ELECTROMAGNETISM – Part II: Magnetic Effects of Current

Learning of electromagnetism is an excellent example of spiral growth of knowledge starting at observation, which in turn invokes experience; exploration of the experience leads to discovery and innovation. Every fresh discovery creates a new set of observations and the cycles has grown like a chain reaction. Basic idea behind this manual is not to reach out readers with a fresh set of discoveries, but to make learning exciting through making the subject matter contextual to human observations and experience. In this back drop this set of chapters, constituting Mentors' Manual would be found different from most of the text books and reference books, and is believed to be considered helpful in igniting fire of learning and exploration.

Discovery of magnetism in ancient times was linked to current electricity by Hans Christian Ørsted in 1819 when he accidently discovered sudden trembling of compass needle near a current carrying wire. This event was an opening of a gateway to Electromagnetism. Series of discoveries by Ampere, Biot, Savart. Faraday and Maxwell, complemented efforts by adding new dimensions to the understanding of the Electromagnetism. Though discovery of magnetism is ancient but, magnetism and electricity are not only inseparable but interactive in nature. The only difference is that in electricity (+)ve and (-)ve electric charges can exist in isolation but in magnetism two poles always exist in pair called dipole. An increasing understanding of atomic structure and association of magnetic field to electric current revolutionized the theory of magnetism leading to electromagnetism. Nevertheless, elaboration of the concepts has been sequenced considering their interdependency.

During development of this text, few figures had to be reduced in size to facilitate space management while placing them along the relevant elaborations. Readers if find that details in figure are not readable, they are requested to zoom it, an advantage with e-Manual.

Magnetism: It was discovered in ancient times based on stones called Lodestone found in the region Magnesia, which could attract iron. These stones were called Magnetite, signifying the place where it was found, and the property to attract iron pieces was called Magnetism. While Aristotle was the First to engage in discussion on magnetism, while around the same era, Sushruta, an ancient Indian surgeon is stated to have

used magnets for surgical purposes. It is around 11th century, Shen Kuo, a Chinese scientist described use of magnetic needle for navigation. It was believed that magnetic property of certain material was associated with tiny magnetic particles, called dipoles, lying in scattered manner, as





shown in the figure, such that net magnetic effect is cancelled. It is only under influence of magnetic field that these dipoles get aligned whereby $-N \rightarrow$ the material starts behaving like magnets. A magnetic piece when

suspended, takes a free position such that its one part is in north direction and is called **North Pole**, and other part in south direction is called **South Pole**. This orientation of the magnetic piece is independent of place of suspension as long as it is in free state.

If two magnets with their North Poles so identified, when brought together, they repel each other, and so also South Poles. But, when North Pole and south Poles are brought together they attract each other. This combination in a formation of closed chain, as shown in the figure above, magnetic property of the magnetic material disappears. Nevertheless, if such material is rubbed with a magnet, repetitively, these dipoles start getting oriented and the magnetic material starts demonstrating magnetic properties. It may be observed in the figure, that in the state of total magnetization -a all dipoles are aligned in parallel open chains, b) interface of North Pole and South Pole of dipoles does not exhibit magnetization, which is experienced only at

ends; One end is North Pole, while the other is South Pole, **c)** Point where magnet leaves contact of the material to be magnetized becomes North Pole; this is attributed to attraction of north poles of the dipoles which aligns them in the direction of motion of



magnet, **d**) at ends of the solid magnetized bar, repulsion of like poles of dipoles causes slight dispersion, **e**) North Pole and South Poles cannot exist in isolation, they exist only in pairs and is unlike electric charges. Since, all materials are not magnetic, and hence this experience occurs only with materials which are inherently magnetic. Elaboration of magnetic property of material and its reasons shall be elaborated separately. Nevertheless, inquisitive readers are welcome to write through <u>Contact Us</u>.

Magnetic Compass is a light magnetic needle suspended over a pivot to move free to position itself in any



direction along 360 degrees, and is enclosed in a circular box of nonmagnetic material. Dial inside the box is marked with N, S, E, W indicating North, South, East and West directions, having a graduation marking of degrees. Box of the needle is gently turned, without disturbing the needle so as to align distinctly marked end to North on the dial, as shown in the figure. This indicates North Direction and other directions

and their angles with respect to North Direction. This instrument is called Magnetic Compass and has been in use since ancient times.

In **magneto-statistics Coulomb's Law** is magnetic equivalent of Coulomb's Law of Electrostatic Forces. Since, magnetic poles do not exist in isolation; net magnetic force at a point is combined effect of forces exerted by both the poles of dipoles. It, therefore, creates difficulties of experimental verification. *Coulomb, in fact pronounced Laws of Force between Electrostatic Charges. Observation and its similarity to magneto-static forces between magnetic poles, except for the proportionality constant, this law is also known as Coulomb's Inverse Square Law of Magnetostatic Forces and is mathematically expressed as \overline{F} = \frac{\mu}{4\pi} \cdot \frac{m_1m_2}{r^2}\hat{r}. Here, \overline{F} is the force vector having unit Newton, m_1 and m_2 are strengths of magnetic poles also referred to as magnetic charge, \hat{r} is unit vector of displacement between two magnetic poles, r is the magnitude of displacement between the Two poles, and \mu = \mu_0 \mu_r is permeability of medium filling the gap between the two magnetic poles, while \mu_0 = 4\pi \times 10^{-7} \text{ H} - \text{m}^{-1} is permeability of free space and \mu_r is relative permeability of the medium and for vacuum or air having its value is 1. Here, unit of \mu_0 and Magnetic Pole strength would be progressively defined as elaboration of interaction between magnetic field and current proceeds, which would make their definition more realistic and scientific. For the present, Unit strength of magnetic pole is defined as that, when Two Like magnetic poles of equal strength are separated by 1m experience a force of repulsion equal to 10^{-7} Newton.*

In this series documents efforts, while elaborating concepts basics including its source and history, to arouse enthusiasm of readers to the fact that the knowledge which is so readily available has to us has taken restless efforts of many scientists, who never had facilities that are privileged to us right from birth. Author has not been able to corroborate association of Law Magnetostatic Forces to Coulomb, just beyond its nature analogous to the law of Electrostatic Forces propounded by Coulomb, readers are requested to update if they have any information on the source of the Inverse law through <u>Contact Us</u>. We would gratefully acknowledge the contribution of readers, with its inclusion in this document in furtherance of knowledge.

Magnetic Field (B) produced by a magnetic pole is defined as Force experienced by a Unit Magnetic Pole, is determined like that of electric field due to electric charge(s); its unit is Newton/Unit_Pole. It is also referred to as *Magnetic Field* at a point or **Magnetic Flux Density** (MFD)at the point. Since magnetic pole does not exist in isolation and hence determination of B due to magnet is driven by vectors \overline{B}_N and \overline{B}_s as shown in the figure. It may be noted from the figure that poles of a magnet lie within its geometry and hence



Magnetic Length (2*l*), distance between North Pole and South Pole of a magnet is shorter than its geometric length (L < 2l). Ratio of Magnetic Length to Geometrical Length is generally $\binom{2l}{L}$ found to be 0.84. In this analysis medium is considered to be air such that $\mu_r = 1$. It gives rise to three cases.

Case I: If $\theta = 0$ or π net value of $B = \frac{\mu_0}{4\pi} \cdot \left(\frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}\right) = \frac{\mu_0(4ml)}{2\pi} \cdot \frac{d}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2-l^2)^2}$ and in a direction $e^{i\sigma}$ for $\theta = 0$ and i.e. $e^{i\pi}$ for $\theta = \pi$, repulsive and attractive is direction of force is attributed to vicinity of North or South Pole.

Case II: If $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ then, $= \theta_2 B = \frac{\mu_0}{4\pi} \cdot \frac{2m \cos \alpha}{d^2 + l^2}$. Geometrically, $\cos \alpha = \frac{l}{\sqrt{d^2 + l^2}}$, hence $B = \frac{\mu_0}{4\pi} \cdot \frac{2m}{d^2 + l^2} \cdot \frac{l}{\sqrt{d^2 + l^2}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^2}$ in a direction $e^{i\pi}$. This formulation introduces a new term *Magnetic Dipole Moment (M)* of a magnet, which always exists like a dipole, M = 2ml.

Case III: Taking a generic case where point P is at an angle $\pm \theta$ with the principal axis of magnet. This will create $\overline{B} = \overline{B_N} + \overline{B_s}$, Taking $d \gg l$, a fair approximation, it would yield $\theta \cong \theta_1 \cong \theta_2$. Accordingly, NP $\cong d - l \cos \theta$ and $SP \cong d + l \cos \theta$ and, further, at point P magnetic field intensity would be $B = \frac{\mu_0}{4\pi} \cdot \left(\frac{m}{(d-l\cos\theta)^2} - \frac{m}{(d+l\cos\theta)^2}\right) = \frac{\mu_0}{4\pi} \cdot \frac{4mdl\cos\theta}{(d^2-l^2\cos^2\theta)^2}$. This equation satisfies Case I for $\theta \to 0$, and also for the Case II when in addition to $d \gg l$ another condition $\theta \to \frac{\pi}{2}$ or $\to \frac{3\pi}{2}$ is satisfied and $B \to 0$, i.e. $\overline{B_N} = -\overline{B_N}$.



