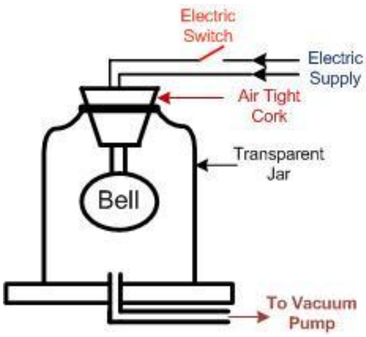
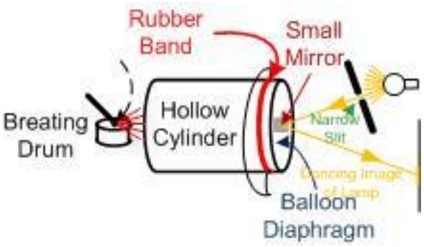
Chapter-VI: Waves and Motion : Sound -Part -II

Dr. Subhash Kumar Joshi

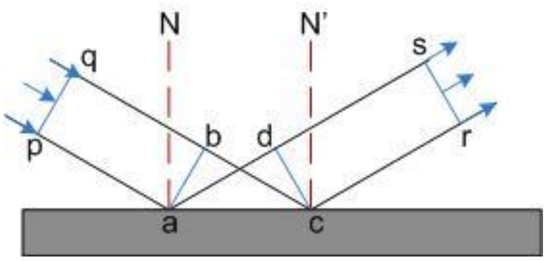
Sound Waves: Sound waves require medium for propagation, and a simple evidence of this fact is shown in an experiment shown in figure. In this with air at atmospheric pressure in the jar, switch is closed to operate the bell, and sound of bell is heard. But, when vacuum pump is operated to evacuate air inside the jar, gradually sound of bell heard outside the jar becomes feeble. ***Sound wave is basically mechanical in nature***. Parameters which regulate velocity of sound wave in solid and fluid, including gases, is elaborated in Part –I. The ***longitudinal nature of sound wave*** can be verified through another simplest experiment as shown in the figure. It comprises of a hollow box with one end open and other end closed with a balloon, tightened with a rubber band. A small piece of mirror is pasted on balloon such that a light ray through a slit creates a image on a wall. As the beating of drum starts at open end of hollow cylinder, the image starts dancing. The simple reason is the pressure waves create a vibration in diaphragm. This vibration changes position of image as per laws of reflection, to be elaborated in light waves. Beating drum sets in longitudinal waves in the form of a pressure wave travelling through air. Velocity of sound waves in different material is shown for comparison.





Frequency of sound waves is ranges from 20 Hz to 20 kHz. Accordingly, waves below 20 Hz are called infrasonic waves and those above 20 kHz are called ultra-sonic waves.

Reflection of sound waves is a usual experience, sound of crackers or beating of drums heard coming in from a direction other than the that of source in acoustic galleries, domes etc. Reflection of sound is simplistically explained with Huygens Wave theory. A wavefront p-q approaching a reflecting surface is shown in the figure.



The wavefront on reaching the reflecting surface is in position a-b. The edge p-a takes another direction along the line 'ad', while the other edge q-b continues its unobstructed travel along line b-c, until it encounters reflecting surface. Since travel a-d and b-c are in the same medium, its velocity shall remain unchanged. After complete reflection the wavefront a-b shall take position c-d. Now, Δabc ≅ Δcda, by RHS theorem since ∠abc = ∠abc=90°, diagonals 'ac' are common and 'ad'='bc' travel distance with uniform velocity in same time. Accordingly, ∠dac=∠bca being corresponding angles, and conversely ∠Nad=∠N'cb. The edges of wavefront remain parallel, travelling in same medium, and therefore, with application of plain geometry ∠Nap=∠Nad, and ∠N'cb=∠N'cr. This is summarized into **Laws of Reflection** as : **i)** *Angle of incidence is equal to angle of reflection*, **ii)** *The incident wave, normal at the point of incidence and the reflected wave are in the same plane.*

