Chapter-VI: Waves and Motion

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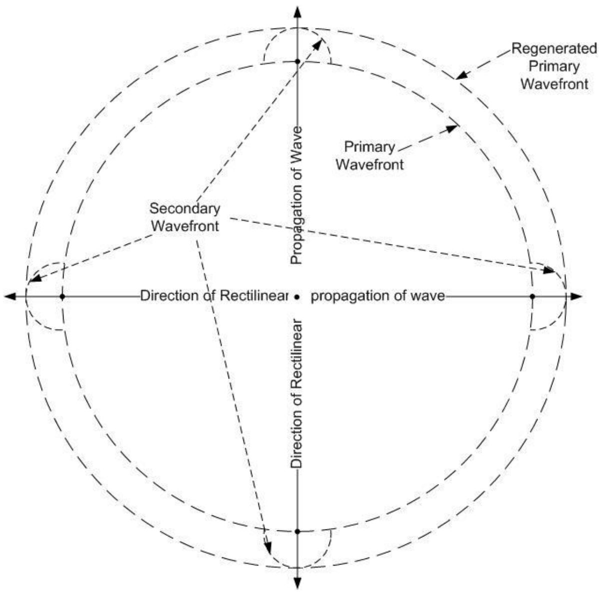
Discovery of correlation between matter and energy has revolutionized understanding of nature. Waves form an indispensable coupling between matter and energy and define boundaries of classical mechanics vis-à-vis relativistic and quantum mechanics. This chapter is, accordingly, positioned in this manual after classical mechanics and heat where existence of waves was introduced. Starting with concept of waves, this chapter is intended to integrate SHM, with concepts of sound and light, which are manifestation of waves in different frequency domain. While elaborating the subject matter electro-magnetic nature of waves is left untouched; it would be incomprehensible without knowledge of electro-magnetism.

Propagation of sound and light through a medium was initially considered to be motion of particles from source to destination. It was **Christiaan Huygens** in 1678 who proposed that rectilinear propagation of light, which was substantiated by **Augustin-Jean Fresnel** in 1816 with his own theory to explain phenomenon of interference in light. The ***Huygens Wave Theory* (HWT)** is explained with a set of postulates that – **a)** light travels like wave propagation away from the source, **b)** the propagation is in the forms of a spherical wave-front in three dimensions, which travel with a uniform velocity in a homogenous medium, **c)** every point on the wave-front in the form of a wave- from acts like a source of light which perpetuates secondary wave-front, **d)** the secondary wave-front, **e)** Envelop of Secondary wave-fronts regenerates new wave-front, which perpetuates rectilinear propagation of the wave. The concept of wavefront can be best visualized by throwing a stone in the midst of a pond or lake and then observing waves propagating to its bank. The HWT successfully explains phenomenon of reflection and refraction, and those involving superposition viz. interference and diffraction. Generation of secondary wave-fronts, taking Four points on a primary wave-front, and a newly regenerated primary wave-front for onward perpetuation in the direction of propagation of wave is are shown in the figure. As we proceed into the journey, use of HWT shall be made while elaborating the above phenomenon.

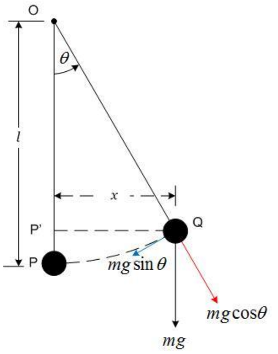


Simple Harmonic Motion (SHM): This is a simple extension of mechanics and very useful in analysis of waves. It would be no exaggeration to state that the SHM is fundamental and most natural motion. *Any periodic motion or vibration, which is also called oscillation can be explained with its constituent SHMs*, which was established by a mathematician ***Joseph Fourier** in 1807*, known as Fourier Analysis, which is a subject matter of higher studies, and hence its elaboration at this point is refrained.

The SHM is best explained with trace of a particle, along a diameter of a circle, which is performing a uniform circular motion. This, however, requires to be substantiated with a pendulum or vibration in a spring, real life visualizations, to appreciate SHM. Accordingly the three types of motions are analysed brought in the table below.

Before, we set on to analyse SHM, its basic premise is – **a)** force on a particle performing SHM is always directed towards its mean position, and **b)** force is always proportional to the displacement of particle from its mean position. With this premise oscillations of Pendulum and Spring shall be analysed. These physical observations are compared with the motion of trace of the particle, performing uniform circular motion i.e. constant angular velocity ω, along diameter of circle. It will be seen that all the three cases are in conformance with the premise of SHM. In respect of oscillation of pendulum and spring, certain assumptions are involved, while motion of trace of a particle performing circular motion, it is an ideal representation of SHM. Accordingly, the latter one shall be extended to further analysis of SHM including time period T. This definition of T shall be applied to the earlier two cases of SHM.

Oscillation of Pendulum



Length of Pendulum=l , and of Arc $PQ = l\theta$

When θ is small, or $x \ll l$,

$\sin\theta \rightarrow \theta = \frac{\text{Arc } PQ}{l} \cong \frac{P'Q}{l}$

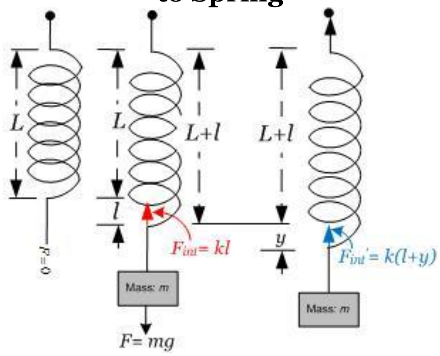
Force $\bar{F} = -mg\frac{P'Q}{l} = -mg\frac{x}{l}$

Acceleration of Mass $a = -\left(\frac{g}{l}\right)x; a \propto x$

This complies with both the premises of SHM

It involves ***assumption:*** θ is small

Oscillation of a Mass attached to Spring



For expansion (l) of spring: $kl = mg$;

Internal force on stretching of spring by additional length (y) : $F' = -(l+y)k$;

