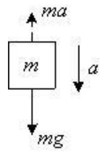
Wave and Motion : Illustration only of Objective and Subjective Questions (Typical)

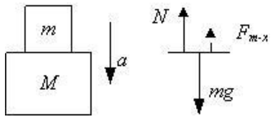
I-01

Combined compression of two masses would create a compression of length l in the spring such that $kl = (M+m)g \Rightarrow l = \frac{(M+m)g}{k}$. This is the equilibrium position.

(a) When the system is set to oscillate around this equilibrium position then acceleration of the combined mass at a displacement x above the equilibrium position the effective compression and effective compressive force shall be $(l-x)$ and $k(l-x)$, respectively. Accordingly acceleration of the combined mass shall be $(M+m)a_x = k(l-x) \Rightarrow a_x = \frac{k}{M+m}\left(\frac{(M+m)g}{k} - x\right) = \left(g - \frac{kx}{M+m}\right)$ towards the equilibrium position i.e. downwards. Thus, considering the pseudo-force, the FBD of the mass m the net force on the mass shall be $F_{m-x} = mg - ma_x = mg - m\left(g - \frac{kx}{M+m}\right) = \frac{mkx}{M+m}$. **Thus, answer of part (a) is** $\frac{mkx}{M+m}$



(b) Since smaller block of mass is resting on the bigger block of mass and hence normal force on the smaller block at this position can be determined with a slight modification in the diagram of case (a) and accounting for normal reaction. Accordingly, $mg = N_{m-x} + F_{m-x} \Rightarrow N_{m-x} = mg - F_{m-x}$ Using the value of F_{m-x} derived at (a) above $N_m = mg - \frac{mkx}{M+m}$. It is seen that as displacement above mean position increases the N_{m-x} also decreases linearly. Hence, N_{m-x} shall be minimum at the highest point. **Thus answer of part (b) is** $N_{m-x} = mg - \frac{mkx}{M+m}$, It is minimum at highest point.

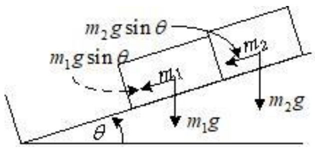


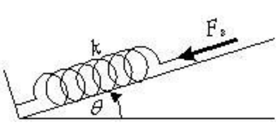
(c) During oscillation at maximum amplitude acceleration is zero. Accordingly taking the value derived at (a) above $a_{x-\max} = 0 = \left(g - \frac{kx_{\max}}{M+m}\right) \Rightarrow \frac{kx_{\max}}{M+m} = g \Rightarrow x_{\max} = \frac{g(M+m)}{k}$. **Hence, answer of part (c) is** $g\frac{(M+m)}{k}$

I-02

The problem is solved progressively case-by-case:

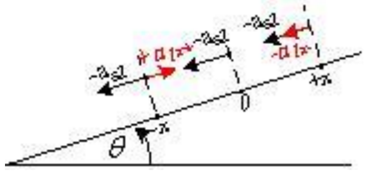
Step (a): Gravitational force of the masses along the plane is $m_1 g$ and $m_2 g$. Both the forces are along the same orientation and hence net force along the inclined surface is $F_s = m_1 g \sin\theta + m_1 g \sin\theta = (m_1 + m_2) g \sin\theta$.





Therefore, compression of the spring would be $l = \frac{F_s}{k} = \frac{(m_1+m_2) g \sin\theta}{k}$. **Thus answer of part (a) is** $\frac{(m_1+m_2) g \sin\theta}{k}$.

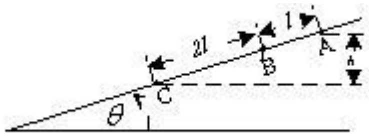
Case (b): It is stated that system is further pushed through $\left(\frac{2}{k}\right)(m_1+m_2)g\sin\theta=2l$. Thus in this new state if equilibrium total compression of the spring is $l+2l=3l=\left(\frac{3}{k}\right)(m_1+m_2)g\sin\theta$.



Further, the mass m_2 is resting against mass m_1 which is fastened to the oscillating end of the spring. Therefore, during oscillation of the combined mass velocity of both the masses remain same. But, they would start separating only when relative acceleration ${}_2a_1\left(=a_2-a_1\right)$ of mass m_2 w.r.t. the mass m_1 is (+)ve i.e. up the slope; when ${}_1a_2$ is (-) ve i.e. down the slope the mass m_2 shall continue to remain in contact with mass m_1 since other end of the spring is fixed. All directions up the slope are marked (+)ve and down the slope are marked (-)ve and accordingly signed values of the accelerations are shown in the figure.

Thus when mass m_2 is at (-)x from position of equilibrium ${}_2a_1=(-)a_{s2}-a_{1x-}<0$. But, when the m_2 is at (+)x from position of equilibrium ${}_2a_1=(-)a_{s2}-(-a_{1x+})$; during this part of motion as long as $x<l$ When, spring stretches to its natural length i.e. $x=l$ determined in case (a) the ${}_2a_1=a_{1x+}-a_{s2}=0$, **and as soon as $x>l$ then ${}_2a_1=a_{1x+}-a_{s2}>0$ and mass m_2 starts separating from mass m_1, this is answer of case (b).**

Case (c): It is stated that after compression stipulated in case (b), identified in the figure by point C, the masses are released and it is required to determine common speed of the two block at time of separation; this position has been derived in case (b) and is identified by point A in the figure. Point B in the figure is the position of the spring under natural compression of the spring caused by the two masses as derived in case (a).

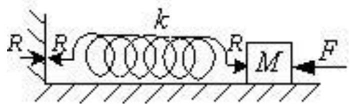


Thus this case (c) is extension of case (a) and (b) from perspective of conservation of energy. Accordingly, $PE_C=TE_A$ …(1), here $PE_C=\frac{9}{2}kl^2$ …(2), because at this state total complression of the spring is $\Delta l_C=l+2l=3l$. At position $TE_A=\Delta PE+\frac{1}{2}Mv^2$ …(3), here the combined mass is $M=m_1+m_2$. And $\Delta PE=Mgh=Mg\left(3l\sin\theta\right)=3Mgl\sin\theta$ …(4), since in reaching point A from point the change of elevation of the combined mass M is $h=3l\sin\theta$. The combining the equations (2), (3) and (4) in equation (1) $\frac{9}{2}kl^2=3Mgl\sin\theta+\frac{1}{2}Mv^2$. Substituting in this equation value of l derived in case (a), $\frac{9}{2}k\left(\frac{Mg\sin\theta}{k}\right)^2=3Mg\left(\frac{Mg\sin\theta}{k}\right)\sin\theta+\frac{1}{2}Mv^2$. This equation solves into $Mv^2=\frac{9}{k}(Mg\sin\theta)^2-\frac{6}{k}(Mg\sin\theta)^2\Rightarrow v^2=\frac{3M}{k}(g\sin\theta)^2\Rightarrow v=\sqrt{\frac{3M}{k}}g\sin\theta$. Substituting the value of M, the velocity of the combined mass is $v=\sqrt{\frac{3(m_1+m_2)}{k}}g\sin\theta$. Thus answer of part (c) is $v=\sqrt{\frac{3(m_1+m_2)}{k}}g\sin\theta$.

N.B.: In this elaboration concept of SHM is applicable only in deriving answer of Part (b), rest is all based on the concept of mechanics.

I-03

Each case is being elaborated step-by-step –



Case (a): In case of equilibrium thje equationof forces on mass M would be $F+R=0 \Rightarrow F=-k(-l)=kl \Rightarrow 10=100\times l \Rightarrow l=0.1\text{m}$. **Thus answer of part (a) is 10 cm.**

Case (b): When some external agent imparts velocity 2m/s to mass M when the system is at state of case (a). Then total energy of the system in this case would be $TE_B = PE_{S-B} + KE_{M-B} = \frac{1}{2}kl^2 + \frac{1}{2}Mv^2$. On substituting the given data $TE_B = \frac{1}{2}100\times(0.1)^2 + \frac{1}{2}\times 1\times 2^2 = 0.5+2 = 2.5\text{J}$. **Hence, answer to part (b) is 2.5 J.**

Case (c): The case (a) forms equilibrium position for the simple harmonic motion (SHM) caused by external agent. In SHM displacement of particle from mean position at any instant t is $x = A\sin\omega t$, here A is amplitude of the oscillation i.e. maximum displacement and ω is uniform angular velocity of the displacement therefore velocity of the particle performing SHM, at any instant, is $v=\frac{dx}{dt}=A\omega\cos\omega t$ and acceleration is $a=\frac{dv}{dt}=-A\omega^2\sin\omega t=-\omega^2 x$. Maximum velocity is at $t=0 \Rightarrow \cos\omega t = 1$i.e. and $v_{\max}=A\omega$…(1) maximum acceleration is when x is maximum i.e. $t=\frac{\pi}{2} \Rightarrow \sin\left(\omega\frac{T}{4}\right)=1 \Rightarrow a_{\max}=A\omega^2$. Thus, velocity at the equilibrium position is $v_{\max}=A\omega=200$ m.s^{-1} and maximum acceleration is maximum compression of spring $a_{\max}=A\omega^2=l\omega^2$. When the mass is imparted velocity of 200 m/s at equilibrium position, as stated in case (b), the kinetic energy gained by mass M is $KE=\frac{1}{2}Mv^2=\frac{1}{2}\times 1\times 2^2=2$J….(2). During SHM of the mass it's KE will be stored in PE when it attains maximum displacement and $PE=\frac{1}{2}kl^2 \Rightarrow PE=\frac{1}{2}\times 100\times l^2=50\times l^2$…(3). As per Law of conservation of energy in this case $PE=KE$,. Accordingly, from equations (2) and (3) $50\times l^2=2 \Rightarrow l=\frac{1}{\sqrt{25}}=0.2$m, and in terms of SHM $A=l=0.2$m…(4). Using this value of $A=0.2$ in equation (1) we get $2=0.2\times\omega \Rightarrow \omega=\frac{2}{0.2}=10$ rad.s^{-1}. Since $\omega=\frac{2\pi}{T} \Rightarrow T=\frac{2\pi}{\omega}=\frac{2\pi}{10}=\frac{\pi}{5}$ sec. **Thus answer of part (c) is $T=\frac{\pi}{5}$.**

Case (d): Amplitude if the oscillation has been derived in case (c) and is $A=0.2$ **m and is part (d) of the answer,**

Case (e): Potential energy of the spring at left extreme position depends upon total compression of spring considering its free state and is $PE_{(d)}=\frac{1}{2}k(x_0+A)^2=\frac{1}{2}\times 100\times(0.1+0.2)^2=\frac{100\times 0.09}{2}=4.5$ J. **Thus answer to part (d) is 4.5 J.**

Case (f): Potential energy of the spring when the block, at rest under force F where equilibrium exists, the compression of the spring is 0.1 m as derived at (a) above. Accordingly, $PE_{(\text{f})}=\frac{1}{2}k{x_0}^2=\frac{1}{2}\times 100\times(0.1)^2=\frac{100\times 0.01}{2}=0.5$ J. **Thus answer of part (f) is 0.5 J.**

Difference in answers in cases (b), (e), and (f) is in accordance with work-energy theorem which takes into account work done by external force together with Law of Conservation of Energy.

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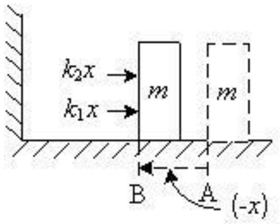
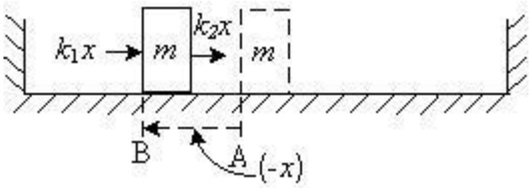
Taking each case independently –

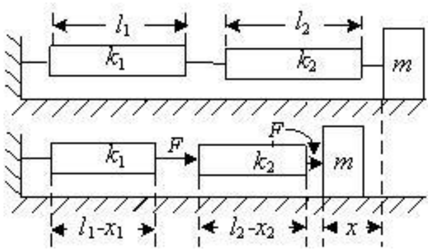


Case (a): Let mass m in the spring-mass system shown in the figure is displaced from position A to position B through a displacement $(-x)$ where $F_x = A\omega^2 x$ ...(1). At this position forces exerted on the mass is $F_x = -k_1(-x) + (-k_1(-x)) = (k_1 + k_2)x = kx$ …(2).. Let v is the velocity imparted by an external agent to set mass at its mean position then maximum displacement A the energy balance equation will be $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow A = \sqrt{\frac{m}{k}}v = \sqrt{\frac{m}{k}}A\omega \Rightarrow \omega = \sqrt{\frac{k}{m}}$, here $v = A\omega$. It leads to $\omega = \sqrt{\frac{k}{m}}$. Accordingly, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$. **Hence answer to part (a) is** $2\pi\sqrt{\frac{m}{k_1 + k_2}}$.



Case (b): Like case (a) the forces on the mass when displaced from its equilibrium position A to position B through a displacement $(-x)$ the forces acting on the mass m due to compression of spring of constant k_1 on the left and by spring of constant k_2 on the right are shown in the figure. Since nature of these forces are linearly additive like that in case and it will have time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$.

Hence answer to part (b) is $2\pi\sqrt{\frac{m}{k_1 + k_2}}$.



Case (c): In this the two springs are connected in series and hence force along each spring will be equal such that $F = k_1x_1 \Rightarrow x_1 = \frac{F}{k_1}$ and $F = k_2x_2 \Rightarrow x_2 = \frac{F}{k_2}$. Thus total displacement $x = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$. It leads to $x = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = F\left(\frac{k_1 + k_2}{k_1k_2}\right) = Fk$, here equivalent spring constant of the combination of the two springs is $k = \left(\frac{k_1 + k_2}{k_1k_2}\right)$ while mass is same. And hence from the derivation at case (a) above, time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1k_2}}$. **Hence answer to part (b) is** $2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1k_2}}$.

