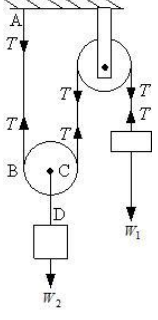



Wave and Motion : Illustrations Vibrations In Strings and Sound Waves–**Objective Questions (Typical)**

I-01	<p>Displacement of a particle executing sine wave in a medium is shown graphically in the figure and is $y = A \sin\left(\omega\left(t - \frac{x}{v}\right)\right)$, here x is the position of particle along direction of propagation of wave is shown on X-axis, and displacement of particle from its mean position is y. Therefore, speed of particle is $v = \frac{dy}{dt} = A\omega \cos\left(\omega\left(t - \frac{x}{v}\right)\right)$.</p> <p>Let x_1 and x_2 are the distances particles having same speed at same instant, say $v_1 = A\omega \cos\left(\omega\left(t - \frac{x_1}{v}\right)\right)$ and $v_2 = A\omega \cos\left(\omega\left(t - \frac{x_2}{v}\right)\right)$. Speed implies that their magnitudes are same irrespective of the direction. Hence, $\omega \cos\left(\omega\left(t - \frac{x_1}{v}\right)\right) = \omega \cos\left(\omega\left(t - \frac{x_2}{v}\right)\right)$. Since velocity is a trigonometric function and hence for $v_1 = v_2$ it leads to $\theta_2 = \theta_1 + n\pi$, where n is an integer. Thus, $\omega\left(t - \frac{x_2}{v}\right) = \omega\left(t - \frac{x_1}{v}\right) + n\pi \Rightarrow \frac{2\pi}{T}\left(t - \frac{x_2}{v}\right) = \frac{2\pi}{T}\left(t - \frac{x_1}{v}\right) + n\pi \Rightarrow \frac{2}{T} \times \frac{x_2}{v} = \frac{2}{T} \times \frac{x_1}{v} - n$. It, further, solves into $\Delta x = x_1 - x_2 = \frac{nvT}{2} = \frac{n\lambda}{2}$. Smallest integer is since 1 and the smallest $\Delta x = \frac{\lambda}{2}$. Hence answer is (c).</p> <p>N.B.: Analytical approach is though longer, leads to correct answer, which by simple observation of graph may lead to interpretation errors.</p>
I-02	<p>Displacement of a particle executing sine wave in a medium is shown graphically in the figure and is $y = A \sin\left(\omega\left(t - \frac{x}{v}\right)\right)$, here x is the position of particle along direction of propagation of wave is shown on X-axis, and displacement of particle from its mean position is y.</p> <p>Let x_1 and x_2 are the position of the particles having Zero displacement at same instant, say $y_1 = A\omega \sin\left(\omega\left(t - \frac{x_1}{v}\right)\right)$ and $y_2 = A\omega \sin\left(\omega\left(t - \frac{x_2}{v}\right)\right)$. It implies $\sin\left(\omega\left(t - \frac{x_1}{v}\right)\right) = \sin\left(\omega\left(t - \frac{x_2}{v}\right)\right)$. Since displacements are Zero $\omega\left(t - \frac{x_2}{v}\right) = \omega\left(t - \frac{x_1}{v}\right) + n\pi \Rightarrow \frac{2\pi}{T}\left(t - \frac{x_2}{v}\right) = \frac{2\pi}{T}\left(t - \frac{x_1}{v}\right) + n\pi$. It, further, leads to $\frac{2}{T} \times \frac{x_2}{v} = \frac{2}{T} \times \frac{x_1}{v} - n \Rightarrow \Delta x = x_1 - x_2 = \frac{nvT}{2} = \frac{n\lambda}{2}$. Since, smallest integer is since 1 and the smallest $\Delta x = \frac{\lambda}{2}$. Hence answer is (c).</p>
I-03	<p>Each of the given options are being examined for the equation given therein -</p> <p>Option (a): It is the case of displacement of particle along X-axis for a single wave propagating along Y-axis, it is the case of a transverse wave.</p> <p>Option (b): It is the case of displacement of particle along X-axis for a single wave propagating along X-axis, it is the case of a longitudinal wave.</p> <p>Option (c): The displacement equation can be written as $x = A \sin ky \cos \omega t = \frac{A}{2} [\sin(ky + \omega t) + \sin(ky - \omega t)]$. Thus it is a combination of transverse Two waves travelling along Y-axis and</p>

	<p>not a single wave.</p> <p>Option (d):The displacement equation can be written as $x = A \cos ky \sin \omega t = \frac{A}{2} [\sin(\omega t + ky) + \sin(\omega t - ky)]$. Thus it is a combination of transverse Two waves travelling along Y-axis and not a single wave.</p> <p>From the above analysis, it is clear that Option (c) and (d) are of combination of Two waves and not a single wave, and are therefore ruled out.</p> <p>The remaining Two options are (a) and (b) out of which Option (b) is ruled out as it is travelling along X-axis as discussed in the analysis.</p> <p>The last option is of a single wave travelling along Y-axis. Hence answer is option (a)</p> <p>N.B.: Here questions asks "...represents a sine wave...", needs to be noted carefully that it asks for a single wave and not a combination of waves. This is the key to right answer.</p>
I-04	<p>The given equation $y = A \sin^2(kx - \omega t) \Rightarrow y = \frac{A}{2} (1 - \cos 2(kx - \omega t)) \Rightarrow y = \frac{A}{2} \left(1 - \cos 2\omega \left(\frac{kx}{\omega} - t\right)\right)$.</p> <p>This equation in final form has magnitude $\frac{A}{2}$ and angular velocity $\omega' = 2\omega$. Since, $\omega' = 2\pi f' \Rightarrow f' = \frac{\omega'}{2\pi}$. It leads to $f' = \frac{2\omega}{2\pi} \Rightarrow f' = \frac{\omega}{\pi}$. Thus, the given equation represents Amplitude $\frac{A}{2}$, frequency $\frac{\omega}{\pi}$ and is matching with option (b). Hence answer is Option (b).</p> <p>N.B.: The given equation has to be analyzed to determine amplitude and frequency and then these values are to be matched with the given options. Therefore, it is a case of straight matching and does not need analysis of each option</p>
I-05	<p>In this case properties of waves given in each option are being checked to determine whether it is a mechanical wave. Accordingly,</p> <p>Option (a): Radio wave is an electromagnetic wave and medium for it propagation is not a necessity. Hence it is not a mechanical wave.</p> <p>Option (b):X-ray is an electromagnetic wave and medium for it propagation is not a necessity. Hence it is not a mechanical wave.</p> <p>Option (c): Light wave is an electromagnetic wave and medium for it propagation is not a necessity. Hence it is not a mechanical wave.</p> <p>Option (d): Sound wave is not an electromagnetic wave and hence medium for it propagation is a necessity. Hence it is a mechanical wave.</p> <p>Thus option (d) is the answer.</p>
I-06	<p>Frequency of the wave is the property of the medium of propagation of the wave. Passing boat is a disturbance and hence wave produced in the pond would characteristic to the water in the pond which in calm state is experiencing frequency ν. Accordingly, passing of boat may change amplitude but not the frequency. Hence, answer is option (a).</p>