Net force : $F'' = F' + mg = ma$

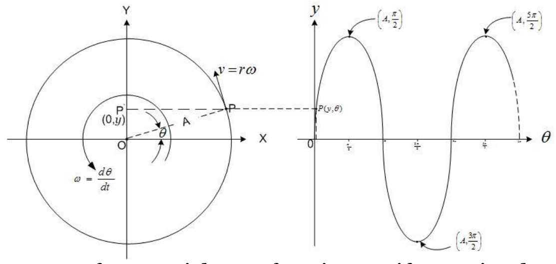
⇨ $ma = -(l+y)k + kl$

⇨ $ma = -ky;\ a = -\left(\frac{k}{m}\right)y;\ a \propto x$

This complies with both the premises of SHM

It involves ***assumption:*** *spring constant k remain uniform*

Motion of Trace of a Particle Performing Circular Motion



Trace of a particle performing uniform circular motion of radius $\boldsymbol{A}$, at $\boldsymbol{\omega}$ rad/sec on y-axis is plotted along with the circular motion.

Displacement from Mean Position: $y = A\sin\theta$

Velocity of the trace: $v = \frac{dy}{dt} = \frac{d}{dt}(\mathrm{A}\cdot\sin\theta)$

⇨ $A\cdot\frac{d}{d\theta}\sin\theta\cdot\frac{d\theta}{dr} = A\omega\cos\theta$

Acceleration of Particle: $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\theta)$

⇨ $-A\omega^2\sin\theta = -\omega^2 y;\ a \propto y$

This complies with both the premises of SHM.

Since, here no assumption is involved, it is ideal representation of SHM.

This analysis is being extended to determine Time Period ***T*** *, and velocity of particle* ***v*** *at a particular displacement* ***y****, for circular motion and applied to oscillation of pendulum and spring*

Displacement of particle from mean position is since a ***Sine*** *function of angular displacenent θ, it is also called a* ***Sinusoidal Wave****.*

$$\omega = \sqrt{\frac{\left(\frac{g}{l}\right)x}{x}} = \sqrt{\frac{g}{l}};\ f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{\sqrt{\left(\frac{g}{l}\right)}} = 2\pi\sqrt{\frac{l}{g}}$$

$$\omega = \sqrt{\frac{\left(\frac{k}{m}\right)y}{y}} = \sqrt{\frac{k}{m}};\ f = \frac{\omega}{2\pi}$$

$$T = \frac{1}{f} = \frac{2\pi}{\sqrt{\left(\frac{k}{m}\right)}} = 2\pi\sqrt{\frac{m}{k}}$$

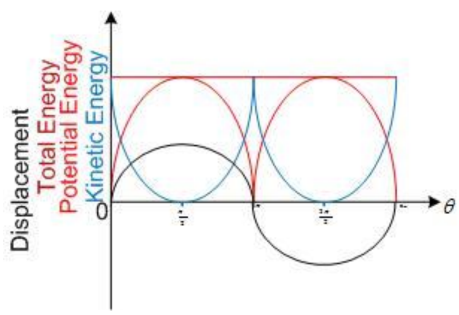
$$\omega = \frac{\text{Acceleration}}{\text{Displacement}} = \sqrt{\frac{a}{y}};\ \omega = 2\pi f = \frac{2\pi}{T}.$$

Here $\boldsymbol{f}$ - is frequency of oscillation (Cycles/Sec).

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

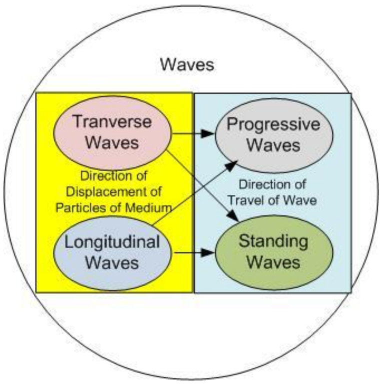
Composition of Energy of a particle performing SHM: Taking that particle is performing SHM in frictionless environment, where there is exchange of energy with external systems. In such a situation energy of particle shall comprise of *Potential Energy (PE)* and *Kinetic Energy (KE),* and the two together shall constitute *Total Energy (TE)* of the Particle.

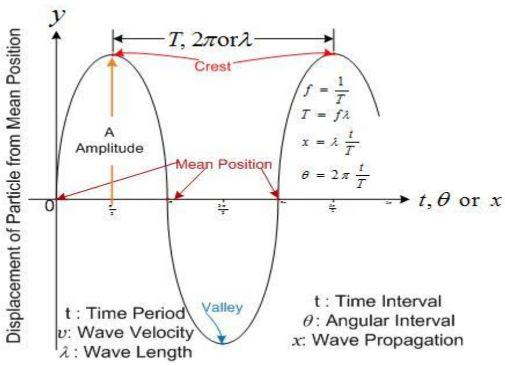
As per definition, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m(A\omega\cos\theta)^2 = \frac{1}{2}mA^2\omega^2\cos^2\theta$, here $\boldsymbol{v}$ – is the instantaneous velocity of the particle of mass $\boldsymbol{m}$**,** $PE = \int_0^y m\cdot a\cdot dy = \int_0^y m\cdot(A\omega^2\sin\theta)\cdot dy$, since $y = A\sin\theta$, hence $dy = A\cos\theta d\theta$. For convenience limits shall be managed at the last step. Accordingly, $PE = \int_0^y m\cdot(A\omega^2\sin\theta)\cdot(A\cos\theta d\theta) = mA^2\omega^2\int_0^y \sin\theta\cos\theta d\theta = \frac{mA^2\omega^2}{2}\int_0^y \sin 2\theta\, d\theta = \frac{mA^2\omega^2}{4}\int_0^\theta \sin u\, du\Big|_{u=2\theta; d\theta=\frac{du}{2}}$. It leads to $PE = \frac{mA^2\omega^2}{4}[-\cos u]_0^\theta = -\frac{mA^2\omega^2}{4}[\cos 2\theta - 1] = -\frac{mA^2\omega^2}{4}[(1-2\sin^2\theta) - 1] = \frac{mA^2\omega^2}{2}\sin^2\theta$. Thus, $TE = PE + KE = \frac{mA^2\omega^2}{2}\sin^2\theta + \frac{mA^2\omega^2}{2}\cos^2\theta = \frac{mA^2\omega^2}{2} = \frac{1}{2}mv^2$; here, $v = A\omega$, which corresponds to tangential velocity of particle performing uniform circular motion, while in case of oscillation of simple pendulum and spring it is velocity of particle at mean position occurring at $\theta = n\pi$, where, n is an integer, PE is ZERO.