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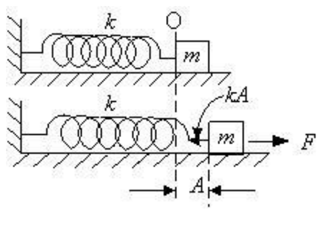
Taking each case sequentially –



Case (a): When the man applies force F as shown in the figure. The spring shall experience elongation until its restraining force is in equilibrium such that $F = kA \Rightarrow A = \frac{F}{k}$. Once it reaches state of equilibrium it will continue to be there unless an external force causes disturbance. But, if the man releases the force the spring-mass system would start oscillating

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with amplitude about its mean position O. **Thus amplitude of the oscillation is** $\frac{F}{k}$. The maximum force on the mass m during oscillation is displacement F from the mean position O and is $F = f_{\max} = m \times a_{\max} \Rightarrow a_{\max} = \frac{F}{m}$. During SHM $a_{\max} = A\omega^2 \Rightarrow \frac{F}{m} = A\omega^2 \Rightarrow \omega = \sqrt{\frac{F}{mA}}$ .It leads to $\frac{2\pi}{T} = \sqrt{\frac{F}{mA}} \Rightarrow T = 2\pi\sqrt{\frac{m}{F} \times \frac{F}{k}} \Rightarrow T = 2\pi\sqrt{\frac{m}{F} \times \frac{F}{k}} = 2\pi\sqrt{\frac{m}{k}}$. **Thus time period of the oscillation is** $2\pi\sqrt{\frac{m}{k}}$ **. Thus combined answer of case (a) is** $\frac{F}{k}$, $2\pi\sqrt{\frac{m}{k}}$

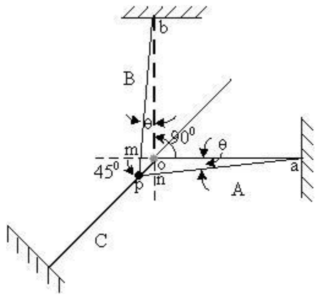
Case (b): The block in equilibrium position is displaced from mean position by A. And hence energy stored in the spring is $PE_S = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{F^2}{2k}$. **Hence answer of case (b) is** $\frac{F^2}{2k}$

Case (c): When block reaches mean position, when spring is unstretched, it's potential energy becomes zero and the whole energy is transferred to the block and hence mass as its kinetic energy, as per principle of conservation of energy would be $(PE_S + KE_m)_{\text{stretched}} = (PE_S + KE_m)_{\text{unstretched}}$. It leads to $\frac{F^2}{2K} + 0 = 0 + KE_m \Rightarrow KE_m = \frac{F^2}{2K}$. **Thus answer 0f part (c) is** $\frac{F^2}{2k}$

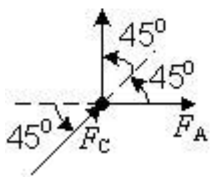
N.B.: The spring shall experience extension until it reaches equilibrium with external force F applied by the man, but the spring-mass system will start oscillating only when the man releases the force.

I-06

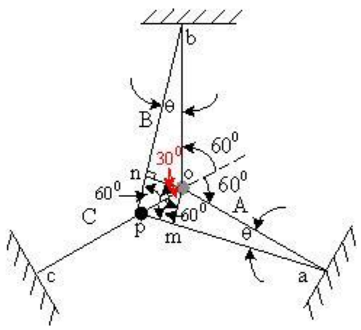
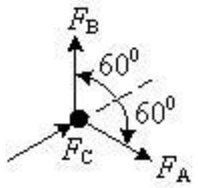
When the block of mass m, represented by a point mass at o, is slightly pushed against spring C such that it's displacement to position is by an amount Δx. The restraining force developed by the spring is $F_C = k\Delta x$. Since the mass is tied to the system of three springs, it will cause stretching of springs A and B, this causes stretching of springs A and B by length n-p and m-p.



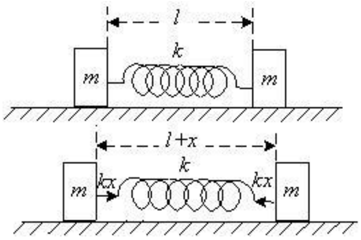
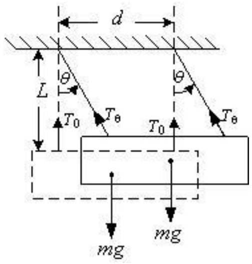
Since displacement $\Delta x <<$ is small hence the angular displacement of spring A by an angle $\theta <<$ and $\angle \text{bom} \to 90^0$ and likewise for spring B the $\angle \text{bon} \to 90^0$. Thus stretching of springs A and B, placed symmetrical to spring C, is by lengths n-p and m-p such that $mp = np = \Delta x \sin 45^0 = \frac{\Delta x}{\sqrt{2}}$

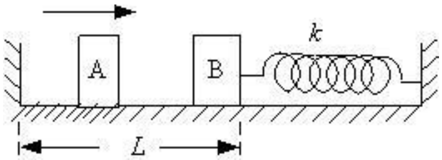


Thus, forces on the mass m have been represented in the diagram such that total force on the mass shall be $F = F_C + F_A \cos 45^0 + F_A \cos 45^0 = k\Delta x + \frac{k\Delta x}{\sqrt{2}}\cos 45^0 + \frac{k\Delta x}{\sqrt{2}}\cos 45^0$. This solves into $F = 2k\Delta x$. Thus acceleration of the mass is for displacement Δx would be $F = ma_{\Delta x} = 2k\Delta x \Rightarrow a_{\Delta x} = \frac{2k\Delta x}{m}$. Since there is no external force acting on the mass m once set into oscillation its amplitude shall remain $A = \Delta x$. Further, in case of SHM $a_{\Delta x} = A\omega^2$, accordingly using the value of $a_{\Delta x}$ determined above, $a_{\Delta x} = \frac{2k\Delta x}{m} = \Delta x\omega^2 \Rightarrow \omega = \sqrt{\frac{2k}{m}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$. Thus answer is $2\pi\sqrt{\frac{m}{2k}}$.

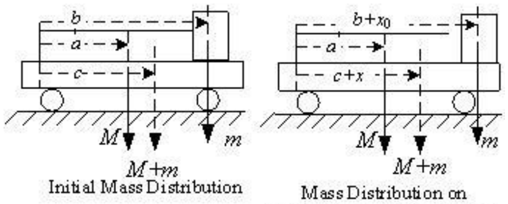
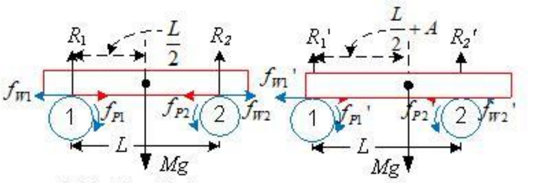
I-07	When the block of mass m, represented by a point mass at o, is slightly pushed against spring C such that it's displacement to position is by an amount Δx. The restraining force developed by the spring is $F_C = k\Delta x$. Since the mass is tied to the system of three springs, it will cause stretching of springs A and B, this causes stretching of springs A and B by length n-p and m-p.  Since displacement $\Delta x <<$ is small hence the angular displacement of spring A by an angle $\theta <<$ and $\angle \text{bom} \rightarrow 90^0$ and likewise for spring B the $\angle \text{bon} \rightarrow 90^0$. Thus stretching of springs A and B, placed symmetrical to spring C, is by lengths n-p and m-p such that  $mp = np = \Delta x \sin 30^0 = \frac{1}{2}\Delta x$ Thus, forces on the mass *m* have been represented in the diagram such that total force on the mass shall be $F = F_C + F_A \cos 60^0 + F_A \cos 60^0 = k\Delta x + k\frac{1}{2}\Delta x\frac{1}{2} + k\frac{1}{2}\Delta x\frac{1}{2}$. This solves into $F = k\Delta x\left(1+\frac{1}{4}+\frac{1}{4}\right) = k\Delta x\left(1+\frac{1}{2}\right) = \frac{3}{2}k\Delta x$. Thus acceleration of the mass is for displacement Δx would be $F = ma_{\Delta x} = \frac{3}{2}k\Delta x \Rightarrow a_{\Delta x} = \frac{3k\Delta x}{2m}$. Since there is no external force acting on the mass *m* once set into oscillation its amplitude shall remain $A = \Delta x$. Further, in case of SHM $a_{\Delta x} = A\omega^2$, accordingly using the value of $a_{\Delta x}$ determined above, $a_{\Delta x} = \frac{3k\Delta x}{2m} = \Delta x\omega^2 \Rightarrow \omega = \sqrt{\frac{3k}{2m}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{2m}{3k}}$. **Thus answer is** $2\pi\sqrt{\frac{m}{2k}}$ **.** **N.B.:** This problem involves careful use of geometry to get to correct answer.
I-08	Taking elaborations of problems at (4) spring constant of series combination of springs having their respective constants k_2 and k_3 is $k_{23} = \frac{k_2k_3}{k_2+k_3}$. And effective spring constant of this series combinations of springs in parallel with the spring having constant k_1 is $k = k_1 + k_{23} = k_1 + \frac{k_2k_3}{k_2+k_3} = \frac{k_1k_2+k_2k_3+k_3k_1}{k_2+k_3}$. A man exerts a constant force F on the spring mass system causing a displacement x the restraining force shall be $F = kx = \left(\frac{k_1k_2+k_2k_3+k_3k_1}{k_2+k_3}\right)x$ and would experience an acceleration towards mean position $F_{\Delta x} = ma_{\Delta x} = k\Delta x \Rightarrow a_{\Delta x} = \left(\frac{k_1k_2+k_2k_3+k_3k_1}{k_2+k_3}\right)\frac{\Delta x}{m}$. When the man releases the force F, the system will be set into SHM with an amplitude A such that $F = kA = \left(\frac{k_1k_2+k_2k_3+k_3k_1}{k_2+k_3}\right)A \Rightarrow A = \frac{F(k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$. **Thus amplitude of oscillation is** $\frac{F(k_2+k_3)}{k_1k_2+k_2k_3+k_3k_1}$ **forms one part of the answer** Acceleration of the mass at this point $a_{\Delta x} = A\omega^2$. Thus equating the two expressions of acceleration $a_{\Delta x}$ we get $\Delta x\omega^2 = \left(\frac{k_1k_2+k_2k_3+k_3k_1}{k_2+k_3}\right)\frac{\Delta x}{m} \Rightarrow \omega = 2\pi f = \sqrt{\frac{k_1k_2+k_2k_3+k_3k_1}{(k_2+k_3)m}}$. Accordingly, **frequency of the oscillation is** $f = \frac{1}{2\pi}\sqrt{\frac{k_1k_2+k_2k_3+k_3k_1}{(k_2+k_3)m}}$ **s is another part of the answer.**

I-09	The three springs are in series and hence one would experience unform force $F = Mg$ and hence alongation in each spring shall be $Mg = k_1\Delta l_1 \Rightarrow l_1 = \frac{Mg}{k_1}$ and hence elastic potential energy stored in the spring $PE_1 = \frac{1}{2}k_1 l_1^2 = \frac{1}{2}k_1\left(\frac{Mg}{k_1}\right)^2 = \frac{1}{2}k_1\left(\frac{M^2g^2}{k_1}\right)^2 = \frac{M^2g^2}{2k_1}$. Likewise for spring with constant k_2 and k_3 the energy would be $PE_2 = \frac{M^2g^2}{2k_2}$ and $PE_3 = \frac{M^2g^2}{2k_3}$ respectively. The three spring system will have effective spring constant $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$. The spring mass system when disturbed through a displacement $A = \Delta x$ will experience an oscillation an acceleration on mass M such that $k\Delta x = Ma_{\max} \Rightarrow a_{\max} = \frac{k\Delta x}{M}$. Chacteristically in SHM $a_{\max} = A\omega^2 = \Delta x\omega^2$. Equating the two expressions of $a_{\max}$ we get $\Delta x\omega^2 = \frac{k\Delta x}{M} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{k}} \Rightarrow T = 2\pi\sqrt{M\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$. **Thus answers are** $\frac{M^2g^2}{2k_1}$, $\frac{M^2g^2}{2k_2}$, and $\frac{M^2g^2}{2k_3}$ **from the above, the time period is** $2\pi\sqrt{M\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$.
I-10	The mass m will experience a gravitational pull $F = mg$ and in turn will cause an elongation l such that $mg = kl \Rightarrow l = \frac{mg}{k}$. If the system in equilibrium is perturbed by a displacement Δx of the mass the acceleration of the system would be $a_{\Delta x} = \omega^2\Delta x$ and $k\Delta x = ma_{\Delta x} \Rightarrow a_{\Delta x} = \frac{k}{m}\Delta x$. Thus equating the two expressions of $a_{\Delta x}$ we get $\omega^2 = \frac{k}{m} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{T}\sqrt{\frac{m}{k}}$. **Hence answer is** $2\pi\sqrt{\frac{m}{k}}$
I-11	The force on the mass m due to gravitation would be mg and it would create an elongation l in the spring such that $mg = kl \Rightarrow l = \frac{mg}{k}$. In this state of equilibrium a small perturbation of the mass is produced in the vertical position of elongated equilibrium. This will set in a SHM assuming that potential energy of the mass remains constant at its mean position. Let at an instant of displacement $x << l \Rightarrow l + x \to l$ from equilibrium position velocity of the mass is v, then as the energy equation of the system would be $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 + mgl = K$, here K is a constant. Differentiating the equation w.r.t. t it gives $mv\frac{dv}{dt} + I\omega\frac{d\omega}{dt} + kx\frac{dx}{dt} + mg\frac{dx}{dt} = 0$. Here $\omega = \frac{v}{r} \Rightarrow \frac{d\omega}{dt} = \frac{1}{r}\frac{dv}{dt}$ and $\frac{dx}{dt} = v$. It simplifies into $mv\frac{dv}{dt} + \frac{I}{r^2}v\frac{dv}{dt} + kxv = 0 \Rightarrow \left(m + \frac{I}{r^2}\right)a = -kx$, where the acceleration is $a = -\left(\frac{k}{m + \frac{I}{r^2}}\right)x$. The