I-07	<p>Velocity of wave propagation in string is $v = \sqrt{\frac{F}{\mu}}$, here F is the tension in the string and μ is mass of string per unit length. The given data is analyzed in the table below –</p> <table border="1" data-bbox="236 331 1450 757"> <thead> <tr> <th>Particulars</th> <th>String-A</th> <th>String-B</th> </tr> </thead> <tbody> <tr> <td>Tension</td> <td>F</td> <td>F</td> </tr> <tr> <td>Density (Same Material)</td> <td>ρ</td> <td>ρ</td> </tr> <tr> <td>Radius of string</td> <td>$2r$</td> <td>r</td> </tr> <tr> <td>Area of cross-section of string</td> <td>$\pi(2r)^2 = 4\pi r^2$</td> <td>$4\pi r^2$</td> </tr> <tr> <td>Mass per-unit length of string (μ)</td> <td>$\mu_A = 4\pi r^2 \rho$</td> <td>$\mu_B = \pi r^2 \rho$</td> </tr> <tr> <td>Velocity of wave</td> <td> $v_A = \sqrt{\frac{F}{\mu_A}} = \sqrt{\frac{F}{4\pi r^2 \rho}}$ $v_A = \frac{1}{2r} \sqrt{\frac{F}{\pi \rho}}$ </td> <td> $v_B = \sqrt{\frac{F}{\mu_B}} = \sqrt{\frac{F}{\pi r^2 \rho}}$ $v_B = \frac{1}{r} \sqrt{\frac{F}{\pi \rho}}$ </td> </tr> </tbody> </table> <p>Accordingly, $\frac{v_A}{v_B} = \frac{\frac{1}{2r} \sqrt{\frac{F}{\pi \rho}}}{\frac{1}{r} \sqrt{\frac{F}{\pi \rho}}} = \frac{1}{2}$. This value matches with that in option (a), Hence, answer is option (a).</p>	Particulars	String-A	String-B	Tension	F	F	Density (Same Material)	ρ	ρ	Radius of string	$2r$	r	Area of cross-section of string	$\pi(2r)^2 = 4\pi r^2$	$4\pi r^2$	Mass per-unit length of string (μ)	$\mu_A = 4\pi r^2 \rho$	$\mu_B = \pi r^2 \rho$	Velocity of wave	$v_A = \sqrt{\frac{F}{\mu_A}} = \sqrt{\frac{F}{4\pi r^2 \rho}}$ $v_A = \frac{1}{2r} \sqrt{\frac{F}{\pi \rho}}$	$v_B = \sqrt{\frac{F}{\mu_B}} = \sqrt{\frac{F}{\pi r^2 \rho}}$ $v_B = \frac{1}{r} \sqrt{\frac{F}{\pi \rho}}$
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I-08	<p>The system is in equilibrium and in that condition $F_2 = 2F_1$. Since strings are of same material and cross-section and $\mu_1 = \mu_2 = \mu$. Hence, velocity of transverse wave in string AB is $v_1 = \sqrt{\frac{F_1}{\mu}}$ and string CB $v_2 = \sqrt{\frac{F_2}{\mu}} = \sqrt{\frac{2F_1}{\mu}}$.</p> <p>Accordingly, $\frac{v_1}{v_2} = \frac{\sqrt{\frac{F_1}{\mu}}}{\sqrt{\frac{2F_1}{\mu}}} = \frac{1}{\sqrt{2}}$. Thus the required ratio derived here matches with option (d). Hence answer is option (d).</p> 																					
I-09	<p>In this question velocity of sound is to be determined in vacuum. Since, sound wave is mechanical wave and would therefore require medium which is absent in vacuum. Hence, sound will not travel in vacuum. Accordingly, answer is option (d).</p>																					
I-10	<p>Propagation of wave on a string has primary requirement that the string is taught position. Since it is a two piece string both the pieces shall carry same force i.e. $F_1 = F_2 = F$. Since the incoming wave of wavelength λ is partially reflected and partially transmitted with a wavelength this can happen only when the extension piece is of higher mass per-unit length i.e. $\mu_1 < \mu_2$ and hence velocity of wave in first piece is $v = \sqrt{\frac{F}{\mu_1}}$ and on second piece is $v' = \sqrt{\frac{F}{\mu_2}}$. Therefore, $\frac{v}{v'} = \frac{\sqrt{\frac{F}{\mu_1}}}{\sqrt{\frac{F}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}}$. With this relation and considering mass per-unit length analyzed above $v > v'$. It is to be noted that frequency (f) of the wave during reflection and transmission does not change. Accordingly, for incident and reflected wave $\lambda = \frac{v}{f}$ and for transmitted wave $\lambda' = \frac{v'}{f}$. This leads to $\frac{\lambda}{\lambda'} = \frac{v}{v'}$, thus $\frac{\lambda}{\lambda'} = \frac{v}{v'}$. Since it has been derived that $v > v'$ using this inequality in the final form of ratio-proportion $\lambda' < \lambda$. This inference is matching with the option (c). Hence answer is option (c).</p> 																					
I-11	<p>During superimposition of waves, displacement of particles in the resultant wave is sum of the displacement of the particle caused by the constituent waves. Thus, $y = y_1 + y_2 \Rightarrow y = a \sin(\omega t - kx) + a \cos(\omega t - kx)$. It leadstoy = $a[\sin(\omega t - kx) + \cos(\omega t - kx)] = a[(\sin \omega t \cos kx - \cos \omega t \sin kx) +$</p>																					

	<p>$(\cos \omega t \cos kx + \sin \omega t \sin kx)$. Combining terms containing $\sin \omega t$ and $\cos \omega t$, we get $y = a[(\cos kx + \sin kx) \sin \omega t + (\cos kx - \sin kx) \cos \omega t]$. This can be rewritten as $y = A[\sin \omega t \cos \delta + \cos \omega t \sin \delta] = A \sin(\omega t + \delta)$. Here, $A \cos \delta = a(\cos kx + \sin kx)$ and $A \sin \delta = a(\cos kx - \sin kx)$. Accordingly, $A = \sqrt{(A \sin \delta)^2 + (A \cos \delta)^2} \dots(1)$ It solves into $(A \cos \delta)^2 = a^2(\cos^2 kx + \sin^2 kx + 2 \cos kx \sin kx) \dots(2)$ And, $(A \sin \delta)^2 = a^2(\cos^2 kx + \sin^2 kx - 2 \cos kx \sin kx) \dots(3)$.</p> <p>Combining (2) and (3) into (1), we get as under-</p> $A = \sqrt{a^2(\cos^2 kx + \sin^2 kx + 2 \cos kx \sin kx) + a^2(\cos^2 kx + \sin^2 kx - 2 \cos kx \sin kx)}$ $\Rightarrow a\sqrt{2(\sin^2 kx + \cos^2 kx)} = a\sqrt{2}.$ <p>This derived value matches with option (b). Hence answer is (b).</p>
I-12	<p>Let is the length L of the wire having Young's modulus Y having cross-sectional area A stretched by a tensile force F, then $Y = \frac{F}{\frac{\Delta l}{L}} \Rightarrow \Delta l = \frac{FL}{AY}$. Since stretching of both the wires is given to be equal hence $\frac{F_A L_A}{A_A Y_A} = \frac{F_B L_B}{A_B Y_B} \dots(1)$. Velocity of wave is $v = \sqrt{\frac{F}{\mu}} \Rightarrow v = \sqrt{\frac{F}{A\rho}}$, since mass per unit length $\mu = A\rho \times 1$. And time taken by a signal generated at one end of a wire to reach other end is $t = \frac{L+\Delta l}{v} \approx \frac{L}{\sqrt{\frac{F}{A\rho}}} \Rightarrow t \approx L\sqrt{\frac{A\rho}{F}} \dots(2)$ In derivation time t for wire is based on the facts that- (a), wire is used for a metallic string, (b) string is used for non-metallic string metallic wire, and (c) value of Y for metallic wires is such that $\Delta l \rightarrow 0$. Accordingly, approximation has been made in derivation (1). Using (1) and (2) required relationship In the relationship of time t for both the wires van be determined</p> <p>combing (1) and (2) $\frac{t_1}{t_2} = \frac{L_A \sqrt{\frac{A_A \rho_A}{F_A}}}{L_B \sqrt{\frac{A_B \rho_B}{F_B}}} \Rightarrow \frac{t_1}{t_2} = \left(\frac{L_A}{L_B}\right) \sqrt{\left(\frac{A_A}{A_B}\right) \left(\frac{\rho_A}{\rho_B}\right) \left(\frac{F_B}{F_A}\right)} \Rightarrow \frac{t_1}{t_2} = \left(\frac{L_A}{L_B}\right) \sqrt{\left(\frac{A_A}{A_B}\right) \left(\frac{\rho_A}{\rho_B}\right) \left(\frac{L_B A_B Y_B}{L_A A_A Y_A}\right)}$. It further solves into $\frac{t_1}{t_2} = \left(\frac{L_A}{L_B}\right)^{\frac{3}{2}} \left(\frac{\rho_A}{\rho_B}\right)^{\frac{1}{2}} \left(\frac{Y_B}{Y_A}\right)^{\frac{1}{2}}$. Though it is given that $Y_A > Y_B$ and $\rho_A > \rho_B$ but, relationship between L_A and L_B is neither known nor could be derived from the given data and hence it is not possible to define relationship between t_1 and t_2 and is provided on option (d). Hence answer is (d).</p>
I-13	<p>Let the two waves be $y_1 = A_1 \sin(k_1 x - \omega_1 t)$ and $y_2 = A_2 \sin(k_2 x - \omega_2 t)$ Net displacement by principle of superimposition is $y = y_1 + y_2 = A_1 \sin(k_1 x - \omega_1 t) + A_2 \sin(k_2 x - \omega_2 t) \dots(1)$ Net velocity shall be $v = \frac{dy}{dt} = -[A_1 \omega_1 \cos(k_1 x - \omega_1 t) + A_2 \omega_2 \cos(k_2 x - \omega_2 t)] \dots(2)$. This also follows principle of superimposition. Kinetic energy of particle of mass m due to first wave is $KE_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (A_1 \omega_1 \cos(k_1 x - \omega_1 t))^2$. It leads to $KE_1 = \frac{1}{2} m A_1^2 \omega_1^2 \cos^2(k_1 x - \omega_1 t) = \frac{1}{4} m A_1^2 \omega_1^2 (1 + \cos 2(k_1 x - \omega_1 t))$. Likewise, kinetic energy of the particle due to second wave is $KE_2 = \frac{1}{4} m A_2^2 \omega_2^2 (1 + \cos 2(k_2 x - \omega_2 t))$. Thus, net kinetic energy as per principle of superimposition will be $KE = KE_1 + KE_2$. This solves into:</p> $KE = \frac{1}{4} m A_1^2 \omega_1^2 (1 + \cos 2(k_1 x - \omega_1 t)) + \frac{1}{4} m A_2^2 \omega_2^2 (1 + \cos 2(k_2 x - \omega_2 t))$ $\Rightarrow \frac{1}{4} m (A_1^2 \omega_1^2 + A_2^2 \omega_2^2) + \frac{1}{4} m (A_1^2 \omega_1^2 \cos 2(k_1 x - \omega_1 t)) + A_2^2 \omega_2^2 \cos 2(k_2 x - \omega_2 t) \dots(3)$ <p>Analyzing equation (2) having Two sinusoidal functions there would be an instance when instantaneous velocity is Zero it is possible to have zero velocity of the particle and it is oscillating equally on (+)ve and (-)ve side of the mean velocity i.e. Zero Value. But, net kinetic energy, though oscillating, is always +ve with a constant bias $\frac{1}{4} m (A_1^2 \omega_1^2 + A_2^2 \omega_2^2)$. These conclusions are supported only by option (b). Hence, answer is option (b).</p>

