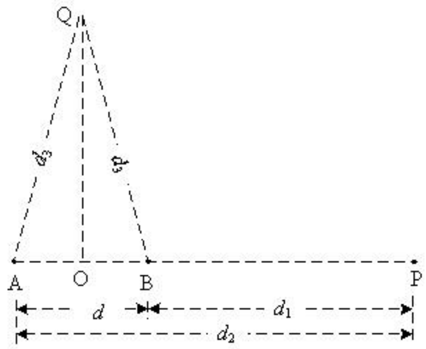
Wave and Motion : Sound – Subjective Questions (Typical)

(Illustrations Only)

I-01	Time taken by sound to travel a distance $t = \frac{s}{v}$. Therefore time taken by sound to travel through the tube of length $s = 1$m shall be $t_t = \frac{s}{v_t}$ and through air in the tube shall be $t_a = \frac{s}{v_a}$. Accordingly, using the given data $t_t = \frac{1}{5200} = 0.19$ ms and $t_a = \frac{1}{330} = 3.03$ ms. Therefore, interval between two blows shall be $\Delta t = t_a - t_t$, or $\Delta t = 3.03 - 0.19 = 2.84$ ms. **Hence, answer is 2.84 ms.**
I-02	Distance covered by reflected sound to reach the person chanting JAI-RAM is $s = 2 \times d \Rightarrow s = 2 \times 80$ m or 160 m. Hence time taken for hearing a chant is $t = \frac{s}{v} = \frac{160}{320} = 0.5$ s. Therefore, a maximum interval of 0.5 s between two chants will create an overlap of two chants to the listener and there would be no distinct reflected sound, called echo, disturbing the listener. Thus **maximum time interval between two chants is 0.5 s.**
I-03	Distance travelled by clapping sound heard by the man clapping is $s = 2 \times d = 2 \times 50.0 = 100$ m. Interval between two claps is $t = \frac{3}{10} = 0.3$ s. Time taken by clapping sound to reach back the man is $t' = \frac{s}{v}$. For merging of echo of the clap essential requirement is $t = t' \Rightarrow 0.3 = \frac{s}{v} \Rightarrow 0.3 = \frac{100}{v} \Rightarrow v = \frac{100}{0.3}$ or $\boldsymbol{v = 333}$ **m/s, is the answer.**
I-04	Wavelength is $\lambda = \frac{v}{f}$. Here, speed of sound is $v = 360$ m/s is constant for all the frequencies. This inverse proportionality of wavelength with frequency implies that $\lambda_{min} = \frac{v}{f_{max}} \Rightarrow \lambda_{min} = \frac{360}{20\times10^3} = 18 \times 10^{-3}$ m or 18 mm and $\lambda_{max} = \frac{v}{f_{min}} \Rightarrow \lambda_{max} = \frac{360}{20} = 18$ m. **Hence, answers are 18 mm, 18 m.**
I-05	Wavelength is $\lambda = \frac{v}{f}$. Here, speed of sound water is $v = 1450$ m/s is constant for all the frequencies. Maximum audible frequency is $f_{max} = 20000$ Hz and minimum frequency is $f_{min} = 20$ Hz This inverse proportionality of wavelength with frequency implies that $\lambda_{min} = \frac{1450}{f_{max}} \Rightarrow \lambda_{min} = \frac{1450}{20\times10^3} = 72.5 \times 10^{-3}$ m or 7.25 cm and $\lambda_{max} = \frac{v}{f_{min}} \Rightarrow \lambda_{max} = \frac{1450}{20} = 72.5$ m. **Hence, answers are 7.25 cm, 72.5 m.**
I-06	**Part (a):** For sound to spread uniformly in all the directions if wavelength $\gg d$. If is also given that $\lambda = 10 \times d$, here $d = 20$ cm or 0.2 m is the diameter of the speaker. Hence, $\lambda = 10 \times 0.2 = 2$ m. It is also given that speed of sound is $v = 340$ m/s. Hence, frequency of sound is $\boldsymbol{f} = \frac{v}{\lambda} = \frac{340}{2} =$ **170 Hz, this is answer of part (a).** **Part (b):** Sound is travels in one direction if $f \ll d$, and given that $\lambda = \frac{d}{10} = \frac{0.2}{10} = 0.02$ m. Hence, corresponding frequency is $\boldsymbol{f} = \frac{v}{\lambda} = \frac{340}{0.02} =$ **17 × 10³ Hz or 17 kH** **Hence, answers are (a) 170 Hz (b) 17 kHz.**
I-07	Velocity of waves changes with medium but its frequency remain unchanged and $\lambda = \frac{v}{f}$. In the instant case frequency of ultrasonic wave is 4.5 MHz. Hence, wavelength in in air it is $\lambda_a = \frac{v_a}{f} = \frac{340}{4.5\times10^6} = 7.6 \times 10^{-5}$ m and in tissue it is $\lambda_t = \frac{v_t}{f} = \frac{1.5\times10^3}{4.5\times10^6} = 3.3 \times 10^{-4}$ m. .

	Hence, answers are $\mathbf{7.6 \times 10^{-5}}$ m, $\mathbf{3.3 \times 10^{-4}}$ m.
I-08	Generic equation of a travelling wave is $y = A\sin\left(\omega t - \frac{x}{\lambda}\right)$ and comparing it with the given equation of wave $A = 6.0 \times 10^{-5}$ m, angular velocity is $\omega = 600$ rad/s, and wavelength is $\lambda = \frac{2\pi}{1.8}$ m. Accordingly, **Part (a):** Required ratio is $\frac{A}{\lambda} = \frac{6.0\times10^{-5}}{\frac{2\pi}{1.8}} = \frac{10.8\times10^{-5}}{2\pi} \rightarrow$ **$\mathbf{1.7 \times 10^{-5}}$, answer of part (a).** **Part (b):** Velocity amplitude is $v_a = A \times \omega$ and speed of wave is $v = f \times \lambda = \frac{\omega}{2\pi} \times \lambda$, Thus the required ratio $\frac{v_a}{v} = \frac{A\times\omega}{\frac{\omega}{2\pi}\times\lambda} = \frac{2\pi A}{\lambda}$. Using the available data $\frac{v_a}{v} = \frac{A\times\omega}{\frac{\omega}{2\pi}\times\frac{2\pi}{1.8}} = 1.8A = 1.8 \times (6.0 \times 10^{-5}) =$ **$\mathbf{1.1 \times 10^{-4}}$, answer of part (b).** **Thus answers are (a) $\mathbf{1.7 \times 10^{-5}}$, (b) $\mathbf{1.1 \times 10^{-4}}$**
I-09	Time period of the wave is $T = \frac{1}{f} = \frac{1}{100} = 0.01$ s or 10 ms. And wavelength $\lambda = \frac{v}{f} = \frac{350}{100} = 3.5$ m = 350 cm. **Part (a):** Relationship of angular shift with time period is $T \rightarrow 2\pi$. Accordingly, at any time $t = 2.5$ ms, phase shift $\theta = \text{mod}\left(\frac{t}{T}\right) \times 2\pi \Rightarrow \theta = \text{mod}\left(\frac{2.5}{1}\right) \times 2\pi$. It leads to $\theta = 0.5\pi = \frac{\pi}{2}$. **Hence, answer of part (a) is $\frac{\pi}{2}$ rad** **Part (b):** Further, relationship between wavelength and angular shift is $\lambda \rightarrow 2\pi$. Accordingly for any distance $x = 10.0$ cm phase shift $\theta = \text{mod}\left(\frac{x}{\lambda}\right) \times 2\pi \Rightarrow \theta = \text{mod}\left(\frac{10}{350}\right) \times 2\pi = \frac{2\pi}{35}$, **Hence answer of part (b) is $\frac{2\pi}{35}$ rad.** **Thus answers are (a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{35}$.**
I-10	The arrangement of sources A and B and points of observation as per part (a) is P and as per part (b) is Q. Here, distance between the sources A and B is $x = 10$. Distance of point P from source B is $d_1 = 20$ cm. Since, P is on the line joining AB and on opposite of source A and hence distance of P from source A is $d_2 = x + d_1 = 10 + 20 = 30$ cm. It is given that point Q is situated on perpendicular bisector of line joining AB and hence by geometry distance from sources is A and B is equal and $d_3 = 20$ cm.  Further it is given that wavelength of the waves from the two sources is $\lambda = 5$ cm. And relationship between wavelength and phase difference is $\lambda \rightarrow 2\pi$. Thus, for any distance x phase shift $\theta = \text{mod}\left(\frac{x}{\lambda}\right) \times 2\pi$. **Part (a):** Phase shift θ_1 in wave from source B is $\theta_1 = \text{mod}\left(\frac{d_1}{\lambda}\right) \times 2\pi = \text{mod}\left(\frac{20}{5}\right) \times 2\pi = 0$. Likewise, phase shift θ_2 in wave from source A is $\theta_2 = \text{mod}\left(\frac{d_2}{\lambda}\right) \times 2\pi = \text{mod}\left(\frac{30}{5}\right) \times 2\pi = 0$. Therefore phase shift in waves arriving at point P is $\Delta\theta_p = \theta_1 - \theta_2 = 0 - 0 = 0$. **Hence, answer of part (a) is Zero.** **Part (b):** Phase shift in waves arriving at point Q both sources is equal $\theta_3 = \text{mod}\left(\frac{d_3}{\lambda}\right) \times 2\pi$. It solves to $\theta_3 = \text{mod}\left(\frac{20}{5}\right) \times 2\pi = 0$. Hence, their difference is also bound to be **zero, this answer of part (b).** **Thus, answers are (a) Zero (b) Zero.**
I-11	Progression of sound wave is gases is an adiabatic process, since rate of undergoing compression and rarefaction of particles of medium participating in wave is so high that it does not have time to exchange heat with the environment. Accordingly, using adiabatic properties of gases speed of sound is $= \sqrt{\frac{\gamma P}{\rho}}$.

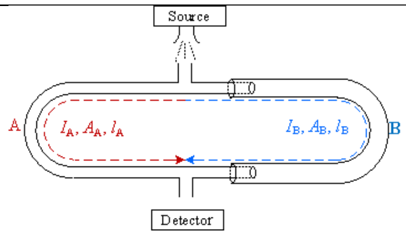
	Here, using given data $\gamma = \frac{C_p}{C_v} = \frac{3.5R}{2.5R} = \frac{7}{5}$, pressure of the gas $P = 1.0 \times 10^5 \text{N/m}^2$, and density $\rho = \frac{m}{V}$ in IS calculates to $\rho = \frac{32\times10^{-3}}{22.4\times10^{-3}} = \frac{32}{22.4}$. Hence, $v = \sqrt{\frac{\left(\frac{7}{5}\right)\times(1.0\times10^5)}{\frac{32}{22.4}}} = \sqrt{\left[\left(\frac{7\times22.4}{5\times32}\right)\times 10\right]} \times 10^2 = 3.13\times10^2 =$ **313 m/s, is the answer.**
I-12	Speed of sound in air is $v = \sqrt{\frac{\gamma P}{\rho}}$ …(1) where, $\gamma = \frac{C_p}{C_v}$, pressure of the gas P and density gas is $\rho = \frac{m}{V}$. Further as per Ideal Gas Equation $pV = nRT \Rightarrow T = \left(\frac{P}{R}\right) \times \frac{V}{n} \Rightarrow T \propto \frac{1}{\rho}$…(2). Since P and R are constants. Therefore, combining equations (1) and (2) we have $v = \sqrt{\frac{\gamma P}{\frac{1}{T}}} \Rightarrow v = \sqrt{\gamma P T} \Rightarrow v \propto \sqrt{T}$…(3) Here, T is temperature in thermodynamic scale i.e. Kelvin. In initial case $T_1 = 10^0\text{C} \Rightarrow T_1 = 273 + 17 = 290$ K and speed of sound $v_1 = 340$ m/s. In case 2 temperature changes to $T_2 = 32^0\text{C} \Rightarrow T_2 = 273 + 32 = 305$ K. Since we are required to determine speed of sound at where only temperature changes, keeping other parameters same, using (3) we have $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow v_2 = \sqrt{\frac{T_2}{T_1}} \times v_1$. Using the available data $v_2 = \sqrt{\frac{305}{290}} \times 340$ or $v_2 = 348.9$ m/s and using principle of SDs **answer is $v_2 = 349$ m/s**.
I-13	Using ideal gas properties and that sound wave is gases travel adiabatically we know that $v \propto \sqrt{T}$. It is given that $T_1 = 273$ K where speed of sound is v_1 and at some other temperature say T_2 K speed of sound is $v_2 = 2v_1$. Thus, using the equation above $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow 2 \times v_1 = \sqrt{\frac{T_2}{273}} \times v_1 \Rightarrow T_2 = 2^2 \times 273 = 1092$ K. The temperature in commonly used scale of centigrade is $\boldsymbol{T_2} = 1092 - 273 =$ **819^0 C**, is the answer.
I-14	Using ideal gas properties and that sound wave is gases travel adiabatically we know that $v \propto \sqrt{T}$…(1). It is given that in space of width d temperature varies linearly from T_1 to T_2 . Therefore, at any distance x from the boundary of temperature T_1 the temperature shall be $T_x = T_1 + \frac{T_2 - T_1}{d} \times x \Rightarrow T_x = \frac{T_1 d + (T_2 - T_1)x}{d}$. Time taken by sound to travel through a thin film of medium of thickness Δx , assuming temperature in the thin film to be uniform, $\Delta t_x = \frac{\Delta x}{v_x}$. Using (1) we have $\frac{v_x}{v_1} = \sqrt{\frac{T_x}{T_1}} \Rightarrow v_x = \sqrt{\frac{T_x}{T_1}} \times v_1$…(2). Therefore, time taken by sound to cross the given space would be $\int_0^t dt_x = \int_0^d \frac{dx}{v_x} \Rightarrow t = \int_0^d \left(\frac{1}{v_1} \times \sqrt{\frac{T_1}{T_x}}\right) dx \Rightarrow t = \frac{1}{v_1} \times \int_0^d \left(\sqrt{\frac{T_1}{\left(\frac{T_1 d + (T_2 - T_1)x}{d}\right)}}\right) dx$. This leads to $t = \frac{\sqrt{T_1 d}}{v_1} \int_0^d \left(\frac{1}{\sqrt{T_1 d + (T_2 - T_1)x}}\right) dx$. Say, $T_1 d + (T_2 - T_1)x = u$ we have $dx = \frac{du}{(T_2 - T_1)}$, $t = \frac{\sqrt{T_1 d}}{v_1} \times \int_0^d \left(\frac{1}{\sqrt{u}}\right) \frac{du}{(T_2 - T_1)}$. It leads to $t = \frac{\sqrt{T_1 d}}{v_1} \times \frac{1}{(T_2 - T_1)} \times \left[2u^{\frac{1}{2}}\right]_0^d$. It, further solves into $t = \frac{\sqrt{T_1 d}}{v_1} \times \frac{\sqrt{d}}{(T_2 - T_1)}\left[T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}}\right]$. This simplifies into $t = \frac{2d \times \sqrt{T_1}}{v_1\left(T_2^{\frac{1}{2}} + T_1^{\frac{1}{2}}\right)\left(T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}}\right)} \times \left(T_2^{\frac{1}{2}} - T_1^{\frac{1}{2}}\right) \Rightarrow t = \frac{2d \times \sqrt{T_1}}{v_1\left(T_2^{\frac{1}{2}} + T_1^{\frac{1}{2}}\right)}$…(3) tGiven that speed of sound at 273 K is $v = 330$ m/s. Hence, using (2) we have $v_1 = v \times \sqrt{\frac{T_1}{273}}$…(4). On combining (3) and (4) we have $t = \frac{2d \times \sqrt{T_1}}{\left(v \times \sqrt{\frac{T_1}{273}}\right)\left(T_2^{\frac{1}{2}} + T_1^{\frac{1}{2}}\right)} \Rightarrow \boldsymbol{t = \frac{2d}{v} \times \frac{\sqrt{273}}{(\sqrt{T_2} + \sqrt{T_1})}}$, **This is the answer of first part in the algebraic form.** Numerical value of the time using the given data is $t = \frac{2\times33}{330} \times \frac{\sqrt{273}}{(\sqrt{310} + \sqrt{280})} \Rightarrow \boldsymbol{t = 0.096}$ **i.e. 96 ms.**

