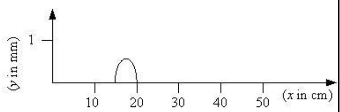
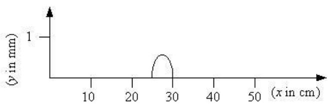
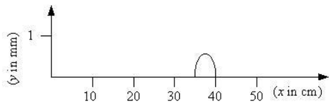
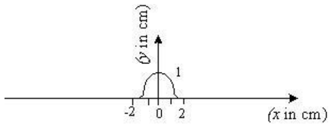
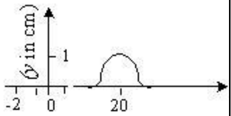
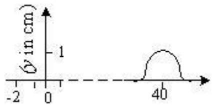
Wave and Motion : Waves in Strings –Subjective Questions (Typical)

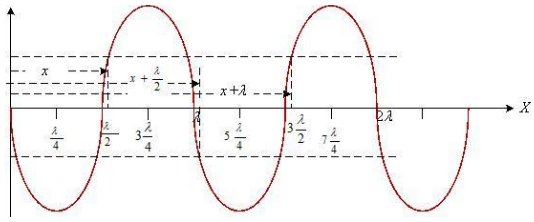
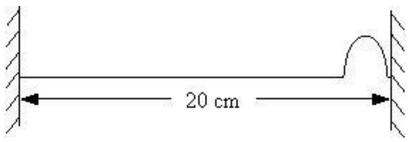
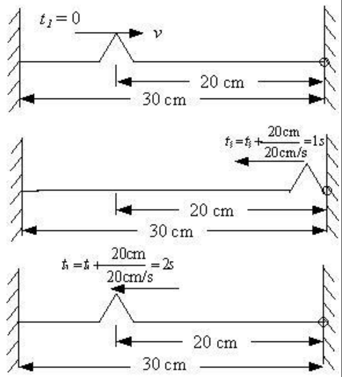
<u>Illustrations Only</u>

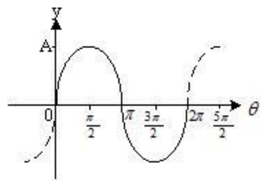
I-01	Since wave is a pulse and it is moving in (-) x direction and hence distance travelled by a pulse which starts at $x = 0$ at $t = 0$ in time $t' = t + \Delta t = 0 + 5 = 5$ s would be $x' = x + \Delta x = 0 + (-v) \times \Delta t = -40 \times 5 = -200$ cm or (-)2 m.
I-02	Each part is being illustrated separately, considering that each particle of the medium performs oscillatory motion about its mean position (in the instant case it is transverse motion along Y-axis)) and while wave transfers energy along the direction of propagation (in the instant case it is X-axis). Accordingly, - **Part (a):** Mathematically $e^p = 1 + p + \frac{p^2}{2!} + \frac{px^3}{3!}$, here since 1 is dimension less and only quantities if same dimension can be added and hence all terms containing p and its exponents must also be dimensionless. Accordingly, $e^{-\left(\frac{x}{a}+\frac{t}{T}\right)^2}$ is dimensionless. Since $[A] = L$ and hence dimensionally $[y] = [A] \Rightarrow [\boldsymbol{A}] = \boldsymbol{L}$. Further, in the given wave equation exponent of e is $\left(\frac{x}{a} + \frac{t}{T}\right)^2$ and it must also be dimensionless. Since the exponent is square of two terms each of them must also be dimensionless, therefore, $\frac{[x]}{[a]} = \frac{L}{[a]} = 1 \Rightarrow [\boldsymbol{a}] = \boldsymbol{L}$. Likewise, $\frac{[t]}{[T]} = \frac{T}{[T]} = 1 \Rightarrow [\boldsymbol{T}] = \boldsymbol{T}$ **Hence, answer is *L, L, T*** **Part (b):** Since wave is travelling along x-axis and hence velocity of the wave is $v = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{d}{dt}\left(Ae^{-\left(\frac{x}{a}+\frac{t}{T}\right)^2}\right) = A\frac{d}{dp}(e^{-p}) \cdot \frac{dp}{dt}\Big\|_{p=\left(\frac{x}{a}+\frac{t}{T}\right)^2} = -Ae^{-\left(\frac{x}{a}+\frac{t}{T}\right)^2} \cdot \frac{dp}{dt} = -y\frac{dp}{dt}$. Now solving for $\frac{dp}{dt}$ we have $\frac{d}{dt}\left(\frac{x}{a} + \frac{t}{T}\right)^2 = \frac{d}{dq}q^2 \cdot \frac{d}{dt}q\Big\|_{q=\frac{x}{a}+\frac{t}{T}} = 2\left(\frac{x}{a} + \frac{t}{T}\right)\frac{d}{dt}\left(\frac{x}{a} + \frac{t}{T}\right) = 2\left(\frac{x}{a} + \frac{t}{T}\right)\left(\frac{1}{a}\frac{dx}{dt} + \frac{1}{T}\right)$. Thus we have $\frac{dy}{dt} = -2y\left(\frac{x}{a} + \frac{t}{T}\right)\left(\frac{1}{a}v + \frac{1}{T}\right)$. Here, $\frac{dy}{dt}$ is the velocity of the particle which is oscillating about its mean position velocity of the wave is $v = \frac{dx}{dt}$. Velocity of the wave remains same far all particles participating in the wave motion and hence when $\frac{dy}{dt} = 0$ the factor $\left(\frac{1}{a}v + \frac{1}{t}\right) = 0$. It leads $v = -\frac{a}{T}$ to. **Thus magnitude of the velocity is $\boldsymbol{\frac{a}{T}}$.** **Part (c):** It has been derived in part (b) that $v = -\frac{a}{T}$. The negative sign indicates that displacement is in (-) ve direction. Hence wave is travelling in (-)ve direction. **Part (d):** The wave function is exponential and hence at $x = 0$ and at $t = 0$ wave-front is at maximum i.e. $y_{max} = A$. Velocity of the wave determined in part (b) is $v = -\frac{a}{T}$. Therefore, at $t = T$ the wave-front will travel $x_1 = v \times T = -\frac{a}{T} \times T \Rightarrow x_1 = -a$. And at $t = 2T$ the wave-front will be at $x_2 = v \times 2T = -\frac{a}{T} \times 2T \Rightarrow x_2 = -2a$. Thus corresponding distances travelled by maximum of the wave at $t = T$ and $t = 2T$ are $-a$ and $-2a$.
I-03	At $t = 0$ the wavefront is at $x_0 = 10$ cm. At $t = 1$ the wavefront is at $x_1 = x_0 + v \times t \Rightarrow x_1 = 10 + 10 \times 1 = 20$ cm.   At $t = 2$ the wavefront is at $x_1 = x_0 + v \times t \Rightarrow x_1 = 10 + 10 \times 2 = 30$ cm.

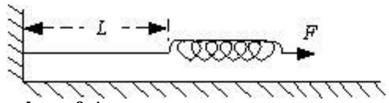
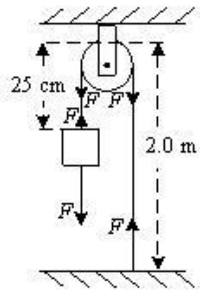
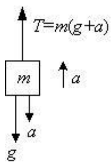
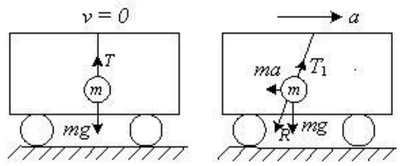
	At $t = 3$ the wavefront is at $x_1 = x_0 + v \times t \Rightarrow x_1 = 10 + 10 \times 3 = 40$ cm. 
I-04	With the given data $y = \frac{0.5^2}{(x-20t)^2+0.5^2} \Rightarrow y = \frac{0.25}{(x-20t)^2+0.25}$. Now at $t = 0$ and $x = 0$, we have $y = \frac{0.25}{0+0.25} = 1$cm; at $x = \pm 1$ cm, we have $y = \frac{0.25}{1+0.25} = \frac{0.25}{1.25} = 0.2$ cm; and $x = \pm 2$ cm, we have $y = \frac{0.25}{4+0.25}$ or $y = \frac{0.25}{4.25} = 0.06$ cm; Accordingly, the wave shape is plotted here.  But at $t = 1$ s the wave would have travelled we have $y = \frac{0.25}{(x-20\times1)^2+0.25}$. Thus at $x = 0$ cm $y = \frac{0.25}{(-20)^2+0.25}$ or $y = \frac{0.25}{400+0.25} \approx 0$, at $x = 10$ cm $y = \frac{0.25}{(10-20\times1)^2+0.25}$; or $y = \frac{0.25}{100.25} \approx 0$at $x = 20$ cm $y = \frac{0.25}{0.25} = 1$. But, along (-)ve direction at $x = -10$ cm $y = \frac{0.25}{(-10-20\times1)^2+0.25} = \frac{0.25}{900.25} \approx 0$; at $x = -20$ the $y = \frac{0.25}{1600.25} \approx 1$. ***This indicates that wave is travelling along (+)x*-direction. Accordingly, shape of the wave is shown here.**  At $t = 2$ s the wave would have travelled we have $y = \frac{0.25}{(x-20\times2)^2+0.25}$. Thus at $x = 30$ cm $y = \frac{0.25}{(-10)^2+0.25}$ or $y = \frac{0.25}{100+0.25} \approx 0$, at $x = 40$ cm $y = \frac{0.25}{(40-20\times2)^2+0.25} = \frac{0.25}{0.25} = 1$ cm Since, it is seen that the wave is travelling along *(+)x*-direction hence progressively amplitudes in this direction have not been calculated. **Thus, shape of the wave is shown here.** 
I-05	Equation of a wave travelling on (+) *x*-direction implies that at any instant *t* the displacement of particle at a distance x must have exited $x = 0$ in phase $f(t') = A\sin\left(\frac{t'}{T}\right)$ where $t' = t - \Delta t$. Here, $\Delta t = \frac{x}{v}$ and *v* is the velocity of the wave. Accordingly, $f(x,t) = f(x,t') = A\sin\left(\frac{t-\Delta t}{T}\right) \Rightarrow f(x,t) = A\sin\left(\frac{t-\frac{x}{v}}{T}\right)$. **Accordingly, $f(x,t) = A\sin\left(\frac{t}{Tv} - \frac{x}{vT}\right)$.**
I-06	Given function is of displacement of a particle from its mean position. Since angle is a dimensionless quantity and so is the sine of the angle. Accordingly,$M^0L^1T^0 = [A]M^0L^0T^0 \Rightarrow [\boldsymbol{A}] = \boldsymbol{L}$. Further, $\left[\sin\left(\frac{x}{a}\right)\right] = M^0L^0T^0$ and $\left[\frac{x}{a}\right] = M^0L^0T^0 \Rightarrow \frac{L}{[a]} = M^0L^0T^0$. **Accordingly, $[\boldsymbol{a}] = \boldsymbol{L}$.** **Thus answer to part (a) is *L, L*.** Given that at $t = 0$ shape of the string is $g(x) = A\sin\left(\frac{x}{a}\right)$ on which a wave is travelling with velocity *v* in (+) *x*-direction. During propagation of wave shape of the wave on a string, which is considered to be elastic, remains unchanged. Therefore, equation of the wave at any point *x* at time *t* would be the wave which existed behind the point by distance $\Delta x = v \times t$ i.e. $x' = x - \Delta x = x - vt$. Therefore, equation of the wave would be $f(x,t) = g(x') = A\sin\left(\frac{x'}{a}\right) \Rightarrow \boldsymbol{f(x,t) = A\sin\left(\frac{x-vt}{a}\right)}$**. This is the answer of Part (b).**
I-07	Given the shape of the string at $t = t_0$ is $g(x,t_0) = A\sin\left(\frac{x}{a}\right)$for a wave travelling with a velocity *v* in (+) *x*-direction. For the wave to be there on the string it would have past a point $x' = x - v(t - t_0)$. Thus, $(x,t) = g(x',t_0) = A\sin\left(\frac{x'}{a}\right)$. **Accordingly, $\boldsymbol{f(x,t) = A\sin\left(\frac{x-v(t-t_0)}{a}\right)}$.**
I-08	Comparing the given equation of wave with the general equation of wave in positive *x*-direction we have $y = (0.10\text{mm})\sin[(31.4\text{m}^{-1})x + (314\text{s}^{-1})t] = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$. Here, pre position the wave to be