	characteristic equation of SHM is $a=-\omega^2 x$. Thus equating the two equation of SHM $\omega^2 x=\left(\frac{k}{m+\frac{I}{r^2}}\right)x$. It leads to $\omega=\frac{2\pi}{T}=\sqrt{\left(\frac{k}{m+\frac{I}{r^2}}\right)}\Rightarrow T=2\pi\sqrt{\left(\frac{m+\frac{I}{r^2}}{k}\right)}$ **Hence answer is** $2\pi\sqrt{\left(\frac{m+\frac{I}{r^2}}{k}\right)}$. **N.B.** The assumption of potential energy of the oscillating mass remains constant at its mean position. This problem can also be solved with consideration of torque and forces s as done in Q-12,
I-12	 The force produced by perturbation x in length of spring. Since both the masses are equal, the relative displacement any of the two masses w.r.t to the other is x accordingly restraining force in the spring is $F=-kx$ and in turn acceleration of the mass will be $ma=-kx\Rightarrow a=\frac{kx}{m}$ …(1) Each mass is oscillating about C.G of the mass system with an amplitude $\frac{x}{2}$. Characteristic equation of spring-mass system is $a=-\omega^2\frac{x}{2}$...(2). Combining the two equations $\omega^2\frac{x}{2}=\frac{kx}{m}$. This leads to $\omega=\sqrt{\frac{2k}{m}}\Rightarrow\frac{2\pi}{T}=\sqrt{\frac{2k}{m}}$. Accordingly the time period is $T=2\pi\sqrt{\frac{m}{2k}}$. **Hence answer is**. $2\pi\sqrt{\frac{m}{2k}}$ **N.B.:** Here it is to be noted that in this case of independent oscillation of the spring-mass system the amplitude of oscillation is w.r.t. CG of the mass system. If this is ignored it will lead to wrong answer. This problem can also be solved with energy considerations as done in Q-11,
I-13	 The rectangular plate is a rigid mass and has been suspended in a horizontal position with Two parallel strings of length l, separated by a distance d. The plate is slightly displaced along its plane as shown in the figure, the supporting strings are displaced through a small angle θ about their fixed points as shown in the figure. Assuming the two strings carry equal tension $T_0=\frac{mg}{2}$. On displacement through the tension in the rwo strings shall be equal such that theit vertical components would balance the wight of the plate. Accordingly, $T_V=T_\theta\cos\theta=\frac{mg}{2}$, the restraining force shall be caused by horizontal component such that $T_H=-T_\theta\sin\theta\approx-\mathrm{T}_\theta\theta$, since for $\theta<<$the $\sin\theta\to\theta$. This leads to $T_H=-\mathrm{T}_\theta\frac{x_\theta}{L}$. Thus proportionate mass $\frac{m}{2}$ supported by each string is accelerated to its mean position would be $a_H=-\frac{2T_\theta x_\theta}{mL}$. In this formulation $\theta<<$ it leads $\cos\theta\to1\Rightarrow T_\theta=\frac{mg}{2}$. Accordingly, $a_H=-\left(\frac{mg}{2}\right)\frac{2x_\theta}{mL}=-\frac{g}{L}x_\theta$ …(1). Further, as per characteristic equation of SHM $a_H=-\omega^2 x_\theta$ …(2). Equating (1) and (2), $\omega^2 x_\theta=\frac{g}{L}x_\theta$. It solves into $\omega=\sqrt{\frac{g}{L}}\Rightarrow\frac{2\pi}{T}=\sqrt{\frac{g}{L}}\Rightarrow T=2\pi\sqrt{\frac{L}{g}}$. **Thus answer is** $2\pi\sqrt{\frac{L}{g}}$. **N.B.:** Actual tension in each string and the corresponding mass shared by each would depend upon point of

	suspension of the plate and this can be determined with the principle of moments. However, in the final result assumption that tension in the both the strings are equal, in turn they share equal mass of the plate is valid since the time period so derived is independent of the mass.
I-14	Amplitude of block of $m_0 = 1$ Kg is given to be $A_0 = 0.1$ m. The spring constant executing the restraining force is given to be $k = 100$ N/m. Accordingly, acceleration of the mass at its amplitude towards mean position would be be $a_0 = -\frac{F_0}{m_0} = -\frac{kA_0}{m_0} = -\frac{100\times 0.1}{1} = -10.0$ m.s^{-2}…(1). Since the spring-mass system is executing SHM and hence as per characteristic equation $a_0 = -\omega_0^2 A_0 = -0.1\omega_0^2$ …(2). Here, ω_0 is the initial angular velocity. Combining (1) and (2) we get $0.1\omega_0^2 = 10.0 \Rightarrow \omega_0^2 = 100 \Rightarrow \omega_0 = 10$ rad/s. The velocity of mass atmean position will be $v_0 = A_0\omega_0 = 0.1\times 10 = 1.0$ m/s. Now when block of mass $m = 3$ kg is gently placed on the m_0 assuming that there is no relative motion between them, i.e. they move together, then as per conservation of momentum let v is the velocity of the combined mass then $(m+m_0)v = m_0 v_0 \Rightarrow v = \frac{m_0 v_0}{m+m_0} = \frac{1\times 1}{1+3} = \frac{1}{4}$ m/s. The kinetic energy of the combined mass of 4 kg at $v = 0.25$ m/s would be $KE = \frac{1}{2}mv^2 = \frac{1}{2}\times 4\times\left(\frac{1}{4}\right)^2 = \frac{1}{8}$ J. This KE, while continuing with SHM in the revised configuration, will be conserved by the spring during maximum compression A, accordingly $\frac{1}{2}kA^2 = KE \Rightarrow A = \sqrt{\frac{2}{k}\times KE} = \sqrt{\frac{2}{100}\times\frac{1}{8}} = \sqrt{\frac{1}{400}} = \frac{1}{20} = 0.05$ m or **amplitude of $A = 5$ cm on part of the answer** The velocity at mean position of SHM $v = A\omega \Rightarrow \omega = \frac{v}{A} = \frac{0.25}{0.05} = 5$ rad/s. From this $\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{5}{2\pi}$ Hz. **Thus $\frac{5}{2\pi}$ Hz, is the answer of another part.**
I-15	 Momentum of the moving block A is $p = mv$. At the instance of collision the block B in equilibrium has zero velocity. Since collision is in equilibrium, velocity of the two masses shall be $2mv' = mv + m\times 0 \Rightarrow v' = \frac{v}{2}$. Mass of both the blocks is m. On collision the kinetic energy of the block A shall get converted in elastic potential energy of the spring at amplitude A of the oscillation of the spring-mass system such that $\frac{1}{2}mv^2 = \frac{1}{2}k(A)^2 \Rightarrow A = \sqrt{\frac{m}{k}}v$. This is in accordance with Law of Conservation of Energy. It is to be noted that once oscillation of the system will involve two parts – (a) One-half of cycle is compression from mean position to reversal back to mean position and it shall take time $\frac{T_s}{2}$, here T_s is time period of spring mass system when two masses are attached. As per characteristic of SHM velocity at equilibrium position is $v' = A\,\omega \Rightarrow \omega = \frac{2\pi}{T_s} = \frac{v'}{A} \Rightarrow T_s = 2\pi\frac{\frac{v}{2}}{\sqrt{\frac{m}{k}}v} = \pi\sqrt{\frac{k}{m}}$. (b) On reaching the state of initial collision, since the masses A and B are not attached, mass attached to the spring B would come to rest while mass A would regain its velocity v in opposite direction, as per Law of

	Conservation of Momentum and as such separation of two masses shall take place at this instant. Since system is frictionless, the mass A on reaching the barrier will experience an elastic collision with it to experience second collision with mass B, and thus complete one cycle. Thus time taken by mass A to seprate from mass B and return back to it will be $T_A = \frac{2L}{v}$ initial velocity will start moving with the velocity of separation, since the system is frictionless. This mass on reaching the stopper will again undergo elastic collision to return to collide with the mass attached to the spring. The time period of the periodic motion comprises of half cycle of SHM as elaborated at (a) i.e. $\frac{T_s}{2}$ and motion of mass A with uniform speed T_A. Accordingly, time period of one this periodic motion would be $T = \frac{T_s}{2} + T_A = \frac{\pi}{2}\sqrt{\frac{k}{m}} + \frac{2L}{v}$. Thus answer is $\frac{\pi}{2}\sqrt{\frac{k}{m}} + \frac{2L}{v}$. **N.B:** While determining time period T_S of spring mass system velocity of velocity at inception of oscillation i.e. $v' = \frac{v}{2}$ has to be considered and not v.
I-16	Motion in each part is being analyzed separately- **Part (a):** ***Along the slope at 45^0*** –Initial velocity along the slope is $u_{45} = 0$ and distance covered by it is $s_{45} = \frac{0.1}{\sin 45} = 0.1\sqrt{2}$ Therefore, time taken to reach bottom of the slope as per second equation of motion is $s_{45} = u_{45}t_1 + \frac{1}{2}(g\cos 45)t_1^2 \Rightarrow 0.1\times\sqrt{2} = \frac{1}{2}\times\frac{10}{\sqrt{2}}\times t_1^2 \Rightarrow t_1 = \sqrt{\frac{4}{100}} = 0.2$ sec. Here, acceleration due to gravity is taken to be $g = 10$ m/s^2 and accordingly acceleration along the slope shall be $a_{45} = g\cos 45$. Velocity of the particle at the bottom of the slope, as First Equation of Motion would be $v_{45} = u_{45} + a_{45}t_1 \Rightarrow v_{45} = \frac{10}{\sqrt{2}}\times 0.2 = \sqrt{2}$ m/s **Part (b):** ***Along the slope at 60^0*** - Since nothing is stated about friction and it is stated that there is no effect of bending near the bottom and therefore the particle will rise to the same height, as per law of Conservation of Energy. But velocities along the slopes shall be $v_{45} = u_{60}$. Time taken to rise to maximum height along the slope would be when its velocity becomes zero. Here,a acceleration up the slope would be $a_{60} = g\cos 30 = 10\times\frac{\sqrt{3}}{2} = 5\sqrt{3}$ m/s. Accordingly, time to rise up the slope would be $0 = u_{60} - 5\sqrt{3}t_2 \Rightarrow t_2 = \frac{\sqrt{2}}{5\sqrt{3}} = 0.163$ s. Time period is the total time taken for the particle to return to the position of start, thus time period is $T = 2(t_1 + t_2) = 2(0.2 + 0.163) = 2(0.363) = 0.726 = 0.73$ s. **N.B.:** The value of T would depend on selection of value of g.

I-17	 Initial Mass Distribution Mass Distribution on Displacement of Mass *m* by x_0 In initial distribution of mass as shown in the figure center of mass from fixed support of the spring be at $c=\frac{aM+bm}{M}$. And when spring is stretched by a length x_0, the revised position of the COM from the fixed support shall be $c+x=\frac{aM+(b+x_0)m}{M}$. Thus displacement of COM with respect to the fixed support shall be $x=\frac{aM+(b+x_0)m}{M+m}-\frac{aM+bm}{M+m}=\frac{x_0m}{M+m}$. Since, during stretching of the spring, there is no external force acting on the system and hence as per First Law of Motion location of COM will remain unchanged as and hence it will cause COM of car to be displaced to left by $x=\frac{x_0m}{M+m}$ and for the block displacement shall be $x_0-x=x_0-\frac{x_0m}{M+m}=\frac{x_0(M+m)-x_0m}{M+m}=\frac{x_0M}{M+m}$. Since it is a spring-mass system, stretching of the spring shall set it in SHM with **amplitudes of block as** $A_m=\frac{x_0M}{M+m}$ **and that of car as** $A_M=\frac{x_0m}{M+m}$ **is one part of the answer.** Potential energy of the spring on stretching by x_0 **is** $PE=\frac{1}{2}kx_0^2$**.** And velocities of both mass in their mean mosition though in opposite directions would be $v_M=A_M\omega=\frac{x_0m}{M+m}\omega$ and that of the block shall be $v_m=A_m\omega=\frac{x_0M}{M+m}\omega$**.** Since SHM is a conservative in energy and hence $PE=\frac{1}{2}Mv_M^2+\frac{1}{2}mv_m^2$. On substituting the values $\frac{1}{2}kx_0^2=\frac{1}{2}M\left(\frac{x_0m}{M+m}\omega\right)^2+\frac{1}{2}m\left(\frac{x_0M}{M+m}\omega\right)^2\Rightarrow k(M+m)^2=(Mm^2+mM^2)\omega^2$. This solves into $k(M+m)^2=Mm(M+m)\omega^2\Rightarrow\omega=\frac{2\pi}{T}=\sqrt{\frac{k(M+m)}{Mm}}\Rightarrow T=2\pi\sqrt{\frac{Mm}{k(M+m)}}$. **Hence, time period of the oscillation is** $T=2\pi\sqrt{\frac{mM}{k(M+m)}}$.
I-18	 Initial Equlibrium On Displacement of Plate by x The system in initial equilibrium is shown where normal reaction of symmetrically placed plate on both the wheels is $R_1=R_2=\frac{Mg}{2}$ and maintains vertical equilibrium with $R_1+R_2=Mg$. Frictional force on wheels would tend to retard their respective rotational motion and at each point of contact of plate with the wheel, the plate will experience reaction of frictional force, such that $f_{P1}=-f_{p2}\Rightarrow f_{P1}+f_{p2}=0$ and thus horizontal equilibrium would be maintained. When plate is displaced by an amount x the vertical reaction on each wheel would change such that $R_2'\times L=Mg\times\left(\frac{L}{2}+A\right)\Rightarrow R_2'=\frac{Mg}{L}\times\left(\frac{L}{2}+A\right)$. Accordingly, $R_1'=Mg-\frac{Mg}{L}\times\left(\frac{L}{2}+A\right)=\frac{Mg}{L}\times\left(\frac{L}{2}-A\right)$. Thus frictional force on plate at the point of contact with wheel 1 would be $f_{P1}'=\mu R_1'=\frac{\mu Mg}{L}\times\left(\frac{L}{2}-A\right)$ and likewise $f_{P2}'=-\mu R_2'=-\frac{\mu Mg}{L}\times\left(\frac{L}{2}+A\right)$. Thus resultant horizontal force on the plate $f_P=f_{P1}'+f_{P2}'$.

