Wave and Motion : <u>Illustrations</u> of Objective and Subjective Questions

I-01

The statement of the student has three components –

(i) Force applied on a particle is $F = -\frac{k}{x}$,

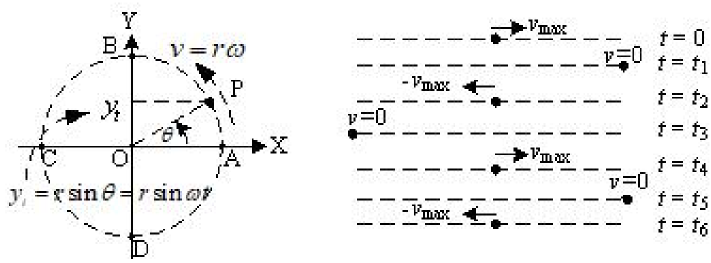
(ii) Particle moved in a SHM

(iii)Information on $k = \text{Constant}$ or $k = f(x)$ is not revealed .

Thus with given (i) for the certainty on (ii) $k = \text{Constant}$ as **option (c) is ruled out**. Further, for SHM necessary condition, taking Newton's Second Law of Motion into consideration, leads to $F = ma = m\frac{d^2x}{dt^2} = -Kx$, here K is a constant. Thus combining NSLM with the given information at (i), $F = -Kx = -\frac{k}{x} \Rightarrow k = Kx^2$. It leads to as x increases k would also increase and is **provided in option (a).** But, it contradicts option (b) stating decrease in k with increase of x. Further, option (d) is contradicting the basic premise given at (ii), and thus option (d) is also ruled out. Accordingly, **answer is option (a).**

I-02

SHM is defined by a mathematical equation $a_y = \frac{d^2y}{dt^2} = -ky$ solution of this differential equation is $y = r\sin\theta = r\sin\omega t$. Conceptually trace of a particle performing uniform circular motion, on one of its diameter satisfies the condition of SHM. And the when particle competes one complete revolution on the circular orbit, it is called one cycle and time taken to complete this one revolution is called time period.

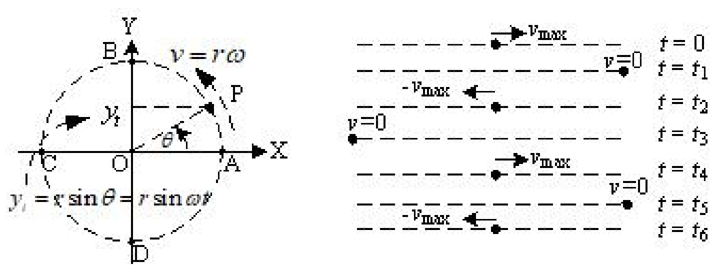


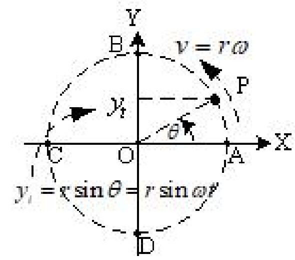
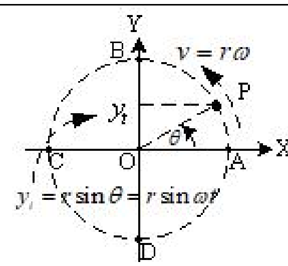
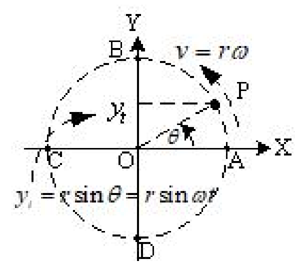
With these definitions during SHM, particle passes through its mean position 'O' twice ove during transition from (-)ve displacement to (+)ve displacement and second time during transition from (+) displacement to (-)ve displacement. This is true for every point in between Two extreme positions which are A and B when $y = y_{max} = +r$ and $y = -y_{max} = -r$. **This rules out option (a), (c) and (d).**

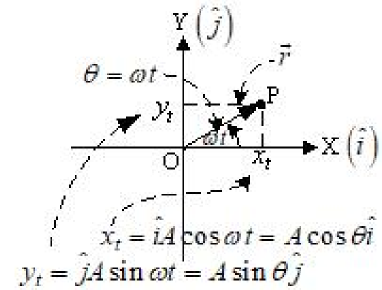
They are the only two points A and B, points of maximum displacement where particle appears only after completing one cycle. And is provided in option (b). **Hence answer is Option (b).**

I-03

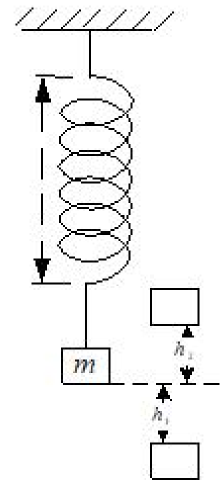
SHM is defined by a mathematical equation $a_y = \frac{d^2y}{dt^2} = -ky$ solution of this differential equation is $y = r\sin\theta = r\sin\omega t$. Conceptually trace of a particle performing uniform circular motion, on one of its diameter satisfies the condition of SHM. And the when particle competes one complete revolution on the circular orbit, it is called one cycle and time taken to complete this one revolution is called time period. Velocity of particle at different values of θ is expressed as $v_t = v_\theta = \frac{d}{dt}r\sin\omega t = r\omega\cos\omega t = r\omega\cos\theta$.Thus maximum absolute value of instantaneous velocity occurs at $\theta = 0, \theta = \pi, \theta = 2\pi, \theta = 3\pi, \ldots$. A close observation would reveal that actual maximum velocity occurs at

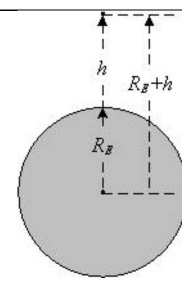


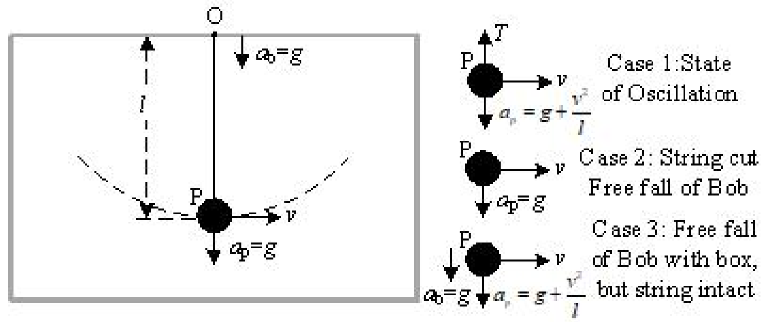
	when the particle traverses back to point A i.e. $v_{\theta=0}=V_{\max};v_{\theta=\pi}=-V_{\max};v_{\theta=2\pi}=V_{\max};v_{\theta=3\pi}=-V_{\max};...$ Accordingly, time period is when particle re-traverses through point A or smallest time between occurrence of $V_{\max}$ and not the $\left\|(v)_{\text{t}-\max}\right\|$. This condition is supported only by option (a). **Hence answer is option (a).**
I-04	When particle P is performing uniform circular motion, its trace on Y-axis performs SHM. The particle completes one cycle or traverses for One time period when it reaches the same point on the perimeter of the circle with same velocity, i.e. direction and magnitude of the velocity is same. Let us consider projection of the particle P when at position A, to complete one cycle it displaces from point O through A to O to D to O. Since displacement is a vector quantity and hence net displacement is $\Delta\vec{y}=r\hat{j}+r\left(-\hat{j}\right)+r\left(-\hat{j}\right)+r\hat{j}=0$. Hence net displacement in one time period matches with that given in option (d). **Hence answer is option (d).** 
I-05	When particle P is performing uniform circular motion, its projection on Y-axis performs SHM. The particle completes one cycle or traverses for One time period when it reaches the same point on the perimeter of the circle with same velocity, i.e. direction and magnitude of the velocity is same. Let us consider projection of the particle P when at position A, to complete one cycle it displaces from point O through A to O to D to O. Since distance is a scalar quantity and hence absolute value of net distance would be $\Delta y=\left\|r\hat{j}\right\|+\left\|r\left(-\hat{j}\right)\right\|+\left\|r\left(-\hat{j}\right)\right\|+\left\|r\hat{j}\right\|$. This solves into $\Delta r=r+r+r+r=4r$. In the instant figure $r\to A$. Hence net distance moved by the particle is $4A$ and is provided in the option (c). **Hence answer is option (c).** 
I-06	As per definition average acceleration is $\vec{a}_{av}=\frac{\vec{v}_{t2}-\vec{v}_{t1}}{t_2-t_1}$, here basic quantity acceleration is a vector. Further, SHM is defined by a mathematical equation $a_y=\frac{d^2y}{dt^2}=-ky$ solution of this differential equation is $y=r\sin\theta$. Accordingly, velocity of projection of the particle performing is $\frac{dy}{dt}=r\frac{d}{dt}\sin\omega t=r\omega\cos\omega t=r\omega\sin\theta$  Further, as per definition projection of a particle completes one time period of when the particle performing SHM rotates through one revolution. Say at time $t_1$corresponding angle $\theta_1=\theta$ then $t_2=t_1+T\to\theta_2=2\pi+\theta_1=2\pi+\theta$. Accordingly, average acceleration is $\vec{a}_{av}=\frac{r\omega\cos\left(2\pi+\theta\right)\hat{j}-r\omega\cos\theta\hat{j}}{\left(t_1+\pi\right)-t_1}=r\omega\frac{r\omega\left(\cos\theta-\cos\theta\right)}{\pi}\hat{j}=0$. This in accordance with only option (d). **Hence answer is option (d).**
I-07	Given that $x=A\sin\omega t+B\cos\omega t$. Taking that $A=R\sin\alpha$ and $B=R\cos\alpha$ the motion of particle can be rewritten as $x=R\sin\alpha\cdot\sin\omega t+R\cos\alpha\cdot\cos\omega t=R\left(\sin\alpha\cdot\sin\omega t+\cos\alpha\cdot\cos\omega t\right)=R\cos\left(\omega t-\alpha\right)$. This equation can be further modified as $x=R\sin\left(\omega t-\alpha-\frac{\pi}{2}\right)=R\sin\left(\omega t-\phi\right)\Big\|_{\phi=\frac{\pi}{2}+\alpha}$. This equation is parallel to equation of SHM where R is the amplitude. Here as per trigonometric identity $A^2+B^2=R^2\sin^2\alpha+R^2\cos^2\alpha=R^2\left(\sin^2\alpha+\cos^2\alpha\right)=R^2\to R=\sqrt{A^2+B^2}$. The derived value of

