Wave and Motion : Vibrations In Strings and Sound Waves– Objective Questions (Typical)

Vibrations in string and sound are the most perceived experience of Simple Harmonic Motion (SHM). Right form our voice to all musical instruments are influenced by it. Moreover at macro scale all high rise structure, transmission line and rope ways have to be made resistant to such vibrations.

In this set of questions some simple problems in respect of vibrations of strings and sound has been incorporated with necessary illustrations involving first principles, to th extent possible.

Solving typical problems on a gradual degree of complexity helps to build power of visualization of concepts that are essential in understanding a problem/n observation and evolving solution/answer. At this stage simpler calculations are being skipped in elaboration, with a hope that reader would be able to decipher intermediate steps.

Competitive examinations and more particularly in real life rarely expose to problems solved. Yet ability to solve such problems one groomed, it enhances competence to handle unknown problems speedily and correctly with a greater degree of clarity and confidence, an essential attribute of thought process needed for success in life.

Mentors' Manual is one of the dimensions of the Gyan Vigyan Sarita through which efforts are being made to reach out to remote teachers through our experience of mentoring unprivileged children who are disconnected from us by virtue of multiple barriers. Direct interaction has been possible through Interactive Online Mentoring Sessions (IOMS) a working model of connecting unprivileged children in a selfless manner. This experience is being disseminated to the teachers spread out by writing of chapters of an open source Mentors' Manual. Simple Harmonic Motion is First of the Three parts of chapter Three covering Sound and Optics..

Science is a subject not to learn but a matter of realization through experiments and its visualization in surrounding. Every student is not equipped either to conduct experiment or an environment for visualization of science in his surroundings. This is where simulation is a technique to verify the concepts and study effect of variation in parameters related to the concept. There are various simulation tools leading to virtual laboratories.

India, growing digital, provides optimism to every student to be able to have an access to virtual laboratory, where without any physical laboratory, involving consumption of equipment and material, it is possible to carry out experiments in an e-environment. There are some excellent videos available on the web either free or on price which provide an experiences of kind in simulation of the concepts, The only problem with this is of sequencing and scaling of concepts and selection of an appropriate video out of a big list of search results. But, it is neither possible nor affordable for a student to first make a survey to select most suitable video and then view it for gaining proficiency in the concepts.

It creates a question, can one wait for suitable virtual labs to become available to each student to gain proficiency in concepts? Definitely not! then the only way to get going on acquiring proficiency in concepts and their applications, soon after learning them, is solving problems of variety. This is a key, have patience and perseverance, to acquire proficiency without consumption of any other resource except time which is available with students. All that they miss is the direction in which they can deploy their efforts. Problem solving in mathematics and physics is inevitable to gain necessary proficiency.

Here, Question Banks include problems from various sources and they are being supported with illustrations. These are not just solutions but an attempt to bring home use of basics involved in solving a problem. In an effort to compile problem there some good text books including those authored by Prof. H.C. Verma and a team of authors Robert Resnick, David Halliday and Kenneth S. Krane and many more. Some objective questions from different examinations have also been included. These questions are graded and authors have attempted to incorporate all concepts covered in the book. Thus it necessitates a student to read each chapter carefully before taking up questions.

In the illustrations to the problems, supported with each question bank, some student may find them to be a bit lengthy and dwelling into basics more than what one requires. Since it targets students, who are in abundance, not directly connected to us, patience of well versed students is requested. Few question with their illustrations are drawn from the set-1, on Waves and Motion : Simple Harmonic Motion, covering and appended here. The complete set of 50 questions is being uploaded as a free web-resource.

This initiative is aimed at to mentor unprivileged children is of a small group of passionate persons is driven with a sense of Personal Social Responsibility (PSR) in a non-organizational, non-remunerative, non-commercial and non-political manner. You are welcome to add value to this initiative by way of suggestion, advising correction or new type of questions. Or any other form that suits to your competence and convenience.