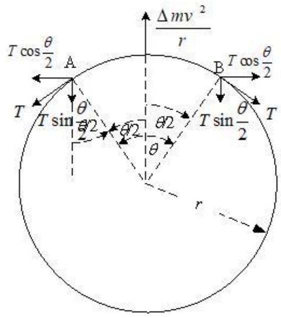
	there at appoint x at time t the distance travelled by it in time t is converted into angular displacement by inserting $\Delta\theta = \frac{2\pi x}{\lambda}$, here λ is wavelength during which the wave subscribes an angular displacement 2π. Thus answers to each part is obtained by comparing the terms in above equation. **Part (a):** We have $(31.4\text{m}^{-1})x = -\frac{2\pi x}{\lambda}$, the (-)ve sign indicates that wave is travelling in a direction opposite to the assumed. **Hence, direction of travel of wave is Negative *x*-direction** . **Part (b):**Amplitude of the wave $A = 0.1 \times 10^{-3} = 10^{-4}$ m and angular velocity $\omega = 2\pi f = 314$ rad/s. Hence, frequency of the wave is $\boldsymbol{f = \frac{314}{2\times 3.14} = 50}$ **Hz**. Velocity of propagation of the wave is $v = \frac{dx}{dt}$ which remains same for all particles of the medium. We have at peak of the wave $\frac{dy}{dt} = 0$. On differentiating the given equation of wave$\frac{dy}{dt} = (0.10\text{mm})\frac{d}{dt}\sin[(31.4\text{m}^{-1})x + (314\text{s}^{-1})t]$, we have $0 = (0.10\text{mm})\cos[(31.4\text{m}^{-1})x + (314\text{s}^{-1})t]\frac{d}{dt}[(31.4\text{m}^{-1})x + (314\text{s}^{-1})t]$. It solves into $\sqrt{1-y^2}\left[(31.4\text{m}^{-1})\frac{dx}{dt} + (314\text{s}^{-1})\right] = 0$. Since, at peak $y = A$, hence this is possible only when $(31.4\text{m}^{-1})\frac{dx}{dt} + (314\text{s}^{-1}) = 0 \Rightarrow \boldsymbol{v} = \frac{314\text{s}^{-1}}{31.4\text{m}^{-1}} = \mathbf{10}$ **m.s^{-1}..** **Thus wavelength is** $\boldsymbol{\lambda} = \frac{v}{f} = \frac{10}{50} = 0.2$ m or **20 cm** **Part (c): Amplitude of the wave** is determined in part (a) and is $\boldsymbol{A = 0.10}$ **mm.** Characteristically, **maximum speed of a portion of string,** as per principle of conservation of energy, while vibrating is at its mean position **is** $\boldsymbol{\frac{dy}{dt}_{max}} = A\omega = (0.10 \times 10^{-3}) \times 314 = \mathbf{3.14}$ **cm/s** **Thus answers are (a) Negative *x*-direction, (b) *v* = 10 m/s, *λ* = 20 cm, *f* = 50 Hz, (c) $\boldsymbol{y_{max}}$ =0.10 mm and maximum velocity of displacement of a portion of string is $\boldsymbol{\frac{dy}{dt}_{max}} = \mathbf{3.14}$ 3.14 cm/s.**
I-09	Generic equation of a wave travelling in (+) x-direction is $y = A\sin(kx - \omega t) = A\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$, or $y = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right) \Rightarrow y = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right)$. Taking each part separately: **Part (a):** With the given that amplitude $A = 0.20$ cm, velocity of the wave $v = 0.20$ m/s and wave-length $\lambda = 2.0$ cm, we have $y = (0.20\text{ cm})\sin\left(\frac{2\pi}{2}x - \frac{2\pi\times 20\times 100}{2}t\right)$, it leads to $\boldsymbol{y = (0.20\text{ cm})\sin\left((\pi\text{cm}^{-1})x - \left(2\pi\times 10^3\text{s}^{-1}\right)t\right)}$**; this is answer of part (a).** **Part (b):** Displacement of particle at $x = 2.0$ cm at time t=0 displacement of particle from its mean position is $y = (0.20\text{ cm})\sin(\pi\text{cm}^{-1})2 = (0.20\text{ cm})\sin 2\pi$, or $\boldsymbol{y = 0.}$ And velocity of particle is $v = \frac{dy}{dt}$ which works out to $v = \frac{d}{dt}(0.20\text{ cm})\sin\left((\pi\text{cm}^{-1})x - \left(2\pi\times 10^3\text{s}^{-1}\right)t\right) = (0.20\text{ cm})\left(2\pi\times 10^3\text{s}^{-1}\right)\cos 2\pi$. It leads to $\boldsymbol{v} = 400\pi \times 1 = \boldsymbol{400\pi}$ **m/s**. **Part (c):** The wave equation can be written in another form as $y = A\,\text{g}(\omega t - kx)$, but the wave being same phenomenon will gave same values. **Thus answers are (a)** $\boldsymbol{y = (0.20\text{ cm})\sin\left[(\pi\text{cm}^{-1})x - (2\pi\times 10^3 s^{-1})\text{t}\right]}$ **(b) Zero, 4*π* m/s (c) No**
I-10	General equation of a wave travelling along x-axis is $y = A\sin(kx - \omega t) = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$ while the given equation is $y = (1.00\text{ mm})\sin\pi\left[\frac{x}{2.0\text{ cm}} - \frac{t}{0.01\text{ s}}\right] \Rightarrow y = (1.00\text{ mm})\sin\left[\frac{\pi x}{2.0\text{ cm}} - \frac{\pi t}{0.01\text{ s}}\right]$. **Part (a):** Comparing the two forms we have $\frac{2\pi}{T} = \frac{\pi}{0.01\text{ s}} \Rightarrow \boldsymbol{T} = 0.02\text{ s} = \mathbf{20\ ms}$. While $\frac{2\pi}{\lambda} = \frac{\pi}{2.0\text{ cm}} \Rightarrow \boldsymbol{\lambda = 4.0}$ **cm**. **Part (b):** Equation of velocity of particle $v_y = \frac{d}{dt}(1.00\text{ mm})\sin\pi\left[\frac{x}{2.0\text{ cm}} - \frac{t}{0.01\text{ s}}\right]$. It leads to a form $v_y = (1.00\text{ mm})\cos\pi\left[\frac{x}{2.0\text{ cm}} - \frac{t}{0.01\text{ s}}\right] \times \frac{d}{dt}\left(\pi\left(\frac{x}{2.0\text{ cm}} - \frac{t}{0.01\text{ s}}\right)\right)$ $\Rightarrow (1.00\text{ mm})\cos\pi\left[\frac{x}{2.0\text{ cm}} - \frac{t}{0.01\text{ s}}\right] \times \left(\pi\left(\frac{1}{2.0\text{ cm}}\frac{d}{dt}x - \frac{1}{0.01\text{ s}}\right)\right)$ $\Rightarrow (-1.00\text{ mm})\cos\pi\left[\frac{1.0\text{ cm}}{2.0\text{ cm}} - \frac{0.01\text{ s}}{0.01\text{ s}}\right] \times \left(\frac{\pi}{0.01\text{ s}}\right)$, since $\frac{d}{dt}x = 0$, as motion at $x = 1.0$ cm (is constant)

