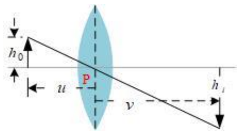
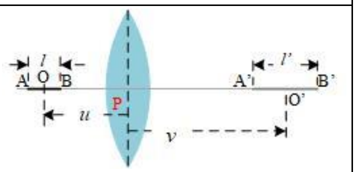
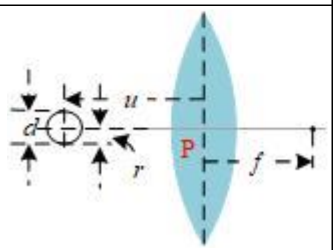
Wave and Motion : Illustration pf Typical Questions

(Rest of Geometrical Optics)

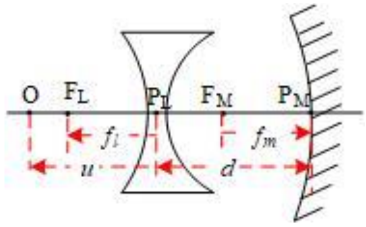
I-01	 Information given in the problem is shown in the figure where ratio of height of image and object $\frac{h_i}{h_0} = 2$. Cartesian sign convention is used, accordingly $u = -18$ cm. Magnifying power of a lens $m = -\frac{h_i}{h_0} = \frac{v}{u}$…(1) Therefore, with the given data $\frac{v}{u} = -2 \Rightarrow v = -2 \times (-18) \Rightarrow v = 36$ cm. Thus, image would be on the side of the lens opposite to the object. Thus using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$…(2). Using the available data $\frac{1}{f} = \frac{1}{36} - \frac{1}{-18} = \frac{1}{12}$…(3) To determine position of the object, using (1) we have $\frac{h_i}{h_0} = 3 \Rightarrow -3 = \frac{v}{u} \Rightarrow v = -3u$…(4). Using (2), (3) and (4) we have $\frac{1}{12} = \frac{1}{-3u} - \frac{1}{u} \Rightarrow \frac{4}{3u} = -\frac{1}{12} \Rightarrow u = -16$cm. **Thus distance of object from lens is 16 cm, is the answer.**
I-02	 Arrangement given in the problem is shown in the figure; convex lens is converging lens. Cartesian sign convention is used. Let O is the centre of the pin of length $l = 2.0$ cm with $u = -11$ cm, Accordingly, corresponding distance for end A of the pin are $u_A = u + \left(-\frac{l}{2}\right)$, it leads to $u_A = -11 + \left(-\frac{2}{2}\right) = -12$ cm . Likewise, $u_B = -11 + \left(\frac{2}{2}\right) = -10$ cm. Further, focal length of the is $f = 11$ cm. A'B' is image of the pin AB. Using the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, distance of image of end from lens $\frac{1}{f} = \frac{1}{v_A} - \frac{1}{u_A} \Rightarrow \frac{1}{v_A} = \frac{1}{f} + \frac{1}{u_A}$. Using the available data $\frac{1}{v_A} = \frac{1}{6} + \frac{1}{-12} \Rightarrow v_A = \frac{6\times12}{12-6} = 12$ cm. Likewise, $\frac{1}{v_B} = \frac{1}{6} + \frac{1}{-10} \Rightarrow v_B = \frac{6\times10}{10-6} = 15$ cm. Thus length of the image is $l' = v_B - v_A \Rightarrow \boldsymbol{l'} = 15 - 12 =$ **3 cm, is the answer.**
I-03	 The sun has in space spherical symmetry and while viewing circular symmetry. Therefore, while calculating magnification. Therefore, while calculating magnification $m = -\frac{h_i}{h_o} = \frac{v}{u}$…(1), we have $h_o = r = \frac{d}{2}$. It leads to $h_o = \frac{1.4\times10^9}{2} = 7.0 \times 10^9$…(2) Focal length of the lens $f = 0.20$ m. Using Lens formula with the available data $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{0.20} + \frac{1}{-1.5\times10^{11}}$. It leads to $v = \frac{0.20\times(1.5\times10^{11})}{1.5\times10^{11}-0.20} \approx 2 \times 10^{-1}$m…(3). Combining (1), (2) and (3) with the available data we have $-\frac{h_i}{7.0\times10^9} = \frac{2,0\times10^{-1}}{1.5\times10^{11}} \Rightarrow h_i = -\frac{7.0\times2.0\times10^8}{1.5\times10^{11}}$. It solves into $h_i = -0.93 \times 10^{-3}$m or -0.93 mm. Here, (-)sign is insignificant due to circular symmetry stated above. **Hence, answer is 0.93 mm.**
I-04	Cartesian sign convention is used. Power of a lens is $P = \frac{1}{f} = 5.0$ D (Diopter). Hence $f = \frac{1}{5} = 0.2$ m or 20 cm…(1), while magnification is $\frac{v}{u} = -\frac{h_i}{h_o} = 4$…(2). It is required to distance of object from the lens. It is assumed that object is placed left of the lens and hence value of u is (-)ve. Using lens formula with

the available data $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{-4u}-\frac{1}{-u}=\frac{1}{20}\Rightarrow\frac{1}{u}-\frac{1}{4u}=\frac{1}{20}\Rightarrow\frac{3}{4u}=\frac{1}{20}$. It leads to $u=15\ cm$. It implies that object is on right of the pole at a distance **15 cm is the answer.**

I-05

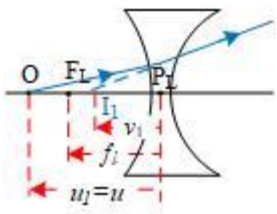
This problem is solved in two methods –

a. Conventional Geometrical Optics. In this two processes refraction through lens and reflection through mirror, both them placed at a distance. Accordingly, transformation of variables is involved. Moreover, use of Cartesian Sign convention requires differentiating between absolute values and signed values of the variables.
b. Using property of reversible traceability in geometrical optics. It involves better understanding of concepts of physics

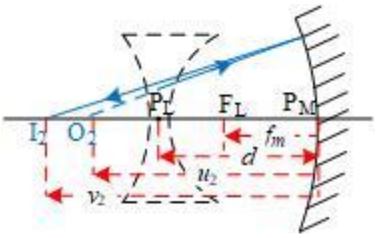


Method (a): The combination of lens and mirror creates optical phenomenon in three stages- **Stage 1:** Image formed by diverging lens, *Stage 2:* image of stage-1 acts as an object for reflection image by concave mirror, **Stage 3:** image of stage-2 act as an object to form refraction image by the lens. The problem is being solved stage wise, using Cartesian Sign convention, as under

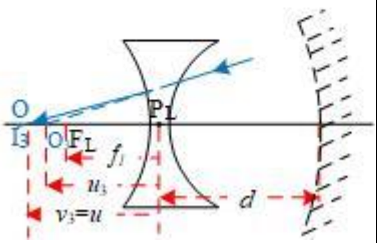
Stage1: Using lens formula with the available data $\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_l}$...(1). In respect of diverging lens all w.r.t. the pole P_L .It leads to $\frac{1}{v_1}=\frac{1}{-f_l}+\frac{1}{-u_1}$, here (-)sign is indicative of its direction.. Thus, $\frac{1}{v_1}=\frac{1}{-20}+\frac{1}{-u_1}$, or we have $v_1=-\frac{20u_1}{20+u_1}$ ...(2), here (-)sign is indicative of its direction, while absolute value of distance of imagew.r.t. P_L is $|v_1|=\frac{20u_1}{20+u_1}$.

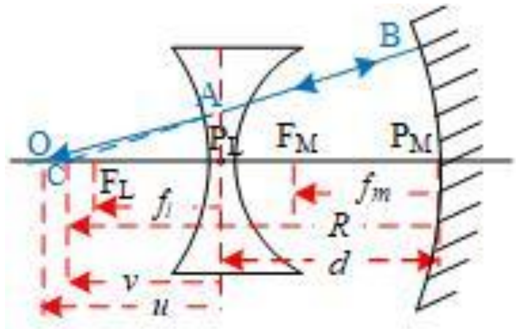
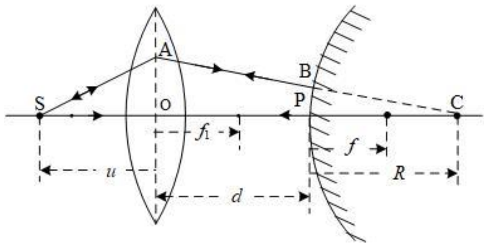
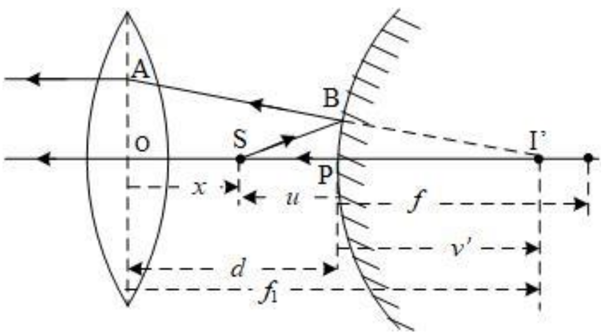