	Thus answers are $\frac{2d}{v} \times \frac{\sqrt{273}}{(\sqrt{T_2}+\sqrt{T_1})}$**, 96 ms.** **N.B:** This an excellent problem involving integration of mathematics with the concepts of physics.
I-15	Speed of sound in liquid is $s = \sqrt{\frac{B}{\rho}}$ where Bulk modulus of elasticity material is $B = \frac{p}{\frac{\Delta V}{V}} \Rightarrow B = \frac{pV}{\Delta V}$ and density of kerosene is given to be $\rho = 800$ kg/m^3. Accordingly, $s = \sqrt{\frac{\frac{pV}{\Delta V}}{\rho}} \Rightarrow s^2\rho\Delta V = pV \Rightarrow \Delta V = \frac{pV}{s^2\rho}$. Substituting the given data change in volume is $\Delta V = \frac{(2.0\times10^5)(1.0\times10^{-3})}{(1330)^2\times800} = 0.14 \times 10^{-6}$ m^3 **or 0.14 cm^3, is the answer.**
I-16	Bulk modulus of elasticity material is $B = \frac{\Delta p}{\frac{\Delta V}{V}} \Rightarrow B = \frac{\Delta pV}{\Delta V}$. It is given that pressure varies between $(1.0 \times 10^5 \pm 14)$ Pa $\Rightarrow \Delta p = 14$ Pa, or . $\Delta p = 14$ N/m^2. Given that wavelength $\lambda = 0.35$ m and maximum displacement i.e. amplitude of vibration of air particles is $\Delta x = 5.5 \times 10^{-6}$ m and maximum angular strain in the medium is $\frac{\Delta V}{V} = \frac{\Delta x}{\frac{\lambda}{2\pi}} \Rightarrow \frac{\Delta V}{V} = \frac{2\pi\Delta x}{\lambda}$. This leads to required bulk modulus of air is $B = \frac{\Delta p}{\frac{2\pi\Delta x}{\lambda}} \Rightarrow B = \frac{\lambda\Delta p}{2\pi\Delta x}$. Substituting, the available data $B = \frac{0.35\times14}{2\pi(5.5\times10^{-6})} = 0.14 \times 10^6 = 1.4 \times 10^5$ N/m^2 is the answer. **N.B.**: This problem solution correlating concept of strain in bulk elasticity applied to waves viz. amplitude corresponding to maximum displacement, and wavelength corresponding to volume of gas under consideration. Since in propagation of sound waves particles of medium keep oscillating about their mean position and hence volume of the medium experiencing bulk deformation in wave is taken corresponding to its amplitude. Factor 2π converts displacement to angle displacement required in bulk modulus.
I-17	It is given that source of frequency 2.0 kHz and power $p = 20$ Wof sound is emitting sound uniformly in all directions. Speed of sound is $v = 340$ m/s in air of density 1.2 kg/m^2. **Part (a):** With the given emission of sound by the source, its intensity at a distance R from the source is $I = \frac{p}{4\pi R^2} = \frac{20}{4\pi 6^2} = 0.044$ W/m^2 or ***I* = 44 mW/m^2, is the answer of part (a).** **Part (b):** Intensity of sound in terms pressure amplitude (ΔP), speed of sound in air $(v)$and density of air (ρ) is of sound wave at a point is $I = \frac{(\Delta P)^2}{2v\rho} \Rightarrow \Delta P = \sqrt{2v\rho I}$. Using the available data we have $\Delta \boldsymbol{P} = \sqrt{2 \times 340 \times 1.2 \times 0.044} = 6$ **N/m^2 is the answer of part (b).** **Part (c):** Relationship of intensity of sound in terms of displacement amplitude (S_0), is $I = \frac{(\omega S_0)^2B}{2v}$. The bulk modulus $B = \frac{p_0}{\frac{\Delta V}{V}} = \frac{p_0}{2\pi\left(\frac{s_0}{\lambda}\right)} = \frac{p_0}{2\pi\left(\frac{s_0 f}{\lambda f}\right)} = \frac{p_0}{\left(\frac{2\pi f s_0}{\lambda f}\right)} = \frac{p_0}{\frac{\omega s_0}{v}} = \frac{vp_0}{\omega s_0}$. Here, p_0 is the pressure amplitude expressed as Δp in [part (b) above. Thus Intensity of sound $I = \frac{(\omega S_0)^2}{2v} \times \frac{vp_0}{\omega S_0}$. It leads to $I = \frac{\omega s_0 p_0}{2}$. Therefore, displacement amplitude $s_0 = \frac{2I}{2\pi f p_0}$. Using the available data we have $\boldsymbol{s_0} = \frac{2\times0.044}{2\times\pi\times2.0\times10^3\times6} = \mathbf{1.2 \times 10^{-6}}$ **m, is the answer of part (c).** **Thus answers are (a) 44mW/m^2 (b) 6.0 pa (c) $\mathbf{1.2 \times 10^{-6}}$m** N.B.: **(1)** Pressure, Power, Pressure Amplitude and Density symbols look similar and therefore they need to be handled carefully. **(2)** Determination of intensity of sound is a culmination of wave equation, displacement and pressure equations of wave, power transmitted by a wave. Moreover, it involves basic principles of mechanics, gas equations, isothermal and adiabatic processes etc. It is a very much involved derivation and hence has been exclusively brought out in Preamble. **(3)** Expression for intensity of sound wave has many variants and choice of a variant in a problem depends upon known variables. This becomes explicit in illustrations of the three parts of this problem.

I-18	Intensity of sound at a distance d from a source of sound of power P is $I=\frac{P}{4\pi d^2}\Rightarrow I\propto\frac{1}{d^2}\Rightarrow\frac{I_2}{I_1}=\left(\frac{d_1}{d_2}\right)^2$. Accordingly, $I_2=I_1\left(\frac{d_1}{d_2}\right)^2$ and using the given data $\boldsymbol{I_2}=(1.0\times10^{-8})\left(\frac{5.0}{25}\right)^2=\mathbf{4.0\times10^{-10}}$ **W/m² is the answer.**
I-19	Intensity of sound at a distance d from a source of sound of power P is $I=\frac{P}{4\pi d^2}\Rightarrow I\propto\frac{1}{d^2}\Rightarrow\frac{I_2}{I_1}=\left(\frac{d_1}{d_2}\right)^2$. Accordingly, $I_2=I_1\left(\frac{d_1}{d_2}\right)^2$ and using the given data $\frac{I_2}{I_1}=\left(\frac{5.0}{50}\right)^2=1.0\times10^{-2}$ W/m². As per definition intensity of sound in decibel is $=10\log_{10}\frac{I}{I_0}$. Accordingly, $\beta_1=10\log_{10}\frac{I_1}{I_0}$ at 5.0 m away source, and $\beta_2=10\log_{10}\frac{I_2}{I_0}$. Therefore, $\beta_2-\beta_1=10\log_{10}\frac{I_2}{I_0}-10\log_{10}\frac{I_1}{I_0}$. This further simplifies into $\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_0}\times\frac{I_0}{I_1}\right)\Rightarrow\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_1}\right)\Rightarrow\beta_2=\beta_1+10\log_{10}\left(\frac{I_2}{I_1}\right)$. Using the given data $\beta_2=40+10\log_{10}(10^{-2})\Rightarrow\beta_2=40+10\times(-2)\Rightarrow\beta_2=40-20\Rightarrow\boldsymbol{\beta_2}=\mathbf{20}$ **dB is the answer.**
I-20	As per definition intensity of sound in dB is $\beta=10\log_{10}\frac{I}{I_0}$. Accordingly, Accordingly, $\beta_1=10\log_{10}\frac{I_1}{I_0}$ at 5.0 m away source, and $\beta_2=10\log_{10}\frac{I_2}{I_0}$. Therefore, $\beta_2-\beta_1=10\log_{10}\frac{I_2}{I_0}-10\log_{10}\frac{I_1}{I_0}$. This further simplifies into $\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_0}\times\frac{I_0}{I_1}\right)\Rightarrow\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_1}\right)$. It is given that $\frac{I_2}{I_1}=2$ then increase in dB is $\Delta\beta=\beta_2-\beta_1=10\log_{10}(2)\Rightarrow\Delta\boldsymbol{\beta}=10\times0.3=\mathbf{3}$ **dB, this is the answer.**
I-21	Intensity of sound at a distance d from the source is $I=\frac{P}{4\pi d^2}$. Further, intensity of sound in dB is expressed as $\beta=10\log_{10}\frac{I}{I_0}$ where $I_0=10^{-12}$ W/m². Thus $\beta=10\log_{10}\frac{\frac{P}{4\pi d^2}}{I_0}\Rightarrow\beta=10\log_{10}\frac{P}{(4\pi d^2)\times I_0}$. Using the given data $120=10\log_{10}\frac{2}{(4\pi d^2)\times10^{-12}}\Rightarrow120=\log_{10}\left(\frac{2}{(4\pi d^2)\times10^{-12}}\right)^{10}\Rightarrow10^{120}=\left(\frac{2}{(4\pi d^2)\times10^{-12}}\right)^{10}$. It leads to $10^{12}=\frac{2\times10^{12}}{(4\pi d^2)}\Rightarrow d^2=\frac{1}{2\pi}\Rightarrow d=\sqrt{\frac{1}{2\pi}}\Rightarrow d=\sqrt{\frac{1}{2\pi}}=0.398$ m or $\boldsymbol{d}=\mathbf{0.40}$ **m or 40 cm.**
I-22	Intensity of sound in dB is expressed as $\beta=10\log_{10}\frac{I}{I_0}$ where $I_0=10^{-12}$ W/m². Given that $\beta_1=50$ dB and $\beta_2=60$ dB. Therefore, $\beta_2-\beta_1=10\log_{10}\frac{I_2}{I_0}-10\log_{10}\frac{I_1}{I_0}\Rightarrow\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_0}\times\frac{I_0}{I_1}\right)=10\log_{10}\left(\frac{I_2}{I_1}\right)$. With the given data $60-50=10\log_{10}\left(\frac{I_2}{I_1}\right)\Rightarrow1=\log_{10}\left(\frac{I_2}{I_1}\right)\Rightarrow\frac{I_2}{I_1}=10^1\Rightarrow\frac{I_2}{I_1}=10$ ...(1) .We know that intensity of sound $I=\frac{p_0^2v}{2B}\Rightarrow I\propto p_0^2$, since for a given medium speed of sound v and bulk modulus is constant. Accordingly, $\frac{I_2}{I_1}=\frac{p_2^2}{p_1^2}$...(2). Comparing (1) and (2) we have $\left(\frac{p_2}{p_1}\right)^2=10\Rightarrow\frac{\boldsymbol{p_2}}{\boldsymbol{p_1}}=\sqrt{\mathbf{10}}$**, is the answer.**
I-23	In a classroom, with 50 students, noise level in absence of a teacher is 50 dB. Average output of sound energy I is same across students, total noise level is $I_1=50I_0$. Accordingly with number of students increasing to 100 the noise level is $I_2=100I_0$. Intensity of sound in dB is expressed as $\beta=10\log_{10}\frac{I}{I_0}$ and hence $\beta_1=10\log_{10}\frac{I_1}{I_0}$ and $\beta_2=10\log_{10}\frac{I_2}{I_0}$. Thus, $\beta_2-\beta_1=10\log_{10}\frac{I_2}{I_0}-10\log_{10}\frac{I_1}{I_0}\Rightarrow\beta_2-\beta_1=10\log_{10}\left(\frac{I_2}{I_0}\times\frac{I_0}{I_1}\right)=10\log_{10}\left(\frac{I_2}{I_1}\right)$. With the given data $\beta_2-50=10\log_{10}\left(\frac{I_2}{I_1}\right)\Rightarrow\beta_2-50=10\log_{10}\left(\frac{100I}{50I}\right)\Rightarrow\beta_2=50+10\log_{10}2\Rightarrow\boldsymbol{\beta_2}=50+10\times0.30=\mathbf{53}$ **dB, is the answer.**
I-24	Maximum sound is detected in when difference in path lengths in the two tubes of the apparatus is $\Delta l=\lambda$ i.e. the waves through two tubes at the detector are in phase. And the sound is minimum when the

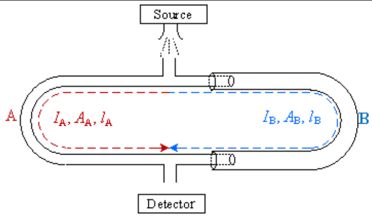
difference $\Delta l' = \frac{\lambda}{2}$, i.e. the two waves are anti-phase. Thus $\Delta l - \Delta l' = 2x \Rightarrow \lambda - \frac{\lambda}{2} = 2x \Rightarrow \lambda = 4x$.

Further, velocity of sound $v = f\lambda \Rightarrow f = \frac{v}{\lambda} \Rightarrow f = \frac{v}{4x}$ Thus, with the available data $\boldsymbol{f} = \frac{340}{4\times(2.5\times10^{-2})} = \mathbf{3.4}$ **kHz is the answer.**



I-25

Maximum sound is detected in when difference in path lengths in the two tubes of the apparatus is $\Delta l = \lambda$ i.e. the waves through two tubes at the detector are in phase. And the sound is minimum when the difference $\Delta l' = \frac{\lambda}{2}$, i.e. the two waves are anti-phase. Thus $\Delta l - \Delta l' = 2x \Rightarrow \lambda - \frac{\lambda}{2} = 2x \Rightarrow \lambda = 4x$.



Further, velocity of sound $v = f\lambda \Rightarrow f = \frac{v}{\lambda} \Rightarrow f = \frac{v}{4x}$.

Part (a): With the given data $f = \frac{330}{4\times(16.5\times10^{-3})} = 5 \times 10^3$. Hz or **5.00 kHz, is the answer of part (a).**

Part (b): Minimum intensity of sound is $I_1 = I$ and its maximum intensity $I_2 = 9I$. Further, we have $I = \frac{\omega^2 s_0^2 B}{2v}$ and $I = \frac{p_0^2 v}{B}$ here ω is angular velocity, s_0 is displacement magnitude, p_0 is pressure amplitude, B is bulk modulus and v is speed of wave. It can be seen that in either form ; likewise $I \propto s_0^2$ and $I \propto p_0^2$ i.e. both displacement amplitude and pressure amplitude are squarely proportional to intensity of sound and in generic form it can be written as $I \propto A^2$. Therefore, we have $\frac{I_2}{I_1} = \frac{A_2^2}{A_1^2} \Rightarrow \frac{A_2}{A_1} = \sqrt{\frac{I_2}{I_1}}$. Using the given data, $\frac{A_2}{A_1} = \sqrt{\frac{9I}{I}}$=3.