	$\Rightarrow (10.0 \text{ cm}) \cos\pi\left[\frac{1}{2}-1\right] = (1.00 \text{ mm}) \cos\frac{\pi}{2} = 0$ $\boldsymbol{v_y = 0}$ **Part (c):** At $t = 0.01$s speed of the particle at $x = 1.0$ cm has been determined in part (b) to be Zero i.e. it is maximum displacement. Further, wave is sinusoidal and hence maximum displacement either (+)/(-) ve would occur at every $\frac{\lambda}{2}$ spacing along the line of propagation . Since the $\lambda = 4$ cm and hence $\frac{\lambda}{2} = 2$ cm. Thus corresponding spacing where velocity of the particle is zero at the instant are $x_1 = x + \frac{\lambda}{2} \Rightarrow x_1 = 1 + 2 = 3$ cm; likewise $x_2 = x_1 + \frac{\lambda}{2} \Rightarrow x_2 = 3 + 2 = 5$ cm and for $x_3 = x_2 + \frac{\lambda}{2} \Rightarrow x_3 = 5 + 2 = 7$ cm. At **all these points speed of particle is Zero.** **Part (d):** Therefore, at instances when speed of particle at $x = 1.0$ cm is to be determined have been rationalized to $t = 0.011\ s$, likewise $t = 0.012\ s$ and $t = 0.013\ s$. Accordingly, speed of the particles: **At $\boldsymbol{t = 0.011\ s}$:** Taking forward derivation in part (b)- $v_y \Rightarrow \frac{d}{dt}\left((1.00 \text{ mm}) \sin\pi\left[\frac{x}{2.0 \text{ cm}} - \frac{t}{0.01 \text{ s}}\right]\right) = (-1.00 \text{ mm}) \cos\pi\left[0.5 - \frac{t}{0.01 \text{ s}}\right] \times \left(\frac{\pi}{0.01 \text{ s}}\right)$ $\Rightarrow 31.4 \times \cos\pi\left[0.5 - \frac{0.011}{0.01}\right] \text{cm.s}^{-1} = 31.4 \times \cos(0.6 \times \pi) \text{ cm.s}^{-1} = -9.7 \text{ cm.s}^{-1}$ **Thus magnitude of speed at all the three instances is 9.7 cm/s** **At $\boldsymbol{t = 0.012\ s}$:** Taking forward derivation at t=0.011 s we have $v_y = 31.4 \times \cos\pi\left[0.5 - \frac{0.012}{0.01}\right] \text{cm.s}^{-1} = 31.4 \times \cos(0.7 \times \pi) \text{ cm.s}^{-1} = -18.455 \text{ cm.s}^{-1}$ **Thus magnitude of speed at all the three instances is 18 cm/s** **At $\boldsymbol{t = 0.013\ s}$:** Taking forward derivation at t=0.011 s we have $v_y = 31.4 \times \cos\pi\left[0.5 - \frac{0.013}{0.01}\right] \text{cm.s}^{-1} = 31.4 \times \cos(0.8 \times \pi) \text{ cm.s}^{-1} = -25.4 \text{ cm.s}^{-1}$ **Thus magnitude of speed at all the three instances is 25 cm/s** **Thus magnitude of speed at the three given instances is 9.7 cm/s, 18cm/s and 25 cm/s.** **N.B.: (1)** In transverse waves particle of medium oscillates about its mean position along X-axis, which is fixed and hence while deriving velocity of particle at any location $v_y = \frac{d}{dt}y(x,t)$, x is a constant and hence term $\frac{d}{dt}x = 0$, wherever it occurs. **(2)** It may not be always necessary to solve each case. Based on data inference of one case can be used for other cases. All that is needed to carefully observe data and the way solution proceeds. This is explicit from solution of part (c) and (d). **(3)** Answer is reported in SDs corresponding to the given data.
I-11	Time taken by particle to move from mean position to extreme position is 5.0 ms, and this is equal to $\frac{T}{4}$ and hence $\frac{T}{4} = 5 \text{ ms} \Rightarrow T = 20 \text{ ms} \Rightarrow \boldsymbol{f} = \frac{1}{T} = \frac{1}{20\times10^3} = \mathbf{50\ Hz.}$ Distance between two consecutive particles at their mean position is $\frac{\lambda}{2}$ and is given to be 2,0 cm. Hence, $\frac{\lambda}{2} = 2.0 \text{ cm} \Rightarrow \boldsymbol{\lambda = 4.0}$ **cm.** Wave speed is $v = \lambda \times f = 4.0 \times 50 = 200$ cm/s and hence ***v* = 2.0 m/s.** **Thus answers are $\boldsymbol{f = 50}$ Hz, $\boldsymbol{\lambda = 4.0}$ cm and *v* = 2.0 m/s.**
I-12	Different quantities required to be determined by close observation of the given figure and are as under – **Part (a):** Peak of the waveform is 1 cm and hence, **amplitude from figure is 1.0 cm** **Part (b):** One complete cycles spans over 4 cm and hence **wavelength ($\boldsymbol{\lambda}$)is 4.0 cm** **Part (c):** The wave number $k = \frac{2\pi}{\lambda}$ and hence $\boldsymbol{k} = \frac{2\pi}{4.0} = \mathbf{1.6}$ **cm^{-1}.** **Part (d):** Frequency of the wave $= \frac{v}{\lambda}$. Using the given value of v and λ determined in part (b) above we have $\boldsymbol{f} = \frac{20}{4} = \mathbf{5}$ Hz. **Thus answer is (a) 1.0 cm (b) 4 cm (c) 1.6 cm^{-1} (d) 5 Hz.**

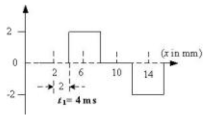
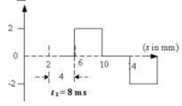
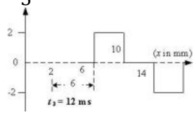
I-13	Given that speed of wave travelling on a string is v = 10 m/s and its time period $T = 20$ ms $= 2.0 \times 10^{-2}$s. Hence, wavelength of the wave is $\lambda = vT = 10 \times (2.0 \times 10^{-2}) = 0.20$m or $\boldsymbol{\lambda = 20}$ **cm. This is answer of Part (a).**  Further, given that displacement of particle at an instant at a particular value of x is $y = 1.5$ cm. The displacement is a vector and it would recur i.e. $y' = y$ at every distance $x' = x + n\lambda\|_{n \in N}$ and would be antiphase i.e. $y' = -y$ at every distance $x' = x + (2n+1)\frac{\lambda}{2}\Big\|_{n \in W}$. Given that $x' = x + 10\ \text{cm}\|_{n=0}$ hence, the displacement is $y' = -y = \mathbf{-1.5}$ **cm. This is answer of Part (b).**
I-14	Speed of a transverse wave on a stretched string is $v = \sqrt{\frac{F}{\mu}}$ m/s, here F is the force on the string in N and mass density of the string is $\mu = \frac{m}{L}$ kg/m. With the given data it calculates to $\mu = \frac{0.005}{0.64}$ kg/m. Accordingly, $v = \sqrt{\frac{8}{\frac{0.005}{0.64}}} = \sqrt{16 \times 64} = 32$ m/s**. Hence, answer is 32 m/s.**
I-15	Velocity of a transverse wave on a stretched string is $v = \sqrt{\frac{F}{\mu}}$ m/s, here F is the force on the string in N and mass density of the string is $\mu = \frac{m}{L}$ kg/m. In IS $\mu = \frac{0.4\times10^{-3}}{1.0\times10^{-2}} = 4.0 \times 10^{-2}$ kg/m. Accordingly, with the given data in the instant case $v = \sqrt{\frac{16}{4.0\times10^{-2}}} = 20$ m/s. The wave-pulse initiated at one end will be reflected at other fixed end, without changing direction of displacement of particles of string. Thus for string to regain its shape after initiation it shall have to travel a distance of d = 40 (=20+20) cm both ways forward and backward. Hence time taken to regain the shape is $\Delta t = \frac{d}{v} = \frac{40\times10^{-2}\text{m}}{20\ \text{m/s}} = 0.02$ s. **Hence, answer of part (a) is $\mathbf{0.02}$ s**. Wave-pulse would travel a distance $x = v \times \frac{\Delta t}{2}$ which calculates to $x = (20\ \text{m/s}) \times \frac{0.2}{2} = 20$ cm, i.e. the wave pulse would travel a distance equal to the length of the string. Accordingly, shape of the string would be as shown in the figure. 
I-16	Initial position of the wave-pulse is shown at $t_1 = 0$ s. The wave with the given velocity travels to the frictionless ring at the other end. Since, the ring does not experience any force to obstruct its transverse motion its overshoots and after reflection at $t_2 = t_1 + \frac{l}{v} \Rightarrow t_2 = 0 + \frac{20\ \text{cm}}{20\ \text{cm/s}} = 1$s, and is shown in the figure.  After reflection the wave starts travelling backwards with same velocity, it reaches its original position $t_3 = t_2 + \frac{L}{v} \Rightarrow t_2 = 1\text{s} + \frac{20\ \text{cm}}{20\ \text{cm/s}} = 2$ s. It is seen that it regains its original shape that was at $t_1 = 0$. Hence, **time taken to regain the shape is 2s, answer of part (a).** Total length of string is given to be 30 cm and the wave pulse initiated at fixed end completes one cycle after return to the same point, following reflection from the free-end attached to the ring, hence total distance covered by wave in $\lambda = 2L = 2 \times 30 = 60$ cm. Hence, time-period of the wave-pulse is $\boldsymbol{T} = \frac{\lambda}{v} = \frac{60}{20} = \mathbf{3}$ **s, is answer of part (b).** Velocity of the wave-pulse is $v = \sqrt{\frac{F}{\mu}}$ m/s. Thus $F = v^2\mu \Rightarrow F = \left(\frac{\frac{20}{100}\text{m}}{\text{s}}\right)^2 \times \frac{0.5\times10^{-3}}{10^{-2}}$. It leads to answer of part (c) $\boldsymbol{F = 2.0 \times 10^{-3}}$**N.**

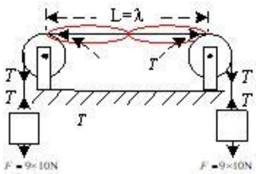
	Thus answers are (a) 2 s (b) 3s (c) $\times 10^{-3}$ N . **N.B.:** The figure given in the question is not reasonably proportional to the length of wire. Making figure reasonably proportionate has been done in the illustration.
I-17	Velocity of a transverse wave in string is $v=\sqrt{\frac{F}{\mu}}$ m/s. Given that two strings of same cross-sectional under tension T have velocities v_1 and v_2 such that $\frac{v_1}{v_2}=2$. Since, both wires carry same tension hence $\frac{v_1}{v_2}=\frac{\sqrt{\frac{T}{\mu_1}}}{\sqrt{\frac{T}{\mu_2}}}$, it leads to $2=\sqrt{\frac{\mu_2}{\mu_1}} \Rightarrow \frac{\mu_2}{\mu_1}=4 \Rightarrow \mu_1=\frac{\mu_2}{4} \Rightarrow \frac{\mu_1}{\mu_2}=\mathbf{0.25}$. **Hence, answer is 0.25.**
I-18	With the given equation of wave $\frac{d}{dt}y=\frac{d}{dt}\left((0.02\text{ m})\sin[(1.0\text{ m}^{-1})x+(30\text{s}^{-1})t]\right)$. Further, $\frac{d}{dt}y=0$, when displacement of a particle is at its peak, but velocity of propagation $v=\frac{dx}{dt}$ remains same across each particle of the medium. Thus $0=(0.02\,m)\cos[(1.0\text{ m}^{-1})x+(30\text{s}^{-1})t]\times\left((1.0\text{ m}^{-1})\frac{dx}{dt}+(30\text{s}^{-1})\right)$. Thus, it leads that factor $\left((1.0\text{ m}^{-1})\frac{dx}{dt}+(30\text{s}^{-1})\right)=0 \Rightarrow \frac{dx}{dt}=\frac{30}{1.0} \Rightarrow v=30$ m/s. Further, $v=\sqrt{\frac{F}{\mu}} \Rightarrow F=\mu\times v^2$. Since given that $\mu=1.2\times10^{-4}$, therefore value of v arrived at above leads to $F=(1.2\times10^{-4})\times(30)^2 \Rightarrow F=0.108\text{ N}\approx 0.11\text{ N}$. **N.B.:** Final result is reported using principle of SDs.
I-19	**Part (a):** Velocity of transverse wave on a string is $v=\sqrt{\frac{F}{\mu}}$. Accordingly, with the given data $v=\sqrt{\frac{90}{0.1}}=\sqrt{900}$. It leads to $\boldsymbol{v}=$ **30 m/s.** Further, it is given that displacement becomes Zero N = 200 times per second, and hence frequency of the wave is $f=\frac{N}{2}=\frac{200}{2}=100$ Hz. Therefore, wavelength of the wave $\lambda=\frac{v}{f}=\frac{30}{100}$. Hence, wavelength is $\boldsymbol{\lambda}=0.3\text{ m}=$ **30 cm.** **Part (b):** General form of equation of a travelling wave with $y=0$ at $x=0$ and $t=0$ is $y=A\sin\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right]$. But, it is given that at $x=0$ and $t=0$ the $y=A$, this is possible when $y=A\sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)+\frac{\pi}{2}\right]$, this can be written as $y=A\cos\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right]$, and on substituting values of wave parameters λ and $T=\frac{1}{f}=\frac{1}{100}$ we have $\boldsymbol{y=A\cos\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]}$, **this is the answer of part (b).**  **Part (c):** Velocity of the particle at x=50 cm at time t = 10 ms is obtained by taking single derivative $v_y=\frac{dy}{dt}$, taking $\frac{dx}{dt}=0$, and extending it to $a_y=\frac{dv_y}{dt}$. Accordingly, $v_y=\frac{d}{dt}A\cos\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]=-A\sin\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]\times\left(-\frac{2\pi}{0.01}\right)$. It leads to $v_y=\frac{2\pi}{0.01}A\sin\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]=\frac{2\pi}{0.01}\times1\times\sin\left[2\pi\left(\frac{50\text{ cm}}{30\text{ cm}}-\frac{0.01\text{ s}}{0.01\text{ s}}\right)\right]$. It solves into $v_y=200\pi\times\cos\frac{4\pi}{3}=-544.14$ cm/s, or $\boldsymbol{v_y}=\mathbf{-5.4}$ **m/s.** And acceleration for the give values is $a_y=\frac{d}{dt}\frac{2\pi}{0.01}\sin\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]$. It solves into $a_y=200\pi\times2\pi\times\cos\left[2\pi\left(\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right)\right]\times\left(-\frac{1}{0.01}\right)=-4\times10^4\times\pi^2\times\cos\left(\frac{2\pi}{3}\right)$. It reduces to $a_y=-19.7\times10^4$ cm/s^2, or $\boldsymbol{a_y}=\mathbf{2.0}$ **km/ s^2.** **Thus part-wise answers are (a) 30 m/s, 30 cm, (b)** $\boldsymbol{y=(1.0\text{ cm})\cos\left[\frac{x}{30\text{ cm}}-\frac{t}{0.01\text{ s}}\right]}$, **and (c) -5.4 m/s, 2.0 km/s^2.**