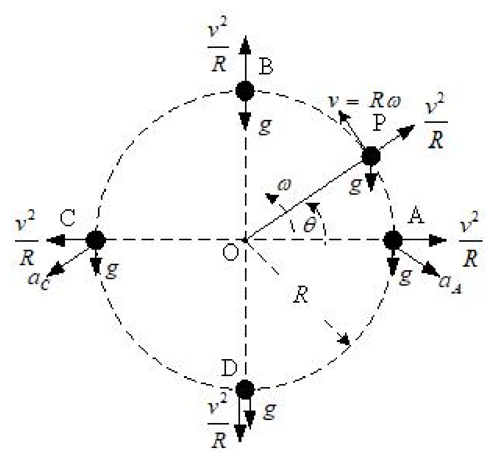
	amplitude is provided in option (d). **Hence answer is option (d).**
I-08	Position vector of the particle P at any instant is shown in the figure where magnitude of the position vector is $r = \left\|\vec{r}\right\|$ and using geometric relationship $r = \sqrt{A^2 \sin^2 \omega t + A^2 \cos^2 \omega t} = A\sqrt{\sin^2 \omega t + \cos^2 \omega t} = A$. This is true for all values of $\theta = \omega t$, i.e. at any instant of time t where A is a constant with centre of the locus at O. This signifies a motion is along a circle of radius A and centre at O. This is specified only in option (c). **Hence answer is Option (c).** 
I-09	For a motion of a particle to be simple harmonic necessary condition is $a_x = \frac{d^2x}{dt^2} = -kx$. While motion of the particle is given to be $x = A + B\sin\omega t \to \frac{d^2}{dt^2}x = \frac{d^2}{dt^2}\left(A + B\sin\omega t\right) = \frac{d}{dt}\left(\frac{d}{dt}\left(A + B\sin\omega t\right)\right)$. It leads to $a_x = \frac{d}{dt}\left(B\omega\cos\omega t\right) = B\omega\frac{d}{dt}\cos\omega t = -B\omega^2\sin\omega t$. Comparing this result with condition of SHM we get $a_x = -kx = -B\omega^2\sin\omega t \Rightarrow -k\left(A + B\sin\omega t\right) = -B\omega^2\sin\omega t$. It leads to – (i) Mean position of the particle is at $\left.\left\|x = A\right\|\right._{t=0}$, (ii) Particle is performing SHM along about point A, (iii) Angular velocity of the SHM is ω and (iv) Amplitude of SHM is B. Thus option (b) is in agreement with the conclusions derived above. **Hence answer is option (b).**
I-10	The Two SHMs shown in the figure above have – (i) Same magnitude judged from the length of the projection of the motion, **hence option (a) is not the answer**. (ii) Both the SHMS start and complete one oscillation at the same instants and hence their Time Period $T \Rightarrow f = \frac{1}{T}$ are the same. **Hence option (b) is not the answer**. (iii) Particle A is starts motion at its mean position in (+)ve direction, while particle B starts from mean position in (-)ve direction. Hence both the SHMS are in different phases. Hence option (c) is the correct answer. (iv) Particle performing SHM $\left(x = A\sin\omega t\right)$ has maximum velocity $v_{\max} = \frac{dx}{dt} = A\omega\cos\omega t = \left.A\omega\right\|_{\omega t = n\pi, n \in W}$. Since amplitude of the Two SHM are same as derived at (i) above and frequency being same as derived at (ii) above, which in turn gives equal angular velocity $\left(\omega = 2\pi f = \frac{2\pi}{T}\right)$ will also have same maximum velocity. Hence option (d) is not the answer. **Thus from above analysis option (c) is the answer.**
I-11	Total mechanical energy of a spring mass system is given to be $E = \frac{1}{2}m\omega^2 A^2$. In the instant case initially $E_1 = \frac{1}{2}m\omega^2 A^2$, when mass is doubled $E_2 = \frac{1}{2}\left(2m\right)\omega^2 A^2$ then as per principle of Conservation of Energy $E_2 = E_1 + \Delta W$ where is external work done on the system. But, in question there is no mention of external work

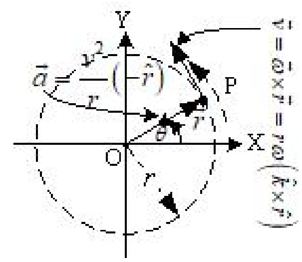
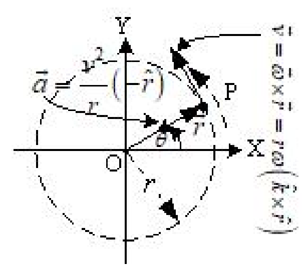
	done on the spring mass system and hence $\Delta W = 0 \Rightarrow E_2 = E_1 \Rightarrow \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(2m)\omega_2{}^2 A^2$ where in latter case angular velocity would adjust to $\omega_2 = \frac{\omega}{\sqrt{2}}$ without change of mechanical energy. **Thus answer would be option (d).**
I-12	During SHM mechanical energy of a particle $E = PE + KE$. At mean position velocity of the particle is maximum $v_{\max} = A\omega$ and hence maximum kinetic energy $KE_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m\omega^2 A^2$ while potential energy is taken to be at reference such that $PE_{\min} = 0$ and at extreme position its potential energy is at $PE_{\max}$ while kinetic energy is at minimum $KE_{\min} = 0$. As per principle of Conservation of energy and an ideal SHM neither there is input or output to system and hence total energy remains fixed at $\frac{1}{2}m\omega^2 A^2$, which is provided only in option (a) **Thus answer is option (a).**
I-13	During SHM particle is performing SHM along X-axis with its mean position at origin. While moving from (+) extreme to (-)ve extreme change is velocity from Zero to Maximum and then back to zero, but while changing its magnitude its velocity vector is in direction $-\hat{i}$ and while moving from (-) extreme to (+)ve extreme change is velocity from Zero to Maximum and then back to zero, but while changing its magnitude its velocity vector is in direction $+\hat{i}$. Energy is a scalar quantity and so also is Kinetic energy $KE = \frac{1}{2}mv^2$. Accordingly, irrespective of the direction of velocity KE energy shall always be (+)ve. Thus while particle traverses from (+)ve extreme to (-)ve extreme through Zero during half cycle of SHM the KE will complete One cycle and therefore in One cycle of SHM the KE will complete Two cycles. According for a SHM of frequency υ the frequency of the energy cycle shall be 2υ. This derivation is supported by only option (c) and **hence answer is option (c)** .
I-14	SHM executed by a particle under restoring force k N.m^{-1} (here it is not per meter length of spring but it is per unit deformation) of a spring has frequency $\omega = \sqrt{\frac{k}{m}} \Rightarrow 2\pi f = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$, here m is mass of the particle and spring is considered to be mass less. However, in problem mass of the particle is not given at the same time spring is also not stated to be mass less and hence m is taken to be mass of the spring which is performing SHM and hence every particle on it will experience same SHM. Let initial time period for full length of the spring having mass m is $T = 2\pi\sqrt{\frac{m}{k}}$. When spring is divided in two equal parts each part will have mass $\frac{m}{2}$. Accordingly, time period of the one part of spring executing SHM will be $T' = 2\pi\sqrt{\frac{m/2}{k}} = 2\pi\sqrt{\frac{m}{k}} \cdot \frac{1}{\sqrt{2}} = \frac{T}{\sqrt{2}}$. This derived value is provided in option (d). **Hence answer is option (d).** **N.B.:** Here assumptions made in the solution are important and have been specially brought out for the answer to be correct.