In case of maximum intensity amplitudes of waves through paths, amplitudes are additive, a case of constructive interference, i.e. $I_2 = I_A + I_B$ and in case of minimum intensity the amplitudes are subtractive, a case of destructive interference, i.e. $I_1 = I_A - I_B$. Accordingly, we have $\frac{A_2}{A_1} = \frac{I_A+I_B}{I_A-I_B} = 3 = k$, this is case of componendo-dividendo and accordingly we have $\frac{A_A}{A_B} = \frac{k+1}{k-1} \Rightarrow \frac{A_A}{A_B} = \frac{3+1}{3-1} = 2$. Hence, ratio of amplitudes of the two waves arriving at the detector is **2, is the answer**.

I-26

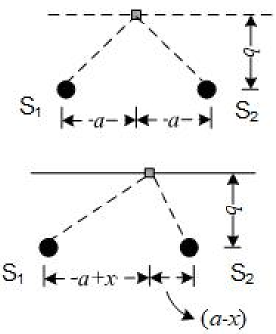
Distance of person from source-1 is $x_1 = 6$ m and source-2 is $x_1 = 6.4$ m. hence $\Delta x = |x_1 - x_2| = 6.4 - 6.0 = 0.4$ m. It is given that speed of sound $v = 320$ m/s. Requirement for constructive interference is $\Delta x = n\lambda$ here $n \in W$and destructive interference is $\Delta x = (2n+1)\frac{\lambda}{2} \Rightarrow \Delta x = (2n+1)\frac{v}{2f}$. With the available data $0.4 = (2n+1)\frac{320}{2f} \Rightarrow f = (2n+1)\frac{320}{0.8} \Rightarrow f = (2n+1)400$. Given that $f_{min} = 500$ Hz and $f_{max} = 5000$ Hz, different values of n for destructive interference are are as under –

(i) $n = 0 \Rightarrow f_0 = 400$ Hz, is invalid $f_0 < f_{min}$
(ii) $n = 1 \Rightarrow f_1 = (2\times1+1)400 = \mathbf{1200}$ **Hz, is valid** $\boldsymbol{f_{min}} < f_1 < \boldsymbol{f_{max}}$
(iii) $n = 2 \Rightarrow f_2 = (2\times2+1)400 = \mathbf{2000}$ **Hz, is valid** $\boldsymbol{f_{min}} < f_2 < \boldsymbol{f_{max}}$
(iv) $n = 3 \Rightarrow f_3 = (2\times3+1)400 = \mathbf{2800}$ **Hz, is valid** $\boldsymbol{f_{min}} < f_3 < \boldsymbol{f_{max}}$
(v) $n = 4 \Rightarrow f_4 = (2\times4+1)400 = \mathbf{3600}$ **Hz, is valid** $\boldsymbol{f_{min}} < f_4 < \boldsymbol{f_{max}}$
(vi) $n = 5 \Rightarrow f_3 = (2\times5+1)400 = \mathbf{4400}$ **Hz, is valid** $\boldsymbol{f_{min}} < f_5 < \boldsymbol{f_{max}}$

Hence answer is 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz and 4400 Hz

I-27

At D Two waves are received one direct from S and other reflected from the card board. Intensity changes, on movement of board along the line joining S and D, from maximum intensity (constructive interference) changes to minimum interference (destructive interference) on displacement of the board by $d = 0.2$ m. The change is path length is caused by the displacement of board through any distance is double the displacement of the board. Thus, in the instant case $\Delta x = 2d = (2n+1)\lambda$, here $n \in W$. With the given data, $2 \times 0.2 = (2n+1)\frac{336}{2f} \Rightarrow f = (2n+1)\frac{336}{0.8}$. Thus, minimum frequency for change of constructive to destructive interference shall be for = 0 . Accordingly, $f = \frac{336}{0.8} =$ **420 Hz is the answer**.

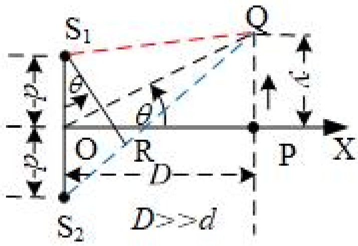
I-28	Path length of wave from S to D is $x_1 = d$ and path length of reflected wave is $x_2 = 2\times\sqrt{\left(\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d\right)^2\right)} \Rightarrow x_2 = 2\times\sqrt{\left(\frac{d^2}{4}+2d^2\right)} \Rightarrow x_2 = 2d\times\sqrt{\frac{9}{4}} = 3d$. Thus $\Delta x = \lvert x_1 - x_2\rvert = 2d$. Further, for constructive interference, $\Delta x = 2d = n\lambda$ and it occurs at wavelength $\lambda = \frac{d}{2}$. Therefore, corresponding value of $2d = n\frac{d}{2} \Rightarrow n = 4$. It is required to determine displacement s of cardboard away from the source when reflected wave becomes anti-phase. Thus new path length is $x_2' = 2\times\sqrt{\left(\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d+s\right)^2\right)}$. Accordingly, revised path difference is $\Delta x' = \lvert x_1 - x_2'\rvert = 2\times\sqrt{\left(\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d+s\right)^2\right)} - d = (2\times n+1)\frac{\lambda}{2}$. With available data we have $2\times\sqrt{\left(\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d+s\right)^2\right)} - d = (2\times 4+1)\frac{d}{4} \Rightarrow 2\times\sqrt{\left(\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d+s\right)^2\right)} = \frac{13}{4}d$. It leads to $\sqrt{\left(\frac{d}{2}\right)^2+\left(\sqrt{2}d+s\right)^2} = \frac{13}{8}d$. Squaring both sides $\frac{d^2}{4}+\left(\sqrt{2}d+s\right)^2 = \frac{169}{64}d^2 \Rightarrow \sqrt{2}d+s = \sqrt{\frac{169}{64}-\frac{1}{4}}\times d$. It leads to $s = \left(\sqrt{\frac{169-16}{64}}-\sqrt{2}\right)d \Rightarrow s = \left(\sqrt{\frac{153}{64}}-1.41\right)d \Rightarrow s = (1.55-1.41)d \Rightarrow \boldsymbol{s = 0.14d}$**, is the answer.**
I-29	Distance of listener from one speaker is $x_1 = 3.2$ m while from the another speaker, as per Pythagoras Theorem is $x_2 = \sqrt{(3.2)^2+(2.4)^2} = 0.8\sqrt{4^2+3^2} \Rightarrow x_2 = 0.8\times 5 = 4\text{m}$. Accordingly,$\Delta x = x_2 - x_1$, or $\Delta x = 0.8$. The frequency range is $f_{\text{min}} = 20$ Hz to $f_{\text{max}} = 20000$ Hz. Minimum intensity of sound is head in case of destructive interference for which $\Delta x = (2n+1)\frac{\lambda}{2}$, which translates in frequency domain as $\Delta x = (2n+1)\frac{v}{2f} \Rightarrow f = (2n+1)\frac{v}{2\Delta x}$. Here $n \in W$ and possible values are 0, 1, 2,3…. Such that $f_{\text{min}} \le f \le f_{\text{max}}$. Thus using the available data on the $f_{\text{min}} \le (2n+1)\frac{v}{\Delta x}$ or $f_{\text{min}} \le (2n+1)\frac{320}{2\times 0.8} \Rightarrow 20 \le 200(2n+1) \Rightarrow 2n+1 \ge \frac{1}{10} \Rightarrow 2n \ge \frac{1}{10}-1 \Rightarrow 2n \ge -\frac{9}{10} \Rightarrow n \le \frac{9}{20}$. Now analyzing the inequality for maximum frequency $200(2n+1) \le f_{\text{max}} \Rightarrow 200(2n+1) \le 20000$. It leads to $2n+1 \le 100 \Rightarrow n \le \text{int}\left(\frac{99}{2}\right) \Rightarrow n \le 49$. **Thus, desired frequencies in answer are** $\mathbf{200(2\boldsymbol{n}+1)}$ **where** $\boldsymbol{n} = \mathbf{0, 1, 2, 3 \ldots 49}$**.** **N.B.: Required frequencies are determined using mathematics of inequalities.**
I-30	 Initial distance of detector from source S_1 be d_1 and of source S_2 be d_2. It will be seen that initial path length of detector from both the sources is $l_1 = l_2 = \sqrt{a^2+b^2}$. When detector shifts through a distance x along a line parallel to the line joining the two sources, the difference in path lengths is $\Delta l' = l_1' - l_2'$. And it works out to $\Delta l' = \sqrt{(a+x)^2+d^2} - \sqrt{(a-x)^2+d^2}$. Here, $a = \frac{20}{2} = 10$ cm, $d = 20$ cm. Accordingly$\Delta l' = \sqrt{(10+x)^2+20^2} - \sqrt{(10-x)^2+20^2}$, It solves into $\Delta l' = \sqrt{100+x^2+20x+400} - \sqrt{100+x^2-20x+400}$ or $\Delta l' = \sqrt{500+x^2+20x} - \sqrt{500+x^2-20x}$…(1) Initially, $\Delta l = l_1 - l_2 = 0$ and both the sources are emitting equal wavelengths $\lambda = 20$ cm which are in same phase hence maximum intensity would occur. But, for first occurrence minimum intensity $\Delta l' = \frac{\lambda}{2} = \frac{20}{2} \Rightarrow \Delta l' = 10$ cm…(2). Equating (1) and (2) we have $10 = \sqrt{500+x^2+20x} - \sqrt{500+x^2-20x}$. Squaring the last expression $100 = (500+x^2+20x)+(500+x^2-20x)+2\sqrt{(500+x^2+20x)(500+x^2-20x)}$. It further leads to $100 = 2(500+x^2)+2\sqrt{(500+x^2)^2-(20x)^2} \Rightarrow \sqrt{(500+x^2)^2-(20x)^2} = -(450+x^2)$. Again squaring this expression $(500+x^2)^2-(20x)^2 = (450+x^2)^2 \Rightarrow (500+x^2)^2-(450+x^2)^2 = (20x)^2$.

It further simplifies into $50(950 + 2x^2) = 400x^2 \Rightarrow 950 + 2x^2 = 8x^2 \Rightarrow 6x^2 = 950 \Rightarrow x^2 = \frac{950}{6}$. It leads to $x = \sqrt{\frac{950}{6}} \Rightarrow x = \pm 12.6$ cm.. **Thus, answer is minimum distance by which detector should be shifted to perceive minimum intensity of sound is 12.6 cm.**

N.B.: Here algebraic solution will become longer and hence numerical values have been used to determine Δl for simplification. It is a good example of optimizing algebraic and numerical solution.

I-31

It is given that $d = |\pm 1.0|$ and $d \ll D$ and hence $y \approx D\theta$, $AB = y \sin\theta$ and $0Q \approx OP$. At P which is equidistant from the given speakers S_1 and S_2 since sound waves would in-phase and hence intensity of the sound would be maximum.



When detector moves through a distance y along a line parallel to S_1 and S_2 deflection of line joining mid of S_1 and S_2 to Q from that of P would be through an angle $\theta \approx \frac{y}{D}$.

In new position Q path length from source S_1 is $l_1 = S_1Q$ and from source S_2 it is $l_2 = S_2Q$, therefore path difference $\Delta l = l_2 - l_1 \approx S_2R = 2d \sin\theta$. With the given information $D \gg$ Geometrically angle $RS_1S_2 \approx QOP = \theta$. Further, with given data wavelength of the sound $\lambda = \frac{v}{f} \Rightarrow \lambda = \frac{330}{600} = \frac{11}{20}$ =0.55 m. Now taking each part separately,

Part (a): For first minimum intensity to occur $\Delta l = \frac{\lambda}{2} \Rightarrow 2d \cdot \sin\theta = \frac{0.55}{2} \Rightarrow \theta = \sin^{-1} 0.1375 \Rightarrow \theta = 7^0 55'$, or $\boldsymbol{\theta} = \mathbf{7.9^0}$**, is the answer of part (a).**

Part (b): For first maximum sound intensity the requirement is $\Delta l = \lambda = 2d \cdot \sin\theta_1 \Rightarrow \sin\theta_1 = \frac{\lambda}{2d}$, or $\theta_1 = \sin^{-1}\frac{0.55}{2} \Rightarrow \theta_1 = 15^0 58'$, , or $\boldsymbol{\theta_1} = \mathbf{16}\ ^{\mathbf{0}}$ **is the answer of part (b).**

Part (c): With this logic maxima would continue to occur as per $\Delta l = n\lambda$ and accordingly for successive maxima

$\boldsymbol{n} = \mathbf{2}$; $\Delta l_2 = 2\lambda = 2d \cdot \sin\theta_2 \Rightarrow \sin\theta_2 = \frac{2\lambda}{2d} = 0.55 \Rightarrow \theta_2 = \sin^{-1} 0.55 \Rightarrow \boldsymbol{\theta_2 = 33^0 20' = 33.3^0}$

$\boldsymbol{n} = \mathbf{3}$; $\Delta l_3 = 3\lambda = 2d \cdot \sin\theta_3 \Rightarrow \sin\theta_3 = \frac{3\lambda}{2d} = 0.825 \Rightarrow \theta_3 = \sin^{-1} 0.825 \Rightarrow \boldsymbol{\theta_3 = 55^0 35' = 55.6^0}$

$\boldsymbol{n} = \mathbf{4}$; $\Delta l_4 = 4\lambda = 2d \cdot \sin\theta_4 \Rightarrow \sin\theta_4 = \frac{4\lambda}{2d} = 1.1$ since for any angle $-1 \le \sin\theta \le 1$ and $\sin\theta_4$ is progressively increasing and maximum intensity would not occur $\Delta l > 3\lambda$

Hence, answer for part (c) is $\mathbf{33.3^0}$ and $\mathbf{55.6^0}$

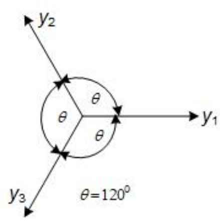
Thus answers are (a) 7.9^0 (b) 16^0 (c) Two at $\mathbf{33.3^0}$ and $\mathbf{55.6^0}$

N.B.: It is seen that-

a) θ is not quite small to be approximated $\sin\theta \approx \theta$
b) $\Delta l = n\lambda = 2d \cdot \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right)$ is a non-linear function and hence displacements for successive maxima cannot be determined linearly $\Delta l = n\lambda = 2d \cdot \theta$

I-32

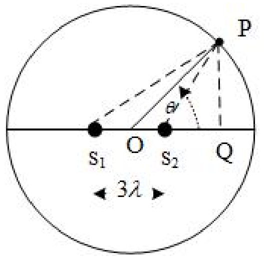
Let, the wave arriving from S_1 at point P is $y_1 = A \sin\left(\omega\left(t - \frac{x}{v}\right)\right)$, then as per given data wave arriving from S_2 at point P is $y_2 = A \sin\left(\omega\left(t - \frac{x}{v}\right) + \frac{2\pi}{3}\right)$ and from S_3 at point P is $y_3 = A \sin\left(\omega\left(t - \frac{x}{v}\right) + \frac{4\pi}{3}\right)$. Thus resultant displacement (which is a vector) would be $y = y_1 + y_2 + y_3 = 0$ and is shown in figure as much as it can be derived trigonometrically. **Thus answer is Zero**.