I-20	Given that spring constant $k = 160$ N/m is stretched by $\Delta l = 1$ cm $= 0.01$ m. Hence, tensile force on the spring is $F = k \times \Delta l = 160 \times 0.01 = 1.6$ N. Accordingly, with the given mass density $\mu = \frac{10\text{ g}}{40\text{ cm}} = \frac{0.01\text{ kg}}{0.4\text{ m}}$. It leads to $\mu = 0.025$ kg/m, velocity of transverse wave in the string is $v = \sqrt{\frac{F}{\mu}} = \frac{1.6}{0.025} = 8$ m/s. Thus time taken wave pulse produced near the wall to reach the string of length $L = 0.4$ m as shown in the figure is $t = \frac{L}{v} = \frac{0.4}{8} \Rightarrow t = 0.05$ s. **Hence, answer is $\boldsymbol{t = 0.05}$ s.** 
I-21	In portion CD of the string $\mu_1 = 8$ g/m $= 8 \times 10^{-3}$ kg/m while force on it is $F_1 = m_1 \times g = 3.2 \times 10$, or $F_1 = 32$ N. Hence velocity in this portion $v_1 = \sqrt{\frac{F_1}{\mu_1}} = \sqrt{\frac{32}{8\times10^{-3}}} = \sqrt{4 \times 10^3} = 63.2$ m/s or **in portion CD velocity of wave is 63 m/s.** In portion AB of the string $\mu_2 = 10$ g/m $= 10 \times 10^{-3}$ kg/m while force on it is $F_2 = (m_1 + m_2)g$. It leads to $F_2 = (3.2 + 3.2) \times 10 \Rightarrow F_2 = 6.4 \times 10 = 64$ N. Hence, velocity in this portion $v_2 = \sqrt{\frac{F_2}{\mu_2}}$ accordingly, $v_2 = \sqrt{\frac{64}{10\times10^{-3}}} = \sqrt{6.4 \times 10^3} = 80$ m/s. i.e. **velocity of wave in portion AB is $\boldsymbol{v_2 = 80}$ m/s.**
I-22	The mass of 2 kg suspended from free end of the string produces a force $F = 2 \times g = 2 \times 10 = 20$ N. Accordingly, velocity of the transverse wave in the string having mass $m = 4.5$ g has linear mass density $\mu = \frac{m}{l_1+l_2} = \frac{4.5\text{ g}}{25\text{ cm} + 2.0\text{ m}} = \frac{4.5\times10^{-3}\text{ kg}}{2.25\text{ m}} = 2 \times 10^{-3}$ kg/m. Accordingly, velocity of transverse wave in the string is $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20}{2\times10^{-3}}} = 100$ m/s. Therefore time taken by the disturbance produced at the floor to reach the pulley is $t = \frac{l_1}{v} = \frac{2}{100} \Rightarrow t = 0.02$ s. **Hence, answer is 0.02 s.**  **N.B.:** In the figure effective length l_1 of string fixed to the floor would terminate at point of contact with the pulley. Therefore length $l_1 \to l_1 - r$, here r is the radius of the pulley. Since, r is not specified and hence a fair assumption $r \ll l_1$ has been made to calculate t.
I-23	Free-body diagram of mass suspended from ceiling of an elevator accelerating upward with $a = 2$ m/s^2 shows that tension in the string is $T = m(g + a) \Rightarrow T = 4(10 + 2) = 48$ N. Linear mass density of the string is $\mu = 19.2 \times 10^{-3}$ kg/m. Accordingly, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{48}{19.2\times10^{-3}}} = 50$ m/s. **Hence, answer is 50 m/s.** 
I-24	When car is at rest its velocity and acceleration are both zero and hence the mass suspended from ceiling will have the string in vertical position and carry a tension $T = mg$. Given that velocity of transverse wave along the string in this case is $v = 60\text{cm/s} \Rightarrow v = 0.6$ m/s. Since, velocity of transverse wave along a string is $v = \sqrt{\frac{gm}{\mu}} \Rightarrow 0.6 = \sqrt{\frac{gm}{\mu}}$…(1)  But, when the car accelerates $v = 62\text{cm/s} \Rightarrow v = 0.62$ m/s while tension in the string is $T_1 = m\sqrt{g^2 + a^2}$. Hence velocity equation of the transverse wave would be $v_1 = \sqrt{\frac{T_1}{\mu}} \Rightarrow 0.62 = \sqrt{\frac{m\sqrt{g^2+a^2}}{\mu}}$…(2) Taking ratios of the final form of (1) and (2) $\frac{0.62}{0.6} = \frac{\sqrt{\frac{m\sqrt{g^2+a^2}}{\mu}}}{\sqrt{\frac{gm}{\mu}}} \Rightarrow \frac{0.62}{0.6} = \frac{\sqrt[4]{g^2+a^2}}{\sqrt{g}} \Rightarrow \frac{g^2+a^2}{g^2} = \left(\frac{0.62}{0.6}\right)^4$. Applying dividend we have $\frac{a^2}{g^2} = \left(\frac{0.62}{0.6}\right)^4 - 1 \Rightarrow a^2 = \left(\left(\frac{0.62}{0.6}\right)^4 - 1\right)g^2 \Rightarrow a^2 = \left(\left(\frac{0.62}{0.6}\right)^4 - 1\right)10^2 = 14 \Rightarrow a = \sqrt{14} = 3.74$, or $a = 3.7$ m/s^2. **Thus, answer using principle of SDs is $\boldsymbol{a = 3.7}$ m/s^2.**

I-25	The rotation of a circular string on a frictionless horizontal plane is a case of hoop-tension T produced in the string. To calculate T, a small element of string AB forming a small angle $\theta \to 0$ at the center is considered. Let, m is the mass if the string. Then mass of the string element $\Delta\theta$ is $\Delta m = \frac{r\theta}{\frac{2\pi r}{m}} = \frac{m\theta}{2\pi}$. Taking free-body diagram of the component of the hoop-tension $T\cos\frac{\theta}{2}$ at ends A and B, being in opposite direction, would cancel out; whereas components $T\sin\frac{\theta}{2}$ at both the ends are unidirectional and hence would balance centrifugal force such that $\frac{\Delta m v^2}{r} = 2T\sin\frac{\theta}{2} \Rightarrow \frac{\left(\frac{m\theta}{2\pi}\right)v^2}{r} = 2T\sin\frac{\theta}{2} \Rightarrow T = \left(\frac{mv^2}{2\pi r}\right)\frac{\frac{\theta}{2}}{\sin\frac{\theta}{2}}$. Since, linear mass density of the string is $\mu = \frac{m}{2\pi r}$, and $\theta \to 0$ hence $\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}} \to 1$. Accordingly, $T = \left(\frac{m}{2\pi r}\right)v^2 = \mu v^2$. Velocity of transverse wave in a taught string is $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu v^2}{\mu}} = v$. **Hence, answer is v.** 
I-26	Each part is be solved separately - **Part (a):** Let us take an element $\Delta x \to 0$ of the rope having linear mass density μ at a height x above its lower end. Therefore, tension in the rope element Δx is $T = (\mu x)g$. Hence, velocity of transverse wave through the element shall be $v = \sqrt{\frac{T}{\mu}} \Rightarrow v = \sqrt{\frac{(\mu x)g}{\mu}} \Rightarrow v = \sqrt{gx}$. **Hence, answer part (a) is $\sqrt{gx}$.** **Part (b):** Time taken by transverse wave to traverse through the rope is $\Delta t = \frac{\Delta x}{v} \Rightarrow \Delta t = \frac{\Delta x}{\sqrt{gx}}$. Therefore time taken by the sudden sideways jerk to reach ceiling through rope of length L works out to $\int_0^T dt = \int_0^L \frac{dx}{\sqrt{gx}} \Rightarrow T = \frac{1}{\sqrt{g}}\int_0^L x^{-\frac{1}{2}}dx \Rightarrow T = \frac{1}{\sqrt{g}}\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^L \Rightarrow T = \frac{2\sqrt{L}}{\sqrt{g}} \Rightarrow T = \sqrt{\frac{4L}{g}}$. **Hence, answer part (a) is $\sqrt{\frac{4L}{g}}$.** **Part (c):** A particle when dropped from ceiling takes time t to drop through a height $L-x$ can be calculated using Second equation of motion as $L - x = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(L-x)}{g}}$…(1) And from solution in part (b) for a pulse generated at bottom to intercept the particle is $t = \sqrt{\frac{4x}{g}}$ …(2). Combining (1) and (2) we have $\sqrt{\frac{4x}{g}} = \sqrt{\frac{2(L-x)}{g}} \Rightarrow \frac{4x}{g} = \frac{2(L-x)}{g} \Rightarrow 4x = 2(L-x) \Rightarrow x = \frac{L}{3}$. **Hence, answer of part (c) is $\frac{L}{3}$ from bottom.**
I-27	Velocity of transverse wave in string A is $v_A = \sqrt{\frac{T_A}{\mu}} = \sqrt{\frac{4.8}{1.2\times10^{-2}}} = 20\ \text{m/s}$ and in string B which has same linear mass density as that of string A, the velocity is $v_B = \sqrt{\frac{7.5}{1.2\times10^{-2}}} = 25\ \text{m/s}$. Let pulses in both the strings meet at a time t_A. Then on pulse would travel on string A a distance equal to $x_A = t_A v_A = t_A \times 20 = 20t_A$. And on string B on which pulse starts at t = 20 ms, then $x_B = (t_B - t)v_A$, it solves into $x_B = (t_B - 20\times10^{-3})25$. Since both pulses meet at $T_A = T_B$, they will have travelled equal distances. Accordingly, $x_A = x_B \Rightarrow 20t_A = (t_A - 20\times10^{-3})25 \Rightarrow 5t_A = 0.500 \Rightarrow t_A = 0.1$ s or **100 ms.** Therefore, distance travelled by waves before they meet is $x_A = 0.1 \times 20 \Rightarrow$ **$x_A = x_B = 2$ m. Hence, answer is 100 ms and 2 m.**