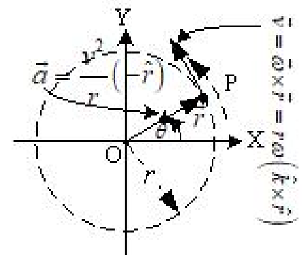
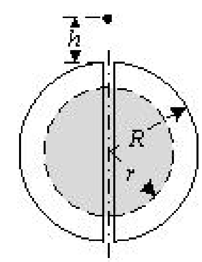
I-15	For two equal masses to attain equal maximum velocities they must stretch through equal lengths and such that net change of potential energy are equal so that as per law of conservation of energy it attains equal maximum velocities during oscillation. Now since masses are stated to be suspended from mass less spring maximum velocity at mean position will have two different values since gravitational force external to spring-mass system is impeding the SHM. Accordingly, in for spring with its constant k_1 the equations evolve into -– (a) during descend from top most position at height h_2 the energy equation will be $\frac{1}{2}mv_{d-\max}^2 = \frac{1}{2}k_1h_2^2 + mgh_2$. This form of equation in this case is since gravitational and spring forces additive. (b) during ascend from bottom most position at depth h_1 the energy equation will be $\frac{1}{2}mv_{a-\max}^2 = \frac{1}{2}k_2h_1^2 - mgh_1$. This form of equation in this case is since gravitational and spring forces subtractive. But, here we have two equations with four variables h_1, h_2, v_a and h_d and they are unsolvable and same is true for other spring having different spring constant k_2. This unsolvable situation in this problem is solved with an assumption that component of gravitational potential energy is considered negligible in comparison to spring potential energy in both the cases. This will make maximum velocity of ascend equal to maximum velocity of descend with respective maximum ascend/descend in two cases as amplitude of oscillation..  Accordingly comparing two equations taking maximum velocities for two springs $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}k_1h_1^2 = \frac{1}{2}k_2h_2^2 \Rightarrow \frac{h_1}{h_2} = \sqrt{\frac{k_2}{k_1}}$. This derived value matches with answer (d), **Hence answer is option (d).** **N.B.:** Here assumption made in the solution is important and have been specially discussed to derive the answer into a solvable form.
I-16	Requirement of SHM is $a_x = \frac{d^2x}{dt^2} = -kx \Rightarrow x = k\omega^2 \sin \omega t$ where $\omega = 2\pi f$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. This relation is satisfied by the restoring force of the spring. But, when the system is taken in an elevator accelerating upward slowly, it will be acting against gravitational force and in turn diminishes its effect on maximum displacement from mean position along Y-axis, but the frequency shall be regulated by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ which is independent of acceleration of the escalator. And hence frequency shall remain constant as provided in option(c) only. **Hence answer is option (c).** **N.B.:** Answered arrived at with proper analysis of concept with its mathematics, it would always lead to correct answer.
I-17	Requirement of SHM is $a_x = \frac{d^2x}{dt^2} = -kx \Rightarrow x = k\omega^2 \sin \omega t$ where $\omega = 2\pi f$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. This relation is satisfied by the restoring force of the spring. But, when the system is taken in a car accelerating horizontally it will be act in a manner to diminish maximum displacement along X-axis in each oscillation , but the frequency shall be regulated by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ which is independent of acceleration of the escalator. And hence frequency shall remain constant as provided in option(c) only. **Hence answer is option (c).** **N.B.:** Answered arrived at with proper analysis of concept with its mathematics, it would always lead to correct

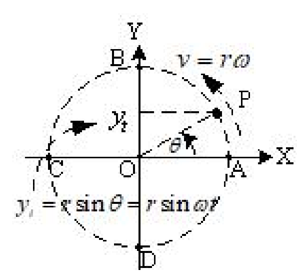
	answer.
I-18	Angular velocity of a pendulum is expressed at $\omega=\sqrt{\frac{g}{l}}=2\pi f \Rightarrow T=\frac{1}{f}=2\pi\sqrt{\frac{l}{g}}$. Accordingly, for a certain pendulum time period on earth is $T_E=2\pi\sqrt{\frac{l}{g_E}}$ and on soon it would be $T_M=2\pi\sqrt{\frac{l}{g_M}}$. Generally, in such problems values of acceleration due to gravity on planets is given. But, in absence of such information in the question from general knowledge (GK) $\frac{g_E}{g_M}\approx 6$. Therefore ratio of time periods will be $\frac{T_M}{T_E}=\frac{\sqrt{\frac{l}{g_M}}}{\sqrt{\frac{l}{g_E}}}=\sqrt{\frac{g_E}{g_M}}=\sqrt{6}\Rightarrow T_M=\sqrt{6}T_E$. Since time period of pendulum on moon is $\sqrt{6}$ times greater and hence the pendulum clock would run $\sqrt{6}$ times slower. This derivation is matching with the option (d). **Hence answer is option (d).** **N.B.:** The acceleration due to gravity is 1.62 m/s^2. This is approximately 1/6 that of the acceleration due to gravity on Earth, 9.81 m/s^2.
I-19	It is given that the clock gives correct time at the equator. The operation of the clock is regulated by spring mass-system where $T=2\pi\sqrt{\frac{m}{k}}$, where m is the mass and k is spring constant and is independent of acceleration due to gravity g, though it changes marginally at equator and poles. Therefore the clock will give correct time at the poles also as it is the equator. **Hence answer is option {d).**
I-20	Time period of the pendulum is $T=2\pi\sqrt{\frac{l}{g}}$ and value of acceleration due to gravity on earth on its surface is $g=\frac{GM_E}{R_E^2}$, here is M_E mass of the earth and R_E is radius of the earth. When object is taken to high altitude say h then effective distance affecting $g\rightarrow g'$ is $R_E\rightarrow R_E+h$. Accordingly, $g'=\frac{GM_E}{(R_E+h)^2}$. It solves into $g'=\frac{GM_E}{R_E^2\left(1+\frac{h}{R_E}\right)^2}=\frac{GM_E}{R_E^2}\left(1+\frac{h}{R_E}\right)^{-2}=g\left(1-2\left(\frac{h}{R_E}\right)-2\times 3\left(\frac{h}{R_E}\right)^2....\right)$. Thus $g'\approx g\left(1-2\left(\frac{h}{R_E}\right)\right)$ To keep correct time $\frac{T}{T'}=\frac{2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l'}{g'}}}=1\Rightarrow\sqrt{\frac{l}{g}}=\sqrt{\frac{l'}{g'}}\Rightarrow l'=g'\frac{l}{g}\Rightarrow l'\propto g'\big\|_{\frac{l}{g}=\text{Constant}}$. Thus as altitude is increased the length of pendulum must be decreased to maintain the correct time. This conclusion is in accordance with the option (c), and **hence answer is Option (c).** 

I-21	It is given that a box with a simple pendulum tied to its ceiling is released for a free fall when bob of the oscillating pendulum is at its lowest point, as shown in the figure. The FBD of the bob is analyzed in three different cases to analyze motion of the bob.  **Case 1:** It is when bob reaches its lowest point as in figure. In this case string remains taught with $T = m\left(g + \frac{v^2}{l}\right)$. This instant is taken to be $t = 0$. **Case 2:** Keeping box in it position at $t = 0$ string is cut allowing bob to make a free fall. In this case at $t = 0^+$, on the bob – (i) along vertical axis initial velocity is $u_{v-0^+} = 0$ but acceleration remains constant at $a_v = g$, (ii) but along horizontal direction acceleration $a_h = 0$ and hence horizontal velocity remains constant at $v_h = v$. This will lead to a projectile motion with a parabolic trajectory. **But, this is not the case to be solved.** **Case 3:** Box is released at $t = 0$ as shown in the figure. Therefore, at $t = 0^+$ acceleration on the bob and the box due to gravity is equal to g .Therefore, net acceleration of the bob w.r.t. box (i.e. as seen from the box) will be $a_{p-b} = a_p - a_b = \left(g + \frac{v^2}{l}\right) - g = \frac{v^2}{l}$. This is the case of a circular motion of a particle experiencing centripetal force balanced by centrifugal force of the string of fixed length l holding the bob on a circular motion about point O. This circular motion is provided only in option (c). **And hence answer is option (c).** **N.B.:** The kind of analysis adds clarity and conviction to answer and leaves no room for the doubt.
I-22	This question requires evaluation of each statement accordingly – **Statement (a):** Acceleration in a SHM is specified as $a = \frac{d^2x}{dt^2} = -kx \Rightarrow x = k\omega^2 \sin \omega t$, here k is constant while constant angular velocity $\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$. Thus, since ω is constant and hence T is also a constant. Here, f is frequency of the SHM and T is the time period of the SHM which is also constant. In a periodic motion its periodicity i.e. T, hence SHM is necessarily periodic. **Thus option (a) is correct.** **Statement (b):** SHM is compared with projection of a particle moving between opposite points of a diameter of a circle, formed by a particle performing uniform circular motion satisfying $a = \frac{d^2x}{dt^2} = -kx$. The kind of motion is compared with the projection of the particle is called *oscillatory since it repeats the motion in every cycle along the same path*. Hence SHM is necessarily oscillatory. **Thus option (b) is correct.** **Statement (c):** A particle performing a similar kind of motion repetitively is called oscillation. This does not specify requirement of uniform periodicity, a necessary condition of SHM a periodic motion. Hence, oscillatory motions are not necessarily periodic. **Thus option (c) is incorrect**. **Statement (d):** A particle performing periodic motion is SHM only if it satisfies $a = \frac{d^2x}{dt^2} = -kx$. A motion with constant periodicity (e.g. a person's daily cycle of sleeping and waking up) is not an oscillation. **Thus option (d) is incorrect**. **Thus option (a) and (b) are the answers.**
I-23	A particle performing moving with a uniform speed $v = R\omega$ along a circular path is a called uniform circular motion. While projection of the particle along any of a diameter of the circle is SHM since its displacement along the diameter from the centre of the circle is

