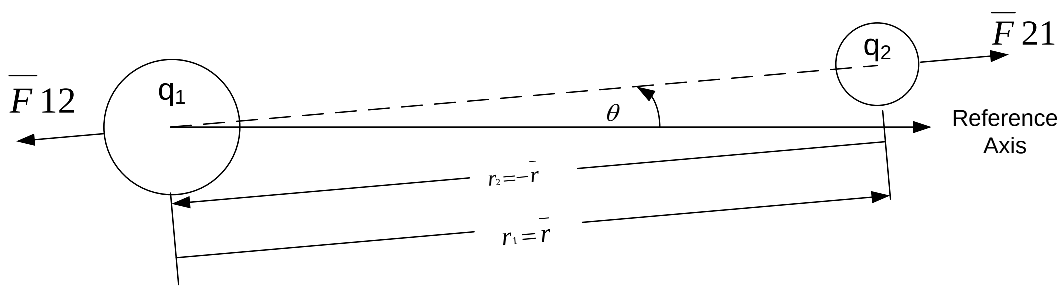
ELETROSTATICS –Part I: BASICS

Presence of **Electric Charge** was first demonstrated by an ancient Greek mathematician Thales of Miletus around 600 BC, by rubbing fur on various substances, such as amber. Thereafter, many scientists continued their discoveries. In 1784, French physicist Charles Augustin de Coulomb published a **law of force** ($\bar{F}$: Vector) **between electric charges** which is mathematically stated as :

$\bar{F} = k_e \frac{q_1 q_2}{r^2}\hat{r} = \frac{q_1 q_2}{4\pi\epsilon_0 r^3}\bar{r}$; a generalized expression, where q_1 and q_2 are two charges at a displacement $\bar{r} = r\hat{r}$ and k_e is the proportionality constant= 8.99×10^9 N m^2 C^{-2} in MKS, while in SI, it is $\frac{1}{4\pi\varepsilon_0}$; where $\varepsilon_0 = 8.85419 \times 10^{-12}$ C^2 m^{-2} N^{-1}. While, in specific case as shown in the figure below force experienced by each charge on account of the other charge is:

$$\bar{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}(-\hat{r}) \text{ =and } \bar{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}(\hat{r})$$; Here, $\hat{r}$ is the Unit vector, while $\bar{r}$ is the radial vector



The charges are of two types +ve and –ve. When Both charges are of same type, either +ve or –ve , then the force between the two charges is repulsive, while in case the two charges are of opposite types then the force is attractive.

Charges are generally bound to opposite charges, but they can exist in isolation and in that case *a charge shall exert a force on any charge around it, until infinity where the force becomes infinitesimally small* $\rightarrow 0$. Presence of this force is since dependent upon the other charge, and it has been rationalised as **Electric field** i.e. Force by an isolated charge, a signed quantity, on another unit +ve charge at a displacement $\bar{r}$:.

$$\bar{E} = \frac{\bar{F}}{q_2} = \frac{q_1}{4\pi\epsilon_0 r^2}\hat{r} \ or \ \frac{q}{4\pi\epsilon_0 r^2}\hat{r}.$$

Whenever, there is force acting on any object, be it a charge, work is required to be done on it in moving against the direction of force. In electrostatics, the amount of work is called ***change of electric potential***. Accordingly, the concept of ***Electric Potential at a point*** becomes essential. This requires remembering concepts of work and energy in mechanics. Accordingly, this concept of electric potential can be better understood when analysed in context of mechanics i.e. gravitational potential and potential energy of a spring. Before, going into analysis of potential energy dimensional analysis of ε_0 is considered essential.

DIMENSIONAL ANALYSIS and UNIT of ε_0: In mechanics dimension of Force= $[MLT^{-2}]$; while dimension of radius $= [L]$, while in electrostatics electrical charge has Coulomb (C) as its unit, while its dimension is derived from definition of current (rate of flow of charge per unit time =$\frac{C}{sec}$) whose effect can be measured. Accordingly $[I]$ is included in fundamental dimensions and thus dimension of charge is $[IT]$. Using these constituent dimensions in expression of electrostatic force:

$[MLT^{-2}] = \frac{[IT]^2}{\varepsilon_0[L^2]}$; here, 4 and π are constants and dimensionless.