Reflection of mechanical waves, synonymous to sound waves at different type of boundary conditions is better explained with mathematical analysis of wave interference. Any incidence wave in an elastic medium is an energy and it continues to perpetuate until it encounter change of medium, eventually change of density. The displacement of particles of medium is mathematically represented as $y_i = A_i \sin(\omega t - k_i x - \varphi_i)$; here $k = \frac{\omega}{v}$, and $\varphi -$ is the phase difference. The frequency (f) of the wave is since decided by the source and hence $\omega = 2\pi f$ remains same. But, velocity of wave $\left(v = \sqrt{\frac{T}{\mu}}\right)$ is characteristic to mass per-unit length density $(\mu)$of medium and tension in the string (T)in it. Accordingly, it is the wavelength $\lambda = \frac{v}{f}$, and not the frequency of wave that changes during propagation of wave through the change in medium. Now, it is pertinent to note that an incident wave encountering change of medium has two possibilities, **a)** *transmission of wave represented by*: $y_t = A_t \sin(\omega t - k_t x - \varphi_t)$ and **b)** *reflection of wave represented by*: $y_r = A_r \sin(\omega t - k_r x - \varphi_r)$.

At this point it is essential to introduce concept of ***impedance*** **(Z)**, *which shall be more elaborately dealt with in current electricity.* $Z = \frac{\text{Force of Wave}}{\text{Velocity of Wave}} = \frac{T}{\sqrt{\frac{T}{\mu}}} = \sqrt{T\mu}$ and boundary there change of impedance.

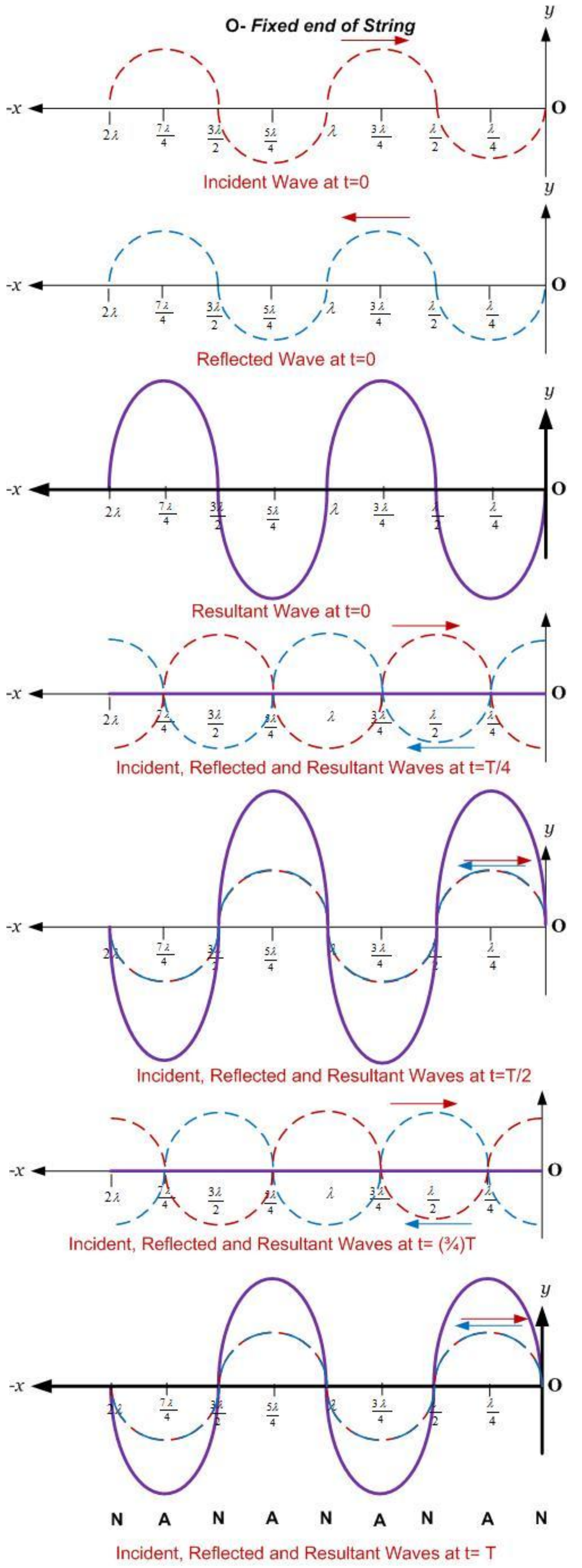
At $x = 0$ the interface of the two strings where tension continues to be unaltered and hence the displacement of the particle from its mean position accordingly, $y(0_-, t) = y(0_+, t)$. This in terms of wave equation at the the boundary shall be: $A_i \sin(\omega t - \varphi_i) + A_r \sin(\omega t - \varphi_r) = A_t \sin(\omega t - -\varphi_t)$. This equality necessitates that $\phi_{i} + \phi_{r} = \phi_{t}$, which leads to $A_i + A_r = A_t$.