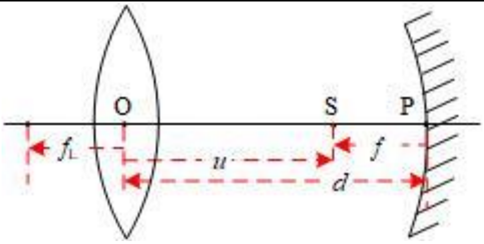
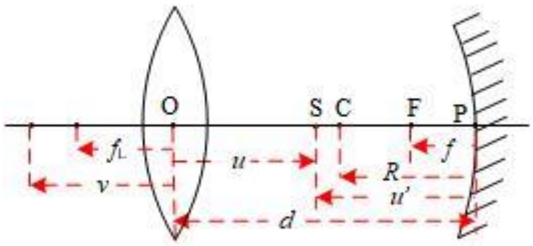
Stage 2: Image in stage 1 becomes object for stage 2, and all distances shall be considered w.r.t. pole of mirror P_M. Further, in this mirror formula shall be used as $\frac{1}{-v_2}+\frac{1}{-u_2}=\frac{1}{-f_m}\Rightarrow\frac{1}{v_2}=\frac{1}{f_m}-\frac{1}{u_2}$. Transforming all distances w.r.t. P_M. In this also all the distances are on left of the pole P_M and hence (-)ve. Thus, it leads to $u_2=d+|v_1|\Rightarrow u_2=5+\frac{20u_1}{20+u_1}=\frac{100+25u_1}{20+u_1}$. Thus, $\frac{1}{v_2}=\frac{1}{10}-\left(\frac{20+u_1}{100+25u_1}\right)=\frac{100+25u_1-200-10u_1}{10\times(100+25u_1)}=\frac{15u_1-100}{10\times(100+25u_1)}\Rightarrow\frac{1}{v_2}=\frac{3u_1-20}{50u_1+200}$. It leads to $v_2=\frac{50u_1+200}{3u_1-20}$is the absolute value of distance of image from P_M.

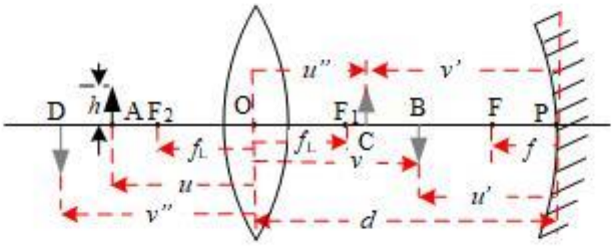
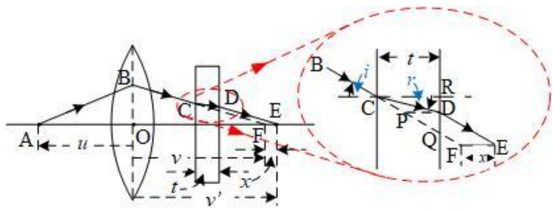


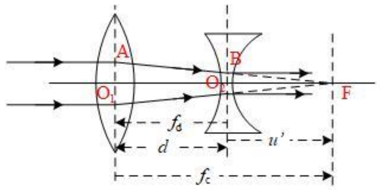
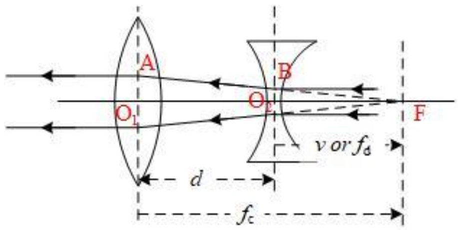
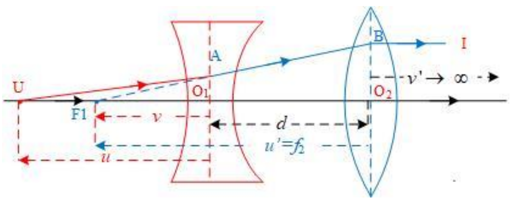
Stage 3: Again there is refraction through diverging lens, therefore distances are transformed w.r.t. where reference to P_L. Assuming that image by mirror I_2 is located at the left of the lens, as shown in the figure of stage-2, hence, physical distance $u_3=(v_2-d)=\frac{50u_1+200}{3u_1-20}-5$. Thus, we have $u_3=\frac{(100+65u_1)-5(3u_1-20)}{3u_1-20}=\frac{(100+65u_1)-(15u_1-100)}{3u_1-20}$. It leads to $u_3=\frac{35u_1+300}{3u_1-20}$. Again using (1) $\frac{1}{v_3}-\frac{1}{u_3}=\frac{1}{f_l}$ with the available we have $\frac{1}{v_3}-\frac{1}{-\frac{35u_1+300}{3u_1-20}}=\frac{1}{-20}\Rightarrow\frac{1}{v_3}=\frac{3u_1-20}{35u_1+300}-\frac{1}{20}$. It is required to determine position of object such that final image falls on itself i.e. $v_3=u_1=u$. Thus, $\frac{1}{-u}=\frac{-3u-20}{-35u+300}-\frac{1}{20}$. It leads to , $\frac{1}{u}=\frac{(60u+400)+(-35u+300)}{20\times(-35u+300)}\Rightarrow\frac{1}{u}=\frac{25u+700}{20\times(-35u+300)}\Rightarrow\frac{1}{u}=\frac{5u+140}{4\times(-35u+300)}\Rightarrow\frac{1}{u}=\frac{5u+140}{-140u+1200}$. It reduces to $5u^2+140u=-140u+1200$. Thus it leads to $u^2+56u-240=0$. It is a quadratic equation where we have $u=\frac{56\pm\sqrt{56^2-4\times1\times(-240)}}{2}$, or $u=\frac{56\pm\sqrt{4196}}{2}=\frac{56\pm64.8}{2}=60.4$ cm or as per SDs **60 cm, is the absolute value, is the answer**



	Method (b): As per principle of reversible traceability Ray from object after refraction if goes along AB then, a ray along BA after refraction would go along AO. Moreover, it is required to obtain position of object such that final image overlaps on the object. This is possible only if refracted ray AB after reflection, retraces it self along BA. This is possible only for radial ray in spherical mirror.  Accordingly, using principle of reversible traceability image of object after refraction shall be at C, the centre of curvature of the mirror. Thus problem is reduced to single refraction such that $v = R - d = 2 \times f_m - d = 2 \times 10 - 5 = 15$cm. Using lens formula we have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-15} - \frac{1}{u} = \frac{1}{-20} \Rightarrow \frac{1}{u} = \frac{1}{-15} - \frac{1}{-20} = -\left(\frac{1}{15} - \frac{1}{20}\right) \Rightarrow \frac{1}{u} = -\frac{1}{60}$, or $\boldsymbol{u} =$ **60 cm is the answer.** **N.B.: (1)** Solution of the problem is beautifully simplified by rightly appreciating the concept of physics going into it.. **(2)** Further, stage-wise solution of problem in conventional manner, though a bit long involves transformation of variables at every stage as reference point toggles $P_L \leftrightarrow P_M$ and associated Cartesian Sign Convention. **(3)** The conventional method is prone to error and hence wherever possible principle of reversible traceability should be used to make crisp and smart solution.
I-06	Reversible traceability of light wave during reflection and refraction is it property. Using the same, if ray SA incident on lens, after refraction is along AC, i.e. radial to the convex mirror, the reflected ray will reverse track along BA, a line segment of CA and on same principle, the refracted ray AS will converge at S, the object. As regards ray along principal axis AO through optical center will travel and reverse track after reflection. Thus both the rays would converge to form the image at S.  Thus, applying Lend formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. Here, we have to determine u, for a lens whose $f_1 = 12$ cm, and $v = d + R = d + 2f$, where distance between lend and mirror is $d = 5$ cm and $f = 7.5$ cm. Thus, we have $v = 5 + 2 \times 7.5 = 20$ cm. Using the available data $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{20} - \frac{1}{u} = \frac{1}{12} \Rightarrow \frac{1}{u} = -\left(\frac{1}{12} - \frac{1}{20}\right) = -\frac{1}{30}$, or $\boldsymbol{u = -30}$ **cm away from lens and on side opposite to the mirror, is the Answer.** **N.B.:** Solution of a problem, apparently complicated, has been simplified using reversible traceability property of light waves.
I-07	The given problem is illustrated in the figure. An converging lens of focal length $f_1 = 25$cm is separated from a diverging (convex) mirror by a distance $d = 15$cm. Focal length of the mirror is $f = 40$ cm.  It is required to determine position of the object between lens and the mirror i.e. u such that reflected ray BA, after refraction through the lens, is parallel to the principal axis, as shown in the figure. This can happen iff intermediate image I' of S formed by the mirror coincides with focal point of the lens. It implies that $f_1 = d + v' \Rightarrow v' = f - d$. Thus, using formula of mirror $\frac{1}{f} = \frac{1}{u} + \frac{1}{v'}$ with the