I-28	Average power of a transverse wave in a string is $P_{av} = 2\pi^2\mu v A^2\nu^2$, here μ is the linear maqss density of the string, $v = 100$ m/s is the velocity of the transverse wave, $A = 0.50$ mm is the amplitude of the wave and $\nu = 100$ Hz is the frequency of the wave. Here, linear mass density is not given by with given velocity of the wave and tension $T = 100$ N keeping the string stretched we can determine $v = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{T}{v^2}$, the formula of average poser gets modified as $P_{av} = 2\pi^2 \frac{T}{v^2} v A^2\nu^2 \Rightarrow$ as $P_{av} = \frac{2\pi^2 T A^2\nu^2}{v}$. Accordingly using the given data $P_{av} = \frac{2\pi^2 100(0.50\times10^{-3})^2 100^2}{100} = 2\pi^2 \times 25 \times 10^{-4} = 493 \times 10^{-4}$ or $P_{av} = 49.3 \times 10^{-3}$W or **answer is 49 mW.**
I-29	Average power of a transverse wave in a string is $P_{av} = 2\pi^2\mu v A^2\nu^2$, here $\mu = 6\ g/m = 6 \times 10^{-3} kg/m$ is the linear mass density of the string, vm/s is the velocity of the transverse wave, $A = 1$ mm $= 10^{-3}$ m is the amplitude of the wave and the frequency of the wave is $\nu = 200$ Hz . Further it is given that tension in the string is $T = 60$ N , thus we can determine $v = \sqrt{\frac{T}{\mu}} \Rightarrow v = \sqrt{\frac{60}{6\times10^{-3}}} = 100$ m/s, Now each part is being solved separately- **Part (a):** Substituting the given and derived data $P_{av} = 2\pi^2 \times (6\times10^{-3}) \times 100 \times (10^{-3})^2 \times 200^2$ or $P_{av} = 473.7 \times 10^{-3}$ W = 0.47 W. **Thus answer of part (a) is 0.47 W.** **Part (b):** Energy is time $E = \int p.dt = P_{av} \times \Delta t$. With the given velocity of the wave time taken by a wave to traverse over a length 2.0 m of the string is $\Delta t = \frac{l}{v} \Rightarrow \Delta t = \frac{2}{100} = 0.02$ s. Hence, energy in the given portion of the string is $\boldsymbol{E} = 0.47$ W × 0.02 s = $\mathbf{9.4}$ **mJ**. **Hence, answers are (a) $\boldsymbol{P_{av}} = \mathbf{0.47}$ W, and (b) 9.4 mJ.**
I-30	Each part of the question is being solved separately – **Part (a)**: Velocity of transverse wave along a string of linear mass density $\mu = 0.01$ kg/m under tensile force $T = 49$ N is $v = \sqrt{\frac{T}{\mu}} \Rightarrow \boldsymbol{v} = \sqrt{\frac{49}{0.01}} = 7\mathbf{0}$ **m/s**. Given that frequency of the wave is $\nu = 440$ Hz and hence wavelength of the wave on the string is $\lambda = \frac{v}{\mu} = \frac{70}{440} = 0.159$ m or $\boldsymbol{\lambda} = \mathbf{16}$ **cm**. **Part (b):** Maximum speed of a particle of string is $V_{max} = A\omega = A(2\pi\nu) = (0.50 \times 10^{-3}) \times 2\pi \times 440 =$ 1.38 m/s or $\boldsymbol{V_{max}} = \mathbf{1.4}$ m/s, this will occur when particle is at its mean position. And maximum acceleration would be $\boldsymbol{a_{max}} = A\omega^2 = A(2\pi\nu)^2 = (0.50 \times 10^{-3}) \times (2\pi \times 440)^2 = \mathbf{3.8}$ **km/s²**. **Part (c):** Average rate of transfer of energy is nothing but average power of the wave $P_{av} = 2\pi^2\mu v A^2\nu^2$. With the given and derived data $P_{av} = 2\pi^2(0.01)(70)(0.50 \times 10^{-3})^2(440)^2 = \mathbf{0.67}$ **W**. **Hence, Part-wise answers are (a) 70 m/s, 16 cm (b) 1.4 m/s, 3.8 km/s (c) 0.67 W.**
I-31	Let displacement equation of two waves are $y_1 = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ and another wave having equal frequencies, wavelengths and amplitudes $A = 4$ mm, but phase difference of $\varphi = 90^0$ the displacement equation is $y_2 = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \frac{\pi}{2}\right] = A\cos\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$. Accordingly, $A' = \sqrt{y_1^2 + y_2^2} = A\sqrt{2}$. Substituting the given value the **resultant amplitude is $\mathbf{4\sqrt{2}}$ mm, is the answer.**

I-32	With the given velocity of the wave $v = 50$ cm/s, the wave will advance along X-axis in time: (i) $t_1 = 4$ ms through a distance $x_1 = vt_1 \Rightarrow x_1 = 50 \times (4 \times 10^{-3}) = 0.2$ cm  (ii) $t_2 = 8$ ms through a distance $x_2 = vt_2 \Rightarrow x_2 = 50 \times (8 \times 10^{-3}) = 0.4$ cm  (iii) $t_3 = 4$ ms through a distance $x_3 = vt_3 \Rightarrow x_3 = 50 \times (12 \times 10^{-3}) = 0.6$ cm 
I-33	Two waves with frequencies $f_1 = f_2 = f$ and wave lengths $\lambda_1 = \lambda_2 = \lambda$ therefore equations of displacement of the two waves shall be $y_1 = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$ and $y_2 = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t+\Delta t}{T}\right)\right]$. Taking solution of each part separately - **Part (a):** Since frequency of both the waves is $f = 100$ Hz $\Rightarrow T = \frac{1}{100} = 0.01$ s $= 10$ ms. Since $T = 0.01$ s $\rightarrow 2\pi$ and hence second wave delayed 0.015 s later than the first wave would correspond $\frac{0.015}{0.01} = \frac{\varphi_1}{2\pi} \Rightarrow \varphi_1 = 3\pi$. **Thus the phase difference is 3π is answer of part (a).** **Part (b):** The wavelength = 2.0 cm → 2π . Hence, starting distance of 4.0 cm of second wave behind the first wave would correspond to $\frac{4}{2} = \frac{\varphi_2}{2\pi} \Rightarrow \varphi_2 = 4\pi$. **Thus the phase difference in part (b) is 4π is the answer.** **Part (c):** In this case resultant amplitude is to be calculated with phase difference taken in each part separately. In Part (a) hence $y_1 = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$ and $y_2{}' = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + 3\pi\right] = -A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$. Hence, the resultant wave is $\boldsymbol{y_{R-1}} = y_1 + y_2{}' \Rightarrow y_{R-1} = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] - A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = \mathbf{0}$. In part (b) $y_1{}' = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + 4\pi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$ and $y_2 = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$. Hence , the resultant wave is $\boldsymbol{y_{R-2}} = y_1{}' + y_2 \Rightarrow y_{R-1} = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] + A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = 2A = \mathbf{4.0}$ **cm. Hence, answer is Zer0 and 4.0 cm.** **Hence answers are (a) 3π (b) 4π (c) Zero, 4.0 mm** **N.B.:** Part (c) has to be read carefully as it asks for resultant wave produced by case (a) and (b) separately and not the combined effect.
I-34	In a stretched string half of wavelength i.e. $\frac{\lambda}{2}$ of fundamental wave is spaced. With the given speed $v = 60$ m/s of the transverse wave on a string of 1 m, time taken to travel the length $\frac{\lambda}{2}$ is $\Delta t = \frac{1}{60}$s. Hence, time taken by the wave to travel its wavelength λ is $T = 2 \times \Delta t \Rightarrow T = 2 \times \frac{1}{60} = \frac{1}{30} \Rightarrow f = \frac{1}{T} = 30$ Hz. **Hence answer is 30 Hz.**
I-35	In a stretched string at fundamental frequency $L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$. Given that fundamental frequency is $f_1 = 100$ Hz and hence time period of the fundamental frequency is $T_1 = \frac{1}{f_1} = \frac{1}{100}$. Let v is the velocity of the wave and time taken by the wave to travel the length L of string is $t = \frac{T_1}{2} = \frac{1}{200}$ Accordingly velocity of the wave of the fundamental frequency is $v = \frac{L}{t} = \frac{2.00}{\frac{1}{200}} = 400$ m/s.