Similar to collision of elastic bodies, behaviour of wave at a boundary satisfies two conditions: **a)** *Continuity of displacement* $(\boldsymbol{y_i + y_r = y_t})$. *This can be represented as* $A_i = A_t + A_r$. This is in conformance with the Newton's Second Law of Motion at a boundary where no external force is coming into play, and **b)** *Conservation of energy in the elastic medium* $(E_i = E_t + E_r)$. Energy of wave is expressed as $E = \frac{1}{2} Z\omega^2 A^2$.

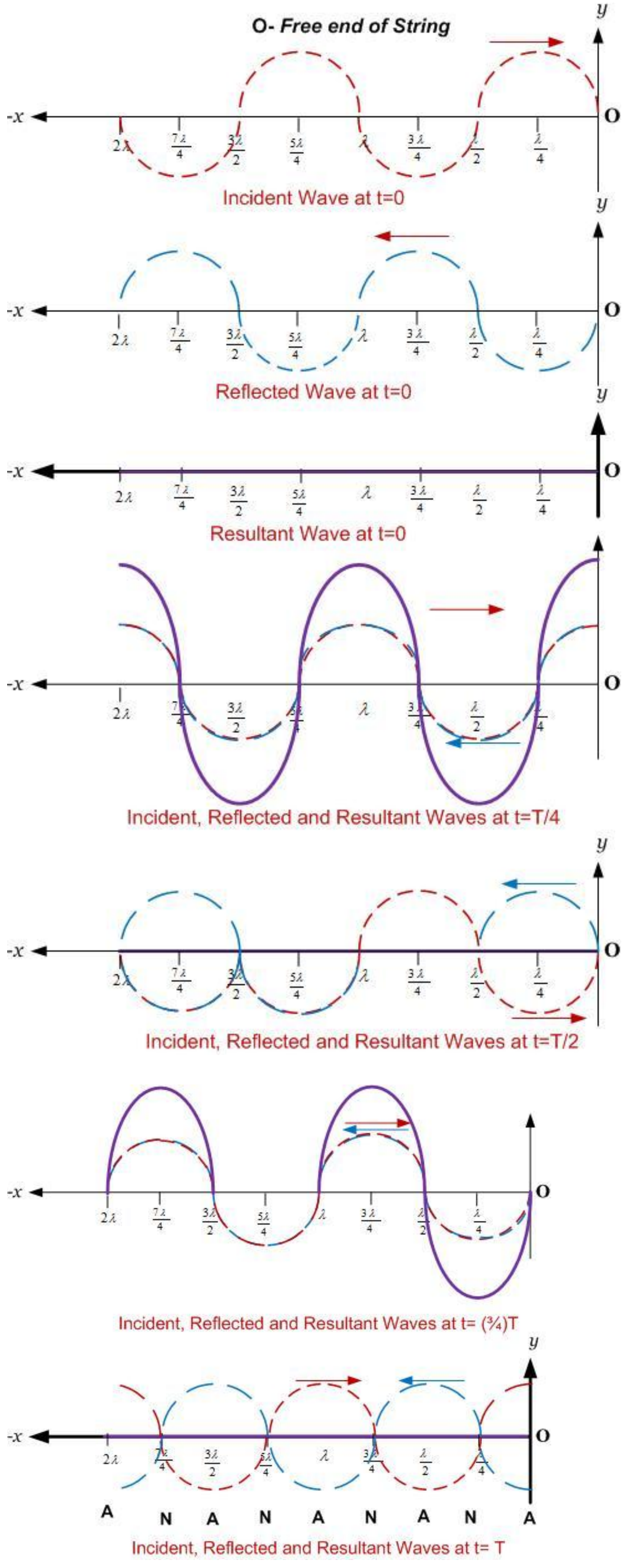
Using these conditions and principle of interference wave equation and principle of shall be determined for the medium carrying incident wave as: $\frac{1}{2} Z_1 \omega^2 A_i^2 = \frac{1}{2} Z_2 \omega^2 A_t^2 + \frac{1}{2} Z_1 \omega^2 A_r^2$; since reflected wave travels in the medium of incident wave and hence $Z_i = Z_r$. Accordingly, $Z_1(A_i^2 - A_r^2) = Z_2 A_t^2$; $Z_1(A_i + A_r)(A_i - A_r) = Z_2 A_t^2$. Combining this energy balance equation with continuity of displacement it leads to $Z_1(A_i - A_r) = Z_2 A_t$, or $(A_i - A_r) = \frac{Z_2}{Z_1} A_t$. This is a case of simple simultaneous equation with two variables A_t and A_r in terms of A_i leading to $A_t = \frac{2Z_1}{Z_1 + Z_2} A_i$, and $A_r = \frac{Z_1 - Z_2}{Z_1 + Z_2} A_i$. Accordingly, different possibilities exist out of which only Two cases are being analysed to understand the phenomenon of wave reflection, and are as under:

