Wave and Motion : Objective and Subjective Questions (Typical)

No of Questions: 50 | **Time Allotted: 6 Hours**

All questions are compulsory

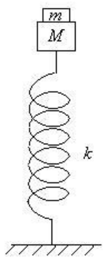
[Note: a. Figures are conceptual only and not to the scale**]**

[b. Solutions may be taken up in Two parts such that Part I- 1 to 25 and Part II- 26-50**]**

[c. It is advised to attempt question under examination conditions**]**

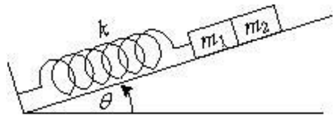
Q-01 A small block of mass m is kept on a bigger block of mass M which is attached to a vertical spring of spring constant k as shown in figure. The system oscillates vertically.

(a) Find the resultant force on the smaller block when it is displaced through a distance x above its equilibrium position.
(b) Find the normal force on the smaller block at this position. When is this force smallest in magnitude?
(c) What can be the maximum amplitude with which the two blocks may oscillate together?



Q-02 The block of mass m_1 shown in figure is fastened to the spring and the block of mass m_2 is placed against it.

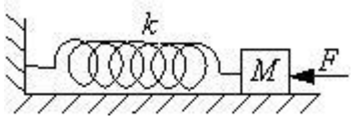
(a) Find the compression of the spring in the equilibrium position.
(b) The blocks are pushed a further distance $\left(\frac{2}{k}\right)(m_1+m_2)g\sin\theta$ against the spring and released. Find the position where the two blocks separate.
(c) What is the common speed of blocks at the time of separation?



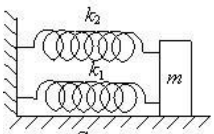
Q-03 In the figure k= 100 N/m, M = 1 kg and F = 10 N.

(a) Find the compression of the spring in the equilibrium position.
(b) A sharp blow by some external agent imparts a speed of 2 m/s to the block towards left. Find the sum of potential energy of the spring and the kinetic energy of the block at this instant.
(c) Find the time period of the resulting simple harmonic motion. Find the amplitude.
(d) Find the amplitude
(e) Write the potential energy of the spring when the block is at the left extreme.
(f) Write the potential energy of the spring when the block is at rest.

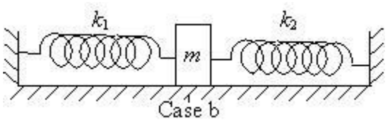
Answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy.



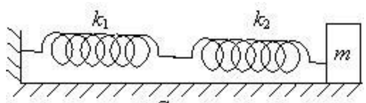
Q-04 Find the time period of the oscillation of mass m in the figure shown in case (a), (b) and (c). What is the equivalent spring constant of the pair of springs in each case?



Case a



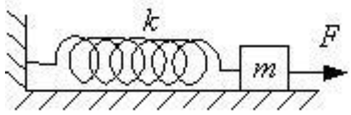
Case b

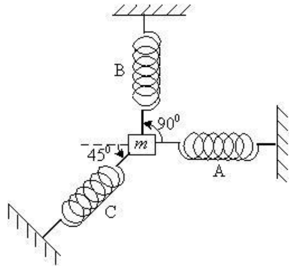
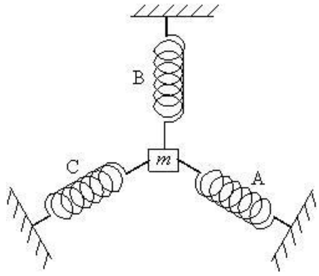
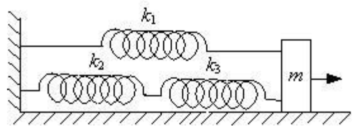
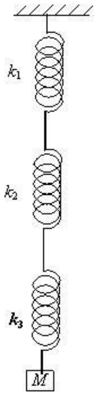
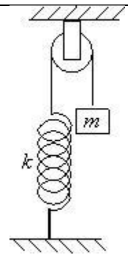
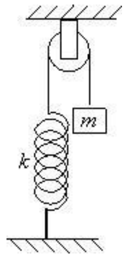