Oscillations are of various kinds: **a)** ***Free Oscillation***, where no external force is applied, trace of motion of planets, satellites electrons in their orbit **b)** ***Damped Oscillations***, whose depletes with passage of time, a swing left unattended, **c)** ***Forced Oscillations***, like swing or clock where regular at regular interval, extra energy is supplied to make up energy lost in each oscillation, **d)** ***Resonant Oscillations***, these occur in a system when its natural frequency is an integral multiple of an oscillation present in the environment. This finds extensive application in musical instruments. **e)** ***Coupled Oscillation***, occur in a system which communicates, exchanges, energy with an external system when it is set into oscillation. This principle is widely used in sound box, speakers.

Waves: Understanding of the SHM is the study of oscillation of a single particle, and is elemental in elaboration of wave, electromagnetic waves, involves medium to either for its existence or its propagation. Classification of waves, based on direction of motion of particles, is in two categories: **a)** ***Longitudinal Wave***, and **b)** ***Transverse Wave***. Further, consideration of propagation of wave creates another classification: **i)** ***Travelling Waves*** - in which every motion of particle perpetuates to the adjoining particle of the medium along the direction of propagation, **ii)** ***Standing waves*** – it is a result of interaction of forward and backward travelling wave, such that all particles of the medium at any point are in same phase, but their amplitude depends upon their position along the wave. While, each of these types of wave is characteristically different in respect of motion of particles of the medium of propagation, and shall be studied with its mathematical and graphical representation in the form of SHM. Elaboration of basic concepts of waves is considered a prerequisite to the understanding of phenomenon of Sound Waves and Light Waves. Accordingly, journey in the subject matter has been structured. Meanwhile, Parameters of wave, common among them, alongwith their mathematical correlation is summarized in the figure and are being defined, and shall find use all along..





Time period (*T*) : *It is the time taken to complete One Cycle.*

Frequency (*f or ν*) : *It is number of cycles in One Second, it is related to* $T = \frac{1}{f}$.

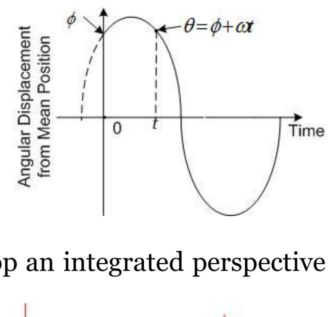
Crest : *It is the point on wave where displacement of particle from mean position is maximum*.

Valley : *It is point on wave where displacement of particle from mean position is minimum*

Wavelength (*λ*) : *It is distance covered by wave in one cycle. Most conveniently is is recorded as distance between two consecutive peaks, as shown in the figure.*

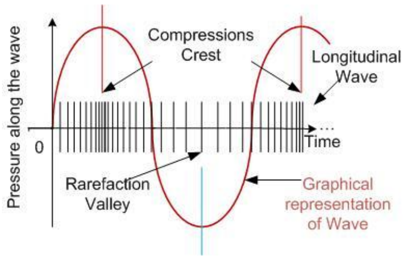
Velocity of wave (*v*) : *It is distance covered by wave in One second, and* $v = \lambda f = \frac{\lambda}{T}$.

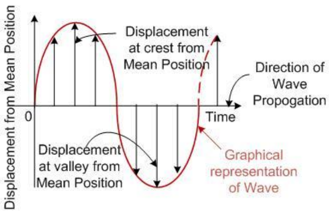
Phase (*θ*): *It is the angular displacement of a particle in a wave from its initial mean position, and* $\theta = \omega t$. *This repeats after every* 2π *angular displacement corresponding to T. In case a particle in a wave, initially displaced from its mean-position by an angle* ϕ, *is set into SHM then its phase after a lapse of time t is* $\theta = \emptyset + \omega t$ *and is shown in the figure.*



Basic concepts of waves are common to Sound wave and Light wave. Accordingly, these concepts are considered a prerequisite to the understanding of phenomenon of Sound Waves and Light Waves, and shall be elaborated before going into Sound and Light waves, to develop an integrated perspective of the two. Accordingly, journey in the subject matter has been structured and is in line with the approach of the Manual.

Longitudinal Wave: In a wave if *particles of medium oscillate, about their mean position, along the direction of wave then it is called longitudinal wave. These oscillating particles create compression ad rarefaction as shown in the figure*. These waves are also represented as Sinusoidal Wave as shown in the figure. These waves are realized in rattling sound of doors and windows during a thunderstorm. Sound

