hence answer of part (d) is 5.3 s.**
I-71	As per Zeroth Law of Thermodynamics in state of thermal equilibrium there is no transfer of heat, and further transfer of heat continues from hotter body to cooler body as long as there is temperature difference between two bodies exist. Therefore, heat maximum loss by a body at temperature θ_1 in an ambient temperature θ_0 is $\Delta Q = ms\Delta t = ms(\theta_1 - \theta_0)$. **Hence answer to part (a) is** $ms(\theta_1 - \theta_0)$**.** Given that as per Newton's Law of cooling $\frac{d\theta}{dt} = -k(\theta - \theta_0)$. When body losses 90% of the maximum determined as part (a) of the answer the remaining heat would be 10% and therefore, $\frac{Q_{10}}{Q_{100}} = \frac{ms(\theta-\theta_0)}{ms(\theta_1-\theta_0)} \Rightarrow \frac{10}{100} = \frac{(\theta-\theta_0)}{(\theta_1-\theta_0)} \Rightarrow \theta = \theta_0 + 0.1 \times (\theta_1 - \theta_0) = 0.1 \times \theta_1 + 0.9 \times \theta_0$. On integrating equation of NLC it leads to $\int_{\theta_1-\theta_0}^{\theta-\theta_0} \frac{d\theta}{(\theta-\theta_0)} = -k\int_0^t t \Rightarrow kt = -\ln\left(\frac{\theta_0 - (0.1\theta_1 + 0.9\theta_0)}{\theta_1 - \theta_0}\right)$. It

	solves into $\Rightarrow kt = -\ln\left(\frac{1}{10}\right) \Rightarrow kt = \ln 10 \Rightarrow t = \frac{\ln 10}{k}$. **Hence answer of part (b) is** $\frac{\ln 10}{k}$. **Thus part-wise answers are (a)** $ms(\theta_1 - \theta_0)$ **(b)** $\frac{\ln 10}{k}$ **N.B**. In this case integral on the L.H.S is w.r.t. θ while the variable is occurring as $(\theta - \theta_0)$. Thus it requires to use limits accordingly which has been $\theta\theta$ done in the illustration**.**

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