I-14	<p>In this question concept of superimposition of displacement caused by constituent waves is being applied to evaluate each of the option.</p> <p>Option (a): The constituent pulses (waves) are travelling on a string in opposite direction. During the period of overlap of pulses, while travelling, instantaneously net displacement on the overlap length of the string principle of superimposition would modify the shape of the pulse. But, on remaining part, only one pulse forward/backward pulse would exist with string in mean position. Thus, pulses will not collide and vanish and pass through each other. Hence this option is wrong.</p> <p>Option (b): Since string is continuous without change in mass per-unit length hence there will be no reflection. Hence this option is wrong.</p> <p>Option (c): From the analysis at option (a) above pulses will pass through each other, but shape will not get modified due to uniform string. Thus this option is also incorrect.</p> <p>Option (d): From elaboration at options (a), (b) and (c) above the pulses will pass through without collision or reflection and loss of pulse shape, since string is uniform. Hence option (d) is correct.</p> <p>Thus answer is option (d).</p>
I-15	<p>By principle of superimposition maximum amplitudes is when they are in phase i.e. $A_{\max} = A_1 + A_2$ and minimum amplitude is when they are out of phase $A_{\min} = A_1 - A_2$. Hence, desired difference in maximum and minimum amplitudes is $\Delta A = A_{\max} - A_{\min} = (A_1 + A_2) - (A_1 - A_2) = 2A_2$. Hence answer is option (b).</p>
I-16	<p>Let the two waves be $y_1 = A \sin(\omega t - kx)$ and other wave be $y_2 = A \sin(\omega t - kx + \delta)$. It is given that the two waves have same amplitude (A) and frequency or angular velocity (ω), but it is silent in respect of phase difference between the two waves. Hence a phase difference y_1 in w.r.t. y_2 is taken to be δ.</p> <p>By principle of superimposition of waves $y = y_1 + y_2 = A \sin(\omega t - kx) + A \sin(\omega t - kx + \delta)$. This solves into $y = A \sin(\omega t - kx) + A[\sin(\omega t - kx) \cos \delta + \cos(\omega t - kx) \sin \delta]$ $\Rightarrow A[(1 + \cos \delta) \sin(\omega t - kx) + \sin \delta \cos(\omega t - kx)] = B \sin(\omega t - kx + \varphi)$</p> <p>This final form is obtained by substituting $B \cos \varphi = A(1 + \cos \delta)$ and $B \sin \varphi = A \sin \delta$. Here, magnitude of the resultant wave is $B = A\sqrt{(1 + \cos \delta)^2 + \sin^2 \delta} \Rightarrow B = A\sqrt{2 + 2 \cos \delta}$. The possible values for angle δ are $0 < \delta < \pi$ and accordingly $-1 < \cos \delta < 1$. This leads $0 < B < 2A$ and this derivation matches with option (d). Hence answer is option (d).</p>
I-17	<p>Let the two waves be $y_1 = A \sin(\omega t - kx)$ and other wave be $y_2 = A \sin\left(\omega t - kx + \frac{2\pi}{3}\right)$. It is given that the two waves have same amplitude (A), frequency or angular velocity (ω), and phase difference between the two waves is given to be $\frac{2\pi}{3}$.</p> <p>By principle of superimposition of waves $y = y_1 + y_2 = A \sin(\omega t - kx) + A \sin\left(\omega t - kx + \frac{2\pi}{3}\right)$. This solves into $y = A \sin(\omega t - kx) + A \left[\sin(\omega t - kx) \cos \frac{2\pi}{3} + \cos(\omega t - kx) \sin \frac{2\pi}{3} \right]$ $\Rightarrow A \left[\left(1 - \frac{1}{2}\right) \sin(\omega t - kx) + \frac{\sqrt{3}}{2} \cos(\omega t - kx) \right] = \left[\frac{A}{2} \sin(\omega t - kx) + \frac{\sqrt{3}A}{2} \cos(\omega t - kx) \right]$</p> <p>This final form is obtained by substituting $B \cos \varphi = A(1 + \cos \delta)$ and $B \sin \varphi = A \sin \delta$. Here, magnitude of the resultant wave is $B = \sqrt{\left(\frac{A}{2}\right)^2 + \left(\frac{\sqrt{3}A}{2}\right)^2} \Rightarrow B = A \sqrt{\frac{1}{4} + \frac{3}{4}} \Rightarrow B = A$. The final derived value of the amplitude of the resultant wave matches with option (a). Hence answer is option (d).</p>
I-18	<p>Maximum wavelength (λ) of string is double the distance (L) between its two fixed ends i.e. $\lambda = 2L$, while velocity if wave on the string is $v = \sqrt{\frac{F}{\mu}}$, here F is tension in the string and μ is mass-density of the string i.e. mass of string per-unit length which is $\mu = \pi d^2 \rho$. Here, d is the diameter of the string and ρ is the density of material of the string.</p>