Work done in Moving a Magnet in Magnetic Field: This derivation is analogues to the one in electric field. Accordingly, a point P is at $r \angle \theta$ from midpoint of a magnet. Despite the fact that in magnets North and South poles coexist, but for the effect of magnetic field each of the end pole was considered in isolation and same is being extended for determining **magnetic potential** at point P. Accordingly, magnetic potential at point P due to north pole would be $V_n = \int_{\infty}^{r_n} \frac{\mu_0}{4\pi} \cdot \frac{m}{x^2} \cdot (-dx) = \frac{\mu_0 m}{4\pi r_n}$ and due to south pole $V_s = -\frac{\mu_0 m}{4\pi r_s}$. Thus net magnetic potential at point P is $V = \frac{\mu_0 m}{4\pi r_n} - \frac{\mu_0 m}{4\pi r_s}$. In case point P is so located that $r \gg 2l$, then $V = \frac{\mu_0 m}{4\pi} \left(\frac{1}{r_n} - \frac{1}{r_s}\right) \approx \frac{\mu_0 m}{4\pi} \left(\frac{1}{r-l\cos\theta} - \frac{1}{r+l\cos\theta}\right)$. It leads to V = $\frac{\mu_0 m}{4\pi} \left(\frac{1}{r_n} - \frac{1}{r_s}\right) \approx \frac{\mu_0 m}{4\pi} \left(\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right) \approx \frac{\mu_0}{4\pi} \cdot \frac{M\cos\theta}{r^2}$. This is since a scalar quantity, like Electric Potential, hence it is called **Scalar Magnetic Potential**. As studies

are advanced in Electromagnetism, which is outside the domain of this Manual, *Vector Magnetic Potential* shall be encountered. Nevertheless, inquisitive readers are welcome to write us through <u>Contact Us</u>.

Magnetic Lines of Force: These are imaginary lines representing direction of magnetostatic force at any

point. There are two methods of drawing *Magnetic Lines of Force (MLF)*: a) Analytical Method - it uses above mathematical formation to determine the direction of B at different points and with best-fit curve joining these points to draw unidirectional lines from north pole to south pole of the magnet. Access of computer has made it much easier and faster. b) Experimental Method - it uses magnetic compass to which is placed near North pole of the magnet and two ends of the compass needle, when reached stationary, are marked. Next compass needle is so positioned that, in stationary state, its south pole coincides with marking of north pole; in this new position North Pole of the magnet is reached, which makes One lines of Force. This process is repeated for many points near the



North Pole of magnet and for every point a new MLF shall appear, as shown in the figure. It will be observed that MLF have following properties -a) Outside magnet they start at North and terminate at South Pole. b) Tangent at any point on MLF, indicates direction of B at that point on the lines of force, c) none of the MLF intersect each other, d) Density of MLF at any point is indicative of MF (*B*) at that point, and is highest near magnetic poles, e) Two MLF never intersect each other, f) MLF, in any region, are parallel and equi-spaced

then MFD in the region is uniform, g) MLF repel each other in a direction perpendicular to them, and is in accordance with repulsion between like polarity, h) MLF along length are like stretched strings and is due to force of attraction between like polarity, and i) MLF form a closed loop which outside magnet emerges from North Pole and terminates at South Pole, while inside magnet, it closes from South Pole to North Pole. This differentiates MLF with to Electrostatic Lines of Force. Since, and is attributed to existences of dipoles as against electric charges which can exist separately as (+)ve and (-)ve charges.

Terrestrial Magnetism: Use of magnetic compass for navigation was based on observation that *magnetic* compass, irrespective of position any where on earth always settles in a direction pointing towards Geographical North. But, why? And answer to this question lead to discovery of **Terrestrial Magnetism**. Magnetic needle always settling in north direction indicates that it is line with the direction of magnetic field as seen during elaboration of MLF. This can happen only when it is behind south of emerging lines of force. All these MLF converge on Magnetic North Pole (MNP) of the Earth and accordingly Magnetic Needle becomes perpendicular to the earth's surface at MNP. Thus if seen that internal magnetism of the earth also acts at as a dipole with its north pole aligned towards Magnetic South Pole (MSP) and south pole aligned to MNP. Nevertheless, anomaly in naming geographic north pole is explained as north seeking pole. The magnetic axis of earth is tilted at an angle of about 11.5° with geographic axis i.e. axis of rotation. Analysis of cause of earth's magnetism would throw discussions out of context that has been built so far, and is refrained at this stage. However, inquisitive readers are requested to write us through *Contact Us*.

There are certain, terms used in context of terrestrial magnetism and are defined for a ready reference.

Angle of Dip or Inclination (δ_B): This the angle made by earth's magnetic field with horizontal direction.



Accordingly, Lines which join points having identical angle of dip called Isogenic Lines. are Further, Geographical Meridian plane passing is a through geographical poles of earth and a point under consideration. Likewise, Magnetic Meridian is a plane passing through magnetic poles of earth and a point under consideration And, line joining points having $\delta_B = 0$ is called Magnetic Equator. Likewise, angle between

geographical axis and magnetic axis of the earth

is called **Angle of Declination** (θ_B). The principle behind two angles is shown in the figure. Horizontal Component of Magnetic Field $B_H = B_E \cos \delta_E$ and Vertical Component of Magnetic Field $B_V = B_E \sin \delta_E$. Accordingly, $\tan \delta_E = \frac{B_V}{B_H}$. An instrument called **Dip Circle** is used for measuring this angle of dip (δ_B) and is shown in the of http://comps.canstockphoto.com/can-stockfigure (Source Picture: photo csp8123680.jpg). In horizontal turn table is levelled using spirit level and vertical frame housing magnetic needle is tuned till it is becomes perfectly vertical and its two ends point (+)90° and (-)90° on the vertical scale. At this position forces due to B_H forming a couple on the needle are zero and accordingly under influence of vertical magnetic field settles vertically. Now, the vertical scale is turned through 90° on the horizontal scale, at this position torques produced by B_H and B_V are in equilibrium and the needle shall settle at an angle be δ_B as shown in the figure.



Magnetic Lines of force of Permanent Magnet placed in Earth's Magnetic Field : Four cases can be created for MLF of a magnet placed in magnetic field of the earth. Case I: South Pole of Magnet aligned towards magnetic North Direction. Case II: North Pole of Magnet aligned towards magnetic North Direction. Case III: North Pole of Magnet aligned towards East Direction and Case IV: North Pole of Magnet aligned towards West Direction. Case IV is diagonal image of Case III. Generally, plotting of MLF is done on a horizontal plane and these invariably interacting magnetic field intensities of permanent magnet and the earth are horizontal components, and are so considered in the following analysis.

Case I: Qualitatively it is seen from the plot of MLF as shown in the figure (it is experimentally verifiable) in the region around points X magnetic field intensities due to magnet $B_M \left(=\frac{\mu_0}{4\pi}\right)$ $\frac{2Md}{(d^2-l^2)^2}$ and horizontal component of earth's magnetic field intensity B_H tend to be in opposite direction. Since, MLF have a property that they do not cross each other, they choose an alternative path. Nevertheless, point at which $\bar{B}_M = -\bar{B}_H$, net *B* shall be zero or a **NULL POINT** of magnetic field intensity and is also called Neutral Point. Here, distance (d) of Null Point from mid-point of the permanent magnet along its axis would depends upon its dipole moment (M), length of magnet (1) and B_{H} . In case length of magnet is small while pole strength of permanent (m) is

sufficiently large, Null Point would occur such that $d \gg l$. Accordingly, $d \cong \left[\left(\frac{\mu_{0M}}{2\pi} \right) \cdot \frac{1}{B_H} \right]^{\frac{1}{3}}$.

Case II: In this case $B_M = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$ and NULL POINT shall be on line perpendicular to axis of

the permanent magnet, on it's midpoint, shown in the figure. In this case also distance (d) of Null Point from mid-point of the permanent magnet along its axis would depends upon its dipole moment (M), length of magnet (l) and B_H . If length of magnet is small while pole strength of permanent (m) is sufficiently large Null Point would occur such that $d \gg l$. Accordingly, null

point shall be at
$$d \cong \left[\left(\frac{\mu_{0M}}{4\pi} \right) \cdot \frac{1}{B_H} \right]^{\frac{1}{3}}$$
.