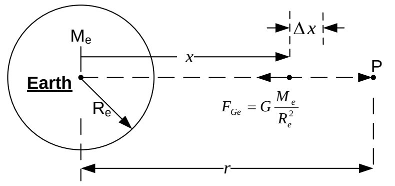
Accordingly, $\varepsilon_0 = \frac{[I^2T^2]}{[MLT^{-2}][L^2]} = \frac{[I^2T^2]}{[ML^3T^{-2}]} = [M^{-1}L^{-3}T^4I^2]$.

Likewise, unit of ε_0 is derived as under: $\varepsilon_0 = \frac{q^2}{4\pi\times F\times r^2}$, thus unit of ε_0 is $\frac{C^2}{Nm^2}$ or $C^2N^{-1}m^{-2}$.

Getting back to concept of Potential Energy in electrostatics is built over that of mechanics as under -

An Integrated View of Field and Potential Energy in Mechanics and Electrostatics

Gravitational Potential



Potential: Work done in moving a unit mass from Earth's surface, against the gravitational pull, up to a point P at a radial distance r :

$$= \sum \Delta w = \int_{R_e}^{r} \left(G \frac{M_e}{x^2} (-\hat{x}) \right) \cdot d\bar{x}$$

$$= GM_e \left[\frac{1}{x} \right]_{R_e}^{r} = GM_e \left[\frac{1}{r} - \frac{1}{R_e} \right]$$

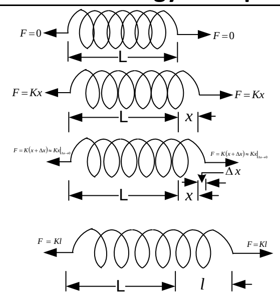
$$= \frac{GM_e}{r} \Big|_{taking, \frac{GM_e}{R_e} = 0}$$

Here, gravitational force, is in direction $-\hat{x}$; while displacement Δx is also in direction $\hat{x}$.

Further, in above derivation, it is assumed that Potential at Earth's surface is Zero and hence the PE at point P, calculated above can be called as relative Potential or Difference in Potential w.r.t. Earth's surface.

The moment mass of the object being moved is considered, other than unity, it becomes **Potential Energy**.

Potential Energy of a spring



Potential Energy: When spring is stretched/ compressed by length x it requires a force in the direction of push/pull= kx.

An incremental pull/push over an infinitesimal length Δx would call upon external work, stored in the form of energy in the system:

$$\Delta W = -k\bar{x} \cdot \Delta\bar{x}$$

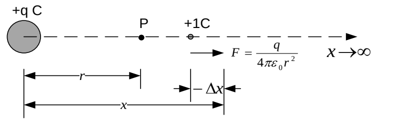
$$W = PE = \int_{x=0}^{l} -k\bar{x} \cdot d\bar{x}$$

$$= -\frac{1}{2} k [x^2]_0^l = -\frac{1}{2} kl^2$$

This is the absolute Potential Energy of the spring when stretched by length *l*. When *l=0,* its PE=0.

It is to be noted that in the above derivation of **Potential Energy** of spring of primary parameter is spring constant k which control restraining force, which is being overcome by external force to cause displacement without acceleration, and there is no role of mass in it.

Electric Potential



Potential: In electrostatic field Force of repulsion → 0 as $x \to \infty$. Hence, it is to appropriate to take electric potential at a point (potential energy of Unit charge) at a distance → ∞ to be = 0.

Accordingly, to determine potential at a point it is logical to move a Unit charge from infinity to the point P. Thus:

$$\Delta W = \bar{F} \cdot (-\Delta\bar{x}) = -\frac{q}{4\pi\varepsilon_0 x^2} \hat{x} \cdot \Delta\bar{x}$$

Potential at P $= \int_{\infty}^{r} -\frac{q}{4\pi\varepsilon_0 x^2} \hat{x} \cdot d\bar{x}$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{x} \right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$

From the above it is concluded that the calculated quantity is absolute potential at Point P, and once potential at any other point due to same charge is known, it is possible to find Potential Difference between the Two points under consideration.