	<p>Fundamental frequency of vibration of string is $f = \frac{v}{\lambda}$, accordingly, $f = \frac{1}{2L} \sqrt{\frac{F}{\pi d^2 \rho}} \Rightarrow f = \frac{1}{2Ld} \sqrt{\frac{F}{\pi \rho}}$.</p> <p>This final expression of frequency is being analyzed to decide its dependency of different variables as under-</p> <p>(i) L - length of string and frequency is inversely proportional to L that makes option (a) to be correct,</p> <p>(ii) F- tension in the string and frequency is directly proportional to square-root of F which makes option (c) to be incorrect.</p> <p>(iii) d- diameter of the string and frequency is inversely proportional to d and not directly proportional to d as stipulated in option (b). Accordingly, option (b) is incorrect.</p> <p>(iv) ρ- density of string and frequency is inversely proportional to square-root of ρ and not proportional to it as stipulated in option (d). Accordingly, option (b) is incorrect.</p> <p>Thus answer is option (a).</p>
I-19	<p>A string will be set into vibration at frequency nf where is $n \in N$ and f is the frequency of the stimulus. In the given question $f = 480\text{Hz}$ and the string will certainly vibrate at 480 Hz and in resonance. This makes option (b) to be correct.</p> <p>As regards frequencies $f_a = 240\text{ Hz}$ and $f_c = 720\text{ Hz}$ provided in option (a) and (c) respectively. Thus, taking remaining options-</p> <p>Option (a): $n_a = \frac{f_a}{f} = \frac{240}{480} = 0.5$ is not an integer and hence option (a) is incorrect.</p> <p>Option (c): $n_c = \frac{f_c}{f} = \frac{720}{480} = 1.5$ is not an integer and hence option (c) is incorrect.</p> <p>Option (d): In presence of a stimulus string of sonometer would certainly vibrate and that vibration would be resonance or not would depend upon fundamental frequency of the sonometer. This makes option (d) to be incorrect.</p> <p>Hence answer is option (b).</p>
I-20	<p>Wire of a sonometer would vibrate at a frequency f of its stimulus or at nf in case of resonance in the sonometer if natural frequency of sonometer f_s is an integer multiple of the stimulus such that $\frac{f_s}{f} = n _{n \in N}$.</p> <p>Since natural frequency of the sonometer is a fraction of the frequency of the stimulus $\frac{f_s}{f} = \frac{410}{480} = 0.85$ hence it will not be set into resonance with the stimulus. Accordingly sonometer string would vibrate only at the frequency of the stimulus $f = 480\text{ Hz}$ making option (b) to be correct and all other options incorrect.</p> <p>Thus answer is option (b).</p>
I-21	<p>If sonometer of length l with wave velocity v vibrates at fundamental frequency when excited by tuning fork of frequency $f = 416\text{ Hz}$. It means wave length of the natural frequency of the sonometer is $\lambda = 2l = \frac{v}{f}$. It implies that $f = \frac{v}{2l}$.</p> <p>Now it is given that other parameters affecting $v = \sqrt{\frac{F}{\mu}}$ remain unchanged and length is changed such that $l \rightarrow l'$ such that $l' = 2l$ accordingly natural frequency of the sonometer would be $f' = \frac{v}{2l'} \Rightarrow f' = \frac{v}{2 \times 2l} = \frac{v}{4l} \Rightarrow f' = \frac{f}{2}$. Since $f' < f$ and hence it will not resonate but string of sonometer would resonate but would vibrate with the frequency of the tuning fork $f = 416\text{ Hz}$. Accordingly, only option (a) is correct.</p> <p>Hence answer is option (a).</p>
I-22	<p>If sonometer of length l with wave velocity v vibrates at fundamental frequency when excited by tuning fork of frequency $f = 416\text{ Hz}$. It means wave length of the natural frequency of the sonometer is $\lambda = 2l = \frac{v}{f}$. Since velocity of the wave is $v = \sqrt{\frac{F}{\mu}}$ where F is the tension and F is the mass of the string per-unit length It implies that $f = \frac{v}{2l} \Rightarrow f = \frac{1}{2l} \times \sqrt{\frac{F}{\mu}}$.</p> <p>It is given that length of sonometer string is changed such that $l \rightarrow l'$ such that $l' = 2l$ accordingly natural</p>

	<p>frequency of the sonometer would be $f' = \frac{1}{2l'} \times \sqrt{\frac{F'}{\mu}} \Rightarrow f' = \frac{1}{2 \times 2l} \times \sqrt{\frac{F'}{\mu}} \Rightarrow f' = \frac{1}{4l} \times \sqrt{\frac{F'}{\mu}}$. Yet natural frequency of sonometer does not change i.e. $f' = f \Rightarrow \frac{1}{4l} \times \sqrt{\frac{F'}{\mu}} = \frac{1}{2l} \times \sqrt{\frac{F}{\mu}} \Rightarrow \sqrt{\frac{F'}{F}} = 2 \Rightarrow F' = 4F$. Using the given value of $F = mg = 4g$ N. Here is the m kg mass supported by the sonometer string. Therefore, we will have $F' = m'g = 4 \times 4g = 16g \Rightarrow m' = 16$ kg. This calculated value matches with that in option (d).</p> <p>Hence answer is option (d).</p>
I-23	<p>Mechanical waves may be-</p> <p>(a) transverse having motion of particles of medium perpendicular to the direction of propagation viz waves in string</p> <p>(b) Longitudinal having motion particles of medium along the direction of propagation viz sound waves.</p> <p>The problem statement is silent on the nature of wave discussed above and hence direction of motion of particles of the medium cannot be said with certainty. According, with the elaboration at (a) above option (d) is correct. Likewise, with the elaboration at (b) above option (c) is correct.</p> <p>Hence answer is option (c) and (d).</p>
I-24	<p>Since nature of wave is transverse along Z-axis motion of particles shall be along a line perpendicular to Z-axis and hence option (a) is ruled out. This can be any line in X-Y plane in accordance with option (d), including X-axis or Y-axis, but orientation of the line about either of X- or Y-axis can be said with certainty and hence options (b) and (c) are ruled out.</p> <p>Thus answer is option (d).</p>
I-25	<p>Polarization of a wave is possible only if direction of motion of particles is perpendicular to the direction of propagation i.e, in transverse waves only can be polarized. The problems states about longitudinal waves which possesses properties –</p> <p>(i) Wavelength is dependent upon available space for oscillation, tension, and mass-density of material. It implies that any of the parameter can regulate wavelength, hence option (a) is incorrect.</p> <p>(ii) Waves do transmit energy and hence option (b) is incorrect.</p> <p>(iii) Wave velocity is dependent upon pressure in the medium and mass density of the medium and hence wave velocity can be changed with moderation of the either of these two parameters and hence option (c) is also incorrect.</p> <p>(iv) The given wave is not transverse and hence as elaborated in the preamble longitudinal waves cannot be polarized hence option (d) is correct.</p> <p>Hence, answer is option (d).</p>
I-26	<p>If cross-sectional area of the medium is sufficiently larger than its length and source of wave produces compression wave along its length, then wave in the solid is longitudinal. Otherwise when cross-sectional area is smaller as compared to length of the solid and if source of wave produces compression, string may not remain in taugt condition and compression would cause bending of solid medium which might hinder progression of longitudinal wave. But, such a medium in taugt condition is a fit case for progression of transverse wave. Since neither the properties of the solid under consideration not its geometry is defined and hence, as per discussions above, wave in the solid may be longitudinal in accordance with option (b) or may be transverse as per option (d).</p> <p>Hence, answer is options (b) and (d).</p>
I-27	<p>Progression of a wave in a gas is through compression and rarefaction since boundaries are fixed; this is characteristic to longitudinal waves and provided in option (a).. Therefore, nature of wave in transverse mode is ruled out thus options (c) and (d) are incorrect. Among options (a) and (b) option (b) is ruled out since it is not optional as elaborated in the beginning.</p> <p>Hence answer is option (a).</p>