Case III and IV: These cases are diagonal images of each other and can be derived from $B_H = \frac{\mu_0}{4\pi}$. $\frac{4mdl\cos\theta}{(d^2-l^2\cos^2\theta)^2}$ both analytically and experimental plot

of lines MLF and is left for reader as an exercise. A conceptual plot of MLF for a permanent magnet with its north pole aligned to east direction is shown in the figure for convenience. Nevertheless, inquisitive readers are welcome to write through **Contact Us.**

Deflection Magnetometer: This is an instrument used to determine NULL POINT of a permanent bar

վարդարությունությունություններ • **1**

placed on a pivots at the centre of a wooden box. This box is covered with glass and a mirror below the compass. A light non-magnetic pointer is so fixed to the compass that it is

perpendicular, and its centre as the pivot. The mirror is useful to remove parallax error in measuring angular position of the needle. This box is placed centrally on a wooden scale. Use if this instrument in Two positions helps to determine ratio $\left(\frac{M}{B_H}\right)$ of Dipole Moment (*M*)

and horizontal component of earth's magnetic field (B_H) . The two positions are called - a) tan-A Position and tan-B Position. Principally, these uses are based on analysis of Magnetic Field Intensity

(B) of a permanent Magnet done for Cases I & II above, for which magnetic lines of forces were conceptualized in presence of horizontal component of earth's magnetic field (B_H) at Cases I & II above. Thus combining the two cases the ratio for the Two positions is summarized below, for verification by the readers. Here, θ is the angular position of the needle in equilibrium of torque produced by M and B_H on the magnetic needle. Thus value of M and B_H determined with this apparatus are relative to each other, and not absolute.



magnet. It has a small magnetic compass







.tan-A Position	tan-B Position
$\frac{M}{B_H} = \frac{2\pi}{\mu_0} \cdot \frac{(d^2 - l^2)^2}{d} \cdot \tan\theta$	$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \cdot (d^2 + l^2)^{\frac{3}{2}} \cdot \tan\theta$

Torque on a Magnet placed in Uniform Magnetic Field: In the dipole shown below, turning moment

on North Pole of strength $+\mathbf{m}$ is $\overline{\Gamma}_{+} = \overline{OA} \times \overline{F}_{+} = \overline{l} \times m\overline{B} = mlB \sin\theta(-\hat{z})$. It is similar to the torque on an electric dipole placed in uniform electric field discussed in Electrostatics. Likewise, turning moment on South Pole ($+\mathbf{m}$) of the dipole would be $\overline{\Gamma}_{-} = \overline{OA} \times \overline{F}_{+} = (-\overline{l}) \times (-m)\overline{B} = lmB \sin\theta(-\hat{z})$. Here,

▶ \overline{l} and $-\overline{l}$ are distances of the poles from the midpoint of the dipole. Thus, **Torque**





on the dipole placed in a uniform magnetic field (\overline{B}) is $\Gamma = \overline{\Gamma}_{-} + \overline{\Gamma}_{-} = -2mlB \sin \theta \hat{z} = -MB \sin \theta \hat{z}$ and it tends reduce the angle of the magnet with the direction of Magnetic Field. Here, $\overline{M} = 2lm\hat{l} = M\hat{l}$. Likewise, when magnetic field is at an angle $+\theta$ with respect to the pole, as shown in the latter of the Two cases above, $\Gamma = MB \sin \theta \hat{z}$ along $+\hat{z}$ direction, still it

tends to the angle. This finds extensive application in Oscillation Magnetometer that follows.

Potential energy (*PE*) of a magnet at a particular angular position (θ) w.r.t. uniform magnetic field (\overline{B}), earlier of the above Two cases, is $U_{\theta} = \int_{0}^{\theta} \Gamma(-d\alpha)$. Since, angle is α is the angle of magnet at an intermediate position, and turning magnet against torque through an angle $\Delta \alpha$ would resolve to $U_{\theta} = \int_{0}^{\theta} (MB \sin \alpha) (-d\alpha)$. Accordingly, it would lead to $U_{\theta} = MB \int_{0}^{-\theta} \sin \theta \, d\theta = MB [-\cos \theta]_{0}^{\theta} = MB(1 - \cos \theta)$. Thus potential energy of a magnet in a position perpendicular to uniform magnetic field is *MB* Joule. Same is true in latter of the above Two cases. Accordingly, a slight perturbation in magnet would set it into oscillation.

Oscillation Magnetometer: Limitation of deflection magnetometer to determine independent values of M and B_H lead to evolution of Oscillation Magnetometer to measure MB_H as shown in the Figure [Source: http://ncerthelp.com/ncertimages/Class12/physics/ch5/ch5physicsno9.jpg]. It has magnet freely suspended from a silk thread, in box openable to place a magnet on a light non-magnetic hanger. Inner bottom of the box has a mirror with a central line marked on it. The box has levelling screw to ensure that suspended magnet in



normal state is on the central line marked N-S called Reference Line . A magnet is placed on the hanger, with untwisted thread. It is aligned to magnetic north, which in turn is aligned to the reference line. In this mean position suspended magnet is deflected about the vertical axis i.e. silk thread with the help of an external magnet. Torsion Head is at the top to which one end of the silk thread is fixed and at its other end hanger is fixed. This sets the suspended magnet into oscillation about the vertical axis. In any position, deflected through an angle θ , it experiences a torque the magnet $\Gamma = |(2l\hat{l}) \times (m\bar{B}_H)| = (2ml)B_H \sin \theta = MB_H \sin \theta \approx MB_H \theta$. Further, as per principles of mechanics, under the influence of the torque (Γ) magnet would experience an angular acceleration such that magnitude of the torque is $\Gamma = I\alpha \rightarrow I\alpha = MB_H\theta \rightarrow \alpha = \frac{MB_H}{I}\theta$. This is a valid case of SHM, which satisfies two basic conditions: **a**) Torque is proportional to the displacement from mean position and **b**) The torque is always directed towards its mean position. Accordingly, angular

acceleration (α) and angular velocity (ω) of the oscillating magnet shall be such that $\alpha = \omega^2 \theta$. Thus, $\omega^2 =$

 $\frac{MB_H}{I} \rightarrow \omega = \sqrt{\frac{MB_H}{I}}$. In this, $I\left(=W\frac{a^2+b^2}{12}\right)$ is Moment of Inertia of the Magnet is its geometric property, where W is the weight, **a** (=2*l*) is its length and **b** is its width. Time Period of Oscillation (*T*) for a magnet is measurable through experiment. Accordingly, to achieve the objective of inventing this instrument the relationship derived above can be written as $MB_H = \frac{4\pi^2}{T^2}I$.

above can be written as $MB_H = \frac{4\pi^2}{T^2}I$. Having determined $\frac{M}{B_H}$ with the help of deflection magnetometer, whichever position, and MB_H with the help of deflection magnetometer, it is possible to determine values of M and B_H separately, in terms of geometric properties with only value of absolute permittivity (μ_0) question. This involves elaboration of electromagnetism to follow.

Dip Circle together with **Deflection Magnetometer** and **Oscillation Magnetometer** work on the basic principle of mechanics i.e. position of least potential under influence of torque couples on a dipole is a state of equilibrium. Accordingly, *measurements through these instruments is susceptible to errors, which largely common in nature* and are tabulated below -

Type of Error	Dip Circle	Oscillation Magnetometer	Deflection Magnetometer
Off-Centers of Needle and Vertical Scale	Possible	Possible, but off-center could be with horizontal circular scale	Not Applicable
Offset in Geometrical and Magnetic Axis of Magnet	Possible	Possible	Possible
$0^0 - 0^0$ of vertical scale is not horizontal	Possible	Not Applicable	Not Applicable
Centre of Mass of the needle does not coincide with the pivot	Possible	Not Applicable	Possible, but off-center could be is with Centre of Mass of Magnet and silk-thread.
Offset in Magnetic and Geometric Centers	Not Applicable	Possible	Possible
Offset in Zero of Linear Scale a center of Circular Scale	Not Applicable	Possible	Not Applicable

Details of the above three instrument involve concepts elaborated earlier. Tangent Galvanometer is an important **Electromagnetic Instrument** used in magnetism. But, it involves concepts of electromagnetism which shall be elaborated a little later.

Gauss's Law in Magnetism: In electrostatics, where (+)q and (-)q electric charges can exist in isolation, $\oint \overline{E} \cdot \overline{ds} = \frac{q}{\epsilon_0}$, here q is the electric charge in space. But, in magnetism, howsoever thin a magnet is North Pole and South Pole cannot exist in isolation, rather they are complementary to each other. It shall be soon elaborated in *Electromagnetism*. Hence, in **magnetism Gauss's Law** reduces to $\oint \overline{B} \cdot \overline{ds} = 0$. It implies that *in any closed surface in a magnetic field, be it uniform or non-uniform the MLF entering the surface is equal to MLF leaving the surface*.

Types of Magnetic Material: There are materials which are non-responsive to static magnetic field and are called **Non-Magnetic** materials. There are certain materials which have qualitatively distinct behavior when placed in static magnetic field and are accordingly classified, as under –

Ferro Magnetic Material: There are materials like iron have relative permeability $\mu_r \gg 1$ and this causes

heavy concentration of MLF within it, when placed in magnetic field as shown in the figure. Dipoles of ferromagnetic materials, tend to get aligned in the direction parallel to external magnetic field as it happen to magnetic compass needle. They are strongly attracted by a magnet.

Paramagnetic Material: There are the material whose relative permeability is just $\mu_r > 1$, **but nearly One**, and this causes feeble concentration of MLF within it, when placed in magnetic field as shown in the figure. Paramagnetic material has a sharp qualitative difference with Ferromagnetic Material, while qualitatively it is same. Like Ferromagnetic materials, these also get aligned in the direction parallel to the external magnetic field. They are weakly attracted by a magnet.