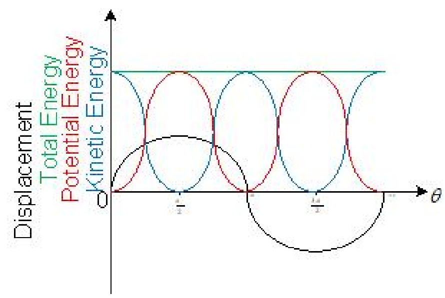
	$x = R\sin\omega t \Rightarrow \frac{d^2x}{dt^2} = -R\omega^2 \sin\omega t \Rightarrow a = \frac{d^2x}{dt^2} = -kx\Big\|_{k=R\omega=v}$. But, the particle itself moves along a circular path and not to-&-from along a line. With this analysis examining each of the given options – **Option (a):** Time period of one cycle of motion $T = \frac{1}{f} = \frac{1}{\frac{v}{2\pi R}} = \frac{2\pi R}{v}$. Since v is constant and hence T is also constant and hence motion is periodic. **Thus option (a) is correct.** **Option (b):** Since it is not moving to-&-fro along a line and hence motion is not oscillatory. **Hence, option (b) is not correct.** **Option (c):** It is the projection of the particle which is satisfying $x = R\sin\omega t \Rightarrow \frac{d^2x}{dt^2} = -kx$ a necessary condition for SHM and not the particle itself. **Hence, option (c) is not correct.** **Option (d):** A body performing angular simple harmonic motion creates torsional restraining force which after body coming to maximum angular displacement is set into angular motion and thus displacement in opposite direction. Thus angular motion is to-&-fro while the particle is stated to be performing circular motion with uniform speed and hence there is no change of direction. **Hence, option (d) is not correct.** **Thus answer is only option (a).**
I-24	A particle under given conditions can be whirled in a vertical circle only when $\frac{v^2}{R} > g$. But, during circular motion its acceleration changes from maximum value $a_D = g + \frac{v^2}{R}$ radial at lowest point D undergoing a departure in oblique direction of a_A and a_A at shown at intermediate points A and C to minimum value $a_B = g + \frac{v^2}{R}$ again in radial direction at point B.  In SHM, centripetal acceleration is always radially outward which is not happening in the instant case hence it is not SHM, **thus option (c) is ruled out.** Since particle uni-directionally on a circular path and not to-&-fro , hence it is not an oscillatory motion. **Thus option (b) is ruled out.** Particle is also not performing angular motion in to-&-fro directions, i.e. clockwise and anti-clockwise alternately. Hence it is also not angular SHM. **This rules out option (d).** But, time period T of the revolution is decided by v, which makes it a periodic motion**. Hence option (a) is correct.** **Hence answer is option (a)**
I-25	If a particle moves in a circular path with continuously increasing speed, it means – (a) Its time of in nth revolution each revolution $T_n = \frac{2\pi R}{v_{n-av}} \Rightarrow T_n \propto \frac{1}{v_{n-av}}$ i.e. time period would continuously decrease in each revolution with decrease in average speed v_{n-av} of the particle in the revolution. Thus ity is not periodic where time of each revolution is constant. **Thus option (a) is ruled out**. (b) Since particle is not moving to-and-fro, requirement for an oscillatory motion, rather it is moving in a circular path hence it is not an oscillatory motion. **Thus option (b) is ruled out**. **(c)** Simple harmonic motion is projection of a particle along one of the diameters of a circle along which it is performing uniform circular motion. Since velocity is not uniform, and hence it violates requirement of SHM. **Thus option (c) is also ruled out.** (d) Since given statement rules out options (a) , (b) and (c) and **hence only option (d) is valid.**

	Thus from the above analysis answer is option (d).
I-26	Torsional pendulum is a circular disc suspended at its centre with a light string. Clock-anticlock wise rotation of the disc produces a corresponding twist in the thread. The angular acceleration of the disc and in turn the end of the thread connected to the disc at any instant is expressed as $\alpha = \frac{d^2\theta}{dt^2} = -\frac{k}{l}\theta \Rightarrow \theta = \theta_{\max} \sin \omega t$. Here, k is torsional constant of the string of length k for an angular twist at then of the tied to the disc. This equation is in variables is parallel to SHM $a = \frac{d^2x}{dt^2} = -kx \Rightarrow x = A \sin \omega t$. Comparing the two equation they have- (a) Time period which makes it a periodic motion. **Hence option (a) is the answer.** (b) The motion is to-&-from along the axis of the thread and hence it is oscillatory. Thus option (b) is correct. (c) Motion is not along a straight line but angular and hence is is not a straight line. **Hence option (c) is not correct.** (d) Since motion is angular and satisfying other conditions of SHM, except motion along a straight line, and hence it is angular simple harmonic. **Thus option (d) is correct.** **Thus answers are options (a), (b) and (d)..**
I-27	Centrifugal force causing circular motion $\vec{F} = m\vec{a}$. Along with all other vector quantities are shown in the figure. Now each of the options is tested for)-) values. **Option (a):** $\vec{F} \cdot \vec{a} = Fa \cos\theta = Fa$, here is $\theta = 0$ angle between vector $\vec{F}$ and $\vec{a}$ both of which are collinear and unidirectional and hence $\cos\theta = \cos 0 = 1$. This leads to the conclusion that value of $\vec{F} \cdot \vec{a}$ is always (+)ve and hence given proposition is always wrong. **Hence option (a) is not the answer.** **Option (b):** $\vec{v} \cdot \vec{r} = vr\cos\theta = vr\cos 90^0 = 0$, since value is Zero scalar value and hence it has no sign, and **hence option (b) is incorrect.** **Option (c):** $\vec{a}.\vec{r} = \left(\frac{v^2}{r}(-\hat{r})\right) \bullet \hat{r} = \frac{v^2}{r} \times r\left((-\hat{r}) \bullet \hat{r}\right) = v^2 \cos 180^0 = -v^2$, and this value is always (-)ve and **hence option (c) is the correct answer.**  **Option (d):** $\vec{F} \times \vec{r} = m\vec{a} \times \vec{r} = m\left(\frac{v^2}{r}(-\hat{r})\right) \times (r\hat{r}) = mv^2(-\hat{r} \times \hat{r}) = mv^2 \sin 0\left(-\hat{k}\right)$ **.** Since $\sin 0 = 0$ and hence magnitude of the vector is Zero, yet as per Right Hand Screw rule direction of the cross product of the vectors is (-)ve since both vectors and are though collinear yet they are anti-direction. **Hence option (d) is the correct answer.** **Thus answer are options (c) and (d)**
I-28	Centrifugal force causing circular motion $\vec{F} = m\vec{a}$. Along with all other vector quantities are shown in the figure. Now each of the options is tested for)-) values.  **Option (a):** $\vec{F} \cdot \vec{a} = Fa \cos\theta = Fa$, here is $\theta = 0$ angle between vector $\vec{F}$ and $\vec{a}$ both of which are collinear and unidirectional and hence $\cos\theta = \cos 0 = 1$. This leads to the conclusion that value of $\vec{F} \cdot \vec{a}$ is always (+)ve and hence given proposition is always right. **Hence option (a) is the correct answer.** **Option (b):** $\vec{v} \cdot \vec{r} = vr\cos\theta = vr\cos 90^0 = 0$, since value is Zero scalar value and hence it has no sign, and **hence option (b) is incorrect.** **Option (c):** $\vec{a}.\vec{r} = \left(\frac{v^2}{r}(-\hat{r})\right) \bullet \hat{r} = \frac{v^2}{r} \times r\left((-\hat{r}) \bullet \hat{r}\right) = v^2 \cos 180^0 = -v^2$, and this value is always (-)ve and **hence option (c) is the incorrect answer.**

	Option (d): $\vec{F}\times\vec{r} = m\vec{a}\times\vec{r} = m\left(\frac{v^2}{r}(-\hat{r})\right)\times(r\hat{r}) = mv^2(-\hat{r}\times\hat{r}) = mv^2\sin 0(-\hat{k})$**.** Since $\sin 0 = 0$ and hence magnitude of the vector is Zero, yet as per Right Hand Screw rule direction of the cross product of the vectors is (-)ve since both vectors and are though collinear yet they are anti-direction. **Hence option (d) is the incorrect answer.** **Thus answer is option (a).**
I-29	Centrifugal force causing circular motion $\vec{F} = m\vec{a}$. Along with all other vector quantities are shown in the figure. Now each of the options is tested for)-) values. **Option (a):** $\vec{F}\times\vec{a} = (m\vec{a})\times\vec{a} = m(\vec{a}\times\vec{a}) = 0$, since angle between two identical vectors is $\theta = 0$ and hence $\sin 0 = 0$. Hence this quantity is always Zero. **Hence option (a) is the correct answer.** **Option (b):** $\vec{r}\times\vec{v} = vr\sin\theta\hat{k} = vr\sin 90^0\hat{k} = vr\hat{k}$**,** since value sin 90^0=1 and neither or is zero is never zero, and **hence option (b) is incorrect.**  **Option (c):** $\vec{a}\times\vec{r} = \frac{v^2}{r}(-\hat{r})\times(r\hat{r}) = -v^2(\hat{r}\times\hat{r})$, The two unit vectors are since identical and hence angle between the two direction vectors $\theta = 0 \Rightarrow \sin\theta = 0$. Hence this value is always Zero. **Thus option (c) is the correct answer.** **Option (d):** $\vec{F}\times\vec{r} = m\vec{a}\times\vec{r} = m\left(\frac{v^2}{r}(-\hat{r})\right)\times(r\hat{r}) = mv^2(-\hat{r}\times\hat{r}) = mv^2\sin 0(-\hat{k})$ **.** Since $\sin 0 = 0$ and hence magnitude of the vector is Zero. **Hence option (d) is the incorrect answer.** **Thus out of given options (a), (c) and (d) are correct.**
I-30	 Acceleration due to gravity near surface of the earth $\vec{g} = -\frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}\hat{r} = -G\left(\frac{4}{3}\pi\rho\right)\vec{R}$ where $h << R$. Thus motion of the particle above the earth is directed towards mean position but is proportional to radius of the earth R and not the distance from mean position $R + h$. But, velocity of the particle when it approaches the tunnel opening, as per third equation of motion is $v = \sqrt{2gh}$. As soon as it enters the tunnel and is at a distance from the center of the circle as per law of gravitation $\vec{g}_r = -\frac{G\left(\frac{4}{3}\pi r^3\right)\rho}{r^2}\hat{r} = -G\left(\frac{4}{3}\pi r\right)\rho\hat{r}$. It leads to $\vec{g}_r = -\frac{4}{3}\pi\rho G\vec{r} = -k\vec{r}$ follows laws of acceleration of SHM. This kind of motion would be reversed when the particle travels after crossing center of the earth. Thus composite motion above the surface of the earth and inside the diametric tunnel is not SHM. **Hence, option (a) is incorrect.** Since motion of the particle is along the line of acceleration passing through the diametric tunnel and hence it is not parabolic**. Hence option (b) is incorrect.** The line of acceleration is since passing through the axis of the tunnel a straight line and **hence option (c) is correct.** Particles since perform composite motion in each cycle under acceleration of the gravity, its time period of the composition remains fixed and hence it is periodic. **Thus option (d) is correct.** **Thus the answers are options (c) and (d).are correct.**
I-31	Acceleration of a particle executing SHM is $\vec{a} = \frac{d^2\vec{x}}{dt^2} = -k\vec{x}$ where $\vec{x}$ is the displacement vector w.r.t. mean position of the particle. Accordingly, analyzing each option – **Option (a):** This is in accordance with relationship for SHM and **hence option (a) is correct**. **Option (b):** Distance at any point of time is from mean position is a scalar quantity and will not have directional