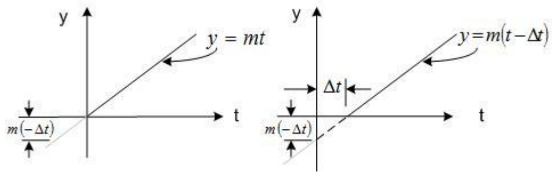




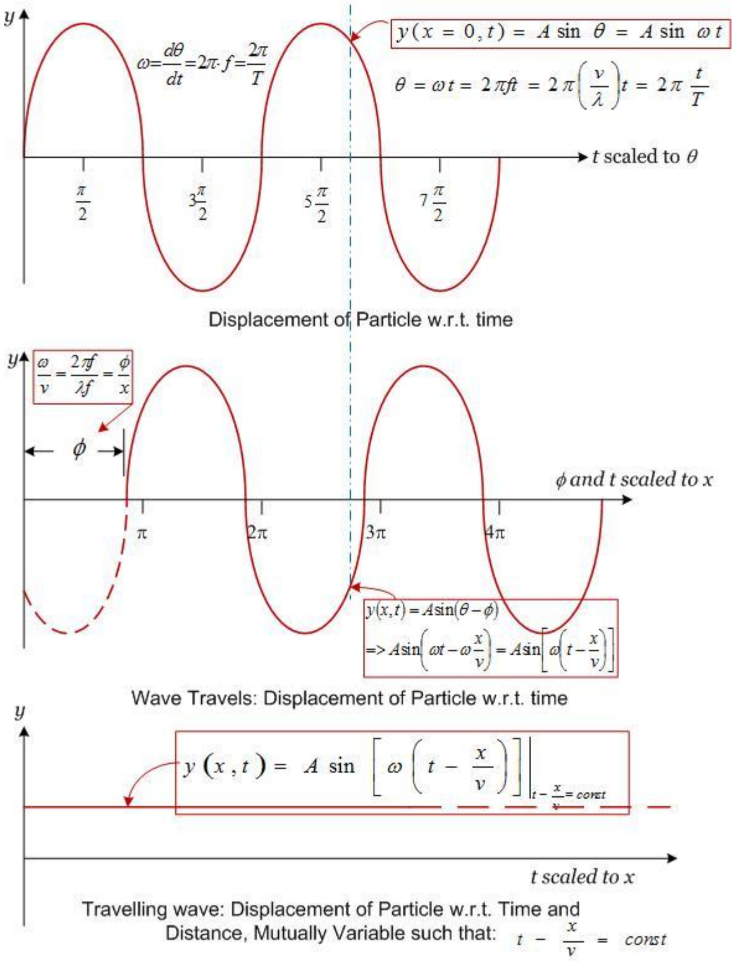
waves are basically Longitudinal Waves.

Transverse Wave: In *this type of wave particles of medium oscillate about their mean position, in a direction perpendicular to the direction of wave*. Waves generated in a water pond by dropping a stone are transverse waves. Likewise, all string-based musical instruments produce transverse waves; so are the Light waves.

Travelling Waves: The SHM elaboration considered oscillation of particles about its mean position and mathematically represented with a sinusoidal function. This is enough to explain the oscillation but in sufficient to represent oscillations that travel from one point to other, called waves, which carries or transfer energy from source to receiver or destination. This is elaborated with a simple straight line function $y = mt$. Here, $\boldsymbol{y}$ is the displacement of particle, from its initial position, at any instance $\boldsymbol{t}$, and $\boldsymbol{m}$, representing slope of line in graph is rate of change of displacement $m = \frac{dy}{dt}$. Let this straight line displacement is travelling, i.e. source is shifting through a displacement $\boldsymbol{a}$, such that $a = v \times \Delta t$. In this new situation displacement of the particle, at slope m, is identical to that having started from initial displacement $-\Delta t$. Accordingly, as per knowledge of Coordinate geometry displacement of particle at any instance is analogous to that at an instance $(t - \Delta t)$.



This logic shall be extended to elaborate travelling SHM called ***travelling or progressive waves*** *expressed as* $y = f(x, t)$ and is elaborated in the figure. First graph shows displacement (y), from mean position $y = A\sin\theta$, here $\theta = \omega t = 2\pi f t = \frac{2\pi}{T}t$ at any instant of time. Next, the wave is taken to be moving along $\boldsymbol{x}$ axis through a distance x corresponding to a phase angle ϕ in time t. Accordingly, displacement of a particle, in accordance with the above example van be represented with a graph below where $y = A\sin\left[\omega t - \omega\frac{x}{v}\right] = A\sin\left[\omega\left(t - \frac{x}{v}\right)\right]$. Taking, variation in $\boldsymbol{x}$ and $\boldsymbol{t}$ such that $\left(t - \frac{x}{v}\right)$ remains constant, the displacement $\boldsymbol{y}$ shall also remain constant. This implies that with passage of time displacement is travelling forward, while particles of medium keep oscillating about their mean position. This is shown in the second graph. The third inference is about progressive displacement of a particle of medium from its mean position while both $\boldsymbol{x}$ and $\boldsymbol{t}$ are changing. It will be seen that when $\left(t - \frac{x}{v}\right)$ remains constant, displacement remains constant, that is with passage of time $\boldsymbol{t}$, displacement moves forward along $\boldsymbol{x}$ with velocity $\boldsymbol{v}$. This nature of travelling wave is shown in the third graph.



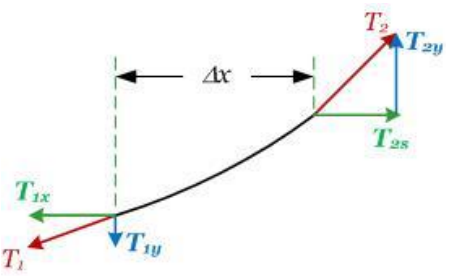
Fourth characteristic of travelling wave comes from its periodicity. At any point on the passage of a the displacement of particle of medium from mean position repeats at an interval $= \frac{2\pi}{\omega} = \frac{1}{f}$, here T - is called ***Time Period*** and it corresponds to angular displacement 2π to complete one oscillation, characteristic to sine function. Likewise, at any instance of time along the passage of time the displacement of particle from its mean position repeats at an interval of $\lambda = \frac{v}{f} = vT$, here λ - is called ***Wavelength***, i.e. distance covered in one Oscillation.

In a wave travelling forward i.e. along ***x***-axis –ve sign appears with $\frac{x}{v}$ and accordingly a general expression of a travelling/progressive wave is $y(x,t) = A\sin\left[\omega\left(t - \frac{x}{v}x\right)\right] = A\sin\left[\omega\left(t - \frac{x}{v}\right)\right]$. If the wave is travelling in a direction along $(-\boldsymbol{x})$ axis, automatically the equation shall take the form $y(x,t) = A\sin\left[\omega\left(t - \frac{(-x)}{v}x\right)\right] = A\sin\left[\omega\left(t + \frac{x}{v}\right)\right]$. Thus, progression of wave perpetuates with time. **This is general expression of a progressive wave** which represents displacement of a particle in the medium as a function of time and position from the source.