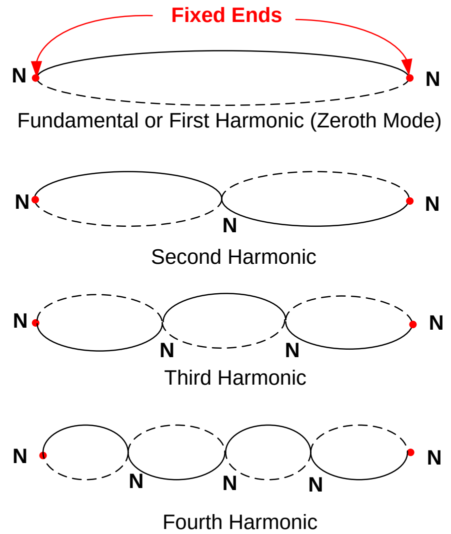
<table>
<tr><th>Case 1 : $Z_2 \to \infty$; Rigid Boundary</th><th>Case 2: $Z_2 \to 0$; Boundary with Rarefied Medium</th></tr>
<tr><td>$A_t = 0, \ A_r = -A_i$</td><td>$A_t = 2A_i; \ A_r = A_i$</td></tr>
<tr><td>Analytical perspective: $y = A_i \sin\left[\omega\left(t - \frac{x}{v}\right)\right] - A_i \sin\left[\omega\left(t + \frac{x}{v}\right)\right]$
$= -2A_i \sin\frac{x}{v} \cos\omega t$ (Using trigonometric identities)
At boundary $(x = 0)$ displacement is zero. Negative sign indicates that resultant wave has a phase shift of 180° w.r.t. incident wave, or conversely, in $-x$ direction, it is constructive.
Mechanics Perspective: This kind of reflection is in conformance with Newton's Third Law of Motion.</td><td>Analytical perspective: $y = A_i \sin\left[\omega\left(t - \frac{x}{v}\right)\right] + A_i \sin\left[\omega\left(t + \frac{x}{v}\right)\right]$
$= 2A_i \cos\frac{x}{v} \sin\omega t$ (Using trigonometric identities)
At boundary $(x = 0)$ displacement is Doubled.
Positive sign indicates that resultant wave is in phase w.r.t. incident wave, it is constructive
Mechanics Perspective: When a pressure wave reaches free boundary, it passes by creating a rarefaction as a reflected wave</td></tr>
<tr><td colspan="2">Magnitude of the resultant displacement is product of two components-
a) Displacement: as function position in the direction of propagation = $-2A_i \cos\frac{x}{v}$
b) Displacement as a function of ωt and is equal to $\sin\omega t$
c) These two components, together, change the combination of incident and reflected wave into a stationary or standing wave, where particles of medium perform SHM, but there is not transfer of energy.</td></tr>
</table>

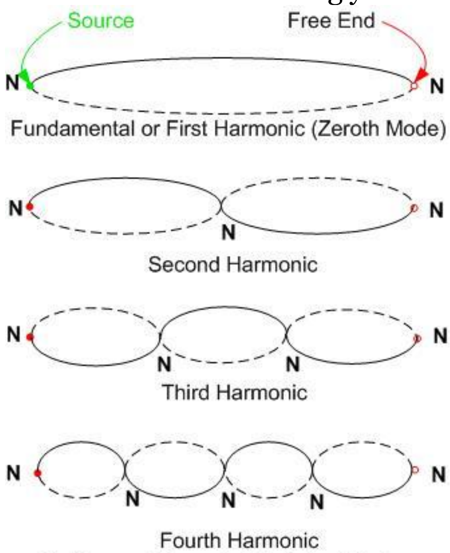
Graphical Representation of the Standing Wave in string with One end Fixed:

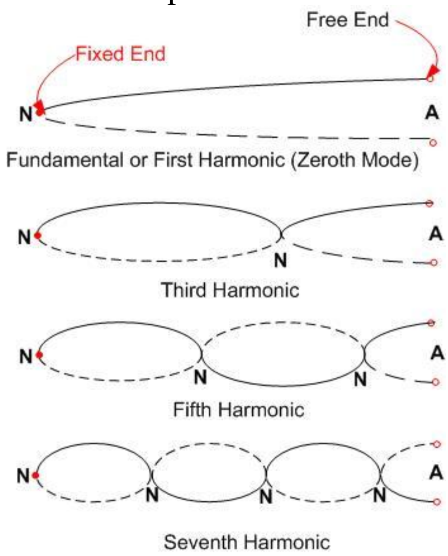


Graphical Representation of the Standing Wave in string with One end Free:



<table>
<tr>
<td>Various Modes of Stationary Waves in Stretched String with Ends Fixed


Stationary Waves in Stretched Strings</td>
<td>Various Modes of Stationary Waves in Stretched String With End Free to Move Along y-axis


Stationary Waves in Stretched Strings With One End Free to Move along y-axis</td>
<td>Various Modes of Stationary Waves in Wind Column with One End Open to Ambient


Stationary Waves in Wind Column With One Fixed and Other Free to Move along y-axis</td>
</tr>
<tr>
<td>Implications: All string based musical instruments, vibrations in stretched rope or wire with fixed ends</td>
<td>Implications: Air column based musical instruments viz, Flute, Shahnai etc. in which One side wind is blown and other is open.</td>
<td>Implications: Jal Tarang (जल तरंग) musical instrument, Sound produced while filling a bottle, closed at one end, from tap.</td>
</tr>
<tr>
<td>Resonant Frequency: $f = n\frac{v}{2L} = \frac{nv}{2L}$; $v = \sqrt{\frac{T}{\mu}}$. Here, n is an integer</td>
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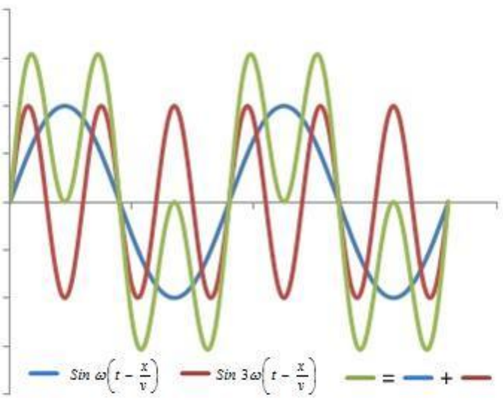
In case of string: $v = \sqrt{\frac{T}{\mu}}$.
In case of air column: $v = \sqrt{\frac{\gamma P}{\rho}}$, as</td>
<td>Resonant Frequency: $f = n\frac{v}{4L} = \frac{nv}{4L}$ and $v = \sqrt{\frac{\gamma P}{\rho}}$; Here, n is an odd number</td>
</tr>
<tr>
<td colspan="3">Modes: Lowest possible resonant frequency is called normal mode, it has $n=1$. All higher modes of resonance are called (n-1) Overtone or nth Harmonic.</td>
</tr>
</table>

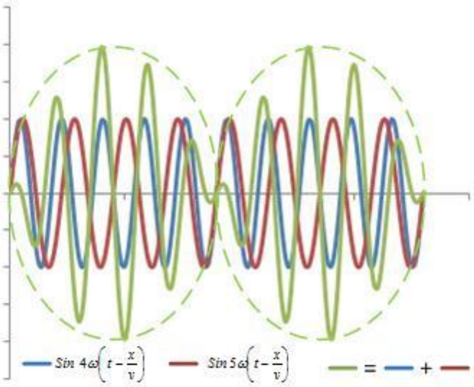
Speed of Sound: Propagation of sound is in space and is unlike rectilinear motion in string, and hence it is referred to as ***Speed of Sound*** and not Velocity of Sound. Velocity of Sound in different typical medium are tabulated below:

Gaseous Medium		Liquid Medium		Solid Medium	
Air at 0°C	331 m/s	Air at 0°C	1402 m/s	Aluminium	6420 m/s
Air at 20°C	343 m/s	Air at 20°C	1482 m/s	Copper	3560 m/s
Helium	965 m/s	Sea Water	1522 m/s	Steel	5941 m/s
Hydrogen	1284 m/s			Granite	6000 m/s
				Vulcanised Rubber	54 m/s

Sound Beats: Analysis of interactions of different waves was of same frequency but with either a phase shift or travelling in opposite direction. But, two waves of different frequencies, with same magnitude when interact with each other it creates a different effect known as ***Beats***, another manifestation if interference, and this too can be determined analytically and also explained graphically.

Let two signals, $y_1 = A \sin \omega_1 \left(t - \frac{x}{v}\right)$ and $y_2 = A \sin \omega_2 \left(t - \frac{x}{v}\right)$ be taken to be travelling along x-axis interfere with each other the resultant wave shall be $y = A \sin \omega_1 \left(t - \frac{x}{v}\right) + A \sin \omega_2 \left(t - \frac{x}{v}\right) = 2A \cos \frac{\omega_1 - \omega_2}{2} \left(t - \frac{x}{v}\right) \sin \frac{\omega_1 + \omega_2}{2} \left(t - \frac{x}{v}\right)$. This resultant wave has three distinct features: **a)** extrinsic amplitude of the envelop signal is 2A , b) Frequency of the envelop signal is $\frac{\omega_1 - \omega_2}{2}$, and **c)** the intrinsic signal of frequency $\frac{\omega_1 + \omega_2}{2}$ has varying amplitude. Interference of Two examples of the kind are shown in figure, one is of frequencies ω and 3ω and other is of frequencies 4ω and 5ω. In the later example where relative frequency is small $\frac{\omega_1}{\omega_2} \to 1$or $\frac{|\omega_1 - \omega_2|}{\omega_2} \to 0$, *variation in amplitude of the signal along its cycle becomes distinct, and it causes variation loudness called* ***Beats***. This is due to gradual variation in displacement or pressure during the cycle as the wave progresses. This superimposition principle is called ***amplitude modulation*** finds application in communication.





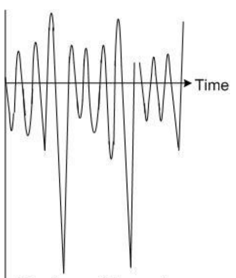
Characterization of Sound: There are certain parameters which are used to characterize sound viz. ***Loudness*** or ***Intensity, Pitch*** or ***frequency, Quality*** or ***waveform***.

Pitch of Frequency of Sound: This helps to discriminate source if sound, viz sound of animals is of low Pitch, while male voice is of medium pitch, and female voice is of high pitch. Likewise, pitch of musical nodes and instruments are also different and help in discrimination by the listener.

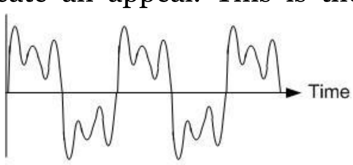
Power of Sound Wave $(= 2\pi^2 \mu v A^2 \mathrm{v}^2)$ is called intensity and, accordingly, ***Reference Intensity of Sound*** has been standardized as $I_0 = 10^{-12} W m^{-2}$, and sound level (β) is expressed in **Decibel (dB)** such that $\beta = 10 \log_{10} \left(\frac{I}{I_0}\right)$. Typical sound levels are shown in the table.

Sound	**Intensity ($W\text{-}m^{-2}$)**	**Sound Level (dB)**
Minimum Hearing Threshold	1×10^{-12}	0
Whisper at 1 m	1×10^{-10}	20
City Street without traffic	1×10^{-9}	30
Office or Classroom	1×10^{-7}	50
Normal Conversation at 1 m	1×10^{-6}	60
Rock Music	1×10^{-1}	110
Maximum Tolerable	1	120
Jet Engine	10	130

Quality or Waveform: As in case of food plain or rice are not relished, unless they are supplemented with vegetable, dal, raita etc. Likewise, plain source having single frequency does not create an appeal. This is the reason each musical instrument is designed to produce a mix of frequencies to create a compound waveform and music-composers use a mix of musical instruments to improve upon quality of their music. Quality (Waveform) of Two musical instruments Trumpet and Clarinet are representative in nature, and similar waveforms, with a difference of degree exist for other musical instruments.