	It will lead to $f_P = \frac{\mu Mg}{L}\times\left(\frac{L}{2}-A\right)-\frac{\mu Mg}{L}\times\left(\frac{L}{2}+A\right)=-\frac{2\mu Mg}{L}A$. Thus net acceleration of the plate is $a_P=\frac{f_P}{M}=-\frac{2\mu g}{L}A$. The (-) sign indicates that this force is restraining force proportional to the displacement, a necessary condition for SHM. Further, as per characterstic equation of SHM $a_P = a = -\omega^2 A$. Combining the two expressions of a_P, it leads to $\omega^2=\frac{2\mu g}{L}\Rightarrow\omega=\sqrt{\frac{2\mu g}{L}}$. Since $\omega=\frac{2\pi}{T}\Rightarrow T=2\pi\sqrt{\frac{L}{2\mu g}}$. **Thus answer is** $2\pi\sqrt{\frac{l}{2\mu g}}$.
I-19	For a simple pendulum $T=2\pi\sqrt{\frac{L}{g}}$. Given that $g=\pi^2$ m/s^2 and $T=2$s. Therefore, $2=2\pi\sqrt{\frac{L}{\pi^2}}\Rightarrow L=1$m. **Hence answer is 1m**.
I-20	On simple harmonic motion displacement of a particle from its mean position is given by $x=A\sin\omega t$. In case of simple pendulum angular displacement is small leading to $\theta\to\sin\theta$. And, $\theta=\omega t$, here angular velocity $\omega=\frac{2\pi}{T}$ …(1) It is given that $\theta=\frac{\pi}{90}\sin\left[(\pi)t\right]\Rightarrow\omega=\pi$ …(2). Combining (1) and (2) $\frac{2\pi}{T}=\pi\Rightarrow T=2$ s. The time period of simple pendulum is given $T=2\pi\sqrt{\frac{l}{g}}$. Using the given value of $g=\pi^2$ and the value of $T=2$ determined above we get $2=2\pi\sqrt{\frac{L}{\pi^2}}\Rightarrow 2=2L\Rightarrow L=1$m. **Hence answer is 1m**.
I-21	Given that $T'=2.04$s, while of a pendulum clock $T'=2$s, Thus each second of the given clock is slow by $\Delta T=T'-2=0.04$ sec or $d=\frac{\Delta T}{T}=\frac{0.04}{2}=0.02$ s/s. Hence, total delay in 24 hours is $D=\Delta T\times 24\times 60\times 60=1728$s, or $D=\frac{1728}{60}=28.8$ minutes slow.. **Hense answer is 28.8 minutes slow.**
I-22	Given that time period of a standard pendulum is $T_1=2$ s at $g_1=9.8$ m.s^{-2}. At another place the pendulum loses i.e. get slower by 24 sec in 24 hours or $\delta=\frac{24}{24\times 60\times 60}=\frac{1}{3600}$ s/s. Time period of pendulum is expressed as $T=2\pi\sqrt{\frac{l}{g}}$. Therefore, in case of change of acceleration due to gravity $\frac{T_1}{T_2}=\frac{2\pi\sqrt{\frac{l}{g_1}}}{2\pi\sqrt{\frac{l}{g_2}}}\Rightarrow\left(\frac{T_1}{T_2}\right)^2=\frac{g_2}{g_1}\Rightarrow g_2=g_1\left(\frac{T_1}{T_2}\right)^2$. With $\delta=\frac{1}{3600}$ s/s, the time period $T_2=T_1(1+\delta)$. Accordingly, $g_2=g_1\left(\frac{T_1}{T_1(1+\delta)}\right)^2\Rightarrow g_2=g_1\left(\frac{1}{1+\delta}\right)^2$. Further, $1+\delta=1+\frac{1}{3600}=\frac{3601}{3600}$, therefore, $g_2=9.8\times\left(\frac{3600}{3601}\right)^2=9.7945=9.745$ m.s^{-2}. Hence answer is 9.745 m.s^{-2}.

N.B.:(a) Using formula for time period of simple pendulum taking ratio of time periods, effect of length which is same in two cases, can be eliminated.
(b) Since value of g_1 is given in Four significant digits, and hence value of g_2 will also have to be given if Four SDs for which calculation result of calculation shall have to be taken in five SDs and then rounded to 4tyh LSD.

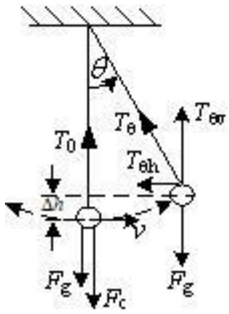
I-23

Time period of a pendulum taking $g = 10\,\text{m/s}^2$ is given by $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{5}{10}} = \sqrt{2}\pi$. Therefore number of oscillations per sec $n = \frac{1}{T} = \frac{1}{\sqrt{2}\pi} = \frac{0.70}{\pi}$. **Thus answer of part (a) is** $\frac{0.70}{\pi}$**.**

As regards part (b) of the problem $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} \Rightarrow \frac{\sqrt{2}\pi}{T_2} = \sqrt{\frac{1.67}{10}} \Rightarrow f_2 = \frac{1}{T_2} = \frac{1}{2\pi}\sqrt{\frac{1.67}{5}} = \frac{1}{2\pi} \times \frac{1}{\sqrt{2.99}}$ **. It leads to** $f_2 = \frac{1}{2\pi\sqrt{3}}$ Hz**. Thus answer of part (a) is** $\frac{1}{2\pi\sqrt{3}}$ **Hz.**

I-24

Figure shows an oscillating pendulum. When $\theta = 0$ the string of pendulum is vertical and angular velocity of the bob at its lowest position be ω. Thus forces on the bob shall satisfy equation $T_0 = F_g + F_c = mg + ml\omega^2 \ldots(1)$ Here, l is the length of the string and m is the mass of the bob. This is the position of maximum tension.



When, bob turns through an angle $\theta = 0$, angular velocity becomes zero and thus there will be no centripetal force. Yet forces on the bob would balance it vertically such that $T_\theta \cos\theta = F_g \Rightarrow T_\theta = mg\cos\theta \ldots(2)$ This is the position of minimum tension.

It is given that $T_0 = 2T_\theta$, therefore, $mg + ml\omega^2 = 2mg\cos\theta \Rightarrow g + l\omega^2 = 2g\cos\theta$ ...(3)

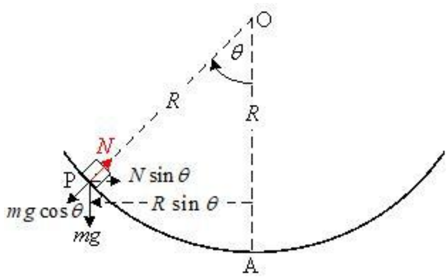
Relationship between l, g and θ can be determined with the principle of conservation of energy such that $\frac{1}{2}m(l\omega)^2 = mg\Delta h$, here $\Delta h = (l - l\cos\theta)$ and it leads to $l\omega^2 = 2g(1 - \cos\theta)\ldots(4)$

Substituting value of $l\omega^2$ from (4) in (3.0.0) $g + 2g(1 - \cos\theta) = 2g\cos\theta \Rightarrow 4\cos\theta = 3 \Rightarrow \cos\theta = \frac{3}{4}$. **Thus angle** $\theta = \cos^{-1}\left(\frac{3}{4}\right)$ **is the answer.**

N.B.: Here the only at θ tension in the string is taken to be $T_\theta = F_g\cos\theta$ **sinceand not** $F_g = T_\theta\cos\theta$ the driving cause is displacement of $F_g = mg$ and result is T_θ**.**

I-25

Let mass of the block be m and when placed at P in a concave smooth surface it will slide down to lowest position A. At this position as shown in the figure restoring force is $F = N\sin\theta = mg\cos\theta\sin\theta = \frac{mg}{2}\sin 2\theta$.



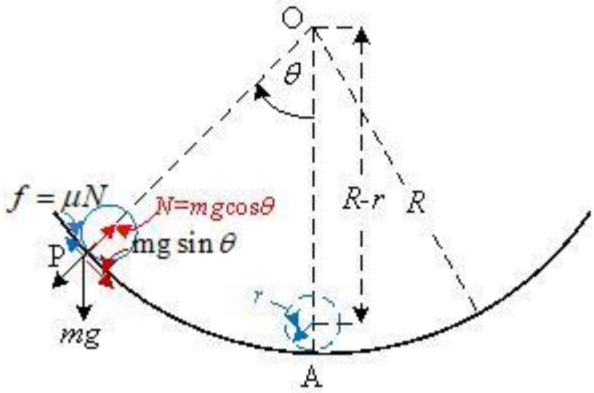
In case of $\theta <<$ it leads to $\sin 2\theta \to 2\theta$. Accordingly, $F = \frac{mg}{2} \times 2\theta = mg\theta \Rightarrow a = g\theta \Rightarrow a = g\frac{PA}{R} \Rightarrow a = \frac{g}{R}PA \ldots(1)$.

Here, PR represents length of the arc.

For SHM $a = -\omega^2 x \Rightarrow a = -\omega^2(-PA) = \omega^2 PA \ldots(2)$.

Combining (1) and (2) $\omega^2 = \frac{g}{R} \Rightarrow \omega = \sqrt{\frac{g}{R}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$ **.Thus answer is** $2\pi\sqrt{\frac{R}{g}}$.

N.B. Here, it is essential to assume $\theta <<$.

I-26	Let ball is displaced through an angle θ about it's mean position of oscillation. Taking $\theta <<$ as given, linear displacement of the COM of the ball, along horizontal direction, would be $x=(R-r)\theta$ …(1) When ball rools down from point P to A, change of height of COM is $\Delta h=(R-r)\cos\theta$ and therefore change of potential energy of the ball w.r.t to mean position A is $PE=mg\Delta h=mg(R-r)\cos\theta$ …(2).  The ball is stated to be rolling without slipping therefore frictional force shall be balanced by downward force due to gravity such that $f=mg\sin\theta$ and hence torque responsible for rolling of the ball shall be $\tau=fr=mgr\sin\theta$. Since ball is accelerating on the curved surface and hence applying parallel axis theorem of moment of inertia of the ball at the curved surface shall be $I=mr^2+I_b$. Moment of inertia of a solid ball is $I_b=\frac{2}{5}mr^2$, therefore, $I=mr^2+\frac{2}{5}mr^2=\frac{7}{5}mr^2$ … (3). Accordingly angular acceleration of the ball $\alpha=\frac{\tau}{\frac{7}{5}mr^2}=\frac{5mgr\sin\theta}{7mr^2}=\frac{5g\sin\theta}{7r}$ …(4). Translational acceleration of the rolling ball would be $a=r\alpha=r\times\frac{5g\sin\theta}{7r}=\frac{5g\sin\theta}{7}=\frac{5g}{7}\theta\Big\vert_{\theta<<}$ …(5) Thus, linear shift x of the COM which is moving along radius $(R-r)$ of the ball shall such that $\theta=\frac{x}{R-r}$ ..(6).. Combining (5) and (6) $a=-\frac{5g}{7}\times\frac{x}{R-r}=-\frac{5g}{7(R-r)}x$ …(7). Here (-)ve sign indicates the acceleration is on a direction opposite to the displacement. As per characteristic equation of SHM $a=-\omega^2 x$ …(8). On combining (7) and (8) it leads to $\omega^2=\frac{5g}{7(R-r)}\Rightarrow\omega=\sqrt{\frac{5g}{7(R-r)}}=\frac{2\pi}{T}\Rightarrow T=2\pi\sqrt{\frac{7(R-r)}{5g}}$. **Hence answer is** $2\pi\sqrt{\frac{7(R-r)}{5g}}$ **.** **N.B.:** This problem involves different concepts of geometry and physics and need to be dealt with carefully to get correct and crisp solution**.**
I-27	Earth's gravitational force on a mass m on the surface of the earth of radius R is $F_R=G\frac{Mm}{R^2}$ and hence acceleration is $g_R=G\frac{M}{R^2}=G\frac{\frac{4}{3}\pi R^3\rho}{R^2}=\left(\frac{4}{3}\pi G\rho\right)R$. With this analogy at a depth of $d=1600$ km where effective distance from COM of the earth shall be $R'=R-d$ the acceleration due to gravity would be $g_d=\left(\frac{4}{3}\pi G\rho\right)(R-d)$. The time period of pendulum $T=2\pi\sqrt{\frac{l}{g}}$, here is the length of the $g=g_R$ t. Accordingly, $T=2\pi\sqrt{\frac{l}{\left(\frac{4}{3}\pi G\rho\right)R}}$ and at depth d the time period $T'=2\pi\sqrt{\frac{l}{\left(\frac{4}{3}\pi G\rho\right)(R-d)}}$. This leads

	to $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{l}{\left(\frac{4}{3}\pi G\rho\right)(R-d)}}}{2\pi\sqrt{\frac{l}{\left(\frac{4}{3}\pi G\rho\right)R}}} \rightarrow T' = T\sqrt{\frac{R}{R-d}}$. Here, $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.4}{10}} = 0.4\pi$. Using these values $T' = 0.4\pi\sqrt{\frac{6400}{4800}}$, it leads to $T' = 0.4\pi\frac{2}{\sqrt{3}} = 1.45\text{s}$. **Hence answer is 1.45 s.** **N.B.:** Since length of the pendulum is given T shall have to be determined for which acceleration due to gravity is taken to be $g = 10$ m.s^{-2}.

