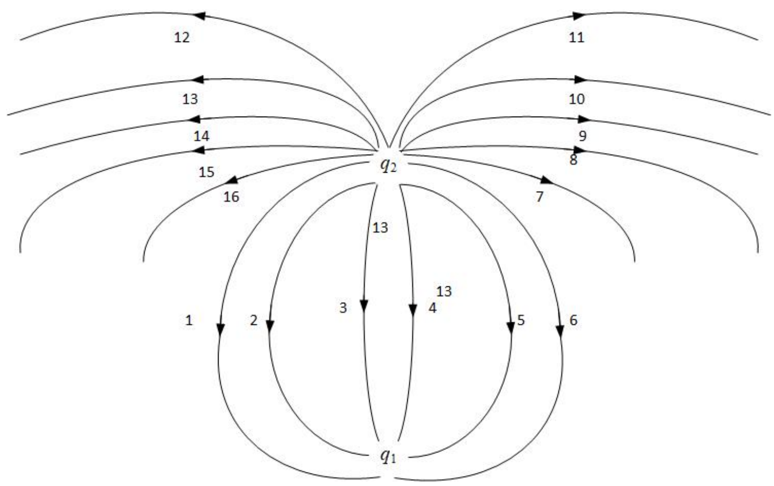
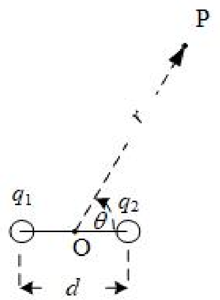
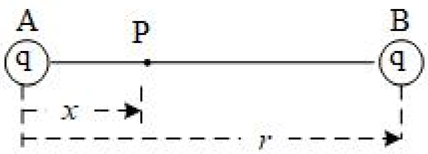
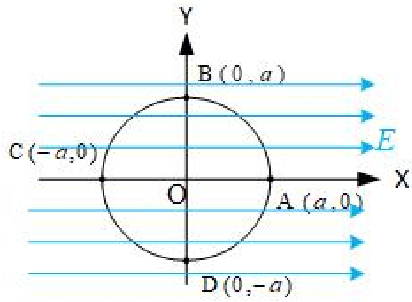
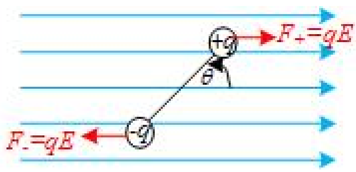
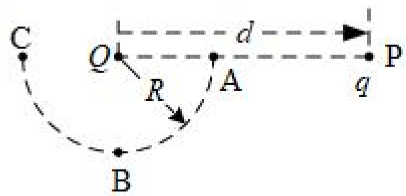
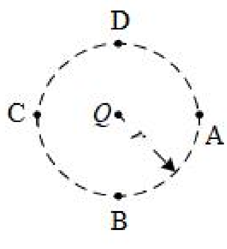
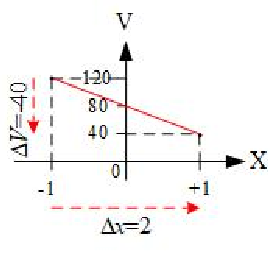
Electromagnetism: Electric Field and Potential – Typical Questions (Illustrations Only)

I-1	Charge of electron and proton are quantitatively equal in magnitude but qualitatively different and hence assigned opposite signs. Therefore, difference in magnitude of their charges cannot be determined arithmetically. Hence, **answer is No.**
I-2	Given is a system of two charged particles and force between them as per Coulomb's Law would be $\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r}$. Here, $4\pi\varepsilon_0$is a constant, q_1 and q_2 are magnitudes of the two charges and their product is (+) ve for like charge and (-)ve for unlike charges, r is the separation between the two charges and $\hat{r}$ is unit direction vector of separation. Thus, the net magnitude of force between the two charges is absolute and based on values q_1 , q_2 and r. Therefore, the only magnitude of the force is $= \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ and there is no other value and hence it can belled as lower limit of the force. Hence, **answer is yes.**
I-3	As per Coulomb's law magnitude of the force between two charged particles A and B having equal charges is $F = \frac{q^2}{4\pi\varepsilon_0 r^2}$. When A is slightly displaced towards B, displacement between the charges would reduce and hence as per mathematical expression, force would also increase but slightly. Thus**, answer to first part is yes.** Now, the displacement is sensed through electrostatic field which is sensed by the other charge at speed of light. Hence, delay would be there, but insignificant. Thus**, answer to first part is no.**
I-4	Gravitational field is generated by mass and results in mutual gravitational force with another mass. While, electric field is produced by electric charge and results into mutual electrostatic force with another charge. But, gravitational field cannot influence any massless charge. Likewise, an electric field cannot influence any uncharged mass. Thus, the two fields are characteristically different and hence cannot be added vectorially. Hence, **answer is no**.