I-33

Path length of S_1 from P is $l_1 = \sqrt{D^2 + x^2}$ and that of S_2 from P is $l_2 = \sqrt{(D - 2\lambda)^2 + x^2}$, and path difference $\Delta l = l_1 - l_2 = D\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}} - (D-2\lambda)\left(1+\left(\frac{x}{D-2\lambda}\right)^2\right)^{\frac{1}{2}}$. Since $D \gg \lambda \Rightarrow D-2\lambda \to D$, it leads to $\Delta l = D\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}} - (D-2\lambda)\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}} \Rightarrow \Delta l = 2\lambda\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}}$. Distance of point P at which intensity of sound is same as that at point O necessitates $I_P = I_O$ is $\Delta l = \lambda \Rightarrow \lambda = 2\lambda\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}}$. It leads to $\left(1+\left(\frac{x}{D}\right)^2\right)^{\frac{1}{2}} = \frac{1}{2}$. It leads to $\frac{D}{\sqrt{D^2+x^2}} = \frac{1}{2} \Rightarrow 4D^2 = D^2 + x^2 \Rightarrow x^2 = 3D^2 \Rightarrow \boldsymbol{x = \sqrt{3}D}$**, is the answer.**

I-34



With the given figure and data combined a more elaborative figure is drawn. Distance of a point P from source S1, source S2 and center of the circular wire O are S1P = l_1, S2P = l_2, OP =r respectively. Then $l_1 = \sqrt{(r\cos\theta + 1.5\lambda)^2 + (r\sin\theta)^2}$ and $l_2 = \sqrt{(r\cos\theta - 1.5\lambda)^2 + (r\sin\theta)^2}$. These two distances are simplified into $l_1 = \sqrt{r^2 + 3r\lambda\cos\theta}$ and $l_2 = \sqrt{r^2 - 3r\lambda\cos\theta}$. Thus for constructive interference $\Delta l = l_1 - l_2 = n\lambda \Rightarrow n\lambda = (r^2 + 3r\lambda\cos\theta)^{\frac{1}{2}} - (r^2 - 3r\lambda\cos\theta)^{\frac{1}{2}}$. Accordingly, $n = \frac{r}{\lambda}\left[\left(1 + 3\frac{\lambda}{r}\cos\theta\right)^{\frac{1}{2}} - \left(1 - 3\frac{\lambda}{r}\cos\theta\right)^{\frac{1}{2}}\right]$. Given that $r \gg \lambda \Rightarrow \frac{r}{\lambda} \ll$ and $\cos\theta \le 1$, hence $3\frac{\lambda}{r}\cos\theta \ll$. Thus both the terms are simplified using Binomial theorem which states that $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4$ and $\sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4$, here when $x \ll$ all terms containing higher order of x can be ignored.

Applying, this binomial approximation we have $n = \frac{r}{\lambda}\left[\left(1 + \frac{1}{2} \times 3\frac{\lambda}{r}\cos\theta\right) - \left(1 - \frac{1}{2} \times 3\frac{\lambda}{r}\cos\theta\right)\right]$, it leads to $n = \frac{r}{\lambda}\left[3\frac{\lambda}{r}\cos\theta\right] \Rightarrow n = 3\cos\theta \Rightarrow \cos\theta = \frac{n}{3}$.

Now that $n \in W$ and $\cos\theta \le 1$ possible values of n are 0, 1, 2, 3. Accordingly, -

- for $\text{n} = 0 \Rightarrow \cos\theta = \frac{0}{3} = 0 \Rightarrow \theta = \frac{\pi}{2} = 90^0$
- for $\text{n} = 1 \Rightarrow \cos\theta = \frac{1}{3} \Rightarrow \theta = 70^0 42' \approx 70.5^0$
- for $\text{n} = 0 \Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = 48^0 42' \approx 48.5^0$
- for $\text{n} = 0 \Rightarrow \cos\theta = \frac{3}{3} = 1 \Rightarrow \theta = 0^0$

These are angular positions of P in 1st quadrant, its vertical images shall occur in 2nd Quadrant with angles $(180-90)^0 = 90^0, (180-70.5)^0 = 109.5^0, (180-48.5)^0 = 131.5^0, (180-0)^0 = 180^0$, Diagonal images shall occur in 3rd quadrant with angles $(180+90)^0 = 270^0, (180+70.5)^0 = 250.5^0, (180+48.5)^0 = 228.5^0, (180+0)^0 = 180^0$. In fourth quadrant angles of the images shall be -90^0, -70.5^0, and 0^0.

Thus answer is 0^0, 48.5^0, 70.5^0, 90^0 and similar points in other quadrants.

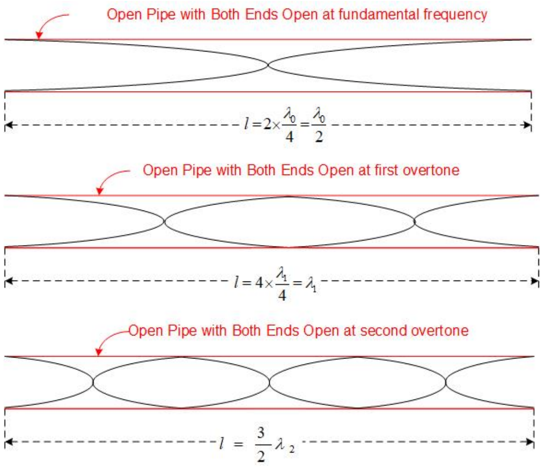
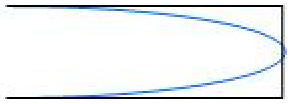
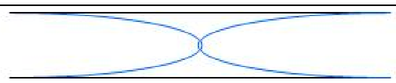
N.B.: This problem involves application of binomial theorem.

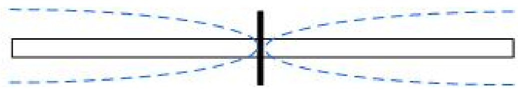
I-35

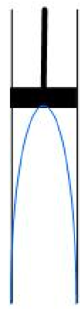
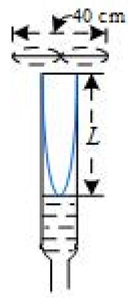
Intensity of sound from a source is $I \propto \frac{p}{r^2}$, here p is power of source and r is the radial distance of point of observation from the source. As shown in the given figure PS1=PS2 and the two sources are coherent i.e. of same frequency and phase and hence waves from both the sources cause constructive interference. Let distance between the two sources be $2d$, given that intensity of sound at P with both the sources on is I_0, therefore, intensity of sound at P from each of the sources is $I_\theta = \frac{p}{(d\sec\theta)^2} = \frac{I_0}{2} \Rightarrow I_0 = \frac{2p}{(d\sec\theta)^2}$, Now taking each part separately -

Part (a): With one source switched $\frac{I_{45}}{I_\theta} = \frac{\frac{2p}{(d\sec 45^0)^2}}{\frac{2p}{(d\sec\theta)^2}} \Rightarrow \frac{I_{45}}{I_\theta} = \left(\frac{\cos 45^0}{\cos\theta}\right)^2 \Rightarrow I_{45} = (\cos 45^0)^2 \times \frac{I_0}{2} = \frac{1}{2} \times \frac{I_0}{2}$,

or $\boldsymbol{I_{45} = \frac{I_0}{4}}$**, is the answer of part (a).**

	Part (b): With one source switched $\frac{I_{60}}{I_\theta}=\frac{\frac{2p}{(d\sec 60^0)^2}}{\frac{2p}{(d\sec\theta)^2}} \Rightarrow \frac{I_{60}}{I_\theta}=\left(\frac{\cos 60^0}{\cos\theta}\right)^2 \Rightarrow I_{60}=(\cos 60^0)^2\times\frac{I_0}{2}=\frac{1}{4}\times\frac{I_0}{2}$, or $\boldsymbol{I_{60}=\frac{I_0}{8}}$ **, is the answer of part (a).** **Thus answers are (a)** $\boldsymbol{\frac{I_0}{4}}$ **(b)** $\boldsymbol{\frac{I_0}{8}}$
I-36	 In case of given organ pipe at fundamental frequency has its length $l=\frac{\lambda_0}{2}\rightarrow\lambda_0=2l$ where anti-nodes are placed at both the open ends and a node at the middle of the pipe. And, $f_0=\frac{v}{\lambda_0}\Rightarrow f_0=\frac{v}{2l}$. Using the given data $f_0=\frac{340}{2\times0.2}\Rightarrow$ $\boldsymbol{f_0=850}$ **Hz**. For First overtone there will be Two nodes in between two antinodes at the ends and thus $l=\lambda_1$. as shown in the figure. Accordingly frequency of first overtone $f_1=\frac{v}{\lambda_1}=\frac{340}{0.20}\Rightarrow \boldsymbol{f_1=1700}$ **Hz.** For second overtone $l=\frac{3}{2}\lambda_2\Rightarrow\lambda_2=\frac{2}{3}l$, accordingly $f_2=\frac{v}{\lambda_2}=\frac{340}{\frac{2}{3}\times0.20}\Rightarrow \boldsymbol{f_2=2550}$ **Hz.** **Hence, answers is 850 Hz, 1700 Hz and 2550 Hz.**
I-37	 Minimum frequency of a closed organ pipe, implies one end closed and other end open, is given to be $f_0=500$ Hz. Since closed pipe shall have node at both ends and an antinode at its middle. It leads to $l=\frac{\lambda_0}{4}$. Further it is given that speed of sound is $v=340$ m/s and $\lambda_0=\frac{v}{f_0}\Rightarrow l=\frac{\frac{v}{f_0}}{4}\Rightarrow l=\frac{v}{2f_0}$. Using the given data $l=\frac{340}{4\times500}=0.17$m or $\boldsymbol{l=}$ **17 cm is the answer.**
I-38	Distance between Two consecutive nodes $d=\frac{\lambda}{2}\Rightarrow\lambda=2d$. Further, frequency of the source $f=\frac{v}{\lambda}=\frac{v}{2d}$. Using the given data, $\boldsymbol{f}=\frac{328}{2\times0.04}=$ **4. 1 kHz is the answer.**
I-39	Separation between a node and next antinode is $\frac{\lambda}{4}=0.25$ m is given and hence $\lambda=1.00$ m. Therefore frequency of the wave with speed $v=340$ m/s would be $f=\frac{v}{\lambda}$ which, with the available data, is $f=\frac{340}{1.00}=$ **340 Hz is the answer**.
I-40	 Fundamental wavelength in a cylindrical tube of length $l=0.50$ m, open at both ends, is $\frac{\lambda_0}{2}=l\Rightarrow\lambda_0=2l=2\times0.50\Rightarrow\lambda_0=1.00$ m. Fundamental frequency of the system is $f_0=\frac{v}{\lambda_0}=340$ Hz. In the given system frequencies of n[th] overtone shall be $f_n=nf_0\Rightarrow f_n=340n$, here $n\in N$. It is required to determine frequencies of overtones $f_{\min}\le f_n\le f_{max}\Rightarrow 1000\le340n\le2000\Rightarrow\frac{1000}{340}\le n\le\frac{2000}{340}$. It leads to $2.9\le n\le5.9$. Hence possible values of $n=3,4$ and 5. Accordingly, $f_3=340\times3=1020$ Hz; $f_4=340\times4=1360$ Hz; $f_5=340\times5=1700$ Hz. **Hence answer is 1020 Hz, 1360 Hz and 1700 Hz**
I-41	In a resonance column open end is antinode and liquid level form node. Let e is the end correction, and first resonance is observed at $l_1=\frac{\lambda}{4}-$e, in second resonance $l_2=\frac{3\lambda}{4}-e$. Therefore, $l_2-l_1=\left(\frac{3\lambda}{4}-e\right)-\frac{3\lambda}{4}-e$. It leads to $l_2-l_1=\frac{\lambda}{2}\Rightarrow\lambda=2(l_2-l_1)$. Further, speed of sound is $v=f\lambda\Rightarrow v=2f(l_2-l_1)$.

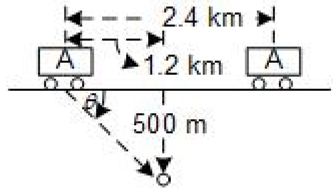
	Part (a): Using the given data $v = 2 \times 400(0.62 - 0.20) \Rightarrow \boldsymbol{v = 336}$ **m/s is the answer.** **Part (b):** At open end antinode is formed and it at a distance e above the open end. Hence with the available data $\boldsymbol{e} = \frac{\lambda}{4} - l_1 \Rightarrow e = \frac{2(l_2 - l_1)}{4} - l_1 \Rightarrow e = \frac{0.84}{4} - 0.20 \Rightarrow e = 0.01$ m or **1cm is answer of part (b)** **Thus answers are (a) 336 m/s (b) 1 cm**
I-42	Let f_{1-0} is the first fundamental frequency of a closed organ pipe P_1 having length l_1; in this case node occurs at closed end and an antinode occurs at open end. Therefore, $\lambda_{1-0} = 4l_1$. Therefore, first overtone wavelength shall be $\lambda_{1-1} = \frac{\lambda_{1-0}}{3}$ and corresponding first overtone frequency shall be $f_{1-1} = 3f_{1-0}$. Accordingly, $f_{1-1} = 3\frac{v}{\lambda_{1-0}} \Rightarrow f_{1-1} = \frac{3v}{4l_1}$…(1) In another open organ pipe P_2 fundamental frequency f_{2-0} of length l_2 where antinodes occur at open ends and a node occurs at the middle. Therefore, $\lambda_{0-2} = 2l_2$ and fundamental frequency of the organ pipe is $f_{2-0} = \frac{v}{\lambda_{0-2}} \Rightarrow f_{2-0} = \frac{v}{2l_2}$…(2) It is given that $f_{1-1} = f_{2-0}$, hence equating (1) and (2) we have $\frac{3v}{4l_1} = \frac{v}{2l_2} \Rightarrow l_2 = \frac{2}{3}l_1 \Rightarrow l_2 = \frac{2}{3} \times 30 \Rightarrow$ $\boldsymbol{l_2 = 20}$ **cm, is the answer.**
I-43	Clamped point of the rod, which is at the middle of it, shall be node while free ends of the rod shall act as antinode. At fundamental frequency the rod length $l = \frac{\lambda_0}{2} \Rightarrow \lambda_0 = 2l$.  Speed of the longitudinal wave in the rod $v = 3.8 \times 10^3$ m/s. Hence, fundamental frequency $f_0 = \frac{v}{\lambda_0} \Rightarrow$ $f_0 = \frac{v}{2l}$. Using the given data $f_0 = \frac{3.8\times10^3}{2\times1} \Rightarrow f_0 = 1.9 \times 10^3$ Hz or $f_0 = 1.9$ kHz. Hence longitudinal standing waves that can be set up in the rod are its higher harmonics $f_n = nf_0\|_{n\in N}$. Since it is required to determine frequencies $f_{\min} \le f_n \le f_{\max}$. With the available data $20 \le n(1.9 \times 10^3) \le 20 \times 10^3$. It, further, leads to $0 \le n \le \frac{200}{19}$. Thus integer values are $n \in N: n \le 10$. **Thus answer is 1.9 $\boldsymbol{n}$ kHz where $\boldsymbol{n \in N: n \le 10}$.**
I-44	Fundamental wavelength of an organ pipe of length l is $\lambda_0 = 2l$, and wavelength $\lambda = \frac{v}{f} \Rightarrow 2l = \frac{v}{f} \Rightarrow l = \frac{v}{2f}$. This inverse proportionality leads to greatest length of the organ pipe $l_{\max} = \frac{v}{2f_{min}}$. Using the given data $l_{\max} = \frac{340}{2\times20} = \boldsymbol{8.5}$ **m is the answer**.
I-45	In an open organ pipe fundamental wavelength $\lambda_0 = 2l$ and fundamental frequency is $f_0 = \frac{v}{\lambda_0} \Rightarrow f_0 = \frac{v}{2l}$. **Part (a):** Using the given data $f_0 = \frac{340}{2\times0.05} = 3.4 \times 10^3$ Hz or **3.4 KHz is the answer if part (a).** **Part (b):** Highest harmonic of the organ pipe must be below within the audible range is $n = \text{int}\left(\frac{f_{max}}{f_0}\right)$. Here, <int> is a function which returns integer quotient. Accordingly, $n = \text{int}\left(\frac{20}{3.4}\right) \Rightarrow \boldsymbol{n = 5}$ **is the answer of part (b)**
I-46	The problem states of a resonance column with a loudspeaker placed near an open end of a resonance column apparatus. It implies that it has a one end closed. In the system it is given that length of air column is $l = \frac{\lambda_0}{4} \Rightarrow \lambda_0 = 4l$. Therefore, fundamental frequency $f_0 = \frac{v}{\lambda_0} \Rightarrow f_0 = \frac{v}{4l}$. Wavelength, at which the air column will resonate are $\frac{\lambda_n}{4} = \frac{1}{(2n+1)} \times \frac{\lambda_0}{4} \Rightarrow \lambda_n = \frac{4l}{(2n+1)} \Rightarrow \lambda_n = \frac{4l}{(2n+1)}$. Hence corresponding frequencies shall be $f_n = \frac{v}{\lambda_n} \Rightarrow f_n = \frac{v}{\frac{4l}{(2n+1)}} \Rightarrow f_n = (2n+1)\frac{v}{4l} \Rightarrow f_n = (2n+1)\frac{320}{0.8\times4}$. It leads to a