	significance attributed by displacement in statement of SHM. **Hence option (b) is incorrect.** **Option (c):** Distance travelled since $t=0$ is cumulatively increases which is in contradiction to displacement provided in statement of SHM**. Hence option (c) is incorrect.** **Option (d):** In statement of SHM it is only acceleration at any point of time is related to displacement from mean position at that instance, Hence relating acceleration to speed is incorrect. **Thus option (d) is incorrect.** **Thus answer is option (a).**
I-32	With given position vector $\vec{r}=\left(\hat{i}+2\hat{j}\right)A\cos\omega t \Rightarrow \vec{v}=\frac{d}{dt}\vec{r}=-\left(\hat{i}+2\hat{j}\right)A\omega\sin\omega t$. It is seen that position vector of particle at any instant along a straight line. **Hence option (a) is correct**. In elliptical motion slope of position vector keeps changing w.r.t. a point on the elliptical trajectory, which is in contradiction to the conclusion in option (a). **Hence option (b) is incorrect.** In the given position vector the multiplier $\cos\omega t$ makes it periodic with time period $T=\frac{2\pi}{\omega}$ for a constant angular velocity ω. Hence motion is periodic **which makes option (c) correct**. Taking acceleration vector from velocity vector derived above $\vec{a}=\frac{d}{dt}\vec{v}=-\left(\hat{i}+2\hat{j}\right)A\omega^2\cos\omega t=-\omega^2\vec{r}$, it is always directed toward mean position and proportional to displacement w.r.t. mean position. Hence motion is SHM. Thus option (d) is correct. Thus answers are options (a), (c) and (d).
I-33	Displacement of a particle x at any instant t is given by $x=x_0\sin^2\omega t \Rightarrow \frac{x_0}{2}\left(1-\cos\left(2\omega t\right)\right)$. Since values of $-1\le\cos\left(2\omega t\right)\le 1$ about mean position $\frac{x_0}{2}$. Accordingly amplitude of SHM is not x_0 **which makes option (a) incorrect.** From the conclusion in respect of option (a), **option (b) is also incorrect**. The SHM would complete one oscillation when $t\to T \Rightarrow 2\omega T=2\pi \Rightarrow T=\frac{\pi}{\omega}$. Thus time period is not $\frac{2\pi}{\omega}$ accordingly **option (c) is incorrect.** Time period calculated while analyzing option (c) is $T=\frac{\pi}{\omega}$ **hence option (d) is correct.** **Thus answer is option (d).**
I-34	Acceleration in SHM is defined as $a=-ky \Rightarrow \frac{d^2y}{dt^2}=-ky$ … **(1),** and displacement and velocity of the projection of particle P on Y-axis at any instant is defined as $y=A\sin\omega t$ … **(2),** and $v=\frac{d}{dt}x=\frac{d}{dt}\left(A\sin\omega t\right)=A\omega\cos\omega t$ **…(3),** where A is the amplitude of SHM, which is nothing but radius of the circular path. Acceleration of the projection is $a=\frac{d}{dt}v=\frac{d}{dt}\left(A\omega\cos\omega t\right)=-A\omega^2\sin\omega t=-\omega^2 y_t$ … **(4)**, such that $k=\omega^2$ with time period T such that $T=\frac{2\pi}{\omega}$ … **(5).**  In the figure displacement, potential energy, kinetic energy and total energy at different angular $\theta=\omega t$ position is shown. Kinetic energy at any instant is $KE_t=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{d}{dt}\left(A\omega\cos\omega t\right)\right)^2=\frac{1}{2}m\left(A\omega\right)^2\cos^2\omega t$. It

solves into $KE_t = \frac{1}{4}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right)$ ... **(6)** Thus kinetic energy at $\theta = 0, \frac{\pi}{2}, \pi, \frac{\pi}{2}$ and 2π … is $\frac{1}{2}m\left(A\omega\right)^2$, 0, $\frac{1}{2}m\left(A\omega\right)^2$, 0, $\frac{1}{2}m\left(A\omega\right)^2$ and 0…. Thus it is seen that Kinetic energy is always (+) with time period $\frac{T}{2}$.



As regards potential energy, unlike KE energy, it is a cumulative effect such that $PE_t = -\int_0^t F_t dy_t$. Here at instant t force $F_t = ma_t = -m\omega^2 y_t$.. Accordingly, $PE_r = -\int_0^x Fdx = -\int_0^x -m\omega^2 \mathrm{y}_t dt = m\omega^2\left[\frac{\mathrm{y}_t^2}{2}\right]_0^t$ it solves into $PE_r = \frac{1}{2}m\omega^2 A^2\sin^2\omega t = \frac{1}{2}m\left(A\omega\right)^2$ …**(7),** since at $t = 0 \rightarrow y_t = 0$ and using equation (2)**.** Thus, PE energy also oscillates with twice the frequency of displacement, in a manner similar to that of kinetic energy with only one difference that it phase displaced such that when PE is maximum, KE is Zero and vice-versa in line with the principle of Conservation of Energy. Accordingly, total energy at any instant is $TE_t = KE_t + PE_t$. This leads $TE_t = \frac{1}{2}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right) + \frac{1}{2}m\left(A\omega\right)^2\sin^2\omega t = \frac{1}{2}m\left(A\omega\right)^2$, using equation (6 & 7). Thus Total energy TE_t is constant at any point of time as shown in the figure.

In light of the above analysis each of the given option is being analyzed –

Option (a): Both PE and KE are pulsating a frequency twice the displacement and both of out of phase and always $PE_t \neq KE_t$. **Hence option (a) is incorrect.**

Option (b): Comparing KE_t and PE_t , and testing them for equality and if it is found to be wrong then the given would be correct. Accordingly taking $\frac{1}{4}m\left(A\omega\right)^2\left(1+\cos\left(2\omega t\right)\right) = \frac{mA^2\omega^2}{4}\left(1-\cos 2\omega t\right)$, it leads to $\cos\left(2\omega t\right) = -\cos\left(2\omega t\right) \Rightarrow 2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4\omega} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4\left(\frac{2\pi}{T}\right)} = \frac{T}{8}$. It leads to corresponding angles such that $T \rightarrow 2\pi \Rightarrow \frac{T}{8} \rightarrow \theta = \frac{2\pi}{8} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{3\pi}{4}$ …. **Hence option (b) is correct.**

Option (c): Average equation (6) KE in any time interval t_1 to t_2 is $KE_{Av} = \frac{\left[\frac{1}{2}m\left(A\omega\right)^2\int_{t_1}^{t_2}\frac{\left(1+\cos\left(2\omega t\right)\right)}{2}dt\right]}{t_2 - t_1}$ **,** since $\cos\left(2\omega t\right) = 2\cos^2\omega t - 1 \Rightarrow \cos^2\omega t = \frac{1+\cos\left(2\omega t\right)}{2}$. It leads to $KE_{Av} = \frac{m\left(A\omega\right)^2}{4\left(t_2 - t_1\right)}\left[1 + \frac{\sin\left(2\omega t\right)}{2\omega}\right]_{t_1}^{t_2}$. This leads to $KE_{Av} = \frac{m\left(A\omega\right)^2}{4\left(t_2 - t_1\right)}\left[\left(t_2 - t_1\right) + \frac{\sin\left(2\omega t_2\right) - \sin\left(2\omega t_1\right)}{2\omega}\right]$ further simplifies into $KE_{Av} = \frac{m\left(A\omega\right)^2}{4}\left[1 + \frac{\sin\left(2\omega t_2\right) - \sin\left(2\omega t_1\right)}{2\omega\left(t_2 - t_1\right)}\right] = \frac{m\left(A\omega\right)^2}{4}\left(1 + D\right)$ … **(8),** here a new term Discriminant

$D=\dfrac{\sin(2\omega t_2)-\sin(2\omega t_1)}{2\omega(t_2-t_1)}$ … **(9)** is being defined which will become clear when average potential energy (PE_{Av}) in an interval is derived. Using equation (7) is $PE_{Av}=\dfrac{\left[\dfrac{1}{2}m(A\omega)^2\int_{t_1}^{t_2}\sin^2(\omega t)\,dt\right]}{t_2-t_1}$. Since $\cos(2\omega t)=1-2\sin^2\omega t\Rightarrow\sin^2\omega t=\dfrac{1-\cos(2\omega t)}{2}$. It further on definite integration of $\cos(2\omega t)$ occurring in derivation of equation (8) simplifies into $PE_{Av}=\dfrac{1}{4(t_2-t_1)}m(A\omega)^2\left[1-\dfrac{\sin(2\omega t)}{2\omega}\right]_{t_1}^{t_2}$. It leads to $PE_{Av}=\dfrac{1}{4}m(A\omega)^2\left[1-\dfrac{\sin(2\omega t_2)-\sin(2\omega t_1)}{2\omega(t_2-t_1)}\right]=\dfrac{1}{4\omega}m(A\omega)^2(1-D)$…**(10).** *Here, discriminant D is same as that in equation (8), and can be considered to be analogous to discriminant in roots of a quadratic equation.*

Thus for $PE_{Av}=KE_{Av}$ it is essential that $D=\dfrac{\sin(2\omega t_2)-\sin(2\omega t_1)}{2\omega(t_2-t_1)}=0\Rightarrow t_2\to t_1$. In such a situation it is an instant and not an interval. Thus this discriminant make $PE_{Av}\neq KE_{Av}$ **Hence option (c) is incorrect.**