The moment charge being moved is considered, other than unity, it becomes **Potential Energy**.

Note: According to basics of mechanics, $W = \bar{F}.\bar{X} = FX\cos\theta$; where θ is angle between vectors $\bar{F}$ (Force) and $\bar{X}$ (Displacement), while W is scalar, a result of . (dot) product. This concept, together with foundation mathematics, shall be applicable in the analysis. Simple formulation of potential seen above and inverse nature of integration and differentiation is used to determine field at a point in a complex situation involving space.

Gravitational Field: Accordingly, the Field $= \frac{d}{dr}\mathrm{PE} = \frac{d}{dr}\left(\frac{GM}{r}\right) = -\frac{GM}{r^2}\hat{r}$.

Conclusions: **(a)** It is to be noted that PE is scalar, while Field is vector, since it is being differentiated w.r.t. r which is a vector. **(b)** (-ve) sign indicates that direction of Field is opposite to $\hat{r}$, unit displacement vector.

Force: Since there is no role of mass in the formulation, and hence force is considered, instead of Field, in case of Gravitational and Electrical phenomenon. Applying differentiation on PE, we get force to keep spring stretched at any length l

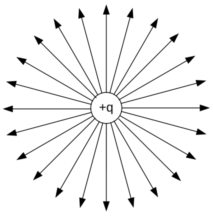
$$= \frac{d}{dl} PE = \frac{d}{dl} \left(\frac{1}{2} kl^2 \right) = kl\,\hat{l}$$

Electric Field: In this case also the field

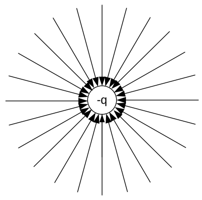
$$= \frac{d}{dr} \left(\frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r} \right) = -\frac{q}{4\pi\varepsilon_0 r^2} \hat{r}$$

Conclusion: **(a)** (-ve) sign indicates that direction of Field is opposite to $\hat{r}$, unit displacement vector. **(b)** Other aspects are similar to that in Gravitational Field and Spring Force.

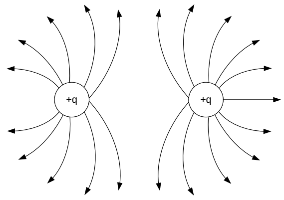
Lines of Electric Force: Graphical representation of electric field, in a Region due to system of charges, is smooth lines or curves such that tangent at every point of it represents direction of Electric Field; these lines are purely imaginary. Shapes of lines of electric force, also called ***Electric Flux,*** for typical arrangement of charges, are shown below.



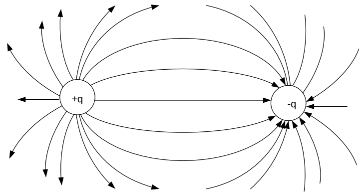
Isolated +ve Charge



Isolated –ve Charge



Two Equal Positive Charges

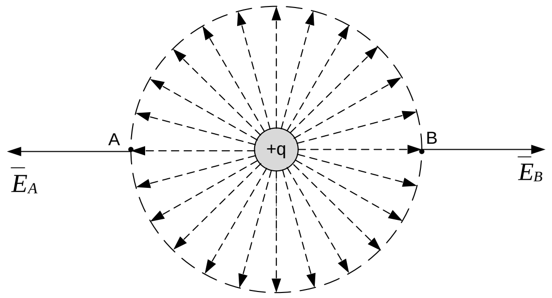


A +ve and –ve Charge

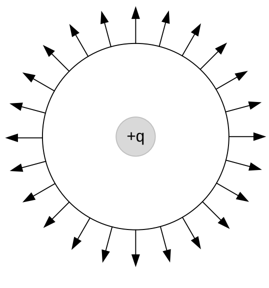
It is observed, in these figures above, that Electric Lines of Force are smooth curves, while electric field at every point is a vector, which can be represented by a tangential straight line, having corresponding magnitude and direction, and is discontinuous and discrete to any point in its neighbourhood.