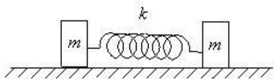
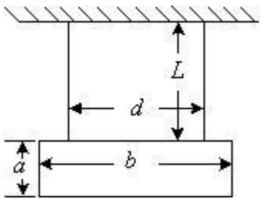
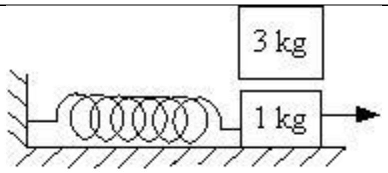
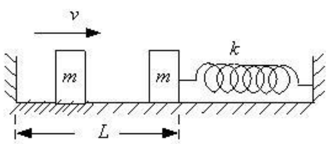
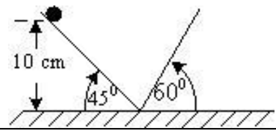
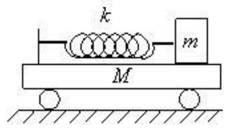
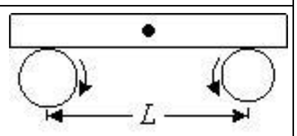
Case c

Q-05 The spring shown in the figure is un-stretched when a man starts pulling on the cord. The mass of the block is m. If the man exerts a constant force F, find –

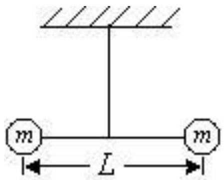
(a) the amplitude and time period of the motion of the block
(b) the energy stored in the spring when the block passes through the equilibrium position and

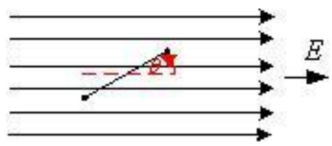
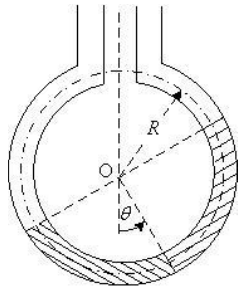
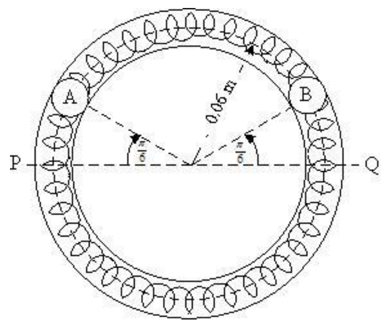


	(c) The kinetic energy of the block at this position.
Q-06	A particle of mass *m* is attached to three springs A, B, and C of equal force constants *k* as shown in the figure. If the particle is pushed slightly against the spring C and released, find the time period of the oscillation. 
Q-07	A particle of mass *m* is attached to three springs A, B, and C of equal force constants *k* as shown in the figure. Angle between each pair of springs is 120^0. If the particle is pushed slightly against the spring C and released, find the time period of the oscillation 
Q-08	Springs shown in the figure are all un-stretched in the beginning when a man starts pulling the block. The man exerts a constant force *F* on the block. Find the amplitude and the frequency of the motion of the block when the force is released. 
Q-09	Find the elastic potential energy stored in each spring shown in the figure when the block is in equilibrium. Also find the time period of vertical oscillation of the block. 
Q-10	The string, the spring and the pulley shown in the figure are light. Find the time period of the mass *m*. 
Q-11	The string, the spring and the pulley shown in the figure are light. If pulley has a moment of inertia *I* about its axis and the string does not slip over it.Find the time period of the mass m . 