Waveform of Trumpet



Waveform of Clarinet

Diffraction of Sound: Propagation of sound **as** Huygens Wave theory is in the form of a spherical wavefront. This wavefrom tends to become plane as it travels away from the source and is substantiated with the principles of geometry radius of curvature $\left(= \frac{1}{r}\right) \to 0 \; as \; r \; \to \infty$. Nevertheless, wavefront changes shapes as and when it encounter obstacle. This is the reason, a person talking on one side of the wall is heard by the other standing across the wall. Likewise, through a small hole it is possible to hear sound across a thick wall. Such bending of sound waves, which otherwise propagate radially in a straight line, is called ***Diffraction***. Diffraction effect gets amplified with increase of frequency, or reduction in wavelength. Audible frequencies ranging from 20 Hz to 20 kHz its wavelength in air ranges from 16 m to 1.6 cm. Sound waves of larger wavelengths are more susceptible to diffraction. *This is more perceivable in light waves and shall be elaborated in that section to follow in a greater length.*

Musical Scale: Musical notes are defined with an Ocatve of Seven Notes as shown in the table. There are different standard notes, nevertheless, multiple of base frequencies are called scales of the notes and is pertinent to learning of music.

Musical Notes in Indian Music	Frequency
सा	256 Hz
रे	288 Hz
ग	320 Hz
म	341.33 Hz
प	384 Hz
ध	426.33 Hz
नी	480 Hz
सा	512 Hz

Acoustics: This is application of principles of sound waves to improve upon quality of audibility, which influenced by reflection of sound causing ***echo***. It must have been experienced that in an empty hall, without furniture and incumbents, cause echo and reverberations. This echo gets reduced with furnishing, curtains and occupancy. Wooden furniture causes lower echo as compared to steel furniture or non-porous objects. This is the reason as to why ceiling, walls, are made of porous or perforated material and location of speakers and seating arrangements are designed and engineered, using the concepts elaborated above in a manner such that ***reverberation of sound*** from different reflecting surfaces is : **a)** at an acceptable and **b)** uniform across the audience. Readers are welcome to raise their inquisitiveness, beyond the contents, through Contact Us.

Summary: *In Part-I concepts of waves that were elaborated have been extended to theory of Sound and Light Waves. Part III of this series shall encompass Geometrical Optics. Theory of Wave and its application into sound and optics is an excellent example of integrating analytical methods into physical phenomenon, which are otherwise difficult to visualize. Nevertheless, readers are welcome to raise their inquisitiveness, beyond the contents, through* Contact Us.

Examples, during elaborations, have been drawn from real life experiences help to build visualization and an insight into the phenomenon occurring around. Solving of problems, is an integral part of a deeper journey to make integration and application of concepts intuitive. This is absolutely true for any real life situations, which requires multi-disciplinary knowledge, in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person. References cited below provide an excellent repository of problems. Readers are welcome to pose their difficulties to solve any-problem from anywhere, but only after two attempts to solve. It is our endeavour to stand by upcoming student in their journey to become a scientist, engineer and professional, whatever they choose to be.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics, representative problems from contemporary text books and Question papers from various competitive examinations shall be drawn to come up with solutions of different type of questions as a dynamic exercise which is contemplated to catalyse the conceptual thought process.

References:

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3. *Resnick, Halliday, Resnick and Krane; Physics (Vol I and II).*
4. *Sears & Zemansky; University Physics with Modern Physics.*
5. *I.E. Irodov; Problems in General Physics*

Author is Coordinator of this initiative **Gyan-Vigyan Sarita**, a non-organizational entity of co-passionate persons who are dedicated to the selfless mission through ***Online Mentoring Sessions*** *(OMS)* to unprivileged children. He had his career as a power engineer, and after superannuation he did his Ph.D, from IIT Roorkee; soonafter this in 2012, he took a plunge into mentoring unprivileged children with Sarthak Prayash an NGO. The endeavour continued in different forms. Currently the thrust area, at the behest of District Administration, is School of Excellence, in Jhabua, a tribal District in MP.

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