Thus, considering a +ve charge, total Electric Field at any point on a Spherical surface of radius *r* with a concentric charge +q is $\frac{q}{4\pi\epsilon_0 r^2}\hat{r}$, as shown in the figure below. Accordingly, Electric Flux out of this surface at two diametrically opposite points, though in opposite directions, as shown in the figure below , does not equate to Zero Flux rather is a summated flux,and is analogues liquid flowing out of a balloon when pressed. This notion was seen by, **Carl Friedrich Gauss** differently and he analysed component of Electric Field ($\bar{E}$) across a surface, taking surface as a vector having direction perpendicular to it. In order to determine overall conclusion of the concept, it calls upon use of Dot Product of vectors ($\bar{E}\cdot\overline{\Delta A}$), Electric Field ($\bar{E}$) at a point and an infinitesimal Area ($\overline{\Delta A}$) having uniform $\bar{E}$ across it; for the purpose of simplicity this is conceptualized for a circle in the figure below; it can safely extrapolated for a sphere in Three Dimension. Thus, the case under consideration leads to mathematical formulation which relates net electric flux out of a closed surface to the charge enclosed within it.

$$\oint \bar{E}.\overline{dA} = \oint \left(\frac{q}{4\pi\epsilon_0 r^2}\hat{r}\right)\cdot dA\bar{r} = \oint\left(\left(\frac{q}{4\pi\epsilon_0 r^2}\right)\cdot dA\right)\cos 0 = \oint\left(\left(\frac{q}{4\pi\epsilon_0 r^2}\right)\cdot dA\right) = \left(\frac{q}{4\pi\epsilon_0 r^2}\right)\oint dA = \frac{q}{4\pi\epsilon_0 r^2}A = \frac{q(4\pi r^2)}{4\pi\epsilon_0 r^2} = \frac{q}{\epsilon_0}$$



Electric Field at Equidistant Diametrically Opposite Points



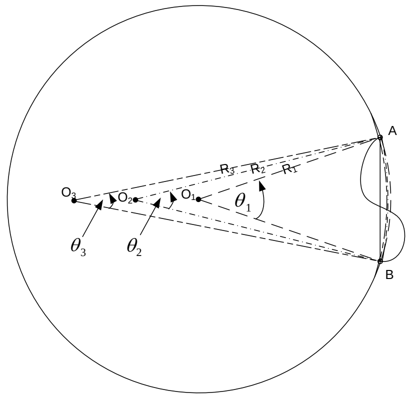
Electric Field at Perimeter of a Circle by Charge at Centre

These findings of Gauss in 1835 took 32 years for him to publish it **in the form of a law** which states that: *The flux of the net electric field through a closed surface equals to net charge enclosed by the surface divided by* ϵ_0 . This law has made a big contribution in understanding of Electric Field and its application, perhaps in these Three-Plus decades, he must spent efforts in verifying possibility of contradictions, if any, to his simplification.

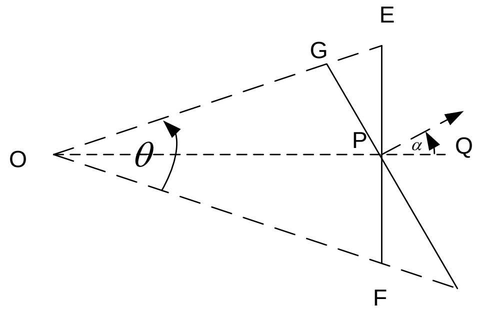
At this point another term Electric Flux Density (D) is introduced. Electric Flux from a charge Q is equal to the charge itself. Thus, it leads to a general expression where, ϵ is permittivity of the medium and specific to vacuum or air, and it is $\oint \bar{D}.d\bar{A} = q = \epsilon\left(\frac{q}{\epsilon}\right) = \epsilon\oint \bar{E}.\overline{dA} = \epsilon\oint(\epsilon\bar{E}).\overline{dA}$. Accordingly, relationship between Electric Field Density and Electric Field Intensity is $\bar{D} = \epsilon\bar{E}$, and is used extensively in higher studies.