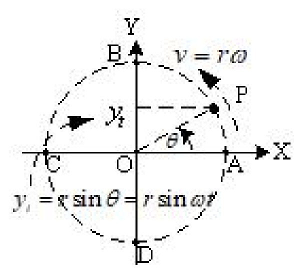
Option (d): It is seen that time period of one cycle of KE and PE is $\dfrac{T}{2}$ and hence interval shall be $t_1=t$ to $t_2=t+\dfrac{T}{2}$. Accordingly, $2\omega t_1=2(2\pi f)t=2\times\dfrac{2\pi}{T}\times t=\dfrac{4\pi}{T}t\Rightarrow\sin(2\omega t_1)=\sin\left(\dfrac{4\pi}{T}t\right)$. Likewise, $2\omega t_2=2(2\pi f)\left(t+\dfrac{T}{2}\right)=2\times\dfrac{2\pi}{T}\times\left(t+\dfrac{T}{2}\right)=2\pi+\dfrac{4\pi}{T}t\Rightarrow\sin(2\omega t_2)=\sin\left(2\pi+\dfrac{4\pi}{T}t\right)=\sin\left(\dfrac{4\pi}{T}t\right)$, since $\sin(2\pi+\theta)=\sin\theta$ it leads to the discriminant substituting these limiting of time in *D* defined in equation (9) it leads to $D=0\Rightarrow PE_{Av}=KE_{Av}=\dfrac{1}{4}m(A\omega)^2$, **hence option (d) is incorrect.**

Thus answer is option (b) and (d)

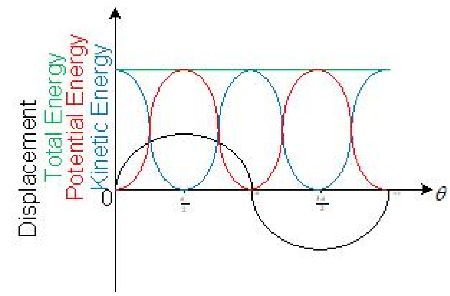
N.B.: Mathematical analysis helps in creating a conviction to the answer and in event of coming across a disagreement; it helps to arrive at logical conclusion and/or its correction, if needed. Thus it helps to enhance intuitive skill. Therefore, approaching answers mathematically is most recommended over intuitive.

I-35

Acceleration in SHM is defined as $a=-ky\Rightarrow\dfrac{d^2y}{dt^2}=-ky$ … **(1),** and displacement and velocity of the projection of particle P on Y-axis at any instant is defined as $y=A\sin\omega t$ … **(2),** and $v=\dfrac{d}{dt}x=\dfrac{d}{dt}(A\sin\omega t)=A\omega\cos\omega t$ **…(3),** where A is the amplitude of SHM, which is nothing but radius of the circular path. Acceleration of the projection is $a=\dfrac{d}{dt}v=\dfrac{d}{dt}(A\omega\cos\omega t)=-A\omega^2\sin\omega t=-\omega^2y_t$ … **(4)**, such that $k=\omega^2$ with time period T such that $T=\dfrac{2\pi}{\omega}$ … **(5).**



In the figure displacement, potential energy, kinetic energy and total energy at different angular $\theta=\omega t$ position is shown. Kinetic energy at any instant is $KE_t = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{d}{dt}\left(A\omega\cos\omega t\right)\right)^2 = \frac{1}{2}m\left(A\omega\right)^2\cos^2\omega t$. It solves into $KE_t = \frac{1}{4}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right)$... **(6)** Thus kinetic energy at $\theta = 0, \frac{\pi}{2}, \pi, \frac{\pi}{2}$ and 2π … is $\frac{1}{2}m\left(A\omega\right)^2$, 0, $\frac{1}{2}m\left(A\omega\right)^2$, 0, $\frac{1}{2}m\left(A\omega\right)^2$ and 0…. Thus it is seen that Kinetic energy is always (+) with time period $\frac{T}{2}$.



As regards potential energy, unlike KE energy, it is a cumulative effect such that $PE_t = -\int_0^t F_t dy_t$. Here at instant t force $F_t = ma_t = -m\omega^2 y_t$.. Accordingly, $PE_r = -\int_0^x F dx = -\int_0^x -m\omega^2 \, y_t dt = m\omega^2\left[\frac{y_t^2}{2}\right]_0^t$ it solves into $PE_r = \frac{1}{2}m\omega^2 A^2\sin^2\omega t = \frac{1}{2}m\left(A\omega\right)^2\sin^2\omega t$ …**(7),** since at $t = 0 \rightarrow y_t = 0$ and using equation (2)**.** Thus, PE energy also oscillates with twice the frequency of displacement, in a manner similar to that of kinetic energy with only one difference that it phase displaced such that when PE is maximum, KE is Zero and vice-versa in line with the principle of Conservation of Energy. Accordingly, total energy at any instant is $TE_t = KE_t + PE_t$. This leads $TE_t = \frac{1}{2}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right) + \frac{1}{2}m\left(A\omega\right)^2\sin^2\omega t = \frac{1}{2}m\left(A\omega\right)^2$, using equation (6 & 7). Thus Total energy TE_t is constant at any point of time as shown in the figure.

Evaluating each of the options –

Option (a): Maximum Kinetic energy from the equation (6) is $KE_{Max} = \frac{1}{4}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right) = \frac{1}{4}m\left(A\omega\right)^2$ when $\sin\omega t = 0$ i.e. $\omega t = n\pi\big|_{n\in W}$ and is depicted graphically also. Likewise, maximum potential energy $PE_{Max} = \frac{1}{2}m\left(A\omega\right)^2\sin^2\omega t = \frac{1}{2}m\left(A\omega\right)^2$, when $\sin\omega t = 1$ i.e. $\omega t = n\frac{\pi}{2}\Big|_{n \text{ is Odd}}$ and is depicted graphically also. Thus $KE_{Max} = PE_{Max}$. **Hence option (a) is correct.**

Option (b): Minimum Kinetic energy from the equation (6) is $KE_{Min} = \frac{1}{4}m\left(A\omega\right)^2\left(1-\sin^2\omega t\right) = 0$ when $\sin\omega t = 1$ i.e. $\omega t = n\frac{\pi}{2}\Big|_{n \text{ is Odd}}$ and is depicted graphically also. Likewise, minimum potential energy $PE_{\text{Min}} = \frac{1}{2}m\left(A\omega\right)^2\sin^2\omega t = 0$, when $\sin\omega t = 0$ i.e. $\omega t = n\pi\big|_{n\in W}$ and is depicted graphically also. Thus $KE_{Min} = PE_{Min}$. **Hence option (b) is correct.**

Option (c): From analysis as option (a) and (b) above $PE_{\text{Min}} = 0$ and $KE_{Max} = \frac{1}{4}m\left(A\omega\right)^2 \neq 0$ since neither of $m, A, \omega \neq 0$ and hence $PE_{\text{Min}} \neq KE_{Max}$, **hence option (c) is wrong.**

Option (d): From analysis as option (a) and (b) above $KE_{\text{Min}} = 0$ and $PE_{Max} = \frac{1}{4}m\left(A\omega\right)^2 \neq 0$ since neither of

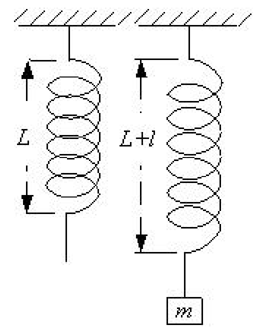
	$m, A, \omega \neq 0$ and hence $KE_{\text{Min}} \neq PE_{Max}$ **, hence option (d) is wrong**. **Thus answer is option (b) and (d).**
I-36	Time taken by a object to fall through a distance h shall be, as per Galileo's Second Equation of motion would be $h = 0 \times t + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$. While for pendulum time period $T = 2\pi\sqrt{\frac{l}{g}}$. Each option is being analyzed here under – **Option (a):** Thus time measured with the pendulum clock in terms of its oscillations is $n = \frac{t}{T} = \frac{\sqrt{\frac{2h}{g}}}{2\pi\sqrt{\frac{l}{g}}} = \frac{1}{2\pi}\sqrt{\frac{2h}{l}}$ is independent gravity which is different on the moon, while other parameters viz. h and l are specific to the apparatus, which is same. Hence, time measured in terms of oscillations of pendulum n remains unchanged. **Hence option (a) is correct**. **Option (b):** Speed of particle is $s = \frac{h}{t}$. It has been derived in (a) that time measured with pendulum clock are same on earth and moon, and height h is parameter of the apparatus which is unchanged. Hence speeds measured will be the same on moon and earth. **Thus option (b) is correct.** **Option (c):** Actual time of fall on earth is $t_E = \sqrt{\frac{2h}{g_E}}$ where g_E is acceleration due to gravity on the earth. Whereas on the moon $t_M = \sqrt{\frac{2h}{g_M}}$ where g_M is acceleration due to gravity on the earth. Since $g_E \neq g_M$ and therefore $T_E \neq T_M$. **Hence option (c) is incorrect.** **Option (d): Actual speed of the object after falling through height** h as per Galileo's Third equation of motion is $v^2 = u^2 + 2gh \Rightarrow v = \sqrt{2gh}$. The speed on the earth would be $v_E = \sqrt{2g_E h}$ while on the moon it is $v_M = \sqrt{2g_M h}$. Using the logic at (c) above therefore $v_E \neq v_M$. **Hence option (d) is incorrect.** **Thus answer is option (a) and (b).**
I-37	Time period for each of the options being analyzed to determine correct answer – **Option (a):** For simple pendulum, time period $T = 2\pi\sqrt{\frac{l}{g}}$, and acceleration due to gravity g on moon is different than that on earth and hence time period would change on the moon. **Thus option (a) is correct**. **Option (b):** A rigid body suspended from a point away from its centre of gravity constitutes physical pendulum instead of a bob at free end of a simple pendulum. Time period in both the cases is dependent on value of acceleration due to gravity g which is different on moon as compared to that on the earth. **Hence option (b) is correct.** **Option (c):** Time period of torsional pendulum is $T = 2\pi\sqrt{\frac{l}{k}}$, where l is length of pendulum and k is torsional constant, while it is independent of acceleration due to gravity. **Hence option (c) is incorrect.** **Option (d):** Time period of spring-mass system is $T = 2\pi\sqrt{\frac{m}{k}}$, where l is mass attached to free end of the

