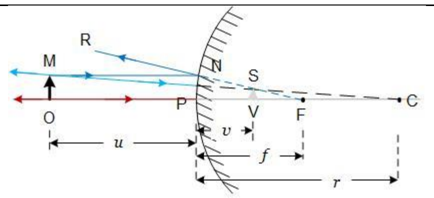
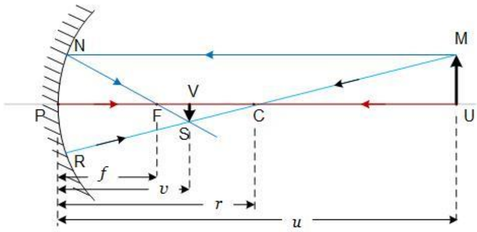
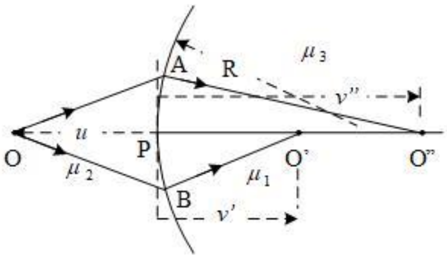
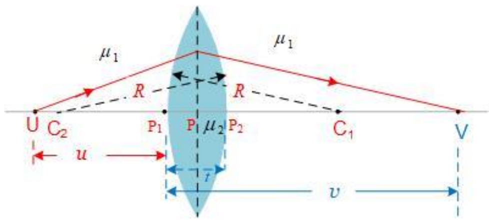
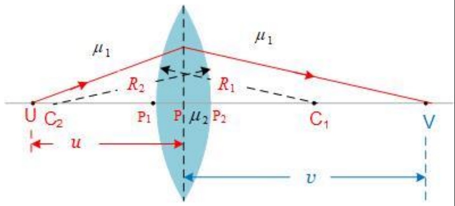
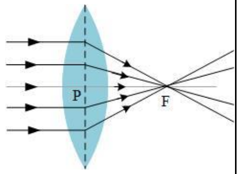
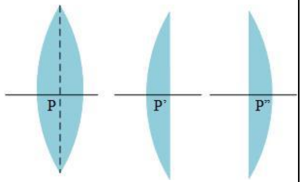
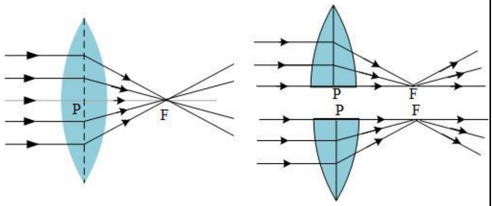
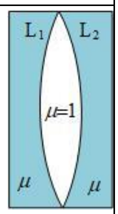
Wave and Motion : Geometrical Optics – Typical Questions

(Illustrations Only)

I-01	Rays incident from a light source are either parallel beam or divergent rays, this depends upon nature of source. Since, each reflected ray will be at angle with the normal equal to angle of its incidence, therefore, parallel ray will remain parallel and divergent ray will remain divergent. In this context each of the option is being analyzed – **Option (a):** Plane mirror forms an image of an object by producing reflected rays backwards. **Hence this option is correct.** **Option (b):** Only divergent rays when tracked backward converge at a point to form a virtual image. While convergent rays on reflection will converge at a point in forwards direction. These reflected rays have no affinity to the normal. **Thus this option is incorrect.** **Option (c):** Angle of reflection is equal to angle of incidence for all rays. Hence they are the divergent rays, which on reflection produced backward would converge at a point behind the mirror to create an image. This phenomenon is independent to angle of incidence. **Hence this option is incorrect**. **Option (d):** Light of different colours have different wavelength and accordingly different refractive index. But law of reflection is equivocally valid for all wavelengths. All rays of different wavelengths with identical angle of incidence will be reflected at the same angle of reflection. Hence only one image would occur. **Thus this option is incorrect.**
I-02	Refractive index is $\mu=\frac{\sin i}{\sin r}$ here i is the angle of incidence in the medium of incident ray and r is the angle of refraction in the medium of which refractive index μ is defined. In case of $\mu<1$ we shall have $i<r$. The limiting value of angle of refraction is $r=90^0$ and corresponding limiting value of angle of incidence called critical angle i_{cr} is $\sin i_{cr}=\mu \Rightarrow i_{cr}=\sin^{-1}\mu$. With this analysis any ray having $i>i_{cr}$ will undergo total reflection. Thus necessary mathematical condition for total reflection is $\mu<1$, which physically implies that ray travelling from optically denser to rarer medium. This condition is provided only in option (b). **Hence option (b) is the answer**.
I-03	For formation of image in geometrical optics in general and from spherical mirror in particular, minimum two rays are considered. One of it is parallel to principal axis i.e. paraxial which after reflection would pass through focal point of the mirror and the other is either passing or appear to be passing through centre of curvature of the mirror, which being normal to the point of incidence on the mirror will return radially after reflection. Point of intersection if these two rays from a point after reflection determine position of the image. Only this condition is stipulated in **option (c), which is the correct**.
I-04	 Formula for reflection from spherical mirrors is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$...(1). Here focal length $f=30$ cm, and distance of object from the mirror is $u=30$ cm. As per Cartesian sign convention all distances along principal axis w.r.t. pole P on the right are (=)ve and on the left are (-)ve. Accordingly, as per figure we have $u=-30$ while focal length remains $f=30$. Thus using the available data in (1) we have $\frac{1}{30}=\frac{1}{-30}+\frac{1}{v}\Rightarrow\frac{1}{v}=\frac{2}{30}$. It leads to $v=15$ cm, a (+)ve value and hence it will be on the right of pole i.e. behind the mirror. This is provided in **option (d), hence is the answer.**

I-05	Figure shows a pair of parallel rays and having angle of incidence i_1 and i_2 respectively. And after reflection angle of reflection be r_1 and r_2 respectively. As per law of reflection for one ray $i_1 = r_1$…(1) and for ray 2 we have $i_2 = r_2$…(2). Subtracting (2) from (1), or vice-versa $i_1 - i_2 = r_1 - r_2$…(3). On a spherical mirror, point of incidence of two rays would be different and hence $i_1 \neq i_2$ and also $r_1 \neq r_2$. Despite in spherical mirrors parallel incidents rays after reflection converge or appear to converge at focal point and therefore they shall not satisfy (3). Yet this is possible only in case of plane mirror provided in option (a). **Hence answer is option (a).**
I-06	The formula of reflection from spherical mirror is $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.  On case of a concave mirror f is always (+)ve and u cannot be (-)ve since that would having an object behind the mirror and contradicts necessary condition of formation of image by a mirror which requires object to be in front of the mirror. Hence, always u is positive. But, if $u < f \Rightarrow \frac{1}{u} > \frac{1}{f} \Rightarrow \frac{1}{v}$ is negative it implies that v is (-)ve i.e. image is virtual. This analysis rules **option (a), (b) and (c) as incorrect**. But certainly image is virtual if object is real and $u < f$, that makes **option (d) as correct**. **N.B.:** For option to be correct, necessary condition is $u < f$, that adds to certainty, but this information is latent in the question.
I-07	Formula for refraction image from a spherical surface is taking μ_1 to be refractive index of entire medium having convex spherical surface on the right and refractive index of material on left of the convex spherical surface is μ_2. Then the formula for position of the image O' is $\frac{\mu_2}{-u} + \frac{\mu_1}{v'} = \frac{\mu_1 - \mu_2}{R}$…(1). Keeping all other parameter same except refractive index of medium convex spherical surface on the right to be medium μ_3 the position of image O'' is $\frac{\mu_2}{-u} + \frac{\mu_3}{v''} = \frac{\mu_3 - \mu_2}{R}$…(2).  Rays emanate radially from object at O and two symmetrical rays OA and OB are taken above principal axis and below principal axis. Accordingly ray OA after refraction would pass through medium of refractive index μ_3 along AO''. This ray together with ray OP would form image O'' as per (3) Likewise, ray OB after refraction into medium of refractive index μ_2 would form image O' as per (2). Since, medium above principal axis and below it are distinctly different and hence $\mu_2 \neq \mu_3$ and hence two distinct images will be formed at O' and O'' as per **option (d), which is correct**.
I-08	Formula for refractive index of a lens is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$…(1). Here, u and v are distances of object and image from P_1. But, in a thin lens of thickness t values of u and v get modified as $u \rightarrow u + t$ and $v \rightarrow u - t$ from P_2. Accordingly, modified formula for the thin lens with new values is $\frac{1}{v-t} - \frac{1}{u+t} = \frac{1}{f}$ and this matches with **option (c), which is correct**. 
I-09	Focal length of double convex lens radii equal magnitude i.e. $R_1 = R_2 = R$ is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Using Cartesian sign convention $R_2 = -R$ while $R_1 = R$. Thus using the available data $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow \frac{1}{f} = 0.5\left(\frac{1}{R} + \frac{1}{R}\right) \Rightarrow \frac{1}{f} = 0.5 \times \frac{2}{R} \Rightarrow f = R$. This matches with **option (b), the answer**. 

I-10	In the problem intensity of light would be maximum at the point of its image i.e. at a distance v from the lens. Formula for a converging lens is $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. Given that, $u=-2f$, it leads to $\frac{1}{v}-\frac{1}{-2f}=\frac{1}{f}$. It leads to $\frac{1}{v}=\frac{1}{f}-\frac{1}{2f}\Rightarrow\frac{1}{v}=\frac{1}{2f}\Rightarrow v=2f$ and is provided in the **option (c), the answer**.
I-11	A beam of light parallel to the principal axis passing through a converging lens woul converge at F. Thus radius of image on a screen placed at any point between P towards F decreases, while there is no loss of light. Therefore, as screen moves **from P to F intensity increases**. But, the rays beyond focal point F diverge and hence radius of image on a screen at any point **beyond F** increases and hence **intensity, light energy per unit area, decreases.** This inference matches with that given **option (d), is correct.** 
I-12	Focal length of a symmetric double convex lens, taking equal radii of curvature equal to R. This enables cutting it in two equal parts perpendicular to principal axis, as shown in the figure. Formula for focal length of the lens can be written as $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)\Rightarrow\frac{1}{f}=(\mu-1)\times\frac{2}{R}$. Power of lens is $P=\frac{1}{f}$. It leads to $P=\frac{2(\mu-1)}{R}=4D\Rightarrow\frac{(\mu-1)}{R}=2D$…(1)  But when the lens is cut in two equal parts one of the radius, corresponding to plane surface, say $R_2=\infty$. Accordingly, for the cut lens $\frac{1}{f'}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{\infty}\right)\Rightarrow\frac{1}{f'}=(\mu-1)\left(\frac{1}{R}-0\right)\Rightarrow\frac{1}{f'}=\frac{(\mu-1)}{R}$. Hence power of the cut lens is $P'=\frac{1}{f'}\Rightarrow P'=\frac{(\mu-1)}{R}$…(2). Combining (1) and (2) we have $P'=2D$, this matches with **option (a), is the answer**.
I-13	Focal length of a symmetric double convex lens, taking equal radii of curvature equal to R. This enables cutting it in two equal parts perpendicular to principal axis, as shown in the figure. Formula for focal length of the lens can be written as $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)$. This leads to $\frac{1}{f}=(\mu-1)\times\frac{2}{R}$. Power of lens is $P=\frac{1}{f}$. It leads to $P=\frac{2(\mu-1)}{R}=4D$…(1)  Now when lens is cut in two equal parts by a plane parallel to principal axis, as shown in the figure, each half of the lens maintains its curvature of both the spherical surfaces and hence its focal length remains unchanged and so also the powers of the lens 4D as per (4). This matches with **the option (c), the answer.**
I-14	Given that two concave lenses L_1 and L_2 have focal lengths f_1 and f_2 respectively. And they are kept in contact with each other with space between them filled with material of $\mu=1$. The combination would look like that shown in the figure.  Now focal length of L_1 is $\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{f_1}=(\mu-1)\left(\frac{1}{\infty}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{f_1}=\frac{(1-\mu)}{R}$. And that of L_2 is $\frac{1}{f_2}=(\mu-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$. It leads to $\frac{1}{f_2}=\frac{(1-\mu)}{R}$. We find that $f_1=f_2=f=\frac{R}{1-\mu}$. Focal length of combination of lenses is $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1\times f_2}$, here d is the separation between two lenses. This separation in the combination, as shown in the figure, is $d=0$. Accordingly, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow F=\frac{f}{2}$, i.e, focal length decreases thus **option (d) is the answer.**

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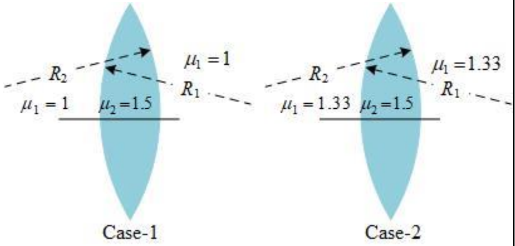
Given are two cases where a lens is held – Case 1: in air ($\mu_1 = 1$), and Case 2: in water ($\mu_1 = 1.33$). Whereas, refractive index of material of the lens is $\mu_2 = 1.5$. Further, focal length of lens is $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Accordingly, using the available data -



Case 1: $\frac{1}{f_1} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = 0.5 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
Case 2: $\frac{1}{f_2} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_2} = 0.13 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

In both the cases focal length remain (+)ve with only difference that in case 2, $f_2 > f_1$, thus a convergent in original remains a convergent lens as long as $\mu_1 < \mu_2$. This agrees with **option (a), the answer.**

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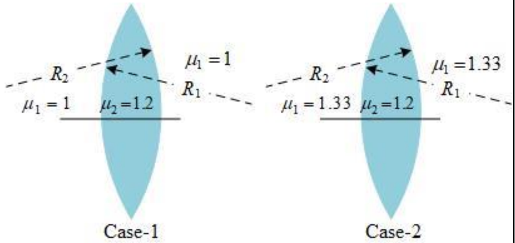
Given are two cases where a lens is held – Case 1: in air ($\mu_1 = 1$), and Case 2: in water ($\mu_1 = 1.33$). Whereas, refractive index of material of the lens is $\mu_2 = 1.2$. Further, focal length of lens is $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Accordingly, using the available data -



Case 1: $\frac{1}{f_1} = \left(\frac{1.2}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = 0.2 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
Case 2: $\frac{1}{f_2} = \left(\frac{1.2}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_2} = (-0.1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

In first case f_1 is (+)ve while in second case f_2 is (-)ve. Thus nature of converging lens would change to diverging lens when dipped in water. This agrees with **option (b), the answer.**

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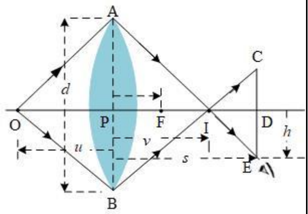
As per lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$. With the given information, as per Cartesian sign convention, object on the left leads to $u = -40$cm while focal point of the convex lens is on its right leads to $f = 20$ cm. Accordingly, using given data $\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \Rightarrow v = 40$cm.