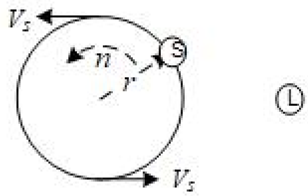
	simplification $f_n = 100(2n+1)$ here $n \in W$ provided $f_{min} \le f_n \le f_{max} \Rightarrow 20 \le 100(2n+1) \le 2000$. It leads to $f_{min} \le f_n \le f_{max} \Rightarrow 0 \le 2n+1 \le 20 \Rightarrow 0 \le 2n \le 20-1 \Rightarrow 0 \le n \le 9$. **Thus answer is $\boldsymbol{f_n = 100(2n+1)}$ where $\boldsymbol{n = 0, 1, 2, 3 \dots 9}$.**
I-47	Resonant frequencies of an open organ pipe are such that $\lambda_0 = 2l \Rightarrow f_0 = \frac{v}{\lambda_0} \Rightarrow f_0 = \frac{v}{2l}$…(1) and resonant frequencies are $f_n = nf_0$…(2). Further, it is given that $f_n = 1944$ Hz and $f_{n+1} = 2592$ Hz.. Using (2) and given data, $f_{n+1} - f_n = 2592 - 1944 = (n+1)f_0 - nf_0 \Rightarrow 648 = f_0$. Further using (1) we have $648 = \frac{324}{2l} \Rightarrow l = \frac{324}{2 \times 648} \Rightarrow l = 0.25$ m or **25 cm is the answer.**
I-48	 The given system is shown in the figure corresponds to a tube with one end closed and other end open. In the system frequency $f = 512$ Hz is fixed by the tuning fork and speed of sound (v) is fixed by the medium which remains same while piston is pulled out through a distance $\Delta l = 0.32$ m, Therefore wavelength shall be fixed $\lambda = \frac{v}{f}$. In the system node shall exist at the piston and antinode shall exist at open end of the tube, therefore, during resonance $l = (2n+1)\frac{\lambda}{4}$ where $n \in N$. Further, during initial resonance $l_i = (2n+1)\frac{\lambda}{4}$ and $l_f = (2n+3)\frac{\lambda}{4}$. Accordingly, for displacement of piston is $\Delta l = l_f - l_i = (2n+3)\frac{\lambda}{4} - (2n+1)\frac{\lambda}{4} \Rightarrow \Delta l = \frac{\lambda}{2} \Rightarrow \lambda = 2\Delta l$. Further, in both the cases medium and frequency of the source remains unchanged and hence speed of sound shall be $v = \lambda f \Rightarrow v = 2\Delta l f$. Using the given data $v = 2 \times 0.32 \times 512 = 327.68$ Hz say **328 Hz is the answer.**
I-49	Now wavelength of waves created by tuning fork is $\lambda = \frac{v}{f} \Rightarrow \lambda = \frac{330}{440} \Rightarrow \lambda = 0.75$m.. In the given system water level in both the arms of U-tube are at same level and form node. The tube protruding above the water level in shorter arm is of length say l_1 producing fundamental vibrations such that $l_1 = \frac{\lambda}{4}$. Accordingly, $\boldsymbol{l_1} = \frac{75}{4} = 18.75$ cm or **18.8 cm**, using principle of SDs. In the another tube it produces first overtones and hence length of the tube shall be $\boldsymbol{l_2} = \frac{3\lambda}{4} = \frac{3 \times 75}{4} = 56.25$ cm or **56.3 cm**, using principle of SDs. **Thus answer is 18.8 cm, 56.3 cm.**
I-50	 Stretched wire when vibrates at its fundamental frequency it has its nodes at fixed ends and middle of wire is antinode. Fundamental frequency of vibration of stretched wire is $f_0 = \frac{1}{2l}\sqrt{\frac{T}{\rho}}$…(1). Here, ρ is mass per unit length and is $\rho = \frac{m}{l}$, where m is mass of the wire and l is length of wire. Using the given data we get $\rho = \frac{4 \times 10^{-3}}{40 \times 10^{-2}} = 10^{-2}$…(2). Given that when the wire vibrates in its second harmonic, it sets vibration in air column in fundamental mode i.e. $f_a = f_1 = 2f_0$…(3). Here, f_a is the frequency of sound vibrations in air, $f_1 = 2f_0$ is the first overtone or second harmonic of the fundamental vibration in wire as given. Fundamental mode of vibration in given air column of length $L = 1$m, with one end closed by water and other end open, has $L = \frac{\lambda_a}{4} \Rightarrow \lambda_a = 4L$ where λ_a is wavelength of sound in air $\lambda_a = \frac{v}{f_a}$. Here, $v = 340$ m/s is speed of sound in air as given. Accordingly, $f_1 = \frac{v}{\lambda_a} \Rightarrow f_1 = \frac{340}{4 \times 1} \Rightarrow f_1 = 85$ Hz…(4) Using value of l and (2) and (4) we have get $T = (0.4)^2 \times (85)^2 \times (10^{-2}) \Rightarrow \boldsymbol{T} = 11.56$ N or **11.6 N is the answer.**
I-51	Given wire has mass per unit length $\rho = \frac{m}{l} = \frac{10 \times 10^{-3}}{30 \times 10^{-2}} = \frac{10^{-2}}{3}$ and vibrating in its fundamental mode. We know that

	$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\rho}}$. Further, the organ pipe, closed at one end, is resonating with the wire in its fundamental mode i.e. $f_0 = \frac{v}{4l'}$. Here, $l' = 0.50$ m is the length of the air column in the pipe. Equating the two expressions of f_0 we have $\frac{1}{2l}\sqrt{\frac{T}{\rho}} = \frac{v}{4l'}$. This leads to $T = \rho\left(\frac{vl}{2l'}\right)^2 \Rightarrow T = \left(\frac{10^{-2}}{3}\right)\left(\frac{340\times0.30}{2\times0.50}\right)^2 \Rightarrow \boldsymbol{T = 347}$ **N, is the answer**.
I-52	Frequency in gaseous medium $f \propto \sqrt{T}$, Therefore, we shall have $\frac{f+\Delta f}{f} = \sqrt{\frac{T+\Delta T}{T}} \Rightarrow 1 + \frac{\Delta f}{f} = \left(1 + \frac{\Delta T}{T}\right)^{\frac{1}{2}}$. Since, $\Delta T \ll T$ and hence neglecting second order terms $\left(\frac{\Delta T}{T}\right)^2$ we have $1 + \frac{\Delta f}{f} = 1 + \frac{1}{2}\cdot\frac{\Delta T}{T} \Rightarrow \boldsymbol{\frac{\Delta f}{f} = \frac{\Delta T}{2T}}$, **proved.**
I-53	Frequency in gaseous medium $f \propto \sqrt{T}$, Therefore, we shall have $\frac{f+\Delta f}{f} = \sqrt{\frac{T+\Delta T}{T}} \Rightarrow 1 + \frac{\Delta f}{f} = \left(1 + \frac{\Delta T}{T}\right)^{\frac{1}{2}}$. Since, $\Delta T \ll T$ and hence neglecting second order terms $\left(\frac{\Delta T}{T}\right)^2$ we have $1 + \frac{\Delta f}{f} = 1 + \frac{1}{2}\cdot\frac{\Delta T}{T} \Rightarrow \boldsymbol{\frac{\Delta f}{f} = \frac{\Delta T}{2T}}$. It is to be noted that here temperature are in thermodynamic scale and hence $T = 273 + 20 = 293$ K. Accordingly, $\Delta f = \frac{f\Delta T}{2T} \Rightarrow \Delta f = \frac{293\times2\Delta T}{2\times293} = 1 \Rightarrow \boldsymbol{f'} = f + \Delta f = 293 + 1 = \mathbf{294}$ **Hz is the answer.** **N.B.: (1)** Temperature in this case are in thermodynamic scale. **(2)** Taking temperatures as at (1) above $\Delta T \ll T$ is valid and hence $\Delta f = \frac{f\Delta T}{2T}$
I-54	The copper rod of length $l = 100$ cm is clamped at $l_1 = 25$ cm one end. Hence length of other free end is $l_2 = l - l_1 = 100 - 25 = 75$ cm. Distance between heaps is $d = 5$ cm. In Kundt's tube heaps are formed in air at nodes where $d = \frac{\lambda_a}{2} \Rightarrow \lambda_a = 2d$. and hence frequency of sound is $f = \frac{v}{\lambda} \Rightarrow f = \frac{v}{2d} = \frac{340}{2\times0.05} = 3400$ Hz. The clamped end serves as node while free ends will serve as antinode at resonant frequencies. From the given geometry wavelength in copper rod is $\lambda_2 = 4l_1$. Since, $\frac{l_2}{l_1} = \frac{0.75}{0.25} = 3$, hence length l_2 will vibrate at Third harmonic. Further, speed of wave is $v = f\lambda$ and hence in copper rod wave will have same frequency but a different wavelength, hence $v_{cu} = f\lambda_2 \Rightarrow v_{cu} = f \times 4l_1 \Rightarrow v_{cu} = 3400 \times 4 \times 0.25 \Rightarrow \boldsymbol{v_{cu} = 3400}$ **m/s is the answer.**
I-55	Given that heaps of powder in Kundt's apparatus are separated by $d = 6.5 \times 10^{-2}$m, hence wavelength of sound in air is $\lambda_a = 2d$. Further the steel rod of length $l = 1.0$ m is clamped at the center. The clamped end shall form node while free ends of length $\frac{l}{2}$ from clamp shall form antinode. When rod vibrates at fundamental frequency $\frac{\lambda_{cu}}{4} = \frac{l}{2} \Rightarrow \lambda_{cu} = 2l$. Frequency being constant, as it depends upon the source, hence speed of sound $v = \lambda f$. It solves into $v_{cu} = 2lf \Rightarrow v_{cu} = 2 \times 1.0 \times 2600 \Rightarrow \boldsymbol{v_{cu} = 5200}$ **m/s**. Likewise speed of sound in air $v_a = 2df \Rightarrow v_a = 2 \times (6.5 \times 10^{-2}) \times 2600 \Rightarrow \boldsymbol{v_a = 338}$ **m/s**. **Hence, answer is 5200 m/s, 338 m/s.**
I-56	Given that $f_1 = 476$ Hz and $f_2 = 490$ Hz, produce $n = 2$ Hz beats at some intermediate frequency f. This is possible when $\lvert f_1 - f\rvert = \lvert f_2 - f\rvert = n \Rightarrow f - f_1 = n \Rightarrow f = 476 + 2 \Rightarrow \boldsymbol{f = 478}$ **Hz is the answer.**
I-57	Given that f_1 Hz, number of beats produced with another tuning fork of frequency $f_2 = 256$ is $n = 4$ Hz. When tuning fork of frequency f_1 is loaded with wax its mass increases its frequency would decrease to f' and it produces beats $n' = 6$ Hz. This data implies that $f_2 > f_1$. Therefore, $f_2 - f_1 = n \Rightarrow f_1 = 256 - 4$, or $\boldsymbol{f_1 = 252}$ **Hz, is the answer.**