I-28

In case (a) particle is projected into the tunnel with a speed of $v_a = \sqrt{gR}$.

In case (b) the particle when released from a height R above the tunnel i.e, at a distance $2R$ from the COM of the earth reduction in potential energy is $\Delta PE = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R} = \frac{mgR}{2}$. Let v is the velocity of the particle when it reaches tunnel then change of kinetic energy would be $\Delta KE = \frac{1}{2}mv_b^2 - 0 = \frac{1}{2}mv_b^2$. Accordingly, As per Law of Conservation of Energy $\frac{1}{2}mv_b^2 = \frac{mgR}{2} \to v_b = \sqrt{gR}$.

In case (c) the particle is projected vertically upward with velocity $\sqrt{gR}$ and therefore, as per Law of Conservation of Energy when it enters the tunnel it will have a downward velocity $v_c = \sqrt{gR}$, it is similar to that of the case (a).

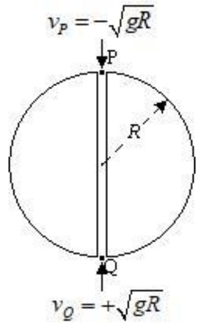
Thus it is seen that velocity of the particle entering the tunnel in each case $v_a = v_b = v_c = v = \sqrt{gR}$…(1). Rest of the problem in each case is same as the tunnel is same.

Acceleration due to gravity at earth's surface $= \frac{GM}{R^2} = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right) = \frac{4}{3}\pi G\rho R$. Since acceleration due to gravity at any point inside earth at a distance x from the earth's center is due to mass inside the sphere of radius x and not the shell outside it and hence on similar lines $g_x = \frac{4}{3}\pi G\rho x$. Accordingly, $\frac{g_x}{g} = \frac{\frac{4}{3}\pi G\rho x}{\frac{4}{3}\pi G\rho R} = \frac{x}{R} \to g_x = \frac{g}{R}x$.

This quantitative relationship can be written with directional sense as $g_x = -\frac{g}{R}x$…(2). Since acceleration vector g_x is toward the centre of the earth while vector x is radially outward. This equation can be compared with characteristic equation of SHM where $a_x = -\omega^2 x$…(3). Comparing equation (2) and (3) $\omega^2 = \frac{g}{R} \to \omega = \sqrt{\frac{g}{R}}$ and time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$…(4)

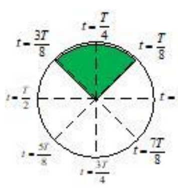
*This is an interesting case of composite motion in Two parts – **(a)** motion inside the tunnel where acceleration is directly proportional to the displacement from mean position and is always directed toward the mean position and therefore SHM, **(b)** motion above the earth's surface where acceleration is inversely proportional to the distance from mean position and is always directed toward the mean position and therefore it follows inverse square law. This situation is converted into an **equivalent SHM** by evolving **velocity-displacement equation (VDE)** using the data of motion of the particle along the diametric tunnel.*

In SHM $x = A\sin\omega t$, and velocity $v_x = \frac{dx}{dt} = A\omega\cos\omega t = A\omega\sqrt{1-\sin^2\omega t} \to v_x = A\omega\sqrt{1-\left(\frac{x}{A}\right)^2} = \omega\sqrt{A^2 - x^2}$…(5); this is being called **VDE** in simple harmonic motion.

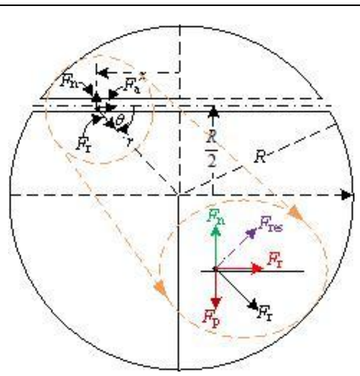


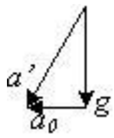
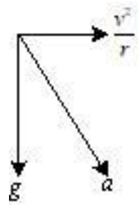
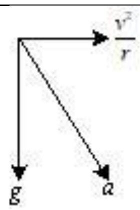
At $x = R$, equation (5) leads to $v = \omega\sqrt{A^2 - R^2}$ accordingly using values of v and ω derived above it leads to $\sqrt{gR} = \sqrt{\frac{g}{R}}\left(\sqrt{A^2 - R^2}\right) \to R^2 = A^2 - R^2 \to A = \sqrt{2}R$. Applying VDE in the instant case $v = A\omega\cos\omega t \to v = \left(\sqrt{2}R\sqrt{\frac{g}{R}}\right)\cos\left(\sqrt{\frac{g}{R}}t\right) = \sqrt{2gR}\cos\left(\sqrt{\frac{g}{R}}t\right)$.

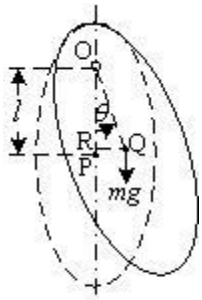
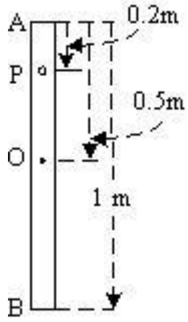
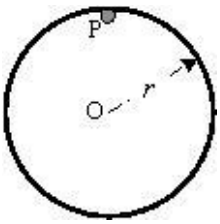
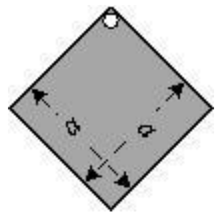
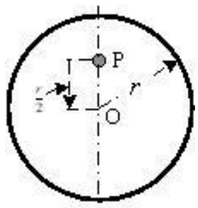
Accordingly, we have $v_p = A\omega\cos\omega t_p \to -\sqrt{gR} = \sqrt{2gR}\cos\omega t_p \to \cos\omega t_p = -\frac{1}{\sqrt{2}}$ .or $\omega t_p = \pi \mp \frac{\pi}{4} \to \frac{2\pi}{T}t_p = \frac{5}{4}\pi, \frac{3\pi}{4} \to t_p = \frac{3}{8}T, \frac{5}{8}T$. Likewise, $v_q = A\omega\cos\omega t_q$, it leads to $\sqrt{gR} = \sqrt{2gR}\cos\omega t_q \to \cos\omega t_q = \frac{1}{\sqrt{2}}$, or $\omega t_p = \mp\frac{\pi}{4} \Rightarrow \frac{2\pi}{T}t_p = \pm\frac{\pi}{4} \Rightarrow t_p = \pm\frac{\pi}{8}$.

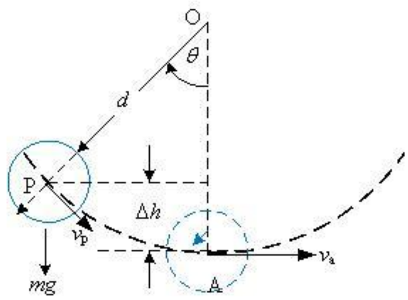
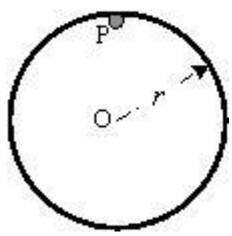


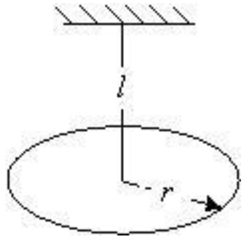
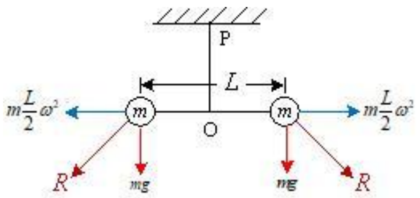
Thus, $\Delta t = t_q - t_p = \frac{3}{8}T - \frac{1}{8}T = \frac{1}{4}T \to \Delta t = \frac{1}{4}\left(2\pi\sqrt{\frac{R}{g}}\right) = \frac{\pi}{2}\sqrt{\frac{R}{g}}$., has been depicted graphically. **Hence answer is** $\frac{\pi}{2}\sqrt{\frac{R}{g}}$.

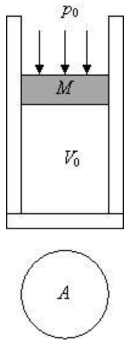
I-29	 Given that, a narrow tunnel dug across earth, like a chord, at a distance $\frac{R}{2}$ from the center of the earth. As per law of gravitation $F_r = \frac{GM_r m}{r^2}$ here $M_r = \frac{4}{3}\pi r^3 \rho$ and $r^2 = x^2 + \left(\frac{R}{2}\right)^2$. It leads to $F_r = \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)m}{r^2} = G\left(\frac{4}{3}\pi\rho\right)mr$. Since mass of earth and radius are more readily available data, therefore, $\frac{4}{3}\pi R^3 \rho = M \to \frac{4}{3}\pi\rho = \frac{M}{R^3}$. Accordingly, $F_R = \frac{GMm}{R^3} r = \frac{GMm}{R^3}\sqrt{x^2 + \left(\frac{R}{2}\right)^2}$. **Thus part (a) of the answer is** $\frac{GMm}{R^3}\sqrt{x^2 + \frac{R^2}{4}}$ **.** From the figure force along the tunnel is $F_a = F_r \cos\theta = \left(\frac{GMm}{R^3} r\right) \times \frac{x}{r} = \frac{GMm}{R^3} x$ and force perpendicular to the tunnel is $F_p = F_r \sin\theta = \left(\frac{GMm}{R^3} r\right) \times \frac{\frac{R}{2}}{r} = \frac{GMm}{2R^2}$. **Hence, answer to part (b) is force along the tunnel and perpendiculars to the tunnel are** $\frac{GMm}{R^3}x$, $\frac{GMm}{2R^2}$ **respectively.** *For clarity forces on the particle are shown in a blown up inset.* Since walls of the tunnel will not let the particle penetrate the surface and the wall will exert a normal reaction equal and opposite to the normal force $F_n = -F_p = -\frac{GMm}{2R^2}$. **Hence, magnitude of the force exerted by the wall is** $\boldsymbol{\frac{GMm}{2R^2}}$ **is an answer to part (c) of the problem.** The resultant force on the particle shall be $\left\|\vec{F}_{res}\right\| = \left\|\vec{F}_n + \vec{F}_a\right\| = \left\|\vec{F}_r\right\| = \frac{GMm}{R^3}\sqrt{x^2 + \frac{R^2}{4}}$. **Hence part (d) of the answer is** $\frac{GMm}{R^3}\sqrt{x^2 + \frac{R^2}{4}}$ **.** The force along the tunnel $F_a = \left(\frac{GMm}{R^3}\right)x$ derived ion part (b) above defines its motion which is characteristically SHM where $\omega^2 = \frac{GMm}{R^3} \to \omega = \frac{2\pi}{T} = \sqrt{\frac{GMm}{R^3}} \to T = 2\pi\sqrt{\frac{R^3}{GMm}}$. **Hence, part (e) of the answer is** $\boldsymbol{2\pi\sqrt{\frac{R^3}{GMm}}}$**.**
I-30	Time period of a simple pendulum under gravity is $T = 2\pi\sqrt{\frac{l}{g}}$. But in the three cases stated in the problem – **Case (a):** relative acceleration when elevator is having an upward acceleration $\vec{a}_e = -\vec{a}_0$ therefore, $\vec{a} = \vec{g} - \vec{a}$ or $\vec{a} = \vec{g} - (-\vec{a}_0) = \vec{g} + \vec{a}_0 \to \|\vec{a}\| = \|\vec{g} + \vec{a}_0\|$. Since, accelerator of the elevator and that of gravity are collinear and hence $a = g + a_0$. Therefore time period in this case shall be $T = 2\pi\sqrt{\frac{l}{a}} = 2\pi\sqrt{\frac{l}{g+a_0}}$. **Hence, answer of part (a) is** $2\pi\sqrt{\frac{l}{g+a_0}}$ **.** **Case (b):** relative acceleration when elevator is having a downward acceleration $\vec{a}_e = \vec{a}_0$ therefore, $\vec{a} = \vec{g} - \vec{a}$ or $\vec{a} = \vec{g} - \vec{a} = \vec{g} - \vec{a}_0 \to \|\vec{a}\| = \|\vec{g} - \vec{a}_0\|$. Since, accelerator of the elevator and that of gravity are collinear and hence $a = g - a_0$. Therefore time period in this case shall be $T = 2\pi\sqrt{\frac{l}{a}} = 2\pi\sqrt{\frac{l}{g-a_0}}$. **Hence, answer of part (a) is** $2\pi\sqrt{\frac{l}{g-a_0}}$ **.** **Case (c):** When accelerator is moving with uniform velocity then $a_0 = 0 \to a = g$ and hence time period shall be $T = 2\pi\sqrt{\frac{l}{g}}$. **Hence answer of part (c) is** $2\pi\sqrt{\frac{l}{g}}$.