Ray diagram as per problem is drawn here where eye is at a distance $s = 60$ cm and diameter of the lens $d = 10$ vm. Now to determine maximum height h below point D on the principal axis, properties of similar triangles will be applied to ΔAPI and ΔIDE we have $\frac{\text{AP}}{\text{PI}} = \frac{\text{ED}}{\text{DI}} \Rightarrow \frac{\frac{d}{2}}{v} = \frac{h}{s-v} \Rightarrow h = \frac{d}{2v}(s - v)$. Using the available data $h = \frac{10}{2\times 40}(60 - 20) \Rightarrow \boldsymbol{h} = \frac{10}{4} =$ **2.5 cm** and is matching with **option (b), the answer.**

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Non convergence of rays of different colours occurs due to different wavelengths if different colours, since refractive index depends upon wavelength of ray. This separation of colours is called chromatic abbertaion provided in **option (d), the answer.**

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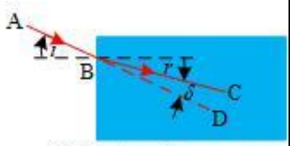
Taking each option separately:

Option (a): A light ray, generally travels in a straight line. But, when a ray AB with large angle of incidence $i \to 90^0$, encounter a reflecting surface as shown in the figure, it deviates from its path through a small fixed angle $\delta = 180^0 - 2i$ and moves along BC as a reflected ray such that $i = r$.**Thus option (a) is correct.**



Refraction phenomenon

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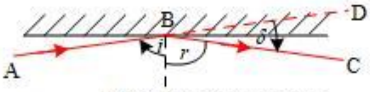
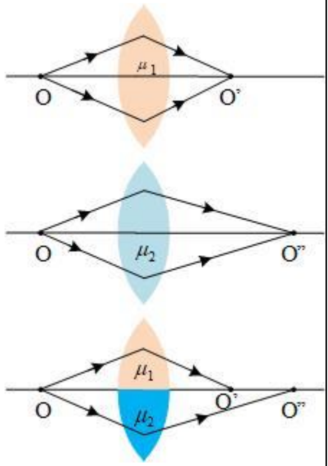
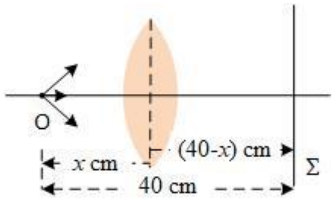
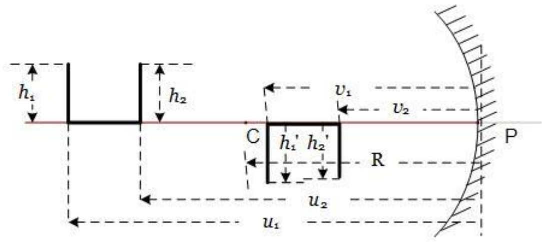
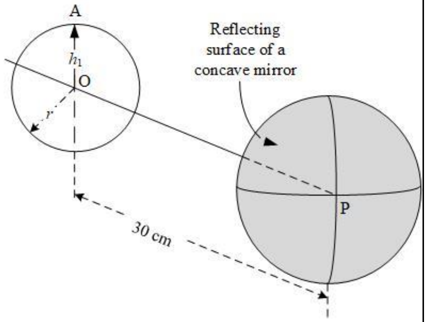
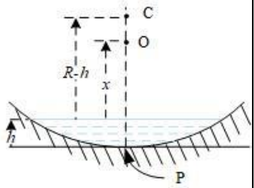
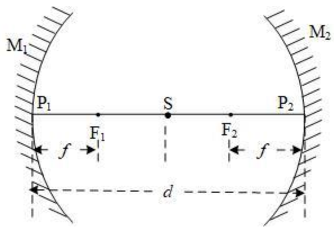
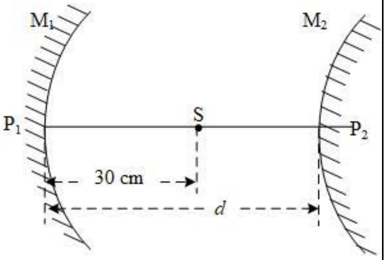
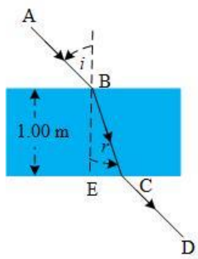
	Option (b): A light ray, generally travels in a straight line. But, when a ray AB with large angle of incidence $i \neq 90^0$, encounter a refracting medium as shown in the figure, it deviates from its path through a small fixed angle $\delta = \|i - r\|$ and moves along BC as a refracted ray such that $i \neq r$. In case the refracting medium is denser $i > r$ as per Snail's Law, and if rarer the $i < r$. **Thus option (b) is correct.**  Reflection phenomenon **Option (c):** In case of diffraction through a narrow slit, it creates bands each having different angle of bending of varying intensity and hence its not have a small fixed angle. **Hence option (c) is incorrect**. **Option (d):** In scattering light spreads in all directions not a small fixed angle**. Hence option (d) is incorrect.** **Thus answer is option (a) and (b).**
I-20	Analyzing each of the given option separately – **Option (a):** A source of light, unless very large produces divergent rays, and if it is large through a small window it emits parallel rays. Thus incident rays from a source are never converging, and hence **option (a) is incorrect**. **Option (b):** Rays from a source may converge either after reflection or refraction and can be imaged on a screen. These rays produce a real image and hence **option (b) is correct**. **Option (c):** Virtual object is a creation by extending divergent rays created by reflection and reflection rays backward. This virtual object on its own would not create an image. Hence, **option (c) is incorrect.** **Option (d):** Formation of virtual image of an object depends upon convergence of reflected or refracted image of an object, which may be real or virtual. Hence, declaring object being virtual can be done with certainty. **Thus option (d) is incorrect.** **Hence option (b) is correct.**
I-21	Analyzing each option separately – **Option (a):** Pole is a geometrical position on a spherical mirror, and hence it is independent if nature of ray hence this option is correct. **Option (b):** Focus is a point of convergence of paraxial rays after reflection. Thus it depends on nature of incident rays on spherical mirror. Hence, **option (b) is incorrect.** **Option (c):** Radius of curvature is dependent on geometry of spherical mirror and has nothing to do with direction of rays. Thus **option (c) is correct**. **Option (d):** Principal axis is is geometrical position on the ray diagram and is independent of direction of rays. Hence, **option (d) is correct.** **Thus correct answers are options (a), (c), and (d).**
I-22	Statement of principal axis, is indirect reference to a case of spherical mirror. Further, extended object implies that it is not a point and it may has a height, which is placed perpendicular to principal axis implies the object is erect. In this context each of the iption is being analyzed separately – **Option (a):** Real image of real object is always inverted. Hence**, option (a) is incorrect.** **Option (b):** Object is placement is defined to be real and hence both object and image cannot be virtual. **This makes option (b) to be incorrect.** **Option (c):** There are positions in concave mirror where real object produces virtual image. And in convex mirror image of a real object is always virtual. **Hence option (c) is correct.** **Option (d):** Virtual object is a virtual image of a real object. This can be created by either concave or convex mirror. Now if another concave mirror is placed infront of such mirror, , both facing each other, such that the distance (d) of the virtual object from second mirror is $d > f$ then a real image would be produced**. Hence option (d) is correct.** **Hence, answers are options (c) and (d).**

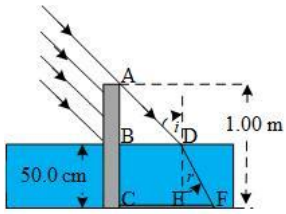
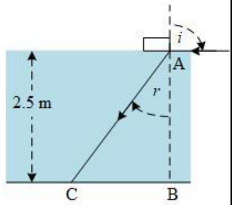
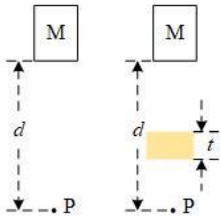
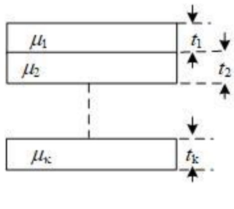
I-23	Real image of a point object on principal axis, due to spherical symmetry is always a point on the principal axis. Hence there will not be any vertical shifting of image. Accordingly, **Options (a) and (b) are incorrect**, while **option (c) is correct**. Now when upper half is painted, only half of the incident rays form the mage, while the other half of it is blocked by black paint. Hence, intensity of the image will decrease. This **option (d) is correct**. **Hence, answers are options (c) and (d).**
I-24	Three lenses with identical geometry are made of material with different refractive indices.  Lens L_1 is made of material with μ_1 refractive index produces image at O' by converging rays from entire aperture as shown in the figure. . Lens L_2 is made of material with μ_2 refractive index produces image at O" by converging rays from entire aperture, as shown in the figure. Lens L_3 is made of material with μ_1 refractive index above a plane passing through principal axis. Accordingly, rays through only upper half aperture produce image at O' as shown in the figure. But the image is of lesser intensity than that when whole lens was made of same material. Likewise, material of the lens below the plane passing through the principal axis is of refractive index μ_2 . Thus, rays passing through the lower half of the lens would produces image at O", but of lower intensity. Since the question is silent on intensity and only position of images is provided in the option. Therefore from the above illustrations images will be formed at O' and O" and **therefore options (a) and (b) are correct.**
I-25	As per lens formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$. As per Cartesian sign convention distance of object from lens is $u=-x$ cm and distance of image formed on the screen Σ is $v=40-x$ cm. Thus for image formed on the screen the equation with the available data is $\frac{1}{f}=\frac{1}{40-x}-\frac{1}{-x}\Rightarrow\frac{1}{f}=\frac{1}{40-x}+\frac{1}{x}$.  Limiting value of position of lens is $0\le x\le 40$. Accordingly, we have $\frac{1}{f}=\frac{40}{(40-x)x}\Rightarrow x^2-40x+40f=0$…(1) It is quadratic equation which leads to $x=\frac{-(-40)\pm\sqrt{(-40)^2-4\times1\times(40f)}}{2}\Rightarrow x=\frac{40\pm4\sqrt{100-10f}}{2}\Rightarrow x=20\pm2\sqrt{100-10f}$ For real value of x, the discriminant $\sqrt{100-10f}$ should be positive it implies that $100-10f>0$. It leads to $100>10f\Rightarrow f<10$. This matches with the **option (a), the correct answer.**
I-26	Formula of formation of image by a spherical having radius of curvature $R=40$ cm and focal length of mirror is $f=\frac{R}{2}$. Further formula of imaged formed by a spherical mirror is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}\Rightarrow\frac{2}{R}=\frac{1}{u}+\frac{1}{v}$. Using the given data $\frac{2}{40}=\frac{1}{30}+\frac{1}{v}\Rightarrow\frac{1}{v}=\frac{1}{20}-\frac{1}{30}\Rightarrow\frac{1}{v}=\frac{1}{60}\Rightarrow v=60$cm. As per Cartesian Sign convention, it would be **60 cm** infront of the mirror i.e. **on the side of the object** *is the answer.*
I-27	Formula for lateral magnification of image is $-\frac{v}{u}=\frac{h_i}{h_0}$…(1) Given that height of image infront of the mirror is 50cm. Since image is infront of the mirror and hence it would real-inverted and therefore as per Cartesian Sign convention $h_i=-50$ cm. Whereas given that height of object is 20 cm; it is placed 5 m away from the mirror and hence $u=500$ cm. Since object on principal axis is erect and hence $h_o=20$ cm, in accordance with sigh convention. Using the given data $-\frac{5}{u}=\frac{-0.5}{0.2}\Rightarrow u=\frac{5\times0.2}{0.5}\Rightarrow \boldsymbol{u=2}$ **m**.

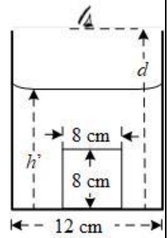
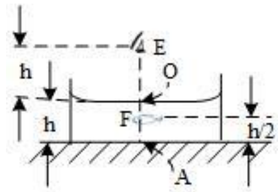
	Now, for focal length of mirror, we have a formula $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Using the given data $\frac{1}{f}=\frac{1}{5}+\frac{1}{2}\Rightarrow\frac{1}{f}=\frac{7}{10}$. It leads to $f=\frac{10}{7}=1.44$ m, say **f = 1.4 m,** using principle of significant digits. **Hence answers are focal length 1.4 m and distance of object from mirror is 2.0 m.**
I-28	From the study of ray diagrams of formation of images by concave mirror having $f=20$ cm if size of image is double the object, there are two possibilities – (a) object is between focal point and center of curvature forming a real image infront of the mirror thus h_0 is (-)ve, and (b) object is between pole of mirror and its focal point, forming virtual image behind the mirror making h_0 (+)ve as per Cartesian sign convention. The formula image formation by spherical mirror is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ …(1). and that of lateral magnification by mirror is $-\frac{v}{u}=\frac{h_i}{h_0}$…(2). Applying the formula for both the cases with the available data we have **-** **Case(a):** Using formula (2) we have $-\frac{v}{u}=\frac{-2h_o}{h_0}\Rightarrow v=2u$…(3) And using (3) in (1) we have $\frac{1}{20}=\frac{1}{u}+\frac{1}{2u}\Rightarrow\frac{2}{3}u=20\Rightarrow \boldsymbol{u=30}$ **cm** **Case(a):** Using formula (2) we have $-\frac{v}{u}=\frac{2h_o}{h_0}\Rightarrow v=-2u$…(3) And using (3) in (1) we have $\frac{1}{20}=\frac{1}{u}-\frac{1}{2u}\Rightarrow 2u=20\Rightarrow \boldsymbol{u=10}$ **cm.** **Thus answers are 10 cm and 30 cm from the mirror**
I-29	In this problem it is essential to recall ray diagram of image formed by convex mirror which is virtual and behind the mirror. Accordingly, as per Cartesian Sigh convention height of object $h_0=1$ cm is (+)ve and so also u is (+)ve. Where has, height of image $h_i=0.6$ cm is (+)ve but position of image being behind the mirror is (-)ve. Further as per sign convention focal length of convex mirror is (-)ve and accordingly we have $f=-7.5$ cm. As per formula of lateral magnification $-\frac{v}{u}=\frac{h_i}{h_0}$, with the available data $-\frac{v}{u}=\frac{0.6}{1}\Rightarrow v=-0.6u$. Further, formula of image formation is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Using the available data $\frac{1}{-7.5}=\frac{1}{u}+\frac{1}{-0.6u}\Rightarrow-\frac{1}{75}=\frac{1}{10u}-\frac{1}{6u}$. It leads to $\frac{1}{75}=\frac{1}{6u}-\frac{1}{10u}\Rightarrow\frac{1}{75}=\frac{2}{30u}\Rightarrow \boldsymbol{u}=\frac{75}{15}=$ **5 cm in front of the mirror, is the answer**.
I-30	A polished ball of bearing can be viewed as a convex surface. Thus its focal length would be $f=-\frac{R}{2}$; it is given that diameter of the ball $D=2R=0.4$ cm and accordingly, focal length of the ball is $f=-\frac{\frac{0.4}{2}}{2}$. It leads to $f=-0.1$ cm. Image of a flame of height $h_o=1.6$ cm placed at a distance 20 cm away from the ball it implies that $u=20$ cm. Formula of formation of image by a spherical mirror is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Using the available data $\frac{1}{-0.1}=\frac{1}{20}+\frac{1}{v}$. It leads to $\frac{1}{v}=-\left(\frac{1}{0.1}+\frac{1}{20}\right)\Rightarrow v=-\frac{20\times0.1}{20.1}\Rightarrow \boldsymbol{v=-0.1}$ **cm or 1 mm inside the ball.** The formula of lateral magnification is $-\frac{v}{u}=\frac{h_i}{h_0}\Rightarrow h_i=-\frac{v}{u}\times h_0$, using the available data $h_i=-\frac{(-0.1)}{20}\times1.6\Rightarrow \boldsymbol{h_i=0.08}$ **mm.** **Thus answers are 1.0 mm inside the ball bearing, 0.08 mm.**
I-31	As per formula of formation of image in a spherical mirror $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. In the instant case it is a convex mirror and hence with the given data $f=-6$ cm, while distance of object from the mirror is $u=7.5$ cm. Accordingly, $\frac{1}{-6}=\frac{1}{7.5}+\frac{1}{v}\Rightarrow\frac{1}{v}=-\left(\frac{1}{6}+\frac{1}{7.5}\right)\Rightarrow \boldsymbol{v}=-\frac{6\times7.5}{13.5}=-\frac{10}{3}$, inside the mirror i.e. **on the side opposite to that of the object, from the mirror.** As regards nature of image using formula of lateral magnification $-\frac{v}{u}=\frac{h_i}{h_0}\Rightarrow h_i=-\frac{-\frac{10}{3}}{7.5}\times3\Rightarrow \boldsymbol{h_i}=\frac{10}{7.5}$ or $\boldsymbol{\frac{10}{3}}$ **cm, since it is a (+)ve the image is erect and virtual..**