SOLID ANGLE: Application of Gauss's Law to different geometrical distribution of Electric Charges involves three dimensional space, and requires understanding of the concept of ***Solid Angle,*** an extension of the concept of ***Plane Angles***, in Plane Geometry, formed by a line defined at a point under consideration is illustrated in Figures below, called ***Angle***. While, in case of Solid Geometry, angle formed by perimeter of an area at a point under consideration which requires consideration of the area into its infinitesimal elements, with touching boundary which on integration forms the given area, is illustrated in the figure below.

Angle $\theta = \frac{\text{Length of Circular Arc, joining Two Points Between which Angle is to be Calculated, having Centre at which Angle is formed}}{\text{Radius of the Arc at which Angle is to be calculated}}$ in Radians (π radians = 180^0), and is self-explanatory in respect of angle formed by points A and B, at O_1, O_2, O_3 and accordingly, radius for each point and corresponding arc is considered. θ , being ratio of two lengths is dimensionless. For radius O_1A, arc is outermost; for radius O_2A, it is the middle arc and for O_3A, it is the inner most arc adjacent to line AB; it is observed that as radius increases, the curvature of corresponding arc reduces and is consistent with the principles of mathematics. But, in case of points E and F, and G and H on a Point O, which are lying on radial OE and extended radial OF respectively, length of the arc, having radius OE with a chord EG is considered.



Angles Formed by Lines Joining Points A and B at Points O_1, O_2 and O_3

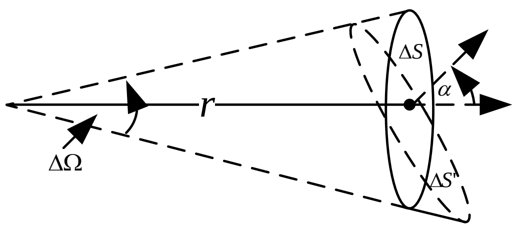


Angles Formed at Point O by Lines AB and AC at Radials OA and O-B-C

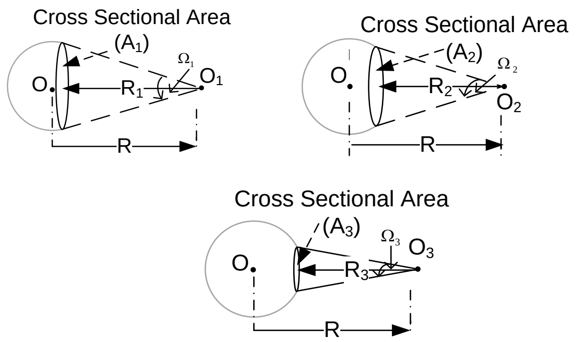
Formation of Angle by a Line on a Point (Concept of Plane Geometry)

While, Sold Angle $(\Omega) = \frac{[\text{Area}]\times\cos\alpha}{[\text{Perpedicular distance of the area from the Point at which Solid anhle is to be calculated}]^2}$; here, α is the angle between area under consideration and the radial. Unit of Ω is radians, while it is dimensionless. Here, In case of an opening in closed areas A_1, A_2 and A_3 the radial and perpendicular to the area are collinear having $\alpha = 0$ or $\cos\alpha = 1$. Accordingly, Solid Angles in the illustrations are:

$\Omega = \frac{\Delta S}{r^2} = \frac{\Delta S' \cdot \cos\alpha}{r^2}$; $\Omega_1 = \frac{A_1}{R_1^2}$; $\Omega_2 = \frac{A_2}{R_2^2}$; and $\Omega_3 = \frac{A_3}{R_3^2}$



Solid Angle Of Areas Enclosed Between Common Conical Surface



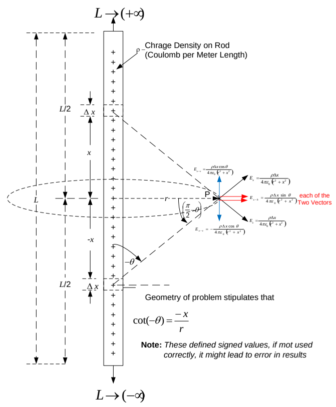
Solid Angles Formed By Slit an Opening In A Closed Surface At A Point Equidistant From the Closed Surface

