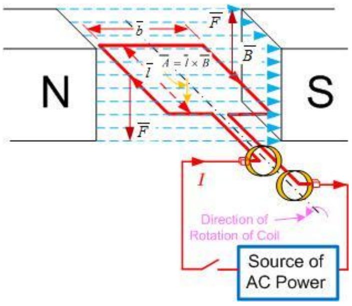
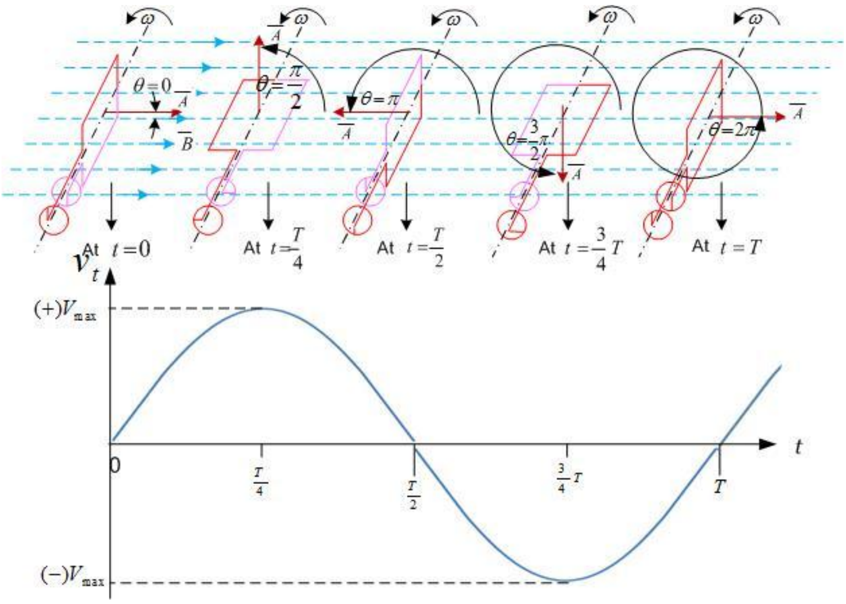
ELECTROMAGNETISM – Part III: Alternating Current and Circuits

Alternating Current has an interesting history of conflict of concepts and convictions between Two great inventors ***Nikola Tesla*** *and* ***Thomson Edison****, who were earlier colleagues,***.** *An international exhibition at Germany, in 1891, where effectiveness of AC in handling power a long distance was demonstrated, Edison was convinced on merits of AC. This event marked* ***End of War of Currents****, and it paved away for further research and development. In present times, despite merits of AC to provide portability and handling of power, there is a threshold beyond which transmission of power in DC mode is preferred. This is an excellent example of theory of Evolution by* **Karl Marx** – ***Thesis ->Antithesis –> Synthesis, which in turn becomes Thesis and cycle continues.*** *As one proceeds corroboration of qualitative aspects and analytical inferences become increasingly important, and that makes the journey thrilling and purposeful. This chapter covers only basics of AC Circuits within the scope of this manual.* Nevertheless, readers inquisitive to know details of AC and DC electrical systems are welcome to write through ***Contact Us***.

Alternating Current: Basics of generator in Electromagnetics, Part-II of Current Electricity, where conversion of mechanical energy into electrical energy has been elaborated. Alternating Current has a specific



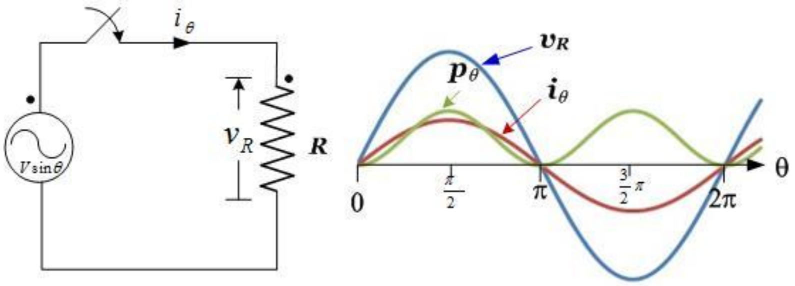
amplitude vs. time variation called sinusoidal waveform, called characteristic waveform, and its generation is at core of AC systems. A coil having N turns if rotates at an angular velocity $\omega = \frac{d\theta}{dt}$ in a uniform magnetic field, its flux linkage changes from $(+)N\emptyset A$ to $(-)N\emptyset A$ and this cycle continues with a periodicity $T = \frac{1}{f} = \frac{2\pi}{\omega}$. Thus according to Faraday-Lenz's Law instantaneous EMF induced in the coil would be, $E_t = -\frac{d\Psi}{dt} = -\frac{d(N\bar{B}\cdot\bar{A})}{dt} = -\frac{d(NBA\cos\theta)}{dt} = -NBA\frac{d(\cos\theta)}{dt}$, since, number of turns (N), Flux Density (B) being uniform and Area (A) , decided by geometry of coil are constant, and hence magnitude of voltage would vary with instantaneous angular position of the coil would be $E_t = NBA\sin\theta\frac{d\theta}{dt} = V_{max}\sin\omega t$. Here, $V_{max} = NBA\omega$, ***Maximum Value*** *of the instantaneous emf or voltage*. This variation of magnitude of voltage at different angular positions of the coil is shown in the figure, for Five discrete positions, with the corresponding graph for continuous variation of voltage w.r.t. time. Accordingly variation of current depending upon characteristic of load, that shall be elaborated a little later. *It is to be noted that unlike DC there are no (+)ve or (-)ve terminals. Both the terminals attain (+)V and (-) V voltages alternately as shown in the figure, and drive current in the circuit. Accordingly, it is called* **Alternating Current.**