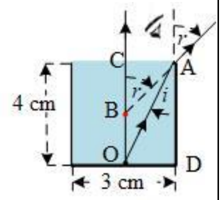
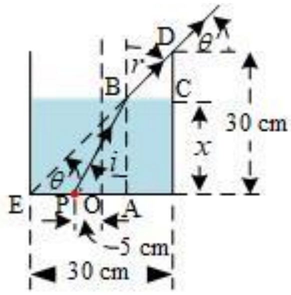
	Thus answers are $\frac{10}{3}$ cm from the mirror on the side opposite to the object, $\frac{10}{3}$ cm, virtual and erect.
I-32	The problem has two parts one is to determine position of image of both arms of U shaped wire and other is to determine height of image of both arms of wire. Accordingly, all parameters of left arm are suffixed with 1 while corresponding parameters of the right arm are suffixed with 2. It is to be noted that concave is so placed that all distances along the principal axis are on the left of pole P and hence as per Cartesian sign convention they are (-)ve.  Therefore, problem of determination of total length is done in three stages- **Stage 1:** *position and height of the left arm of wire,* **Stage 2:** *position and height of the left arm of wire,* **Stage 3:** *length of the base of the U shaped wire*. Stage wise solution is as under – **Stage 1:** $f=-\frac{R}{2}=-\frac{20}{2}\Rightarrow f=-10$ cm. Distance of object $u_1=-(30+10)=-40$ cm. Therefore, we have $\frac{1}{f}=\frac{1}{u_1}+\frac{1}{v_1}$. Using the available data, $\frac{1}{-10}=\frac{1}{-40}+\frac{1}{v_1}\Rightarrow \frac{1}{v_1}=\frac{1}{40}-\frac{1}{10}\Rightarrow v=\frac{40\times10}{10-40}\Rightarrow$ $\boldsymbol{v_1=-\frac{40}{3}}$ **cm**. Therefore, from formula of linear magnification $-\frac{v_1}{u_1}=\frac{h_1'}{h_1}\Rightarrow h_1'=-\frac{-10}{-40}\times\frac{40}{3}\Rightarrow \boldsymbol{h_1'=-\frac{10}{3}}$ **cm.** **Stage 2:** $f=-10$ cm. Distance of object $u_2=-30$ cm. Therefore, $\frac{1}{f}=\frac{1}{u_2}+\frac{1}{v_2}$. Using the available data, $\frac{1}{-10}=\frac{1}{-30}+\frac{1}{v_2}\Rightarrow \frac{1}{v_2}=\frac{1}{30}-\frac{1}{10}\Rightarrow v_2=\frac{30\times10}{10-30}\Rightarrow \boldsymbol{v_2=-15}$ cm Therefore, from formula of linear magnification $-\frac{v_2}{u_2}=\frac{h_2'}{h_2}\Rightarrow h_2'=-\frac{-15}{-30}\times10\Rightarrow \boldsymbol{h_2'=-5}$ **cm**. **Stage 3:** Length of image *L*' is sum of absolute values of height of two arms and separation between the two arms. Thus $\boldsymbol{L'}=\lvert h_1'\rvert+\lvert h_2'\rvert+\lvert v_1-v_1\rvert=\frac{10}{3}+5+\left(15-\frac{40}{3}\right)=\frac{10}{3}+5+\frac{5}{3}=$ **10 cm is the answer.**
I-33	During shaving enlarged and erect image is required. Based on study of ray diagrams for image formation in spherical mirrors, this can happen only with the concave lens is so held that face of the man is between pole and focal point of the lens. Given that $\frac{h\prime}{h}=1.4$ and as per formula of linear magnification $-\frac{v}{u}=\frac{h\prime}{h}$. It leads to $v=-\left(\frac{h\prime}{h}\right)u$. Using the given data $v=-(1.4)\times25=-35$ cm. Thus using formula of image formation $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}\Rightarrow\frac{1}{f}=\frac{1}{25}+\frac{1}{-35}\Rightarrow \boldsymbol{f}=\frac{25\times35}{35-25}=$ **87.5 cm is the answer**.
I-34	Given that focal length of a concave mirror is $f=7.6$ m. Diameter of the moon is $d_m=3.450\times10^6$m, while distance of moon from earth $u_m=(3.8\times10^5)\times10^5\Rightarrow u_m=3.8\times10^8$m. As per Cartesian sign convention both u and f are (=)ve and as per formula of imaging in spherical mirror $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. Accordingly, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u_m}\Rightarrow,\frac{1}{v}=\frac{1}{7.6}-\frac{1}{3.8\times10^8}\Rightarrow v=\frac{7.6\times(3.8\times10^8)}{3.8\times10^8-7.6}\Rightarrow v\approx7.6$ cm. Now as per linear magnification formula $-\frac{v}{u}=\frac{d_i}{d_m}\Rightarrow d_i=-\frac{v}{u}\times d_m$. Since face of the moon has circular symmetry and hence (-) sign here is not significant Accordingly, $d_i=-\frac{7.6}{3.8\times10^8}\times3.450\times10^6$, it leads to 6.9×10^{-3} m or **diameter of image of moon is 6.9 cm.**

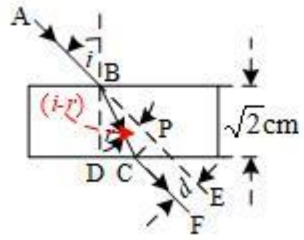
I-35	The problems is conceptualized as object OA placed at a distance PO $= 30$ cm. from a concave mirror. Height of object $r = h_1 = 2.0$ cm. Focal length of the mirror *f* is 20 cm. Grayed surface of the mirror is non-reflecting surface. Thus as per Cartesian sign convention we have $u = -30$ cm and $f = -20$ cm. We are required to determine radius of the circle formed by the image.  Therefore, as per image formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{-20} = \frac{1}{-30} + \frac{1}{v} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{20} \Rightarrow v = \frac{20\times30}{(20-30)} \Rightarrow v = -60$ cm, it implies that image would be formed in front of the mirror. Now, radius of the circle of image h_2 can be determined using formula of lateral magnification $-\frac{v}{u} = \frac{h_2}{h_1}$, it leads to $h_2 = \left(-\frac{v}{u}\right) \times h_1$. Using the available data $h_2 = \left(-\frac{-60}{-30}\right) \times 2.0 = -4.0$ cm. This implies that it would be an inverted image such that at every position of the particle on the circle, its image would be at the position determined but diametrically opposite on a circle of **radius 4.0 cm** with its centre on principal axis at a distance 60 cm from pole. **Thus answer is 4.0 cm.**
I-36	The experimental setup of a concave mirror of radius *R* filled with water of refractive index μ upto a height h is shown in the figure. Let object is placed at O at a height x above the water surface. In concave mirror image is formed at the place of object only when object is at centre of curvature of concave mirror. Therefore, in the experiment optical distance *d*' of the object placed at O must be equal to *R*.  Physical distance of O above water surface is *x* and distance of centre of curvature from water surface is $x' = R - h$ and serves as apparent distance to satisfy the stipulated condition of image. Thus wile equating the optical path, $\frac{Read\ distance\ from\ water\ surface}{Apparent\ distance\ from\ water\ surface} = \frac{1}{\mu} \Rightarrow \frac{x}{R-h} = \frac{1}{\mu}$. It solves into distance of object above water surface $\boldsymbol{x = \frac{R-h}{\mu}}$ **is the answer**.
I-37	Given that both the mirrors have focal length *f*. Position image formed by each mirror shall be determined such that they coincide to form a single image as desired. The problem can be solved using pre-knowledge of ray diagrams of concave mirrors, Second method is using more generic equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ …(1) and is used here.  Taking S as origin as Cartesian sign convention for mirror M_1, focal distance mirror $u_1 = \frac{d}{2}$, focal length is$f_1 = f$, while for mirror M_2, focal distance mirror $u_2 = -\frac{d}{2}$, focal length is $f_2 = -f$. Therefore, using (1) distance of image from P_1 formed by M_1 is $\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} \Rightarrow \frac{1}{v_1} = \frac{1}{f} - \frac{1}{\frac{d}{2}} \Rightarrow \frac{1}{v_1} = \frac{1}{f} - \frac{2}{d}$. It leads to $v_1 = \frac{fd}{d-2f}$. Next is image formed by mirror M_1 acts as an object for mirror M_2. Accordingly,$u_2 = d - \frac{fd}{d-2f}$. It leads to $u_2 = \frac{d^2-2fd-fd}{d-2f} \Rightarrow u_2 = \frac{d^2-3fd}{d-2f}$. Accordingly, $\frac{1}{v_2} = \frac{1}{f_{21}} - \frac{1}{u_2} \Rightarrow \frac{1}{v_2} = \frac{1}{-f} - \frac{1}{\frac{d^2-3fd}{d-2f}}$. It further solves to $\frac{1}{v_2} = \frac{d-2f}{d^2-3fd} - \frac{1}{f} \Rightarrow v_2 = \frac{f(d^2-3fd)}{(df-2f^2)-(d^2-3fd)}$. Taking it ahead, $v_2 = \frac{f(3fd-d^2)}{2f^2+d^2-4df}$. As per sign convention, v_2 is (-)ve and hence condition to be satisfied is $v_1 - v_2 = d$. Accordingly, $d = \frac{fd}{d-2f} - \frac{f(3fd-d^2)}{2f^2+d^2-4df}$. Therefore, $1 = \frac{f}{d-2f} - \frac{f(3f-d)}{2f^2+d^2-4df}$. It is further solved as under –

	$$(d-2f)(2f^2+d^2-4df)=f(2f^2+d^2-4df)-\ (d-2f)\times f(3f-d)$$ $$\Rightarrow (2f^2+d^2-4df)(d-3f)=(df-2f^2)(d-3f)$$ $$\Rightarrow 2f^2+d^2-4df=df-2f^2 \Rightarrow d^2-5df+4f^2=0$$ Factorizing the last form, $(d-4f)(d-f)=0$. It implies possible values of separation between the two mirrors is $d=4f$ or f **N.B.:** Deliberately algebraic solution has been brought out at illustration. It may give a satisfaction to students that in case a student accidently chooses it, it needs to be pursued with confidence rather than abandoning it for a quick and short answer. Otherwise in this case rays diagram gives quick and correct solution.
I-38	Given that focal length of both the mirrors have focal lengths $f=20$cm, while distance of source S from concave mirror is $d=30$. Cartesian sign convention is used taking origin as S, and all distances are measured from pole of mirror under consideration and accordingly (+) or (-) signs are attributed. Two reflections, one from each mirror, are required to be considered using formula $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ …(1), and are as under –  **Mirror M$_1$**: Taking distances w.r.t. P$_1$, $f_1=f=20$ cm and $u_1=30$ cm. Accordingly, position of the image using (1). $\frac{1}{20}=\frac{1}{30}+\frac{1}{v_1}\Rightarrow\frac{1}{v_1}=\frac{1}{20}-\frac{1}{30}$, it leads to $v_1=\frac{20\times30}{30-20}\Rightarrow v_1=60$ cm w.r.t. P$_1$ infront of M$_1$. **Mirror M$_2$**: Taking distances w.r.t. P$_2$, $f_2=f=20$ cm and $u_2=-(d-30)=30-d$ cm. Accordingly, position of the image using (1). $\frac{1}{20}=\frac{1}{30-d}+\frac{1}{v_2}\Rightarrow\frac{1}{v_2}=\frac{1}{20}-\frac{1}{30-d}$, it leads to $v_2=\frac{20\times(30-d)}{30-20-d}$. It leads to $v_2=\frac{600-20d}{10-d}$ cm w.r.t. P$_2$, and it would be behind M$_1$. Now, considering the given conditions that the images coincide, hence $v_1=v_2+d\Rightarrow 60=\frac{600-20d}{10-d}+d$, it leads to $60=\frac{(600-20d)+(10d-d^2)}{10-d}\Rightarrow 600-60d=600-20d+10d-d^2\Rightarrow d^2-50d=0\Rightarrow$ **$d=50$ cm is the separation of mirrors is answer of part (a).** Position of image of source caused by single reflection from M$_2$ it is inside or behind the reflecting surface of the mirror, is virtual and at a distance **10 cm, is answer of part (b).** **Answers are (a) 50 cm (b) 10 cm inside the diverging mirror i.e. virtual image.**
I-39	Given the angle of incidence $i=45^0$ and angle of refraction $r=30^0$, the refracted ray BC crosses slab of thickness BE $=d=1.00$ m.  Refractive index of the medium $\mu=\frac{\sin i}{\sin r}=\frac{v}{v_i}$ …(1), here $v=3\times10^8$ m/s is velocity of light in vacuum and v_i is velocity of light in ice slab. Time taken by light to cross the slab is $t=\frac{\text{BC}}{v_i}$…(2). Here, $\frac{\text{BE}}{\text{BC}}=\cos r\Rightarrow \text{BC}=\frac{\text{BE}}{\cos r}$ …(3). Combining (1), (2) and (3) we have $t=\frac{\frac{\text{BE}}{\cos r}}{\frac{v}{\mu}}\Rightarrow t=\frac{\text{BE}}{\cos r}\times\frac{\mu}{v}\Rightarrow t=\frac{\text{BE}}{\cos r}\times\mu\times\frac{1}{v}$. It further leads to $t=\frac{\text{BE}}{\cos r}\times\frac{\sin i}{\sin r}\times\frac{1}{v}$. Using the given data, $t=\frac{1.00}{\cos 30^0}\times\frac{\sin 30^0}{\sin 30^0}\times\frac{1}{v}\Rightarrow t=\frac{1.00}{\frac{\sqrt3}{2}}\times\frac{\frac{1}{\sqrt2}}{\frac{1}{2}}\times\frac{1}{3\times10^8}$. It leads to $t=\frac{1.00\times2\times\sqrt2}{3\times\sqrt3}\times10^{-8}\Rightarrow t=0.544\times10^{-8}\Rightarrow$ $\boldsymbol{t}=5.44\times10^{-9}$s, or **5.44 ns is the answer**. **N.B.:** Numerical part of the problems done at the ends apparently makes it a long calculation and algebraic part tedious. But it has following advantages – (a) Mostly in last stage many values are cancelled, incidentally in this it is not so, (b) Calculation at each stage makes it apparently simple, but may lead to error creeps in while considering significant digits (c) It is easier to apply SDs at last stage based on given data.