TYPICAL QUESTIONS WITH ILLUSTRATION

Question 01: A sine wave is travelling in a medium. The minimum distance between two particles having same speed is –

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{34}$ (c) $\frac{\lambda}{2}$ (d) $\frac{\lambda}{4}$

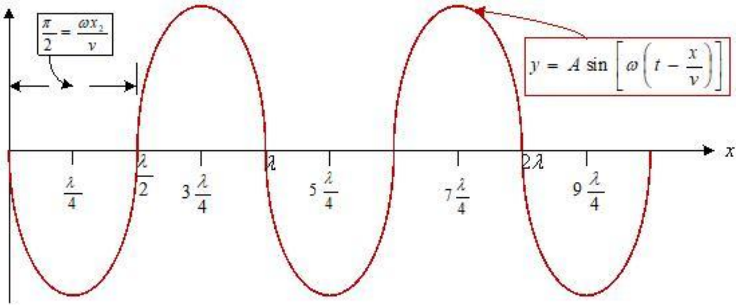

Illustration-01: Displacement of a particle executing sine wave in a medium is shown graphically in the figure and is $y = A\sin\left(\omega\left(t-\frac{x}{v}\right)\right)$, here x is the position of particle along direction of propagation of wave is shown on X-axis, and displacement of particle from its mean position is y. Therefore, speed of particle is $v = \frac{dy}{dt} = A\omega\cos\left(\omega\left(t-\frac{x}{v}\right)\right)$.
Let $x_1$and x_2 are the distances particles having same speed at same instant, say $v_1 = A\omega\cos\left(\omega\left(t-\frac{x_1}{v}\right)\right)$ and $v_2 = A\omega\cos\left(\omega\left(t-\frac{x_2}{v}\right)\right)$. Speed implies that their magnitudes are same irrespective of the direction. Hence,$\omega\cos\left(\omega\left(t-\frac{x_1}{v}\right)\right) = \omega\cos\left(\omega\left(t-\frac{x_2}{v}\right)\right)$. Since velocity is a trigonometric function and hence for$|v_1| = |v_2|$ it leads to$\theta_2 = \theta_1 + n\pi$, where n is an integer. Thus, $\omega\left(t-\frac{x_2}{v}\right) = \omega\left(t-\frac{x_1}{v}\right) + n\pi \Rightarrow \frac{2\pi}{T}\left(t-\frac{x_2}{v}\right) = \frac{2\pi}{T}\left(t-\frac{x_1}{v}\right) + n\pi \Rightarrow \frac{2}{T}\times\frac{x_2}{v} = \frac{2}{T}\times\frac{x_1}{v} - n$. It, further, solves into $\Delta x = |x_1 - x_2| = \frac{nvT}{2} = \frac{n\lambda}{2}$. Smallest integer is since 1 and the smallest $\Delta x = \frac{\lambda}{2}$. **Hence answer is (c).**

N.B.: Analytical approach is though longer, leads to correct answer, which by simple observation of graph may lead to interpretation errors.

Question-03: Which of the following equations represents a sine wave travelling along Y-axis?
(a) $x = A\sin(ky - \omega t)$ (b) $x = A\sin(kx - \omega t)$ (c) $x = A\sin ky\cos\omega t$ (d) $x = A\cos ky\sin\omega t$

Illustration-03: Each of the given options are being examined for the equation given therein -

Option (a): It is the case of displacement of particle along X-axis for a single wave propagating along Y-axis, it is the case of a transverse wave.

Option (b): It is the case of displacement of particle along X-axis for a single wave propagating along X-axis, it is the case of a longitudinal wave.

Option (c): The displacement equation can be written as $x = A\sin ky\cos\omega t = \frac{A}{2}[\sin(ky+\omega t) + \sin(ky-\omega t)]$. Thus it is a combination of transverse Two waves travelling along Y-axis and not a single wave.

Option (d): The displacement equation can be written as $x = A\cos ky\sin\omega t = \frac{A}{2}[\sin(\omega t+ky) + \sin(\omega t-ky)]$. Thus it is a combination of transverse Two waves travelling along Y-axis and not a single wave.

From the above analysis, it is clear that Option (c) and (d) are of combination of Two waves and not a single wave, and are therefore ruled out.
The remaining Two options are (a) and (b) out of which Option (b) is ruled out as it is travelling along X-axis as discussed in the analysis.
The last option is of a single wave travelling along Y-axis. **Hence answer is option (a)**

N.B.: Here questions asks "….represents a sine wave….", needs to be noted carefully that it asks for a single wave and not a combination of waves. This is the key to right answer.