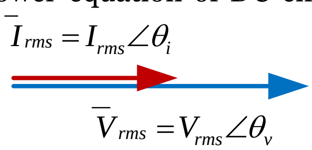
Parametric Quantities of an AC waveform: An AC waveform has Three parametric quantities – **a)** Maximum Value, **b)** Average Value, **c)** Effective Value , and **d)** Form Factor, each of them is being elaborated. **Maximum Value** *is peak of value (unsigned) of the electrical quantity, be it voltage or current*, and is important from the point of maximum mechanical stress that can occur in use of an electrical device. This is pertinent from the point of Ampere's Force Law, which stipulates, dependence of force between two current carrying conductors on the magnitude of the current. Qualitatively average value of voltage, as visible from the waveform over complete cycle is Zero, nevertheless, it is defined over half-time period of the cycle from $t=0 \rightarrow \theta=0$ to $t=\frac{T}{2} \rightarrow \theta=\pi$ and mathematically **Average Value** $V_{av}=\frac{\int_0^{\pi} V_{max}\sin\theta d\theta}{\pi}=\frac{V_{max}[-\cos\theta]_0^{\pi}}{\pi}=\frac{V_{max}}{\pi/2}$. *DC measuring instruments, which are not based on the principle of power content of electrical quantity are capable of measuring Average Value.* Concept of ***Effective Value*** comes from heating effect, which as per definition of electrical power is proportional to square of the voltage or current and, therefore, effect of signed value of the electrical quantity vanishes over complete cycle. Accordingly, can be taken over any period, $t \rightarrow \theta$ to $t=t+\frac{T}{2} \rightarrow \theta=\theta+\pi$. Therefore, mathematically, $V_{eff}^2=\frac{1}{\pi}\int_{\theta}^{\theta+\pi}(V_{max}\sin\theta)^2 d\theta=\frac{V_{max}^2}{\pi}\int_{\theta}^{\theta+\pi}\sin^2\theta\, d\theta=\frac{V_{max}^2}{2\pi}\int_{\theta}^{\theta+\pi}(1-\cos 2\theta)d\theta$. On integration it leads to $V_{eff}^2=\frac{V_{max}^2}{2\pi}\left[\theta-\frac{\sin 2\theta}{2}\right]_{\theta}^{\theta+\pi}=\frac{V_{max}^2}{2}$, or $V_{eff}=\frac{V_{max}}{\sqrt{2}}$, and it leads to eventually it is square-root of mean of the square of the electrical quantity and hence, this ***Effective Value*** *is also called* ***Root Mean Square (RMS) Value*** *of the voltage or current.* ***Concept of rms value*** *of voltage and current would become more explicit, a little later, when voltage, current and power of AC circuits is elaborated.* Ratio of RMS value and average value of an electrical quantity, is called **Form Factor** (*FF*) and is representative of degree of distortion of actual waveform of electrical voltage and current from Sinusoidal waveform. *Form Factor for sinusoidal waveform is* $FF=\frac{V_{rms}}{V_{av}}=\frac{V_{max}/\sqrt{2}}{V_{max}/\frac{\pi}{2}}=\frac{\pi}{2\sqrt{2}}=1.11$, *and it changes on distortion in waveform.* Readers inquisitive to know details of AC and DC electrical systems are welcome to write through ***Contact Us***.

AC Circuits: These are classified into – **a)** VR circuit having a sinusoidal voltage source connected across resistance only, **b)** VR circuit having a sinusoidal voltage source connected across inductance only, **c)** VR circuit having a sinusoidal voltage source connected across capacitance only, and **d)** VR circuit having a sinusoidal voltage source connected across combination of resistance, inductance and capacitance, there three electrical elements in turn can be in series, parallel and hybrid formations.

VR Circuit: A circuit having an AC source ($v_t=V\sin\omega t \rightarrow v_{\theta}=V\sin\theta$) with a resistance R connected across it through a switch, as shown in the figure. In this analysis, transients in electrical circuits during switching-in/off of circuit are ignored and steady-state condition, when current waveform has stabilized have been considered. Instantaneous voltage across resistance, as per Ohm's Law shall be $v_{\theta}=i_{\theta}R$. As per Kirchhoff's Voltage Law (KVL) $V\sin\omega t-i_tR=0 \rightarrow V\sin\theta=i_{\theta}R$, therefore, current through resistance (R) shall be $i_{\theta}=\frac{V}{R}\sin\theta$. Thus current shall follow the same waveform as that of voltage with 0^+ crossing at the same instance, with a difference only in magnitude which is decided by $\frac{V}{R}$. Thus it is seen that value of resistance (R) remains unchanged. These voltage and current waveforms are called to be in-phase, as elaborated in Waves and Motion chapter.