Solid Angles Formed By An Area At a Point Outside It

With these basic concepts, mathematical representation of Gauss's law is:

$\frac{q}{\epsilon_0} = \oint \bar{E} \cdot \overline{dS}$; where $\oint$ represents integral over a closed surface; while $\bar{E} \cdot \overline{dS}$ represents dot product of vectors Electric field ($\bar{E}$) at a point and $\overline{dS}$ is the vector of infinitesimal area whose direction is perpendicular to the area. Thus, Gauss's law is an integral form of Coulomb's law. The only constraint in application of the Gauss's law is that the closed surface does not pass through the charge. In case surface being considered in Gauss's law passes through charge, $r \to 0$ and consequently as per Coulomb's law $\bar{E} \to \infty$ making the integral indeterminate. A typical case of linear charge distribution is taken for determination of Electric field at a point located at a distance r from the axis.

Electric Field as Per Coulomb's Law



N.B.: May please zoom for a clear view

Electric Field (Vector) is in horizontal direction $(\hat{r})$ having magnitude:

$$E_p = \int_{-\infty}^{+\infty} E_{x-h}\, dx = \int_{-\infty}^{+\infty} \frac{\rho \sin\theta}{4\pi\epsilon_0 (r^2 + x^2)} dx = \frac{\rho}{4\pi\epsilon_0} \int_{-\infty}^{+\infty} \frac{\sin\theta}{(r^2 + x^2)} dx$$

Let, $-x = r\cot(-\theta)$; $dx = r\,\mathrm{cosec}^2\,\theta d\theta$, and using limit as $x \to -\infty$ then $\theta \to 0$. Likewise, as $x \to \infty$ then $\theta \to -\pi$. It is to be noted that angle in anti-clock-wise direction is +ve and clock-wise direction is –ve (Sign convention). Here, for the purpose of angle reference axis is taken same as that of the line charge.

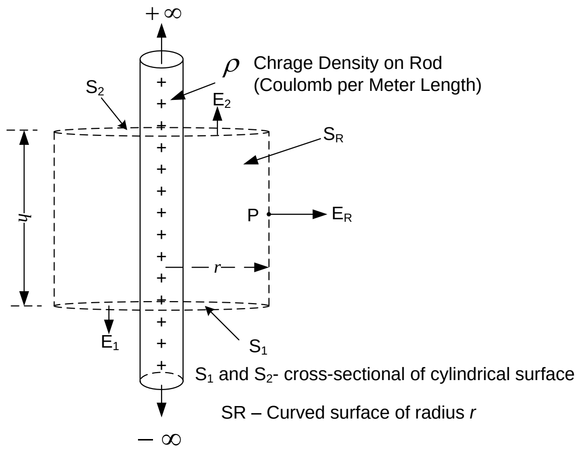
Accordingly, using property of Definite Integral for an even function:

$$E_p = \frac{\rho}{4\pi\epsilon_0} \int_0^{-\pi} \frac{\sin\theta}{r^2 + r^2\cot^2\theta} r\,\mathrm{cosec}^2\,\theta d\theta$$

$$= \frac{\rho}{4\pi\epsilon_0} \int_0^{-\pi} \frac{\sin\theta}{r^2(1+\cot^2\theta)} r\,\mathrm{cosec}^2\,\theta d\theta = \frac{\rho}{4\pi\epsilon_0 r} \int_0^{-\pi} \sin\theta d\theta$$

$$= -\frac{\rho}{4\pi\epsilon_0 r} [\cos\theta]_0^{-\pi} = -\frac{\rho}{4\pi\epsilon_0 r}[\cos(-\pi) - \cos 0] = \frac{\rho}{2\pi\epsilon_0 r}$$