Question-04: The equation $y = A\sin^2(kx - \omega t)$ represents a wave motion with –

(a) Amplitude A, frequency $\frac{\omega}{2\pi}$
(b) Amplitude $\frac{A}{2}$, frequency $\frac{\omega}{\pi}$

(c) Amplitude A, frequency $\frac{\omega}{4\pi}$
(d) Do not represent a wave motion

Illustration 04: The given equation $y = A\sin^2(kx - \omega t) \Rightarrow y = \frac{A}{2}(1 - \cos 2(kx - \omega t)) \Rightarrow y = \frac{A}{2}\left(1 - \cos 2\omega\left(\frac{kx}{\omega} - t\right)\right)$. This equation in final form has magnitude $\frac{A}{2}$ and angular velocity $\omega' = 2\omega$. Since, $\omega' = 2\pi f' \Rightarrow f' = \frac{\omega'}{2\pi}$. It leads to $f' = \frac{2\omega}{2\pi} \Rightarrow \boldsymbol{f'} = \frac{\boldsymbol{\omega}}{\boldsymbol{\pi}}$. Thus, the given equation represents Amplitude $\frac{A}{2}$, frequency $\frac{\omega}{\pi}$ and is matching with option (b). Hence answer is Option (b).
N.B.: The given equation has to be analyzed to determine amplitude and frequency and then these values are to be matched with the given options. Therefore, it is a case of straight matching and does not need analysis of each option

Question 29: A wave is represented by an equation: $y = 0.001\text{mm}\sin[(50\text{s}^{-1})t + (2.0\text{m}^{-1})x]$
(a) The wave velocity is 100 m/s
(b) The wavelength is 2.0 m
(c) The frequency is $\frac{25}{\pi}$ Hz
(d) The amplitude is 0.001 mm

Illustration 29: The given wave $y = 0.001\text{mm}\sin[(50\text{s}^{-1})t + (2.0\text{m}^{-1})x] \Rightarrow y = A\sin\omega\left(t - \frac{x}{v}\right)$.
Equating the coefficients we have:
(i) Given that $A = 0.001$mm. **This makes option (d) correct.**
(ii) Given that $\omega = 50$ s^{-1}, since $\omega = \frac{2\pi}{f} \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow f = \frac{50}{2\pi} = \frac{25}{\pi}$ Hz. **This makes option (c) correct.**
(iii) Given that $2 = \frac{\omega}{v} \Rightarrow \frac{50}{v} = 2 \Rightarrow v = \frac{50}{2} = 25$ m.s^{-1}. This calculated value of wave velocity does not match with that given at option (a). **Hence, option (a) is incorrect**.
(iv) Since $\lambda = \frac{v}{f}$ m. Using values of and derived at (ii) and (iii) above we have $\Rightarrow \lambda = \frac{25}{\frac{25}{\pi}} = \pi$m. This calculated value does not match with that given option (b). Hence option (b) is incorrect.
Thus answer is option (c) and (d).
N.B.: The problems involves comparison of given equation of wave with the generic equation of a travelling wave. Hence, attempting this problem taking each option sequentially would be incorrect.

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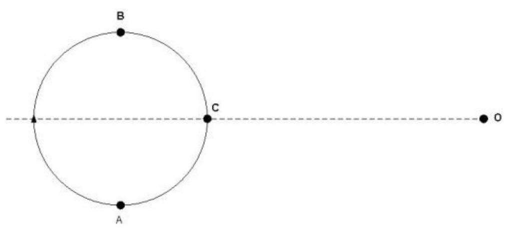
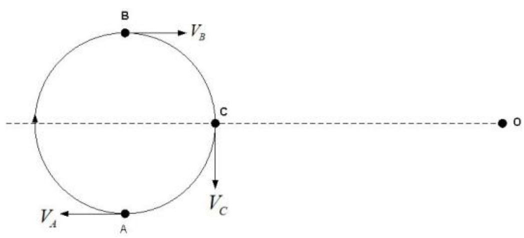
Question 48: A small source of sound moves on a circle in clockwise direction as shown in the figure and an observer is sitting at O. Let υ_1, υ_2 and υ_3 be the frequencies heard when the source is at A , B and C respectively. Then