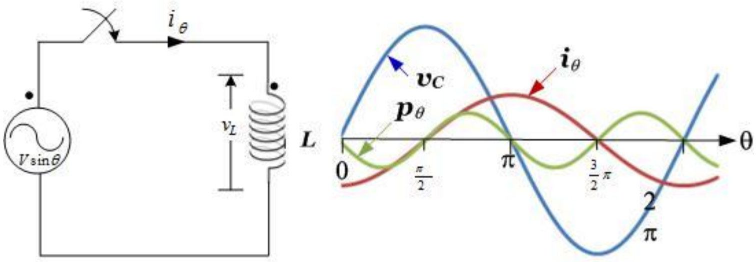


Instantaneous power in the ***VR circuit*** is $p_{\theta}=v_{\theta}i_{\theta}=(V\sin\theta)\left(\frac{V}{R}\sin\theta\right)=\frac{V^2}{R}\sin^2\theta$. Thus ***average power*** *utilized by the resistance* works out to be $P=\frac{1}{2\pi}\cdot\frac{V^2}{R}\int_0^{2\pi}\sin^2\theta\, d\theta=\frac{V^2}{2\pi R}\int_0^{2\pi}\frac{1}{2}(1-\cos 2\theta)d\theta$. On integration it leads to $\frac{V^2}{4\pi R}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{2\pi}=\frac{V^2}{4\pi R}\cdot 2\pi=\frac{V^2}{2R}=\frac{V}{\sqrt{2}}\cdot\frac{\frac{V}{\sqrt{2}}}{R}$. This is evident from the waveform of p_{θ} which is on (+)ve over the complete cycle. It leads to $\boldsymbol{P}=V_{rms}\cdot I_{rms}$. ***Thus effective power of a resistive circuit is product of rms values of voltage and current***. This is analogues to power equation of DC circuit where instead of V_{DC} and I_{DC}, values used are V_{rms} and I_{rms} , respectively.

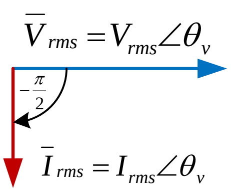


Voltage and current in AC circuit can also be represented as vectors as shown in the figure, accordingly for in-phase current across resistor, it complies with power equal to dot product of Voltage and Current, analog of Force and Displacement, respectively. Thus, $P=\bar{V}_{rms}\cdot\bar{V}_{rms}=(V_{rms}\angle\theta_v)\cdot(I_{rms}\angle\theta_i)=V_{rms}V_{rms}\cos(\theta_v-\theta_i)$. It leads to $P=V_{rms}V_{rms}|_{\theta_v=\theta_i}$.

V-L Circuit: Analysis of V-L circuit in AC with a voltage source ($v_t = V\sin\omega t$) is identical to that of VR circuit except that voltage equation, as per KVL, takes a different form due to inductive element (L). Voltage drop across inductance and Faraday-Lenz's Law $v_L = L\frac{di}{dt}$, and this absorbs (-)ve sign since it is voltage drop which tends to oppose the current through it, and accordingly as per KVL, $v_t - L\frac{di_t}{dt} = 0$. Accordingly, the equation becomes $\frac{di_t}{dt} = \frac{V}{L}\sin\omega t \rightarrow i_t = \frac{V}{L}\int \sin\omega t\, d\theta t = -\frac{V}{\omega L}\cos\omega t$. It leads to $i_t = \frac{V}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)\Big|_{X_L=\omega L}$. It leads to Three inferences – **a)** *current waveforms lags behind the voltage waveform by an angle* $\frac{\pi}{2}$ *radians (=90$^{o.}$)*, or conversely voltage leading current by an angle *90^o*, called ***Phase Angle*** *of current w.r.t. voltage in VL circuit*, **b)** *instead of resistance (R) another factor* ***Inductive Impedance*** ($X_L = \omega L$)*comes into play*. In this, L is dependent on geometric and magnetic properties of the ***Inductance,*** *while* $\omega (= 2\pi f)$, depends upon frequency of the AC voltage, **c)** *Impedance of inductor is* ***directly proportional to the frequency***.

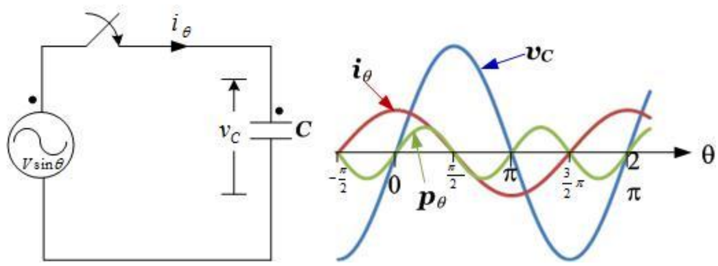


Instantaneous power in the ***V-L circuit*** is $p_t = v_t i_t = (V\sin\omega t)\left(\frac{V}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)\right) = \frac{V^2}{X_L}\sin\omega t\cos\omega t$. Thus ***average power*** *utilized by the inductance* works out to be $P = \frac{1}{2\pi}\cdot\frac{V^2}{X_L}\int_0^T \sin 2\omega t\, dt = \frac{V^2}{4\pi R}[\cos 2\omega t]_0^T = 0$. It leads to a conclusion that, though instantaneous power of an inductor is not Zero, but power consumption by an inductor over a cycle is Zero. This is evident from the waveform of instantaneous power (p_θ), which completes one cycle during a period $\frac{T}{2} = \pi$, leading to the above analytical conclusion. A qualitative justification for Zero power of inductor is that inductor exchanges energy stored in the form of magnetic field alternately with the source. Nevertheless, *since current is drawn by the inductor, product of V_{rms} and I_{rms} is called Apparent Power* $P_{app} (= V_{rms}\cdot I_{rms})$.