I-40	Pictorial representation of the problem shows that Pole AC of length 1.00 m rests on the bed of swimming pool filled upto a height BC $= 50.0$ cm. Sunlight is at an angle $i = 45^0$ after refraction in water routes along DE. 
I-41	This is a case of reverse of critical angle of refraction, since light nearly parallel to surface of the liquid in lighter medium refracts along AC into a denser medium of refractive index $\mu = \frac{4}{3}$ which is2.5 m deep. Hence, $\mu = \frac{\sin i}{\sin r} = \frac{4}{3} \Rightarrow \sin r = 1 \times \frac{3}{4}$, it leads to $\sin r = \frac{3}{4}$. Trigonometrically we have $\tan r = \frac{\sin r}{\sqrt{1-\sin^2 r}}$. Accordingly we have $\tan r = \frac{\frac{3}{4}}{\sqrt{1-\left(\frac{3}{4}\right)^2}} = \frac{3}{\sqrt{7}} = 1.13$.  The diagram depicts that point C is on the edge of the shadow which is away from the point B, right below the A, in the direction of the light ray by a distance BC $=$ AB $\tan r = 2.5 \times 1.13 = \mathbf{2.83}$ **m shifted from the position directly below the wood is the answer.**
I-42	Let d is the distance, optical path length, of microscope M from object P in focus in air. When a glass slab of refractive index μ and thickness t is introduced between M and P, the optical path length changes to $d' = (d - t) + \frac{t}{\mu} \Rightarrow d' = d - \left(1 - \frac{1}{\mu}\right)t$. Distance between P and M depends upon size of P and hence it remains fixed. Therefore, to keep P in focus of M the latter shall have to be shifted by Δd such that $d' + \Delta d = d \Rightarrow \Delta d = d - d' = d - \left(d - \left(1 - \frac{1}{\mu}\right)t\right) \Rightarrow \Delta d = \left(1 - \frac{1}{\mu}\right)t$. Using the given data $\Delta d = \left(1 - \frac{1}{1.5}\right) \times 2.1 = \mathbf{0.70}$ **cm away from the object, is the answer.** 
I-43	Height of water layer of refractive index $\mu_1 = 1.33$ in a vessel is $h_1 = 20$ cm and that of oil layer having refractive index $\mu_1 = 1.30$ above the water $h_2 = 20$ cm.. We know that $\frac{\text{Real Depth }(h)}{\text{Apparent Depth }(d)} = \mu \Rightarrow d = \frac{h}{\mu}$. Accordingly, for water $d_1 = \frac{h_1}{\mu_1}$ and for oil $d_2 = \frac{h_2}{\mu_2}$. Apparent depth of the bottom of the vessel is $d = d_1 + d_2 = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$. Using the available data we have $d = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{20}{1.33} + \frac{20}{1.30} \Rightarrow d = \mathbf{30.4}$ **cm depth of vessel when viewed from the top.**
I-44	Physical distance of point P from eye is $d = d_1 + t_1 + d_2 + t_2 + d_2 + t_2$. But with transparent slabs placed between the eye and point P apparent distance, as per figure, is $d' = d_1 + \frac{t_1}{\mu_1} + d_2 + \frac{t_2}{\mu_2} + d_2 + \frac{t_3}{\mu_3}$. Using the given data $d = 1.0 + 0.4 + 1.0 + 0.3 + 1.0 + 0.2 = 3.9$cm. Whereas, the transparent slabs inserted the $d' = 1.0 + \frac{0.4}{1.4} + 1.0 + \frac{0.3}{1.3} + 1.0 + \frac{0.2}{1.2} \Rightarrow d' = 3.0 + 0.29 + 0.23 + 0.17 = 3.69 = 3.7$cm. Thus image of P is displaced from original position by $\Delta d = d - d' = 3.9 - 3.7 = \mathbf{0.2}$ **cm above P, is the answer.**
I-45	Refractive index of a medium is $\mu = \frac{\text{Real Thickness}}{\text{Apparent Thickness}}$. Taking k slabs the real thickness $t = t_1 + t_2 + \cdots t_k = \sum_{i=1}^{k} t_i$. While each of the i[th] slab has refractive index μ_i has apparent thickness $t_i' = \frac{t_i}{\mu_i}$. Accordingly, total apparent thickness $t' = t_1' + t_2' + \cdots t_k'$. It leads to $t' = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \cdots \frac{t_k}{\mu_k} = \sum_{i=1}^{k} \frac{t_i}{\mu_i}$.  Therefore, equivalent refractive index of the composite slab is $\mu = \frac{t}{t'} \Rightarrow \boldsymbol{\mu} = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k} \frac{t_i}{\mu_i}}$ **is the answer**.

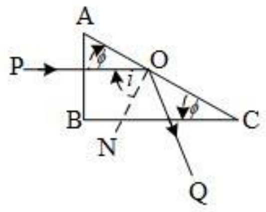
I-46	 Volume of water $V = 800\pi$ cm^3. Volume of glass cylinder of diameter 8 cm and height 8 cm is $V' = \left(\frac{\pi 8^2}{4}\right) \times 8 = 128\pi$ cm^3. Thus height h' of water surface above the bottom of a cylinder of diameter 12 cm can be arrived at $\left(\frac{\pi 12^2}{4}\right) \times h' = V + V'$. It resolves with values arrived at above $\left(\frac{\pi 12^2}{4}\right) \times h' = 800\pi + 128\pi \Rightarrow 36h' = 928 \Rightarrow h' = 25.8$ cm. Let *d* is the real height of the eye above the bottom of the empty cylindrical vessel under the glass. But, when water of volume *V* is poured in the vessel and glass cylinder of volume *V*' is placed in water, as shown in the figure, height of eye above the bottom of the glass cylinder will be $d' = \frac{8}{\mu_g} + \frac{h'-8}{\mu_w} + (d - h')$. Here, refractive index of glass is $\mu_g = 1.5$ and refractive index of water is $\mu_w = 1.33$. Therefore, change in depth of the bottom of the vessel below the glass cylinder is $\Delta d = \left\|d - \left(\frac{8}{1.5} + \frac{h'-8}{1.33} + (d - h')\right)\right\|$. It solves to $\Delta d = h' - \frac{h'}{1.33} + \frac{8}{1.33} - \frac{8}{1.5} \Rightarrow \Delta d = 25.8 \times \left(1 - \frac{1}{1.33}\right) + 8 \times \left(\frac{1}{1.33} - \frac{1}{1.5}\right) \Rightarrow \Delta d = 7.08$ cm or **7.1 cm above the bottom.**
I-47	 We know that refractive index of water w.r.t. air is: $${}_a\mu_w = \frac{\text{Real Distance in water}}{\text{Distance in water as apparent from air}} = \mu,$$ and refractive index of air w.r.t. water is: $${}_w\mu_a = \frac{\text{Real Distance in air}}{\text{Distance in air as apparent from}} = \frac{1}{{}_a\mu_w} \Rightarrow \quad {}_w\mu_a = \frac{1}{\mu}.$$ This principle shall be applied to find distances in the given problem. **Part (a):** Images of eye E in air as seen by the fish F in water shall be (i) direct and (ii) in mirror placed below the pot. Each of the case is being analyzed separately – (i) *Direct:* distance *d*' = (direct distance FO in water) PLUS (apparent distance OE of eye in air) $\Rightarrow d' = \frac{h}{2} + \mu \times h = \boldsymbol{h\left(\mu + \frac{1}{2}\right)}$ **above itself is the answer for image (i) in part (a)** (ii) *In Mirror*: distance *d*"= (distance of image of fish in the mirror) PLUS (distance of eye behind the fish) $\Rightarrow$ 2 × FA + [(direct distance FO in water) PLUS (apparent distance OE of eye in air)] Since fish and distance FA is in water, i.e. no change of medium and hence both physical and apparent distances shall be same. But, distance of eye behind the fish shall be equal to *d*' determined at **a(i)** above. Accordingly, $d"=2\times\frac{h}{2} + d'=h+h\left(\mu+\frac{1}{2}\right) \Rightarrow d" = \boldsymbol{h\left(\mu+\frac{3}{2}\right)}$ **below itself is the answer for image (ii) in part (b)** **Part (b):** Images of fish F in water as seen by the fish E in water shall be (i) direct and (ii) in mirror placed below. Each of the case is being analyzed separately – (i) *Direct:* distance *d*' = (direct distance EO in air) PLUS (apparent distance OF of Fish in water) $\Rightarrow d' = h + \frac{h}{2} \times \frac{1}{\mu} = \boldsymbol{h\left(1 + \frac{1}{2\mu}\right)}$ **below itself is the answer for image (i) in part (b)** (ii) *In Mirror*: distance *d*"= 2 ×(distance of image of eye the mirror) MINUS (distance of fish from the eye) $\Rightarrow$ 2 × [(direct distance EO in air) PLUS (apparent distance OA in air)] MINUS (distance of fish from eye in b(i) above)

	Since fish and distance FA is in water, i.e. no change of medium and hence both physical and apparent distances shall be same. But, distance of eye behind the fish shall be equal to *d*' determined at **a(i)** above. Accordingly, $d"=2\times\left(h+\frac{h}{\mu}\right)-d'=2h\left(1+\frac{1}{\mu}\right)-h\left(1+\frac{1}{2\mu}\right)\Rightarrow d"=h\left(1+\frac{3}{2\mu}\right)$ **below itself is the answer for image (ii) in part (b)** **Hence answers are-** **(a)** $\boldsymbol{h\left(\mu+\frac{1}{2}\right)}$ **above itself,** $\boldsymbol{h\left(\mu+\frac{3}{2}\right)}$ **below itself** **(b)** $\boldsymbol{h\left(1+\frac{1}{2\mu}\right)}$ **below itself and** $\boldsymbol{h\left(1+\frac{3}{2\mu}\right)}$ **below itself**
I-48	The system given in the problem is shown in the figure with object O placed at bottom of the cylindrical vessel. The vessel is filled with water of refractive index ${}_a\mu_w=1.33$ it is for rays incident from air, refracting in water. In the instant case rays from object are refracting into water and hence ${}_w\mu_a=\frac{1}{{}_a\mu_w}=\frac{1}{1.33}$.  With the given geometry of the system shown in the figure, from ΔOAD we have $\tan i=\frac{OD}{AD}=\frac{1.5}{4}=\frac{3}{8}\Rightarrow\cot i=\frac{8}{3}$. Therefore from trigonometry, $\sin i=\frac{1}{\sqrt{1+\cot^2 i}}=\frac{1}{\sqrt{1+\left(\frac{8}{3}\right)^2}}=\frac{3}{\sqrt{73}}=0.35$. As per Snell's Law, $\mu=\frac{\sin i}{\sin r}\Rightarrow\sin r=\frac{\sin i}{{}_w\mu_a}=\frac{0.35}{\frac{1}{1.33}}=0.467$. Again trigonometrically from ΔABC we have $\tan r=\frac{\mathrm{AC}}{\mathrm{BC}}\Rightarrow\mathrm{BC}=\frac{\mathrm{AC}}{\tan r}=\mathrm{AC}\times\cot r$. And $\cot r=\frac{\sqrt{1-\sin^2 r}}{\sin r}$. It leads to apparent width $\mathrm{BC}=1.5\times\frac{\sqrt{1-(0.467)^2}}{0.467}=\mathbf{2.84}$ **cm is answer of part (a).** Ratio of real depth to apparent depth is $=\frac{\mathrm{OC}}{\mathrm{BC}}=\frac{4}{2.84}=\mathbf{1.41}$ **is answer of part (b).** **Thus answers are (a) 2.84 cm (b) 1.41**
I-49	The system given in the problem is shown in the figure with particle P placed at bottom of the cylindrical vessel at a distance PO = 5 cm …(1), from the centre O. The vessel is filled upto height x with water of refractive index ${}_a\mu_w=1.33$ to be able to see the particle along ED when the vessel is empty. Refracting index of water from air to water ${}_a\mu_w$, becomes ${}_w\mu_a=\frac{1}{{}_a\mu_w}=\frac{1}{1.33}$ …(2) for rays regacting from water to air. Therefore, the ray from particle along PB, after reflection is aligned to ED making angle $\theta=\tan^{-1}\frac{30}{30}=\tan^{-1}1$, or $\theta=45^0$, and hence geometrically angle of refraction at B is $r=90^0-\theta=45^0$.  Therefore, from Snell's Law ${}_w\mu_a=\frac{\sin i}{\sin r}\Rightarrow\frac{1}{1.33}=\frac{\sin i}{\sin 45^0}\Rightarrow\sin i=\frac{\frac{1}{\sqrt{2}}}{1.33}=\frac{1}{1.33\times\sqrt{2}}=0.532$. For achieving this angle i , vessel is required to be filled upto height x such that in ΔPAB, $\tan i=\frac{\mathrm{PO+OA}}{x}$, and $\tan i=\frac{\sin i}{\sqrt{1-\sin^2 i}}=\frac{0.532}{\sqrt{1-(0.532)^2}}=0.628$.Therefore, $0.628=\frac{5+\mathrm{OA}}{x}\Rightarrow 0.628\times x=5+\mathrm{OA}$ …(3) And in ΔPAB, $\tan\theta=\tan 45^0=\frac{\mathrm{AB}}{\mathrm{EO+OA}}\Rightarrow 15+\mathrm{OA}=x$…(4)

	Eliminating OA from (3) and (4), we have $0.628\times x=5+(x-15)\Rightarrow(1-0.628)x=10\Rightarrow x=$ 26.88 cm say **26.9 cm is the answer.**
I-50	Given that light ray is incident at B at an angle $i=45^0$ on a plate of thickness $\sqrt{2}$ cm, or refractive index $\mu=2$ after refraction passes along BC at angle r and again after refraction at point C on the other face of the plate refracts along CD. As per Snell's Law $\mu=\frac{\sin i}{\sin r}$. Since, there is air on the both sides of the plate and hence ray CD emergent from the plate remains parallel to projected incident ray AE, but displaced by AE $=d$.  Geometrically in ΔBCP, we have $\mathrm{CP}=\mathrm{BC}\times\sin(i-r)$ …(1), and in ΔBDC, we have $\mathrm{BC}=\mathrm{BD}\times\sin r$…(2). Using given data $\sin r=\frac{\sin i}{\mu}=\frac{\sin 45^0}{2}=\frac{1}{\sqrt{8}}$. …(3).Therefore, $\cos r=\sqrt{1-\sin^2 r}=\sqrt{1-\left(\frac{1}{2\sqrt{2}}\right)^2}=\sqrt{\frac{7}{8}}$…(4) . Further, $\sin 45^0=\cos 45^0=\frac{1}{2}$. From (1) and (2), shift in the path of ray is $d=\mathrm{CP}=\left(\frac{\mathrm{BD}}{\cos r}\right)\times\sin(i-r)$. It leads to $$d=\frac{\mathrm{BD}}{\cos r}\times(\sin i\cos r-\cos i\sin r).$$ Using the available data $\mathrm{d}=\sqrt{2}\times\frac{1}{\sqrt{\frac{7}{8}}}\times\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{7}}{\sqrt{8}}-\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{8}}\right)=\frac{4}{\sqrt{7}}\times\frac{\sqrt{7}-1}{4}\Rightarrow d=1-\frac{1}{\sqrt{7}}=0.62$. cm. **N.B.:** This illustration demonstrates nicely benefit of exercising patience to defer calculation until last step.
I-51	Refractive index of fibre is $\mu_f=\frac{v}{v_f}=1.72$, here v_f is velocity of light in fibre and v is the velocity of light in vacuum and refractive index of glass is index of fibre is $\mu_g=\frac{v}{v_g}=1.50$, here v_g is velocity of light in glass. Therefore, refractive index of fibre w,r.t. glass is ${}_f\mu_g=\frac{v_f}{v_g}=\frac{v_f}{v}\times\frac{v}{v_g}=\frac{1}{\mu_f}\times\mu_g$. Using the available data ${}_f\mu_g=\frac{1}{1.72}\times 1.5=0.87$. Further for critical angle $\mu=\frac{\sin i_c}{\sin 90^0}\Rightarrow\mu=\sin i_c$…(1). Here i_c is the critical angle of incidence for total reflection. Moreover, total reflection occurs when light travels from denser medium (having higher refractive index) to rarer medium (having lower refractive index). Since, $\mu_f>\mu_g$ and therefore, it is a valid case for total refraction. Refractive index of light travelling from fibre to glass coating is $\mu={}_f\mu_g=0.87$. Therefore, using (1) we have $\sin i_c=0.87$. Hence critical angle is $\boldsymbol{i_c=\sin^{-1}(0.87)}$ **is the answer**.

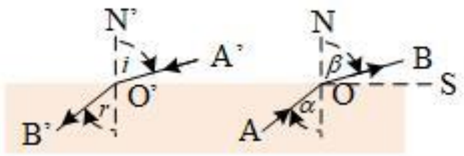
I-52

Incident ray PO is normal to the surface of the prism along AB and hence it will travel un refracted intercept surface of the prism along AC. At point C it makes an angle of incidence $i = 90^0 - \emptyset$ and is subject to refraction if $i < i_c$, here i_c is critical angle. Thus as angle $\emptyset$ angle i decreases until it reaches a value equal to critical angle such that $\frac{1}{\mu} = \frac{\sin i_c}{\sin 90^0} \Rightarrow \sin i_c = \frac{1}{\mu}$. Using the available data $\sin(90^0 - \emptyset_c) = \frac{1}{1.5} \Rightarrow \cos \emptyset_c = \frac{2}{3}$. It leads to $\emptyset_c =$ **$\cos^{-1}\frac{2}{3}$, is the answer.**

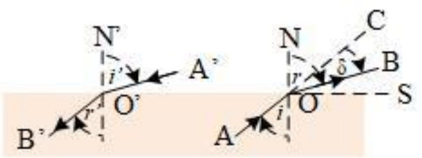


I-53

Refractive index of glass for light entering into from air into it is given to be $\mu = 1.5$ and as per Snell's Law it leads to $\mu = \frac{\sin i}{\sin r}$, as shown in the diagram. Refraction is a reversible phenomenon and hence, when a light ray is refracted from glass to air i.e. denser to lighter medium $\mu' = \frac{\sin \alpha}{\sin \beta} = \frac{1}{\mu} \Rightarrow \frac{\sin \alpha}{\sin \beta} = \frac{1}{1.5} = \frac{2}{3}$. Since $\mu' < 1$, and hence increase in angle of refraction β in air is in a proportions greater than increase in angle of incidence α in glass. This phenomenon of refraction will continue until angle of refraction $\beta \to 90^0$. Hence, **maximum angle of refraction is 90^0 is the answer.**



I-54

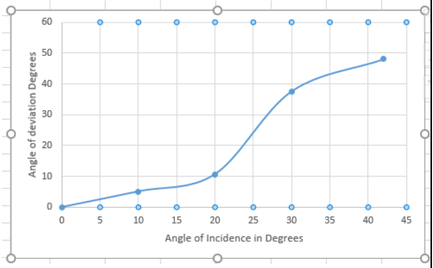


In the figure two ray diagrams of ray refracting from air to glass with refractive index μ such that as per Snell's Law $\mu = \frac{\sin i'}{\sin r'}$. The refraction is a reversible phenomenon and hence ray diagrams with ray from glass to air is shown in the figure such that $i' = r$ and $r' = i$ which leads to $\frac{1}{\mu} = \frac{\sin i}{\sin r}$…(1). And angle of deviation of refracted ray PB from the incident ray is $\delta = r - i$.