Diamagnetic Material: These materials are in contrast to Ferromagnetic and Diamagnetic Materials, since they distract MLF unlike these materials. This attributed to its characteristic difference where $\mu_r < 1$. Unlike Ferromagnetic and Paramagnetic material they get aligned in a direction perpendicular to the external magnetic field.

Electromagnetism: Observation of deflection of a magnetic needle when brought near a current carrying current by **Hans Christian Oersted** (pronounced as Ørsted), in 1819, was a *great beginning to discovery of inseparable and interactive nature of electric current and magnetic field and known as* **Electromagnetism**. This would lead to redefining of unit and dimensions of strength of magnetic pole (**m**) and absolute permeability (μ_0) in terms of Fundamental dimensions. It is the most awaited and most happening topic in physics which was deliberately kept in abeyance till proper context and concepts are built.

Ørsted Experiment: An accidental observation of jerks in magnetic near a current carrying conductor by Hans Christian Ørsted_in 1819. By then, sufficient knowledge had been gained about magnetism. Accordingly, Ørsted's observation provided a good reason to explore *relationship between electric current and magnetism* and a new stream of discoveries called **Electromagnetism** Started.

Ørsted found that, for a straight wire carrying a steady (DC) current -



Direction of

a. Current

b. Advancing of Screw

- The magnetic field lines encircle the current-carrying wire
- The magnetic field lines lie in a plane perpendicular to the wire
- If the direction of the current is reversed, the direction of the magnetic force reverses.
- The strength of the field is directly proportional to the magnitude of the current.
- The strength of the field at any point is inversely proportional to the distance of the point from the wire.

Ampere's Swimming Rule : Investigation into magnetic behavior of current lead to discovery of André-Marie Ampère, in 1820 about the direction of magnetic field created by a current carrying conductor. It states that= "if we imagine a man is swimming along the wire in the direction of current with his face always turned towards the needle ,so that the current enters through his feet and leaves at his head, then the

towards the needle ,so that the current enters through his feet and leaves at his head, then the north pole of magnetic needle will be deflected towards his left hand". It is graphically shown in the figure. It is also known as **SNOW Rule**. It is



graphically shown in the figure. It states that in a conductor carrying electric current flowing from South to North and it is placed over the conductor then north pole of the magnetic needle would align along West. A simplification to this rule was made by **James Clerk Maxwell** and is



known as Maxwell's Right Hand Thumb (or Grip) Rule according to it if a current carrying conductor is held in right hand with its thumb pointing towards the direction of current then the folding of fingers is in the direction of magnetic field created by the current, as shown in the figure. A another version of it, a **Cork** Screw Rule according which states when a screw is rotated in cork, direction of rotation of screw and magnetic field is same, while direction of travel of screw is the direction of current. Maxwell's Right Hand Screw Rule is considered to be simplest to apply.

Biot-Savart's Law: A major step towards quantification of magnetic field at a point near a current carrying conductor was made, through a mathematical statement, by Jean-Baptiste Biot and Félix Savart and is known as Biot-Savart's Law in 1820. Accordingly, magnetic field intensity at a point P due to a current (I) through an element of infinitesimal length (dl) is given by $d\overline{B} =$ $\frac{\mu_0}{4\pi}I\frac{d\bar{l}\times\hat{r}}{r^2}$, in vector form. Here, $d\bar{B}$ is the elemental Magnetic Field Density (MFD) Vector, $d\bar{l}$ is the element vector in the direction of the current (I) through the conductor, **r** is distance of the point under consideration from the element of the conductor under consideration, and \hat{r} is the direction vector of the point w.r.t. the conductor element. It resolves into $d\bar{B} = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \sin \theta(-\hat{k})$. Here, direction vector $(-\hat{k})$ is in accordance with the rule of cross-product of vectors and is in conformance with the Maxwell's Right-Hand Thumb Rule.

This Law has been extended to Two cases a) determination of magnetic field at any point across a long straight conductor and **b**) along axis of a coil. Both the cases are elaborated below.



Magnetic Field Due to Current in a Straight Wire: Conceptually, this analysis is similar to that done for determining electric field intensity E due to a conductor carrying uniform charge density, but difference is in mathematical formation, due to nature of the problem. Accordingly, extending Biot-Savart's Law, MFD due to a small conductor element of Δx as shown in the figure $d\bar{B} = \frac{\mu_0}{4\pi} I \frac{dy \times \hat{r}}{r^2}$. It leads to magnitude $= \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} dy \sin \theta$. . Therefore integrated effect of a straight long conductor $B = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} dy \sin \theta$. $\frac{1}{r^2}\sin\theta dy$. It involves three variables, which is simplified by substituting $-y = d \cot \theta \rightarrow dy = (d \csc^2 \theta) d\theta$. Using the same trigonometric $B = \int_{0}^{-\pi} \frac{\mu_{0l}}{4\pi d} \sin \theta \, d\theta \rightarrow \frac{\mu_{0l}}{4\pi d} [-\cos \theta]_{0}^{\pi} = \frac{\mu_{0l}}{2\pi d} = \mu_{0} H.$ Here, *H* is called **Magnetic Flux Intensity** (MEI)

Arc Length: $\Delta l = r \Delta \theta$

2dB cost

(MFI).

Magnetic Field Due to Current in a Circular Loop along its Axis: A piece of straight conductor carrying current (I), in clockwise direction, when shaped into a circular loop field is required to be determined at a point P located at a distance (d) from thee entre of the loop along its axis. Each of the element A and B, of the circular current carrying loop, of length $\Delta l = r \Delta \theta$, has carrying current flowing through it in opposite din opposite directions. Since, these elements A and B are placed diametrically placed opposite they will produce MFDs $d\bar{B}_A$ and $d\bar{B}_B$ at point P which symmetrically displaced by an angle α w.r.t. axis OP, Thus components of the MFDs perpendicular to axis OP would cancel out and net field would be directed towards center O in accordance with





 $B = \frac{\mu_0 Ir}{2(r^2 + d^2)} \cdot \frac{r}{\sqrt{r^2 + d^2}} = \frac{\mu_0 Ir^2}{2(r^2 + d^2)^{\frac{3}{2}}}.$ Thus MFD at point O, when $d \to 0$, $B = \frac{\mu_0 I}{2r}$ and shall be along $-\hat{i}$ or it will act like south pole with MLF entering the loop. Determination B at any point off center of the loop becomes a

complex mathematical formation, nevertheless inquisitive readers are welcome to write through *Contact Us*.

Likewise for current in clockwise direction shall be along \hat{i} i.e. it will act like north pole with MLF leaving the loop. In this derivation width of loop has no significance and is in **conformity with the premise**, that North-South pole always coexist. An easy to apply anecdote to remember the magnetic pole created by a loop with current seen to be flowing in the coil, either clockwise and anticlockwise direction of current, is shown in the Figure.

Ampere's Circuital Law : It is an extension of Bio-Savart's Lawm propounded by Ampere in 1823 to quantitatively relates MFD (B) around a current source(s). It states that line integeral of Magnetic Flux Density in a closed path is proportional to current encircled inside the

closed path. It is mathematically expressed as $\oint_C \bar{B} \cdot d\bar{l} = \mu_0 I_{enc}$. A closed path is traced around a system of current carrying conductor, as shown in the figure, such that net current inside the loop is $I = I_1 + I_2 + I_3 - I_3 + I_4 = I_1 + I_2 + I_4$. Taking a small element of the loop of length vector $d\bar{l}$ at point P where let MFD is $\overline{B_p}$, then as



per Ampere's Circuital Law $\oint_C \overline{B_p} \cdot d\overline{l} = \oint_C B_p \cos \theta \, dl = \mu_0 I$. As per *Biot-Savart's* Law MFD at a distance r from the axis of a long conductor carrying current (I) is

 $B = \frac{\mu_0 I}{2\pi r}$. This B is always tangential to the circular path of length $2\pi r$ and satisfies Ampere's Circuital Law.