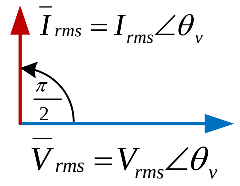
Representing voltage and current vectors in V-L circuit, analogous to that in V-R circuit, current vector is shown in figure lagging, the voltage vector, by a phase difference of $\frac{\pi}{2}$, in accordance with the above stated analytical inference. Accordingly, power being equal to dot product of Voltage and Current vectors $P = \bar{V}_{rms}\cdot\bar{V}_{rms} = (V_{rms}\angle\theta_v)\cdot(I_{rms}\angle\theta_i)$. It leads to $P = V_{rms}V_{rms}\cos\left(\frac{\pi}{2}\right)\Big|_{\theta_v-\theta_i=\frac{\pi}{2}} = 0$.

V-C Circuit: Analysis of V-C circuit in AC is identical to that of V-L, circuit except that voltage equation, as KVL, is in context of capacitive element (C). Voltage across capacitor $v_c = v_\theta = \frac{q_\theta}{C}$ and as per KVL $V\sin\theta - v_c = 0 \rightarrow V\sin\theta = \frac{q_\theta}{C}$. In differentiating w.r.t. t it leads to $(\omega C)V\cos\theta = i_\theta$ since $i_\theta = \frac{d}{dt}q_\theta$. It $i_\theta = \frac{V}{\frac{1}{\omega C}}\sin\left(\theta + \frac{\pi}{2}\right) = \frac{V}{X_C}\sin\left(\theta + \frac{\pi}{2}\right)\Big|_{X_C=\frac{1}{\omega C}}$. In time domain the equation becomes $i_t = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$. It leads to Three inferences – **a)** *current waveforms leads the voltage waveform by an angle* $\frac{\pi}{2}$ *radians (=90$^{o.}$)*, or conversely voltage lagging current by an angle *90^o*, called ***Phase Angle*** *of current w.r.t. voltage in VL circuit*, **b)** *Like inductive impedance, in case of capacitor another factor called* ***Capacitive Impedance*** ($X_C = \frac{1}{\omega C}$)*comes into play*. In this, C is dependent on geometric and magnetic properties of the ***Capacitance***, *while* ***ω*** $\rightarrow f$, depends upon frequency of the AC voltage, **c)** *Impedance of capacitor is* ***inversely proportional to the frequency***.



Instantaneous power in the ***V-C circuit*** is $p_t = v_t i_t = (V \sin \omega t)\left(\frac{V}{X_C}\sin\left(\omega t+\frac{\pi}{2}\right)\right) = \frac{V^2}{X_L}\sin \omega t \cos \omega t$. Thus ***average power*** *utilized by the capacitance* works out to be $P = \frac{1}{2\pi}\cdot\frac{V^2}{X_L}\int_0^T \sin 2\omega t\, dt = \frac{V^2}{4\pi R}[\cos 2\omega t]_0^T = 0$, and is equivalent to that derived for inductor. It leads to similar conclusion that, though instantaneous power of an capacitor is not Zero, but power consumption by an capacitor over a cycle is Zero. Capacitors exchanges energy stored in the form of electric field is alternately with the source. This is evident from the waveform of instantaneous power (p_θ), which completes one cycle during a period $\frac{T}{2} = \pi$, leading to the above analytical conclusion.

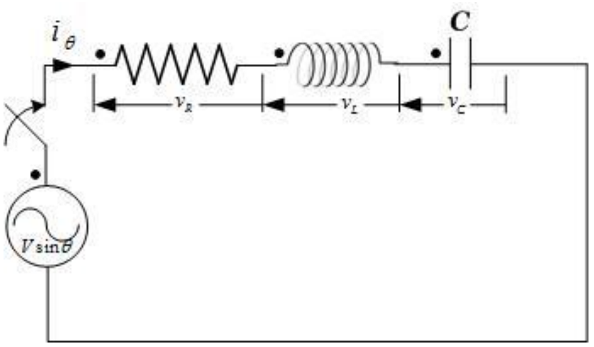
Since, ***phase difference*** *between current with respect to voltage is (+) $\frac{\pi}{2}$, while in inductor it is (-) $\frac{\pi}{2}$, the instantaneous power waveforms in inductor and capacitor are anti-phase. This can be observed by comparing waveforms of p_t of V-L and V-C circuits shown above. Thus it forms an interesting case of having Resistance (R), Inductance (L) and Capacitance (C) in varying proportions i.e. RLC circuit, generally encountered in AC circuits, and shall be elaborated a little later.*



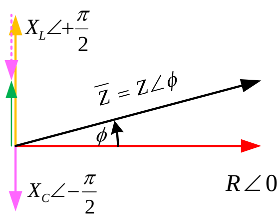
Representing voltage and current vectors in V-C circuit, on the lines of V-L circuit, current vector is, shown in figure, leading the voltage vector by a phase difference of $\frac{\pi}{2}$, in accordance with the above stated analytical inference. Accordingly, power being equal to dot product of Voltage and Current vectors $P = \bar{V}_{rms}\cdot\bar{V}_{rms} = (V_{rms}\angle\theta_v)\cdot(I_{rms}\angle\theta_i) = V_{rms}V_{rms}\cos\left(\frac{\pi}{2}\right)\Big|_{\theta_v-\theta_i=\frac{\pi}{2}} = 0$.