But this is valid until $r = 90$ with a critical angle of incidence $\sin i_c = \frac{\sin 90^0}{\mu} \Rightarrow i_c = \sin^{-1}\frac{1}{\mu}$. With the given data we have $i_c = \sin^{-1}\frac{1}{1.5} = \sin^{-1} 0.6\acute{6} = \sin^{-1} 0.67 \Rightarrow i_c = 42^0$

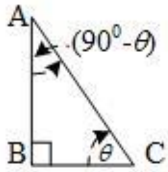
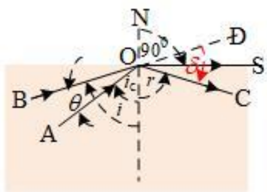
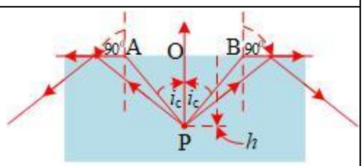
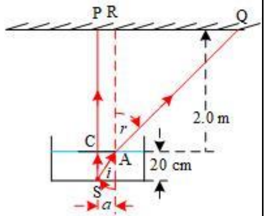
Accordingly, calculations of δ for discrete values of $i = 0^0, 30^0, 45^0, 60^0, 90^0$ in the table shown below –

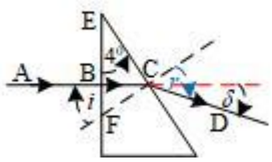
i	$\sin i$	$\sin r = \mu \sin i$	r	$\delta = r - i$
0^o	0	0	0	0
10^0	0.17	0.26	$15^0 6'$	$5^0 6'$ (5.1^0)
20^0	0.34	0.51	$30^0 42'$	$10^0 42'$ (10.7^0)
30^o	0.5	0.75	$48^0 36'$	$37^0 36'$ (37.6^0)
42^o	0.67	1.0	90^0	48^0

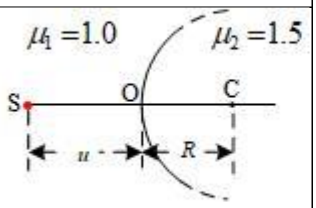
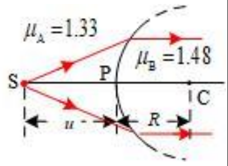
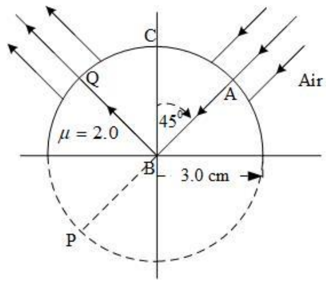
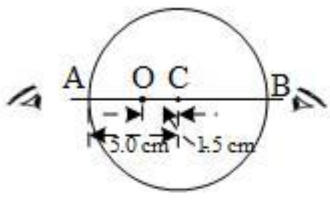


Using the table , sketch of variation of $i - \delta$ is shown in the chart.

N.B.: calculations angles from their sin values determined from Snell's Law require use of Trigonometric Tables, an integral part of booklet on Log Tables

I-55	Problem states that there is a composite medium of glass having refractive index ${}_a\mu_g = 1.50 = \frac{3}{2}$ and water with ${}_a\mu_w = 1.33 = \frac{4}{3}$. Therefore, refractive for light passing from glass to water is ${}_g\mu_w = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{\frac{4}{3}}{\frac{3}{2}} = \frac{8}{9}$…(1) A normal to the interface of the two medium $(i = 0)$passes without refraction and therefore $\delta_{min} = 0$. But, for a ray of light passing from denser to rarer medium, as is given in the problem, the rays undergoes refraction for angle of incidence $0 \le i \le i_c$, here i_c is called critical angle of incidence at which refracted ray becomes tangential to the interface i.e. $r = 90^0$ beyond which refraction is not possible. Accordingly, ${}_g\mu_w = \frac{\sin i_c}{\sin 90^0}$. It leads to $\sin i_c = {}_g\mu_w$…(2) Angle of deviation $\delta = r - i$, therefore at i_c we have $\delta_{max} = 90 - i_c$…(3). A simplified presentation of result is made using a bit of trigonometry using figure where $\sin\theta = \frac{\text{AB}}{\text{AC}}$ and $\cos(90^0 - \theta) = \frac{\text{AB}}{\text{AC}} = \sin\theta \Rightarrow 90^0 - \theta = \cos^{-1}(\sin\theta)$…(4). In the instant case $\theta = i_c$ and hence combining(1), (2), (3) and (4) we have $\delta = \cos^{-1}(\sin i_c) = \cos^{-1}\frac{8}{9}$. **Thus, for angle of deviation in the range $0 \le \delta \le \cos^{-1}\frac{8}{9}$ there are two possible angle of incidence.**  **N.B.:** Mathematics is a great tool of simplification of problem and answer, and it has been used here.
I-56	Refractive index of glass is given to be ${}_a\mu_g = 1.5 = \frac{3}{2}$. Further it is stated light falls on glass and refracts into air a rarer medium and hence ${}_g\mu_a = \frac{1}{{}_a\mu_g} = \frac{2}{3}$. At boundary condition of refraction, critical angle of incidence ray AO is refracted along OS such that ${}_g\mu_a = \frac{\sin i_c}{\sin 90^0} \Rightarrow \sin i_c = {}_g\mu_a = \frac{2}{3} = 0.67$. Accordingly, we have $i_c = \sin^{-1} 0.67 = 41.8^0$. An incident ray BO at an angle $i = i_c + \theta\|_{\theta>0}$ is reflected along OC at an angle $r = i$, as shown in the figure. Thus reflected ray is deviated from BD through an angle $\delta = 180^0 - 2i$. Using the given value of deviation $\delta = 90^0$ it leads to $\boldsymbol{i = \frac{180-90}{2} = 45^0}$ **is the answer**. 
I-57	Refractive index of glass is given to be ${}_a\mu_w = \mu$. Further it is stated light falls on glass and refracts into air a rarer medium and hence ${}_w\mu_a = \frac{1}{{}_a\mu_w} = \frac{1}{\mu}$. At boundary condition of refraction, critical angle of incidence i_c ray PA with normal at A is refracted is along surface of water such that ${}_w\mu_a = \frac{\sin i_c}{\sin 90^0} \Rightarrow \sin i_c = \frac{1}{\mu} \Rightarrow i_c = \sin^{-1}\frac{1}{\mu}$. Any ray of light having angle of incidence $i > i_c$ will undergo total reflection and will not be visible to an observer in the air. Thus light that escapes through a circular area of radius r with its centre at O just above the point source P where $r = h\tan i_c$. **This illustration forms answer of part (a)**, *while algebraic expression is only for academic importance.*  The value of $\boldsymbol{i_c = \sin^{-1}\frac{1}{\mu}}$ **derived above is the answer of part (b).**
I-58	Given that refractive index of water, it implies for light entering from air, is glass is given to be ${}_a\mu_w = \frac{4}{3}$. Therefore, for light from a source inside water at a depth $h = 0.2$ m entering air would be ${}_w\mu_a = \frac{1}{{}_a\mu_w} = \frac{3}{4}$. Therefore, for a ray SA touching inner radius of the ring, $a = 0.15$ m, making an angle of incidence i will be refracted along AQ with an angle of refraction r, such that ${}_w\mu_a = \frac{\sin i}{\sin r}$. From ΔSAC we have $\tan i = \frac{a}{h} \Rightarrow \sin i = \frac{\tan i}{\sqrt{1+\tan^2 i}} \Rightarrow \sin i = \frac{a}{\sqrt{a^2+h^2}} = \frac{15}{\sqrt{15^2+20^2}} = \frac{3}{5}$. As per Snell's Law ${}_w\mu_a = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{{}_w\mu_a} = \frac{\frac{3}{5}}{\frac{3}{4}} = \frac{4}{5}$. With this analysis answers for each part are being determined. 

	Part (a): Radius of the shadow of the ring is $a' = \text{PR} + \text{RQ} = a + \text{AR} \times \tan r$. Again trigonometrically $\tan r = \frac{\sin r}{\sqrt{1-\sin^2 r}} = \frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = \frac{4}{3}$. Therefore, using the available data, $a' = 0.15 + 2.0 \times \frac{4}{3} =$ 2.**8 m is the answer of part (a) considering SDs.** **Part (b):** Maximum value of a_{max} for which shadow of the ring would be formed is determined by critical angle of incidence such that $\sin i_c = {}_w\mu_a = \frac{3}{4}$. Therefore, $\tan i_c = \frac{\sin i_c}{\sqrt{1-\sin^2 i_c}} = \frac{\frac{3}{4}}{\sqrt{1-\left(\frac{3}{4}\right)^2}} = \frac{3}{\sqrt{7}}$. …(1). And, geometrically $\tan i_c = \frac{a_{max}}{h} \Rightarrow \boldsymbol{a_{max}} = h \times \tan i_c = 20 \times \frac{3}{\sqrt{7}} = \mathbf{22.7}$ **cm is the answer.** **N.B.:** It is to be noted that here calculations of $\sin i$ and $\tan r$ have been made where they occur first to simplify calculations. This it is purely a conscious judgement as to where calculations to be deferred till end or to be done intermittently. This sense of judgement is evolved with practice.
I-59	 Path of light is reversible, and it is known that minimum deviation in light ray occurs when ray of light inside the prism is symmetrical to the two surfaces intercepted by the light at point say Q and R as shown in the figure. Thus, geometrically $\delta_{min} = 2(i - r)$…(1). Further, normal N and N' are at angle 90^0 at to the respective refractive surfaces of the prism at Q and R, and satisfy condition of a circumscribing circle through vertex of the prism and Q, O and R. Therefore, $A + \angle\text{QOR} = 180^0$…(2). Further, in ΔOQR we have $2r + \angle\text{QOR} = 180^0$ …(3). Equations (2) and (3) lead to $A = 2r$…(4) Given that refractive index of the material of prism is $\mu = 1.732$ and ,therefore, as per Snell's Law we have $\mu = \frac{\sin i}{\sin r}$…(5). Using (1) and (2) $\mu = \frac{\sin\left(\frac{\delta_{min}+A}{2}\right)}{\sin\frac{A}{2}}$…(6). Given that the prism is equilateral and hence $A = 60^0$. Using the available data $1.732 = \frac{\sin\left(\frac{\delta_{min}}{2}+30^0\right)}{\sin 30^0} \Rightarrow \sin\left(\frac{\delta_{min}}{2} + 30^0\right) = 1.732 \times \frac{1}{2} = 0.866$. It leads to $\frac{\delta_{min}}{2} + 30^0 = \sin^{-1} 0.866 = 60^0 \Rightarrow \frac{\delta_{min}}{2} = 60^0 - 30^0 = 30^0 \Rightarrow$ $\boldsymbol{\delta_{min} = 60^0}$ **. is the answer of first part.** Now using (5) and (6) we have $i = \frac{\delta_{min}+A}{2} \Rightarrow i = \frac{60^0+60^0}{2} = \mathbf{60^0}$**is the angle of incidence** for minimum deviation, **the answer of second part.**
I-60	 Given that refractive index of prism is $\mu = 1.5 = \frac{3}{2}$. Ray of light AB perpendicular to the surface of the prism at B passes through prism along BC without deviation. But, at point C, it makes an angle i with the normal. In ΔEFC, the $\angle\text{FEC} = 4^0$, and $\angle\text{ECF} = 90^0$ therefore, $\angle\text{EFC} = 180^0 - \angle\text{ECF} - \angle\text{FEC} \Rightarrow \angle\text{EFC} = 86^0$. Likewise, In ΔBFC, the $\angle\text{CBF} = 90^0 \Rightarrow \angle\text{BCF} = 180^0 - 90^0 - \angle\text{BFC} \Rightarrow \angle\text{BCF} = 90^0 - 86^0 = 4^0$. As per Snell's Law, $\mu = \frac{\sin i}{\sin r}$. At point C the ray is negotiating glass to air surface and it will be $\mu' = \frac{1}{\mu} = \frac{\sin i}{\sin r} \Rightarrow \sin r = \mu \times \sin i$. Using the available data $\sin r = 1.5 \times \sin 4^0$. It leads to $\sin r = 1.5 \times 0.0698 = 0.1047 \Rightarrow r = \sin^{-1} 0.1047 = 6^0$. Further, from diagram $\delta = r - i \Rightarrow \boldsymbol{\delta} = 6^0 - 4^0 = \mathbf{2^0}$ **is the answer.**
I-61	Given that angle of prism $A = 60^0$, a ray of deviates thorough an angle 30^0. It is required to find limiting value of refractive index μ. Since, $\mu = \frac{\sin\left(\frac{\delta_{min}+A}{2}\right)}{\sin\frac{A}{2}}$ where for δ_{min} depends upon μ and vice-versa. There one relation with Two variables and hence by taking $\delta_{min} = 30^0$, we can determine limiting value of refractive index. Accordingly, using the available data $\mu = \frac{\sin\left(\frac{30^0+60^0}{2}\right)}{\sin\frac{60^0}{2}} \Rightarrow \mu = \frac{\sin 45^0}{\sin\frac{30^0}{2}} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$ is the