	available data we have $\frac{1}{40}=\frac{1}{u}+\frac{1}{25-15}\Rightarrow\frac{1}{u}=\frac{1}{40}-\frac{1}{10}\Rightarrow\frac{1}{u}=-\frac{3}{40}$ cm. Here (-)ve sign implies that $u=$ 13.13 cm left of P towards the lens. Therefore, geometrical distance of object from the lens is $x=d-u=15-13.13\Rightarrow \boldsymbol{u=1.67}$ **cm from the lens is the answer.**
I-08	The geometry of the system is shown in the figure. Given that convex (Converging) lens of focal length $f_L=15$ cm is at a distance $d=50$ cm from the converging (concave) mirror of focal length $f=10$ cm. A point source is placed between the lens and mirror at a distance $u=40$ cm from the lens. Thus distance of S from the mirror $u'=d-u\Rightarrow x=50-40=10$ cm. Thus $x=f=10$ cm, or S is at focal point of the mirror. Hence, reflected rays as per mirror formula will create image  at $\frac{1}{u\prime}+\frac{1}{v\prime}=\frac{1}{f}\Rightarrow\frac{1}{-10}+\frac{1}{v\prime}=\frac{1}{-10}$, here signed values of and are (-) as per Cartesian Sign Convention. This leads to $\frac{1}{v\prime}=0\Rightarrow v'=\infty$, or reflected rays are parallel to the principal axis. These reflected ray are incident from (+)ve direction and hence where $u_1=v'=\infty$, here, $f_1=-15$ cm . Using available data in the Lens formula $\frac{1}{f_1}=\frac{1}{v_1}-\frac{1}{u_1}\Rightarrow\frac{1}{-15}=\frac{1}{v_1}-\frac{1}{\infty}\Rightarrow v_1=-15$ cm. This (-)ve sign implies that the image would be formed by reflected rays at **15 cm on the left focal point of the lens.** The other image is formed exclusively by lens where, as per Cartesian Sign convention w.r.t. optical center of lens O is $u=+40$ cm and $f=-15$ cm. Thus as per lens formula $\frac{1}{f_1}=\frac{1}{v_2}-\frac{1}{u}\Rightarrow\frac{1}{-15}=\frac{1}{v_2}-\frac{1}{40}$. It leads to $\frac{1}{v_2}=\frac{1}{40}-\frac{1}{15}\Rightarrow\frac{1}{v_2}=\frac{3-8}{120}=-\frac{5}{120}\Rightarrow v_2=-24$. The (-)ve sign signifies that the **other image is 24 cm on the left of the lens.** **Hence answers are 15 cm and 24 cm on the left of the lens, away from the mirror.**
I-09	The system is shown in the figure where a converging (convex) lend of focal length $f_L=15$ and a converging (concave) mirror of focal length $f=10$ cm are placed at a distance $d=50$ cm with common principal axis. It is required to find position of object S such that image formed by lens and mirror are at same place. Position of image formed by lens is determined using formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$…(1)  But, image formed by mirror is in two stages: **Stage 1-** By reflection from concave mirror and **Stage 2-** image formed by lens of image in stage 1. Thus, if image formed by mirror is at any point other than S, leading to different value u', the two images cannot be coincident. Thus, primary requirement for two images to be at same place is S and S' coincide. This as per mirror formula $\frac{1}{v}+\frac{1}{u'}=\frac{1}{f}\Rightarrow\frac{1}{R}+\frac{1}{u'}=\frac{1}{f}\Rightarrow\frac{1}{2f}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{u'}=\frac{1}{f}-\frac{1}{2f}\Rightarrow\frac{1}{u'}=\frac{1}{2f}\Rightarrow\frac{1}{u'}=\frac{1}{R}$, here $u'=R=2f$ for a mirror. It implies that S and C are coincide. Accordingly, $u=d-u'\Rightarrow u=d-2f$. Thus it leads to $u=50-2\times 10\Rightarrow \boldsymbol{u=30}$ **cm from lens towards the mirror is the answer.** **N.B.:** In this problem numerical part is very little, all that drives to correct answer is inferences from logic of reflection and refraction and corresponding formula.

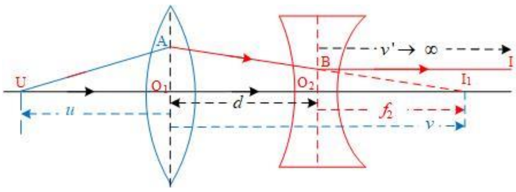
I-10	Given a system, a shown in the figure, of converging (convex) lens of focal length $f_L = 15$ and a converging (concave) mirror of focal length $f = 10$ cm. Both are placed at a distance $d = 50$ cm with common principal axis. It is required to find position of a pin of length $h = 2.0$ cm placed at A, such that $u = 30$ cm from lens, farther away from mirror, i.e. left of lens.  It is required to find location and size of final image. The final image will be formed in three stages: **Stage 1:** Image B by the lens as per formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$…(1) , in this focal point F_1 will come into play, In accordance to sign convention As per Cartesian sign convention $u = -30$ and $f_L = 15$. Thus position of image would be $\frac{1}{v} = \frac{1}{f_L} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-30} = \frac{1}{30}$., A real inverted image **Stage 2:** This image B is at a distance from the mirror $u' - d - v \Rightarrow u' = 50 - 30 = 20$ cm. This becomes object for image formed by mirror. It is seen that $u' = 20 = 2 \times f = 2 \times 10 = R$. Where, is radius of curvature of mirror. Accordingly, as per lens formula $\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f}$. As per sign convention, it leads to $\frac{1}{v\prime} + \frac{1}{-R} = \frac{1}{-f} \Rightarrow \frac{1}{v\prime} - \frac{1}{2f} = -\frac{1}{f} \Rightarrow \frac{1}{v'} = -\frac{1}{f} + \frac{1}{2f} \Rightarrow \frac{1}{v'} = -\frac{1}{2f} \Rightarrow \frac{1}{v'} = \frac{1}{-R}$. Thus B and C would coincide such that the image is real and inversion of the inverted image, i.e. erect image. **Stage 3:** Since, image formed in stage 2 coincides with image in stage 1, hence u"=d-v'$\Rightarrow u" = 50 - 20$ or $u" = 30$ cm. And $u"= v = u$. Therefore, based on the principle of reversible traceability on geometrical optics **final image would be at the point of object i.e. A, but inverted**. Further, as per magnification formula $\frac{h\prime}{h} = -\frac{v}{u} = -1$, hence **size of the final image will be same as that of the object.** **Thus answer is final image is at the object itself, of the same size.**
I-11	This problem involves solution in two stages and are illustrated below. **Stage 1:** It is determination of position of image using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$…(1) Here, as per sign convention $u = -30$ cm and $f = 15$ cm; in this case focal point on the right of optical centre O will come into play and $f = 15$ is positive. Accordingly, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-30} = \frac{1}{30} \Rightarrow v = 30$ cm.  **Stage 2:** A glass plate of thickness $t = 1$ cm would affect optical path, as shown in the inset in the figure, to cause a shift by FE a distance $+x$ in the image; this is being formulated. Geometrically, shift $x = \text{FE} = \text{PD}$. In ΔPDE, the angle ∠DPQ = i, and $\tan i = \frac{\text{DQ}}{\text{PD}} = \frac{\text{RQ}-\text{RD}}{x}$, it leads to $x = \frac{\text{RQ}-\text{RD}}{\tan i}$...(2). In ΔCRQ we have $\tan i = \frac{\text{RQ}}{\text{CR}} \Rightarrow \text{RQ} = t \cdot \tan i$...(3), in ΔCRD $\tan r = \frac{\text{RD}}{\text{CR}} \Rightarrow \text{RD} = \text{t} \cdot \tan r$ ...(4). Combining, (2), (3) and (4) we have $x = \frac{t\cdot \tan i - t\cdot \tan r}{\tan i} \Rightarrow x = \left(1 - \frac{\tan r}{\tan i}\right) t$. ..(6) The lens formula is approximation of $i \ll$ and consequent $r \ll$ and this leads to $\sin i \approx \tan i$ and likewise, $\sin r \approx \tan r$. Thus, (6) is transformed to $\frac{\tan r}{\tan i} \approx \frac{\sin r}{\sin i} = \frac{1}{\mu}$, here refractive index of glass plate is given to be $\mu = 1.5 = \frac{3}{2}$. Therefore, using (6) with the available data $x = \left(1 - \frac{2}{3}\right) \times 1 = 0.33$cm Combining results of stage 1 & 2, we have position of final image $v' = v + x \Rightarrow v' = 30 + 0.33 \Rightarrow x =$ **30.33 cm, is the answer from the lens towards the glass plate.**