(a) $\upsilon_1 > \upsilon_2 > \upsilon_3$ (b) $\upsilon_1 = \upsilon_2 > \upsilon_3$
(c) $\upsilon_2 > \upsilon_3 > \upsilon_1$ (d) $\upsilon_1 > \upsilon_3 > \upsilon_2$

Illustration 48: Frequency heard by an observer is $f' = f.\left(\frac{V - V_o}{V - V_s}\right)$ where, V is the velocity of sound, V_s velocity of source, V_o velocity of the observer, f is the frequency of the sound produced by the source and f' is the frequency heard by the observer. Given that $V_o = 0$ for observer and for source at A, B and C velocities of source w.r.t. observer are $V_A = V_S$, $V_B = V_S\cos\pi = -V_S$ and $V_C = V_S\cos\frac{\pi}{2} = 0$, respectively.



Accordingly, frequency heard by the observer when source is at A is $\upsilon_1 = f\left(\frac{V}{V - V_A}\right) = f\left(\frac{V}{V - V_S}\right)$, when source is at B it is $\upsilon_2 = f\left(\frac{V}{V - V_B}\right) = f\left(\frac{V}{V - (-V_S)}\right) = f\left(\frac{V}{V + V_S}\right)$ and $\upsilon_3 = f\left(\frac{V}{V - V_C}\right) = f\left(\frac{V}{V - 0}\right) = f\left(\frac{V}{V}\right)f$.
A close observation of the mathematical expressions of υ_1, υ_2, and υ_3 reveals that only denominator is changing and it is maximum for υ_1 and hence it would minimum; the denominator is minimum for $\upsilon_2$and it would maximum, and for υ_3 it in between the two

values for $v_1$and v_2 and v_3 would be in between the two. Accordingly, $v_2 > v_3 > v_1$. These conclusions match with those given in option (c).
Hence answer is option (c).

Question 55: The displacement y of a particle executing periodic is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin 1000t$. This expression may be considered to be a result if superimposition of ……… Independent harmonic motions
(a) Two (b) Three (c) Four (d) Five

Illustration 55: This problem requires simplification of trigonometric equation of motion $y = 2\left(2\cos^2\left(\frac{1}{2}t\right)\right)\sin 1000t$. It simplifies into $y = 2(\cos t + 1)\sin 1000t = 2\sin 1000t + 2\sin 1000t\cos t$. As per trigonometric identity $\sin A\cos B = \frac{\sin(A+B)+\sin(A-B)}{2}$ we have $y = 2\sin 1000t + 2\left(\frac{\sin 1001t + \sin 999t}{2}\right)$. This, further simplifies into $y = \sin 1001t + 2\sin 100t + \sin 999t$. Thus displacement equation is superimposition three independent SHMs. This conclusion **matches with option (b).**
Hence answer is option (b).
N.B.: This answer can also be arrived at by simple observation of the equation and order of sinusoidal terms.

Question 56: An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that half of its volume is submerged. The new fundamental frequency in Hz is
$300\left(\frac{2\rho-1}{2\rho}\right)^{\frac{1}{2}}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{\frac{1}{2}}$ (c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

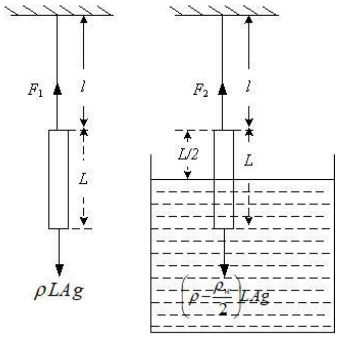


Illustration 56: This is the case of a string of length l kept taught by a force equal to the weight of the object suspended from it. In this at fundamental frequency $\frac{\lambda}{2} = l \Rightarrow \lambda = 2l$, and $f_0 = \frac{v}{\lambda} = \frac{v}{2l}$. Here, velocity of the wave in the string is $v = \sqrt{\frac{F}{\mu}} \Rightarrow f_0 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$. In the problem this fundamental frequency changes with, initially the force, is weight of the object, $F_1 = AL\rho g$ is weight of the object and it would leads to $f_{01} = \frac{1}{2l}\sqrt{\frac{AL\rho g}{\mu}}$…(1) Later, when half-length of the force is reduced due to buoyancy when half-length of the object is immersed $F_2 = AL\rho g - A\frac{L}{2}\rho_w g$. This simplifies into $F_2 = \left(\rho - \frac{\rho_w}{2}\right)ALg$. Thus, $f_{02} = \frac{1}{2l}\sqrt{\frac{\left(\rho-\frac{\rho_w}{2}\right)ALg}{\mu}}$…(2). Accordingly, using (1) and (2) we have $\frac{f_{02}}{f_{01}} = \frac{\frac{1}{2l}\sqrt{\frac{\left(\rho-\frac{\rho_w}{2}\right)ALg}{\mu}}}{\frac{1}{2l}\sqrt{\frac{AL\rho g}{\mu}}} \Rightarrow$