	Given that the string is stretched with a tensile force $F = 160\ N$. Further we know that $v = \sqrt{\frac{F}{\mu}} \Rightarrow \mu = \frac{F}{v^2}$. Substituting the given and derived values $\mu = \frac{F}{v^2} = \frac{160}{400^2} = 0.001$ kg/m or $\boldsymbol{\mu} = \mathbf{1.00}$ **g/m is the answer.**
I-36	In case of a wire stretched between two ends its linear mass density is $\mu = \frac{m}{L} = \frac{4.0\times10^{-3}}{80\times10^{-2}} \Rightarrow \mu = 5 \times 10^{-3}$ kg/m. Further, it is given that the wire is stretched with a force $T = 50$ N. Therefore, velocity of the wave is $v = \sqrt{\frac{F}{\mu}} \Rightarrow v = \sqrt{\frac{50}{5\times10^{-3}}} = 100$ m/s. In stretched wire its length is half-of-wavelength of the fundamental transverse wave its length $L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$ and $T_1 = \frac{\lambda_1}{v} = \frac{2L}{v} \Rightarrow f_1 = \frac{1}{T_1} = \frac{v}{2L} = \frac{100}{2\times0.8} = 62.5$ Hz. Hence frequency of the fourth harmonic of the fundamental frequency is $\boldsymbol{f_4} = 4f_1 = 4 \times 62.5 = \mathbf{250}$ **Hz** and wavelength of the fourth harmonic $\boldsymbol{\lambda_4} = \frac{\lambda_1}{4} = \frac{2L}{4} = \frac{2\times80}{4} = \mathbf{40}$ **cm**. Hence, **answers are 250 Hz, 40 cm.**
I-37	Linear mass density of the string is $\mu = \frac{m}{L}$ and with given data $\mu = \frac{6\times10^{-3}}{90\times10^{-2}} = 6.67 \times 10^{-3}$ kg/m. Fundamental frequency is given to be $\nu = 261.63$ and velocity of the wave is $v = \nu \times \lambda = \nu \times 2L = 261.63 \times 2 \times 0.9$, it calculates to 471 m/s. Velocity of transverse wave on a string is $v = \sqrt{\frac{F}{\mu}}$, hence tension in the string shall be $F = \mu v^2 = (6.67 \times 10^{-3}) \times (471)^2 = 1480$ N. **Hence, answer is 1480 N.** **N.B.:** In case concept of SDs is applied on intermediate results, answer could be different but it would be in the same range.
I-38	Distance between bridges $L = 1.50\ \text{m} = \frac{\lambda}{2} \Rightarrow \lambda = 2L = 2 \times 1.50 = 3.00$ m. Given that frequency of the second harmonic is $f_2 = 256$ Hz and hence $f_1 = \frac{f_2}{2} = \frac{256}{2} = 128$ Hz . Hence, speed of the transverse wave on the wire is $\boldsymbol{v} = f_1 \times \lambda = 128 \times 3.00 = \mathbf{384}$ **m/s. Thus answer is 384 m/s.**
I-39	 Conceptual representation of a string vibrating stretched between two pulleys with a node at its mid-point is shown in the figure. Thus, wavelength of the transverse wave on the string is $\lambda = L = 1.5$ m. The string as shown on the figure will be stretched with a force $F = 9\text{kg} \times 10\frac{\text{m}}{\text{s}^2} = 90$ N and linear mass density $\mu = \frac{m}{L}$ this with the given data is $\mu = \frac{12\times10^{-3}}{1.5} = 8 \times 10^{-3}$ kg/m. Accordingly, velocity of the transverse string $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{90}{8\times10^{-3}}} = 112.5$ m/s. Further, $v = f\lambda \rightarrow f = \frac{v}{\lambda} = \frac{112.5}{1.5} = 75$ Hz. **Hence, answer is 70 Hz.**
I-40	In case of a string of linear mass density $\mu = \frac{40\times10^{-3}}{1} = 0.04$ kg/m of length L with one end fixed and the other attached to a tuning fork and string vibrates in four loops. Since two loops form one wavelength and hence with four loops along the length of wire $L = 2\lambda \Rightarrow \lambda = \frac{L}{2}$. Since, frequency of wave is that of the tuning fork $f = 128$ Hz. Accordingly, velocity of the wave is $v = f\lambda = f \times \frac{L}{2}$. Thus, with the given data $v = 128 \times \frac{1}{2} = 64$ m/s. Further, we know $v = \sqrt{\frac{F}{\mu}} \Rightarrow F = \mu v^2 \Rightarrow \boldsymbol{F} = 0.04 \times (64)^2 = 183.8$ N or **164 N**. **Hence, answer is 164 N.**
I-41	Wire is resonant at two frequencies $f_1 = 240$ Hz and $f_2 = 320$. Hence, maximum fundamental frequency of the wire having resonant frequencies $f_1 = 240 = 80 \times 3$ Hz and $f_2 = 320 = 80 \times 4$, shall be HCF of the two. Since, HCF is 80 and hence **80 Hz is the answer of part (a).** Length (L) of string fixed at both ends is $L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$, where λ is wavelength of the transverse wave on

	the string. Given that speed of the wave is $v = 40$ m/s and hence since $2L = \lambda = \frac{v}{f} = \frac{40}{80} = 0.50$ m. It resolves to $L = \frac{0.50}{2} = 0.25$ m or **25 cm is the answer of part (b).** **Thus answers are (a) 80 Hz (b) 25 cm.**
I-42	When distance between two consecutive nodes for frequency f_1 is $\frac{\lambda_1}{2} = 2$ cm $\Rightarrow \lambda_1 = 4$ cm $= 40$ mm . Where as when distance between two consecutive nodes for frequency f_2 is $\frac{\lambda_2}{2} = 1.6$ cm $\Rightarrow \lambda_2 = 3.2$ cm $=$ 32 mm. In this case LCM is the minimum length which will allow both the frequencies f_1 and f_2 on the string. Since, $\lambda_1 = 40 = (2^3 \times 5)$ mm and $\lambda_2 = 40 = 2^5$ mm . Accordingly, have requisite wavelength is $\lambda = 2^5 \times 5 = 160$ mm. Minimum length of string for this wavelength is $L = \frac{\lambda}{2} \Rightarrow L = \frac{160}{2}$ mm $= 8.0$ cm. **Hence, answer is 8.0 cm.**
I-43	Given that velocity of transverse wave is $v = 220$ m/s with three loops of wavelength $\frac{\lambda}{2}$ accordingly length of wire is $L = 3 \times \frac{\lambda}{2}$ of frequency $f = 660$ Hz. Hence $\lambda = \frac{v}{f} \Rightarrow \lambda = \frac{220}{660} = \frac{1}{3}$ m. Hence wavelength of the string $L = 3 \times \frac{1}{3} = 0.5 -$ m or 50 cm. **Thus answer of part (a) is 50 cm.** Given the maximum amplitude the generic form of equation the standing wave on the string is $y = A \sin\left[2\pi \frac{x}{\lambda}\right] \times \cos[(2\pi f)t]$. Substituting the given and derived data $y = (0.5 \text{ cm}) \sin[(6\pi \text{ m}^{-1}) \times x] \times \cos[(1320\pi \text{s}^{-1})t]$, Rationalizing all units of length to cm we have $\boldsymbol{y = (0.5 \text{ cm}) \sin\left[(0.06\pi \text{ cm}^{-1}) \times x\right] \times \cos\left[(1320\pi \text{s}^{-1})t\right]}$ **is solution of part (b)**
I-44	For a guitar for $f_1 = 196$ Hz the length of string of a guitar is $l_1 = 30$ cm $= 0.3$ m. Velocity of transverse wave on the string shall be constant for the same string but $\lambda_1 = 2l_1$ and $\lambda_1 = \frac{v}{f_1} = \frac{l_1}{2} \Rightarrow f_1 l_1 = 0.3 \times 196$ or $f_1 l_1 = 58.8$ m.Hz, which is a constant for the string under consideration. Therefore, for all the given frequencies on the string of guitar $f_1 l_1 = f_2 l_2 = f_3 l_3 = f_4 l_4 = f_5 l_5$. Accordingly, (i) $l_2 = \frac{58.8}{f_2} = \frac{58.8}{220} = 0.267$ m or 26.7 cm. (ii) $l_3 = \frac{58.8}{f_3} = \frac{58.8}{247} = 0.238$ m or 23.8 cm. (iii) $l_4 = \frac{58.8}{f_4} = \frac{58.8}{262} = 0.224$ m or 22.4 cm. (iv) $l_5 = \frac{58.8}{f_5} = \frac{58.8}{294} = 0.200$ m or 20.0 cm. **Hence, lengths of strings corresponding to the given frequencies are 26.7 cm, 23.8 cm, 22.4 cm, and 20.0 cm is the answer.**
I-45	Frequency of the nth harmonic of a fundamental frequency $f_1 = 200$ Hz is $f_n = nf_1$. Given that $f_{max} = 14 \times 10^3$ Hz. Hence, highest harmonic is integer quotient $n = \text{int}\left(\frac{f_{max}}{f_1}\right) = \text{int}\left(\frac{14\times10^3}{200}\right) = 70$. Hence, highest harmonic that is audible and can be played on the string is **70^{th}, this is the answer.**
I-46	Each part is being solved separately – **Part (a):** With the given resonant frequencies f_1 =90, f_2 =150, and f_3 =210 Hz, highest fundamental frequency is HCF of the three. We have $f_1 = 90 = 2 \times 3^2 \times 5$, $f_2 = 150 = 2 \times 3 \times 5^2$ and for the third frequency $f_3 = 210 = 2 \times 3 \times 5 \times 7$. Accordingly, fundamental frequency is the HCF $\boldsymbol{f} = 2 \times 3 \times 5 = \mathbf{30}$ **Hz.** **Part (b):** The order of harmonic of the given frequencies are - **(i)** For $f_1 = 90$ Hz we have $n_1 = \frac{90}{30} = 3$ i.e. **3^{rd} harmonic.** **(ii)** For $f_2 = 150$ Hz we have $n_2 = \frac{150}{30} = 5$ i.e. **5^{th} harmonic**. **(iii)** For $f_2 = 210$ Hz we have $n_3 = \frac{210}{30} = 7$ i.e. **7th harmonic**.

Part (c): The order of harmonic of the given frequencies are -

(i) Overtone of $f_1 = 90$ Hz is $O_1 = n_1 - 1 = 3 - 1 = 2$ i.e. **2nd overtone.**

(ii) Overtone of $f_2 =$ 150 Hz is $O_2 = n_2 - 1 = 5 - 1 = 2$ i.e. **4th overtone.**

(iii) Overtone of $f_3 =$ 210 Hz is $O_3 = n_3 - 1 = 7 - 1 = 6$ i.e. **6th overtone.**

Part (d): Given length of the string $L = 80$ cm $= 0.80$ m and for fundamental frequency $\lambda_1 = 2L$. Hence speed of transverse wave on the string is $\boldsymbol{v} = f_1\lambda_1 = 2f_1L = 2 \times 30 \times 0.8 =$ **48 m/s.**

Hence, answers are (a) 30 Hz (b) 3rd, 5th and 7th (c) 2nd, 4th and 6th (d) 48 m/s.

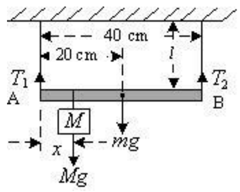
I-47

Let distance between the two supports be L. Ratio of tensions in the two wires are $\frac{F_1}{F_2} = \frac{2}{1}$, ratio of radius is $\frac{r_1}{r_2} = \frac{3}{1}$ and ratio of densities is $\frac{\rho_1}{\rho_2} = \frac{1}{2}$. Linear mass density of wire is $\mu = \pi r^2$ and hence its ration for the two wires is $\frac{\mu_1}{\mu_2} = \frac{\pi r_1^2 \rho_1}{\pi r_2^2 \rho_2} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{r_1^2}{r_2^2}$. Further, wavelength of a transverse wave on a string stretched between two fixed ends is $\lambda = 2L$ and hence for the two strings under consideration $\lambda_1 = \lambda_2 = 2L$.

Velocity of a transverse wave is $= \sqrt{\frac{F}{\mu}}$, while frequency of the transverse wave is $f = \frac{v}{\lambda} \Rightarrow v = f\lambda \Rightarrow v = 2fL$. Hence, $\frac{v_1}{v_2} = \frac{2f_1L}{2f_2L} \Rightarrow \frac{v_1}{v_2} = \frac{f_1}{f_2}$. Accordingly, $\frac{f_1}{f_2} = \frac{\sqrt{\frac{F_1}{\mu_1}}}{\sqrt{\frac{F_2}{\mu_2}}} \Rightarrow \frac{f_1}{f_2} = \sqrt{\left(\frac{F_1}{F_2}\right) \times \left(\frac{\mu_2}{\mu_1}\right)} \Rightarrow \frac{f_1}{f_2} = \sqrt{\left(\frac{F_1}{F_2}\right) \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{\rho_2}{\rho_1}\right)}$.