I-58	Frequency of a source is $f=\frac{v}{\lambda}$, With the given wavelengths $\lambda_1=32$ cm and $\lambda_2=32.2$ cm speed of sound $v=350$ m/s we have $f_1=\frac{v}{\lambda_1}\Rightarrow f_1=\frac{350}{0.32}=1094$ Hz and $f_2=\frac{v}{\lambda_2}\Rightarrow f_2=\frac{350}{0.322}=1087$. Therefore number of beats $n=f_1-f_2\Rightarrow \boldsymbol{n}=1094-1087=$ **7 per second is the answer.**
I-59	A tuning fork for frequency f_1 Hz creates beats $n=5$ per second with another tuning fork of frequency f_2 Hz. Frequency f_2 resonates with a closed organ pipe of $l=0.40$ m at its fundamental mode. Since, length of closed organ pipe $l=\frac{\lambda}{4}\Rightarrow\lambda=4l$. Speed of sound is $v=320$ m/s and hence $f_2=\frac{v}{\lambda}=\frac{320}{4\times0.40}=200$. Further, it is stated that when first tuning fork is slightly loaded with wax number of beats decrease; this is effect of decrease in frequency of the first form due to increase in its mass. This can happen only if $f_1>f_2$ therefore $f_1-f_2=n\Rightarrow f_1=f_2+n$. Using the available data $\boldsymbol{f_1}=200+5=$ **205 Hz, is the answer.**
I-60	Frequency of vibration of a stretched string $f=\frac{1}{2l}\sqrt{\frac{T}{\rho}}$, here l is length of the string, ρ is mass per unit length and T is tension in the string. In piano l and ρ are same and it is only T that affects frequency. Hence, we can say that $f\propto\sqrt{T}$. It leads to $\frac{f_2}{f_1}=\sqrt{\frac{T_2}{T_1}}\Rightarrow\frac{606}{600}=\sqrt{\frac{T_2}{T_1}}\Rightarrow\frac{T_2}{T_1}=(1+0.01)^2$. Since $0.01\ll$ hence the binomial expression can be approximated to $\frac{T_2}{T_1}=1+2\times0.01$. **It leads to $\frac{T_2}{T_1}=1.02$ is the answer.**
I-61	Given that tuning fork has frequency $f_1=256$ Hz. Let f_2 is the frequency of wire of length $l=0.25$ m vibrating in fundamental mode. In case of stretched wire $f_2=\frac{1}{2l}\sqrt{\frac{T}{\rho}}\Rightarrow f_2\propto\frac{1}{l}$…(1). Further, frequency of beats $n=\|f_1-f_2\|$…(2). When length of wire is reduced then as per (1) there is increase in f_2 and as per (2) it would leads to reduction in n only when $f_1>f_2$. It is given that initially $n=4\Rightarrow f_2=f_1-4=252$ Hz ...(3) It is required to determine length of wire l', without changing other parameters, at which new frequency $f_2'=f_1$. Using (1) $\frac{f_2}{f_2'}=\frac{l'}{l}\Rightarrow l'=\left(\frac{f_2}{f_2'}\right)l$. Using available data $l'=\left(\frac{f_2}{f_1}\right)l\Rightarrow l'=\left(\frac{252}{256}\right)25\Rightarrow l'==24.60$ cm. Hence length of wire is shortened by $\Delta\boldsymbol{l}=l-l'\Rightarrow\Delta l=25-24.60=$ **0.40 cm is the answer**.
I-62	This is a case of Doppler's effect where apparent frequency to the scooter driver is $f'=f\left(\frac{V-V_0}{V-V_s}\right)$…(1) In this case observer is scooter driver who is approaching the policeman, while source of sound is policeman. Therefore relative velocity of the source w.r.t. observer is $V_s=\frac{36\times10^3}{3.6\times10^3}\Rightarrow V_s=10$ m/s while $V_o=0$. Thus using the available data $f'=2000\left(\frac{340}{340-10}\right)\Rightarrow\boldsymbol{f'}=2000\left(\frac{34}{33}\right)=$ **2.06 kHz is the answer**.
I-63	It is given that frequency of sound $f=2400$ Hz. Car. The source of sound is moving with a velocity $V_s=\frac{18\times10^3}{3.6\times10^3}\Rightarrow V_s=5$ towards an observer standing on the road infront of the car i.e. $V_o=0$. Speed of sound in air is $V=340$ m/s. Hence, as per Doppler's effect apparent frequency is $f'=f\left(\frac{V-V_0}{V-V_s}\right)$. Using the given data $f'=2400\left(\frac{340}{340-5}\right)\Rightarrow\boldsymbol{f'}=2400\times\frac{340}{335}=$ **2436 Hz, is the answer.**
I-64	Given that car sounding whistle of frequency $f=1250$ Hz is moving with a speed 72 kmph or $V_s=\frac{72\times10^3}{3.6\times10^3}\Rightarrow V_s=20$ m/s. Velocity of sound is $V=340$ m/s. The problem is on Doppler's effect and it has two parts as under – **Part (a):** Observer is standing infront of the car i.e. source is moving towards observer and hence V_s is (+)ve. Accordingly, $f'=1250\left(\frac{340}{340-20}\right)\Rightarrow\boldsymbol{f'}=1250\left(\frac{34}{32}\right)=$ **1328 Hz is answer of part (a).** **Part (b):** The observer is standing behind the car i.e. source is moving away from the observer and hence

	V_s is (-)ve. Accordingly, $f' = 1250\left(\frac{340}{340-(-20)}\right) \Rightarrow \boldsymbol{f'} = 1250\left(\frac{34}{36}\right) = \mathbf{1181}$ **Hz is answer of part (a).** **Thus answers are (a) 1328 Hz (b) 1181 Hz**
I-65	A train making whistle is approaching platform with a speed of 54 kmph or $V_s = \frac{54\times10^3}{3.6\times10^3} \Rightarrow V_s = 15$ m/s and frequency apparent to the observer at the platform is of frequency $f' = 1620$ Hz. Speed of sound is 332 m/s. As per Doppler's Effect $f' = f\left(\frac{V}{V-V_s}\right) \Rightarrow f = f'\left(\frac{V-V_s}{V}\right)$. It is required to determine frequency of whistle apparent to an observer standing on platform after the train has crossed the platform. It implies that source is moving away from the platform and hence in this part V_s becomes (-)ve. Accordingly, apparent frequency to the observer is $f'' = f\left(\frac{V}{V-(-V_s)}\right) \Rightarrow f'' = f'\left(\frac{V-V_s}{V}\right)\left(\frac{V}{V+V_s}\right)$. Thus, using the available data we have $f'' = 1620\left(\frac{332-15}{332}\right)\left(\frac{332}{332+15}\right) \Rightarrow f'' = 1620 \times \frac{317}{332} \times \frac{332}{347} \Rightarrow \boldsymbol{f''} = 1620 \times \frac{317}{347} = \mathbf{1480}$ **Hz, is the answer.**
I-66	**Insert Figure** When bat is flying at $V_b = 6$ m/s, between two walls X and Y, producing the waves of frequency produced by the bat is $f = 4.5 \times 10^4$ Hz. Velocity of sound in air is $V = 330$ m/s. It is important to note that during reflection there is no change of frequency. As per Doppler's effect $f' = f\left(\frac{V-V_o}{V-V_s}\right)$ Apparent frequency of waves interacting between wall-Y and the bat is analyzed in two parts – (i) *Frequency f_Y perceived by the wall-Y in front of the bat:* in this case $V_o = 0$ and $V_s = V_b = 6$ m/s. Accordingly, using the available data $f_Y = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_Y = f\left(\frac{330}{330-6}\right) \Rightarrow f_Y = f\left(\frac{330}{324}\right)$ (ii) *Frequency f_Y' of the wave reflected from wall-Y as perceived by the bat:* in this case bat is the observer and travelling against the direction of the reflected wave and hence $V_o = -6$ m/s, while . the wall-Y has acting as a source has $V_s = 0$. Therefore, $f_Y' = f_Y\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_Y' = f_Y\left(\frac{330-(-6)}{330}\right)$. It leads to $f_Y' = f\left(\frac{330}{324}\right)\left(\frac{336}{330}\right) \Rightarrow f_Y' = f\left(\frac{336}{324}\right)$ Hz. Likewise, apparent frequency of waves interacting between wall-X and the bat is analyzed in two parts – (i) *Frequency f_X perceived by the wall-X behind the bat:* in this case $V_o = 0$ and $V_s = -V_b = -6$ m/s. Accordingly, using the available data $f_X = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_X = f\left(\frac{330}{330-(-6)}\right) \Rightarrow f_X = f\left(\frac{330}{336}\right)$ (ii) *Frequency f_X' of the wave reflected from wall-X as perceived by the bat:* in this case bat is the observer and travelling the direction of the reflected wave and hence $V_o = 6$ m/s, while . the wall-X has acting as a source has $V_s = 0$. Therefore, $f_X' = f_X\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_X' = f_X\left(\frac{330-6}{330}\right)$. It leads to $f_X' = f\left(\frac{330}{336}\right)\left(\frac{324}{330}\right) \Rightarrow f_X' = f\left(\frac{324}{336}\right)$ Hz. Thus, frequency of beats is $n = \lvert f_Y' - f_X'\rvert \Rightarrow n = \left\lvert f\left(\frac{336}{324}\right) - f\left(\frac{324}{336}\right)\right\rvert \Rightarrow n = (4.5\times10^4)\left\lvert\frac{336}{324} - \frac{324}{336}\right\rvert$. It solves to $\boldsymbol{n} = 3.3 \times 10^3 = \mathbf{330}$ **Hz is the answer.**
I-67	This is case of Doppler's Effect according to which apparent frequency is $f' = f\left(\frac{V-V_o}{V-V_s}\right)$. Given that speed of sound is $V = 330$ m/s, speed of source of sound (bullet train) is $V_s = 220$ m/s, and since bullet train passes a person and $V_s \gg V_o$ and hence by approximation $V_o = 0$. It involves two cases - **Case 1:** Now when train approached the person, direction of train and sound is same and hence V_s is (+)ve, accordingly apparent frequency while approach is $f_a = f\left(\frac{V}{V-V_s}\right) \Rightarrow f_a = f\left(\frac{330}{330=220}\right) \Rightarrow f_a = 3f$. **Case 2:** Now when train separating i.e. moving away approached the person, directions of train and sound are opposite and hence V_s is (-)ve, accordingly apparent frequency while approach is $f_s = f\left(\frac{V}{V-(-V_s)}\right) \Rightarrow f_s = f\left(\frac{330}{330-(-220)}\right) \Rightarrow f_s = 0.6f$.

	Accordingly fractional change is $\Delta f = \frac{f_a - f_s}{f_a} \Rightarrow \Delta f = \frac{3f - 0.6f}{3f} \Rightarrow \boldsymbol{\Delta f} = \frac{2.4}{3} = \mathbf{0.8}$ **is the answer.**
I-68	The position man with respect to the two trains is shown in the figure. Accordingly, angle made by line of travel of sound by the trains is such that $\tan\theta = \frac{0.5}{1.2} \Rightarrow \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+\left(\frac{0.5}{1.2}\right)^2}}$, it leads to $\cos\theta = 0.92$.  Accordingly velocity source-A of sound (the train) in the direction of the observer is $V'_{sA} = V_{sA}\cos\theta = V_s\cos\theta$ and that of source-B of sound (the train) in the direction of the observer is $V'_{sB} = V_{sB}\cos(\pi - \theta) = -V_s\cos\theta$. Here, for uniformity of units $V_s = \frac{72\times1000}{3600} = 20$ m/s As per Doppler's effect $f' = f\left(\frac{V-V_o}{V-V_s}\right) = f\left(\frac{V}{V-V_s}\right)$, since it is given that man is standing i.e. $V_o = 0$, and sound always travels radially and hence V remains unaffected. Accordingly, apparent frequency of train A to the person is $f_A = f\left(\frac{V-V_o}{V-V_s}\right) = f\left(\frac{V}{V-V'_{sA}}\right) \Rightarrow f_A = 500\left(\frac{340}{340-20\times0.92}\right) \Rightarrow \boldsymbol{f_A} = 528.6 \approx \mathbf{529}$ **Hz.** Likewise, apparent frequency of train B to the person is $f_{AB} = f\left(\frac{V}{V-V'_{sB}}\right) \Rightarrow f_A = 500\left(\frac{340}{340-(-20\times0.92)}\right) \Rightarrow$ $\boldsymbol{f_A} = 474.2 \approx$ **474 Hz. Hence answer is 529 and 474 Hz.**
I-69	4 beats are heard by the violinist playing $f = 440$ Hz note standing near the track. The violinist in the train is also playing $f = 400$ Hz note which appears to the violinist near the track $(V_o = 0)$ is $f' = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow$ $f' = f\left(\frac{V}{V-V_s}\right)$. Therefore, number of beats heard by the violinist standing near the track $n = \lvert f - f' \rvert =$ $\left\lvert f - f\left(\frac{V}{V-V_s}\right)\right\rvert \Rightarrow n = f\left(\frac{V_s}{V-V_s}\right) \Rightarrow \frac{V}{V_s} = \frac{f+n}{n} \Rightarrow V_s = V\frac{n}{f+n}$. Using the given data $V_s = 340\frac{4}{440+4} =$ 3.06 m/s. It leads to $\boldsymbol{V_s} = \frac{3.06*3600}{1000} = \mathbf{11}$ **kmph, is the answer.**
I-70	As per Doppler's effect apparent frequency $f' = f\left(\frac{V-V_o}{V-V_s}\right)$. In system velocity of source is $V_s = 0$ accordingly $\Rightarrow f' = f\left(\frac{V-V_0}{V}\right)$. Now in respect of source in the front, velocity of observer is against direction of velocity of sound hence it shall be (-)ve, therefore $f' = \frac{256(332-(-3.0))}{332} \Rightarrow f' = 258.3 \approx 258$ Hz. As regards source behind the man velocity of observer is in direction of velocity of sound hence it shall be (+)ve, therefore $f'' = \frac{256(332-3)}{332} \Rightarrow f'' = 253.7 \approx 254$ Hz. Hence, frequency of beats $n = \lvert f'' - f' \rvert$. Using the values derived above $n = \lvert 258.3 - 253.7 \rvert \Rightarrow \boldsymbol{n} = \mathbf{4.6}$ **Hz is the answer**.
I-71	Source infront of the standing listener i.e. $V_0 = 0$, the source is moving in the direction sound travelling to the listener and V_s is (+)ve. Therefore, apparent frequency as per Doppler's effect is $f_f = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_f =$ $\frac{fV}{V-V_s}$. Using the available data $f_f = \frac{512\times330}{330-5.5} = 520.68$ Hz. Likewise, the other source behind the observer it is moving against the direction of sound from it travelling to the listener and hence V_s is (-)ve . Accordingly, $f_b = f\left(\frac{V}{V-(-V_s)}\right) \Rightarrow f_b = \frac{fV}{V+V_s}$. Again using the available data $f_b = \frac{512\times330}{330+5.5} = 503.61$. Hence, frequency of beats $n = \lvert f_f - f_b \rvert$. Using the values derived above $n = \lvert 520.68 - 503.61 \rvert \Rightarrow \boldsymbol{n} =$ $17.07 \approx 17.1$ Hz is the answer. *Since number of beats is less than 20 Hz i.e. beyond the audible range of frequency, beats heard by the listener is* **Nil, is the answer.**