V-RLC Circuit: In this all the three elements, resistance, inductance and capacitance are there, in various formations. Initially, a simplest case of series combination of the three elements is shown in figure, for analysis. *In elaboration of V-L and V-C circuit current lagging or leading voltage, respectively, or conversely voltage leading or lagging current, respectively was stated. Relevance of this statement shall become explicit in forgoing analysis.*

In series combination of V-RLC current flowing through each element, in vector form, is $\bar{I} = I\angle\theta_i$, accordingly



voltage across resistance (R) shall be $\bar{V}_R = \bar{I}R$, $\bar{V}_R = (I\angle\theta_i)R\angle 0 = IR\angle\theta_i$, and is in accordance with the inference in VR circuit. Likewise, vectorially, voltage across inductance (L) is $\bar{V}_L = (I\angle\theta_i)\left(\omega L\angle\frac{\pi}{2}\right) = IX_L\angle\left(\theta_i+\frac{\pi}{2}\right)$, this too is in conformity of phase-difference between current and voltage in VL circuit. Similarly, voltage across capacitance (C) is $\bar{V}_C = (I\angle\theta_i)\left(\frac{1}{\omega C}\angle-\frac{\pi}{2}\right)$. Or, $\bar{V}_C = IX_C\angle\left(\theta_i-\frac{\pi}{2}\right)$, this too goes along with the inference of phase-difference between current and voltage in VL circuit. This analytical resemblance with inference brought out in individual circuit would not have been possible without considering $R\angle 0$ as reference vector and inductance and capacitance as vectors leading $\left(X_L\angle+\frac{\pi}{2}\right)$and lagging $\left(X_C\angle-\frac{\pi}{2}\right)$ w.r.t reference vector. Thus, source voltage vector $\bar{V}$, as per KVL, eventually shall be



$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$. It leads to $\bar{V} = \bar{V}_R = IR\angle\theta_i + IX_L\angle\left(\theta_i+\frac{\pi}{2}\right) + IX_C\angle\left(\theta_i-\frac{\pi}{2}\right) = I\angle\theta_i Z\angle\phi$. Here, magnitude of total impedance of RLC series combination is $Z = \sqrt{R^2+(X_L-X_C)^2}$, and phase-angle $\phi = \tan^{-1}\left(\frac{X_L-X_C}{R}\right)$ a complex quantity $\bar{Z} = R + j(X_L - X_c)$. It is to be noted that *in analysis of AC circuits while representing of complex quantity* $\boldsymbol{j}$ *is used instead of* $\boldsymbol{i} = \sqrt{-1}$ *as in algebra. This difference is attributed to electrical nature of inductance and capacitance drawing current lagging or leading by an angle $\frac{\pi}{2}$, elaborated above, and not the algebraic meaning, despite mathematical resemblance between them.* Thus complex form of electrical quantities is conceptually different from complex numbers, despite identical mathematical treatment. This facilitates representation of complex value of electrical element in Cartesian and exponential form and for resistance it is $\bar{R} = R + j0 = Re^{j0}$, for inductance it is $\bar{X}_L = 0 + jX_L = X_L e^{j\frac{\pi}{2}}$, for capacitance $\bar{X}_C = 0 - jX_C = X_C e^{-j\frac{\pi}{2}}$ and for total impedance of the series combination it is $\bar{Z} = R + j(X_L - X_c) = Ze^{j\phi}$. Accordingly, for determination of phase-angle of current vector $I\angle\theta_i$ in reference to source voltage $\bar{V} = V\angle 0$ is by simple division of complex quantities and it is $I\angle\theta_i = \frac{\bar{V}}{\bar{Z}} = \frac{Ve^{j0}}{Ze^{j\phi}} = \frac{V}{Z}e^{-j\phi}$. Conversely, if voltage vector is $V\angle\theta_v$, then current vector is $I\angle\theta_i = \frac{\bar{V}}{\bar{Z}} = \frac{Ve^{j\theta_v}}{Ze^{j\phi}} = \frac{V}{Z}e^{-j\phi}$Thus magnitude of current $I = \frac{V}{Z}$. In this case $\theta_i = \theta_v - \phi$, and power drawn by the circuit $p = \bar{V} \cdot \bar{I} = VI\cos\phi$. This angle ϕ is called ***Impedance Angle*** and also as ***Power Factor*** of the circuit. *This is also expressed as simple product of complex voltage and current as* $\boldsymbol{p = \bar{V}\bar{I}^*} \equiv \boldsymbol{\bar{V} \cdot \bar{I}}$, *here, voltage is* $\bar{V} = Ve^{j\theta_v}$ *and* ***Conjugate Current*** $\bar{I}^* = Ie^{-j\theta_i}$, *is taken instead of current* ($\bar{I} = Ie^{j\theta_i}$), *for mathematical validity.*