Q-12	Consider the situation shown in the figure. Show that of the blocks are displayed slightly in opposite directions and released, they will execute simple harmonic motion. Calculate the time period. 
Q-13	A rectangular plate of side *a* and *b* is suspended from a ceiling by the parallel strings if length *L* each as shown in the figure. The separation between the strings is *d*. The plate is displaced slightly in its plane keeping the strings tight. Show that it will execute simple harmonic motion. Find the time period. 
Q-14	 A block of $m_0 = 1\,\text{kg}$ block is executing simple harmonic motion of amplitude 0.1 m on a smooth horizontal surface under the restoring force of a spring of spring constant 100 N/m. A block of mass 3 kg is gently placed on it the instant it passes through mean position. Assuming that the two blocks move together, find the frequency and the amplitude if the motion,
Q-15	The left block in the figure moves at a speed *v* towards the right block placed in equilibrium. All collisions to rake place are elastic and the surfaces are frictionless. Show that the motions of the two blocks are periodic. Find the time period of these periodic motions. Neglect the widths of the blocks. 
Q-16	Find the time period of the motion of the particle shown in the figure. Neglect the small effect of the bend near the bottom. 
Q-17	All the surfaces shown in the figure are frictionless. The mass of the car is *M*, that of the block is *m* and the spring has spring constant *k*. Initially, the car and the block are at rest and the spring is stretched through a length x_0 when the system is released. (a) Find the amplitude of the simple harmonic motion of the block and of the car as seen from the road. (b) Find the time period(s) of the two simple harmonic motions. 
Q-18	A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions as shown in the figure. The separation between the wheel is *L*. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released. 
Q-19	A pendulum having time period equal to two seconds is called a seconds pendulum. Those used in pendulum clocks are of this type. Find the length of a second pendulum at a place where $g = \pi^2\,\text{m.s}^{-2}$.
Q-20	The angle made by the spring of a simple pendulum with the vertical depends on time as $\theta = \frac{\pi}{90}\sin\left[\left(\pi s^{-1}\right)t\right]$, Find length of the pendulum if $g = \pi^2\,\text{m.s}^{-2}$.
Q-21	The pendulum of a certain clock has time period 2.04 s. How fast or slow does he clock run during 24 Hours?
Q-22	A pendulum clock giving correct time at a place where $g = 9.800\,\text{m.s}^{-2}$ is taken to another place where it loses 24 seconds during 24 hours. Find the value of *g* at new place.
Q-23	A simple pendulum is constructed by hanging a heavy ball by a 5.0 m long string. It undergoes small oscillations. (a) How many oscillations does it make per second?

	(b) What will be the frequency if the system is taken on the moon where acceleration due to gravitation of the moon is 1.67 $m.s^{-2}$?
Q-24	The maximum tension in the string of an oscillating pendulum is double of the minimum tension. Find the angular amplitude.
Q-25	A small block oscillates back and forth on a smooth concave surface of radius *R* as shown in the figure. Find time period of oscillation. 
Q-26	A spherical ball of mass *m* and radius *r* rolls without slipping on a rough concave surface of large radius *R*. It makes small oscillations about the lowest point. Find the time period.
Q-27	A simple pendulum of length 40 cm is taken inside a deep mine. Assume for the time being that the mine is 1600 km deep. Calculate the time period of pendulum there. Radius of the earth = 6400 km.
Q-28	Assume that a tunnel is dug across the earth (radius = *R*) passing through its center. Find the time a particle takes a cover the length of the tunnel if- (a) It is projected into the tunnel with a speed of $\sqrt{gR}$, (b) It is released from a height R above the tunnel (c) It is thrown vertically upward along the length of tunnel with a speed of $\sqrt{gR}$.
Q-29	Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance *R*/2 from the earth's center where *R* is the radius of the earth. The wall of the tunnel is frictionless. (a) Find the gravitational force exerted by the earth on a particle of mass *m* placed in the tunnel at a distance *x* from the center of the tunnel. (b) Find the component of this force along the tunnel and perpendicular to the tunnel. (c) Find the normal force exerted by the wall on the particle. (d) Find the resultant force on the particle (e) Show that the motion of the particle in the tunnel is simple harmonic and find the time period..
Q-30	A simple pendulum of length *l* is suspended through the ceiling of an elevator. Find the time period of small oscillations of the elevator – (a) Is going up with an accelerations a_0 (b) Is going down with an acceleration a_0 and (c) Is moving with a uniform velocity.
Q-31	A simple pendulum of length 1feet suspended from the ceiling of an elevator takes $\frac{\pi}{3}$ seconds to complete one oscillation. Find acceleration of the elevator.
Q-32	A simple pendulum fixed in a car has a time period of 4 seconds when the car is moving uniformly on a horizontal road. When the accelerator is pressed, the time period changes to 3.99 seconds. Making an approximate analysis, find the acceleration of the car.
Q-33	A simple pendulum of length *l* is suspended from the ceiling of a car moving with a speed *v* on a circular horizontal road of radius *r*. (a) Find the tension in the string when it is at rest with respect to the car. (b) Find the time period of small oscillation.