I-72	 As per geometry velocity of the source of frequency $f = 500$ Hz rotating in circle of radius $r = \frac{100}{\pi}$ at a speed $n = 5.0$revolutions per second. Therefore velocity of the source is $v = 2\pi rn$ and in SI units $v = r\omega \Rightarrow v = r(2\pi n) \Rightarrow v = \left(\frac{100}{\pi} \times \frac{1}{100}\right)(2\pi \times 5) \Rightarrow v = 10$ m/s. As per Doppler's effect is $f' = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f' = \frac{fV}{V-V_s}$ since observer is stationary. That sound travels radially and apparent frequency would be maximum when source is travelling towards the listener i.e. V_s is (+)ve and thus $f_{max} = \frac{fV}{V-V_s}$. Using the available data $f_{max} = \frac{500\times332}{332-10} = 515.53$ Hz, say $\boldsymbol{f_{max}} = \mathbf{516}$ **Hz.** Likewise, source is travelling away from the listener i.e. V_s is (-)ve and thus $f_{min} = \frac{500\times332}{332-(-10)}$Using the available data $f_{min} = \frac{500\times332}{342} = 485.4$ Hz, say $\boldsymbol{f_{min}} = \mathbf{485}$ **Hz**. Hence, **answer is 485 and 516 Hz.**
I-73	 Speed of the two trains approaching towards each other @ 90 kmph**.** One of the train is source and hence its velocity is velocity of the source $V_s = \frac{90\times1000}{3600} = 25$ m/s. The other train, the observer, is also travelling with same speed but against velocity of sound and hence $V_0 = -25$ m/s . Therefore, as per Doppler's effect apparent frequency by the observer train **is** $\boldsymbol{f'} = 500\left(\frac{350-(-25)}{350-25}\right) = 576.9 \approx$ **577 Hz, is the answer.**
I-74	As per Doppler's effect $f' = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f' = \frac{f(V-V_o)}{V}$ where policeman the source of sound of frequency $f = 16$ kHz is stationary and hence $V_s = 0$. Since, the observer is moving against the sound and hence V_o is (-)ve. Taking the pleas of the driver to be correct $f' > 20$ kHz, and using the available data $\frac{16(330-(-V_o))}{330} > 20 \Rightarrow 330 + V_o > 330 \times \frac{20}{16} \Rightarrow V_o > 330 \times \left(\frac{5}{4} - 1\right) \Rightarrow V_o > \frac{330}{4} \Rightarrow V_o > 82.5$ m/s. This minimum value of V_0 translates to **297 kmph is answer of part (a).** Maximum attainable speed of with ordinary cars is about 150 kmph hence plea of the driver is **impractical, is answer of part (b).** **Thus answers are (a) 297 kmph (b) Impractical**
I-75	First car (source of sound) is moving at 108 kmph and hence $V_s = \frac{108\times1000}{3600} = 30$ m/s. The car in the front (observer) is moving at 72 kmph and hence $V_o = \frac{72\times1000}{3600} = 20$ m/s. Both of them are since in the direction of the sound and hence V_s and V_o shall have (+)ve values in the formula of Doppler's effect $f' = f\left(\frac{V-V_o}{V-V_s}\right)$. Thus, using the available data $\boldsymbol{f'} = 800\left(\frac{330-20}{330-30}\right) = 826.7 \approx$ **827 Hz is the answer.**
I-76	In this case of Doppler's effect $f' = f\left(\frac{V-V_o}{V-V_s}\right)$ both the submarines are approaching each other. Velocity of the submarine creating sound signal is $V_s = 36$ kmph $\Rightarrow V_s = \frac{36\times1000}{3600} = 10$ m/s, is (=)ve since it is moving in the direction of the sound. The second submarine approaching the source is observer and travelling against direction of sound $V_o = 54$ kmph $\Rightarrow V_0 = \frac{54\times1000}{3600} = 15$ m/s shall be (-)ve. **Part (a):** Using the available data apparent frequency of the observer submarine is $f' = 2000\left(\frac{1500-(-15)}{1500-10}\right)$, it leads to $f' = 2033.56 \approx$ **2034 Hz is answer of part (a).** **Part (b):** When the signal of frequency $f'$is reflected by the second submarine, it velocity becomes source velocity in the direction of reflected sound and shall be (+)ve. Thus new source velocity $V_s' = 15$ m/s. But, the first submarine in this case become observer travelling against direction of reflected sound and hence it shall be (-)ve of value $V_o' = 10$. Therefore, again as per Doppler's effect $f" = f'\left(\frac{V-V_o'}{V-V_s'}\right)$, using the available data $f" = 2034\left(\frac{1500-(-10)}{1500-15}\right) = 2067.8 \approx$ **2068 Hz, is answer of**

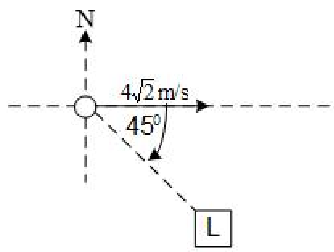
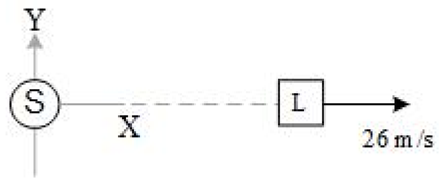
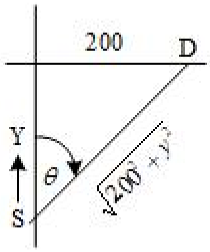
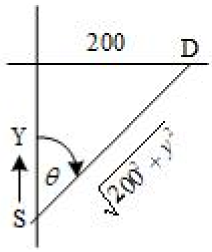
	part (b). **Thus answers are (a) 2034 Hz (b) 2068 Hz**
I-77	Given frequency of sound $f = 800$ Hz. As per Doppler's Effect frequency perceived by an observer in line of motion of source having velocity V_s when it is approaching the detector with velocity $V_o = 0$; at in this duration V_s shall be (+)ve. Accordingly, $f' = f\frac{V}{V-V_s}$. When, source moves away from the detector its velocity becomes against velocity of sound reaching detector and hence V_s shall be (+)ve. Hence , $f" = f\frac{V}{V-(-V_s)} = f\frac{V}{V+V_s}$. Bandwidth of the frequency detected by the detector is $\Delta f = f' - f" = f\frac{V}{V-V_s} - f\frac{V}{V+V_s}$. Using the available data $8 = 800\left(\frac{340}{340-V_s} - \frac{340}{340+V_s}\right)$. It solves into $\frac{1}{34000} = \frac{2V_s}{(340)^2-V_s^2} \Rightarrow V_s^2 + 68000V_s - (340)^2 = 0$. This being quadratic equation we have $V_s = \frac{-68000\pm\sqrt{(68000)^2-4\times1\times(-(340)^2)}}{2} \Rightarrow V_s = \frac{-68000\pm680\times\sqrt{10000+1}}{2} \Rightarrow V_s = 340(\sqrt{10001} - 100)$.. It solves into $V_s = 340(100.005 - 100) = 340 \times 0.005 \Rightarrow V_s = 1.7$ m/s. Given that source of sound is oscillating in SHM with an amplitude $A_s = 0.17$ m and in SHM $v = A\omega$. Accordingly, in the instant case $V_s = A_s\omega_s \Rightarrow 1.7 = 0.17\omega_s \Rightarrow \omega_s = \frac{1.7}{0.17} = 10$ rad/s. Further, $\omega = 2\pi f \Rightarrow \omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{10} = 0.628$. Thus *time period of the oscillation of the source is* $\boldsymbol{T = 0.63}$**s, is the answer.**
I-78	Velocity of biker as shown in the figure is $u = 4\sqrt{2}$ m/s. At the time oh making sound pulse it is aligned towards the standing observer $(V_o = 0)$ at 45^0. Therefore, velocity of source $V_s = u\cos 45^0$, it leads to $V_s = 4\sqrt{2} \times \frac{1}{\sqrt{2}} = 4$ m/s. Given that velocity of sound $V = 334$ m/s and frequency of the source $f = 1670$ Hz, apparent frequency, as per Doppler's effect $f' = f\frac{V}{V-V_s} \Rightarrow f' = 1650\frac{334}{334-4} \Rightarrow f' =$ **1670 Hz is the answer.** 
I-79	This is a problem of Doppler's effect where apparent frequency is $f' = f\left(\frac{V-V_o}{V-V_s}\right)$ and given that velocity of source of frequency $f = 660\ Hz$ is given to be $v = 26$ m/s and observer is moving along a line $x = 336$ m i.e. the line is parallel to Y-axis. Velocity of sound is $V = 330$ m/s. Sound since travels radially and hence V remains constant, velocity of source $V_s = 0$ since it fixed at origin, but velocity of observer w.r.t. sound is changing in each of the three cases and that would affect apparent frequency as under –  **Case(a):** In this case velocity of observer w.r.t. sound is $V_o = v\cos(-\theta) = v\cos\theta$, here, with the given data $\tan\theta = \frac{336}{140} \Rightarrow \cos\theta = \frac{1}{1+\tan^2\theta} \Rightarrow \cos\theta = \sqrt{\frac{1}{1+\left(\frac{336}{140}\right)^2}} \Rightarrow \cos\theta = 0.38$. It is shown in the figure that V_o is against velocity of sound and hence it will be (-)ve. Accordingly, using the available data $f'_a = f\left(\frac{V-(-V_o)}{V}\right) \Rightarrow f'_a = f\left(\frac{V+V_o}{V}\right) \Rightarrow f'_a = 660\left(\frac{330+26\times0.38}{330}\right) =$ **680 Hz, is answer of part (a).** **Case (b):** In this case $\theta = 90^0$ and hence $V_o = v\cos 90^0 = 0$ and hence $f'_b = f\left(\frac{V-0}{V}\right) \Rightarrow f'_b = f = 660$ Hz is answer of part (b). **Case(c):** In this case velocity of observer w.r.t. sound is $V_o = v\cos(-\theta) = v\cos\theta$, here, with the given data $\tan\theta = \frac{-140}{338} \Rightarrow \cos\theta = \frac{1}{1+\tan^2\theta} \Rightarrow \cos\theta = \sqrt{\frac{1}{1+\left(\frac{336}{140}\right)^2}} \Rightarrow \cos\theta = 0.38$. It is shown in the

	figure that V_o is in the direction of the velocity of sound and hence it will be (+)ve. Accordingly, using the available data $f_c' = f\left(\frac{V-V_o}{V}\right) \Rightarrow f_c' = 660\left(\frac{330-26\times0.38}{330}\right) =$ **640 Hz, is answer of part (c).** **Thus answers are (a) 680 Hz (b) 660 Hz (c) 640 Hz.**
I-80	This is the case of Doppler's effect and apparent frequency $f' = f\left(\frac{V-V_o}{V-V_s}\right)$ is required to be determined where velocity of train $v_t = 108$ kmph which equates to $v_t = \frac{108\times1000}{3600} = 30$ m/s. Velocity of sound is $V = 340$ m/s and its frequency is $f = 500$. Taking each part separately – **Part (a):** Passenger (the observer) is sitting inside train which is blowing whistle. Thus velocities of passenger and whistle with respect to train are Zero and hence $f' = f\left(\frac{V-0}{V-0}\right) \Rightarrow \boldsymbol{f' = f =}$ **500 Hz, is answer of part (a).** **Part (b):** Observer is standing on the ground near the track and train (the source) has just passed the observer, therefore, velocity of source shall be against direction of sound and it shall be (-)ve. Accordingly, $f' = f\left(\frac{V-V_o}{V-(-V_s)}\right) \Rightarrow f' = 500\left(\frac{340-0}{340-(-30)}\right) \Rightarrow \boldsymbol{f' =}$ **459 Hz is the answer of part (b).** **Part (c):** When wind is blowing with a velocity $V_w = 36$ kmph $\Rightarrow V_w = \frac{36\times1000}{3600} = 10$ m/s, which is medium of the propagation of sound effective velocity of sound is $V' = V \pm V_w$; the V_w is additive when wind is blowing in the direction of propagation of sound and subtractive when it is blowing in opposite direction. Accordingly, taking effect of blowing wind formula of Doppler's effect gets slightly modified as $f' = f\left(\frac{V'-V_o}{V'-V_s}\right)$. This part has two cases same as that of part (a) and part (b) respectively. And they shall be solved with identical data except that V is replaced with V'. Accordingly – **Case 1:** Apparent frequency for the passenger is $f' = f\left(\frac{V'-0}{V'-0}\right) \Rightarrow \boldsymbol{f' = f =}$ **500 Hz is answer of part (c-1).** **Case 2:** Apparent frequency of the observer standing near the track is $f' = f\left(\frac{V'-V_o}{V'-(-V_s)}\right) \Rightarrow f' = 500\left(\frac{(340-10)-0}{(340-10)-(-30)}\right) \Rightarrow \boldsymbol{f' =}$ **458 Hz is answer of part (c-2).** **Thus answers are (a) 500 Hz (b) 459 Hz (c-1) 500 Hz and (c-2) 458 Hz.**
I-81	This problem is on Doppler's effect where apparent frequency is $f' = f\left(\frac{V-V_o}{V-V_s}\right)$, In the problem dog on the ground is just notional and has no significance since it is neither source nor observer. Given that velocity of cyclist (source) is $V_c = 12\ kmph \Rightarrow V_c = \frac{12\times1000}{3600} = \frac{10}{3}$ m/s. Frequency of source $f = 1600$ Hz.. Taking both parts separately – **Part (a):** Since wall is the observer and hence $V_0 = 0$, and source is moving in the direction of sound and hence $V_s = V_c$ is (+)ve. Accordingly, using the available data $f' = 1600\left(\frac{330}{330-\frac{10}{3}}\right) \Rightarrow \boldsymbol{f' = 1616}$ **Hz, is answer of part (a).** **Part (b):** Reflected whistle has wall as source of frequency $\boldsymbol{f'}$ and the cyclist as an observer travelling against direction of propagation of sound. Therefore, in this case $V_s = 0$ and $V_0 = -V_c$. Accordingly, apparent frequency to the cyclist will be $f" = f'\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f" = 1616\left(\frac{330-(-\frac{10}{3})}{330}\right) \Rightarrow \boldsymbol{f" = 1632}$ **Hz is answer of part (b).** **Answers are (a) 1616 Hz (b) 1632 Hz**.
I-82	Speed of signal is $V = 330$ m/s and man standing on the road producing signal of frequency $f = 1600$ Hz has $V_s = 0$. The sound is reflected from the car and returns to the man. It is required to determine apparent