Extension of Ampere's Circuital Law (ACL) practical applications has been through Torroids and Sloenoid



and is elaboreated below. Torroid is a circular core of Ferromagnetic material having relative ermeability μ_0 over it circular loops in series are wound with one inlet and outlet for current I. A circular path of radius r is taken inside to the torroid. Then as per ACL, $\oint_C \overline{B} \cdot d\overline{l} = 2\pi r B = \mu(NI) =$

 $\mu_0\mu_r(2\pi rn)I$. It leads to $\oint_C \bar{B} \cdot d\bar{l} = (2\pi r)B = \mu_0\mu_rNI$, here total number are turns $N = 2\pi r n$, where **r** is the radius of the circuital path chosen and \boldsymbol{n} is number of

turns per-unit length on the toroid. Accordingly, **MFD inside the toroid** $B = \frac{\mu NI}{2\pi r}$ Now, taking a path of radius r_p in air inside the inner radius of the toroid, applying ACL to determine B_p along the path $\oint_C \overline{B_p} \cdot d\overline{l} = 2\pi r_p B_p = \mu_0 (0 \cdot I) = 0$, it leads to $B_p = 0$,



can be considered to be perpendicular to the axis of the solenoid. In absence of this assumption

there is fringing of B at ends of a solenoid, which would make it quite complicated for evolving solution at this stage, and hence the assumption is carried through. In analysis of the idealized solenoid having n turns per unit length is idealized such that it is placed on a ferromagnetic core of permeability $(\mu = \mu_0 \mu_r)$ having uniform cross-section (A) and mean circutal length (L) but solenoid having n turns per meter length spread over a length (l) such that total number of N (= nl), applying ACL $\oint_C \overline{B} \cdot d\overline{l} = \overline{B} \cdot \overline{L} = BL = \mu(nl)I = \mu NI$.





driving flux is called **Magneto-Motive Force (MMF)** which is mathematically expressed as $NI = \emptyset \left(\frac{l}{\mu A}\right)$, is anlogus to Ohms Law in Current Eletricity; here MMF (NI) corresponds to EMF, Magnetic Flux (\emptyset) corresponds to Electric Current, Reluctance $(\Re = \frac{l}{\mu A})$ corresponds to Resistance (R), and permittivity (μ) is analogus to conductivity (σ) which is reciprocal of resitivity ($\rho = \frac{1}{\sigma}$). Solenoid has extensive application in electromagnets, electrical devices and machines, subject matter that shall be introduced a little later and advanced studies in engineering. Unit of MMF, generally expressed as H, is A-m⁻¹. Magnetic

Susceptibility of Magnetic Materials: In **magnetic circuits** presence of magnetic material influences its behaviour. The permeability (μ) has two components – **a**) absolute permeability (μ_0), already defined earlier and **b**) **susceptibility** (χ). These are related through an equation which defines characteristic of material and it is $\mu = \mu_0(1 + \chi)$. Value of χ for three basic class of magnetic materials classified earlier are- **i**) Ferromagnetic Materials $\chi \gg 1$, **ii**) Diamagnetic materials $\chi > 1$, for **iii**) Diamagnetic Materials $\chi < 0$.

Magnetizing Characteristics of Ferro-magnetic Material: It is the buildup of MFD (B) on application

of MMF, a typical plot of the magnetizing characteristic is shown in figure. It depends upon point of start of magnetization based on residual magnetism in the magnetic material. This residual magnetism depends upon magnetic history of the magnetic specimen and can be better understood taking magnetization of a totally demagnetized ferromagnetic material shown by Point 'O' on B-MMF curve, in figure. This curve is also referred to as B-H curve since, $MMF \propto H$. As MMF is increased, magnetic induction increases almost linearly upto point 'B', it is known as **Knee Point**. Beyond Knee Point incremental increase in *MMF* for increase in **B** is non-linearly high. Beyond Point 'C' practically there is no increase in **B**, despite increase in *MMF*. On reaching point 'C' decreasing *MMF* takes a path above the curve OAC, such that when *MMF* = **0**, there is



magnetic induction, called **Residual Magnetism, Remnance or Retentivity** identified as 'D' a point of intersection of magnetization curve with ordinate i.e. **B**-axis. Moving forward on the (–)ve MMF-axis, it is at point 'E', **B** reduces to Zero, and is called **Coercive Force or Coercivity**. Saturation point 'F' on (-)ve induction is shown in figure, after reaching that increase in MMF traces magnetization curve a paths FGHC, and this cycles repeats, unless magnetization is abruptly changed, which make the retracing of magnetizing characteristic plot different from the earlier. The loop CDEFGHC is called **Hysteresis Loop** and are under the loop is called **Hysteresis Loss**.

Force Between Two Current Carrying Conductors: Force between Two current carrying conductors can be classified in Two categories, First when conductors are parallel and Second when there is angular shift between the Two conductors. This again leads to Two cases -a) Current in the Two conductor is in same direction and **b**) Current in Two Conductors is in opposite direction. A qualitative analysis reveals that the parallel conductors carrying current in opposite direction have unidirectional flux in the inter-conductor region and thus tends to increase the MLF, which have as per their property do not intersect each other. Thus in order to adjust to uniform density of MLF, they act as tight stings of a bow and trying to push the conductors away like arrows. This results into a force of repulsion on the conductors as shown in the figure. Extending the same logic to conductors carrying current in same direction it is seen that conductors experience of attraction due to normalize MLF density in inter-conductor region is feeble due cancellation magnetic flux due to Two



conductors in opposite directions. *Quantitative analysis of this observation was made by Ampere in 1825 and Gauss in 1833 and to calculate force between two parallel conductors carrying current. As known as Ampere's Force Law.* According to the Law *force per*

unit length experienced by two parallel conductors,

¹carrying currents I_1 and I_2 , separated by a distance r is equal to $\frac{F}{L} = B \cdot I_2 - \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2}{r}$ N-m⁻¹. Explanation for the



interrelation between force and two current conductors is due interactive nature of magnetic field and current. It has been derived from Biot-Savrat's Law that MFD at a distance **r** from a straight conductor carrying current I_1 is $B_1 = \frac{\mu_0 I_1}{2\pi r}$ and force experience by another conductor carrying current I_2 , placed in this magnetic field,

force experienced by the conductor per unit length is $F/l = B_1 I_3$ and in vector form $\frac{\overline{F}}{l} = \overline{I} \times \overline{B}$. It was used to define unit of current Ampere. In SI unit $\mu_0 = 4\pi 10^{-7}$ N-A⁻². Accordingly, in SI unit when Two conductors each carrying **One unit Current**, causes force between the conductors is 10^{-7} N, with dimension I as one of the Fundamental Dimensions.

Further, once unit of current is defined in measurable units of mechanics, units equivalent of Charge (Q) which is One Coulomb equal to One Ampere-Second having Dimension [IT], where [I] is one of the fundamental dimensions; Flux Density (B) is Tesla having dimension $[MI^{-1}T^{-2}]$ and is equal to Weber/per m²; Magnetic Flux (ϕ) has a unit Weber having dimension $[ML^2I^{-1}T^{-2}]$; Magnetic Pole Strength (m) as Ampere-Meter having dimension [IM], Permittivity of space (ε_0) has a unit Coulomb/Newton/Meter² $(C^2N^{-1}M^{-2})$ having dimension $[M^{-1}L^{-3}T^4I^2]$. Maxwell through his Electromagnetic Equations unified permittivity (μ_0) and permeability (ε_0) into velocity of light such that $\mu_0 \varepsilon_0 = \frac{1}{c^2}$. Since Maxwell's Electromagnetic theory is outside domain of this manual, therefore any inquisitiveness of the readers together verify dimension of each of the above electro-magnetic quantity are welcome through Contact Us.

Force On a Moving Charge: This is a stage where independent interaction of magnetic field and electrical charges has been established and a new point of exploration is created to know behaviour of a moving charged particle in presence of both the magnetic field and electrostatic field. This integrated behaviour was propounded by **Hendrik Lorentz** in 1896 and is known as **Lorentz Force Equation** and it is $\overline{F} = q\overline{E} + q\overline{v} \times \overline{B}$. The Force (in Newton) has two components – **a**) component of Electrostatic Force (= qE) pronounced by Coulomb's Inverse Square Law, where E is the electric field intensity created by a system of charges, and **b**) component of magnetostatic force created by magnetic field intensity on a moving charge (= $q\overline{v} \times \overline{B}$) as per Ampere's Force Law, here **q** is the charge on the moving particle (in Coulomb) and \overline{v} is the velocity vector of the charged particle in m/sec. In this component of magnetostatic force definition of current ($\overline{I} = q\overline{v}$) is used. This magnetic field can be due to permanent magnets, terrestrial magnetism, electromagnets or a current carrying conductor as discussed earlier. Thus this development is attributed to cumulative contributions of many scientists. This integration by Lorentz created a path for invention of Cyclotron by Ernest O. Lawrence in 1932 to invent cyclotron to accelerate charged particles and a new era in nuclear physics.

Electromagnetic Induction: Another big leap in Electromagnetism was provided by *Michael Faraday* through experimental verification of induction of voltage and in turn current through relative change in linkage of flux with a coil in 1831 through an experiment as shown in the figure. Whenever, circuit supplying current to one coil on a toroid was closed or interrupted, current was induced current in the other coil wound



on the toroid. Direction of the current induced in second coil while closing switch supplying current to the first coil was opposite to that while interrupting. Incidentally, in 1832 **Joseph Henry** independently discovered the phenomenon but, publication of observation by Faraday precedes that of Henry and accordingly it is known as **Faraday's Laws of Induction** which states that whenever magnetic field through closed circuit changes an

electromotive force (EMF) is induced. in any closed circuit is equal to negative rate of change of the magnetic flux enclosed by the circuit. It is mathematically expressed as $E = \left|\frac{d\Psi}{dt}\right| = \left|\frac{d(N\emptyset)}{dt}\right|$, here \emptyset is the flux linking N turns of the coil. This law was silent in respect of direction of EMF which was complemented by Emil Lenz in, 1834, stating that direction of induced in coil by changing flux, called flux linkage, as per Faraday's Law of Induction, is such that it will create a magnetic field that opposes the change. De facto it is electrical versions of Newton's Third Law of Motion reaction is equal and opposite to the action. Thus complete version of Faraday's Law is $E = -\frac{d\Psi}{dt} = -\frac{d(N\emptyset)}{dt} = -\frac{d(N\emptyset)}{dt}$ $\frac{d}{dt} \oint N\overline{B} \cdot d\overline{S}$, here, (-)ve sign is attributed to Lenz and thus it can also be called Faraday-Lenz's Law, with a necessary condition that flux linkage must be time variant. This law became an integral part of Electromagnetic Theory propounded by Maxwell, much talked about in this text, but in a different form which is outside scope of this manual and inquisitive readers are welcome to write through <u>Contact Us</u>.