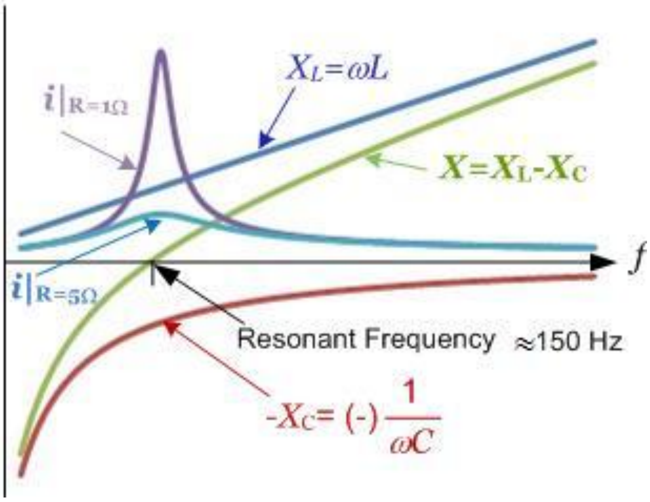
This case of RLC combination leads to three situations **a)** Total Resistive Impedance or Load occurs when $X_L = X_C$. Since impedances X_L and X_C electrically out of phase, they cancel each other and $Z\angle\phi = R\angle 0$ and therefore power factor of load is Unity ($\cos 0 = 1$), i.e. $\emptyset = 0$ **b)** Inductive Load, when $X_L > X_C$ and this case inductance is partially cancelled by capacitance and $Z\angle\phi = \sqrt{R^2 + (X_L - X_c)^2}\angle\tan^{-1}\frac{X_L - X_c}{R}$,where $\phi = \tan^{-1}\frac{X_L - X_c}{R}$, i.e. $0 < \emptyset < \frac{\pi}{2}$, **c)** Capacitive load , when $X_L < X_C$ and this case capacitance is partially cancelled by inductance and $Z\angle\phi = \sqrt{R^2 + (X_L - X_c)^2}\angle\tan^{-1}\left(-\frac{X_L - X_c}{R}\right)$,where $\phi = \tan^{-1}\left(-\frac{X_L - X_c}{R}\right)$, , i.e. $0 > \emptyset > -\frac{\pi}{2}$ or , $\emptyset$ is –ve in the range.

Summary of AC Circuits

V-R Circuit	V-L Circuit	V-C Circuit	V-RLC Circuit
$\boldsymbol{\bar{I}} = \frac{\boldsymbol{\bar{V}}}{\boldsymbol{\bar{R}}} = \frac{\boldsymbol{V}\angle\theta_v}{\boldsymbol{R}\angle 0} = \frac{\boldsymbol{V}}{\boldsymbol{R}}\angle\theta_v$	$\boldsymbol{\bar{I}} = \frac{\boldsymbol{\bar{V}}}{\boldsymbol{\bar{X}_L}} = \frac{\boldsymbol{V}\angle\theta_v}{\boldsymbol{X_L}\angle\frac{\pi}{2}}$ $= \frac{\boldsymbol{V}}{\boldsymbol{X_L}}\angle\left(\theta_v - \frac{\pi}{2}\right)$	$\boldsymbol{\bar{I}} = \frac{\boldsymbol{\bar{V}}}{\boldsymbol{\bar{X}_C}} = \frac{\boldsymbol{V}\angle\theta_v}{\boldsymbol{X_C}\angle -\frac{\pi}{2}}$ $= \frac{\boldsymbol{V}}{\boldsymbol{X_C}}\angle\left(\theta_v + \frac{\pi}{2}\right)$	$\boldsymbol{\bar{I}} = \frac{\boldsymbol{\bar{V}}}{\boldsymbol{\bar{Z}}} = \frac{\boldsymbol{V}\angle\theta_v}{\boldsymbol{Z}\angle\phi}$ $= \frac{\boldsymbol{V}}{\boldsymbol{Z}}\angle(\theta_v - \phi)$
Current in phase with voltage	Current in lagging voltage at an angle $\frac{\pi}{2}$	Current in leading voltage at an angle $\frac{\pi}{2}$	Current could be lagging voltage at an angle ϕ. • ϕ is (+)ve if $X_L > X_C$ • ϕ is (-)ve if $X_L < X_C$ • ϕ is Zero if $X_L = X_C$
$\bar{Z} = R + j0 = Re^{j0}$	$\bar{Z} = 0 + jX_L = X_L e^{j\frac{\pi}{2}}$	$\bar{Z} = 0 - jX_C = X_C e^{-j\frac{\pi}{2}}$	$\bar{Z} = R + j(X_L - X_C)$ $= \sqrt{R^2 + (X_L - X_c)^2}e^{j\phi}$; $\phi = \angle\tan^{-1}\frac{X_L - X_c}{R}$
Power Factor=1	**Power Factor =0**	**Power Factor =0**	**Power Factor =cos ϕ**

Resonance and Damping: Behaviour of a typical series of a RLC circuit at different frequencies is shown in the figure, where $L = 10$mH and $L = 100$μF. Variation of X_L, X_C and X with frequency over a range from 50 Hz to 500 Hz has been plotted. Corresponding variation of current with Two values of resistance $R = 1\Omega$ and $R = 5\Omega$, from a source of 50V, with variable frequency, has also been shown in the figure. Equivalent reactance of the circuit is $X = X_L - X_C$, where variation in $X_L = \omega L = 2\pi fL$; it is proportional to frequency, while variation in $X_C = \frac{1}{2\pi f}$; it is inversely proportional to frequency, and being anti-phase to X_L is shown in