I-28	<p>Each option is being analyzed independently:</p> <p>Option (a): When a wave passes through a region, all particles of the medium constituting the region shall have same frequency hence option (a) is incorrect.</p> <p>Option (b): With the given phase difference π wave equation of particle A is $y_a = A \sin(\omega t - kx)$ and for the particle B it is $y_b = A \sin(\omega t - kx + \pi) \Rightarrow y_b = -A \sin(\omega t - kx)$. Since, it is required to decide upon direction of movement and hence for particle A: $v_a = \frac{d}{dt} y_a = A \omega \cos(\omega t - kx)$.</p> <p>Likewise for particle B $v_b = \frac{d}{dt} y_b = \frac{d}{dt} (-A \sin(\omega t - kx)) = -A \omega \cos(\omega t - kx)$. It is seen that magnitudes of rate of movement of particles A and B are same but have opposite sign and hence $v_a = -v_b$. Thus, option (b) is correct.</p> <p>Option (c): SHM of particles A stated in the problem can be represented $y_a = A \sin(\omega t - kx)$ and the particle B having a phase difference π can be represented as either $y_b = A \sin(\omega t - kx + \pi)$ which can be also expressed as $y_b = A \sin\left(\frac{2\pi}{T}t - kx + \pi\right) \Rightarrow y_b = A \sin\left(\pi\left(\frac{2t}{T} - 1\right) - kx\right)$ taking displacement of particle from mean position at any point along the wave. Phase difference π can also occur at same instant along x when $y_b = A \sin\left(\omega t - k\left(x + (2n \pm 1)\frac{\lambda}{2}\right)\right)$. This requires separation $(2n \pm 1)\frac{\lambda}{2}$ where $n \in N$. Hence, option (c) is incorrect.</p> <p>Option (d): Displacement is a vector quantity but its magnitude is scalar and $y_a = A \sin(\omega t - kx)$ for particle A and for particle B it is $y_b = A \sin\left(\frac{2\pi}{T}t - kx + \pi\right) \Rightarrow y_b = -A \sin(\omega t - kx)$ it leads to $y_b = A \sin(\omega t - kx)$. Thus, $y_a = y_b$. Hence option (d) is correct.</p> <p>Hence answer is options (b) and (d).</p>
I-29	<p>The given wave $y = 0.001\text{mm} \sin[(50\text{s}^{-1})t + (2.0\text{m}^{-1})x] \Rightarrow y = A \sin \omega \left(t - \frac{x}{v}\right)$.</p> <p>Equating the coefficients we have:</p> <p>(i) Given that $A = 0.001\text{mm}$. This makes option (d) correct.</p> <p>(ii) Given that $\omega = 50 \text{ s}^{-1}$, since $\omega = \frac{2\pi}{f} \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow f = \frac{50}{2\pi} = \frac{25}{\pi} \text{ Hz}$. This makes option (c) correct.</p> <p>(iii) Given that $2 = \frac{\omega}{v} \Rightarrow \frac{50}{v} = 2 \Rightarrow v = \frac{50}{2} = 25 \text{ m.s}^{-1}$. This calculated value of wave velocity does not match with that given at option (a). Hence, option (a) is incorrect.</p> <p>(iv) Since $\lambda = \frac{v}{f} \text{ m}$. Using values of f and v derived at (ii) and (iii) above we have $\Rightarrow \lambda = \frac{25}{\frac{25}{\pi}} = \pi \text{ m}$. This calculated value does not match with that given option (b). Hence option (b) is incorrect.</p> <p>Thus answer is option (c) and (d).</p> <p>N.B.: The problems involves comparison of given equation of wave with the generic equation of a travelling wave. Hence, attempting this problem taking each option sequentially would be incorrect.</p>
I-30	<p>In standing wave on a string clamped end is a node and free end is anti-node. Thus minimum distance between the node and anti-node is $= \frac{\lambda}{4}$. depending upon the frequency of the wave on the string. In case of increase of frequency standing wave on the string shall have to satisfy conditions of node and anti-node. Accordingly, the length of the string shall be $l = (2n + 1)\frac{\lambda}{4}$ where $n \in N$. Hence, the answer is option (a).</p> <div data-bbox="933 1612 1516 1780" style="text-align: right;"> </div>
I-31	<p>Each option is being analyzed independently:</p> <p>Option (a): In travelling wave in a string displacement of each particle is $y_a = A \sin(\omega t - kx)$ about the mean position. Since length under is not defined to be wave length and hence for any small of</p>

string energy would change with time. **Hence option (a) is incorrect.**

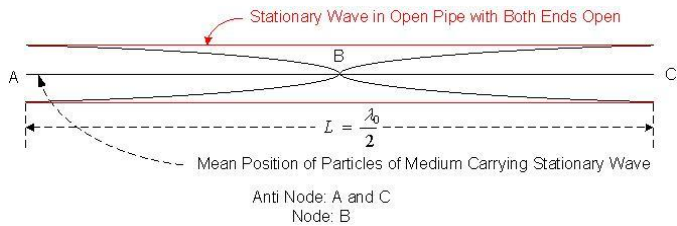
Option (b): In standing wave on a string each particle performs SHM with different amplitude and displacement of particle is expressed as $y_x = 2A \cos kx \sin \omega t$ and actual amplitude at any position x comes to be $A_x = 2A \cos kx$. Energy of a particle on string is constant since they do not transfer energy and it only undergoes transformation between potential and kinetic energy as per principle of conservation of energy. **Hence Option (b) is correct.**

Option (c): From analysis at option (a) above energies in travelling wave are continuously transferred from one part to the other and hence it cannot be same/. **This makes option (c) incorrect.**

Option (d): As per discussions at option (b) energy of each small part in standing wave on a string remains constant, but across all small parts is not equal because each particle of string has different amplitude. **This makes option (d) to be incorrect.**

Thus answer is option (b).

I-32 Each of the given option is being analyzed separately:



Option (a): Particles in segment of string ABC are in-phase and while particles in segment CDE are anti-phase. This makes option (a) to be incorrect.

Option (b): From analysis at option (a) above anti-node B and D are out of phase. This makes option (b) to be incorrect.

Option (c): It is seen that anti-node B in segment ABC and anti-node F in segment EFG are alternate antinodes. Likewise, anti-node D in segment CDE and anti-node H in segment GHI are alternate anti-nodes. This sequence follow all long the standing wave. **This makes option (c) to be correct.**

Option (d): All particle in segment ABC are in phase with anti-node B. Likewise, all particle in segment CDE are in phase with anti-node D. And this sequence follows. **Thus option (d) is correct.**

Thus answer is option (c) and (d).

I-33 Sound is a mechanical wave with oscillation of particles about their mean position in longitudinal manner. Thus as particles oscillate the pressure in the space would oscillate due to change in density of gas molecules as per ideal gas equation $pV = nRT \Rightarrow p = \left(\frac{RT}{V}\right)n$. **Thus, statement (A) is correct.**

Sound as per Huygens principal travels in spherical manner and particles of medium oscillate about their mean position. Hence, the complete spherical layer would oscillate in time and this makes **statement B to be correct.** These conclusions in respect of **statement A and B match with that given at option (a).**

Hence, option (a) is correct.

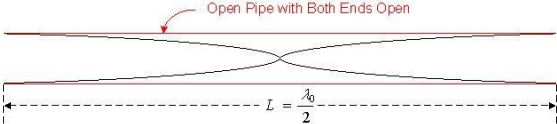
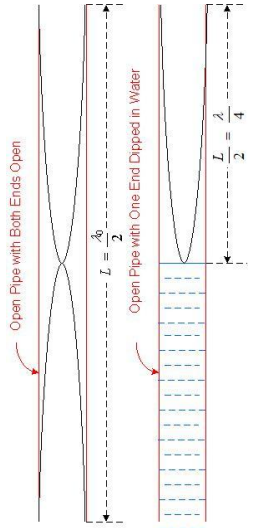
I-34 Clapping produces sound of a particular frequency $f_n = \frac{\omega_n}{2\pi}$ and amplitude p_{0n} depending on speed of clapping and pressure exerted during clapping, respectively. These two specific parameters are included in option at (d) and hence it best describes the clapping sound and that makes **Option (d) to be correct.**

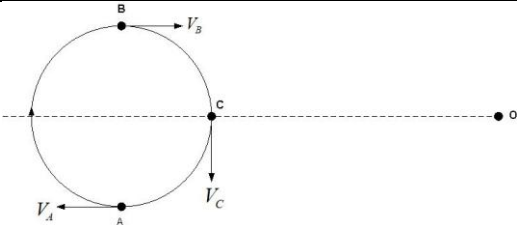
Option (a) is generic expression for any frequencies and amplitude unlike those discussed above. Thus while it describes the sound but is not best representation of clapping sound. This makes **option (a) to be incorrect.**

Discussions in respect of option (a) are applicable to option (b) and (c) also. Moreover, taking generic equation of sound $p = p_0 \sin(kx - \omega t) \Rightarrow p_0(\sin kx \cos \omega t - \cos kx \sin \omega t)$. Thus expression in option (b) for clapping sound is possible only when $\cos kx = 0$ it implies $x = (2n \pm 1) \frac{\lambda}{4k} \Big|_{n \in N}$ which is not stated in the problem. Likewise, equation in option (c) is possible only when $\sin kx = 0$ it implies $x = n \frac{\lambda}{2k}$ which is not stated in the problem. Hence, **options (b) and (c) cannot best describe sound produced by clapping.**

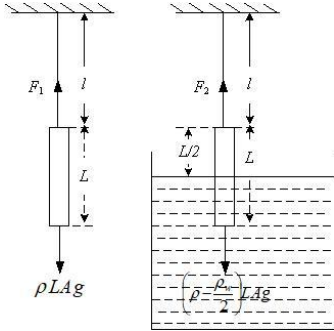
Hence option (a) is the answer.