This principle with above broad classification has found widest application in growth of science. It is pertinent to recall a conversation of Faraday, when he demonstrated **Dynamo**, a device to convert mechanical energy into electrical energy, based on this principle of electromagnetic induction, he was asked by a curious person – "*What is use of this new device* (*dynamo*)". Faraday, very humbly replied – "*it has the same use as that of a new born child*". *That dynamo has tuned out to be (Great)*ⁿ *Grandfather of the energy sources that are supplying electricity to support the technological development on this earth.*

Faraday's Laws are applicable to changing magnetic field w.r.t. time for whatever reason be it – **Case I:** changing MMF, **Case II:** shifting coil in magnetic field, **Case III:** rotation of coils causing angular displacement between coil surface and /or, **Case IV:** changing of area intercepting magnetic field and has found application over a very wide range. This broad classification is conceptualized in table below, with analytical elaboration to follow.

Cas	e I	Case II		Case III		Case IV	
		Direction of Magnet		N	B T T	X X X X X X X X X X X X X X X X X X X	x x x x x x x x x x x x x x x x x x x
Cause	Effect	Cause	Effect	Cause	Effect	Cause	Effect
Current in	Induced	Motion	Induced	Rotationa	Induced	Translationa	Induced
Coil on the	current in	of Coil	current in	l motion	current in	l motion of	current in
right is	coil on the	away	coil is	of coil in	coil tending	conducting	loop tends
increasing	left is	from	clockwise,	uniform	to	link in	to
through	tending to	magnet,	tending to	magnetic	compensat	uniform	compensat
rheostat. It	compensate	tends to	compensat	field,	e decrease	magnetic	e decrease
tends to	increase in	decrease	e decrease	tends to	in flux	field,	in flux
increase flux	flux-linkage	in flux	in flux-	decrease	linkage of	tending to	linkage of
linkage of	of coil.	linkage	linkage of	flux	coil.	decrease flux	loop.
coil.		of coil.	coil.	linkage of		linkage of	-
				coil.		loop.	

Inductance: Biot-Savart-Ampere's Law together with Faraday-Lenz's Law has introduced **Inductance** a Third property of electrical element, after *Resistance and Capacitance*. Inductance is classified in Two Categories – **a**) **Self-inductance** of an electrical due to flux created by the current flowing through itself, and **b**) **Mutual Inductance** of an electrical element is due linkage of in another electrical element. This *Inductance in accordance with Faraday-Lenz law come into play only when current causing the flux changes*, which states that $E = \frac{d\Psi}{dt} = \frac{dLi}{dt} = L\frac{di}{dt}$. This inductance for an electrical element depends upon the geometry of the element is fixed, as long as geometry of the element remains unchanged. Determination of inductance is outside domain of this manual, nevertheless for simple geometry its being elaborated to strengthen basic concept.

In an idealized solenoid on a toroid $\boldsymbol{B} = \frac{\mu NI}{2\pi r}$, therefore total flux linking the N turns of the toroid $\phi = \int \bar{B} \cdot d\bar{A}$. Thus $\Psi = N \int \bar{B} \cdot d\bar{A} = N \left(\frac{\mu NI}{2\pi r}\right) A = (n2\pi r) \left(\frac{\mu(n2\pi r)I}{2\pi r}\right) A = \mu n^2 lAI$. Here, \boldsymbol{n} is number of turns of solenoid spread over per unit length of toroid, \boldsymbol{l} is the mean length of the toroid and \boldsymbol{A} is the area of cross-section of toroid such that magnitude $\boldsymbol{A} >> \boldsymbol{l}$. Accordingly, flux linkage per ampere of current, is called $\boldsymbol{L} = \frac{\Psi}{I} = \mu n^2 lA$. Since, this inductance of solenoid on toroid is due to flux created by solenoid, when current flows through it, is linking it's own turns and hence it is called **Self-inductance**. Likewise, when flux produced by one coil called primary coil, links other coil, called secondary coil, as shown in Case I in elaboration of Faraday's Laws of Induction. In this case $\Psi = N_1 k_1 \phi_2 = k_1 N_1 (k_2 N_2 I_2) = k N_1 N_2 I_2$, here k_1 and k_2 depend upon geometry of primary and secondary coils and properties of magnetic circuit, while N_1 and N_2 number of turns of the respective coils. Thus, $M = \frac{\Psi}{l} = kN_1N_2$, where $k = k_1k_2$ a geometrical-cum-magnetic constant of pair of the Two coils, and M is called their *Mutual inductance*. Unit of Inductance is Henry and its dimension is $[ML^2I^{-2}T^{-2}].$

With this definition of inductance, voltage across an inductance is $V_L = L \frac{dI_L}{dt}$, accordingly power in inductor at any point of time is $p_L = V_L I_L = \left(L \frac{dI_L}{dt}\right) I_L = L \left(I_L \frac{dI_L}{dt}\right)$, and **energy in inductor** $E = \int p_L dt = L \int I_L dI_L = L \int I_L dI_L dI_L$ $\frac{1}{2}LI_L^2$. It is to be noted that energy in the inductor is stored in the form of magnetic field and is attributed to current through the inductor. This is analogous to energy in capacitor $\left(=\frac{1}{2}CV^2\right)$ where energy stored in the form of electric field and is attributed to voltage across the capacitor. Accordingly a comparison of inductor and capacitor is as under -

Parameter	Inductor	Capacitor		
Constant Value	L : depends upon geometry of	C: depends upon geometry of the		
	electrical element	electrical element		
Unit	Henry (H)	Farad (F)		
Dimension	$[ML^2I^{-2}T^{-2}]$	$[I^2T^4IM^{-1}L^2]$		
Voltage across the element	$E = L \frac{dI}{dt}$	$V = \frac{Q}{C}$		
Current across the element	Ι	$I = \frac{dQ}{dt} = C \frac{dv}{dt}$		
Energy across the element	$E = \frac{1}{2}LI^2$	$E = \frac{1}{2}CV^2$		

Combination of resistance (R), inductance (L) and capacitance (C) has found wide application in electrical systems and hence influence of these three elements, in various combinations, is extremely useful in analysis and their application. Accordingly, three generic combinations are a) RL circuits, b) RC circuits and c) RLC circuits when supplied from battery are being taken up initially.

RL Circuits: There are two cases in this circuit - a) inductance is fully energized, i.e. no current is flowing through and b) inductance is energized i.e. current is flowing through it. Taking **First Case**, when switch is closed, as per Kirchhoff's Voltage Law (KVL) $= iR + L\frac{di}{dt}$. Accordingly, $\int_{0+}^{i} \frac{1}{E-iR} di = \int_{0}^{t} \frac{dt}{L}$. Taking an intermediate variable u = E - iR it leads to du = -Rdi. Substituting u it resolves into $-\frac{1}{R}\int_{0+u}^{i} du = \frac{1}{L}\int_{0}^{t} dt$. It leads to $\int_{0+u}^{i} du = -\frac{R}{L}\int_{0}^{t} dt$, which on integration is $[\log(E - iR)]_{0}^{i} = -\frac{R}{L}t$, or $\log \frac{E-iR}{E} = -\frac{R}{L}t \rightarrow \frac{E-iR}{E} = e^{-\frac{R}{L}t}$. It simplifies into $I = \frac{E}{R}$ $I = I = \frac{E}{R}$ $I = \frac{E}{R}$. Thus the expression of current reduces to $I = \frac{E}{R}$. Thus the expression of current reduces to $I = \frac{1}{R} (1 - e^{-\frac{t}{T}})$, where $T = \frac{L}{R}$ is the **Time Constant of RL** *circuit*. Likewise, when current (I_0) flowing through *inductor is interrupted* accordingly voltage equation would be $0 = iR + L\frac{di}{dt}$. through and **b**) inductance is energized i.e. current is flowing through it.

interrupted accordingly voltage equation would be $0 = iR + L\frac{di}{dt}$, since there is no voltage source in the circuit. Further, set different set of boundary conditions would exist, such that at $t = 0_+$ current through the circuit is I, and as $t \to \infty$, $i \to 0$, since after switching of the circuit there is no source of current. Accordingly, $\int_{l_0}^{i} \frac{di}{i} = -\frac{R}{L} \int_{0}^{t} dt$. On integration in leads to $\log\left(\frac{i}{l_0}\right) = -\frac{R}{L}t \to i =$

 $I_0 e^{-\frac{R}{L}t}$ and in its standard form $i = I_0 e^{-\frac{t}{T}}$. In the *i-t curve* shown in the figure $I_0 = I$ has been taken. A comparison of the current through RL circuit when switched ON and OFF is shown in a table below.