Q-34	The ear-ring of a lady shown in the figure has a 3 cm long light suspension wire. (a) Find the time period of small oscillation if the lady is standing on the ground. (b) The lady now sits in a merry-go-round moving at 4 m/s in a circle of radius 2 m. Find the time period of the small oscillations of the ear-ring. 
Q-35	Find the time period of small oscillations of the following systems. (a) A meter stick suspended through the 20 cm mark (b) A ring of mass *m* and radius *r* suspended through a point on its periphery. (c) A uniform square plate of edge *a* suspended through a corner. (d) A uniform disc of mass m and radius suspended through a point *r*/2 from the center.
Q-36	A uniform rod of length *l* is suspended by an end and is made to undergo small oscillations. Find the length of the simple pendulum having the time period equal to that of the rod..
Q-37	A uniform disc of radius *r* is to be suspended through a small hole made in the disc. Find the minimum possible time period of the disc for small oscillations. What should be the distance of the hole from the center for it to have minimum time period?
Q-38	A hollow sphere of radius 2 cm is attached to an 18 cm long thread to make a pendulum. Find the time period of oscillation of this pendulum. How does it differ from the time period calculated using the formula for a simple pendulum?
Q-39	A closed circular wire hung on a nail in a wall undergoes small oscillations of amplitude 2^0 and time period 2 s. Find – (a) the radius of the circular wire, (b) the speed of the particle farthest away from the point of suspension as it goes through its mean position, (c) the acceleration of this particle as it goes through its mean position (d) the acceleration of this particle when it is at an extreme position.
Q-40	A uniform disc of mass *m* and radius r is suspended through a wire attached to its center. If the time period of the torsional oscillations be *T*, what is the torsional constant of the wire?
Q-41	Two small balls, each of mass *m*, are connected by a light rigid rod of length *L* as shown in the figure. The system is suspended from its center by a thin wire of torsional constant *k*. The rod is rotated about the wire through an angle θ_0 and released. Find the tension in the rod as the system passes through the mean position. 
Q-42	A particle is subjected to two simple harmonic motions of the same period in the same direction. The amplitude of the first motion is 3.0 cm and that of the second is 4.0 cm. Find the resultant amplitude if the phase difference between the motions is – (a) 0^0, (b) 60^0, (c) 90^0.
Q-43	Three simple harmonic motions of equal amplitude A and equal time period in the same direction combine. The phase of the second motion is 60^0 ahead of the first and phase of the third motion is 60^0 ahead of the second. Find amplitude of the resultant motion.
Q-44	A particle is subjected to two simple harmonic motions given by $x_1 = 2.0\sin(100\pi t)$ and $x_2 = 2.0\sin\left(120\pi t + \frac{\pi}{3}\right)$, where *x* is in centimeter and time *t* is in second. Find the displacement of the particle at – (a) $t = 0.0125$ and (b) $t = 0.025$

Q-45	A particle is subjected to two simple harmonic motions, one along X-axis and the other on a line making an angle of 45^0 with X-axis. The two motions are given by (a) $x = x_0 \sin \omega t$ and (b) $s = s_0 \sin \omega t$. Find the amplitude of the resultant motion.
Q-46	An Ideal gas is condensed in a vertical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional areas A. Atmospheric pressure is p_0 and when piston is in equilibrium the volume of the gas is V_0. The piston is now displaced slightly from the equilibrium position. Assuming that the system is completely isolated from the surroundings, show that the piston executes simple harmonic motion and find the frequency of the oscillation.
Q-47	A thin ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having a negative charge 1×10^{-6} C is placed on the axis at a distance of 1 cm from the center of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of the oscillation.
Q-48	A point particle of mass M attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particles carry charges $+q$ and $-q$ respectively. The arrangement is held horizontally in a region of uniform electric field E such that the rod makes a small angle θ (say 5 degrees) with the field direction. Find an expression for the minimum time needed for the rod to become parallel to the field after it is set free. 
Q-49	Two non-viscous, incompressible and immiscible liquids of densities ρ and 1.5ρ are poured into the two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in the figure. Each liquid occupies one fourth of the circumference of the tube. (a) Find the angle θ that the radius to interface makes with the vertical in equilibrium position. (b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations. 
Q-50	Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the form of a circle as shown in the figure. The pipe is fixed in horizontal plane. The centers of the ball can move in a circle of radius 0.06 m. Each spring has a natural length of $0.06\,\pi$ m and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \frac{\pi}{6}$ rad with respect to the diameter PQ of the circle (as shown in the figure) and released from rest – (a) Calculate the frequency of oscillation of the ball B (b) Find the speed of ball A when A and B are two ends of diameter PQ. (c) What is the total energy of the system, 

—00—