Using the given and derived data we have $\frac{f_1}{f_2} = \sqrt{\left(\frac{2}{1}\right) \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{1}\right)} = \sqrt{\left(\frac{2}{3}\right)^2} = \frac{2}{3}$ or $f_1 : f_2 :: 2:3$. **Hence answer is 2:3.**

I-48



Let length of wires supporting the horizontal rod of mass m from a horizontal ceiling be l at both ends A and B. The bare uniform rod would produce equal tension $T = \frac{mg}{2}$ in both the strings. But, with an external mass M the tension in wires at end A and B would change to T_A and T_B respectively. Let tuning fork excites wire on the right end B at its fundamental frequency f_B. Since, vibration on the wire on the left end A is first overtone and hence $f_A = 2f_B \Rightarrow \frac{f_A}{f_B} = \frac{2}{1}$…(1). Hence, wavelengths of fundamental waves through both the identical wires having linear mass density μ are $\lambda_B = l$ and wavelengths of first overtone is $\lambda_A = l$. Thus we have $\frac{\lambda_A}{\lambda_B} = 1$…(2).

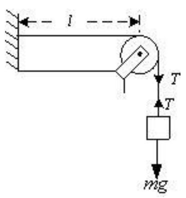
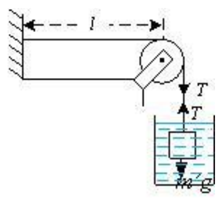
Speed of transverse wave on on a stretched string $v = \sqrt{\frac{T}{\mu}}$, and hence $\frac{v_A}{v_B} = \frac{\sqrt{\frac{T_A}{\mu}},}{\sqrt{\frac{T_B}{\mu}},} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{T_A}{T_B}}$…(3) Since, $v = f\lambda$ and hence $\frac{v_A}{v_B} = \frac{f_A\lambda_A}{f_B\lambda_B} \Rightarrow \frac{v_A}{v_B} = \frac{f_A}{f_B} \times \frac{\lambda_A}{\lambda_B}$. Using equations (1) and (2) $\frac{v_A}{v_B} = 2 \times 1 = 2$…(4).

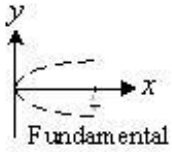
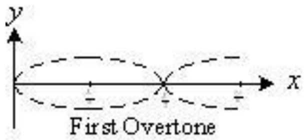
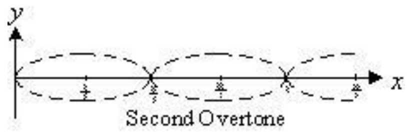
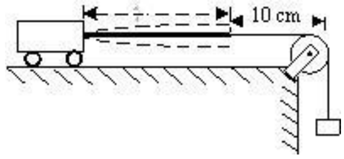
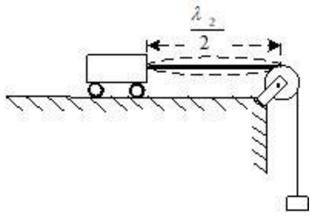
Combining (3) and (4) we have $\sqrt{\frac{T_A}{T_B}} = 2 \Rightarrow T_A = 4T_B$…(5)

Next is to determine tensions T_1 and T_2 which are under vertical equilibrium with total weight such that $T_A + T_B = (m + M)g$ …(3). Using the given data $T_A + T_B = (1.2 + 4.8)10 \Rightarrow T_A + T_B = 60$ N…(6).
Using equations (5) and (6) we have $5T_B = 60 \Rightarrow T_B = 12$ N and $T_A + 12 = 60 \Rightarrow T_A = 48$ N.
Now considering rotational equilibrium, moments at end A we have $T_B \times L = mg \times \frac{L}{2} + Mg \times x$… (7), Substituting the given and derived values, $12 \times 0.4 = 1.2 \times 10 \times \frac{0.4}{2} + 4.8 \times 10 \times x \Rightarrow 4.8 = 2.4 + 48x$, or $x = \frac{2.4}{48} = 0.05$ m or $\boldsymbol{x} =$ **5 cm. is the answer**.

N.B.: Wire at both ends with equal length shall have half of the wavelength of waves generated at both ends. Yet the velocity of the transverse waves in the two wires supported by tensions in the wires of same linear mass density would be decide the fundamental frequency and first overtone as stipulated in the wire.

I-49	Mass density in IS is $\mu = \pi r^2 \rho$ and hence for steel wire $\mu_S = \pi A_S \rho_S = 1 \times 7.8 \times 10^{-3} = 7.8 \times 10^{-3}$ kg/m and for aluminum wire $\mu_A = A_A \rho_A = 3 \times 2.6 \times 10^{-3} = 7.8 \times 10^{-3}$ kg/m. Further, velocity of transverse wave through a stretched string is $v = \sqrt{\frac{F}{\mu}}$. It is seen that both the wires carry same tension $F = 40$ N and have same linear mass density $\mu = \mu_A = \mu_B = 7.8 \times 10^{-3}$ Hence, velocity of wave in steel wire is $v = v_S = v_A = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{40}{7.8 \times 10^{-3}}} = 71.6$ m/s. With the change of medium velocity of a wave changes but frequency remains unchanged and let frequency of the tuning fork is f. Thus, the transverse wave through the two wires in series shall have same wavelengths $\lambda_A = \lambda_S = \frac{v}{f} = \lambda$, since $v = \lambda f$ and moreover for minimum frequency, leading to minimum loops of stationary wave, over a given length requires maximum wavelength which will be HCF(80cm,60cm)=20 cm. Since each loop is of half the wavelength and hence $\frac{\lambda}{2} = 20 \Rightarrow \lambda = 40$ cm or 0.4 m. Hence minimum frequency of tuning fork causing vibration in the wire that satisfies the condition of node at the joint is $f = \frac{v}{\lambda} = \frac{71.6}{0.4} = 179$ Hz. **Hence answer is 180 Hz.** **N.B.:** Though calculated value is 179 Hz yet the answer is reported as 180 Hz i.e. 2 SDs and is in line with the SDs of the given data.
I-50	Illustration of each part is done separately- **Part (a):** Given that string is vibrating at fundamental frequency and hence length L of the string constitutes half wavelength i.e. $L = \frac{\lambda}{2} \Rightarrow \boldsymbol{\lambda = 2L}$ **.** And wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$. Thus wave number $\boldsymbol{k} = \frac{\pi}{L}$**. Thus answers of part (a) are** $\boldsymbol{2L}$ **,** $\boldsymbol{\frac{\pi}{L}}$ **..** **Part (b):** Equation of a standing wave is $y = A \sin\left(\frac{2\pi x}{\lambda}\right) \times \sin\left(\frac{\omega t}{T}\right) \Rightarrow y = A \sin\left(\frac{2\pi x}{2L}\right) \times \sin\left(\frac{2\pi t}{T}\right)$. It leads to $\boldsymbol{y = A \sin\left(\frac{2\pi x}{2L}\right) \times \sin(2\pi v t)}$**, here frequency** $\boldsymbol{v = \frac{1}{T}}$**, is answer of part (b).** **Thus answers are (a)** $\boldsymbol{2L}$ **,** $\boldsymbol{\frac{\pi}{L}}$ **(b)** $\boldsymbol{y = A \sin\left(\frac{\pi x}{L}\right) \times \sin(2\pi v t)}$**.**
I-51	Illustrations of the solution of each part are as under – **Part (a):** Given that string is fixed at both ends is vibrating on its first overtone. The span of the wire L for fundamental frequency is $\lambda_1 = 2L = 2 \times 2 = 4$m. Since, $v = f\lambda$, hence at first overtone frequency would double and in turn wavelength $\boldsymbol{\lambda_2} = \frac{\lambda_1}{2} = \frac{4}{2} = \mathbf{2}$ **m**. Accordingly, for a given velocity of 200 m/s, frequency would be $\boldsymbol{f_2} = \frac{v}{\lambda_2} = \frac{200}{2} = \mathbf{100}$ **Hz. Hence answers of part (a) are 2 m, 100 Hz .** **Part (b):** Equation of a standing wave is $y = A \sin\left(\frac{2\pi x}{\lambda_2}\right) \times \sin(2\pi f t) \Rightarrow y = A \sin\left(\frac{2\pi x}{2}\right) \times \sin(2\pi \times 100 \times t)$. It leads to $\boldsymbol{y = A \sin\left((\pi m^{-1})x\right) \times \sin\left(200\pi s^{-1}\right) t}$**, here frequency** $\boldsymbol{v = \frac{1}{T}}$**, is answer of part (b).** **Thus answers are (a) 2 m, 100 Hz (b)** $\boldsymbol{(0.5\ cm) \sin[(\pi\ \mathrm{m}^{-1})x] \times \cos[(200\pi\ \mathrm{s}^{-1})t]}$**.**
I-52	Generic equation of stationary wave on a string is $y = A \sin\left(\frac{2\pi x}{\lambda_3}\right) \times \cos(2\pi f t)$. This equation is being compared with the given equation $y = (0.4\text{ cm}) \sin[(0.314\text{cm}^{-1})x] \cos[(600\pi s^{-1})t]$to find answers to each part of the question. **Part (a):** We have from two equations $2\pi f_3 = 600\pi \Rightarrow \boldsymbol{f_3} = \frac{600}{2} = \mathbf{300}$ **Hz is answer of part (a)** **Part (b):** The string is vibrating with Third harmonics and hence it will have three loops with nodes at fixed supports at ends and two in between separated by a distance $\frac{\lambda_3}{2}$. Since, $\frac{2\pi}{\lambda_3} = 0.314 \Rightarrow \frac{2}{\lambda_3} = \frac{1}{10}$. Hence, $\frac{\lambda_3}{2} = 10$. Thus nodes will occur at $x = 0, 10$ cm, 20 cm and 30 cm. Hence answer of part (b) is $x = 0, 10$ cm, 20 cm and 30 cm **Part (c):** First node is at $x = 0$ and with three nodes in between fourth node is at $x = 30$ cm. **Hence, length of the string is 30 cm, is answer of part (c).**