This expression is being extended to a differential equation of this time and position varying phenomenon known as Wave Equation. Accordingly, velocity of particles of medium performing SHM w.r.t. time and position are expressed as $v'_y = \frac{\partial}{\partial t}y(x,t) = \omega A\cos\left[\omega(t - \frac{x}{v})\right]$, and $v''_y = \frac{\partial}{\partial x}y(x,t) = \frac{\omega}{v}A\cos\left[\omega(t - \frac{x}{v})\right]$, respectively. It is to be noted with a caution that that this velocity of the particle and the velocity of travelling wave (v). Taking it forward, acceleration of the particle w.r.t time and position is $a'_y = \frac{\partial^2}{\partial t^2}y(x,t) = -\omega^2 A\sin\left[\omega(t - \frac{x}{v})\right]$ and $a''_y = \frac{\partial^2}{\partial x^2}y(x,t) = -\frac{\omega^2}{v^2}A\sin\left[\omega(t - \frac{x}{v})\right]$, respectively. This leads to $\frac{a'_y}{a''_y} = \frac{\frac{\partial^2}{\partial t^2}y(x,t)}{\frac{\partial^2}{\partial x^2}y(x,t)} = \frac{=-\omega^2 A\sin\left[\omega(t-\frac{x}{v})\right]}{-\frac{\omega^2}{v^2}A\sin\left[\omega(t-\frac{x}{v})\right]} = v^2$. It is to be noted with a caution that that this velocity of the particle and the velocity of travelling wave (v). It is most convenient to express a dynamic process as differential equation. Accordingly, $\boldsymbol{\frac{\partial^2}{\partial t^2}y(x,t) = v^2\left(\frac{\partial^2}{\partial x^2}y(x,t)\right)}$, and in its complementary form as $\boldsymbol{\frac{\partial^2}{\partial x^2}y(x,t) = \frac{1}{v^2}\left(\frac{\partial^2}{\partial t^2}y(x,t)\right)}$, called ***Wave Equation***. *Discovery of One Dimensional Wave Equation by* **Jean le Rond d'Alembert**, *in 1746, followed by* **Leonhard Euler** *Three Dimensional Wave Equation within a decade, was a great leap in discovery of physical systems and processes. Discovery of wave equation later helped to generalize transfer of energy through wave over a broad spectrum, right from mechanical vibrations to sound and electromagnetic radiation.* ***Here, analysis is confined to One Dimensional waves.***

Velocity of Wave: In this wave equation velocity of wave is a parameter which rationalizes acceleration of particles of medium w.r.t. time and position from the source. In strings based musical instruments transverse waves are established. While, in gases waves are where longitudinal. In both the cases, velocity of waves are governed by different phenomenon and are being elaborated separately.

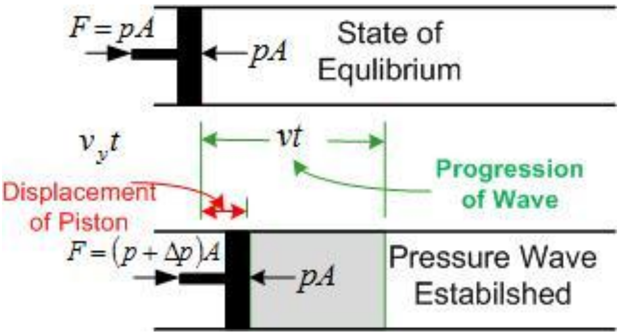
Velocity of Wave in String: Strings are so made that their mass per unit length (μ) is uniform and is valid in normal state of rest. When string is set to transverse wave, along its length, non-uniform extension will take place and thus influence uniformity of μ . Looking at graphical representations of waves it might be perceived to be rigidity of a metal string keeps significant, but in reality it is quite small and for all practical purposes it is considered to be uniform. Now it needs to be explored as to how Tension (T) and μ play role in velocity of wave. Consider an element Δx having Tensions T_1 and T_2 at its Two ends, which goes in to decide shape of the waveform. Since, the wave propagation is transverse and hence displacement, velocity and accelerations of the particles of string along length shall not exist, Accordingly, $T_{1x}=T_{2x}$ and it complies with Newton's Third Law of Motion. But, the nature of wave demanding transverse motion of particles of string will utilize difference in transverse components of tension such that $T_{2y} - T_{1y} = (\mu\cdot\Delta x)\frac{\partial^2 y}{\partial t^2}$, as per Newton's Second Law of Motion. Looking at the tensions over element Δx of the string, $T_{2y} - T_{1y} = T\left(\frac{T_{2y}}{T_{2x}} - \frac{T_{1y}}{T_{1x}}\right)\Big|_{T_{2x}=T_{1x}=T} = T\left(\frac{\left(\frac{\partial y}{\partial x}\Big|_{x\to x+x} - \frac{\partial y}{\partial x}\Big|_{x\to x}\right)}{\Delta x}\right)\Delta x = T\cdot\Delta x\frac{\partial^2 y}{\partial x^2}$.



Comparing it with Wave Equation, which can be written as $\boldsymbol{\frac{\frac{\partial^2}{\partial t^2}y(x,t)}{\frac{\partial^2}{\partial x^2}y(x,t)} = v^2}$, the equation evolved here for string lead to a similar form: $T_{2y} - T_{1y} = (\mu\cdot\Delta x)\frac{\partial^2 y}{\partial t^2} = (T\cdot\Delta x)\frac{\partial^2 y}{\partial x^2}$. It leads to $\frac{\mu\frac{\partial^2 y}{\partial t^2}}{T\frac{\partial^2 y}{\partial x^2}} = 1$, or $\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$. Comparing this equation with the wave equation, velocity of wave in a string is: $v = \sqrt{\frac{T}{\mu}}$. This is also expressed as $v = \sqrt{\frac{Y}{\rho}}$ based on dimensional equality of $\sqrt{\frac{T}{\mu}} = \sqrt{\frac{Y}{\rho}}$.