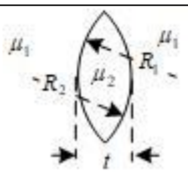
	limiting value of refractive index. But, point of contention to be ascertained is whether it is maximum or minimum. This depends upon $f" = \frac{d^2}{d\delta_{min}{}^2}\mu$. If $f"$ is (-)ve then it is maximum value and if $f"$ is (+)ve then it is minimum value. Accordingly, $f" = \frac{d^2}{d\delta_{min}{}^2}\left(\frac{\sin\left(\frac{\delta_{min}+A}{2}\right)}{\sin\frac{A}{2}}\right) = \frac{1}{\sin\frac{A}{2}}\frac{d}{d\delta_{min}}\left(\frac{1}{2}\times\cos\left(\frac{\delta_{min}+A}{2}\right)\right) = \frac{1}{2\sin\frac{A}{2}}\left(-\frac{1}{2}\times\sin\left(\frac{\delta_{min}+A}{2}\right)\right)$. It is found that $f" = (-)\frac{1}{4}\mu$ is (-) hence, **limit of refractive index is $\mu \leq \sqrt{2}$ is the answer.**
I-62	Formation of image due to spherical refracting medium is expressed by a formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R.}$...(1) In this Cartesian sign convention applies to u, v and R. In this from the figure $u = -25$cm and $R = 25$ cm, while, μ_1 and μ_1 are absolute values. Using the available data in (1) we have $\frac{1.5}{v} - \frac{1.0}{-25} = \frac{1.5-1.0}{20} \Rightarrow \frac{1.5}{v} = \frac{1.0}{40} - \frac{1.0}{25}$. It leads to $\frac{1.5}{v} = -\frac{3}{200} \Rightarrow v = \frac{1.5\times200}{3} = -$**100 cm is on the side of source from the pole O is the answer.** 
I-63	Formation of image due to spherical refracting medium is expressed by a formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R.}$...(1). In this Cartesian sign convention applies to u, v and R. In this from the figure u is to be determined and $R = 30$ cm, while, $v = \infty$ since refracted rays become paraxial, i.e,. parallel to principal axis, and parallel lines are assumed to meet at ∞. While refractive indices are absolute, and given to be $\mu_1 = \mu_A = 1.33$ and $\mu_2 = \mu_B = 1.48$. Thus, using the available data $\frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48-1.33}{30.} \Rightarrow \frac{1.33}{u} = -\frac{0.15}{30}$. It leads to $u = -\frac{4}{3}\times\frac{30}{0.15} = -$**267 cm away from the pole on the principal axis is the answer**. 
I-64	We know that for a medium having refractive index μ w.r.t. air critical angle of incidence i_c is $\frac{1}{\mu} = \frac{\sin i_c}{\sin 90^0} \Rightarrow \sin i_c = \frac{1}{2} \Rightarrow i_c = \sin^{-1}\frac{1}{2} = 30^0$. Given that rays are incident at an angle $i = 45^0$. Solving each part separately –  **Part (a):** In this case $i > i_c$ and it is case of total reflection. Hence rays would be reflected from plain surface of the hemisphere. Hence answers is incident **rays are reflected, is the answer of part (a).** **Part (b):** Considering it as a case of refraction in a spherical medium we have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$. Since incident rays are parallel and hence $u = \infty$. Using the Cartesian sign convention with principal axis along BA and taking A as origin we have $R = -3.0$ and the available data $\frac{2}{v} - \frac{1}{\infty} = \frac{2-1}{-3.0} \Rightarrow \frac{2}{v} = -\frac{1}{3.0} \Rightarrow v = -6.0$ cm. Since, incident rays are reflected as derived in part (a), the plane surface of the hemisphere will act as mirror, and **virtual image would be formed at P on the surface of the sphere assumed to be complete, diametrically opposite to A.** **Part (c):** Since the plane surface of the hemisphere is acting as a mirror and combining conclusions in part (a) and (b), **real image of the source at Q on surface of the hemisphere behind BC, like mirror image of A on BC, but it is real image. This is the answer of part (c).** **Part (d):** Final ray diagram if rays emerging out of the hemisphere is drawn in the figure above
I-65	Given that refractive index of glass sphere kept in air is $\mu = 1.5$. And formula for image in spherical medium is $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$. Here $\mu_1 = 1.5$ is the refractive index of spherical medium in which object is placed and $\mu_2 = 1.0$ is refractive index of medium surrounding the sphere. In solving the problem Cartesian Sign convention is used. Solving each part separately – 

Part (a): When observer is on the left and reference is taken as A, it leads to $R = 5.0$ cm and $u = R - 1.5 = 3.5$ cm. Accordingly, $\frac{1}{v} - \frac{1.5}{3.5} = \frac{1-1.5}{5} \Rightarrow \frac{1}{v} = \frac{1.5}{3.5} - \frac{0.5}{5}$. It leads to $\frac{1}{v} = \frac{23}{70} \Rightarrow v \approx 3$cm. Hence, image is **2 cm left to the centre of the sphere, is the answer of part (a)**.

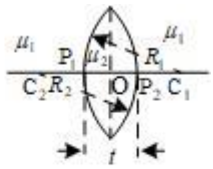
Part (b): When observer is on the right and reference is taken as B, $u = -(R + 1.5) = -6.5$ cm, and $R = -5.0$ cm. Accordingly, $\frac{1}{v} - \frac{1.5}{6.5} = \frac{1-1.5}{-5} \Rightarrow \frac{1}{v} = \frac{1.5}{6.5} - \frac{1}{10}$. It leads to $\frac{1}{v} = -\frac{17}{130} \Rightarrow v \approx 7.6$cm. Hence, position of image w.r.t. centre C is $7.6 = 5 =$ **2. 6 cm left to the centre of the sphere, is the answer of part(b)**.

(a) **Thus answers are (a) 2 cm left to the centre, (b)2.6 cm left to the centre**

I-66



Statement of variables in the problem are depicted in the figure on the left side. Here $\mu_2 = \mu = 1.5, \mu_1 = 1.0$, $R_1 = R_2 = R = 10$ cm and thickness of lens in the middle is $t = 5$ cm. Since, it is not the case of a thin lens and hence refraction from each surface as a boundary of the two medium is being analyzed separately using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ …(1), for which geometrical details are shown in the figure on right side. Cartesian sign convention is assigned to distances used in determi8ning position of image.



Since lens is symmetrical and hence object can be placed on either side for determining the final image. Taking object placed on the left side of the lens.

Image by Left Surface of Lens: Geometry of the lens for this case is shown in the figure with P_1 as a reference pole. Using (1) with the available data $\frac{1.5}{v_1} - \frac{1.0}{\infty} = \frac{1.5-1.0}{10}$. It leads to $\frac{1.5}{v_1} = \frac{0.5}{10} \Rightarrow v_1 = 30$ cm. Distance of image I_1 from O is $d_1 = P_1C_1 - P_1O$. It solves to $d_1 = v_1 - \frac{t}{2} = 30 - \frac{5}{2} \Rightarrow d_1 = 27.5$ cm from O.

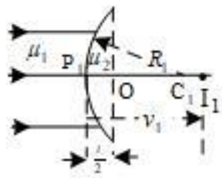
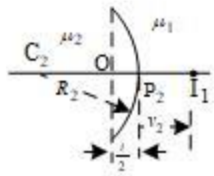


Image by Right Surface of Lens: Geometry of the lens for this case is also shown in the figure with P_2 as a reference pole. The image I_1 formed by left surface of the lens becomes source for the right surface and thus in this case $u_2 = OI_1 - P_2I_1 = d_1 - \frac{t}{2}$. Using d_1 determined above $u_2 = 27.5 - \frac{5}{2} = 25$ cm, while in this case $R = R_2 = -10$ cm and v_2 is the distance if image from P_2.



Here it is to be noted that position of image I_1 was arrived at material of refractive index on right of the left surface of the lens is of refractive index $\mu = 1.5$. It implies that object is embedded inside the glass. Therefore, refraction at right surface of the lens is arrived at by reversing the refractive indices on both sides of the right surface such that $\mu_1 = 1.5$ and $\mu_2 = 1.0$ is the refractive index on the side of the lens facing source. Accordingly, using the available data in (1) we have $\frac{1}{v} - \frac{1.5}{25} = \frac{1.0-1.5}{-10} \Rightarrow \frac{1}{v} = \frac{1.5}{25} + \frac{0.5}{10}$. It leads to $\frac{1}{v} = \frac{5.5}{50} \Rightarrow v = 9.1$ cm from the farther face of the lens.

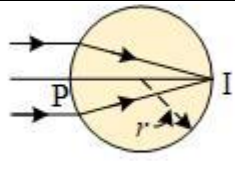
N.B.: (1) Here formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ cannot be applied since thickness of the lens is significant w,r.t. to radius of its spherical surfaces. And hence refraction from each surface is considered separately to determined position of image as a combined effect.

(2) Consideration in change of refractive indices while determining image formed by surface of lens farther from the source is important and need to be noted.

(3) This problem can be extrapolated with different radius of curvatures for both the spherical surfaces and different refractive indices for medium on each side of the lens

I-67

Given is a narrow beam (pencil) of parallel light incident on a solid transparent sphere of radius r. It is required to determine refractive index of the sphere in two conditions as under –



Condition (a) – At the surface of the sphere: Since beam is incident normally from left hand side with P as pole, it can be focused only on a point I diametrically opposite to O. It implies $v = 2r$. Since beam is parallel $u = \infty$.

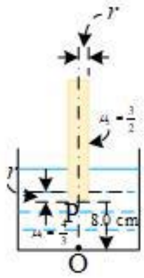
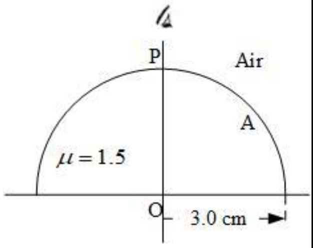
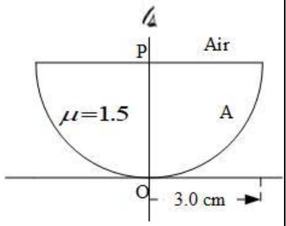
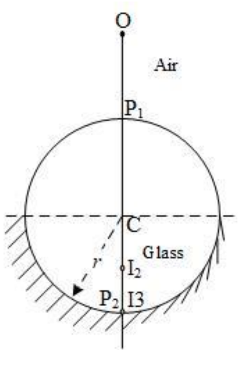
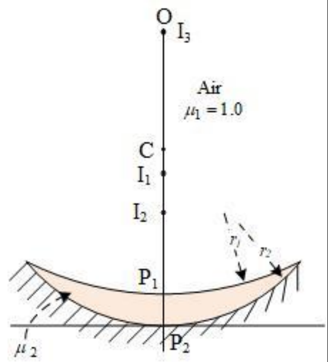
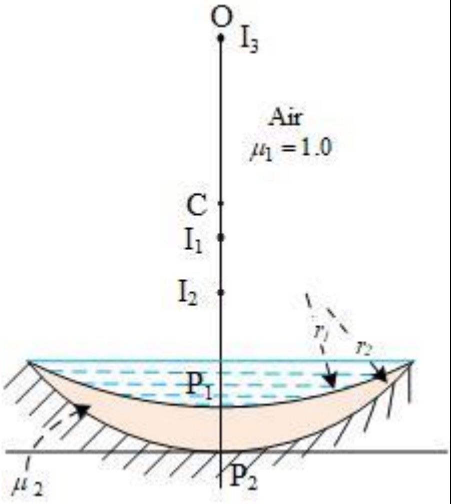
	Using formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, here $\mu_2=\mu$ refractive index of the sphere w.r.t. to its surrounding medium, $\mu_1=1$ is refractive index of medium surrounding the sphere, $R=r$. Using the available medium $\frac{\mu}{2r}-\frac{1}{\infty}=\frac{\mu-1}{r}\Rightarrow\mu\times r=(\mu-1)\times 2r\Rightarrow\mu=2\mu-2\Rightarrow\boldsymbol{\mu}=$ **2, is the answer of part (a).** **Condition (b)** – At the centre of the sphere: It implies $v=r$. Using formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$. Using the available medium $\frac{\mu}{r}-\frac{1}{\infty}=\frac{\mu-1}{r}\Rightarrow\mu\times r=(\mu-1)\times r\Rightarrow r=0$ it implies that sphere ceases to exist, that contradicts the proposition. However, of refractive index is large such that $\mu\to\mu-1$, then image will be close to centre. Hence **it is not possible isthe answer of part (b).** **Thus answers are (a) 2 , (b) not possible.**
I-68	Given system is shown in the figure where a glass rod having refractive index $\mu_2=1.50=\frac{3}{2}$ has a hemispherical end is having radius $r=1.0$ cm. This end is dipped in water having refractive index $\mu_1=\frac{4}{3}$ such that edge of the hemisphere is $d=8.0$ cm above the object O being observed . Using Cartesian sign convention with P as reference point distance between lowest point of hemisphere P (taken as a pole) is $u=-d=-7.0$ cm. Formula for refraction at a spherical surface is $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, here $R=r=1.0$ cm. Using the available data $\frac{\frac{3}{2}}{v}-\frac{\frac{4}{3}}{-8.0}=\frac{\frac{3}{2}-\frac{4}{3}}{1.0}\Rightarrow\frac{3}{2v}+\frac{4}{21.0}=\frac{1}{6.0}\Rightarrow\frac{3}{2v}=\frac{1}{6.0}-\frac{1}{6.0}\Rightarrow\frac{3}{2v}=0\Rightarrow$ $v=\infty$ **is the answer.** 
I-69	System as described in the problem is shown in the figure. Using Cartesian sign convention and taking point as pole, taking refractive index of air is $\mu_1=1.0$ and refractive index of glass $\mu_2=1.5$, $R=-3.0$. Since letters are placed near the centre and observer looks at them vertically $u=-3.0$. Using the formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with the available data $\frac{1.5}{v}-\frac{1.0}{-3.0}=\frac{1.5-1.0}{-3.0}$. it leads to $\frac{1.5}{v}=-\frac{0.5}{3.0}-\frac{1.0}{-3.0}\Rightarrow\frac{1.5}{v}=-\frac{1.5}{3.0}\Rightarrow v=-3.0$. Since numerically $u=v$ and hence **no shift is observed is the answer**. 
I-70	System as described in the problem is shown in the figure. Using Cartesian sign convention and taking point as pole, taking refractive index of air is $\mu_1=1.0$ and refractive index of glass $\mu_2=1.5$, $R=-3.0$ cm. Since letters are placed near the centre and observer looks at them vertically $u=-3.0$ cm. Since plane surface of the hemisphere is toward the observer and hence $R_1=\infty$ and spherical surface is towards to the letters being observed $R_2=3.0$ cm. This is like a plano-convex lens. Therefore, using the formula $\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ and the available data $\frac{1}{v}-\frac{1}{-3.0}=\left(\frac{1.5}{1.0}-1\right)\left(\frac{1}{\infty}-\frac{1}{3.0}\right)\Rightarrow\frac{1}{v}=-\frac{1}{3.0}-\frac{0.5}{3.0}\Rightarrow\frac{1}{v}=-\frac{1.5}{3.0}\Rightarrow v=-2$ cm from pole and is used to identify position of image in the figure. It implies that image is at height $h=-2-(-3)=$ **1 cm above the page, is the answer.** 
I-71	It is case of refraction-reflection-refraction. Using Cartesian sign convention with the available data each of the stage are solved sequentially - **Stage 1 – Refraction:** First is refraction of rays from object O placed with $u_1=2r$. Radius of the spherical surface is $R=-r$, refractive index glass sphere is $\mu_2=\mu=1.5$, it is placed in air with $\mu_1=1.0$. Since, it is refraction through only one surface the formula is $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ …(1) Using the given data $\frac{1.5}{v_1}-\frac{1}{2r}=\frac{1.5-1.0}{-r}\Rightarrow\frac{1.5}{v_1}=\frac{1}{2r}-\frac{0.5}{r}\Rightarrow\frac{1.5}{v_1}=0\Rightarrow v_1=\infty$. Thus it will produce an image I_1 at infinity, it implies a beam of parallel rays. **Stage 2 - Reflection:** This is a case of concave-mirror reflection from silvered surface, as shown in figure for which pole is P_2. It has focal length $f=\frac{r}{2}$. 