	frequency heard by the man. This problem, as per Doppler's effect where $f' = f\left(\frac{V-V_o}{V-V_s}\right)$, is solved in two stages as under – **Stage 1:** Apparent frequency for the car where car is travelling in thet direction of sound with a velocity $V_c = 72$ kmph $\Rightarrow V_c = \frac{72\times1000}{3600} = 20$ m/s and hence $V_0 = V_c$. Hence, apparent frequency for the car is $f' = 1600\left(\frac{330-20}{330}\right) = 1600\left(\frac{310}{330}\right)$ Hz. **Stage 2:** The car becomes the source of reflected sound of frequency f' travelling against the direction of the sound and hence $V_s = -V_c$ while the man being the observer $V_0 = 0$. Accordingly, it leads to $f"=f'\left(\frac{330}{330\text{-}(-20)}\right)=1600\left(\frac{310}{330}\right)\left(\frac{330}{350}\right) \Rightarrow f" = 1600\left(\frac{310}{350}\right) \Rightarrow f" = \mathbf{1417}$ **Hz is the answer**.
I-83	Given that speed of car $V_c = 54$ kmph $\Rightarrow V_c = \frac{54\times1000}{3600} = 15$ m/s. Car emits sound of frequency $f = 400$ Hz and speed of sound is $V = 335$ m/s. Solving each part separately – **Part (a):** Wave length of the sound emitted by the car in the front will be $\lambda = \frac{V_{s-c}}{f}$, here velocity of sound w.r.t. car , both in same direction, is $V_{s-c} = V - V_c = 335 - 15 \Rightarrow V_{s-c} = 320$ m/s. Hence, $\lambda = \frac{320}{400} = 0.8$ m or $\boldsymbol{\lambda} =$ **80 cm, is the answer of part (a).** **Part (b):** As per Doppler's Effect apparent frequency $f' = f\left(\frac{V-V_o}{V-V_s}\right)$, where f is frequency of the sound perceived by the cliff with $V_0 = 0$ and $V_s = V_c$ in the direction of sound. Therefore, $f' = 400\left(\frac{335}{335-15}\right) \Rightarrow f' = \frac{400\times335}{320}$. Therefore, wavelength of the reflected wave is $\lambda' = \frac{V}{f'}$. It leads to $\lambda' = \frac{335}{\frac{400\times335}{320}} \Rightarrow \lambda' = \frac{320}{400} \Rightarrow \lambda' = 0.8$ m or $\boldsymbol{\lambda'} =$ **80 cm, is answer of part (b).** **Part (c):** For persons sitting in the car is the observer and moving against direction of the reflected sound and hence $V_o = -V_c$ and $V_s = 0$, while velocity of sound remains V. Hence apparent frequency for a person sitting in the car shall be $f" = f'\left(\frac{V-(-V_c))}{V}\right) \Rightarrow \boldsymbol{f}" = \left(\frac{400\times335}{320}\right)\left(\frac{335+15)}{335}\right) = 437.5$ Hz **.** Since last digit is 5 and last-but-one digit is an odd number therefore as per principle of Significant Digits, $\boldsymbol{f}" =$ **437 Hz is answer of part (c).** **Part (d):** No of beats per second heard by person sitting in car $n = f" - f = 425 - 400 = 25$ per second. Since frequency of beats is more than maximum perceivable limit of 15 Hz and hence **beats cannot be heard, is answer of part (d).** **Answers are (a) 80 cm (b) 80 cm (c) 438 Hz (d) Beats cannot be heard.**
I-84	Frequency of the signal $f = 400$ Hz produced by an operator sitting at base camp ($V_s = 0$) The signal is reflected back by car moving at a speed V_c towards source of sound i.e. velocity of car is against speed of sound and hence it shall be (-)ve, accordingly $V_o = -V_c$Thus frequency perceived at car as per Doppler's effect is $f' = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f' = 400\left(\frac{324+V_c}{324}\right)$. For the operator to be hearing reflected sound the car is a source of frequency f' moving in the direction of sound and hence it is (+)ve and in this case $V_s = V_c$. While, the operator sitting at base camp becomes observer such that $V_o = 0$. Hence, apparent frequency of the reflected sound to the operator is $f" = f'\left(\frac{V-V_o}{V-V_s}\right)$. Using the available data $410 = 400\left(\frac{324+V_c}{324}\right)\left(\frac{324}{324-V_c}\right) \Rightarrow 41(324 - V_c) = 40(324 + V_c)$. It leads to $81V_c = 324 \Rightarrow$ $\boldsymbol{V_c} =$ **4 m/s or 14.4 kmph is the answer.**
I-85	Listener stands at Q, 330 and hence $V_o = 0$, velocity of sound $V = 330$ m/s. Source of sound moving along X-axis is with a velocity $v = 22$ m/s continuously emits sound of 2.0 kHz and at $t = 0$, the source crosses the origin P. It has three parts – **Part (a):** Time taken by sound at P to reach the listener at a distance d is $t = \frac{d}{dV}$ since sound travels radially. Hence, $\boldsymbol{t} = \frac{330}{330} = 1$ **second, is the answer of part (a).** **Part (b):** Since angle between velocity of source at P and path of travel of sound PQ is 900, hence velocity of source along the line is $V_s = v\cos 90^0 = 0$, hence apparent frequency to the listener as per

	Doppler's effect is $f' = f\frac{V-V_0}{V-V_s} \Rightarrow f' = 2.0 \times 10^3 \times \frac{330-0}{330-0} \Rightarrow \boldsymbol{f' = 2.0}$ **is the answer of part (b).** **Part (c):** At the instance when sound emitted by source at P reaches the listener at Q is $x = v \times t = 22 \times 1$ or $\boldsymbol{x = 22}$ **m is the answer of part (b).`** **Thus answer are (a) $\boldsymbol{t = 1}$ second (b) 2.0 kHz (c) at $\boldsymbol{x = 22}$ m.**
I-86	 Let at t=0, the position of the source moving along Y-axis is $(0, y)$ with velocity $v = 22$ m/s, a pulse of sound produced at this position reaches listener situated at (660 m,0) when the source reaches the origin as shown in the figure; this implies $V_0 = 0$. Thus time (t) taken by source and sound pulse to travel the distance SO and SL is same and hence as per geometry $t = \frac{y}{v} = \frac{y}{22} = \frac{\sqrt{y^2+(660)^2}}{330} \Rightarrow (15y)^2 - y^2 = (660)^2 \Rightarrow 14 \times 16 \times y^2 = (660)^2$. It leads to $y = \frac{660}{\sqrt{224}}$. At the position or instance of creation of sound pulse, velocity of source along the velocity of sound $V_s = V\cos\theta = 22 \times \frac{y}{\sqrt{y^2+(660)^2}} \Rightarrow V_s = 22 \times \frac{\frac{660}{\sqrt{224}}}{\sqrt{\left(\frac{660}{\sqrt{224}}\right)^2+(660)^2}} \Rightarrow V_s = 22 \times \frac{1}{\sqrt{225}} \Rightarrow V_s = \frac{22}{15}$. As per Doppler's effect apparent frequency $f' = f\left(\frac{V-V_0}{V-V_s}\right)$. Using the available data $f' = 4000\left(\frac{330}{330-\frac{22}{15}}\right) \Rightarrow f' = 6017.9$ or $\boldsymbol{f' = 6018}$ **Hz, is the answer.** **N.B.:** Calculation of velocity of source along the line of travel of sound pulse at the instance of emission is to be taken into account, is key to the solution.
I-87	 When source gets closest to it the sound of frequency $f = 1200$ Hz reaching the detector was emitted by source moving along the line with a speed $v = 170$ m/s at $(-)t$ as shown in the figure where $t = \frac{x}{v} = \frac{\sqrt{x^2+200^2}}{V} \Rightarrow \frac{x}{170} = \frac{\sqrt{x^2+200^2}}{340} \Rightarrow 2x = \sqrt{x^2+200^2} \Rightarrow 3x^2 = 200^2 \Rightarrow x = \frac{200}{\sqrt{3}}$m. Therefore, angle made by line of travel of sound with the line of motion of source is such that $\cos\theta = \frac{x}{\sqrt{x^2+200^2}} \Rightarrow \cos\theta = \frac{\frac{200}{\sqrt{3}}}{\sqrt{\left(\frac{200}{\sqrt{3}}\right)^2+200^2}} = \frac{200}{200\times 2}$, It leads to $\cos\theta = \frac{1}{2}$. Therefore, velocity of source along the line of travel of sound is $V_s = v\cos\theta = 170 \times \frac{1}{2}$ or $V_s = 85$ m/s. The source is since stationary hence $V_o = 0$. Taking solution part-wise – **Part (a):** As per Doppler's effect, apparent frequency for the detector is $f' = f\left(\frac{V-V_0}{V-V_s}\right) \Rightarrow f' = 1200\left(\frac{340}{340-85}\right) \Rightarrow \boldsymbol{f' = 1600}$ **Hz is answer of part (a).** **Part (b):** Detector would detect frequency 1200 Hz when velocity of source at the time of emission of sound is perpendicular to the line of travel OQ i.e. $\alpha = 90^0$. Time taken by sound to travel distance OQ is $t' = \frac{OQ}{V} = \frac{200}{340}$ during which source would have travelled a distance PR equal to $x' = v \times t'$ it leads to $x' = 170 \times \frac{200}{340} = 100$ m. Therefore, distance QR is $d' = \sqrt{x'^2 + 200^2}$ it solves into $d' = \sqrt{100^2+200^2} = 100\sqrt{5} \Rightarrow \boldsymbol{d' = 224}$ **m is the answer of part (b).**
I-88	When an object tied to a string is whirled vertically condition of minimum velocity for the string to just remain tight is when particle moves in a circular trajectory of radius $r = 1.6$ m is $\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{rg} \Rightarrow v = \sqrt{1.6 \times 10} \Rightarrow v = 4$ m/s. It is given that observer is at a large distance aligned to the center of circle and hence velocity of source at its highest position A $V_{SA} = 4$ m/s. At the lowest position B velocity of source V_{SB} shall be determined with the principle of conservation of energy $\frac{1}{2}mV_{SB}^2 = \frac{1}{2}mV_{SA}^2 + mgh$. It leads to $V_{SB}^2 = V_{SA}^2 + 2gh \Rightarrow V_{SB}^2 = 4^2 + 2 \times 10 \times 2r \Rightarrow V_{SB}^2 = 16 + 2 \times 10 \times 2 \times 1.6 \Rightarrow V_{SB}^2 = 80$. It

	solves into $V_{sB} = 8.9$ m/s. As per Doppler's effect apparent frequency is $f' = f\left(\frac{V-V_0}{V-V_s}\right)$. Now, taking each part separately – **Part (a):** Maximum apparent frequency to the observer denominator of the expression of Doppler's Effect is minimum. This would occur when V_{sB} which greater than V_{sA} is in the direction of the sound and accordingly using the available data $f' = f\left(\frac{V-V_0}{V-V_{sA}}\right) \Rightarrow f' = 500\left(\frac{330}{330-8.9}\right) \Rightarrow \boldsymbol{f'} = \mathbf{514\ Hz}$ **is the answer of part (a).** **Part (b):** In respect of positions C and D when source is at the height of the center as per conservation of energy velocity of source would be $\frac{1}{2}mv'^2 = \frac{1}{2}mV_{sA}^2 + mgr \Rightarrow v'^2 = 4^2 + 2 \times 10 \times 1.6 \Rightarrow v'^2 = 16 \times 3 \Rightarrow v' = 4\sqrt{3} = 6.9$ m/s. Not it is has two cases with observer situated vertically above center of the circle. **Case 1:** When source is at C, as shown in the figure velocity of source is in the direction of the propagation of sound and hence it is positive or $V_{sC} = 6.9$ m/s and hence apparent frequency shall be $f'' = f\left(\frac{V-V_0}{V-V_{sC}}\right) \Rightarrow f'' = 500\left(\frac{330}{330-6.9}\right) \Rightarrow \boldsymbol{f''} = \mathbf{511\ Hz.}$ **Case 2:** When source is at D, as shown in the figure velocity of source is against the direction of the propagation of sound and hence it is negative or $V_{sD} = -6.9$ m/s and hence apparent frequency shall be $f''' = f\left(\frac{V-V_0}{V-V_{sD}}\right) \Rightarrow f''' = 500\left(\frac{330}{330-(-6.9)}\right) \Rightarrow \boldsymbol{f'''} = \mathbf{490\ Hz.}$ **Thus answers are (a) 514 Hz (b) 490 Hz and 511 Hz**
I-89	This problem involves application of Doppler's effect but not of the standard formulation, rather instance diagram shown since velocity he source is constantly changing with uniform acceleration. Let D is the distance between the source and time period the first cycle of the sound wave emitted by the source at $t = 0$ to reach the observer $t_1 = \frac{D}{V}$. Time period of the wave is $T = \frac{1}{f}$ and before emitting the second cycle the source having velocity expressed by Second equation of Motion $d = \frac{1}{2}aT^2\Big\vert_{u=0}$ and distance required to travel by the second cycle emitted at $t = T$ to reach the source is $D' = D - d$ and time taken in this travel is $t_2 = T + \frac{D\prime}{V} \Rightarrow t_2 = T + \frac{D-d}{V}$. Thus time period of the wave apparent to the observer is duration between two successive cycles just after the source starts, at which apparent frequency is required to be determined, and it is $T' = T + (t_1 - t_2) \Rightarrow T' = \left(T + \frac{D-d}{V}\right) - \frac{D}{V} \Rightarrow T' = T - \frac{d}{V} \Rightarrow T' = T - \frac{1}{2V}aT^2 \Rightarrow T' = \frac{1}{f} - \frac{1}{2V}a\frac{1}{f^2} \Rightarrow T' = \frac{1}{f} - \frac{a}{2Vf^2} \Rightarrow T' = \frac{2Vf-a}{2Vf^2}$. Therefore, frequency of the wave apparent to the observer situated at a large distance from observer is $f' = \frac{1}{T\prime} \Rightarrow f' = \frac{1}{\frac{2Vf-a}{2Vf^2}} \Rightarrow f' = \frac{2Vf^2}{2Vf-a}$ is the answer. **N.B.:** This solution involves application of Doppler's Effect from its basic principle. Here, deliberately frequency is represented by f instead of ν and velocity by V instead of v as generally used to avoid confusion to students not very much conversant with difference between notations ν and v.
I-90	Frequency of the signal f_0 Hz produced by a stationary source ($V_s = 0$) The signal is reflected back by two cars moving at a speeds v_1 and v_2 respectively. Given that difference in frequency of the reflected sound from the two cars Δf is 1.2% of f_0. It is required to determine difference in speeds of the two cars. This problem is solved be determining apparent frequency f_1'' of car-1 first and using it to determine f_2'', to reach end result. **Car-1:** Initially velocity of car approaching the source is against the speed of sound and hence it shall be (-) ve, accordingly $V_o = -v_1$. Thus frequency apparent to the car as per Doppler's effect is $f' = f\left(\frac{V-V_o}{V-V_s}\right) \Rightarrow f_1' = f_0\left(\frac{330-(v_1)}{330}\right) \Rightarrow f_1' = f_0\left(\frac{330+v_1}{330}\right)$. The source perceiving the reflected sound the car is a source of frequency f_1' moving in the direction of sound and hence it is (+)ve and in this case $V_s = v_1$. While, the operator being stationary $V_o = 0$. Hence,

	apparent frequency of the reflected sound to the operator is $f_1'' = f_0\left(\frac{330+v_1}{330}\right)\left(\frac{330}{330-v_1}\right) \Rightarrow f_1'' = f_0\left(\frac{330+v_1}{330-v_1}\right)$. **Car-2:** In case of Car-2 the process is same as that of Cae-1 except that v_1 is replaced with v_2, accordingly, we shall have $f_2'' = f_0\left(\frac{330+v_2)}{330-v_2}\right)$. Thus $\Delta f = \frac{\|f_1''-f_2''\|}{f_0} \times 100 \Rightarrow 1.2 = \frac{\left\|f_0\left(\frac{330+v_1}{330-v_1}\right)-f_0\left(\frac{330+v_2}{330-v_2}\right)\right\|}{f_0} \times 100 \Rightarrow 1.2 = \left\|\left(\frac{330+v_1}{330-v_1}\right)-\left(\frac{330+v_2}{330-v_2}\right)\right\| \times 100$. It is given that $v_1 \ll 330$ and $v_2 \ll 330$ and hence $\frac{v_1v_2}{330^2} \ll$ and hence ignored. It further solves into $\frac{1.2}{100} = \left\|\left(\frac{330+v_1}{330-v_1}\right)-\left(\frac{330+v_2}{330-v_2}\right)\right\| \Rightarrow \frac{1.2}{100} = \left\|\frac{2\times330\times(v_1-v_2)}{330^2}\right\|$. Let $\Delta v = \|v_1 - v_2\|$ Accordingly, $\frac{1.2}{100} = \frac{2\Delta V}{330} \Rightarrow \Delta V = \frac{1.2\times330}{200}$. It leads to $\Delta V = 1.98$ m/s or 7.1 kmph. Since answer is asked in nearest integer and hence **difference in speeds of cars is 7 kmph is the answer.** **N.B.:** In this problem approximation plays an important role in simplification of solution.

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