RC Circuits: There are two cases in this circuit - a) a discharged capacitor is charged, i.e. no voltage across the

capacitor, and **b**) an energized capacitor is discharged. Taking **First Case**, when switch is closed, as per Kirchhoff's Voltage Law (KVL) $E = iR + \frac{q}{c} = R \frac{dq}{dt} + \frac{q}{c} \rightarrow \frac{EC-q}{c} = R \frac{dq}{dt} \rightarrow \frac{1}{EC-q} dq$. Integrating both sides, $\log(EC - q) = K - \frac{1}{RC}t$, here **K** is *integration constant.* It leads to $EC - q = e^{K - \frac{1}{RC}t}$. At $t = 0_+, q = 0$, $EC - 0 = e^K$ since capacitor is fully discharged. It transforms the equation into $EC - q = ECe^{-\frac{1}{RC}t}$, or, $q = EC\left(1 - e^{-\frac{1}{RC}t}\right) = EC\left(1 - e^{-\frac{t}{T}}\right)$, here **T** is the **Time Constant of RC circuit** is T = RC. This equation is analogues

A C Capacitor Discharging into when inductor is switched in the first case, and so will be the charge build-up curve w.r.t time (t). Accordingly, a capacitor in RC circuit would be 63% charged at its Time Constant. Likewise, when Capacitor as per KVL, $0 = iR + \frac{q}{c} \rightarrow R \frac{dq}{dt} = -\frac{q}{c}$. Accordingly, $\log q = -\frac{t}{RC} + K$, or $q = e^{K - \frac{t}{RC}}$. Let, at t = 0, charge on capacitor is Q_0 , then $Q_0 = e^K$ accordingly the capacitor discharge equation transforms into $q = Q_0 e^{-\frac{t}{RC}}$. This equation is also analogues to current discharge equation of RL circuit, and capacitor of a RC circuit shall retain

37% charge at its time constant.

Behaviour of RL and RC Circuit When Switched ON and OFF						
Parameter		RL Ci	ircuit	RC Circuit		
		Switch ON Switch OFF		Switch ON	Switch OFF	
Initial Current at $(t = 0_+)$		i = 0	$i = I_0$	q = 0	$q = Q_0$	
Instantaneous Current		$i = I\left(1 - e^{-\frac{t}{T}}\right)$	$i = I_0 e^{-\frac{t}{T}}$	$q = EC\left(1 - e^{-\frac{t}{T}}\right)$	$q = Q_0 \left(1 - e^{-\frac{t}{T}} \right)$	
At Saturation i.e. at $t \to \infty$		$I = \frac{E}{R}$	$I = \frac{E}{R} \qquad \qquad I = 0 \qquad \qquad Q = EC$		Q = 0	
Poheviour	at $t = 0_+$	Open Circuit	Short Circuit	Short Circuit	Open Circuit $(i=0)$	
Denaviour	at $t \to \infty$	Short Circuit	Open Circuit $(i = 0)$	Open Circuit $(i=0)$	Short Circuit	
Time Constant		$T = \frac{L}{R}$		T = RC		
Current at $t = T$		$i_T = 0.63 I$	$i_T = 0.37 I_0$	$q_T = 0.63 Q$	$q_T = 0.37 Q_0$	

The behavior of RL and RC circuits, elaborated above is summarized below.

RLC Circuits: Having learnt about three basic electrical elements R, L and C and RL, RC circuit, there shall be an obvious curiosity to know about behavior of a circuit having all the three elements are there in the circuit forming an RLC circuit. This also has Two possibilities – **a)** Switching On of the RLC Circuit and **b)** Switching of RLC Circuit. According to KVL, voltage equation in **First Case** would be $E = iR + L\frac{di}{dt} + \frac{q}{c} \rightarrow R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{c}$. Rearranging the

voltage equation, $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E$ is a Second Order Differential Equation and is outside the scope of this manual. Likewise, in **Second Case**, the voltage equation is of the form $\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$, and is again a Second Order Differential Equation. Nevertheless, inquisitive readers are welcome to write through **Contact Us**. RLC Circuit in Second case, by ignoring resistance takes a form $\frac{d^2q}{dt^2} = -\frac{1}{LC}q = 0$, and is of oscillatory nature where charge on capacitor and in turn energy stored in it in the form of electric field reversibly



transferred to inductor in the form of magnetic field and this cycle continues at a frequency $f = \frac{1}{2\pi\sqrt{LC}}$, and is called natural frequency of the oscillatory LC circuit. This is analogous to SHM motion where interchange of potential energy into kinetic energy and vice-versa continues. In LC circuit, resistance R acts, like friction in SHM, to damp the oscillation magnitude, without affecting its natural frequency (f).

Application of Electromagnetism: Electromagnetism has found widest application from industry, domestic, medical, communication and name the field it is there. Nevertheless, limiting these applications to the scope of this manual three basic applications are being elaborated **a**) *Electric Motors* for conversion of electrical energy to mechanical energy, b) Generator for conversion of mechanical energy to electrical energy, c) Galvanometer for measurement of electrical quantities, d) Eddy Currents and e) Induction Coil. These three applications have found utility with many of its variants which are again outside scope of this manual, which is aimed at reinforcing basic concepts.

Electric Motor: Basic laws of electromagnetism discussed so far are being extended to application Ampere's



Force Law seen qualitatively has been brought out in figure. This was generalized by John

Ambrose Fleming in late 19th Century as Fleming's Left Hand Rule (FLHR) and is quite

explicitly shown in the figure. FLHR is anecdote directional relevance of Ampere's Force Law, applicable to force on current carrying conductor placed in a magnetic field. This rule is used to explain direction of rotation Electric



S



Motor. A conceptual representation of electric motor is also shown in the figure.

Force on both the section of coil each conductor of length \overline{l} are carrying equal current I and in uniform magnetic field \overline{B} , Accordingly, force on the conductor as per Amperes Force Law $\frac{\overline{F}}{\overline{l}} = \overline{I} \times \overline{B} \to \overline{F} = l\overline{I} \times \overline{B} = I\overline{l} \times \overline{B}$, here interchange of scalar and vector quantity $l \leftrightarrow \overline{l}$ is valid because current is along the conductor and there is no change in vector form of equation. Accordingly, Two equal, opposite and parallel forces \overline{F} separated by a distance \overline{b} , width of the coil form a couple, and similar to the electric and magnetic dipole and thus, $\overline{T} = \overline{b} \times \overline{F} = \overline{b} \times I\overline{l} \times \overline{B} = I((\overline{b} \times \overline{l}) \times \overline{B}).$

Here, area of the coil $\overline{A} = \overline{l} \times \overline{B}$ and accordingly torque equation transforms into $\overline{\Gamma} = (I\overline{A}) \times \overline{B} = \overline{\mu} \times \overline{B}$, where $\bar{\mu} = I\bar{A}$ is called **Magnetic Dipole Moment** or **Magnetic Moment** of current loop. In case coil has multiple turns (**n**) then $\bar{\mu} = nI\bar{A}$. Since, Torque $\bar{\Gamma}$ is a vector product, as coil turns, torque on the coil, which is maximum when face of the coil is along the magnetic field, which in turn has an angle θ between vectors \overline{A} and \overline{B} equal to 90° since $\sin \theta|_{\theta=90^\circ} = 1$. But, as coils turns, in every rotation θ Ν varies $0^{0} \leq \theta \leq 360^{0}$. This sinusoidal variation, similar to magnitude of a wave, is smoothened by making a magnetic flux radial with innovation as shown in the figure. Here, poles are provided with identical arc shaped cover with a circular core, to make the flus concentric and perpendicular to both

the magnetic surfaces. Inside the core, MLFs complete there path from north pole to south pole. Coil on the core are embedded into it in motors, unlike that shown in the figure, to make the core rotate about its axis, and transfer mechanical energy through shaft at the axis of the core like axle of the paddle of a bicycle. While in *galvanometer*, used for measurement, to be elaborated a little later, the core is fixed, while coils made of fine wire or mounted on a former and detached from the core.

Generator: It is an extension of Case 2, 3 and 4 of Faraday's Law of Electromagnetic Induction. It is a device used to convert mechanical energy into electric energy by changing linkage of magnetic flux w.r.t. a coil mounted on a shaft, coupled to a mechanical device called **prime mover** not shown in the figure.



charges due to protons embedded in nucleus and electrons as (-)ve charges some of which keep revolving around nucleus as bound charges and some of it are like *free electron* cloud performing Brownian motion inside volume of the conductor. Thus when conductor is moved in the magnetic field it experiences electrostatic and electromagnetic forces as per *Lorentz's Force Equation*. Such drift of electrons continues till an equilibrium is produced between Electrostatic Force and Electromagnetic force and thus potential difference between two ends of the conductor is called *EMF*. When, two ends of the conductor are closed through a resistance, the EMF drives current through circuit, which in turns equilibrium of constituents of the Two Forces. This in turns induces



drift of electrons to maintain the *EMF* corresponding to *B*, angular

EMF corresponding to *B*, angular velocity of coil, area of cross-section of coil and its instantaneous position of coil, in accordance with Faraday's Law of Induction. It is to be noted that forces on electrons (-)ve charge and their drift is opposite to the notional direction of forces and current, which is defined for (+)ve charges. Accordingly, correlation between, motion, flux and current is defined by Fleming known as **Fleming's Right Hand Rule (FRHR)**; this in fact is a colloloray of FLHR. A generator producing alternating current of sinusoidal

waveform, known as **Alternator**, is consptualized in the figure. Aleternating Current and circuits shall be elaborated in next section.