I-12	There are two possibilities in the given system - **Case 1:** beam incident on the combination of lenses is first on convex (converging) lens having $f_c = 20$ cm, and **Case 2:** beam is incident on the combination of lenses is first on concave (diverging) lens having $f_d = 10$ cm. Both the cases, with beam travelling parallel to the principal axis of the combination of lenses, are illustrated separately. **Case 1:** radius of the incident beam $r = \mathrm{AO_1} = \frac{5.00}{2} = 2.50$ cm. We have lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ …(1).  In case of converging (convex) lens, the parallel beam has $u = \infty$ and $f = 20$ cm. Hence, $\frac{1}{v} - \frac{1}{\infty} = \frac{1}{20} \Rightarrow v = 20$, cm, i.e. at F. This, converging beam is intercepted by diverging (concave) lens of focal length $f_d = -10$ cm with respect to O_2. The lens is placed at a distance $d = 10$ cm from O_1. Thus, $u' = f_c - d = 20 - 10 \Rightarrow u' = 10$ cm. Again, using (1) again, $\frac{1}{v'} - \frac{1}{u'} = \frac{1}{f_d} \Rightarrow \frac{1}{v'} - \frac{1}{10} = \frac{1}{-10}$. It leads to $\frac{1}{v} = 0 \Rightarrow v = \infty$. Therefore, the emerging beam from the concave lens, is again parallel to the Principal axis. It is seen from figure that $\Delta \mathrm{AO_1Q}$ and $\Delta \mathrm{BO_2F}$ are similar and, therefore, $\frac{\mathrm{AO_1}}{\mathrm{BO_2}} = \frac{f_c}{u'}$. Using the available data, $\frac{2.5}{\mathrm{BO_2}} = \frac{20}{10}$, therefore, radius of the emerging beam is $BO_2 = \frac{2.50}{2} = 1.25$ **or diameter is** $= 2 \times 1.25 =$ **2.5 mm is the answer in this case 1.** **Case 2:** For simplicity the same conceptual arrangement as in case 1 has been used, by simply changing direction of rays reversed such that incident beam is first encountered by concave (diverging) lens, and accordingly, analysis is undertaken.  Using (1) on concave (diverging) lens having $f_d = 10$cm which intercepts incident cylindrical beam from right side i.e. $u = \infty$, we have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_d} \Rightarrow \frac{1}{v} - \frac{1}{\infty} = \frac{1}{10} \Rightarrow \frac{1}{v} - \frac{1}{\infty} = \frac{1}{10} \Rightarrow v = 10$cm. Thus image F acts virtual image for convex lens, for which, $u' = d + v = 10 + 10 = 20 = f_c$. Thus, intermediate virtual source at F, the focal point of convex lens, will create a parallel beam. It is seen from figure that $\Delta \mathrm{AO_1Q}$ and $\Delta \mathrm{BO_2F}$ are similar and, therefore, $\frac{\mathrm{AO_1}}{\mathrm{BO_2}} = \frac{f_c}{fd}$. Using the available data, $\frac{\mathrm{AO_1}}{2.5} = \frac{20}{10}$, therefore, radius of the emerging beam is $\mathrm{AO_1} = 2.5 \times 2 = 5.0$ **or diameter is** $= 2 \times 2.5 = 10$ mm or **1 cm is the answer in this case 2.** **Thus, answer is 1.0 cm if the light is incident from the side of concave lens and 2.5 mm if it is incident from the side of the convex lens.**
I-13	Given is a divergent (concave) lens of focal length $f_1 = 20$ cm and a converging (convex) lens of focal length $f_2 = 30$ cm are placed apart at a distance $d = 15$ cm. It is required to find location of object for image formed by the combination of lenses is at ∞. This problem, based on reversible traceability of light rays during refraction, can be treated as incident rays are parallel, position of image is effective focal point of the combination of lenses.  This problem has two cases: Case 1: Concave lens facing the object, and case 2: convex lens facing the object. **Case 1:** This case is solved in two stages the using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$…(1); **Stage 1:** image F_1, formed by concave lens facing the object U, and **Stage 2:** image F_1 in stage 1 acts as an intermediate object for convex lens, to form the final image at infinity.

Stage 1: As per (1), we have $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}\Rightarrow\frac{1}{u}=\frac{1}{v}-\frac{1}{f_1}$…(2) Here, for divergent lens $f_1=-20$ cm, as per Cartesian Sign Convention are w.r.t. optical center O_1, the lens under consideration.

Stage 2: Since it is required that the emergent ray from the lens-combination is parallel to principal axis, this is possible only if, F_1 is at focal point of convex lens having optical center O_2. Since lenses are separated by $d=15$ cm, hence geometrically $v=f_2-d\Rightarrow v=30-15\Rightarrow v=15$cm…(3). Combining (2) and (3), along with sign convention $\frac{1}{u}=\frac{1}{-15}-\frac{1}{-20}\Rightarrow\frac{1}{u}=\frac{1}{20}-\frac{1}{15}$, we have $\frac{1}{u}=\frac{3-4}{60}\Rightarrow\frac{1}{u}=-\frac{1}{60}$, i.e. **60 cm away from the diverging lens**.

Case 2: This case is again solved in two stages similar to that in case 1, where image I_1 is formed by convex lens facing the object U, in stage 1. In stage 2 stage the divergent (concave) lens intercepts the converging rays of stage 1. Since emergent rays are required to be parallel, leading to final image at infinity, this will happen only if I_1 coincides with the focal point of divergent lens.



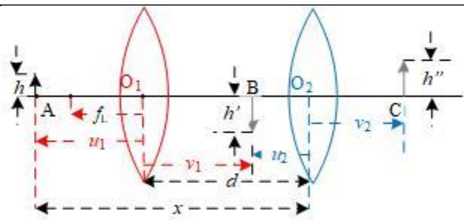
Stage 1: As per (1), we have $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}\Rightarrow\frac{1}{u}=\frac{1}{v}-\frac{1}{f_1}$ …(2). Here, $f_1=30$ cm is focal length of divergent lens and is (+)ve as per Cartesian Sign Convention are w.r.t. optical center O_1, the lens under consideration.

Stage 2: Since it is required that the emergent ray from the lens-combination is parallel to principal axis, this is possible only if, I_1 is at focal point of divergent (concave) lens having optical center O_2. As per Cartesian sign convention f_2 is (+)ve. Since lenses are separated by $d=15$ cm, hence geometrically $v=d+f_2\Rightarrow v=15+20\Rightarrow v=35$cm…(3). Combining (2) and (3), along with sign convention $\frac{1}{u}=\frac{1}{35}-\frac{1}{30}$. It leads to $\frac{1}{u}=\frac{1}{5}\times\left(\frac{1}{7}-\frac{1}{6}\right)\Rightarrow\frac{1}{u}=\frac{1}{5}\times\left(\frac{6-7}{42}\right)\Rightarrow\frac{1}{u}=-\frac{1}{210}\Rightarrow$ $\boldsymbol{u=-210}$**, is the answer**.

Thus answer is 60 cm from diverging lens or 210 cm from converging lens.

I-14

Given are two convex lenses of focal lengths $f_1=10$ cm and $f_2=5$ cm. An object of height $h=5$mm is placed in front of lens of f_1 at a distance $u=15$ cm. The two lenses are placed at a distance $d=40$ cm, as shown in the figure. Distance of lens f_2 from object at A is $x=55=u_1+d=40+15$ cm; this is however, a redundant information.



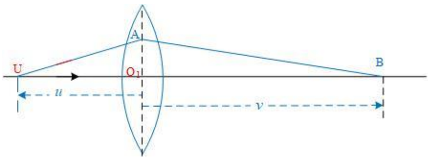
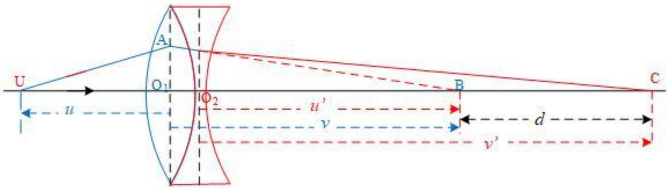
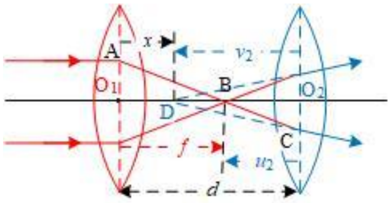
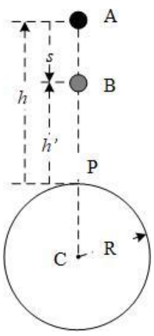
Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$…(1), position of image by lens f_1 is $\frac{1}{v_1}=\frac{1}{f_1}+\frac{1}{u_1}$. ..(2). Here, using Cartesian Sign Convention, for a converging lens focal length is (+)ve and but $u_1=-15$ cm, therefore, $\frac{1}{v_1}=\frac{1}{10}+\frac{1}{-15}\Rightarrow\frac{1}{v_1}=\frac{1}{30}\Rightarrow v_1=30$ cm at B. this as per magnification formula is $\frac{h'}{h}=\frac{v}{u}\Rightarrow h'=\frac{h\times v}{u}\Rightarrow h'=\frac{5\times30}{-15}=-10$ mm. The (-) sign indicated that it is real inverted image.