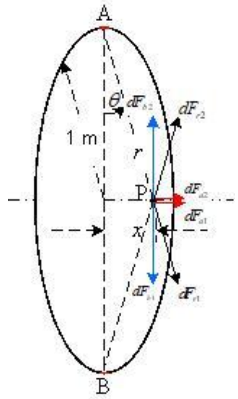
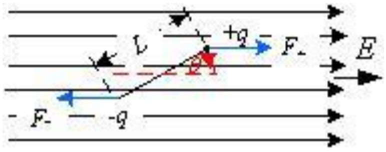
I-31	For a simple pendulum of length l feet $T = 2\pi\sqrt{\frac{l}{a}}$ s, and it is given that $T = \frac{\pi}{3}$s and in FPS $g = 32$ ft.s^{-2}, therefore, $\frac{\pi}{3} = 2\pi\sqrt{\frac{1}{a}} \to \left(\frac{1}{6}\right)^2 = \frac{1}{a} \to a = 36$ Ft.s^{-2}. Since, $a > g$ it is possible only when. relative acceleration when elevator is having an upward acceleration $\vec{a}_e = -\vec{a}_0$ therefore, $\vec{a} = \vec{g} - \vec{a}$ or $\vec{a} = \vec{g} - (-\vec{a}_0) = \vec{g} + \vec{a}_0 \to |\vec{a}| = |\vec{g} + \vec{a}_0|$. Since, acceleration of the elevator and that of gravity are collinear and hence $a = g + a_0$. It leads to $a_0 = a - g = 36 - 32 = 4$ ft.s^{-2}. **Hence, answer is 4 ft.s^{-2}.**
I-32	For a simple pendulum in a car either stationary or moving with a uniform velocity $T = 2\pi\sqrt{\frac{l}{a}}$ where $T = 4$s and $a = g$. But when car is accelerating $T' = 2\pi\sqrt{\frac{l}{a'}}$ where $T = 3.99$s. Therefore, it leads to $\frac{T}{T'} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l}{a'}}} = \sqrt{\frac{a'}{g}} \to a' = \left(\frac{T}{T'}\right)^2 g = \left(\frac{4}{3.99}\right)^2 g$. From the vector diagram $a'^2 = g^2 + a_0^2$, it implies that $a_0^2 = a'^2 - g^2 =\to a_0^2 = a'^2 - g^2 = \left(\left(\frac{4}{3.99}\right)^4 - 1\right) g^2 = (4^2 + 3.99^2)(4^2 - 3.99^2)\frac{g^2}{3.99^4}$. It further approximates to $a_0^2 \approx (4^2 + 4^2)(4+4)(4-3.99)\frac{g^2}{4^4} = (2 \times 4^2)(2 \times 4) \times 0.01 \times \frac{g^2}{4^4} \to a_0^2 \approx (0.1 \times g)^2$. It solves into $a_0 = \frac{g}{10}$. **Hence the answer, acceleration of car, is $\frac{g}{10}$.** 
I-33	Acceleration of the bob of a simple pendulum is resultant of acceleration due to gravity g and centrifugal acceleration $a_c = \frac{v^2}{r}$ due to circular motion with a velocity v on a track of radius r. Since, both and are perpendicular to each other and hence effective acceleration $a = \sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{g^2 + \frac{v^4}{r^2}}$. Therefore, tension in the string of the pendulum having a bob of mass m shall be $F = ma$. Thus answer to part (a) is ma where $a = \left[g^2 + \frac{v^4}{r^2}\right]^{\frac{1}{2}}$.  And time period of the pendulum shall be $T = 2\pi\sqrt{\frac{l}{a}}$ **, hence answer to part (b) is $2\pi\sqrt{\frac{l}{a}}$ with the value of *a* as defined in part (a).**
I-34	The ear-ring is similar to a simple pendulum and hence its time period shall be $T = 2\pi\sqrt{\frac{l}{g}}$, taking $g = 10$ m.s^{-2} the time period shall be $T = 2\pi\sqrt{\frac{0.03}{10}}0.344$s say 0.34 s considering SDs and **hence answer to part (a) is 0.34 s**.  When the lady is in a merry go round the effective acceleration $a = \sqrt{10^2 + \left(\frac{4^2}{2}\right)^2}$. This calculates to $a = \sqrt{196} \approx 10\sqrt{2}$. Accordingly, time period $T = 2\pi\sqrt{\frac{0.03}{10 \times 1.414}} = 0.289$ say 0.30 s considering SDs and **hence answer to part (a) is 0.30 s**.

I-35	This is case of oscillation of physical bodies and can be conceptualized from the figure shown here. Let a body of mass m and moment of inertia (MOI) I is hanging from a point O such that it is above P, the center of gravity (CG), by a length l . When it is hanging in a steady state its centre of gravity (CG) is at point P. When body is set into oscillation and is inclined by an angle θ with the vertical line passing through O, it will experience a torque $\tau = mg \times RQ_{length} = mgl\sin\vartheta$…(1). Considering the situation from the point of view of rotational dynamics $\tau = I\alpha$ …(2). Combining the (1) and (2) we get $I\alpha = mgl\sin\vartheta$. Since for small amplitude oscillation when $\sin\theta \to \theta\|_{\theta<<}$. Accordingly, $I\alpha = mgl\theta$, it leads to $\alpha = \frac{mgl}{I}\theta$ ...(3). Equation (3) is comparable to translational SHM where $a = \omega^2 x$…(4) , comparing (3) and (4) $a \to \alpha$ and $x \to \theta$ and therefore $\omega^2 = \frac{mgl}{I} \to \omega = \sqrt{\frac{mgl}{I}} = \frac{2\pi}{T}$. It leads to $= 2\pi\sqrt{\frac{I}{mgl}}$. Accordingly, for a physical body $\alpha = \omega^2\theta$ …(5) and time period of SHM of any physical body is $T = 2\pi\sqrt{\frac{I}{mgl}}$. …(6). This concept is applicable all cases in this problem.   **Case (a):** Moment of inertia a uniform bar of mass m and length $L = 1$m about its centre O is $I = \frac{mL^2}{12} = \frac{m}{12}$. Since the bar is hanging from point P, above by $l = 0.5 - 0.2 = 0.3$m and hence moment of inertia of the bar about P, by parallel axis theorem is $I_P = I + ml^2$, it simplifies into $I_O = \frac{m}{12} + \frac{9m}{100} = \frac{52m}{300}$, therefore time period would be $T = 2\pi\sqrt{\frac{\frac{52m}{300}}{m\times10\times0.3}}$. It reduces to $T = 2\pi\sqrt{\frac{\frac{52m}{300}}{m\times10\times0.3}} = 2\pi\sqrt{\frac{52}{900}} =$ 1.51s. **Hence answer of part (a) is 1,51 s.** **Case (b):** Moment of inertia of a circular ring about its centre O os $I = mr^2$ and therefore its MI about point of  hanging P is $I_P = I + mr^2 = 2mr^2$. Therefore, its time period would be $T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$. **Hence answer of part (b) is $2\pi\sqrt{\frac{2r}{g}}$s.**  **Case (c):** In case of a square plate using perpendicular axis theorem MI about its center O is $\boldsymbol{I = \frac{ma^2}{12} + \frac{ma^2}{12} = \frac{ma^2}{6}}$. Accordingly, MA about the point P shall be $I_P = \frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{3}ma^2$. Therefore, time period of SHM shall be $T = 2\pi\sqrt{\frac{\frac{2}{3}ma^2}{mg\left(\frac{a}{\sqrt{2}}\right)}}$. **Accordingly, $\boldsymbol{T = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}}}$ is the answer of part (c).** **Case (d):** In this case the disc will swing like a pan across its surface unlike that in case case (b) above. Accordingly, MI about point P is $I_P = \frac{I_O}{2} + m\left(\frac{r}{2}\right)^2 = \frac{mr^2}{2} + \frac{mr^2}{4} = \frac{3mr^2}{4}$. And hence time period $T = 2\pi\sqrt{\frac{\frac{3mr^2}{4}}{mg\frac{r}{2}}} = 2\pi\sqrt{\frac{3r}{2g}}$. **Thus, $\boldsymbol{T = 2\pi\sqrt{\frac{3r}{3g}}}$ is the answer of part (c).**  **N.B.:** In this problem MI of an object about different points of its plane have been very nicely articulated.
I-36	Time period of a pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$, here L is length of the string of a pendulum. But, for a physical object $T = 2\pi\sqrt{\frac{I}{mdg}}$. It is known that for a inform rod $I = \frac{ml^2}{12}$ and its CG is at the center and hence its distance from point of suspension is $d = \frac{l}{2}$, hence CG at the point of suspension $I_P = I + md^2 = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{3}$. Accordingly, for both the time periods to be equal $2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{\frac{ml^2}{3}}{mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2l}{3g}}$. It implies that $\frac{L}{g} = \frac{2l}{3g} \to L = \frac{2l}{3}$.

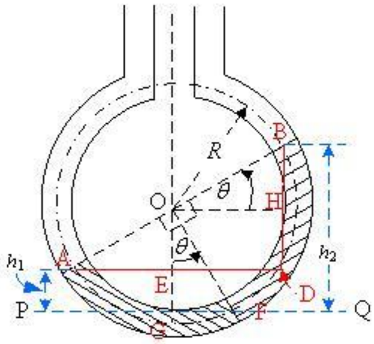
	Thus equivalent length of simple pendulum $\frac{2l}{3}$ is the answer.
I-37	Moment of inertia of a circular disc about an axis perpendicular to the its plane passing througj its center O is $I_O = mr^2$. Since disc is diametrically symmetrical and hence about vertical diameter as per perpendicular axis theorem $I = \frac{I_o}{2} = \frac{mr^2}{2}$. Let point of suspension P be at a distance *x* vertically above the center O, therefore, we have $I_P = I + mx^2 \to I_P = \frac{mr^2}{2} + mx^2$. Thus time period on suspension at P is $T = 2\pi\sqrt{\frac{I_P}{mgx}} = 2\pi\sqrt{\frac{\frac{mr^2}{2}+mx^2}{mgx}}$. Necessary condition for minimum value of T is $\frac{dT}{dx} = 0$ or $\frac{dT^2}{dx} = 0 \to 4\pi^2\frac{d}{dx}\left(\frac{r^2}{2x} + x\right) = 0 \to -\frac{r^2}{2x^2} + 1 = 0$. It leads to $x = \frac{r}{\sqrt{2}}$., and $T = 2\pi\sqrt{\frac{\frac{mr^2}{2}+m\left(\frac{r}{\sqrt{2}}\right)^2}{mg\frac{r}{\sqrt{2}}}} = 2\pi\sqrt{\frac{\frac{r^2}{2}+\frac{r^2}{2}}{g\frac{r}{\sqrt{2}}}} = 2\pi\sqrt{\frac{r\sqrt{2}}{g}}$. **Thus answer is time period and distance from center are $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$ and $\frac{r}{\sqrt{2}}$, respectively.**
I-38	 Moment of inertia of a hollow sphere is $I = \frac{2}{3}mr^2$ and MI of the sphere about tnhe point of suspension is $I_P = I + md^2 = \frac{2}{3}mr^2 + m(0.02 + 0.18)^2 = m\left(\frac{2}{3} \times 0.0004 + 0.04\right) = m\left(\frac{0.0008+0.12}{3}\right) = m\left(\frac{0.1208}{3}\right)$. Let angular displacement of the pendulum at P is θ. Then potential energy of the pendulum w.r.t the mean position A is $PE = mg\Delta h = mgR(1 - \cos\theta)$, here $R = d + r = 0.02 + 0.18 = 0.2$m and kinetic energy at P be $KE = \frac{1}{2}I\omega^2$. As per conservation of energy thotal energy is always constant and hence $mgR(1 - \cos\theta) + \frac{1}{2}I\omega^2 = \text{Const}$.....(1). On differentiating (1) $mgR\sin\theta \cdot \omega + I\omega\alpha = 0 \to \alpha = -\frac{mgR}{I}\theta$…(2). Here, $\omega = \frac{d\theta}{dt}$ and$\alpha = \frac{d\omega}{dt}$ and for SHM $\theta \ll$ hence $\sin\theta \to \theta$. The characteristic equation of anglar SHM is $\alpha = -\omega^2\theta$ accordingly $\omega = \sqrt{\frac{mgR}{I}}$. It leads to $\omega = \frac{2\pi}{T} = \sqrt{\frac{mgR}{I}} \to T = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{m\left(\frac{0.1208}{3}\right)}{m\times10\times0.2}} \to T = 2\pi \times \sqrt{\frac{0.1208}{6}} = 0.89$ s.. **Thus one part of the answer is 0.89 s.** Taking it as a simple pendulum with a point mass attached to a 0.18 m thread $T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{0.18}{10}} = 0.84$ s. Thus error in percentage is $\frac{0.89-0.84}{0.89} \times 100 = 4\%$. **Thus another part of the answer is 4%**
I-39	 **Case (a):** In this case moment of inertia of the ring about the nail P is $I = mr^2 + mr^2 = 2mr^2$. Time period of oscillation about the nail is $T = 2\pi\sqrt{\frac{I}{mgr}}$. Given value of $T = 2$ s. we have $2 = 2\pi\sqrt{\frac{2mr^2}{mgr}} \to \sqrt{r} = \frac{1}{\pi^2}\sqrt{\frac{10}{2}} = 0.5$m**. Hence answer of part (a) is 50 cm**. **Case (b):** Speed of the particle farthest away from p is diametrically opposite and hence $d = 2r = 1.0$ m. Since, given is the angular amplitude is 2^0 where velocity becomes zero velocity at mean position from conservation of energy would be $mgd(1 - \cos 2^0) = \frac{1}{2}I\omega^2 \to \omega = \sqrt{\frac{2gd}{I}(1 - \cos 2^0)}$. It solves into $\omega = \sqrt{\frac{4gr}{2r^2}(1 - \cos 2^0)} = \sqrt{\frac{2g}{r}(1 - \cos 2^0)} = \sqrt{\frac{2\times10}{0.5}(1 - \cos 2^0)} = 0.11$ rad/s. Hence velocity of the farthest point is $v = d\omega = 2r\omega = 2 \times 0.5 \times 0.11 = 0.11$ m/s. **Therefore, answer of part (b) is 11 cm/s.** **Case (c):** Acceleration of the particle in part (b) while passing through mean position shall be centripetal in