Understanding of waves in strings, as seen in musical instruments, is with its ends fixed called ***Node***, which has no motion, be it transverse. Therefore, definition of wave where $v = \lambda f$. The number of Nodes between the fixed ends and length of wire (L) would decide pitch length and in turn frequency of wave. In case there are no nodes between the fixed ends, the length of wires it constitutes half pitch length $L = \frac{\lambda}{2}$. Accordingly, $v = 2L \cdot f = \sqrt{\frac{T}{\mu}}$,. It leads to natural frequency of vibration of string as $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$.

Velocity of Wave in Fluids: Progression of wave in fluid is conceptualized in One Dimension in the figure, where travel of piston at velocity $\boldsymbol{v_y}$ in time $\boldsymbol{t}$ establishes longitudinal pressure wave in fluid which travels a distance vt, in corresponding time, such that velocity of wave is v. Beyond the distance of travel of wave, medium remains at equilibrium state. Considering, bulk elasticity of the medium ($\boldsymbol{B}$), it leads to $B = \frac{\Delta p}{v_y t/_{vt}} = \frac{v\Delta p}{v_y}$. Accordingly, $\Delta p = B\frac{v_y}{v} => F = A\Delta p = B\frac{Av_y}{v}$, and per Newton's Second Law of Motion impulse $F = m\Delta v_y = \rho(Av)v_y$, thus a generalized expression comes to $F = B\frac{Av_y}{v} = \rho(Av)v_y$, or $\boldsymbol{v} = \sqrt{\frac{\boldsymbol{B}}{\boldsymbol{\rho}}}$.

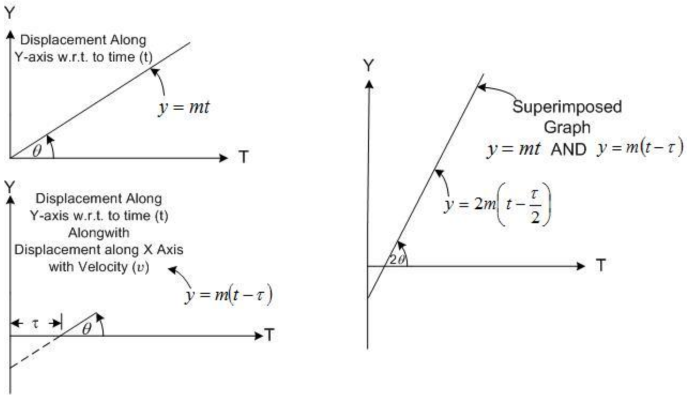


Velocity of Wave in Gas: Gases are highly compressible as compared to liquids**.** Therefore, *Newton* assumed that temperature of gas remains constant and accordingly used Boyle's Law pv = Const. to investigate velocity of wave in gases. Differentiating Boyle's Equation as : $p\partial v + V\partial p = 0$, or $B = -\frac{\partial p}{\left(\partial V/_V\right)} = p$. Using this value of B, Newton redefined velocity of wave in gases as $= \sqrt{\frac{p}{\rho}}$ ***;*** this is known as ***Newton's Equation of Velocity of Wave in Gases.*** This equation is comparable to velocity of wave in strings.

In case of gases which are highly compressible, and the velocity of wave is quite high, the progression of pressure wave is an Adiabatic process where medium has no time to exchange heat with the environment, either during compression or rarefaction. Accordingly, in this case instead of Boyle's Law equation, and comply with $\boldsymbol{pV^{\gamma}} = \textbf{Constant}$ as per ***Poisson's Law***, covered in Heat and Thermodynamics. Thus Laplace, suggested a correction based on Poisson's Law whose logarithm is $\log p + \gamma \log V = Const.$ Differentiating this log-equation w.r.t. $\boldsymbol{t}$ leads to $\frac{1}{p}\frac{dp}{dt} + \frac{\gamma}{V}\frac{dV}{dt} = 0$, or $B = \frac{dp}{dV/_V} = \gamma p$. Accordingly, the corrected equation $v = \sqrt{\frac{\gamma P}{\rho}}$ is known as Newton-Laplace Equation *of velocity in gases.*

Energy and Power in Wave: In wave represented by $y(x,t) = A\sin\left(\omega t - \frac{\omega}{v}x\right)$. In a string tension is always along the its length and hence $\frac{\partial y}{\partial x} = \frac{T_y}{T_x} = -A\omega\cos\left(\omega t - \frac{\omega}{v}x\right)$. Accordingly, $T_y = -TA\omega\cos\left(\omega t - \frac{\omega}{v}x\right)$, here $T_y = T$ which is uniform along length of string. Thus, instantaneous power $P(x,t) = T_y\frac{\partial y}{\partial t} = T\left[A\left(\frac{\omega}{v}\right)\cos\left(\omega t - \frac{\omega}{v}x\right)\right] \cdot \left[A\omega\cos\left(\omega t - \frac{\omega}{v}x\right)\right]$. It leads to $P(x,t) = \frac{TA^2\omega^2}{v}\cos^2\left(\omega t - \frac{\omega}{v}x\right) = \frac{TA^2\omega^2}{v}\left[\frac{1-\cos 2\left(\omega t - \frac{\omega}{v}x\right)}{2}\right]$. It comprises of Two Terms, One is a constant and it contains parameters characteristic to wave and independent of variables $\boldsymbol{x}$ and $\boldsymbol{t}$. And, the other term is a cosidal trigonometric function, which is time and place variant, averages to Zero over a cycle. Thus, average power of wave $\boldsymbol{P_{av}}$ is represented as $= \frac{1}{2} \cdot \frac{TA^2\omega^2}{v} = \frac{1}{2} \cdot \frac{\boldsymbol{T v A^2\omega^2}}{\boldsymbol{v^2}} = \frac{1}{2} \cdot \boldsymbol{\mu v A^2 \omega^2}\Big|_{v=\sqrt{\frac{T}{\mu}}}$. This expression of average power is be represented in terms of frequency (ν) as $\boldsymbol{P} = \boldsymbol{2\pi^2 \mu v A^2} \mathrm{\nu}^2$.