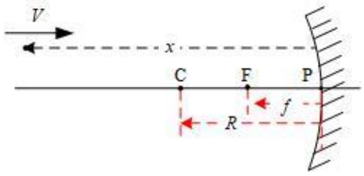
This image at B acts as object for lens at O_2. As per geometry $u_2=d-v_1$, and sign convention we have $u_2=d-40+30=-10$ cm. It is observed that magnitudes $u_2=2f_2$. In such condition again using (2) we have $\frac{1}{v_2}=\frac{1}{f_2}+\frac{1}{u_2}\Rightarrow\frac{1}{v_2}=\frac{1}{5}+\frac{1}{-10}\Rightarrow\frac{1}{v_2}=\frac{1}{10}$. Thus for final image $v_2=$ **10 cm at C i.e. from second lens is answer of Part (a).**

In this case again using magnification formula $h'=\frac{h'\times v_2}{u_2}\Rightarrow h''=\frac{-10\times10}{-10}\Rightarrow$ $\boldsymbol{h''}$**=10 cm, is size of final image is answer of part (c)**. Here, (+) sign indicates that the **final image is erect and real, answer of part (b).**

Size of image

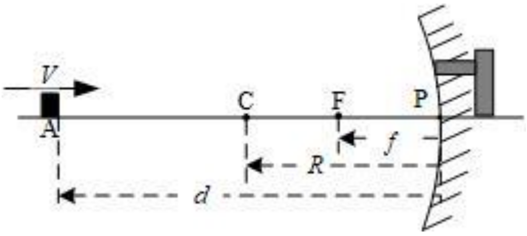
Thus answers are (a) 10 cm from the second lens further away, (b) erect and real (c) 10 mm.

I-15	Given that a convex (convergent) lens with optical center O_1, produces an image at a distance of $v = 30$ cm on the other side of an object placed at a distance of $u = -15$ cm as shown in the figure. Therefore, as per lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15}$, or $\frac{1}{f_1} = \frac{1}{30} + \frac{1}{15} = \frac{1}{10} \Rightarrow \boldsymbol{f_1 = 10}$ **cm, is one part of the answer.**  Now, a concave (divergent) lens of focal length f_2 is placed in contact with the convex lens as shown in the figure. Therefore, image at B will act as an intermediate image for formation of final image at C. It is stated that $d = 30$ cm. Here, it is assumed that both the lenses are thin and, therefore, distance between O_1 and O_2 is negligible, such that $v \approx u'$ and $v' = u' + d$.  Thus, with the given data $v' = 30 + 30$ cm. Again, applying lens formula, with the available data to concave lens $\frac{1}{f_2} = \frac{1}{v\prime} - \frac{1}{u'} \Rightarrow \frac{1}{f_2} = \frac{1}{60} - \frac{1}{30} \Rightarrow \frac{1}{f_2} = -\frac{1}{30} \Rightarrow \boldsymbol{f_2 = -30}$ **cm for concave lens**. Here, (-)ve sign to the focal length is indicative that the lens is divergent. **10 cm for convex lens and 60 cm for concave lens.**
I-16	Given that a pair of two identical convex (converging) lenses have focal length $f = +10$ cm are separated by $d = 15$ cm. A light beam parallel to the principal axis from left, as shown in figure would converge at B the focal point of the lens with optical center O_1.  Since, optical center of another lens is at O_2, the distance of B, is $BO_2 = d - f = 15 - 10 = 5$ cm. In accordance with sign convention $u_2 = -5$ cm. Applying lens formula $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} = \frac{1}{10} + \frac{1}{-5} \Rightarrow \frac{1}{v_2} = \frac{1}{10} - \frac{1}{5} \Rightarrow \frac{1}{v_2} = -\frac{1}{10} \Rightarrow v_2 = -10$ cm at D, with divergent rays emerging out of the lens system. **Thus part (a) is proved.** Position of final image at D above is virtual with $v_2 = -10$ cm. Thus its location is $x = d + v_2$, which using available data is $x = 15 + (-10) =$ **5 cm from the first lens towards the second lens**, **is the answer of part (b).** Focal length of the combination of a pair of lenses f_c is expressed as $\frac{1}{f_c} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$. In the instant case $f_1 = f_2 = f = 10$. Thus using the available data $\frac{1}{f_c} = \frac{1}{10} + \frac{1}{10} - \frac{15}{10\times10} \Rightarrow \frac{1}{f_c} = \frac{5}{100} \Rightarrow \boldsymbol{f_c = 20}$ **cm, is the answer of part (b).** **Thus answers are (a) Proved (b) 5 cm from the first lens towards the second lens (c) 20 cm**
I-17	Let initially when the ball is dropped $u = 0$, from a height h above a transparent sphere of transparent material having refractive index h. Here, the experiment is conducted on earth wherein fall of body is governed by equations of motion where acceleration is $a = -g$ and is radial along CA, which is perpendicular to the earth's surface, i.e. direction of motion under gravity.  Therefore, in time t the ball, as per second equation of motion, will fall through a height $s = ut + \frac{1}{2}(-g)t^2 \Rightarrow s = -\frac{gt^2}{2}$. Thus effective distance of the ball from the sphere, taken vectorially, is $h' = h + s \Rightarrow h' = h - \frac{gt^2}{2}$. ..(1). It is required form speed of ball as a function of time t such that $t < \sqrt{\frac{2h}{g}}$. This implies from (1) that until ball reached the surface of the sphere. As regards formation of image, it is an optical phenomenon, and determination of speed of image, as required, involves concepts of mechanics–cum-optics.

	Formation of image, in this case, is refraction through spherical surface where $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ …(2). In this case given that relative refractive index of material of sphere, in the medium surrounding the sphere, is $\mu=\frac{\mu_2}{\mu_1}$…(3). Further, $u=h'$ …(4) distance of the ball from pole P of the spherical surface as (+)ve, as per Cartesian Sign convention, the reference point for optics, and correspondingly v is the distance of image of the sphere at an instant t from P and R has (-)ve numerical value. Accordingly, combining (2) and (3) we have $\frac{\mu_2}{\mu_1}\times\frac{1}{v}-\frac{1}{u}=\frac{\frac{\mu_2}{\mu_1}-1}{R}\Rightarrow\frac{\mu}{v}=\left(\frac{1}{u}+\frac{\mu-1}{R}\right)$…(5). Equation (5) can be transformed into a form $v=\frac{\mu^2Ru}{\mu R+u(\mu-1)}$ …(6). Therefore, to determine speed of image differentiate (5) w.r.t. t we get $\frac{d}{dt}\left(\frac{\mu}{v}\right)=\frac{d}{dt}\left(\frac{1}{u}+\frac{\mu-1}{R}\right)$. Here, both u and v are functions of t, while $\frac{\mu-1}{\mu R}$ is a constant of the system. Thus $\mu\frac{d}{dv}\left(\frac{1}{v}\right)\left(\frac{dv}{dt}\right)=\frac{d}{du}\left(\frac{1}{u}\right)\left(\frac{dv}{dt}\right)$. This solves into $-\frac{\mu}{v^2}\left(\frac{dv}{dt}\right)=-\frac{1}{u^2}\left(\frac{du}{dt}\right)$. ..(7). Here ,speed of image is $V=\frac{d}{dt}v$ and thus $V=\frac{1}{\mu}\left(\frac{v}{u}\right)^2\times\frac{du}{dt}$….(8) Combining (1) and (4), $\frac{du}{dt}=\frac{d}{dt}h'=\frac{d}{dt}\left(h-\frac{gt^2}{2}\right)\Rightarrow\frac{du}{dt}=-\frac{g}{2}\frac{dt^2}{dt}=-\frac{g}{2}\times 2t\Rightarrow\frac{du}{dt}=-gt$…(9), the (-) Sign indicates the ball moving toward the center of the sphere. Combining (9) and (6) in (8) we have, $V=\frac{1}{\mu}\left(\frac{\frac{\mu Ru}{\mu R+u(\mu-1)}}{u}\right)^2\times(-gt)\Rightarrow V=-\frac{gt}{\mu}\times\frac{\mu^2R^2}{(\mu R+u(\mu-1))^2}$. It simplifies into $V=-\frac{\mu R^2gt}{(\mu R+u(\mu-1))^2}$. Here, (-) sign indicates that direction of image is towards the center of the sphere. **N.B.:** Here elaboration of concept of differential calculus is considered essential for those students who have not been sufficiently exposed to it, and keeping in consideration especially the students of bio-science. We regret our inability to illustrate in greater details, as it would shift focus of the illustration from physics to mathematics.
I-18	Given system is depicted in the figure. Distance of object from concave mirror x is large; C is the center of curvature of the mirror having radius R and F is the focal point of the mirror such that $R=2f$. As per Cartesian Sign convention both x and R are (-)ve w.r.t. pole P, and hence f is also (-).  Position of image as per formula of spherical mirrors is $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$. As per Cartesian Sign convention w.r.t. pole P, $u=-x$ and $f=-\frac{R}{2}$. Hence, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$. Using the available data $\frac{1}{v}=\frac{1}{-\frac{R}{2}}-\frac{1}{-x}\Rightarrow\frac{1}{v}=\frac{1}{x}-\frac{2}{R}$…(1). Distance of image can be also written as $v=\left(\frac{Rx}{R-2x}\right)$ ...(2). Here, it is given that the object is moving along the principal axis with a constant speed $V=\frac{dx}{dt}$. Therefore, speed of the image is $\frac{d}{dt}v$ is obtained by differentiating both sides of (1) w.r.t. t. Accordingly, $\frac{d}{dt}\left(\frac{1}{v}\right)=\frac{d}{dt}\left(\frac{1}{x}-\frac{2}{R}\right)\Rightarrow\frac{d}{dv}\left(\frac{1}{v}\right)\times\frac{dv}{dt}=\frac{d}{dx}\left(\frac{1}{x}\right)\times\frac{dx}{dt}\Rightarrow\left(-\frac{1}{v^2}\right)\times\frac{dv}{dt}=\left(-\frac{1}{x^2}\right)\times V\Rightarrow\frac{dv}{dt}=v^2\frac{V}{x^2}$…(3). Thus with (2), $\frac{dv}{dt}=v^2\frac{V}{x^2}\Rightarrow\frac{dv}{dt}=\left(\frac{Rx}{R-2x}\right)^2\frac{V}{x^2}\Rightarrow\frac{dv}{dt}=\frac{R^2V}{(R-2x)^2}$ or $\boldsymbol{\frac{R^2V}{(2x-R)^2}}$ **is the answer**. **N.B.:** Here elaboration of concept of differential calculus is considered essential for those students who have not been sufficiently exposed to it, and keeping in consideration especially the students of bio-science. We regret our inability to illustrate in greater details, as it would shift focus of the illustration from physics to mathematics.