I-35	<p>Velocity of sound in fluids is expressed as $v = \sqrt{\frac{\gamma P}{\rho}}$, here $\gamma = \frac{C_p}{C_v}$ is ratio of specific heat capacities of medium at constant pressure (C_p) and at constant volume (C_v). This ratio for water is $\gamma = 1$ since water is incompressible. While this ratio for air is $\gamma = 1.4$. In view of this pressure of medium applies to air and is not relevant for water. Densities of air and water $\rho_w \gg \rho_a$. The bulk modulus of elasticity though specified does not appear for velocity of sound in fluids.</p> <p>Thus, above discussion leads to conclusion that velocity of sound in water and air cannot be compared; this conclusion matches with option (d).</p> <p>Hence answer is option (d).</p>
I-36	<p>Velocity of sound in gases is expressed as $v = \sqrt{\frac{\gamma P}{\rho}}$ and it applies to air also. As per Ideal Gas Equation (IGE) $pV = nRT \Rightarrow p \propto T$ and hence with change of temperature, pressure of the gas would change and hence it would influence velocity of sound. This makes option (c) to be correct. As regards other options they are analyzed here under –</p> <p>Displacement amplitude: Amplitude of vibrations transmitted in air is decided by the source of sound i.e. tuning fork, and hence it would not depend upon the temperature. Thus change in this parameter of wave influenced by change in option (a) is incorrect. Thus option (a) is wrong.</p> <p>Frequency: Frequency of vibrations transmitted in air is decided by the source of sound i.e. tuning fork, and hence it would not depend upon the temperature. Thus change in this parameter of wave influenced by change in option (b) is incorrect. Thus option (b) is wrong.</p> <p>Time Period: Time period of wave in sound is $T = \frac{1}{f}$. As discussed above since frequency is not influenced by temperature and hence time-period would also remain unchanged. Thus option (d) is wrong.</p> <p>Hence correct answer is option (c).</p>
I-37	<p>Wave is a SHM motion whose characteristic parameter is frequency and remains unchanged during reflection and refraction sufficiently explained by Huygens Wave Theory. In this context each of the parameter given in the option is being analyzed –</p> <p>Wave Number: it is a parameter that signifies number of wavelengths per unit length $\tilde{\nu} = \frac{1}{\lambda}$. And $\lambda = \frac{v}{f}$ where v is the velocity of wave and f is frequency of wave. Velocity of wave is different for different medium and it would change during refraction and despite frequency being characteristic to a wave, remaining unchanged during refraction, the wave number would change. Hence, option (a) is incorrect.</p> <p>Wavelength: Taking discussions above on Wave Number wavelength would change with change of velocity of sound wave during refraction from air to water. Hence, option (b) is incorrect.</p> <p>Wave Velocity: Taking discussions on Wave Number and wavelength, velocity of sound wave is different in air and water and hence it would change during refraction. Hence, option (c) is incorrect.</p> <p>Frequency: Frequency of wave is a parametric constant of wave and would not change during refraction. Hence, option (d) is correct.</p> <p>Thus answer is option (d).</p>
I-38	<p>Speed of sound as per Newton-Laplace equation is $v = \sqrt{\frac{K_s}{\rho}}$ where K_s is coefficient of stiffness signifying elastic property of the medium and ρ is the density of the medium which is mass per-unit volume and signifies inertia of the medium. Thus speed of sound depends upon elastic property of medium and its inertia property which matches with that given in option (c). Hence, option (c) is the answer.</p>
I-39	<p>Two sound waves in same medium will have same velocity. It is given that $\lambda_1 = 2\lambda_2$ but $A_1 = A_2$. Power of a wave is intensity (I) which is average power transmitted per unit area and is expressed as $I = \frac{p_0^2 v}{2B}$. Here, $B = \rho v^2$ and hence $I = \frac{p_0^2 v}{2\rho v^2} = \frac{p_0^2}{2\rho v}$. Here, p_0 is the pressure amplitude which is given to be same for both the waves and expression of I is independent of wavelength. Hence, $P_1 = P_2$, this makes option (a) to be correct.</p>

	Thus answer is option (a).
I-40	<p>Energy is scalar and follows laws of conservation of energy. Accordingly, Option (a), (b) and (c) are ruled out. The only effect of interference of the two waves would be redistribution of energy in space which remains constant with time in accordance with option (d).</p> <p>Hence, answer is option (d).</p>
I-41	<p>Organ pipe is open at both ends and hence the ends would have antinodes having maximum displacement and middle of the organ pipe shall have node with minimum displacement with maximum variation of pressure.</p> <p>Hence answer is option (b)</p> 
I-42	<p>An organ pipe with both ends open can contain stationary sound waves; characteristically, sound waves are longitudinal waves. Both these properties of waves in organ pipe are contained only in option (a).</p> <p>Hence answer is Option (a).</p>
I-43	<p>A cylindrical tube with fundamental frequency ν, has say a length L. The tube will have anti-nodes at both the open ends thus $L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$. Let velocity of the wave in the tube be v then frequency $\nu = \frac{v}{\lambda} \Rightarrow \nu = \frac{v}{2L} \dots(1)$</p> <p>The tube when dipped in water to half of its length there are two changes;</p> <ol style="list-style-type: none"> Free length changes to $L' = \frac{L}{2}$, and It becomes a tube with one end closed forming node, while open end remains anti-node. Thus new fundamental wavelength would be $\frac{\lambda'}{4} = L' = \frac{L}{2} \Rightarrow \lambda' = 2L$. <p>Since medium is same and hence velocity of the wave in the medium would remain same. Hence, new fundamental frequency would be $\nu' = \frac{v}{\lambda'} = \frac{v}{2L} \dots(2)$</p> <p>Comparing (1) and (2) above. It is seen that fundamental frequency remains unchanged i.e. $\nu = \nu'$. This conclusion matches with that given in option (c).</p> <p>Hence answer is option (c).</p> 
I-44	<p>Phenomenon of beats occur when two waves of same amplitude with marginal difference of frequency interfere with each other such that $\Delta\omega = \omega_1 - \omega_2 \approx \frac{\omega_1 + \omega_2}{2} = \omega$. The resultant wave at a point x at any instant is $p(t, x) = p_0 \sin(\omega_1 t - kx) + p_0 \sin(\omega_2 t - kx) \Rightarrow p(t, x) = 2p_0 \cos\left(\frac{\Delta\omega}{2} \left(t - \frac{x}{v}\right)\right) \sin \omega \left(t - \frac{x}{v}\right)$.</p> <p>The phenomenon of beats is considered with only $\Delta\omega$ and it has nothing to do with mechanics of wave be it transverse or longitudinal. This matches with the option (c).</p> <p>Hence answer is option (c).</p>
I-45	<p>Phenomenon of beats occur when two waves of same amplitude with marginal difference of frequency interfere with each other such that $\Delta\omega = \omega_1 - \omega_2 \approx \frac{\omega_1 + \omega_2}{2} = \omega$. The resultant wave at a point x at any instant is $p(t, x) = p_0 \sin(\omega_1 t - kx) + p_0 \sin(\omega_2 t - kx) \Rightarrow p(t, x) = 2p_0 \cos\left(\frac{\Delta\omega}{2} \left(t - \frac{x}{v}\right)\right) \sin \omega \left(t - \frac{x}{v}\right)$.</p> <p>This expression gives $\Delta\omega = \omega_1 - \omega_2 \dots(1)$. Here, $\Delta\omega$ is the beat frequency.</p> <p>Frequency of vibration of string is $f_0 = \frac{1}{2L} \sqrt{\frac{F}{\mu}} \Rightarrow f_0 \propto \sqrt{F} \dots(2)$. thus fundamental frequency of increase with increase of tension.</p> <p>Let, $\omega_1 = 2\pi f_1$ where f_1 is the frequency of the tuning fork and $\omega_2 = 2\pi f_2$ where f_2 is the frequency of the string.</p> <p>It is given that with increase of tension beat frequency reduces. This can happen only when $f_1 > f_2$, else</p>