	spring and k is torsional constant, while it is independent of acceleration due to gravity. **Hence option (d) is correct.** **Thus answer is option (a) and (b).**
I-38	For a particle executing SHM its displacement from mean position at any point of time t is $x = A\sin(\omega t+\phi)$ where A is amplitude of SHM, ϕ is phase displacement depending upon initial angular displacement of the particle at $t=0$, and ω is angular velocity such that $\omega = 2\pi f = \frac{2\pi}{T}$, here f is frequency of SHM and T is the time period of SHM. Accordingly, using the given data the displacement equation becomes $5 = 10\sin\left(\frac{2\pi}{6}\times 0+\phi\right) \Rightarrow \sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$. Accordingly equation of displacement w.r.t. would be $x = 10\sin\left(\frac{2\pi}{6}t+\frac{2\pi}{6}\right)$. Thus answer of part (a) of the problem is $x = 10\sin\left[\frac{2\pi}{6}t+\frac{\pi}{6}\right]$cm. Acceleration of particle executing SHM is $a = \frac{d^2x}{dt^2} = \frac{d}{dt}v = \frac{d}{dt}\left(\frac{d}{dt}x\right) = \frac{d}{dt}\left(\frac{d}{dt}A\sin(\omega t+\phi)\right)$. It leads to $a = -A\omega^2\sin\omega t = -A\left(\frac{2\pi}{T}\right)^2\sin\left(\frac{2\pi}{T}\times t+\phi\right)$. On substituting the given data $a = -10\left(\frac{2\pi}{6}\right)^2\sin\left(\frac{2\pi}{6}\times 4+\frac{\pi}{6}\right)$. This leads to $a = -10\left(\frac{2\pi}{6}\right)^2\sin\left(\frac{9}{6}\pi\right) = -10\left(\frac{\pi}{3}\right)^2\sin\left(\frac{3}{2}\pi\right) = -10.97(-1) = 11$. **Thus answer of part (b) is 11 cm.s^{-2}.** Thus part-wise **answers are (a)** $x = 10\sin\left[\frac{2\pi}{6}t+\frac{\pi}{6}\right]$**cm, (b) 11 cm.s^{-2}**
I-39	For a particle executing SHM its displacement from mean position at any point of time t is $x = A\sin(\omega t+\phi)$ where A is amplitude of SHM, ϕ is phase displacement depending upon initial angular displacement of the particle at $t=0$, and ω is angular velocity such that $\omega = 2\pi f = \frac{2\pi}{T}$, here f is frequency of SHM and T is the time period of SHM. Accordingly, at any instance velocity of the particle is $v = \frac{d}{dt}x = A\omega\cos(\omega t+\phi)$ and acceleration of the particle is $a = \frac{d^2x}{dt^2} = \frac{d}{dt}v = \frac{d}{dt}\left(A\omega\cos(\omega t+\phi)\right) = -A\omega^2\sin(\omega t+\phi)$. Using the given data at an instant, displacement is $x = A\sin(\omega t+\phi) = 2$ cm … (1), velocity is $v = A\omega\cos(\omega t+\phi) = 1$ m.s^{-1}… (2) and acceleration is $a = -A\omega^2\sin(\omega t+\phi) = 10$ m.s^{-2} … (3). Taking ratio equations (1) and (3) $\frac{x}{a} = \frac{A\sin(\omega t+\phi)}{A\omega^2\sin(\omega t+\phi)} = \frac{1}{\omega^2} \Rightarrow \frac{x}{a} = \frac{1}{\left(\frac{2\pi}{T}\right)^2} \Rightarrow T = 2\pi\sqrt{\frac{x}{a}}$ s. Using the given data $T = 2\pi\sqrt{\frac{0.02}{10}} = 0.28$ s. **Thus answer of part (b) of the problem is 0.28 s.** Amplitude of oscillation A is maximum displacement. From trigonometric identity $\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi) = 1$. Substituting value of $\sin(\omega t+\phi) = \frac{x}{A}$ and $\cos(\omega t+\phi) = \frac{v}{A\omega}$ from

	equation (1) and (2) $\left(\frac{x}{A}\right)^2+\left(\frac{v}{\omega A}\right)^2=1 \Rightarrow A=\sqrt{x^2+\left(\frac{vT}{2\pi}\right)^2}$. Substituting given data a value of T derived for part (b) of the solution $A=\sqrt{(0.02)^2+\frac{1\times 0.28}{2\pi}}=0.049$ m or equal to 4.9 cm. **Thus answer of part (a) of the problem is 4.9 cm.** **Thus answer of parts are (a) 4.9 cm and (b) 11 cm.s^{-2}.** **N.B.:** (a) Care is required to check units; displacement is given in cm while base unit of displacement in velocity and acceleration is meter. (b) It is not necessary that each part of the problem is independent and only solution of later part at times forms data for previous part.
I-40	For a particle executing SHM its displacement from mean position at any point of time t is $x=A\sin(\omega t+\phi)$ where A is amplitude of SHM, ϕ is phase displacement depending upon initial angular displacement of the particle at $t=0$, and ω is angular velocity such that $\omega=2\pi f=\frac{2\pi}{T}$, here f is frequency of SHM and T is the time period of SHM. Accordingly, at any instance velocity of the particle is $v=\frac{d}{dt}x=A\omega\cos(\omega t+\phi)$ and acceleration of the particle is $a=\frac{d^2x}{dt^2}=\frac{d}{dt}v=\frac{d}{dt}\left(A\omega\cos(\omega t+\phi)\right)=-A\omega^2\sin(\omega t+\phi)$. Kinetic energy at any point is given by $KE_x=\frac{1}{2}mv_x^2=\frac{1}{2}m\left(A\omega\cos(\omega t)\right)^2=\frac{1}{2}m(A\omega)^2\cos^2(\omega t)$. It leads to $KE_x=\frac{1}{2}m\omega^2\left(A^2-\left(A\sin(\omega t)\right)^2\right)=\frac{1}{2}m\omega^2\left(A^2-x^2\right)$. And potential energy $PE_x=-\int_0^x Fdx=-\int_0^x madx$ $PE_x=-\int_0^x m\left(-\omega^2x\right)dx=m\omega^2\left[\frac{x^2}{2}\right]_0^x=\frac{1}{2}mA^2x^2$. Thus distance from mean position where $PE_x=KE_x$ is $\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\omega^2\left(A^2-x^2\right)\Rightarrow 2x^2=A^2\Rightarrow x=\frac{A}{\sqrt{2}}$. On substituting the given value of amplitude we get $x=\frac{10}{\sqrt{2}}=5\sqrt{2}$ cm. **Hence answer is $5\sqrt{2}$ cm.**
I-41	For a particle executing SHM its displacement from mean position at any point of time t is $x=A\sin(\omega t+\phi)$ where A is amplitude of SHM, ϕ is phase displacement depending upon initial angular displacement of the particle at $t=0$, and ω is angular velocity such that $\omega=2\pi f=\frac{2\pi}{T}$, here f is frequency of SHM and T is the time period of SHM. Accordingly, at any instance velocity of the particle is $v=\frac{d}{dt}x=A\omega\cos(\omega t+\phi)$, and acceleration of the particle is $a=\frac{d^2x}{dt^2}=\frac{d}{dt}v=\frac{d}{dt}\left(A\omega\cos(\omega t+\phi)\right)=-A\omega^2\sin(\omega t+\phi)$. Each of the terms has a sinusoidal multiplier whose maximum value is ONE accordingly $x_{Max}=A$, $v_{Max}=A\omega$ and $a_{Max}=A\omega^2$. Thus from given data $\frac{a_{Max}}{v_{Max}}=\frac{A\omega^2}{A\omega}=\omega\Rightarrow\omega=\frac{50}{10}=5$ rad.s^{-1}. Accordingly, amplitude of

	oscillation is $A=\frac{v_{Max}}{\omega}=\frac{10}{5}=2$ cm.s^{-1}. Now, since $v=v_{Max}\cos\omega t=v_{Max}\sqrt{1-\sin^2\omega t}=v_{Max}\sqrt{1-\left(\frac{x}{A}\right)^2}\Rightarrow\left(\frac{x}{A}\right)^2=1-\left(\frac{v}{v_{Max}}\right)^2\Rightarrow x=A\left(1-\left(\frac{v}{v_{Max}}\right)^2\right)^{\frac{1}{2}}$. Using the given and derive value of *A*, we get $x=2\left(1-\left(\frac{8}{10}\right)^2\right)^{\frac{1}{2}}=\pm1.2$. Hence answer is ±1.2 cm from the mean position. **N.B.:** Since in final expression exponent is $\frac{1}{2}$ and hence answer is assigned sign $\pm$.
I-42	Comparing the equation of displacement of a particle performing SHM $x=A\sin(\omega t+\phi)$ with the given equation $x=(2.0)\sin\left[(100)t+\frac{\pi}{6}\right]$. Taking solution of each part separately- **Part (a):** Comparing the parameters $A=2.0$, $\omega=2\pi f=\frac{2\pi}{T}=100$ rad.s^{-1}. It leads to $T=\frac{2\pi}{100}=0.063$ s. And in spring-mass system $T=2\pi\sqrt{\frac{m}{k}}\Rightarrow k=m\left(\frac{2\pi}{T}\right)^2$ N.m-1. On substituting data we get $k=\left(10\times10^{-3}\right)\left(\frac{2\pi}{\frac{2\pi}{100}}\right)^2=\left(10\times10^{-3}\right)\times10^4=100$ N.m-1. **Thus answers to part (a) are 2.0 cm, 0.063 s, 100 N.m^{-1}.** **Part (b):** Since phase difference is given to be $\phi=\frac{\pi}{6}$ and hence at $t=0$ position is $x=(2.0)\sin\left[(100)\times0+\frac{\pi}{6}\right]=(2.0)\sin\frac{\pi}{6}=1.0$ cm, velocity $v=\frac{dx}{dt}=2\times100\times\cos\frac{\pi}{6}=100\sqrt{3}$ m.s^{-1}, and acceleration $a=2\times(100)^2\sin\frac{\pi}{6}=10^4$ m.s^{-2}. **Hence answers to part (b) are 1.0 cm, 1.73 m.s^{-1}, 100 m.s^{-2}.**
I-43	Motion of particle starting at $t=0$ is defined by $x=5\sin\left(20t+\frac{\pi}{3}\right)$. Taking solution of each part separately – Part (a): For particle to come to rest its velocity should be $v=0\Rightarrow\frac{dx}{dt}=5\times20\cos\left(20t+\frac{\pi}{3}\right)=0$. Nearest value of angle for its cosine to be equated to zero is $\left(20t+\frac{\pi}{3}\right)=\frac{\pi}{2}\Rightarrow20t=\frac{\pi}{2}-\frac{\pi}{3}$. It solves into $t=\frac{\pi}{6\times20}=\frac{\pi}{120}$ s. **Hence answer of part (a) is $\frac{\pi}{120}$ s.** Part (b): For acceleration of particle to be zero $a=0\Rightarrow\frac{dv}{dt}=5\times(20)^2\sin\left(20t+\frac{\pi}{3}\right)=0$. Nearest value of