I-19

The given system is depicted in figure the block of mass m at A, at a distance $-d$, from the mirror of radius R w.r.t. its pole P is moving towards the mirror with a velocity V . The block makes elastic collision with the mirror with its stand of mass m, at rest on the table. Eventually, on applying principle of conservation of momentum and conservation of energy to system of the block and the mirror, after collision the block will come to rest while, mirror will be set into motion with velocity V, while the block will come to rest on the table.

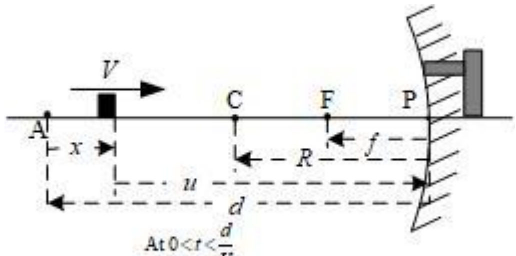


Time at which block will reach mirror and collide with it is $t = \frac{d}{V}$. It is required to find velocity of the image: **(a)** pre-collision i.e. at a time $t < \frac{d}{V}$ and **(b)** post collision i.e. at a time $t < \frac{d}{V}$.

As per mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, here focal length f is constant and distance of object from pole of mirror. Thus, distance of image from mirror v is $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{u-f}{uf} \Rightarrow v = \frac{uf}{u-f}$…(1). Therefore, $\frac{d}{dt}\left(\frac{1}{v}\right) = \frac{d}{dt}\left(\frac{1}{f} - \frac{1}{u}\right) \Rightarrow \frac{d}{dv}\left(\frac{1}{v}\right) \times \frac{dv}{dt} = -\frac{d}{du}\left(\frac{1}{u}\right) \times \frac{du}{dt}$. It leads to $\left(-\frac{1}{v^2}\right) \times \frac{dv}{dt} = -\left(-\frac{1}{u^2}\right) \times \frac{du}{dt}$. Thus, we have $\frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \frac{du}{dt} \Rightarrow \frac{dv}{dt} = -\left(\frac{uf}{u-f} \times \frac{1}{u}\right)^2 \frac{du}{dt}$. It solves into $\frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2 \frac{du}{dt}$…(2)

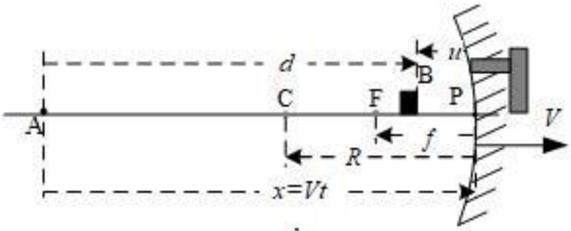
Reference point for distances in case of mirrors is its pole. But, in the instant case pole is static during $0 < t < \frac{d}{V}$ i.e. pre-collision and during post collision it is in motion with constant velocity for $t > \frac{d}{V}$.

Case (a): $0 < t < \frac{d}{V}$: In this case since P is static and hence as per normal convention it is taken as reference point, and accordingly $u = -d + x$…(3) and $\frac{du}{dt} = \frac{d(Vt)}{dt} = V$ …(4) while $f = -\frac{R}{2}$…(5). Using (3), (4) and (5) in (2) velocity of image w.r.t. P is $\frac{dv}{dt} = -\left(\frac{-\frac{R}{2}}{(x-d)-\left(-\frac{R}{2}\right)}\right)^2 V$. It simplifies into $\frac{dv}{dt} = -\left(\frac{R}{2(d-vt)-R}\right)^2 V$ **is the answer of part (a).**



At $0<t<\frac{d}{V}$

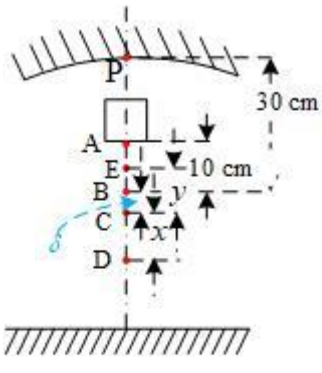
Case (b): $t > \frac{d}{V}$: In this case since P is moving with mirror and hence A, starting point of the block, is taken as reference point. Accordingly, velocity of the image is arrived at by taking $u = (x - d) \Rightarrow Vt - d$…(6), while focus w.r.t. P is at $f = -\frac{R}{2}$ as per (5) Accordingly, using (2), (5) and (6) $\frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2 V \Rightarrow \frac{dv}{dt} = -\left(\frac{-\frac{R}{2}}{(Vt-d)-\left(-\frac{R}{2}\right)}\right)^2 V$. It resolves into the velocity of image w.r.t. A as $\frac{dv}{dt} = -\frac{R^2}{[2(d-Vt)-R]^2}V$ Therefore, absolute velocity of image is $V_{Img} = V + \frac{dv}{dt} \Rightarrow V_{Img} = V - \frac{R^2}{[2(d-Vt)-R]^2}V \Rightarrow \boldsymbol{V_{Img} = V\left[1 - \frac{R^2}{[2(d-Vt)-R]^2}\right]}$ **is the answer of part (a).**



At $t > \frac{d}{V}$

N.B.: **(a)** This problem involves principles of optics together with principle of collision and relative motion as well.

(b) Velocity of image is absolute and accordingly in Part (b) P is in motion with the mirror and hence point A which is static in both the cases is taken as reference. Since in part (a) both point A and P are static and hence P is taken as reference as per convention.