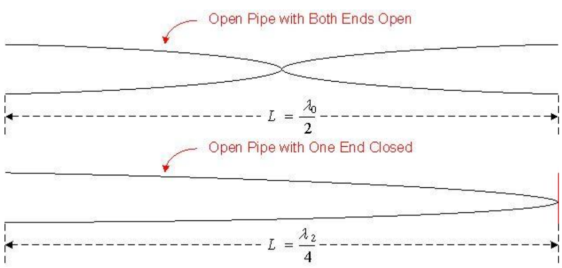
$f_{02} = f_{01}\sqrt{\frac{2\rho-\rho_w}{2\rho}}$. Given that $f_{01} = 300$Hz and using CGS $\rho_w = 1$ we have

$f_{02} = 300\sqrt{\frac{2\rho-1}{2\rho}}$. This derived value **matches with option (a) and hence it correct.**

N.B.: Since $\frac{2\rho-\rho_w}{2\rho}$ the ratio is of quantities having same unit, and $\rho_w = 1$ in CGS , it leads to simplification of calculations.

Question 60: An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that resonance again occurs in nth harmonic. Choose correct option
(a) $n = 3,\ f_2 = \frac{3}{4}f_1$ (b) $n = 3,\ f_2 = \frac{5}{4}f_1$
(c) $n = 5,\ f_2 = \frac{5}{4}f_1$ (d) $n = 5,\ f_2 = \frac{3}{4}f_1$

Illustration 60: Frequency is $f = \frac{v}{\lambda}$ where v is the velocity of the sound in a medium and λ is the wavelength at that velocity.



Natural frequency of an organ pipe open at both ends is $f_0 = \frac{v}{\lambda_0}$, since in this length of the pipe constitutes half wavelength $L = \frac{\lambda_0}{2} \Rightarrow \lambda_0 = 2L$. It leads to $f_{01} = \frac{v}{2L}$. Given that $f_1 = 2f_{01} = \frac{v}{L}$.
But, when organ pipe is closed at one end then $L = \frac{\lambda_2}{4} \Rightarrow \lambda_2 = 4L$ and hence $f_{02} = \frac{v}{\lambda_2} = \frac{v}{4L}$ and its nth harmonics is $f_2 = (2k+1)f_{02}|_{k\in N} \Rightarrow f_2 = (2k+1)\frac{v}{4L} \Rightarrow f_2 = \left(\frac{2k+1}{4}\right)\frac{v}{L} \Rightarrow f_2 = \left(\frac{2k+1}{4}\right)f_1$. It is given that that $f_2 > f_1$for which minimum value of $k = 2$. Accordingly, $f_2 = \left(\frac{2\times2+1}{4}\right)f_1 \Rightarrow \boldsymbol{f_2 = \frac{5}{4}f_1}$. Further, order of harmonic of f_2 with organ pipe closed at one end would be $f_2 = \left(\frac{2\times2+1}{4}\right)f_1$ substituting value of f_1 we have $f_2 = \frac{5}{4}\times\frac{v}{L} \Rightarrow f_2 = 5\times\frac{v}{4L} \Rightarrow f_2 = 5\times f_{02}\big|_{n=5}$. Thus, condition of $f_2 > f_1$ is satisfied at 5th harmonic i.e. at $\boldsymbol{n = 5}$. Thus, $n = 5$ and $f_2 = \frac{5}{4}f_1$ match with those given at option (d).
Hence, answer is option (d).

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Problems are meant to be solved; every solution open doorway to new problems.
This is an endless journey to discovery of nature.
We are, what we are, because of rigorous efforts of countless persons.

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