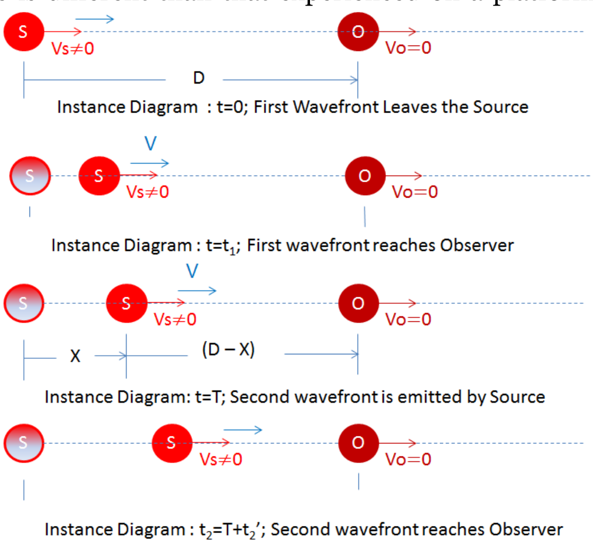
Principle of superimposition of Waves: A simple case of an object moving vertically with a constant velocity in an inertial frame of reference shown in the figure as (y,t) graph $y = mt$. In the graph below, another object moves vertically with the same constant velocity, but after a lapse of time τ. Like a wave, considering second function to be continuous, the (y,t) graph is plotted for $t > 0$. Summation of the two functions $y = mt + m(t-\tau) = 2m\left(t - \frac{\tau}{2}\right)$, in another (y,t) is synonymous to superimposition of two functions, and is easy to graph. But, in case of complex functions like wave functions, superimposition is best represented mathematically.



Accordingly, two wave functions $y_1 = A_1 \sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$ and $y_2 = A_2 \sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$ are considered to elaborate superimposition of waves. It could lead to multiple cases where in the Two wave functions with different – **a)** Amplitudes (A), **b)** Wave lengths (λ), **c)** velocity (v), and **d)** wave travel during initial phase shift. In most of the problems of wave superimposition that are encountered at this stage are for two waves with identical, Amplitude, velocity and frequency or wave length and accordingly simplistic mathematical analysis of waves travelling in opposite directions such as: $y = A\sin\frac{2\pi}{\lambda}(vt + x) + A\sin\frac{2\pi}{\lambda}(vt - x)$. Using trigonometric identities it reduces into $y = \left[A\sin\frac{2\pi}{\lambda}(vt + x) + \sin\frac{2\pi}{\lambda}(vt - x)\right] = 2A\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi vt}{\lambda}\right) = A_x \sin\left(\frac{2\pi vt}{\lambda}\right)$. Here, Amplitude of wave function at every point corresponding to $\left(\frac{2\pi x}{\lambda}\right)$, along the pitch is $A_x = 2A\cos\left(\frac{2\pi x}{\lambda}\right)$. And, displacement of each particle (y) from its mean position at any instant (t) is in same phase $y \propto \sin\left(\frac{2\pi vt}{\lambda}\right)$, where proportionality constant is A_x. This is a special case of ***Standing wave or Stationary wave*** and finds extensive application in *Sound Waves*. Further, *analysis of superimposition shall be dealt with as resonance of sound waves in strings and air column, while in reflection, refraction, interference and diffraction common to sound and light waves*, in Part II.

A generic analysis of periodic wave function was suggested by **Joseph Fourier**, in 1807, in the form of a series of sinusoidal functions : $A_x = \frac{A_0}{2} + \sum_{n=1}^{N} A_n \cdot \sin\left(\frac{2\pi nx}{T} + \varphi_n\right)$. Here, parameters of the waveform are, A_0 – is the bias from mean position of the periodic waveform, A_1- is the amplitude of the sinusoidal waveform of frequency as that of the periodic waveform; this is called fundamental frequency (f), A_n – is the amplitude of sinusoidal wave form of frequencies multiple of fundamental frequency (nf) and are called harmonics, n- is called the order of harmonic, and φ_n – is the phase shift from the initial in respect of each harmonic. Determination of these parameters of the frequencies constituting a non-sinusoidal periodic function was suggested by Fourier and is known as ***Fourier Analysis***; this is inverse of superposition of sinusoidal waveforms. Elaboration of Fourier Analysis is outside the scope of this document, nevertheless, *readers are welcome to raise their inquisitiveness through* ***Contact Us***.

Doppler Effect in an Inertial Frame : Shrill of a train while arriving at platform and while leaving is different. Likewise, shrill of a horn of a Train being chased by a vehicle is different than that experienced on a platform. This is being analysed in three different cases; **a)** Source moving towards a stationary Observer, **b)** an Observer moving away from a stationary source, **c)** Both Source and Observer moving in one direction, with Observer ahead of Source. The results of the analysis in three cases have been generalized, at the end.



<u>Case 1: Source moving towards a stationary Observer</u>

Standard notations being used in the analysis, elaborated in figure, are as under-

V- Velocity of Sound ; *Vs* – Velocity of Source ≠ 0; *Vo* – Velocity of Observer =0,
T – Time period of Sound ; *t* - an instance in the analysis,

t_1 – an instance when **First wave-front** emitted by source at t=0 reaches observer $= \frac{D}{V}$.
X – distance moved by Source during $= T \cdot Vs$, when Source emits **Second wave-front**
t_2'- is the time taken **Second wave-front** to reach the Observer $= \frac{D-X}{V}$
f- Frequency of Sound; f' – Apparent Frequency of Sound

Therefore, effective Time Period for the Observer-

$$T' = t_2 - t_1 = (T + t_2') - t_1 = \left(T + \frac{D-X}{V}\right) - \frac{D}{V} = \left(T + \frac{D-TVs}{V}\right) - \frac{D}{V} = T\left(1 - \frac{Vs}{V}\right)$$