I-20	 Let bullet be of mass m receding with velocity V, and the gun is of mass M. After the bullet is fired the gun acquires velocityV_g. The gun-bullet system is at rest at $t = 0^-$ i.e. pre-firing and post firing at $t \geq 0^+$, as per conservation of momentum velocities of the system shall be $M \times 0 + m \times 0 = mV + MV_g \Rightarrow V_g = -\frac{m}{M}V$ i.e. in a direction opposite to the bullet. Therefore, velocity of the bullet w.r.t. to gun vis-à-vis mirror fitted on the gun as shown in the mirror is $V_r = V - V_g$, it implies that $V_r = V\left(1 - \left(-\frac{m}{M}\right)\right) = \left(1 + \frac{m}{M}\right)V \Rightarrow V_r = \frac{M+m}{M}Vt$;, Therefore, separation of bullet at any time $t > 0$ from the mirror is $u = V_r \times t \Rightarrow u = \frac{M+m}{M}Vt$. ..(1) As per mirror formula, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$…(2). Hence, $v = \frac{uf}{u-f}$…(3) Here, u is distance of image from the mirror, f is focal length of the concave (converging) mirror and v is distance of image of bullet from the mirror. Here, both u and f are (+)ve as per Cartesian Sign Convention are (+)ve . Here, v will take appropriate sign determined by (3). Thus, separation of bullet and its image will be $x = u - v$…(4). It is required in the question to determine velocity of separation of bullet and its image, just after bullet is fired i.e. $t \to 0^+$. Therefore, distance of bullet from the mirror from (1) would be $u = \left(\frac{M+m}{M}V \times t\right) \Rightarrow u \to 0^+$. For this infinitesimally small value of u, from (3) , we have $v = -uf \times (f-u)^{-1}\vert_{u\to 0^+}$...(5). This is a problem of limit involving expansion $(f-u)^{-1}\vert_{u\to 0^+} = [f^{-1} + (-1)f^{-2}u^{-1} \ldots.]u \to 0^+$. This is an infinite converging series where excluding first terms all other terms are negligible. Accordingly, it solves into $(f-u)^{-1}\vert_{u\to 0^+} = f^{-1}$…(6). Combining, (5) and (6),$v = -uf \times f^{-1} \Rightarrow v = -u$ …(7). Combining (4) and (7) $x = u - (-u) = 2u$. Therefore, velocity of separation is of bullet w.r.t. its image is $V_S = \frac{dx}{dt} = \frac{d}{dt}2u \Rightarrow V_S = 2\frac{d}{dt}\left(\frac{M+m}{M}Vt\right) \Rightarrow V_S = 2\left(\frac{M+m}{M}V\right)\frac{d}{dt}t \Rightarrow \boldsymbol{V_S = 2\left(\frac{M+m}{M}V\right)}$**, is the answer**. **N.B.:** It is essential to carefully note last part of the question "…just after the gun was fired". This statement turns this problem of opto-mechanics (Optics and Mechanics) into a problem involving application of concepts of Limits in mathematics. If the nuance is missed problem becomes more complicated and in turn leads to a longer solution. Placing limits as $t \to 0^+$ will eventually reduce the result the one obtained above. But, appreciating application of limit **in initial stage itself** on equation (3) will avoid all complications and thus possibilities of error. This solution very simple and crisp, yet it appears to be lengthy because of illustration of associated concepts for a clear understanding.
I-21	 This problem has Four parts which lead to length in which image of mass oscillates. Cartesian sign convention is used taking pole of mirror P, as shown in the figure, as a reference. **Part a:** Mass is dropped from A at a height $h = 10$ cm above B, the free end of the spring. Mean position of the mass C when it sticks on spring, and is determined by equilibrium of forces on the mass. It is subjected to two forces $F_g = m(-g)$ and Restraining force of the spring $F_s = k\delta$, here $g = 0{,}05$ kg is mass of the block under consideration, $g = 10$ m/s^2 is acceleration due to gravity, x is compression length if the spring and $k = 500$ N/m is spring constant. At equilibrium $F_g + F_s = 0 \Rightarrow -mg + (k-)\delta = 0 \Rightarrow \delta = -\frac{mg}{k} \Rightarrow \delta = -\frac{0.05\times 10}{500} = (-)1 \times 10^{-3}$m or (-)0.1 cm compression. Thus mean position B from the pole of the mirrors is PC = PB + BC = 30 + 0.1 = 30.1 cm. **Part b:** Maximum displacement of mass(x) from its equilibrium C, determined in part (a), to D is governed by *principle of conservation of energy* (PCE). Let, loss of potential of the mass dropped from a state of rest at A until it reaches D, the maximum compressed position of the spring is $\Delta PE_m = mg(h + \delta + x)$. This energy is absorbed by the spring change as $\Delta PE_m = \frac{1}{2}k(\delta + x)^2$. Thus as per PCE $\Delta PE_m = \Delta PE_m \Rightarrow mg\big(h + (\delta + x)\big) = \frac{1}{2}k(\delta + x)^2$. Let $z = \delta + x$ and accordingly, $kz^2 - 2mgz - 2mgh = 0$ is a quadratic equation. Using the available data we have $500 \times z^2 - 2 \times (50 \times 10^{-3}) \times 10 \times z - 2 \times (50 \times 10^{-3}) \times 10 \times 0.10 = 0$. It solves into

$500 \times z^2 - z - 0.1 = 0$. Thus, $z = \frac{1 \pm \sqrt{1-4\times500\times(-0.1)}}{2\times500} \Rightarrow z = 0.001 \times \left(1 \pm \sqrt{201}\right)$. It solves into $z = 0.001 \times (1 \pm 14{,}1)$. Thus possible values are $z = 0.015$ m and $z = (-)0.013$ m. Since compression cannot be negative acceptable value is $z = 1.5$ cm.

Thus maximum displacement from equilibrium position C to D is $\alpha = z - \delta \Rightarrow \alpha = 1.5 - 0.1 = 1.4$ cm.

Part c: At D, KE of the mass reduces to ZERO, but restraining force is maximum and against direction of displacement. This will set the mass into oscillation about its equilibrium position C, where its KE will be maximum. After reaching D, the spring starts recoiling during which it acquires maximum velocity at C and comes to rest at position of maximum displacement E. This looks like a SHM, but really not since the motion of mass is biased by gravitational force F_g which is constant and unidirectional i.e. always uniformly directed towards the ground unlike SHM for which basic requirements are **(i)** restraining force at any position of displacement is always directed towards equilibrium position, also called mean position, and **(ii)** the restraining force is proportional to its displacement from mean position.

But, the point of consideration is that for displacement of $\delta = 1$ mm from B to C, the equilibrium position having a displacement $F_g \leq F_s$, and sooner the displacement of mass getting attached to free end of spring is $z > \delta$ the F_s increasingly predominates F_g. Thus $\theta = \frac{\delta}{z} = \frac{1}{15} \Rightarrow \theta \approx 7\%$ of maximum displacement.

Therefore, despite this motion not being ideally SHM, is approximated to SHM, within limits if observation, the essence of Physics, involved in the problem.

Thus mass is deemed to oscillate with SHM between E to D over a length$2\alpha = 2 \times 1.4 = 2.8$ cm.

Part d: Since length in which image of the mass oscillates, value of u corresponding to the position D $u_1 = d + z \Rightarrow u_1 = 30 + 1.5 = 31.5$ cm, and to the position E is $u_2 = d + \delta - \alpha$. This leads to $u_2 = 30 + 0.1 - 1.4 \Rightarrow u_2 = 28.7$ cm.

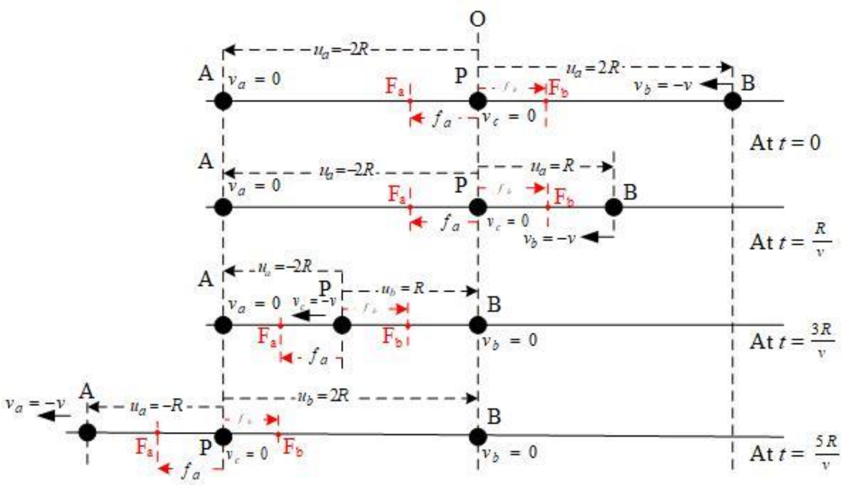
Position of image as per lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$. Here as per Sign convention, taking P as a reference, both the values of u are negative and so also $f = 12$ is also negative.

Thus, corresponding position of images are $\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u_1} \Rightarrow \frac{1}{v_1} = \frac{1}{-12} - \frac{1}{-31.5} \Rightarrow \frac{1}{v_1} = \frac{1}{31.5} - \frac{1}{12}$, or $v_1 = \frac{12\times31.5}{12-31.5} = -19.4$ cm and $\frac{1}{v_2} = \frac{1}{f} - \frac{1}{u_2} \Rightarrow \frac{1}{v_2} = \frac{1}{-12} - \frac{1}{-28.7} \Rightarrow \frac{1}{v_2} = \frac{1}{28.7} - \frac{1}{12} \Rightarrow v_2 = \frac{12\times28.7}{12-28.7}$, or $v_2 = -20.6$. Thus length in which image oscillates is $l = |v_2 - v_1|$ is an absolute value and accordingly $l = |-20.6 - (-19.4)| \Rightarrow \boldsymbol{l = 1.2}$ **cm is the answer.**

N.B.: *Discussion at (c) above make this problem to be one of the finest where application of physics, approximation, rounding of numbers and significant digits are involved.* **.** It reminds that in physics concepts find application in an interdisciplinary, manner.