Electric Field as per Gauss's Law



Dot (·) product in Gauss's Law provides for Component of Electric Flux orthogonal to the surface under consideration. The cylindrical surface, has enclosed within a conductor having charge density ρ Coulomb per meter, has cylindrical curved surface and two crosses-sectional plane surfaces. Therefore, according to Gauss's Law:

$$\frac{\rho h}{\epsilon_0} = \oint \bar{E} \cdot \overline{ds} = \int E_1 \cdot ds_1 + \int E_2 \cdot ds_2 + \int E_R \cdot ds_R$$

Uniform geometry creates a radial field across cylindrical surface and across cross-sectional surfaces $E_1 = E_2 = 0$ and hence,

$$\frac{\rho h}{\epsilon_0} = \int E_R \cdot ds_R = E_r(2\pi r);$$

$$E_p = E_r = \frac{\rho h}{2\pi r \epsilon_0}$$

These results conform to those obtained for Electric Field at point 'P' using Coulomb's Law.

Convertibility of Electric Potential into Electric Field: $V_r = \int_{\infty}^{r} \bar{E}.d\bar{x}$; conversely, $\frac{d}{d\bar{x}} V_r = \bar{E}_x$ for a radial field, which is created by a concentric charge. This logic can be extended to determine field in space. Accordingly, in Cartesian coordinates (3D) :

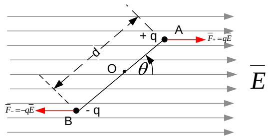
$\frac{d}{d\bar{x}} V = -\bar{E}_x$; $\frac{d}{d\bar{y}} V = -\bar{E}_y$; $\frac{d}{d\bar{z}} V = -\bar{E}_z$; where, $\bar{E} = \bar{E}_x + \bar{E}_y + \bar{E}_z$ and $\Delta V = \frac{\partial}{\partial \bar{x}} V \cdot \Delta\bar{x} + \frac{\partial}{\partial \bar{y}} V \cdot \Delta\bar{y} + \frac{\partial}{\partial \bar{z}} V \cdot \Delta\bar{z}$. This in 2D plane where along $\hat{z}$, V is constant ($\frac{\partial}{\partial \bar{z}} V = 0$) , this reduces to - $\Delta V = \frac{\partial}{\partial \bar{x}} V \cdot \Delta\bar{x} + \frac{\partial}{\partial \bar{y}} V \cdot \Delta\bar{y}$.

This equation in polar coordinates is : $\Delta V = \frac{\partial}{\partial \bar{r}} V \cdot \Delta\bar{r} + \frac{\partial}{\partial \bar{\theta}} V \cdot (r\Delta\bar{\theta}) = -\bar{E}_r \cdot \Delta\bar{r} - \bar{E}_\theta \cdot (r\Delta\bar{\theta})$.

Thus in the form of partial derivatives (where while considering variation of one parameter other is taken to be constant): $\frac{\partial}{\partial \bar{r}} V = -\bar{E}_r$, and $\frac{\partial}{\partial \bar{\theta}} V = -r\bar{E}_\theta$.

In polar coordinates, $r\Delta\bar{\theta}$ is used instead of $\Delta\bar{\theta}$, unlike Cartesian coordinates, since $\Delta\bar{\theta}$ is only direction while $r\Delta\bar{\theta}$ is displacement.

This concept is best applied in case of Electric Dipole, which is a pair of equal and opposite charges displaced through a small distance.



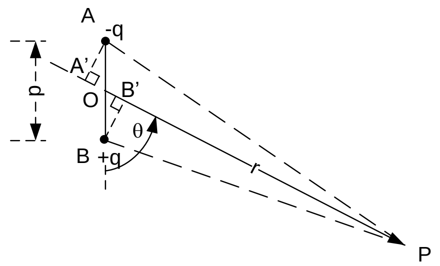
In the dipole shown below, $\bar{p} = q\bar{d}$, here, $\bar{d}$ is a vector joining –ve to +ve charge and $\bar{p}$ is called dipole moment. And this dipole in a uniform electric field, in x-y plane, will experience a torque:

$$\hat{\tau} = \overline{OA} \times \bar{F}_+ + \overline{OB} \times \bar{F}_- = \overline{OA} \times q\bar{E} + \overline{OB} \times \left(q(-\bar{E})\right) = 2q(\overline{OA} \times \bar{E}) = -(qd\,\mathrm{E}\sin\theta)\hat{z} = -(pE\sin\theta)\,\hat{z}$$

This is in line with principles of mechanics.