	nature and shall have a magnitude $\alpha = \omega^2 d = 2\omega^2 r = 2 \times 0.11^2 \times 0.5 = 1.2 \times 10^{-2}$m.s^{-2} . **Thus answer of part (c) is 1.2 cm.s^{-2}.** **Case (d):** Given the time period $T = 2$s, $\omega = \frac{2\pi}{T} = \pi$ rad/s. At the displacement equal to angular amplitude $\theta = 2^0$ magnitude of the angular acceleration is $\alpha = \omega^2\theta \rightarrow \alpha = \pi^2 \times \left(2 \times \frac{\pi}{180}\right) = \frac{\pi^3}{90}$ rad/s^{-2}. Since it has been asked to determine acceleration it implies translational acceleration which is same as tangential acceleration and hence $a = \alpha d = \frac{\pi^3}{90} \times 1 = \frac{\pi^3}{90} = 0.34$m/s^{-2}. **Hence, answer of part (d) is 34 cm/s^{-2}.** **N.B.:** This problem requires clear understanding of distinction in SHM of a simple pendulum and a rigid body.
I-40	Time period of a torsional pendulum is $T = 2\pi\sqrt{\frac{I}{k}}$...(1) here torsional constant of wire is given tho be k and moment of inertia of a uniform disc of radius r is $I = \frac{mr^2}{2}$ ...(2) and hence combining (1) and (2) $T = 2\pi\sqrt{\frac{\frac{mr^2}{2}}{k}} \rightarrow T^2 = 4\pi^2 \times \frac{mr^2}{2k} \rightarrow k = \frac{2\pi^2 mr^2}{T^2}$. **Thus answer is** $\frac{2\pi^2 mr^2}{T^2}$ 
I-41	Moment of Inertia of two small balls of mass m separated by a light rigid rod of length about its center O is $I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$...(1) Torsional energy stored in the suspension wire $TE = \frac{1}{2}k\theta^2$...(2) and when the rod passes through its mean position it will be converted into Kinetic Energy $KE = \frac{1}{2}I\omega^2$...(3) such that $\frac{1}{2}I\omega^2 = \frac{1}{2}k\theta^2 \rightarrow \omega = \sqrt{\frac{k}{I}}\theta$...(4).  The centripetal force on the rod would be $F_c = m\left(\frac{L}{2}\right)\omega^2 \rightarrow F_c = m\left(\frac{L}{2}\right)\left(\frac{k}{I}\right)\theta^2 = m\left(\frac{L}{2}\right)\left(k \times \frac{2}{mL^2}\right)\theta^2 = \frac{k}{L}\theta^2$. In addition gravitational force of the balls is $F_g = mg$. It is seen from the figure that both F_c and F_g are orthogonal and both the balls are attached to the rod, while being symmetrical to the wire. Hence, resultant force on the rod that supports the balls is $R = \sqrt{F_c^2 + F_g^2} = \sqrt{\left(\frac{k}{L}\theta^2\right)^2 + (mg)^2} = \sqrt{\frac{k^2}{L^2}\theta^4 + m^2g^2}$. **Thus answer is** $\sqrt{\frac{k^2}{L^2}\theta^4 + m^2g^2}$ **N.B.:** Since magnitude of resultant motion is always (+) ve hence correct representation in radical form and not in exponential form..
I-42	The two SHM can be expressed as $x_1 = A_1 \sin\omega_1 t$ and $x_2 = A_2\sin(\omega_2 t + \theta)$. It is given that $A_1 = 3.0$ cm and $A_2 = 4.0$ cm same period i.e. $\omega_1 = \omega_1 = \omega$, Therefore, $x = x_1 + x_2 = 3.0\sin(\omega t) + 4.0\sin(\omega t + \theta)$...(1) Further there are three cases – **Case (a) $\theta = 0^0$:** Using the value of θ in (1), $x = 3.0\sin(\omega t) + 4.0\sin(\omega t) = 7.0 \sin(\omega t)$ **Hence, amplitude is 7.0 cm, is answer of part (a)** **Case (b) $\theta = 60^0$:** Using the value of θ in (1), $x = 3.0\sin(\omega t) + 4.0\sin(\omega t + 60^0)$ This can be written as $x = 3.0\sin(\omega t) + 4.0(\sin(\omega t)\cos 60^0 + \cos(\omega t)\sin 60^0) = 3.0\sin(\omega t) + 4.0 \times \frac{1}{2}\sin\omega t + 4.0 \times \frac{\sqrt{3}}{2}\cos\omega t$. It leads to $x = 5.0\sin\omega t + 2\sqrt{3}\cos\omega t = A\sin\omega t\cos\alpha + A\cos\omega t\sin\alpha = A\sin(\omega t + \alpha)$, here, $A\cos\alpha = 5.0$ and $A\sin\alpha = 2.0\sqrt{3}$. It implies that $A = \sqrt{(A\sin\alpha)^2 + (A\cos\alpha)^2} = \sqrt{25 + 12} = \sqrt{37} = 6.08$. **Considering SDs answer to part (b) is 6.1 cm** **Case (c) $\theta = 90^0$:** Using the value of θ in (1), $x = 3.0\sin(\omega t) + 4.0\sin(\omega t + 90^0)$ This can be written as $x = 3.0\sin(\omega t) + 4.0(\sin(\omega t)\cos 90^0 + \cos(\omega t)\sin 90^0) = 3.0\sin\omega t + 4.0\cos\omega t$. It leads to $x = A\sin\omega t\cos\alpha + A\cos\omega t\sin\alpha = A\sin(\omega t + \alpha)$, here, $A\cos\alpha = 3.0$ and $A\sin\alpha = 4.0$ It implies that $A = \sqrt{(A\sin\alpha)^2 + (A\cos\alpha)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.0$. **Hence, answer of part (c) is 5.0 cm.**

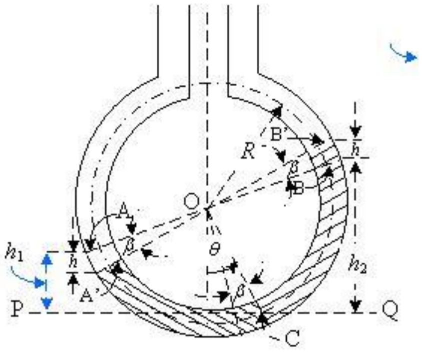
I-43	Let, First SHM be $x_1 = A\sin\omega t$, second SHM be $x_2 = A\sin(\omega t + 60) = A(\sin\omega t\cos 60 + \cos\omega t\sin 60)$ and $x_2 = A\sin(\omega t - 60) = A(\sin\omega t\cos 60 - \cos\omega t\sin 60)$. Therefore, resultant of three SHM shall be – $x_1 = A\sin\omega t$ $x_2 = A\sin\omega t\cos 60 + A\cos\omega t\sin 60$ $x_3 = A\sin\omega t\cos 60 - A\cos\omega t\sin 60$ $x = A(\sin\omega t + 2\sin\omega t\cos 60)$ $= A\left(\sin\omega t + 2\sin\omega t\frac{1}{2}\right)$ $= 2A\sin\omega t$. **Thus amplitude of the resultant shall be 2A and is answer of part (c).**
I-44	Resultant displacement of the two given SHMs shall be $x = x_1 + x_2 = 2.0\sin(100\pi t) + 2.0\sin\left(120\pi t + \frac{\pi}{3}\right)$. Both cases are solved separately here under - **Case (a)** $t = 0.0125$**:** Therefore, $x = 2.0\sin(100\times\pi\times 0.0125) + 2.0\sin\left(120\times\pi\times 0.0125 + \frac{\pi}{3}\right)$. This simplifies into $x = 2.0\sin(1.25\pi) + 2.0\sin\left(1.5\pi + \frac{\pi}{3}\right) = 2.0\sin\left(\pi + \frac{\pi}{4}\right) + 2.0\sin\left(\pi + \left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right)$. This further simplifies to $x = 2.0\left[\left(\sin\pi\cos\frac{\pi}{4} + \cos\pi\sin\frac{\pi}{4}\right) + \left(\sin\pi\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) + \cos\pi\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right)\right]$. In next stage it is $x = 2.0\left[-\frac{1}{\sqrt{2}} - \sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right] = -2.0\left[\frac{1}{\sqrt{2}} + \sin\frac{\pi}{2}\cos\frac{\pi}{3} + \cos\frac{\pi}{2}\sin\frac{\pi}{3}\right] = -2.0\left[\frac{1}{\sqrt{2}} + \frac{1}{2}\right] = -2.0\frac{1+\sqrt{2}}{2} = -2.41\text{cm}$. **Thus answer of part (a) is 2.41 cm** **Case (b)** $t = 0.025$**:** Therefore, $x = 2.0\sin(100\times\pi\times 0.025) + 2.0\sin\left(120\times\pi\times 0.025 + \frac{\pi}{3}\right)$. This simplifies into $x = 2.0\sin(2.5\pi) + 2.0\sin\left(3.0\pi + \frac{\pi}{3}\right) = 2.0\sin\left(2\pi + \frac{\pi}{2}\right) + 2.0\sin\left(2\pi + \left(\pi + \frac{\pi}{3}\right)\right)$. This further simplifies to $x = 2.0\left[\left(\sin 2\pi\cos\frac{\pi}{2} + \cos 2\pi\sin\frac{\pi}{2}\right) + \left(\sin 2\pi\cos\left(\pi + \frac{\pi}{3}\right) + \cos 2\pi\sin\left(\pi + \frac{\pi}{3}\right)\right)\right]$. In next stage it is $x = 2.0\left[1 + \sin\left(\pi + \frac{\pi}{3}\right)\right] = 2.0\left[1 + \sin\pi\cos\frac{\pi}{3} + \cos\pi\sin\frac{\pi}{3}\right] = 2.0\left[1 - \frac{\sqrt{3}}{2}\right] = -(2 - 1.732) - 0.368$ cm. **Using significant digits answer of part (b) is 0.37 cm.** **N.B.:** This requires to use standard table of trigonometric ratios, which get memorized without extra efforts just with problem solving .
I-45	This is a case of two SHMs of with same period and phase but with – (a) different magnitudes and displaced physically by an angle 45^0. Thus magnitude resultant motions shall be on lines similar to that of addition of vectors such that $R = \sqrt{x_0^2 + s_0^2 + 2x_0s_0\cos 45} = \sqrt{x_0^2 + s_0^2 + 2x_0s_0\frac{1}{\sqrt{2}}} = \sqrt{x_0^2 + s_0^2 + \sqrt{2}x_0s_0}$. **Hence answer is** $\sqrt{x_0^2 + s_0^2 + \sqrt{2}x_0s_0}$ **.** **N.B.:** Since magnitude of resultant motion is always (+) ve hence correct representation in radical form.
I-46	 The system as enumerated in the problem has been exhibited in the figure. The system is completely isolated from the surrounding it means there is no heat exchange and hence the process is adiabatic. When system is in equilibrium pressure on the gas in the cylinder $p = p_0 + \frac{Mg}{A}$...(1) Let piston is moved by a distance x from position of equilibrium (i.e. mean position)outwards then change in volume is $dv = A\cdot x$...(2) In the given conditions of thermal isolation it shall behave as per equation $v^\gamma = \text{Const}$. Taking partial derivative of the equation $d(pv^\gamma) = v^\gamma\cdot dp + p\cdot\gamma v^{\gamma-1}\cdot dv = 0 \rightarrow dp = -\gamma\frac{p}{v}dv$...(3) Combining (1), (2) and (3) we get $dp = -\gamma\frac{p_0 + \frac{Mg}{A}}{V_0}A\cdot x\Big\rvert_{dv\rightarrow 0}$ ...(4) Taking net force on the piston,