	Part (d): With the derivation in part (b), $\frac{\lambda_3}{2} = 10 \Rightarrow \boldsymbol{\lambda_3 = 20}$ **cm**. Accordingly, velocity of wave with derivation in part (a) and in this part $v = f_3\lambda_3 = 300 \times 20 = 6000$ cm/s **or** $\boldsymbol{v = 60}$ **m/s**. Thus answer of part (d) is $\boldsymbol{\lambda_3 = 20}$ **cm** and $\boldsymbol{v = 60}$ **m/s.** **Thus answers are (a) 300 Hz (b) 0, 10 cm, 20 cm, 30 cm (c) 30 cm (d) 20 cm. 60 m/s.**
I-53	Generic equation of stationary wave on a string is $y = A\sin\left(\frac{2\pi x}{\lambda}\right) \times \cos(2\pi f t)$. While the given equation is $y = (0.4\text{ cm})\sin[(0.314\text{cm}^{-1})x]\cos[(600\pi s^{-1})t]$. Comparing the two equations we have $\frac{2\pi}{\lambda} = 0.314 \Rightarrow \frac{2}{\lambda} = \frac{1}{10}$ or $\frac{\lambda}{2} = 10$cm. In case of standing wave smallest length of the string is $\frac{\lambda}{2} = 10$ cm. **Hence, answer is 10 cm.**
I-54	Given that wire of length $L = 40\text{ cm} = 0.40\text{ m}$ and mass $m = 3.2\text{ g} = 3.2 \times 10^{-3}$ kg (linear mass density $\mu = \frac{m}{L} = \frac{3.2\times10^{-3}}{0.4} = 8 \times 10^{-3}$ kg/m. Wire is stretched to length 40.05 cm = 0.5005 m with a force F and at its fundamental mode of vibration at 220 Hz $\frac{\lambda}{2} = 0.4005 \Rightarrow \lambda = 0.801$m. Hence, velocity of the wave is $v = f\lambda \Rightarrow v = 220 \times 0.801 = 176.22$ m/s. Accordingly, from equation $v = \sqrt{\frac{F}{\mu}} \Rightarrow F = \mu v^2 \Rightarrow F = \left(8 \times 10^{-3}\right)(176.22)^2 = 248.4$ N. With given data elongation of wire is $\Delta l = 50.05 - 40 = 10.05$ hence strain is $\frac{\Delta l}{L} = \frac{0.05}{40} = 1.25 \times 10^{-3}$ and stress in the wire $\frac{F}{A} = \frac{248.4}{1\times10^{-6}} = 2.48 \times 10^{8}$ N/m^2. Accordingly Youmg's Modulus is $Y = \frac{\frac{F}{A}}{\frac{\Delta l}{L}}$. Hence, $Y = \frac{2.48\times10^{8}}{1.25\times10^{-3}} = 1.98 \times 10^{11}$ N/m^2. **Thus answer is** $\boldsymbol{Y = 1.98 \times 10^{11}}$ **N/m^2.**
I-55	Initially when the block is hanging in air tension in the string $T = mg$ and the string of length l and linear mass density μ vibrates in tenth harmonic f_{10} unison with tuning fork of frequency f i.e. $f_{10} = 10f$. It implies that $l = 10 \times \frac{\lambda_{10}}{2} = 5\lambda_{10}$. The velocity of the wave in the string $v = \sqrt{\frac{mg}{\mu}}$. It is given that string is vibrating in its 10th harmonic hence $f_{10} = 10f_1 \Rightarrow f_{10} = v_{10} = \sqrt{\frac{mg}{\mu}}$. It solves into $f_{10} \times \lambda_{10} = \sqrt{\frac{mg}{\mu}} \Rightarrow f_{10} \times \left(\frac{l}{5}\right) = \sqrt{\frac{mg}{\mu}}$. Considering that object has volume V and density ρ the relation can be written as $\frac{f_{10}l}{5} = \sqrt{\frac{V\rho g}{\mu}}$ …(1)  Now when solid is completely immersed in water let reduced mass due to buoyancy is m' and string vibrates in at a frequency $f_{11} = 11f_1$ and $l = 11 \times \frac{\lambda_{11}}{2} = 5.5\lambda_{11}$. With other parameters viz. f_1, l, μ and g remaining unchanged we will have $v_{11} = f_{11} \times \lambda_{11} = \sqrt{\frac{m'g}{\mu}} \Rightarrow \frac{f_{11}l}{5.5} = \sqrt{\frac{m'g}{\mu}}$…(2)  Now it requires to determined m' for solid of volume V and of density ρ such that $m = \rho V$. But, when solid is immersed in water having relative density $\rho_w = 1$, because of buoyancy the relative mass of solid would be $m' = (\rho - 1)V$. Accordingly, equation (2) can be rewritten as $\frac{f_{11}l}{5.5} = \sqrt{\frac{(\rho-1)Vg}{\mu}}$…(3) Combining (1) and (3) we have $\frac{\frac{f_{10}l}{5}}{\frac{f_{11}l}{5.5}} = \frac{\sqrt{\frac{V\rho g}{\mu}}}{\sqrt{\frac{(\rho-1)Vg}{\mu}}}$ …(4) Since in both the cases, despite change of velocity due to change in tension, string is resonating with the same tuning fork and hence Now, that tuning fork is same and hence $f_{10} = f_{11}$. Using this identity in (4) we have $\frac{\frac{f_{10}l}{5}}{\frac{f_{10}l}{5.5}} = \frac{\sqrt{\frac{V\rho g}{\mu}}}{\sqrt{\frac{(\rho-1)Vg}{\mu}}}$, or $\frac{5.5}{5} = \sqrt{\frac{\rho}{\rho-1}} = 1.1 \Rightarrow \frac{\rho}{\rho-1} = 1.21 \Rightarrow 1.21(\rho - 1) = \rho \Rightarrow 0.21\rho = 1.21 \Rightarrow \rho = \frac{1.21}{0.21} = 5.76$ g/cm^3 or $\boldsymbol{\rho = 5.8 \times 10^3}$ **kg/m^3 is the answer.** **N.B.:** Relation of resonating frequency with that of the tuning fork is the key consideration and leads to conclusion that with decrease in tension velocity decreases but frequency increases. This can be realized in skipping rope game.

I-56	Linear mass density of rope of length $L = 2.00$ m and mass $m = 80$ g is $\mu = \frac{m}{L} = \frac{80\times10^{-3}}{2.00} = 4.0\times10^{-2}$ kg/m. Tension in the rope is $F = 256$ N . Hence velocity of transverse wave along the string is $v = \sqrt{\frac{F}{\mu}}$. Accordingly, $v = \sqrt{\frac{256}{4.0\times10^{-2}}} = 80$ m/s.Since rope has one end fixed and the other end being tied to a light string acts like a free to move vertically and $L = \frac{\lambda}{4} \Rightarrow \lambda = 4L = 4\times2 = 8$m. Since, $v = f\lambda$ and hence fundamental frequency is $f_1\lambda_1 = 80 \Rightarrow \boldsymbol{f_1} = \frac{80}{8} = \mathbf{10}$ **Hz**. This fundamental frequency is Zero[th] overtone and thus $\boldsymbol{O_0} = \boldsymbol{f_1} = \mathbf{10}$ **Hz**   Since anti-node occurs at the free end tied to a light string hence of first overtone will occur at a frequency when next anti-node occurs at free end. Accordingly, it is $\boldsymbol{O_1} = 3f_1 = 3\times10 = \mathbf{30}$ **Hz** and frequency of second overtone will occur at a next frequency when anti-node recurs at free end, thus $\boldsymbol{O_2} = 5f_1 = 5\times10 = \mathbf{50}$. **Hence, anser of part (a) is 10 Hz, 30 Hz and 50 Hz.**  Wavelength of fundamental frequency has been determined above $\boldsymbol{\lambda} = \mathbf{8.00}$ **m.** Extending the logic of frequency of overtones, wavelength for first overtone is $\boldsymbol{\lambda_{01}} = \frac{8}{3} = \mathbf{2.67}$ **m.** and $\boldsymbol{\lambda_{02}} = \frac{8}{5} = \mathbf{1.60}$**m. Thus answer of part (b) is 8.00m, 2.67 m and 1.60 m.** **Thus answers are (a) 10 Hz, 30 Hz, 50 Hz (b) 8.00 m, 2.67 m, 1.60 m.** **N.B.:** In case of overtones in a string with both ends fixed and with one end free the logic is different and needs to be used appropriately.
I-57	In the initial position of movable support joint of heavy string with that of the light string is 10 cm from the pulley. The joint will act as a free end of the heavy string, whose other end is fixed to a support. Thus joint will be anti-node. With the given loading in the system, let v is the speed of the transverse wave in the string of length L. Hence, $L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$. Therefore, lowest frequency is $f_1 = \frac{v}{4L} = 120$ Hz as per the given data.   Now when the truck is moved towards the pulley such that joint is on the pulley making the joint to be fixed end $L = \frac{\lambda_2}{4} \Rightarrow \lambda_2 = 2L$. Since loading pattern on the string does not change hence velocity would also remain unchanged. Accordingly, $f_2 = \frac{v}{2L} \Rightarrow f_2 = 2\times\frac{v}{4L} = 2\times f_1 \Rightarrow f_2 = 2\times120 =$ 240 Hz. **Thus answer is 240 Hz.**
I-58	Resultant wave is $y = y_1 + y_2 = 4\sin(2x-6t) + 3\sin\left(2x-6t-\frac{\pi}{2}\right) = 4\sin(2x-6t) + 3\cos(2x-6t)$. Taking $4 = A\cos\theta$ and $3 = A\sin\theta$ we have $y = A\cos\theta\sin(2x-6t) + A\sin\theta\cos(2x-6t)$. It leads to $y = A\sin[(2x-6t)+\theta]$. Thus, amplitude of the resultant wave is $A = \sqrt{A^2\sin^2\theta + A^2\cos^2\theta} \Rightarrow A = \sqrt{3^2+4^2}$ or $\boldsymbol{A} = \mathbf{5}$ **is the answer.**
I-59	Given is a string of length $l = 20$ cm $= 0.20$ m of mass $m = 1.0\,g = 1.0\times10^{-3}$ kg is stretched with a force $F = 0.5$ N. Hence, velocity of transverse wave is $v = \sqrt{\frac{F}{\mu}}$. The linear mass density of the string $\mu = \frac{m}{L} \Rightarrow \mu = \frac{1.0\times10^{-3}}{0.2} = 5\times10^{-3}$. Therefore, velocity $v = \sqrt{\frac{0.5}{5\times10^{-3}}} \Rightarrow v = 10$ m/s. Therefore, wavelength of the transverse wave is $\lambda = \frac{v}{f} = \frac{10}{100} = 0.1$ m or 10 cm. In a wavelength two nodes occur and distance between successive nodes is 5 cm. **Thus, answer (in cm) is 5.**

I-60	General expression of a transverse wave is $y = A\sin\left[\omega t + \frac{2\pi x}{\lambda} \pm \varphi\right]$ where A is amplitude, ω is the angular velocity, λ is the wave length, x is distance travelled at any instant , and φ is initial phase angle at $x = 0$ and $t = 0$. In this case maximum transverse velocity, given in the problem is $v_{max} = 3$ m/s, is when particle is at mean position is $v_{max} = A\omega = 3$ m/s. And maximum transverse acceleration, given in the problem is $a_{max} = 90$ m/s^2, is when particle is at maximum displacement and it is $a_{max} = A\omega^2 = 90$ m/s$^{2.}$ Accordingly angular velocity is $\omega = \frac{A\omega^2}{A\omega} \Rightarrow \omega = \frac{a_{max}}{v_{max}} = \frac{90}{3}$, or $\omega = 2\pi f = 30$ rad/s $\Rightarrow f = \frac{30}{2\pi}$. Further, wavelength of the transverse wave of velocity $v = 20$ m/s is $\lambda = \frac{v}{f} \Rightarrow \lambda = \frac{20}{\frac{15}{\pi}} = \frac{4\pi}{3}$. Accordingly, coefficient of x is $\frac{2\pi}{\lambda} = \frac{2\pi}{\frac{4\pi}{3}} = \frac{3}{2}$. Since, nothing is specified about initial condition, the phase difference is taken to be φ. Thus constructing the wave equation from the derived values of ω, λ and $\boldsymbol{\varphi}$ initial phase difference we have $y = 0.1\sin\left[30t \pm \frac{3}{2}x \pm \varphi\right]$. Since, the wave travels along $\pm x$ from the point of initiation and hence $\boldsymbol{y = 0.1\sin\left[30t \pm \frac{3}{2}x \pm \varphi\right]}$ **is the answer**.

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