I-22

This problem involves principle of collision for two collisions, one at $t_1 = \frac{2R}{v}$ between B and C, at the other at $t_2 = \frac{4R}{v}$ between A and C. Accordingly, position of the two objects at three given instance $t_a = \frac{R}{v}$, $t_b = \frac{3R}{v}$ and $t_c = \frac{5R}{v}$ would determine position of their images. Analysis simplified with conceptual representation on line diagrams for each case as under –



Case (a) at $t = 0$: This is the base case for reference of analysis of the cases below. The analysis of position of images of A and B formed by two concave mirrors facing them would be made using mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow v = \frac{fu}{u-f}$, for which and would be assigned signs as per Cartesian Sign convention w.r.t. O, the original position of mirror-stand system, and accordingly distances are marked in the figure for each instance and tabulated below. Here, collisions are stated to be elastic and hence principle of conservation of momentum across bodies negotiating momentum during collision having same mass m. Thus, at each stage of collision, colliding body comes to a state of rest and collided body is set into motion with velocity $(-v)$, as per sign convention. Initially, in each case position of image is determined w.r.t. P, and then position of image w.r.t. O, as desired is obtained, and shown in the table below –

Case	**Time stamp**	**For A**				**For B**			
		Image with respect to (w.r.t.) P			**Image w.r.t. O (x)**	**Image with respect to (w.r.t.) P**			**Image w.r.t. O (x)**
		$\boldsymbol{u_a}$	$\boldsymbol{f_a}$	$\boldsymbol{v_a = \frac{f_a u_a}{u_a - f_a}}$	**OP** $\boldsymbol{+ v_a}$	$\boldsymbol{u_b}$	$\boldsymbol{f_b}$	$\boldsymbol{v_a = \frac{f_b u_b}{u_b - f_b}}$	**OP** $\boldsymbol{+ v_b}$
(a)	$t = \frac{R}{v}$	$-2R$	$-\frac{R}{2}$	$\frac{\left(-\frac{R}{2}\right)(-2R)}{(-2R) - \left(-\frac{R}{2}\right)} = -\frac{2R}{3}$	$x = \frac{2R}{3}$	R	$\frac{R}{2}$	$\frac{\left(\frac{R}{2}\right)(R)}{(R) - \left(\frac{R}{2}\right)} = R$	R
(b)	$\boldsymbol{t} = \frac{3R}{v}$	$-R$	$-\frac{R}{2}$	$\frac{\left(-\frac{R}{2}\right)(-R)}{(-R) - \left(-\frac{R}{2}\right)} = -R$	$x = (-R) + (-R) = -2R$	R	$\frac{R}{2}$	$\frac{\left(\frac{R}{2}\right)(R)}{(R) - \left(\frac{R}{2}\right)} = R$	$-R + R = 0$
(c)	$t = \frac{5R}{v}$	$-R$	$-\frac{R}{2}$	$\frac{\left(-\frac{R}{2}\right)(-R)}{(-R) - \left(-\frac{R}{2}\right)} = -R$	$x = (-2R) + (-R) = -3R$	$2R$	$\frac{R}{2}$	$\frac{\left(\frac{R}{2}\right)(2R)}{(2R) - \left(\frac{R}{2}\right)} = \frac{2R}{3}$	$x = -2R + \frac{2R}{3} = -\frac{4R}{3}$

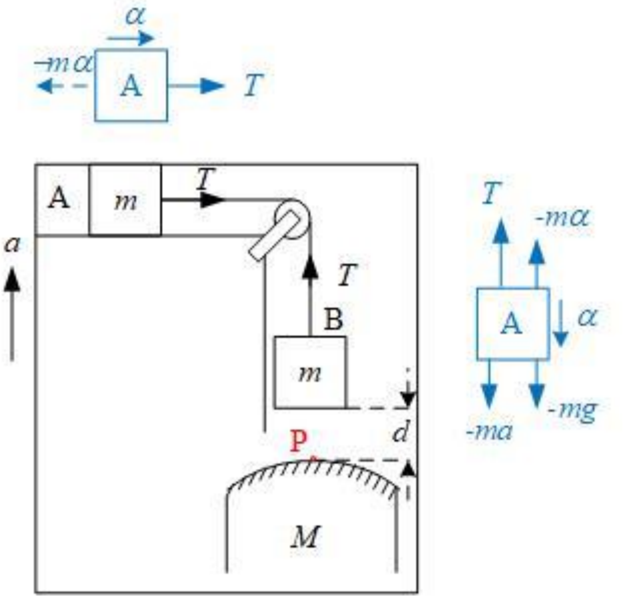
N.B.:1. It is a good example of change of position of reference point pole of mirrors P. But, position of image is required to be determined w.r.t. O, position of mirror-stand system at $t = 0$.
2. Mirror formula is valid for distances w.r.t pole of the mirror,
3. Mirrors system is considered to be thin such that poles of both the mirrors coincide at P.at.
4.It reminds that in physics concepts find application in an interdisciplinary, manner.

I-23

Given system is illustrated in the figure here with tension in the string, acceleration of masses and free body diagram of the masses. Mass B due gravitational force $F_g = -mg$ and pseudo force due to acceleration of the lift $F_s = -ma$. Thus tension in the string when mass B is in the lift is in dynamic equilibrium of with a constant acceleration α downward is, $T - (g + a - \alpha) = 0$. It leads to $T = m(g + a - \alpha)$…(1).

Likewise, for mass A, only horizontal forces will account for with constant acceleration α towards the pulley, such that $T - m\alpha = 0 \Rightarrow T = m\alpha$…(2).

In this set of (1) and (2) only T and α are unknown. Accordingly, $m(-\alpha) = m(-g + a + \alpha) \Rightarrow 2\alpha = -g - a \Rightarrow \alpha = \frac{-g-a}{2}$ Using the available data $\alpha = \frac{-10-2}{2} \Rightarrow \alpha = -6$ m/s^2.



Therefore distance travelled by the block in given time $t = 0.2$ s, Cartesian sign convention is adopted for all distances, velocities and acceleration starting from state of rest, as per Second Equation of Motion is $s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2} \times (-6) \times 0.2^2 = -0.12$ m or $(-)12$ cm.

Given that initial distance of block is $d = 42$ cm, while object travels downward $s = -12$. Therefore, at $t = 0.2$ s distance of block from mirror is $u = d + s = 42 + (-12) \Rightarrow u = 30\ cm$. Therefore distance of image from the mirror as per lens formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \Rightarrow v = \frac{fu}{u-f}$. In case of convex mirror with given data $f = -12$ cm, accordingly, $v = \frac{-12\times30}{30-(-12)} \Rightarrow v = -8.57$cm.

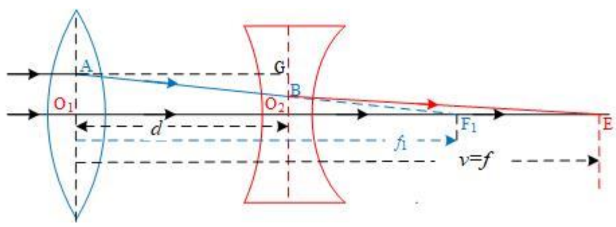
N.B.: (1) This problem is though a simple application of mechanics. But it reminds of the fact that as we move towards real-life problems, application of concepts of physics become more integrated and involved. Thus it is training of mind to think inclusively rather than exclusive to topic under consideration.

(2) Cartesian sign convention is used with each of the distance, velocity and acceleration.

APPENDIX

Focal Length of Combination of Two Lenses

Given is a divergent (concave) lens of focal length $f_1$and a converging (convex) lens of focal length $f_2$are placed apart at a distance d. It is required to find location of object for image formed by the combination of lenses is at ∞. This problem, based on reversible traceability of light rays during refraction, can be treated as incident rays are parallel, position of image is effective focal point of the combination of lenses.



This problem is solved in two stages the using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$…(1); **Stage 1:** image formed by convex lens facing the object, and **Stage 2:** image in stage 1 acts as an intermediate object for concave lens, to form the final image.

Stage 1: As per (1)**,** we have $\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{\infty} + \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{f_1} \Rightarrow v_1 = f_1$…(2) In this all distances are as per Cartesian Sign Convention are w.r.t. optical center O_1, the lens under consideration.

Stage 2: In stage image formed in stage 1 acts as object. But, in this stage reference point for distances as per sign convention is O_2, the optical center of the lens under consideration. Accordingly, using (2) we have $u_2 = v_1 - d \Rightarrow u_2 = f_1 - d$…(3). Applying (1), $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{u_2} + \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{f_1-d} + \frac{1}{f_2}$…(3). Since this is final image at E is formed by lens-combination and hence maintaining reference point as O_1, distance of final image is $v = f$. Here, f is the effective focal length of the lens-combination. Thus, $v_2 = v - d = f - d$ …(4). Combining (3) and (4) we have $\frac{1}{f-d} = \frac{1}{f_1-d} + \frac{1}{f_2} \Rightarrow \frac{1}{f-d} = \frac{f_2+(f_1-d)}{(f_1-d)f_2} \Rightarrow f - d = \frac{f_1f_2-f_2d}{f_2+f_1-d}$. This algebraic expression is further resolved into $f = d + \frac{f_1f_2-f_2d}{f_2+f_1-d} \Rightarrow f = \frac{f_2d+f_1d-d^2+f_1f_2-f_2d}{f_2+f_1-d}$. This leads to $f = \frac{f_1f_2+f_1d-d^2}{f_2+f_1-d}$…(5). Taking its reciprocal, to bring the equivalent focal length in format of (1) we have $\frac{1}{f} = \frac{f_1+f_2-d}{f_1f_2+f_1d-d^2}$…(6). In case the $d \ll f_1$, $d \ll f_2$ then $f_2d \ll f_1f_2$ and $d^2 \ll f_1f_2$, and thus $(f_1f_2 + f_1d - d^2) \rightarrow f_1f_2$. Accordingly, (6) leads to $\frac{1}{f} \approx \frac{f_2+f_1-d}{f_1f_2} \Rightarrow \frac{1}{f} \approx \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1f_2}$…(7). The formula (7) is generally used.

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