	we get $F = A\cdot dp = -\gamma\frac{A^2p_0+AMg}{V_0}x$. This can be written as $F = -Kx$ …(5) here, $K = \frac{A^2p_0+AMg}{V_0}$ is a constant. This equation (5) satisfies necessary condition of SHM : (a) Force is proportional to displacement from mean position, (b) Negative sign indicated that force is always directed towards mean position. In SHM characteristic equation of acceleration is $a = -\omega^2x$ hence $a = \frac{F}{M} = -\frac{K}{M}x \rightarrow a = -\frac{A^2p_0+AMg}{MV_0}x$. Hence, $\omega^2 = \frac{A^2p_0+AMg}{MV_0} \rightarrow \omega = \sqrt{\frac{A^2p_0+AMg}{MV_0}} = 2\pi f$ or frequency of oscillation is $f = \frac{1}{2\pi}\sqrt{\frac{A^2p_0+AMg}{MV_0}}$. **Hence answer is $\frac{1}{2\pi}\sqrt{\frac{A^2p_0+AMg}{MV_0}}$**
I-47	 Given yjat charge on the ring of radius 1m is $q = 1\times10^{-5}$ C and hence charge on an elements A and B of length dl is $dq = \frac{q}{2\pi\times1}dl = \frac{q}{2\pi}dl$. The distance of particle P of mass $m = 0.9\text{ g} = 9.0\times10^{-4}$kg is placed at $x = 1\text{ cm} = 1\times10^{-3}$m carries negative charge i.e. $q_p = 1\times10^{-6}$C. Therefore, force due to elemental charge at A on the charge on particle P would be $dF_{r1} = \frac{dq\times q_p}{4\pi\varepsilon_0 r^2} = \frac{\frac{q}{2\pi}dl\times q_p}{4\pi\varepsilon_0(1+x^2)} \approx \frac{qdl\times q_p}{2\pi(4\pi\varepsilon_0)}\Big\vert_{x\ll1}$. Vectorially $d\vec{F}_{r1} = d\vec{F}_{a1} + d\vec{F}_{b1}$, here, magnitude of axial component $dF_{a1} = dF_{a1}\sin\theta$ and $dF_{b1} = dF_{a1}\cos\theta$. Similar forces shall be produced by elemental charge at B, placed on the ring in a diametrically opposite position, except that $d\vec{F}_{b1} = -d\vec{F}_{b1}$ and hence would cancel out. But, the axial force, as is clear from the figure are in additive $dF_a = dF_{a1}\sin\theta + dF_{a2}\sin\theta \approx 2dF_{a1}\theta$, since $\theta \ll$. Using the given data $dF_a = 2\frac{(1\times10^{-5})dl\times(-1\times10^{-6})}{2\pi}\times(9\times10^9)\times\frac{x}{1} = -\frac{9x}{\pi}\times10^{-2}\times dl$. This is the combined effect of elements A and B with cumulative length $2dl$. Therefore, on integration net force due to total charge would be $F = dF_a\times\frac{2\pi\times1}{2} = -\frac{9x}{\pi}\times10^{-2}\times\pi = -9\times10^{-2}\times x$ and acceleration of the particle would be $a = \frac{F}{m} = -\frac{9\times10^{-2}\times x}{9\times10^{-4}} = -100x$. Characteristic equation of acceleration in SHM is $a = -\omega^2x \rightarrow \omega^2 = 100$. It implies that $\omega = \sqrt{100} = 10 = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{10} = 0.628 \approx 0.6$s. **Hence, considering SDs in the given data answer is 0.6 s**.
I-48	 Given that system of charged masses and non-conducting rod connecting them is placed horizontally. It implies in absence of electric field it will remain where it is placed. Placing it in electric field is like its superimposition on gravitational equilibrium. Thus $+q$ would experience a force $F_+ = +qE$ and charge $-q$ would experience a force $F_- = -qE$. This forms a force couple with a torque such that $\vec{\tau} = (\overrightarrow{qL\sin\theta})\times\vec{E} = -qLE\theta\hat{k}\vert_{\theta\ll}$…(1) Moment of inertia of the masses of two charges at the end of the rod is $I = M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$…(2) Thus as per rotational mechanics angular acceleration of the system is $\tau = I\alpha \rightarrow \alpha = \frac{\tau}{I}$…(3). Combining (1), (2) and (3) we have $\alpha = -\frac{qLE\theta}{\frac{ML^2}{2}} = -\frac{2qE}{mL}\theta$. Characteristic equation of angular acceleration in SHM is $\alpha = -\omega^2\theta$, hence $\omega^2 = \frac{2qE}{mL} \rightarrow \omega = \sqrt{\frac{2qE}{mL}} = \frac{2\pi}{T}$. Accordingly, time period of the oscillation is $T = 2\pi\sqrt{\frac{mL}{2qE}}$. The system is at maximum displacement at an angle $\theta$with the electric field while direction of electric field is mean position during SHM of the system. Hence, time taken by the system to align wuth the electric field from initial position will be$t = \frac{1}{4}T = \frac{1}{4}2\pi\sqrt{\frac{mL}{2qE}} = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$ s. **Hence answer is $\frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$ s.**
I-49	In a state of equilibrium horizontal liquid above line PQ balance each other by way of equal pressure exerted by liquid in both the limbs, as shown in the figure, such that $p + h_1\rho_1 = p + h_2\rho_2 \rightarrow \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} = \frac{\rho}{1.5\rho} = \frac{2}{3}$…(1). Here, is the atmospheric pressure above the liquid surface in both the limbs of the tube and would cancel out.

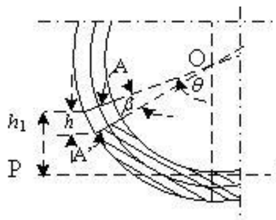
The angle of the diameter forming surfaces of the two liquids is $\tan\theta = \frac{BD}{AD} = \frac{h_2 - h_1}{2R\cos\theta} \rightarrow \sin\theta = \frac{h_2-h_1}{2R}$Mass of liquid of density $\rho_1$is $h_1 = OG - OE = R\cos\theta - R\sin\theta = R(\cos\theta - \sin\theta)$…(2). Also, we have $h_2 = h_1 + BH + HD = h_1 + BH + HD = R(\cos\theta - \sin\theta) + 2R\sin\theta$, it solves to $h_2 = R(\cos\theta + \sin\theta)$.
Combining (2) and (3) in (1) we have $\frac{R(\cos\theta - \sin\theta)}{R(\cos\theta + \sin\theta)} = \frac{2}{3} \rightarrow \cos\theta = 5\sin\theta$. It leads to $\tan\theta = \frac{1}{5} \rightarrow \theta = \tan^{-1}\frac{1}{5}$...(4).**Thus answer of part (a) is $\tan^{-1}\frac{1}{5}$**



Now liquid in equilibrium is displaced by a small angle β i.e. in anticlockwise direction. It results in fall of height of liquid column in left limb by h such that $h_1^{'} = h_1 - h$. Since the liquids are incompressible there will be consequential rise in height of liquid column on right by height by h such that $h_2^{'} = h_2 + h$. Net pressure difference at level of PQ is $h_2^{'} - h_1^{'} = (h_2 + h) - (h_1 - h) = (h_2 - h_1) + 2h$.



Since liquid in the two limbs are of different densities and hence change of pressure on the left due to fall of level is $\Delta p_L = \rho_1 g(-h) = -1.5\rho g h$ and change of pressure on the right limb due to rise of level $\Delta p_R = \rho_2 g\rho(h) = g\rho h$. Therefore, $\Delta p = \Delta p_R - \Delta p_L = g\rho h - (-1.5g\rho h) = 2.5\rho g h$.



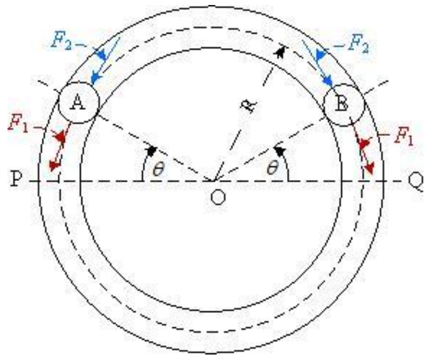
Let A is the cross-section of the tube, the torque about O is $\vec{\tau} = \vec{R} \times \vec{F}$, it solves into $\vec{\tau} = \vec{R} \times (\overline{\Delta p A}) = -2.5\rho g h A R\hat{k}$. Here, $h = R(\sin\theta - \sin(\theta - \beta))$, as shown in the figure geometrically. Further, $\sin\theta - \sin(\theta-\beta) = \sin\theta - (\sin\theta\cos\beta - \cos\theta\sin\beta)$. Since, β is small hence $\sin\theta - \sin(\theta - \beta) = \sin\theta - (\sin\theta - \beta\cdot\cos\theta) = \beta\cdot\cos\theta$. Thus, $\vec{\tau} = -2.5\rho g(R\beta\cos\theta)AR\hat{k} = -\left(\frac{5}{2}\rho g A R^2\cos\theta\right)\beta\hat{k}$. Ot leads to $\tau = -K\beta$ …(5).

Here, in the expression of torque angular displacement β is in clockwise direction and unit vector $(-\hat{k})$ indicates torque is in anticlockwise direction and thus it satisfies conditions of SHM. Here, coefficient $K = 2.5\rho g A R^2\cos\theta$ …(6) is a constant including θ determined in part (a) on the answer.

In rotatory motion angular acceleration α is such that $\tau = I\alpha$ …(7). In the given system $I = m_1R^2 + m_2R^2$ since mass of liquid is distributed about O at a distance R, and $m_1 = \frac{\pi}{4}RA\rho_1 = 1.5 \times \frac{\pi}{4} \times RA$ and $m_2 = \frac{\pi}{4}RA\rho_2 = \frac{\pi}{4}RA$. Thus, moment of inertia $I = \left(1.5RA\rho\left(\frac{\pi}{2} - \theta\right)\right)R^2 + \left(RA\rho\left(\frac{\pi}{2} + \theta\right)\right)R^2 = \frac{5\pi}{4}\rho A R^3$…(8).

Thus combining (5), (6), (7) and (8) we have $\left(\frac{5\pi}{4}\rho A R^3\right)\alpha = -\left(\frac{5}{2}\rho g A R^2\cos\theta\right)\beta$. Here, $\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{5}{\sqrt{26}}$, using the value $\tan\theta = \frac{1}{5}$ arrived at (4). Thus, $\alpha = -\frac{\frac{5}{2}\rho g A R^2 \times \sqrt{\frac{25}{26}}}{\frac{5\pi}{4}\rho A R^3}\beta = -0.98 \times \frac{2g}{\pi R}\beta = -0.624\frac{g}{R}\beta$. Thus. As per characteristic equation of SHM $= -\omega^2\beta$, accordingly $\omega^2 = 0.624\frac{g}{R} \rightarrow \omega = \sqrt{0.624\frac{g}{R}} = \frac{2\pi}{T}$or, $T = \frac{2\pi}{\sqrt{0.624\frac{g}{R}}} = 2\pi\sqrt{\frac{R}{0.624g}}$. Taking $g = 10$ m.s^{-2}, we have $T = 2\pi\sqrt{\frac{R}{6.24}} = 2.5\sqrt{R}$ s. **Hence answer of part (b) is $\mathbf{2.5\sqrt{R}}$ s.**

N.B.: Here $h_1 = R(1 - \sin\theta)$ therefore when θ undergoes a small change say β then $\Delta h_1 = -(R\cos\theta)\,\beta$. Therefore, change in height of C where $\theta \rightarrow \frac{\pi}{2}$, then $\Delta h_C \rightarrow -\left(R\cos\frac{\pi}{2}\right)\,\beta = 0$. Accordingly, in the elaborations above, changes in level of C, when liquid in equilibrium displaced by small angle β, have been ignored.

I-50	Initial length of both springs $L_0 = 0.6\pi = \pi R$ since radius of the circle on which balls move is $R = 0.6$ m.. It also implies that the balls are point masses.  When each of the balls is moved towards each other by an angle θ one spring is stretchered by$\Delta l = \alpha R = 2 \times \theta \times R = 2R\theta$ rad, while other spring is also compressed by $\Delta l = \alpha R = 2 \times \theta \times R = 2R\theta$ Thus, restoration force exerted by stretched spring $F_1 = k \times \Delta l = 2kR\theta$ N and likewise the spring getting compressed exerts force $F_2 = k \times \Delta l = 2kR\theta$ N. Both of these forces are unidirectional, hence net force on the balls is $F = F_1 + F_2 = 4kR\theta$. Thus, torque exerted by the force about the center of the path O would be $\vec{\tau} = \vec{R} \times \vec{F} = -4kR\theta \cdot R = -4kR^2\theta$ N.m. ..(1). Here, the angular acceleration is proportional to angular displacement and (-) sign indicates that it is in a direction opposite to the angular displacement and thus it satisfies conditions of SHM of the balls. Now, moment inertia of the ball about the center O is $I = mR^2$ and in rotatory motion $\tau = I\alpha = mR^2\alpha$…(2). Thus combining (1) and (2) we have $mR^2\alpha = -4kR^2\theta \rightarrow \alpha = -\frac{4kR^2}{mR^2}\theta \rightarrow \alpha = -\frac{4k}{m}\theta$…(3) Here, the angular acceleration is proportional to angular displacement and (-) sign indicates that it is in a direction opposite to the angular displacement and thus it satisfies conditions of SHM of the balls. Characteristic equation if SHM is $\alpha = -\omega^2\theta$…(4). Combining (3) and (4) $\omega^2 = \frac{4k}{m} \rightarrow \omega = 2\sqrt{\frac{0.1}{0,1}} = 2$ rad.s^{-1}. Hence, frequency of oscillation of the spring is $\omega = 2\pi f \rightarrow 2 = 2\pi f \rightarrow f = \frac{1}{\pi}$ Hz. **Thus answer of part (a) is $\frac{1}{\pi}$ Hz.** When masses are positioned at $\theta = \frac{\pi}{2}$ energy stored in compressed spring $PE_1 = \frac{1}{2}k\left(\left(R\frac{\pi}{6}\right) \times 2\right)^2 = \frac{k\pi^2R^2}{18}$ and energy stored in stretched spring is $PE_2 = \frac{1}{2}k\left(\left(R\frac{\pi}{6}\right) \times 2\right)^2 = \frac{k\pi^2R^2}{18}$. Hence, total potential energy in the two springs is $PE_{\frac{\pi}{2}} = PE_1 + PE_2 = 2 \times \frac{k\pi^2R^2}{18} = \frac{k\pi^2R^2}{9}$. At this position both the masses are released from state of rest and hence $KE_{\frac{\pi}{2}} = 0$. Thus total energy of the system is $TE = KE_{\frac{\pi}{2}} + KE_{\frac{\pi}{2}} = \frac{k\pi^2R^2}{9} = \frac{0.1\times\pi^2\times0.06^2}{9} = 3.95 \times 10^{-4}$J or 4.0×10^{-4}J. **Thus answer of part (c) is $\mathbf{4.0 \times 10^{-4}}$J.** At $\theta = 0$ when the two ball A and B are two ends of diameter PQ, springs are at natural length and hence $PE_0 = 0$ and $KE_0 = TE = 4.0 \times 10^{-4} = 2 \times \left(\frac{1}{2}mv^2\right) \rightarrow v = \sqrt{\frac{4.0\times10^{-4}}{0.1}} = \sqrt{40} \times 10^{-2} = 6.3 \times 10^{-2}$m.s^{-1}. **Hence, answer of part (b) is $\mathbf{6.3 \times 10^{-2}}$m.s^{-1}.** **N.B.:** In this velocity of the masses at diametrically opposite positions i.e. $\theta = 0$, when the ball are released from speed cannot be determined from equations of motion since acceleration is changing with change in length of spring. Hence, principle of conservation of energy has been applied.

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Problems are meant to be solved; every solution open doorway to new problems.
This is an endless journey to discovery of nature.
We are, what we are, because of rigorous efforts of countless persons.

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