	Image I_1 in stage-1 becomes object O_2 with $u_2 = \infty$. Thus using formula of spherical mirror $\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f}$ with the available data it leads to $\frac{1}{\infty} + \frac{1}{v_2} = \frac{1}{\frac{r}{2}} \Rightarrow v_2 = \frac{r}{2}$ and produce an image I_2 as shown in the figure. **Stage 3 – Refraction:** In this case again pole is P_1 and I_2 becomes object with $u_3 = -P_1I_2 = -(P_1P_2 - P_2P_1)$. It leads to $u_3 = -P_1I_2 = -\left(2r - \frac{r}{2}\right) \Rightarrow u_3 = -\frac{3r}{2}$ and again $R = -r$. Though, again refraction occurs through non-polished spherical surface but object is inside glass sphere. Therefore, while using equation (1) we will have $\mu_1 = 1.5$ and $\mu_2 = 1.0$ and therefore $\frac{1.0}{v} - \frac{1.5}{-1.5r} = \frac{1.0-1.5}{-r}$. It leads to $\frac{1.0}{v} = \frac{0.5}{r} - \frac{1}{r} \Rightarrow \frac{1.0}{v} = -\frac{0.5}{r} \Rightarrow v = -2r$. **Thus final image I_3 will occur at P_2, right at the reflecting surface,** as shown in the figure **Thus answer is at the reflecting surface of the mirror.**
I-72	It is case of refraction-reflection-refraction It is required to determine distance of pin O from the lens such that image coincides the object in two parts **(a)** When lens is made of glass$\left(\mu = \frac{3}{2}\right)$, **(b)** lens is a thin hollow shell filled with water $\left(\mu = \frac{4}{3}\right)$.  Each of the part shall have to solved in three stages using Cartesian sign convention and available data **Part (a) Lens is of glass :** **Stage 1 – Refraction:** First is refraction of rays from object O placed w.r.t P_1 with $u_1 = x$ to be determined. Radius of the spherical surface being concave is $R = r_1 = 60$ cm, refractive index glass sphere is $\mu_2 = \mu = 1.5$. It is placed in air. Since, it is refraction through spherical surface the formula is $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ …(1) Using the given data $\frac{1.5}{v_1} - \frac{1}{x} = \frac{1.5-1.0}{60} \Rightarrow \frac{1.5}{v_1} = \frac{1}{x} + \frac{1}{120} \Rightarrow \frac{1}{v_1} = \frac{1}{1.5x} + \frac{1}{180}$…(2) Thus it will produce an image I_1 at a distance v_1 w.r.t. P_1. **Stage 2 - Reflection:** Given that it is thin concavo-convex lens and hence P1 and P2 are assumed to coincide i.e. $P_1P_2 \approx 0$. Given that convex surface of the silver polished and hence it will act as a concave mirror and image I_1 formed in stage-1 will act as object such that $u_2 = v_1$, while, $f_2 = \frac{r_2}{2}$. Thus using formula of spherical mirror $\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2}$ ..(3). With the available data and (2) it leads to $\left(\frac{1}{1.5x} + \frac{1}{180}\right) + \frac{1}{v_2} = \frac{2}{20} \Rightarrow \frac{1}{v_2} = \frac{1}{10} - \left(\frac{1}{1.5x} + \frac{1}{180}\right)$. It leads to $\frac{1}{v_2} = \frac{17}{180} - \frac{1}{1.5x}$ …(4) produce an image I_2 as shown in the figure. **Stage 3 – Refraction:** In this case again pole is P_1 and I_2 becomes object with $u_3 = v_2$…(5) and again $R = r_1$. Unlike stage 1, in this stage refraction occurs through non-polished spherical surface into air. Therefore, while using equation (1) we will have values of μ_1 and μ_2 will interchange such that now we have $\mu_1 = 1.5$ and $\mu_2 = 1.0$ and therefore $\frac{1.0}{v_3} - \frac{1.5}{u_3} = \frac{1.0-1.5}{r_1}$. It is given that final image coincides O whose distance from P_1 is $v_3 = x$. Thus using available data and equations (4) and (5) we have $\frac{1.0}{x} - \frac{1.5}{1} \times \left(\frac{17}{180} - \frac{1}{1.5x}\right) = \frac{1.0-1.5}{60} \Rightarrow \frac{1}{x}(1.0 + 1.0) = \frac{1.5\times17}{180} - \frac{1.0}{120}$. It further solves into $\frac{2.0}{x} = \frac{1}{60}(0.5 \times 17 - 0.5) \Rightarrow \frac{2.0}{x} = \frac{0.5\times16}{60} \Rightarrow x = \frac{60\times2.0}{8} = 15$ cm. **Thus answer of part (a) is 15 cm from the lens.**

Part (b) Concave part is filled with water: It implies that the lens is placed in water. It changes nature of problems with combination of (a) Plano-convex lens of refractive index $=\frac{4}{3}$ (b) Concave-convex lens of refractive index $=\frac{3}{2}$ and (c) a concave mirror.

Accordingly, number of stages to solve the problem increases with: *Stage 1*- Refraction through Plano-convex lens, *Stage 2* – refraction through Concave-convex lens, *Stage 3* – Reflection through concave mirror, *Stage 4* – refraction through Concave-convex lens, *Stage 5*- Refraction through Plano-convex lens.



But, taking the two lenses: (a) lens of water with focal length f_a and (b) lens of glass with focal length f_b as composite lens such that $\frac{1}{f_c}=\frac{1}{f_a}+\frac{1}{f_b}$ ...(6). This requires first to determine focal length of each of the lenses where, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

Focal length of Water lens: Here, $R_1=\infty$ and $R_2=60$ cm. Accordingly, $\frac{1}{f_a}=\left(\frac{4}{3}-1\right)\left(\frac{1}{\infty}-\frac{1}{60}\right)\Rightarrow\frac{1}{f_a}=-\frac{1}{180}$ …(7)

Focal length of Glass lens: Here, $R_1=60$ and $R_2=20$ cm. Accordingly, $\frac{1}{f_b}=\left(\frac{3}{2}-1\right)\left(\frac{1}{60}-\frac{1}{20}\right)$. It leads to $\frac{1}{f_b}=-\frac{1}{60}$ …(8)

Focal length of composite lens: Using (6) we have $\frac{1}{f_c}=-\frac{1}{180}-\frac{1}{60}=-\frac{4}{180}\Rightarrow\frac{1}{f_c}=-\frac{1}{45}$…(9)

Now going ahead with solving each stage -

Stage 1 – Refraction: Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ …(10) with the given data $\frac{1}{v_1}-\frac{1}{x}=-\frac{1}{45}\Rightarrow\frac{1}{v_1}=\frac{1}{x}-\frac{1}{45}$…(11)

Thus it will produce an image I_1 at a distance v_1 from the lens.

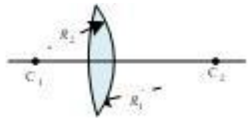
Stage 2 - Reflection: Since $u_2=v_1$,…(12), while, $f_2=\frac{r_2}{2}$. Thus using formula of spherical mirror $\frac{1}{u_2}+\frac{1}{v_2}=\frac{1}{f_2}$ ..(3). With the available data and (12) it leads to $\left(\frac{1}{x}-\frac{1}{45}\right)+\frac{1}{v_2}=\frac{2}{20}\Rightarrow\frac{1}{v_2}=\frac{1}{10}-\left(\frac{1}{x}-\frac{1}{45}\right)$. It leads to $\frac{1}{v_2}=\frac{11}{90}-\frac{1}{x}$ …(13) produce an image I_2 as shown in the figure.

Stage 3 – Refraction: In this case $u_3=v_2$ and again applying (10) with value of f_c distance of image is determined $\frac{1}{v_3}-\left(\frac{11}{90}-\frac{1}{x}\right)=-\frac{1}{45}$. Since it is given that final image coincides with O and hence $v_3=x$. Accordingly, $\frac{1}{x}-\left(\frac{11}{90}-\frac{1}{x}\right)=-\frac{1}{45}\Rightarrow\frac{2}{x}=\frac{11}{90}-\frac{1}{45}\Rightarrow\frac{2}{x}=\frac{9}{90}\Rightarrow\boldsymbol{x=20}$ **cm.**

Thus answers are (a) 15 cm , (b) 20 cm .

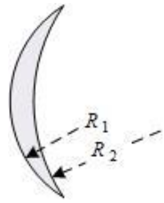
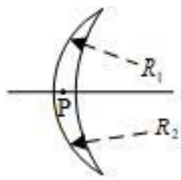
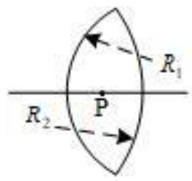
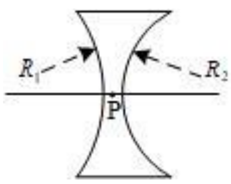
N.B.: (1) This is a good example of underlying concepts of physics and is brought out stage-wise illustration of the solution. Direct use of formula is creates a burden of remembering it with different variants, and is prone to error either in use of its correct form, variables with appropriate sign convention. **(2)** Such questions involving multiple stages and options are seldom asked in examination. Yet in practice solving them helps to build patience and confidence of problem solving with added advantage of making concepts intuitive.

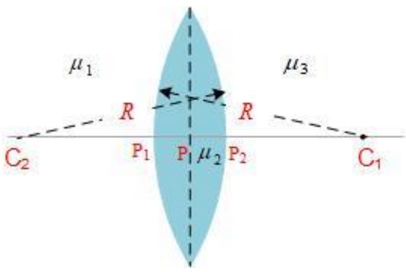
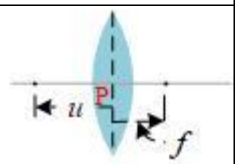
I-73

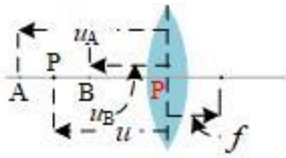
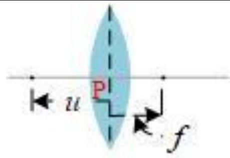
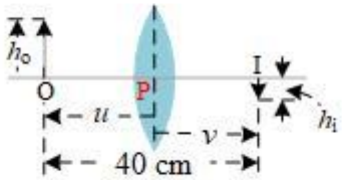
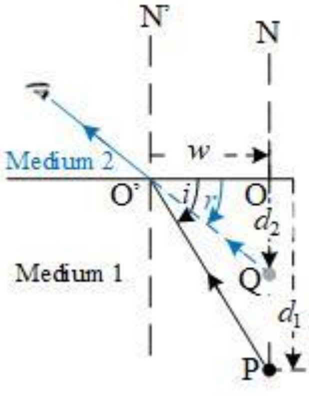


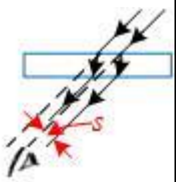
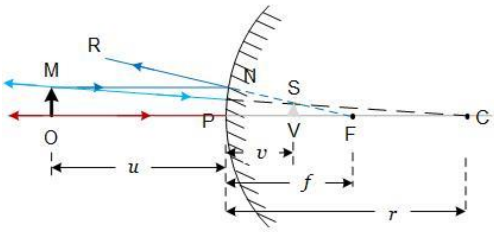
Geometry of the lens is shown in the figure. Given that focal length of the lens $f=25$ cm, refractive index of the medium w.r.t. to air in its surrounding is $\mu=1.5$ and $R_1=2R_2$ …(1). Since thickness of the lens is not given and hence safely the lens can be considered to be thin. Focal length of a thin lens is $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ …(2) Using the available data we have $\frac{1}{25}=(1.5-1)\left(\frac{1}{2R_2}-\frac{1}{-R_2}\right)\Rightarrow\frac{1}{25}=-\frac{1.5}{2R_2}\Rightarrow R_2=-25\times0.75=-18.75$. Here, (-) sign depicts the lens is double convex. Thus absolute value of $\boldsymbol{R_2}$ **is 18.75 cm** and using (1) we have $\boldsymbol{R_1}=2\times18.75=\boldsymbol{37.5}$ **cm.**

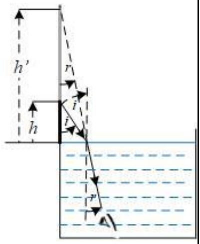
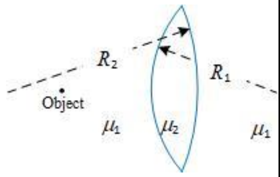
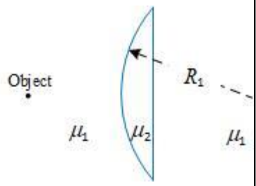
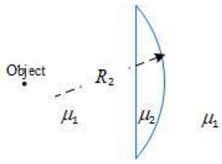
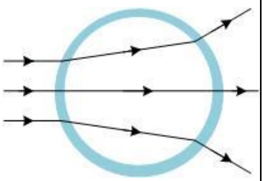
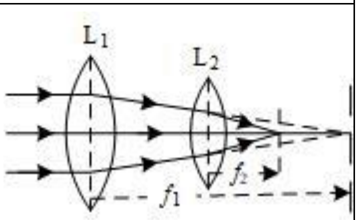
Thus answer is 18.75 cm, 37.5 cm.

I-74	It is given that both radii are (+)ve and hence it implies that it is a convexo-concave lens as shown in the figure. Since thickness of the lens is not given and hence safely the lens can be considered to be thin. Further, it is given that $R_1 = +20$ cm and $R_2 = +30$ cm. Focal length of a thin lens is $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(1) . Question is in two parts with lens kept in air and water and each part is solved separately.  **Part (a) – Lens kept in air:** Refractive index of the material of lens is given to be $\mu_2 = 1.6$ and that of air is $\mu_1 = 1.0$. Using the available data we have $\frac{1}{f} = \left(\frac{1.6}{1.0} - 1\right)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow \frac{1}{f} = 0.6 \times \frac{1}{60}$. It leads to $\boldsymbol{f = 100}$ **cm is the answer of part (a).** **Part (b) – Lens kept in water:** Refractive index of water is $\mu_1 = 1.33 = \frac{4}{3}$. Using the available data we have $\frac{1}{f} = \left(\frac{1.6}{\frac{4}{3}} - 1\right)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow \frac{1}{f} = (1.2 - 1) \times \frac{1}{60} \Rightarrow \frac{1}{f} = \frac{0.2}{60}$. It leads to $\boldsymbol{f = 300}$ **cm is the answer of part (b).** **Thus answers are (a) 100 cm and (b) 300 cm.**
I-75	Three cases are conjured for the two radii. Further Cartesian sign convention is used to determine focal lengths. It is given that the two radii are 20 cm and 30 cm, while formula for focal length of a thin lens is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(1), here, $\mu = 1.50$ is refractive index if material of lens while R_1 is radius of curvature of spherical surface on the left of the pole P and R_2 is radius of curvature of spherical surface on the right of pole P. Signed values of the two radii will be used. **Case 1 - Convexo-concave Lens:** Taking out of the given data based in the figure both the radii are positive such that $R_1 = +20$ cm and $R_2 = +30$ cm. Using the available data in (1) we have $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow \frac{1}{f} = 0.5 \times \frac{1}{60}$. It leads to $f = 120$ cm. Keeping the same formation of the lens if both the radii or interchanged ($R_1 \leftrightarrow R_2$) then $f = -120$. Thus focal length in this formation is $\boldsymbol{\pm 120}$ **cm.**  **Case 2 - Convex-Convex Lens:** In this formation while $R_1 = +20$ cm the $R_2 = -30$ cm. Using the available data in (1) we have $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-30}\right)$. It leads to $\frac{1}{f} = 0.5 \times \frac{5}{60}$. It leads to $f = 24$ cm. Keeping the same formation of the lens if both the radii or interchanged ($R_1 \leftrightarrow R_2$) then $f = -24$. Thus focal length in this formation is $\boldsymbol{\pm 24}$ **cm.**  **Case 3: Conacvo-Conacve Lens:** In this formation while $R_1 = -30$ cm the $R_2 = +20$ cm. Using the available data in (1) we have $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-30} - \frac{1}{20}\right)$. It leads to $\frac{1}{f} = -0.5 \times \frac{5}{60}$. It leads to $f = -24$ cm. Keeping the same formation of the lens if both the radii or interchanged ($R_1 \leftrightarrow R_2$) then $f = +24$. Thus focal length in this formation is $\boldsymbol{\pm 24}$ **cm, it is same as in case 2.**  **Thus answer is $\boldsymbol{\pm 24}$ cm and $\boldsymbol{\pm 120}$ cm.**