	<p>frequency of beats would increase. Among the frequencies given in the options it is only option (a) where $f_1 > f_2$ that makes it correct answer. Hence answer is option (a).</p>
I-46	<p>Frequency of train whistle sound is a function of $v_{p,t}$ relative velocity $v_{p,t}$ of passenger w.r.t.train. Passenger in the train moves with the velocity of the train and hence, relative velocity $v_{p,t}$ of passenger w.r.t. is zero. Hence, frequency heard by a passenger is frequency of the whistle . This matches with the option (d). Hence, answer is option (d).</p>
I-47	<p>In Doppler's effect effective frequency is $f' = f \cdot \left(\frac{V-V_o}{V-V_s}\right) \Rightarrow \frac{f'}{f} = \frac{V-V_o}{V-V_s} \Rightarrow 1 - \frac{f'}{f} = 1 - \frac{V-V_o}{V-V_s} \Rightarrow \frac{\Delta f}{f} = \frac{V_o-V_s}{V-V_s} \Rightarrow \frac{\Delta f}{f} = \frac{V_{o,s}}{V-V_s}$. Here, V is the velocity of sound, V_s velocity of source, V_o velocity of the observer, f is the frequency of the sound produced by the source, $V_{o,s}$ is the relative velocity of the observer w.r.t. source, f' is the frequency heard by the observer and Δf is the change in frequency heard due to $V_{o,s}$. Separation between source and observer is affected by the relative velocity $V_{o,s}$ and this influences separation between the source and the observer. This inference matches with that given in option (d). Hence, answer is option (d).</p>
I-48	<p>Frequency heard by an observer is $f' = f \cdot \left(\frac{V-V_o}{V-V_s}\right)$ where, V is the velocity of sound, V_s velocity of source, V_o velocity of the observer, f is the frequency of the sound produced by the source and f' is the frequency heard by the observer. Given that $V_o = 0$ for observer and for source at A, B and C velocities of source w.r.t. observer are $V_A = V_s$, $V_B = V_s \cos \pi = -V_s$ and $V_C = V_s \cos \frac{\pi}{2} = 0$, respectively.</p>  <p>Accordingly, frequency heard by the observer when source is at A is $v_1 = f \left(\frac{V}{V-V_A}\right) = f \left(\frac{V}{V-V_s}\right)$, when source is at B it is $v_2 = f \left(\frac{V}{V-V_B}\right) = f \left(\frac{V}{V-(-V_s)}\right) = f \left(\frac{V}{V+V_s}\right)$ and $v_3 = f \left(\frac{V}{V-V_C}\right) = f \left(\frac{V}{V-0}\right) = f \left(\frac{V}{V}\right) f$.</p> <p>A close observation of the mathematical expressions of v_1, v_2, and v_3 reveals that only denominator is changing and it is maximum for v_1 and hence it would minimum; the denominator is minimum for v_2 and it would maximum, and for v_3 it is in between the two values for v_1 and v_2 and v_3 would be in between the two. Accordingly, $v_2 > v_3 > v_1$. These conclusions match with those given in option (c). Hence answer is option (c).</p>
I-49	<p>While speaking to a friend frequency and amplitude are similar across them and hence options (a) and (c) are incorrect. But, velocity of sound wave is dependent upon medium and its conditions at that time. Hence, wave velocity is unique that makes option (d) to be correct. Further, wavelength $\lambda = \frac{v}{f}$, here though velocity v is unique, the frequency f is not unique and ruled out for option (a) and hence option (b) is also not correct. Hence answer is (d).</p>
I-50	<p>Sound wave of source are stated to be of constant frequency (f) and amplitude (A). Yet it is stated that temperature of medium (T) is increasing while pressure (P) remains constant.</p> <p>Speed of sound in air is $v = \sqrt{\frac{\gamma P}{\rho}}$. As per IGE, $PV = nRT \Rightarrow V \propto T$ and hence for same mass of gas density would reduce. Since in the given expression for velocity, density ρ is in the denominator velocity of sound would increase this makes option (c) to be correct. Further, wavelength $\lambda = \frac{v}{f} \Rightarrow \lambda \propto v$ and frequency is regulated by source of constant frequency and from conclusion for option (c) wavelength would also increase to make option(a) to be correct. Since, frequency is stated to be constant the option (b) would go incorrect and time period of wave $= \frac{1}{f}$ this will also make option (d) to be incorrect. Thus answer is (a) and (c).</p>

I-51	<p>In case the organ pipe has one end closed then first overtone occurs at thrice the fundamental frequency of the overtone which is given to be $f_0 = 200$ Hz and hence $f_{1-c} = 3f_0 = 600$ Hz. If the organ is open at both ends then first overtone occurs at frequency double of the fundamental frequency f_0 and hence $f_{1-o} = 2f_0 = 400$. Since, nature of organ pipe is not stated hence with certainty first overtone frequency cannot be stated to be 400 Hz, that makes option (a) to be incorrect.</p> <p>However, in wake of uncertainty on nature of organ pipe, and discussion first overtone frequency may be 400 Hz or 600 Hz. This makes options (b) and (c) to be correct.</p> <p>As regards option frequency 600 Hz stated to be an overtone frequency in option (d), it is correct since it is not specific for overtone number and therefore applies to third overtone for organ pipe with one end closed and first overtone with both ends open. Thus option (d) is correct.</p> <p>Thus answer is options (b), (c) and (d).</p>	
I-52	<p>This problem involves application of Doppler's effect, and each of the option is being analyzed separately –</p> <p>Option (a): Frequency of source is since a constant it would not change and hence option (a) is incorrect.</p> <p>Option (b): Once sound wave is emitted by the source its velocity is regulated by the medium which is stated to be same. Hence velocity sound in the medium would not change. This makes option (b) to be incorrect.</p> <p>Option (c): As per Doppler's effect frequency of the sound waves perceived by the observer f' increases due to motion of source towards the observer, while velocity of the wave in the medium is constant as discussed at option (b) above. Hence $\lambda = \frac{v}{f'} \Rightarrow \lambda \propto \frac{1}{f'}$. Here, λ is wavelength and hence with the increase of f' wavelength would decrease. Thus, option (c) is correct.</p> <p>Option (d): Amplitude of the wave motion of the particles of the medium is controlled by the source and since there is no resonance to affect amplitude, amplitude would not change. Thus option (d) is incorrect.</p> <p>Thus answer is option (c).</p>	
I-53	<p>Velocity of wind if in the direction of movement of sound, as is stated in the problem, then effective velocity of sound $V' = V + V_w$ where V is velocity of sound in still air, V_w is velocity of wind in the direction of sound from source to listener. As per Doppler's Effect $f' = f \cdot \left(\frac{V - V_o}{V - V_s} \right)$ here velocity of listener $V_o = 0$. Thus, the revised expression of frequency perceived by the listener is $f' = f \left(\frac{V + V_w}{V + V_w - V_s} \right)$. Observation of this final form reveals that incremental effect due to V_w in numerator is greater than the denominator and hence change of frequency perceived by the listener, but it is not asked. But frequency of the sound in medium remains unchanged, Accordingly, option (a) is correct.</p> <p>Since velocity of sound is dependent on the medium and remains unchanged and hence option (b) is incorrect.</p> <p>Frequency (f) of sound in medium is same as that produced by the source. But, with the effective velocity of sound $V' = V + V_w$ due to blowing of wind number effective wavelength would be $\lambda' = \frac{V'}{f} = \frac{V + V_w}{f}$ would also change. This makes option (c) to be incorrect.</p> <p>Time period $T = \frac{1}{f}$ and since frequency of sound in the medium remains unchanged as concluded for option (a). Hence, time period would also not change and thus option (d) is correct.</p> <p>Thus answer is option (a) and (d).</p>	
I-54	<p>Stationary wave is formed when two identical waves travelling in opposite directions, and at node displacement is zero. One of the wave is given to be $y_1 = a \cos(kx - \omega t)$ and hence other identical wave in opposite direction shall have to be $y_2 = Ka \cos(-kx - \omega t) = Ka \cos(kx + \omega t)$, and for node to exist at a certain point $x = 0$ it has to satisfy $y_{1x} + y_{2x} = a \cos(kx - \omega t) + Ka \cos(kx + \omega t) = (1 + K) \cos \omega t = 0$. Since, $-1 < \cos \omega t < 1$ undergoes cosidal variation with time. Hence for node to exist at $x = 0$ the necessary condition is $1 + K = 0 \Rightarrow K = -1$, accordingly $y_2 = -a \cos(kx + \omega t)$, this is as per equation in option (c). Hence answer is option (c).</p>	