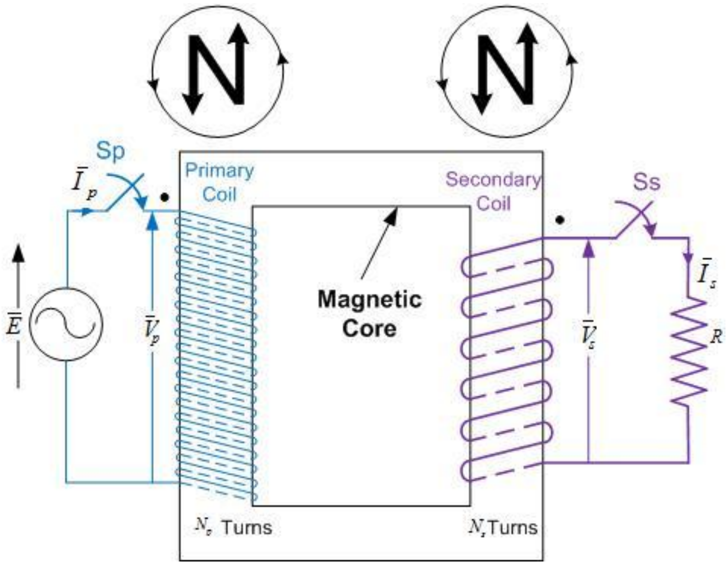
VI[th] quadrant. The point of intersection of X with frequency axis is $2\pi f L = \frac{1}{2\pi f C} \rightarrow f = \frac{1}{2\pi\sqrt{LC}}$ and is called **Frequency of Resonance**. Adjusting value of inductance or capacitance in the circuit to obtain any desired resonant frequency is called **Tuning of Circuit**. *At resonant frequency, there is cyclic exchange of energy in magnetic field in inductance is completely with the energy in electric field of capacitor, and no energy is exchanged by either the inductor or the capacitor with the source.* Nevertheless, energy dissipated by resistance due to flow of current through it is supplied by the source, and eventually power factor of the circuit at resonant frequency is Unity. In the instant case resonant frequency is ≈ 150 Hz.



It is to be noted that magnitude of current increases sharply with the decrease of resistance R in the circuit. Hypothetically, in the event of $R \rightarrow 0$, current drawn by the circuit would tend to ∞, and capable of damage to the circuit.

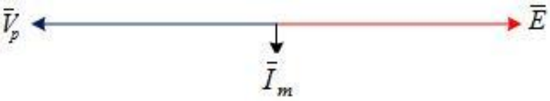
Resonant circuits have wide application in not only in electronic and communication, but also in operation and safety of power system.

Transformer: This is the most versatile equipment in electrical systems, which helps in transfer of power from point of generation, at low voltage, to a distant point at high voltage. Since power $P = VI$, therefore, with the increase of voltage there is decrease of current. This low current offers advantage of saving in cost of material, and power loss $(= I^2R)$. *Transformer is a static device and an excellent application of the principle of mutual inductance.* An ideal Two-winding transformer is shown in the figure mounted on a magnetic core. The winding connected to source is called **Primary Coil**, and the other connected to load (R) us called **Secondary Coil**. Principle of transformer is analyzed in three cases.



Case I: Primary coil is disconnected from source,(switch **Sp** is open). In this case since there is no flow current, there would be no flux and therefore it is just an inactive core coil assembly, where switch connecting load (**Ss**) is open or closed does not make any difference.

Case II: Switch '**Ss**' is open and switch '**Sp**' is closed and transients have settled. In this case applying KVL, loop voltage equation would be $\bar{E} - \bar{V}_p = \bar{E} = \bar{I}_m \bar{X}_p = 0$, here, I_m is the magnetizing current and vectorially it can be expressed as shown in the figure. Magnetizing current is generally very small, while magnitude of Primary voltage is equal to source voltage i.e. $V_p \approx E$. It leads to $V_p = -\frac{d\Psi}{dt} = -N_p\frac{d\emptyset}{dt}$. Since E is sinusoidal and hence corresponding ∅ would also be sinusoidal. Accordingly, coefficient of maximum ∅, which corresponds E, would vary between in the range of -1 to 1. Moreover, in transformer, primary coil is mounted on a magnetic core, closed in shape, it would have very small of coil reluctance ($\Re$), considered to be negligible and conversely relative permeability is very large. Thus, current (I_m) through the coil would be negligible since $\emptyset = \frac{N_p I_m}{\Re}$. In any case $I_m \neq 0$, for V_p to exist, otherwise it would reduce to Case I. Ideally, magnetic flux ∅ would take a closed path through magnetic core, and hence entire flux would also link with



the secondary coil. Accordingly, voltage induced in secondary coil would be $V_p = -N_s\frac{d\emptyset}{dt}$. In ideal transformer, resistance of primary coil is negligible as compared to its inductance and hence I_m would lag behind applied voltage by an angle 90^0, and is shown in the figure. Voltage equation for primary and secondary coils have a common factor (-)$\frac{d\emptyset}{dt}$, which is constant since voltage of source E is constant. Thus, it leads to an important inference $\frac{V_p}{V_s} = \frac{N_p}{N_s}$ called ***Transformation Ratio (TR)***, and $\bar{V}_p$ and $\bar{V}_s$ are in phase.

Case III: Switch **Ss** is closed on resistance R, and transients have settled down. Since, load connected to the secondary coil is resistive, it would draw a current $\bar{I}_s = \frac{\bar{V}_s}{R}$ in phase with $\bar{V}_p$. In analysis of transformer, voltage of secondary coil is normalized by taking $TR = 1$. This eventually leads to $V_p \rightarrow V_s$, and, therefore, to avoid confusion coil voltages are represented as V. In an additional modification in vector diagram for this case I_m is ignored being negligible. Once current is supplied to R by secondary coil, as per Lenz's Law it would tend to cause it generation i.e. $\emptyset$. This would in turn reduce $\emptyset$ in the core and as a consequence, V_p would also reduce. This would cause an increase in current in the primary coil in accordance with voltage equation $\frac{\bar{E}-\bar{V}_p}{\bar{X}_p} = \bar{I}_p$. This increase would correspond to cancellation of flux by current in secondary coil and to restore flux in the core to its initial value $\emptyset$. Since, $\emptyset = \frac{MMF}{\Re}$, this can happen only if $N_pI_p = N_sI_s$, and *is called* ***MMF balance*** *caused by current through windings of transformer*. It leads to $\frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{TR}$. Using the TR determined in case II it turns to $\frac{V_p}{V_s} = \frac{I_s}{I_p} = TR$, or, $V_pI_p = V_sI_p = S$. Here, **S** is called **Capacity of Transformer,** an AC equipment**,** and its unit is **Volt-Ampere or Kilovolt-Ampere** and not Watts or Kilowatt.