Galvanometer: Galvanometer is a sensitive device used for measurement of electrical quantitities based on principle of electric motor, which produces mechanical torque when electric current is supplied through it.Vibration Moving Coil Galvanometer, shown in the figure. Considering the requirement of sensitivity, which is totally an application area is left out while elaborating basic concepts of physics. In brief, it has a light moving coil assembly, made of thin wire mounted on a former, between narrow space magnetic poles, in the shape of circular, and a fixed magnetic core at its center. The coil suspended with the help of an elastic phosphor bronze wire and a spring of conducting material resting on a jewel bearing which is nearly frictionless, to keep it suspended concentrically along the axis of magnetic field. Whenever, current is



Conceptually, it is similar to that in case of electric motor, with only one difference that in motor rotation of coil is produced when current is passed through, while in generator, rotation of coil produces an EMF capable of driving current in the circuit closing the coil, as shown in the figure. Conductor, as discussed in current electricity consists of (+)ve



supplied through the coil, having a rating of a few milli-amperes, it experiences a deflecting torque($\tau_d \propto i$), as a motor, while the phosphor bronze wire and the spring exert a controlling torque($\tau_c \propto \theta$). This leads to eventually, $i \propto \theta$, and current through the galvanometer is calibrated with the angle of deflection, determined geometrically, by observing shift in image of a light beam caused by reflection of mirror, as shown in the figure. This instrument being very delicate cannot handle current and voltages of the order experienced in scientific experiments and various application. In view of this extending range of galvanometer to enable its use as **a**) **Ammeter**, for measurement of current in circuit, and **b**) **Voltmeter**, for measurement of voltage across two terminals is being consolidated in a comparative manner.

Particulars	Ammeter	Voltmeter	
Purpose	Measure Current (<i>I</i>) in amperes in a circuit	Measure Potential Difference (<i>V</i>) in volts across two terminals in a circuit	
Constraint	$I \gg I_g$, here, I_g rated current of galvanometer	Current through Galvanometer on direct connection $\frac{V}{R_g} \gg I_g$, here, R_g is resistance of galvanometer	
Problem Statement	Reduce current through galvanometer $i_l \leq I_g$, where i_l is the limiting value of current for desired range of instrument.	Reduce current through galvanometer $i_l < I_g$, where i_l is the limiting value of current for desired range of instrument.	
Remedy	Use a low resistance R_p in parallel to galvanometer such that $R_p \ll R_g$	Use a low resistance R_s in series galvanometer such that $R_s \gg R_g$	
Schematic Diagram	$ \begin{array}{c} I \\ i_{p} \\ Galvanometer \\ R_{p} \end{array} $	+ve	
Equivalent Circuit	$ \begin{array}{c} I \\ i_{I} \\ R_{g} \\ R_{g} \end{array} $	+ve V $v_g R_g$ i_I i_I $v_g R_g$ i_I i_I $v_g R_g$ i_I i_I $v_g R_g$ i_I	
Calculation on Current Limiting Resistance	 It is solving simple parallel combination of resistances for the given condition i_l ≤ I_g	 It is solving simple series combination of resistances for the given condition <i>i_l</i> ≤ <i>I_g</i> <i>i_L</i> = <i>V</i>/<i>R_s</i> + <i>R_g</i> Hence, for Rated Current (<i>I_R</i>), <i>v_L</i> = <i>R_sI_R</i>/<i>R_s</i> + <i>R_g</i> ≤ <i>I_gR_g</i> Equivalent resistance of Voltmeter <i>R_V</i> = <i>R_s</i> + <i>R_g</i> 	

Tangent Galvanometer is a special purpose variant of *Galvanometer* shall be discussed at the end in Section on Instrumentation.

Eddy Currents: A time varying magnetic field ($\Phi(t)$) of any current carrying coil when intercepted by any external conductive material, it acts like virtual conductive rings forming secondary coil. As per *Faraday's*

Laws of Induction current produced in these secondary coils is called **Eddy Current**. This eddy current is a disadvantage due additional loss of energy and consequent heating and is pertinent to AC circuits, which shall be elaborated in a separate part of this Chapter on Electromagnetics. Determination of eddy current losses, is also beyond scope of this text. The eddy currents have been effectively used in electrical instruments for damping a qualitative illustration of this is shown in the figure. A conductive disc, rotating at its passing through center 'O', has one magnet placed away from the axis with its field



perpendicular to the plane of the disc. When disc is rotated in clockwise direction (cause of action), virtual radial stands of the disc intercept magnetic field and produce an emf across it as per FRHR. The current so produced, in turn intercepts magnetic field, producing a torque on each of the stand as per FLHR.

Induction Coil: Is a simple device of a solenoid wound over a closed magnetic circuit to act as source of high voltage when circuit is interrupted $e = L \frac{di}{dt}$ and act as short-circuit when current through it becomes steady $e = \mathbf{0}|_{i=\text{constant}}$. Further, the Eddy current effect enhances it utility in creating heating. In alternating circuits, inductance has a different behavior which shall be elaborated in next section.

Comparison of Electrostatics, Magnetostatics and Magnetism: It is time to vouch that these three fields are so interconnected that it is impossible to think of either of them in isolation. Nevertheless, an effort has been made to elaborate these topics in rhythmic manner to bring them to a point where, with understanding of vector calculus it shall be possible to explore Electromagnetic Wave Theory and Maxwell's Wave Equations, which are outside scope of this manual. Nevertheless, inquisitive readers are welcome to write through <u>Contact Us</u>.

Electrostatics	Magnetostatics	Electromagnetism
$\bar{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r} \mathbf{N}$ (Coulomb's Law)	$\bar{F} = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2} \hat{r} \mathbf{N}$ (Coulomb's Law)	$\overline{F} = q\overline{E} + q\overline{v} \times \overline{B} \mathbf{N}$ (Lorentz's Force Equation)
Electric Field Intensity at a Point: $\overline{E} = \frac{\overline{F}}{q}$, here \overline{F} is the force experienced by a charge q placed in the Electric Field	Magnetic Field Density at a Point: $\overline{B} = \mu \overline{H}$	Here, $\overline{\mathrm{E}}$ and $\overline{\mathrm{B}}$ as defined in electrostatics and Magnetostatics
Electric Field Density at a Point: $\overline{D} = \varepsilon \overline{E}$	Magnetic Field Density at a Point: $\overline{B} = \frac{\overline{F}}{m}$, here \overline{F} is the force experienced by a pole of strength m placed in the Magnetic Field	
Permittivity of vacuum: $\varepsilon_0 = 8.854 \dots \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$	Permeability of vacuum: $\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$	$\mu_0 \varepsilon_0 = \frac{1}{c^2}$ c=2.99792458 ms ⁻¹ (Velocity of Light) (Maxwell's Wave Equation)

Summary: As one advances into journey into Physics, he finds increasing integration of Mathematics into Physics. The advent of current electricity has opened a new era of not only nature but transforming world through technological developments.

Understandings of Electrostatics, and Current Electricity, in earlier three parts, has been extended into basic concepts of Electromagnetism. Understanding Electromagnetism is slightly different from other topics, where one could observe the phenomenon. In this topic everything happen, but nothing is visible. But, verification of phenomenon of current electricity is through observations of its effects. At this point, some of the inter-related topics are referred to but their elaboration has been deferred, till related concepts are covered. Nevertheless, readers are welcome to raise their inquisitiveness, beyond the contents, through

<u>Contact</u> <u>Us</u>. Likewise, any suggestion or correction considered essential by the readers are welcome; it would be gratefully acknowledged and incorporated suitably

Solving of problems, is an integral part of a deeper journey to make integration and application of concepts intuitive. This is absolutely true for any real life situations, which requires multi-disciplinary knowledge, in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person. References cited below provide an excellent repository of problems. Readers are welcome to pose their difficulties to solve any-problem from anywhere, but only after two attempts to solve. It is our endeavour to stand by upcoming student in their journey to become a scientist, engineer and professional, whatever they choose to be.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics, representative problems from contemporary text books and Question papers from various competitive examinations shall be drawn and supported with necessary guidance to evolve solutions as a dynamic exercise which is contemplated to accelerate the conceptual thought process.

References:

- 1. NCERT; PHYSICS, Text Book for Class XI (Part I and II), and Exemplar Problems.
- 2. भौतिक शास्त्र, कक्षा ११ एवं १२,, मध्य प्रदेश पाठ्यपुस्तक निगम, 2016
- 3. S.L.Loney; The Elements of Statistics and Dynamics: Part 1 Statics and Part 2 Dynamics.
- 4. H.C. Verma; Concepts of Physics, (Vol 1 & 2).
- 3. Resnick, Halliday, Resnick and Krane; Physics (Vol I and II).
- 4. Sears & Zemansky; University Physics with Modern Physics.
- 5. *I.E. Irodov; Problems in General Physics.*

Author is Coordinator of this initiative Gyan-Vigyan Sarita. *e-Mail ID*: subhashjoshi2107@gmail.com