I-76	 The geometry of the lens and refractive indices of the medium are shown in the figure. Though magnitude of radii of both the convex surfaces is same refractive index of medium on both sides of the lens are different as also shown in the figure. Therefore, focal length of the kens would be determined in two stages for each of the parts, using formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ …(1), as under- **Part (a) Incident beam is from medium of Refractive index μ_1:** **Stage 1- Refraction across left surface:** Since incident rays are a parallel beam $u_1=\infty$ and radius of curvature $R_1=R$. Accordingly, image I_1 is formed due to left convex surface , using the available data $\frac{\mu_2}{v_1}-\frac{\mu_1}{\infty}=\frac{\mu_2-\mu_1}{R}\Rightarrow\frac{\mu_2}{v_1}=\frac{\mu_2-\mu_1}{R}$…(2) **Stage 2- Refraction across right surface:** In this case image I_1 formed by left surface becomes object and thus $u_2=v_1$…(3) Further, since C_2 is on the left of P and hence radius of curvature $R_2=-R$.Accordingly, final image I_2 is formed due to right convex surface. Thus, $\frac{\mu_3}{v_2}-\frac{\mu_2}{u_2}=\frac{\mu_3-\mu_2}{-R}$…(4). Combining (2) and (3) we have $\frac{\mu_2}{u_2}=\frac{\mu_2-\mu_1}{R}$…(5). Further, adding (4) and (5) we have $\frac{\mu_3}{v_2}-\frac{\mu_2-\mu_1}{R}=\frac{\mu_3-\mu_2}{-R}\Rightarrow\frac{\mu_3}{v_2}=\frac{2\mu_2-\mu_1-\mu_3}{R}$. It leads to $\frac{1}{v_2}=\frac{2\mu_2-\mu_1-\mu_3}{\mu_3 R}$…(6). This final image is caused by a parallel incident beam and final image I_2 is formed at its focal point and hence using (6) $f=v_2=\frac{\mu_3 R}{2\mu_2-\mu_1-\mu_3}$ **…(7) is the answer of part (a)** **Part (b) Incident beam is from medium of Refractive index μ_3:** This case is optical inversion of medium such that $\mu_1\leftrightarrow\mu_3$…(8). Accordingly, using (7) and (8) we have $f=\frac{\mu_1 R}{2\mu_2-\mu_1-\mu_3}$ **is the answer of part (b).** **Thus answers are (a)** $\frac{\mu_3 R}{2\mu_2-\mu_1-\mu_3}$ **(b)** $\frac{\mu_1 R}{2\mu_2-\mu_1-\mu_3}$
I-77	 We know that $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$…(1). Using Cartesian Sign convention, with the given geometry distance of point object (u) from the lens is (-)ve, while focal distance of convex lens (f) id (+)ve. With the given data both parts of the problem are solved using (1) as under – **Part (a):** $\frac{1}{v}-\frac{1}{-9.8}=\frac{1}{10}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{1}{9.8}\Rightarrow\frac{1}{v}=\frac{9.8-10}{98}\Rightarrow v=-\frac{98}{0.2}=-\mathbf{490}$ **cm**. (-) sign is indicative of the fact that the image is **on the side of the object**. Signs of *u* and *v* are since same the image is **virtual**. Hence answer of the part (a) is **490 cm on the side of the object, virtual** **Part (b):** $\frac{1}{v}-\frac{1}{-10.2}=\frac{1}{10}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{1}{10.2}\Rightarrow\frac{1}{v}=\frac{10.2-10}{102}\Rightarrow v=\frac{102}{0.2}=\mathbf{510}$ **cm**. (+) sign is indicative of the fact that the image is **on the other side of the object**. Signs of *u* and *v* are since opposite the image is **real**. Hence answer of the part (a) is **510 cm on the side of the object, real.** **Thus answers are (a) 490 cm on the side of the object, virtual , (b) 510 cm on the other side, real**
I-78	Spherical mirrors and lenses have circular symmetry and hence while magnification of slide of size (35mm × 23mm) on a screen of size (2 m × 2 m) if larger side of slide i.e. 35 mm = 35×10^{-3} m is magnified within the limit of 2m of size of screen, the small side will automatically remain on the screen. Formula for magnification is $m=-\frac{h_i}{h_0}$…(1). Here, h_0 is the height of object and h_i is the height of the image.. Further, relating magnification to distances object $m=-\frac{h_i}{h_0}=\frac{v}{u}$…(2). Thus using the given data and (2) $m=-\frac{2}{35\times10^{-3}}=\frac{v}{u}$…(3). It is required to find focal length of the lens when the screen is placed at a distance of $v=10$ m from the lens and therefore $=-\frac{2}{35\times10^{-3}}=\frac{10}{u}\Rightarrow u=-\frac{350\times10^{-3}}{2}=-175$ mm. We have equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. Using the available data $\frac{1}{-175}-\frac{1}{10\times10^3}=\frac{1}{f}\Rightarrow f=-\frac{175\times(10\times10^3)}{10175}\Rightarrow \boldsymbol{f}=\mathbf{172}$ **mm, is the answer.**

I-79	Given that a particle say O is executing SHM with its mean position at a distance 20 cm from the lens, as shown in the figure. Cartesian sign convention is used. Amplitude of SHM is $A = 1.0$ cm with it maximum displacement positions points A and B such that $u_A = -20 - 1 = -21$ cm and $u_B = -20 + 1 = -19$ cm.  Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$ with the given focal length $f = 12$ cm, the position of image for two extreme positions A and B, are determined. For image of at position A, we have $\frac{1}{v_A} = \frac{1}{f} + \frac{1}{u_A} \Rightarrow$, $\frac{1}{v_A} = \frac{1}{12} + \frac{1}{-21}$. It leads to $v_A = \frac{12\times21}{21-12} = 28.0$ cm. For image of at position B, we have $\frac{1}{v_B} = \frac{1}{f} + \frac{1}{u_B} \Rightarrow$, $\frac{1}{v_B} = \frac{1}{12} + \frac{1}{-19}$. It leads to $v_B = \frac{12\times19}{19-12} = 32.6$ cm. Therefore amplitude of oscillation of image would be $A_i = \frac{\lvert v_A - v_B\rvert}{2} = \frac{\lvert 28.0-32.6\rvert}{2} \Rightarrow A_i = \frac{4.6}{2} = \mathbf{2.3}$ **cm is the answer.**
I-80	This problem has two parts. Part (a) is graphical in nature and with the illustrations in the Mentors' Manual and reference books are explicit and can be referred to by the readers. Therefore, part (b) is being solved. Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$. Referring to the figure and the given data $u = -5.0$ cm and focal length $f = 8.0$ cm we have $\frac{1}{v} = \frac{1}{8.0} - \frac{1}{5.0} \Rightarrow \boldsymbol{v} = \frac{5.0\times8.0}{5.0-8.0} = -\mathbf{13.3}$ **cm., i.e. towards the object, erect and virtual image is the answer of part (b).** 
I-81	Cartesian Sign convention is used for the geometry as shown in the figure. Further, magnification is expressed as $m = -\frac{h_i}{h_0} = \frac{v}{-u} \Rightarrow \frac{h_i}{h_0} = \frac{v}{u}$ …(1) . Given that $h_0 = 2.00$ cm and for inverted image $h_i = 1.00$ cm. Accordingly, $\frac{1}{2} = \frac{v}{u}$, it leads to $u = 2v$…(2)  Further, as per lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{-2v} \Rightarrow f = \frac{2}{3}v$…(3). With the given separation between object (O) and image (I), $v + u = 40.0 \Rightarrow v + 2v = 40 \Rightarrow v = \frac{40}{3}$…(4) . Combining (3) and (4) leads to $f = \frac{2}{3}\times\frac{40}{3} = \frac{80}{9} \Rightarrow \boldsymbol{f} = \mathbf{8.89}$ **cm**. And distance of pin from lens, using (2) and (4) is $\boldsymbol{u} = 2\times\frac{40}{3} = \mathbf{26.7}$ **cm.** **Thus answers are 8.89 cm and 26.7 cm.** **N.B.: Answers are based on principle of significant digits.**
I-82	Given the formula of refractive index is $\mu = \frac{\text{Real depth}}{\text{Apparent Depth}}$ when viewed in the figure is $\mu = \frac{d_1}{d_2}$. The position of the object P in medium 1 at depth d_1 when viewed from medium 2 is along the normal N and above the object at u at a depth d_2. Thus, horizontal distance of the object from the point of emergence of ray is remains w. Therefore, with given formula $\mu = \frac{\mathrm{d}_1}{\mathrm{d}_2} \Rightarrow \mu = \frac{\frac{\mathrm{d}_1}{w}}{\frac{\mathrm{d}_2}{w}}$. It leads to $\mu = \frac{\tan i'}{\tan r'}$. Here, i' is the angle of incident ray PO with the interface of two mediums and $r'$is the angle of emergent ray which when tracked backward OQ leads to apparent position of the object Q.  When object is viewed quite from the normal N such that the point of emergence of ray from the object O' moves away from O leading to increase in w, while real depth remains d_1 remains unchanged. This leads to $i' \ll \Rightarrow \tan i' \approx i'$ and also $r' \ll\ll \Rightarrow \tan r' \approx r'$. Thus $\mu = \frac{\tan i'}{\tan r'} \to \frac{i'}{r'}$, but $\frac{\tan i'}{\tan r'} \neq \frac{i'}{r'}$. Hence, the given formula approximates but is not

	valid when object is viewed made by incident ray with the normal at the point of incidence O, on the interface of the two mediums and likewise r is the angle made by the refracted ray with the normal. When position of viewing is quite close to the normal i<< and likewise r<< and correspondingly $\sin i \approx i$ and $\sin r \approx r$. Thus relationship becomes $\mu = \frac{i}{r} \neq \frac{\sin i}{\sin r}$. **Hence, the formula is not valid.**
I-83	In prism none inclined surfaces are not parallel, but the cross-sectional of prism may or may not be parallel. Thus rays from an object in a medium outside the prism where refractive indices of material of prism and the medium outside the prism are different and deviation is inevitable.. **Thus to have undeviated ray is possible when prism is placed in a medium having refractive index same as that of the material of prism.**
I-84	Both glass and diamond are transparent material but their refractive indices are $\mu_g \approx 1.5$ and $\mu_d \approx 2.4$. From the relation $\mu = \frac{\sin 90^0}{\sin \theta_{cr}} \Rightarrow \sin \theta_{cr} = \frac{1}{\mu} \Rightarrow \theta_{cr} = \sin^{-1}\left(\frac{1}{\mu}\right)$. Therefore, their critical angle are $\theta_{cd-g} \approx \sin^{-1}\left(\frac{1}{1.5}\right)$ and $\theta_{cd-d} \approx \sin^{-1}\left(\frac{1}{2.4}\right)$ respectively and numerically $\theta_{cd-d} < \theta_{cd-g}$. Light, is entering the crystals of both glass and diamond equally but, in glass greater proportion of it would pass through as refracted rays, and lesser proportion of the incident rays would experience total reflection. Conversely, proportion of light undergoing total reflection in diamond is greater and contributes to greater shining.
I-85	When refractive index fluctuates so also the distance of separation also changes. But the eye relates to position of object in line with the refracted rays and hence irrespective of the object being stationary it appears twinkling similar to that of the stars. 
I-86	Image formed by a plane mirror is not due to actual convergence of rays from an object. Rather, the image is due to backtracking of reflected rays and thus it cannot be taken on a screen. Hence answer is No.
I-87	Burning of paper is caused by real convergence of rays. Since virtual image is apparent convergence of rays created by extending rays against direction of propagation. Hence paper will **not burn** when it is placed at the position of virtual image. But when paper is placed at the position of real image, it is the place where rays are really converging and hence paper **will burn.** When image is real it is real convergence of rays, irrespective of object being real or virtual, hence paper **will burn.**
I-88	Virtual image is backward convergence of divergent rays. Therefore, using lens divergent rays are made to **create a real by using lenses in camera.**
I-89	Show object in a size within vision of the driver and erect. This is achieved using a convex mirror where virtual image, in erect form, is formed within its focal length. As against this plane mirror though produces erect image, but it is of size and distance equal to the object. Thus image goes out of vision of the driver.
I-90	Formula for image in spherical mirror is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$…(1). Here, u is distance of object from pole, v is the distance of image from the pole, f is distance of focus from the pole. Using the Cartesian sign convention, the equation can be written as $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$…(2) Here, $v < f$, while $u \gg$ but f is constant for a spherical mirror. Hence, $u > v$…(3). 

	Since speed are to be compared and hence differentiating (2) we have $\frac{d}{dt}\left(-\frac{1}{u}\right)+\frac{d}{dt}\left(\frac{1}{v}\right)=\frac{d}{dt}\left(\frac{1}{f}\right)$. It leads to $\frac{1}{u^2}\times\frac{du}{dt}-\frac{1}{v^2}\times\frac{dv}{dt}=0\Rightarrow\frac{du}{dt}=\left(\frac{u}{v}\right)^2\frac{dv}{dt}$…(4). Combining (3) and (4) we have $\frac{du}{dt}>\frac{dv}{dt}$, i.e. **image move slower than object.**
I-91	Situation given in the problem is shown in the figure. A ray of light from top of the man standing on the edge of the pool makes an angle i with the normal at the interface of water and air. After refraction it makes angle r with the normal and seen by the person swimming in the pool. The refracted ray when traced back the height of the man appears to be h'. It is seen that $r<i$ and $h<h'$. The man on the edge **appears to be taller** to the man swimming inside the pool. 
I-92	Equation is given to be $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}\Rightarrow\frac{\mu_2}{v}-\frac{\mu_1}{u}=0\|_{R\to\infty}\Rightarrow\frac{\mu_2}{v}=\frac{\mu_1}{u}\Rightarrow\frac{\boldsymbol{\mu_1}}{\boldsymbol{\mu_2}}=\frac{\boldsymbol{u}}{\boldsymbol{v}}$. Here, u is the real depth of the object inside spherical surface and v is the apparent depth of the object. Thus, $\frac{\textbf{Real Depth}}{\textbf{Apparent Depth}}=\frac{\boldsymbol{\mu_1}}{\boldsymbol{\mu_2}}$
I-93	Lens formula for a spherical lens, shown in the figure, is $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. Position image is determined from formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$. For a given object position of image depends upon focal length of the lens, of a given material, which in turn is dependent upon radius of curvature. In question Two cases are given as under –  **Case 1:** Convex surface having radius of curvature $R_1=R$ faces the object as shown in the figure, while for plane surface $R_2=\infty$. Accordingly, focal length of the lens would be $\frac{1}{f_1}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R}-\frac{1}{\infty}\right)\Rightarrow\frac{1}{f_1}=\left(\frac{\mu_2}{\mu_1}-1\right)\times\frac{1}{R}$.  **Case 2:** Plane surface faces the object and in this case $R_1=\infty$ and $R_2=R$ and accordingly focal length would be $\frac{1}{f_2}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{\infty}-\frac{1}{R}\right)\Rightarrow\frac{1}{f_2}=-\left(\frac{\mu_2}{\mu_1}-1\right)\times\frac{1}{R}$.  In the two case $f_2=-f_1\Rightarrow f_2\neq f_1$ and **hence** $\boldsymbol{v_2\neq v_1}$
I-94	A converging lens would always converge a parallel beam of light, while diverging lens would always diverge a parallel beam. Since, the question states that beam is parallel and is diverging while passing through the lens and hence **with certainty the lens is divergent one.**
I-95	A water drop resembles to a convex-convex lens where refractive index of the medium surrounded by the water is $\mu>1$. But, bubble is a hollow sphere of water with air inside and outside it. Thus rays passing it encounter a combination of convex-concave AND Concave-Convex lens. Accordingly, the ray diagram indicates that the **bubble is diverging lens**. 
I-96	The arrangement as described in the problem is shown in the figure. Lens L_2 of focal length f_2 is placed such that it is between the lens L_1 and it focal point such that $f_2>f_1$. This has two effects – (a) the beam of light received by L_1 is focused and becomes narrower, thus the combination **requires lesser aperture**, (b) during (a) there is no loss of light and thus **intensity of beams** (light per unit area) **increase**. 

I-97	Reflection phenomenon is about travel of light in same medium and hence change of medium has no effect on reflection. Secondly, focal length of a spherical mirror depends upon its radius of curvature which is dependent upon geometry of mirror. Accordingly, dipping of spherical mirror in water will **not change its focal length.**
I-98	Formula of focal length of lens is $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$. Here R_1 and R_2 de[end upon geometry of the lens while μ_2 is refractive index of material of the lens, all these three parameters are characteristic to the lens. But, μ_1 is the refractive index of the medium surrounding the lens. Thus when lens is dipped in water, refractive index of the medium changes and hence **focal length of the lens would change**.
I-99	Unlike refraction phenomenon, in reflection phenomenon is equally valid for all wavelengths of light. Therefore , constituent wavelengths of incident light would undergo identical reflection. Hence there would **not be any chromatic aberration in reflection**.
I-100	Chromatic aberration occurs when incident light ray constitutes multiple wavelengths. But laser light is since monochromatic, and hence there will **not be any chromatic aberration**.

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