Hence, apparent frequency : $\boldsymbol{f'} = \frac{1}{T\prime} = \frac{1}{T\left(1-\frac{Vs}{V}\right)} = \boldsymbol{f} \cdot \frac{\boldsymbol{V}}{\boldsymbol{V-Vs}}$

Inference: f ' is equal to (f)x(Ratio of Velocity of sound w.r.t. Observer to Velocity of Sound w.r.t Source

<u>Case 2: An Observer moving away from a stationary source</u>:

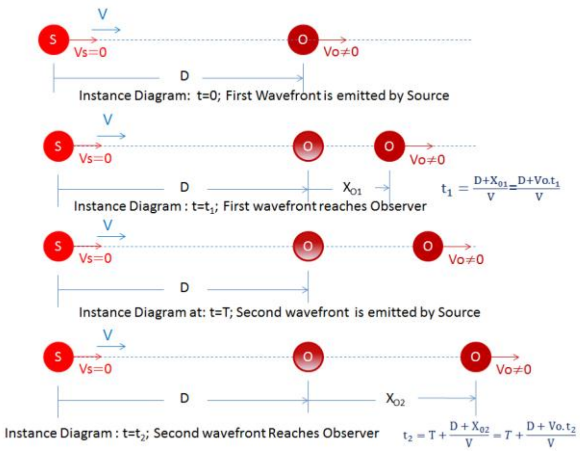
Vs=0 and $Vo \neq 0$, and is elaborated in figure.

From the above-

$t_1\left(1 - \frac{Vo}{V}\right) = \frac{D}{V}$; $t_1 = \frac{D}{V-Vo}$; and $t_2\left(1 - \frac{Vo}{V}\right) = T + \frac{D}{V}$; $t_2 = \frac{TV+D}{V-Vo}$

In this case Apparent Time Period for the Observer is –

$t' = t_2 - t_1 = \frac{TV}{V-Vo}$; or $f' = \frac{1}{t\prime} = \frac{1}{T}\left(\frac{V-Vo}{V}\right) = f\left(\frac{V-Vo}{V}\right)$



Inference: f ' is equal to (f)x(Ratio of Velocity of sound w.r.t. Observer to Velocity of Sound w.r.t Source ***(Same as in case 1)***

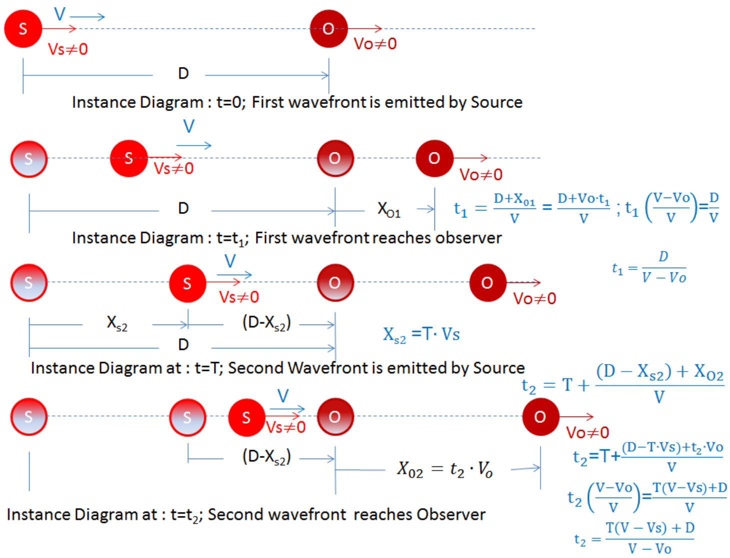
<u>Case 3: Both Source and Observer moving in one direction, with Observer ahead of Source</u>

Figure below specifies each instance and specially t1 and t2 when First and Second Wave-front emitted by Source reach Observer, respectively alongwith relationships of related variables, as shown in the figure. Accordingly, apparent Time Period would be -

$$T' = t_2 - t_1 = \frac{T(V - Vs) + D}{(V - Vo)} - \frac{D}{(V - Vo)} = T\frac{(V - Vs)}{(V - Vo)}$$

Or, $f' = \frac{1}{T\prime} = \frac{1}{T} \cdot \left(\frac{V-Vo}{V-Vs}\right) = f.\left(\frac{V-Vo}{V-Vs}\right)$

Inference : f ' is equal to (f)x(Ratio of Velocity of sound w.r.t. Observer to Velocity of Sound w.r.t Source ***(Same as in case 1)***



General Inference on Doppler's Effect : *In case of source and/or observer moving, apparent frequency to the observer is natural frequency of source multiplied by ratio of relative velocity of sound w.r.t. Observer to the relative velocity of sound w.r.t. the Source.* Manifestation of Doppler Effect in Light is change of Colour, called Doppler Shift and shall be elaborated in Part-II.

Summary: *Initially the concepts of waves discussed above are applicable in analysis of Sound Waves and Light Waves, with distinct boundary of frequencies. Accordingly, these concepts shall be used to elaborate commonalities in respect of various phenomena like reflection, refraction, interference, diffraction and polarization common to light and shall be included in Part II of this article, in 3rd Quarterly e-Bulletin due on 1st April'17. Light waves are a narrow part of electromagnetic waves, which is outside scope of this manual. Nevertheless, readers are welcome to raise their inquisitiveness through* ***Contact Us***.

Examples have been drawn from real life experiences help to build visualization and an insight into the phenomenon occurring around. A deeper journey into the problem solving would make integration and application of concepts intuitive. This is absolutely true for any real life situations, which requires multi-disciplinary knowledge, in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person. References cited below provide an excellent repository of problems. Readers are welcome to pose their difficulties to solve any-problem from anywhere, but only after two attempts to solve. It is our endeavour to stand by upcoming student in their journey to become a scientist, engineer and professional, whatever they choose to be.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics, representative problems from contemporary text books and Question papers from various competitive examinations, it is contemplated to come up with solutions of different type of questions as a dynamic exercise to catalyse the conceptual thought process.

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