I-55	<p>This problem requires simplification of trigonometric equation of motion $y = 2 \left(2 \cos^2 \left(\frac{1}{2} t \right) \right) \sin 1000t$. It simplifies into $y = 2(\cos t + 1) \sin 1000t = 2 \sin 1000t + 2 \sin 1000t \cos t$. As per trigonometric identity $\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$ we have $y = 2 \sin 1000t + 2 \left(\frac{\sin 1001t + \sin 999t}{2} \right)$. This, further simplifies into $y = \sin 1001t + 2 \sin 1000t + \sin 999t$. Thus displacement equation is superimposition three independent SHMs. This conclusion matches with option (b). Hence answer is option (b). N.B.: This answer can also be arrived at by simple observation of the equation and order of sinusoidal terms.</p>
I-56	<p>This is the case of a string of length l kept taut by a force equal to the weight of the object suspended from it. In this at fundamental frequency $\frac{\lambda}{2} = l \Rightarrow \lambda = 2l$, and $f_0 = \frac{v}{\lambda} = \frac{v}{2l}$. Here, velocity of the wave in the string is $v = \sqrt{\frac{F}{\mu}} \Rightarrow f_0 = \frac{1}{2l} \sqrt{\frac{F}{\mu}}$.</p> <p>In the problem this fundamental frequency changes with, initially the force, is weight of the object, $F_1 = AL\rho g$ is weight of the object and it would leads to $f_{01} = \frac{1}{2l} \sqrt{\frac{AL\rho g}{\mu}} \dots (1)$ Later, when half-length of the force is reduced due to buoyancy when half-length of the object is immersed $F_2 = AL\rho g - A \frac{L}{2} \rho_w g$.</p> <p>This simplifies into $F_2 = \left(\rho - \frac{\rho_w}{2} \right) ALg$. Thus, $f_{02} = \frac{1}{2l} \sqrt{\frac{\left(\rho - \frac{\rho_w}{2} \right) ALg}{\mu}} \dots (2)$. Accordingly, using (1) and (2)</p> <p>we have $\frac{f_{02}}{f_{01}} = \frac{\frac{1}{2l} \sqrt{\frac{\left(\rho - \frac{\rho_w}{2} \right) ALg}{\mu}}}{\frac{1}{2l} \sqrt{\frac{AL\rho g}{\mu}}} \Rightarrow f_{02} = f_{01} \sqrt{\frac{2\rho - \rho_w}{2\rho}}$. Given that $f_{01} = 300\text{Hz}$ and using CGS $\rho_w = 1$ we have $f_{02} = 300 \sqrt{\frac{2\rho - 1}{2\rho}}$. This derived value matches with option (a) and hence it correct.</p> <p>N.B.: Since $\frac{2\rho - \rho_w}{2\rho}$ the ratio is of quantities having same unit, and $\rho_w = 1$ in CGS, it leads to simplification of calculations.</p> 
I-57	<p>As per Hooke's Law $Y = \frac{\text{Stress}}{\text{Strain}}$, and $\text{Stress} = \frac{\text{Force (F)}}{\text{Area (A)}}$ and $\text{Strain} = \frac{\text{Elongation } (\Delta l)}{\text{Unstretched length (l)}}$. Thus, $Y = \frac{Fl}{A\Delta l}$ it leads to $F = \Delta l \left(\frac{AY}{l} \right)$. Speed of wave in a string $v = \sqrt{\frac{F}{\mu}} \Rightarrow v \propto \sqrt{F}$. Accordingly, in two cases $v_1 \propto \sqrt{F_1}$; it leads to $v_1 \propto \sqrt{x \left(\frac{AY}{l} \right)} \Rightarrow v_1 \propto \sqrt{x}$ when extension in length of string is x and $v_2 \propto \sqrt{1.5x \left(\frac{AY}{l} \right)} \Rightarrow v_2 \propto \sqrt{1.5x}$</p> <p>This is further rationalized into $\frac{v_2}{v_1} = \frac{\sqrt{1.5x}}{\sqrt{x}} \Rightarrow v_2 = \sqrt{1.5} v_1$. Given that $v_1 = v$ we have $v_2 = 1.22v$, this derivation matches with option (a). Hence, answer is option (a).</p>
I-58	<p>As per Doppler's effect frequency heard by an observer is $f' = f \cdot \left(\frac{V - V_o}{V - V_s} \right)$ where, $V = 340 \text{ m.s}^{-1}$ is the velocity of sound, V_s velocity of source, $V_o = 0$ velocity of the observer who is stationary and f is the frequency of the sound produced by the source. In the problem velocity of source change from $V_{s1} = 34 \text{ m.s}^{-1}$ to $V_{s2} = 17 \text{ m.s}^{-1}$.</p> <p>Accordingly, $f_1 = f \cdot \left(\frac{340}{340 - 34} \right) \Rightarrow f_1 = \frac{340}{306} f$ and $f_2 = f \cdot \left(\frac{340}{340 - 17} \right) \Rightarrow f_2 = \frac{340}{323} f$. Hence, $\frac{f_1}{f_2} = \frac{\frac{340}{306} f}{\frac{340}{323} f} \Rightarrow \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$.</p> <p>This ratio matches with that given at option (d). Hence, answer is option (d).</p>
I-59	<p>This is a problem of refraction of sound while travelling from source in water to the observer in air. In refraction phenomenon frequency remains constant and due to change of velocity it the e wavelength that</p>

undergoes change. Thus frequency of sound produced by source $f = 600$ Hz would be heard by the observer. **This matches with that given in option (d).**
Hence answer is option (d).

I-60

Frequency is $f = \frac{v}{\lambda}$ where v is the velocity of the sound in a medium and λ is the wavelength at that velocity. Natural frequency of an organ pipe open at both ends is $f_0 = \frac{v}{\lambda_0}$, since in this length of the pipe constitutes half wavelength $L = \frac{\lambda_0}{2} \Rightarrow \lambda_0 = 2L$. It leads to $f_{01} = \frac{v}{2L}$. Given that $f_1 = 2f_{01} = \frac{v}{L}$.

But, when organ pipe is closed at one end then $L = \frac{\lambda_2}{4} \Rightarrow \lambda_2 = 4L$ and hence $f_{02} = \frac{v}{\lambda_2} = \frac{v}{4L}$ and its n^{th} harmonics is $f_2 = (2k + 1)f_{02} |_{k \in \mathbb{N}} \Rightarrow f_2 = (2k + 1) \frac{v}{4L} \Rightarrow f_2 = \left(\frac{2k+1}{4}\right) \frac{v}{L} \Rightarrow f_2 = \left(\frac{2k+1}{4}\right) f_1$. It is given that that $f_2 > f_1$ for which minimum value of $k = 2$. Accordingly, $f_2 = \left(\frac{2 \times 2 + 1}{4}\right) f_1 \Rightarrow f_2 = \frac{5}{4} f_1$. Further, order of harmonic of f_2 with organ pipe closed at one end would be $f_2 = \left(\frac{2 \times 2 + 1}{4}\right) f_1$ substituting value of f_1 we have $f_2 = \frac{5}{4} \times \frac{v}{L} \Rightarrow f_2 = 5 \times \frac{v}{4L} \Rightarrow f_2 = 5 \times f_{02} |_{n=5}$. Thus, condition of $f_2 > f_1$ is satisfied at 5th harmonic i.e. at $n = 5$. Thus, $n = 5$ and $f_2 = \frac{5}{4} f_1$ match with those given at option (d).
Hence, answer is option (d).