Taking a case where a point is asymmetrically placed with respect to the two charges of Dipole, as shown in the figure below, and potential and field is to be determined.

Electric Potential at a Point P by a Dipole



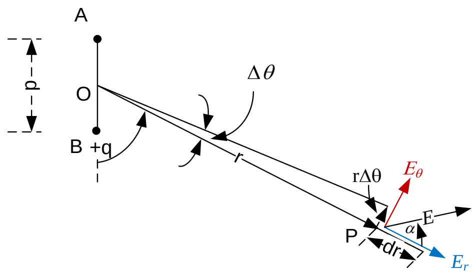
OP line joining centre of O and Point P at a distance r, which makes an angle θ with the line of dipole, charges separated by a distance d, is extended backwards and perpendiculars AA' and BB' are drawn on it. By geometry, $A'P = OP + A'O = r + \frac{d}{2}\cos\theta$, and $B'P = OP - OB' = r - \frac{d}{2}\cos\theta$. Since $d << r$ and hence $AP \approx A'P = r + \frac{d}{2}\cos\theta$ and $BP \approx A'P = r + \frac{d}{2}\cos\theta$

Therefore, Potential at Point P is:

$$V = \frac{q}{4\pi\epsilon_0\left(r - \frac{d}{2}\cos\theta\right)} - \frac{q}{4\pi\epsilon_0\left(r + \frac{d}{2}\cos\theta\right)} = \frac{1}{4\pi\epsilon_0}\left(\frac{qd\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\right) \approx \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}.$$

Here, p is the dipole moment. In case, P is symmetrically placed where AP=BP, $\theta = \frac{\pi}{2}$, and $\cos\theta = 0$, makes V=0; this result is consistent with the definition of potential symmetrically placed w.r.t two equal and opposite charges would cancel out.

Electric Field at a Point P by a Dipole



Having determined Potential at a point P, electric field can be determined using partial derivatives in system of polar coordinates elaborated above, using its two components along $\hat{r}$ and $\Delta\hat{\theta}$, as under-

$$\bar{E}_r = \left(-\frac{dV}{dr}\right) = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}\text{; and } \bar{E}_\theta = \frac{1}{r}\left(-\frac{dV}{d\theta}\right) = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

Accordingly, resultant field at point P is :

$$\bar{E} = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}\sqrt{1 + 3\cos^2\theta}$$

An angle of $\bar{E}$ w.r.t. radial OP is: $\alpha = \tan^{-1}\left(\frac{\bar{E}_\theta}{\bar{E}_r}\right) = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$.

Note: It is essential to remember sign conventions, and use it correctly, in absence of which error would creep in mathematical derivation.

Assignment: This basic definition of Electric Field and Electric Potential can be extended to any system charges in any configuration, using above concepts and are covered in text books of physics either as illustration, solved/unsolved examples. Some of typical configurations are listed below as assignment:

a) Discrete charges at

(i) Vertices of an equilateral triangle,

(ii) Vertices of a square

b) Uniformly distributed charge on a -

(i) Straight thin conductor **(already covered in illustration, above)**

(ii) Circular ring conductor

(iii) Hollow sphere conductor

(iv) Solid sphere dielectric

c) Pair of opposite electric charges (Dipole)

Understanding of the concept, alone will not help in making it intuitive, unless it is followed with extensive problem solving.

References:

1. *NCERT; PHYSICS, Text Book for Class XI and XII (Part I and II),*
2. *H.C. Verma; Concepts of Physics, (Vil 1 & 2).*
3. *Resnick, Halliday, Resnick and Krane; Physics (Vol I and II)*
4. *Sears & Zemansky; University Physics with Modern Physics.*