An important point needs elaboration is determination of the notional polarity of primary and secondary coils represented by a dot (·) in the figure of transformer . Notional polarity of the primary coil is the point of input, while notional polarity of the secondary coil depends upon the direction of its winding, relative to the primary coil. In the given configuration, seeing from the top, direction of winding of primary coil is in anti-clockwise direction, while secondary coil is in clock-wise direction . Hence, notional current through primary coil would produce North Pole at the top. In order to maintain MMF Balance, current through secondary coil shall have to be such that create flux which opposes flux created by current in primary winding. This is possible only when direction current through of secondary coil is also anti-clockwise. Eventually with this direction of current in secondary coil current would leave it from top and enter from bottom. This leads to node of secondary coil on the top as shown with the dot (·). This concept of node is extremely useful in analysis of Mutli-Winding Transformers, which is outside the scope of this manual. Readers inquisitive readers are welcome to write through ***Contact Us***.

Types of Transformer: Transformers are of various types and this classification depends upon their transformation ratio, voltage rating, construction, and uses. At this stage classification is made on transformation ratio and its uses.

In case $\boldsymbol{TR > 1}$ it would lead to secondary voltage, across secondary coil proportionately lower than primary voltage, across primary coil, and is called ***Step-Down Transformer***. Whereas, for $TR < 1$ it will provide secondary voltage higher than primary voltage and is called ***Step-Up Transformer***.

Transformers find application from electronic devices to electrical network used for supply of power all across the country and even trans-countries. In this context, transformer used for converting voltage level of electrical power, critical application and most widely in use, are called **Power Transformers**.

Efficiency of Transformer: As per definition of efficiency (η), it is ratio of out power to input power, and is equally valid for transformer also and for an ideal transformer it should be one, since this idealization is based on coils having Zero or negligible resistance. Nevertheless, this assumption is not valid in practical cases and hence $\eta = \frac{V_sI_s\cos\emptyset_s}{V_pI_p\cos\emptyset_p}$, which involves impedance loads $\bar{Z} = R + jX$, where $R \neq 0$, and $X \neq 0$, and so also both primary and secondary coils have resistances are non-Zero. This analysis is beyond scope of this manual. Nevertheless, inquisitive readers welcome to know more about it through ***Contact Us***

Some interesting questions: Transformers form the core of electrical network and every beginner confronts two conceptual questions – **a)** *Can transformer connected to AC Circuit? and* **b)** *Can step-up transformer be used infinitely, in a cascaded formation, to compensate voltage drop in supply circuit?*

Answers to both the questions are NO. In first case reason is that transformer is an inductive device which acts as short-circuit to DC supply. Therefore, after the transients settle down, it would offer a resistance corresponding to that of its primary coil. In turn it would draw a dangerously very high current. Reason for the second case is that a transformer, as the name suggests, transfers power from primary to secondary coil, through magnetic linking provided by the core. Thus power drawn from the base source, is dependent on its capacity. Moreover, it is not possible to have an ideal network with Zero Resistance-Zero Loss and therefore, it cannot breach power capacity of the source. Along with this, secondary current reflected on primary would tend to add up with every stage of cascading of transformer. Eventually, at some stage either it would exceed VA capacity of either feeding transformer or the base source of power. Thus it would prohibit infinite cascading.

Summary: *This part concludes Electricity and Magnetism, with AC Circuits and transformers; the concepts most widely in use in day-to-day life.Understandings of Electricity and Magnetism needs to be reviewed since phenomenon elaborated in this are totally conceptual and not available for direct observation, It is only effects and there inferences that goes to firm up the concepts. Inquisitive readers are welcome to raise their quarries, doubts or suggestions and keenness beyond the contents, through* ***Contact Us***.

Solving of problems, is an integral part of a deeper journey to make integration and application of concepts intuitive. This is absolutely true for any real life situations, which requires multi-disciplinary knowledge, in skill for evolving solution. Thus, problem solving process is more a conditioning of the thought process, rather than just learning the subject. Practice with wide range of problems is the only pre-requisite to develop proficiency and speed of problem solving, and making formulations more intuitive rather than a burden on memory, as much as overall personality of a person. References cited below provide an excellent repository of problems. Readers are welcome to pose their difficulties to solve any-problem from anywhere, but only after two attempts to solve. It is our endeavour to stand by upcoming student in their journey to become a scientist, engineer and professional, whatever they choose to be.

Looking forward, these articles are being integrated into Mentors' Manual. After completion of series of such articles on Physics, representative problems from contemporary text books and Question papers from various competitive examinations shall be drawn and supported with necessary guidance to evolve solutions as a dynamic exercise which is contemplated to accelerate the conceptual thought process.

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Author is Coordinator of this initiative Gyan-Vigyan Sarita. ***e-Mail ID***: *subhashjoshi2107@gmail.com*