	angle for its cosine to be equated to zero is $\left(20t+\frac{\pi}{3}\right)=\pi \Rightarrow 20t=\pi-\frac{\pi}{3} \Rightarrow t=\frac{2\pi}{20\times 3}=\frac{\pi}{30}$ s. **Hence answer of part (a) is** $\frac{\pi}{30}$ **s.** Part (c): For maximum speed $v=v_{Max} \Rightarrow \cos\left(20t+\frac{\pi}{3}\right)=1 \Rightarrow 20t+\frac{\pi}{3}=\pi \Rightarrow t=\frac{\pi}{30}$. **Hence answer of part (a) is** $\frac{\pi}{30}$ **s.**
I-44	Motion of particle starting at $t=0$ is defined by $x=2.0\cos\left(50\pi t+\tan^{-1}0.75\right)$. Taking solution of each part separately – Part (a): For particle to come to rest its velocity should be $v=0 \Rightarrow \frac{dx}{dt}=-5\times 50\sin\left(50\pi t+\tan^{-1}0.75\right)=0$. Nearest (+)ve value of angle for its cosine to be equated to zero is $\left(50\pi t+\tan^{-1}0.75\right)=\pi \Rightarrow t=\frac{1}{50\pi}\left(\pi-\tan^{-1}0.75\right)=\frac{3.14-0.64}{50\pi}\Big\vert_{\tan^{-1}0.75=0.64}=\frac{2.5}{50\pi}=\frac{1}{20\pi}=1.6\times 10^{-2}$. **Hence answer of part (a) is** $\frac{1}{20\pi}=1.6\times 10^{-2}$ **s.** Part (b): For acceleration of particle to be zero $a=a_{\max} \Rightarrow \frac{dv}{dt}=5\times(50)^2\cos\left(50t+\tan^{-1}0.75\right)$. For the required solution, it leads to $50t+\tan^{-1}0.75=\pi \Rightarrow t=\frac{1}{50\pi}\left(\pi-\tan^{-1}0.75\right)$. On further solving $t=\frac{3.14-0.64}{50\pi}\Big\vert_{\tan^{-1}0.75=0.64}=\frac{2.5}{50\pi}=\frac{1}{20\pi}=1.6\times 10^{-2}$. **Hence answer of part (a) is** $\frac{1}{20\pi}=1.6\times 10^{-2}$ **s.** Part (c): For particle to come to rest second rime its velocity should be $v=0 \Rightarrow \frac{dx}{dt}=-5\times 50\sin\left(50\pi t+\tan^{-1}0.75\right)=0$. Nearest (+)ve value of angle for its cosine to be equated to zero is $\left(50\pi t+\tan^{-1}0.75\right)=2\pi \Rightarrow t=\frac{1}{50\pi}\left(2\pi-\tan^{-1}0.75\right)$. It's solution leads to $t=\frac{6.28-0.64}{50\pi}\Big\vert_{\tan^{-1}0.75=0.64}=\frac{5.64}{50\pi}=3.6\times 10^{-2}$. **Hence answer of part (a) is** 3.6×10^{-2} **s.** **Thus answers to each part are (a)** 1.6×10^{-2} **s, (b)** 1.6×10^{-2} **s, and (c)** 3.6×10^{-2} **s.** **N.B.: (1)** Since answer to part **(c)** cannot be expressed with rational coefficient of π hence answers to all parts are expressed in scientific notation to maintain coherency across all answers. **(2)** Care should be taken to notice that equation of SHM is cosine function unlike generally y=used sign function. It affects the answers abruptly.
I-45	Equation of displacement in SHM is $x=A\sin(\omega t+\phi)$. Let at t_1 displacement is $\frac{A}{2}$ and hence using the given equation $\frac{A}{2}=A\sin(\omega t_1+\phi) \Rightarrow \omega t_1+\phi=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$. Likewise, at t_2 displacement is A and hence using the given equation $A=A\sin(\omega t_2+\phi) \Rightarrow \omega t_2+\phi=\sin^{-1}(1)=\frac{\pi}{2}$. Therefore, time taken for displacement to

	reach from half amplitude to amplitude is $\omega t_2 - \omega t_1 = \frac{\pi}{2} - \frac{\pi}{6} \Rightarrow \frac{2\pi}{T}\left(t_2 - t_1\right) = \frac{\pi}{3} \Rightarrow \frac{2\Delta t}{T} = \frac{1}{3} \Rightarrow \Delta t = \frac{T}{6}$ s. **Hence answer is $\frac{T}{6}$ s.**
I-46	Pendulum clock completes one side swing in 1s, and hence its time period is $T = 2$ s. And time period of a spring mass system $T_s = 2\pi\sqrt{\frac{m}{k}}$. For the two time periods to be same $2 = 2\pi\sqrt{\frac{m}{0.1}} \Rightarrow m = \frac{0.1}{\pi^2} \approx 1.0\times10^{-2}$ kg or ≈ 10 gm. **Hence answer is ≈ 10 g.**
I-47	A block of mass m hanging from a vertical spring of length L, having constant k, in a state of equilibrium, will experience a stretch $l = \frac{mg}{k}$. Time period of the spring-mass system is $T_S = 2\pi\sqrt{\frac{m}{k}}$. Where as time period of simple pendulum is $T_P = 2\pi\sqrt{\frac{l_P}{g}}$ f pendulum is equal to stretch of spring, It is given that time period of oscillation of spring mass system is equal to that of the simple pendulum i.e. $T_S = T_P \Rightarrow 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l_P}{g}} \Rightarrow l_p = \frac{mg}{k}$. It is seen that with given conditions $l = \frac{mg}{k} = l_P$. **Hence proved.** 
I-48	A spring mass system hanging vertically when the mass is at its bottom most position it experience net acceleration $a_B = g - a_{Max}$ and when it is at its highest position net acceleration is $a_H = g + a_{Max}$. Maximum acceleration $a_{Max} = A\omega^2 = A\left(\frac{2\pi}{T}\right)^2$. Using the given data $a_{Max} = 0.1\times\left(\frac{2\pi}{0.314}\right)^2 = 0.1\times\left(20\right)^2 = 40$ m.s^{-2}. Taking acceleration due to gravity $g = 10$ m.s^{-2}, net acceleration when block is at the highest position is $a_H = 10 + 40 = 50$ m.s^{-2}. Thus net force acting on the block at highest position is countered by the spring and is $F_{Max} = m\times a_H = 0.5\times50 = 25$ N. **Hence answer is 25 N.**
I-49	In a state of equilibrium the vertical spring-mass system having mass $m = 2$ kg has a time period $T = 4$ s. Since time period of SHM of the spring-mass system is $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 4 = 2\pi\sqrt{\frac{2}{k}} \Rightarrow \left(\frac{2}{\pi}\right)^2 = \frac{2}{k} \Rightarrow k = \frac{\pi^2}{2}$. The system undergoes stretching by a length l such that $mg = kl \Rightarrow l = \frac{mg}{k}$, here k is the spring constant. In the process potential energy undergoes – (1) storage in the $PE_S = \frac{1}{2}kl^2$, and (ii) loss of potential energy due to descend of mass by height l due to expansion of spring potential energy . Thus total potential energy stored in the system $PE = PE_S + PE_M = \frac{1}{2}kl^2 - mgl$ due to descend of mass by height l due to expansion of spring. Using the value k of derived above, the given data and $g = 10$ m.s^{-2}, it leads

	to $PE=\frac{1}{2}(k)\left(\frac{mg}{k}\right)^2-mg\left(\frac{mg}{k}\right)=\frac{1}{2}\times\frac{(mg)^2}{k}=\frac{1}{2}\times\frac{(2\times10)^2}{\frac{\pi^2}{2}}=\frac{400}{\pi^2}=40.5$ J. **Using the principle of SDs the answer is 40 J.**
I-50	From the given data potential energy of the spring $PE_S=\frac{1}{2}kl^2\Rightarrow 5=\frac{1}{2}k(0.25)^2\Rightarrow k=160$ N.m^{-1}. The block fastened at free end oscillates such that its time period is $T=\frac{1}{n}=\frac{1}{5}=0.2$ s. Time period of a spring-mass system is $T=2\pi\sqrt{\frac{m}{k}}\Rightarrow m=k\left(\frac{T}{2\pi}\right)^2=160\left(\frac{0.2}{2\pi}\right)^2=0.16$ kg. **Hence answer is 0.16 kg.**

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