I-5	Phonograph record is cleaned by rubbing it with an insulating cloth. In the process of friction transfer of energy causes transfer of charge causing both the cloth and record getting electrostatically charged. Thus the charged record causes **electrostatic induction** of charge on the nearby gust particles and it leads to **force of attraction**.
I-6	Force on a charge q_1 due to another charge q_2 as per Coulomb's law is $\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r}$ where r is the separation between the two charges. This force is a vector quantity. Thus, each of the nearby charge produces an independent force $\vec{F_i} = \frac{q_1 q_i}{4\pi\varepsilon_0 {r_i}^2}\hat{r}_i$ on the charge q_1 under consideration. Therefore, net force on the charge q_1 is $\vec{F} = \sum_{i=2}^{N}\vec{F}_i$. But, the force on a charge q_1 due to another charge q_2 does not depend on nearby charges. Hence, **answer is no**.
I-7	Electric lines of force from a charge emanate uniformly in space around the charge radially over a solid angle 4π. Thus, quantification of electric lines of force from a charge q is $4\pi q$, and it has **nothing to do with the fact whether 4π is an integer or not.**
I-8	Electric field at a point is perpendicular to the corresponding equipotential surface at that point. Thus, two equipotential surfaces will have two perpendiculars in the direction of the corresponding electric fields at the point of intersection.

	But, at any point there is single electric field resultant of electric fields produced by charges around and is $\vec{E_i} = \sum_{j\neq i} \frac{q_i q_j}{4\pi\varepsilon_0 {r_{ij}}^2} \hat{r}_{ij}$. Hence, at any point there cannot be two equipotential surfaces. Therefore, **two equipotential surfaces cannot cut each other is the answer.**
I-9	Motion of a particle in electric field is a result of negotiation of velocity of a particle at a point and acceleration due to electric field at that point. As a result of this a particle placed at rest in an electric field will accelerate tangential to the electric lines of force, navigating constantly to trace a line of force. Hence answer to first part is Yes. As regards charges on a pair of parallel surfaces placed lines of force are parallel. But, when geometry of charged bodies change electric lines of force of initially diverge on increase of distance from (+)ve charge and then again converge at (-)ve charge creating curved line io force, such that resultant electric field at every point on the lines of force is tangential.
I-10	Since electric lines of force are emanating for $\boldsymbol{q_2}$ hence it **is (+)ve** and entering into $\boldsymbol{q_1}$ hence it is **(-)ve** Electric lines of force emanating or entering a charge are 4π times the charge $\phi = 4\pi q \Rightarrow \frac{\phi_1}{\phi_2} = \frac{4\pi q_1}{4\pi q_2} = \frac{q_1}{q_2}$...(1) It is seen from the figure that the lines of force numbered 1 to 6 only emanating from charge q_2 are entering into charge q_1 i.e. $\phi_1 = 6$. While all the lines of force numbered 1 to 18 are emanating from charge q_2 and $\phi_2 = 18$. Lines of force numbered 7 to 18 are terminating on unknown charges. Therefore, using the available data in (1) $\frac{q_1}{q_2} = \frac{6}{18} \Rightarrow \boldsymbol{\frac{q_1}{q_2} = \frac{1}{3}}$ **is the answer.** 
I-11	Work done by electric field in moving a charge from A to B is $W_{AB} = \int_{r_A}^{r_B} \vec{E}q.d\vec{x} \Rightarrow W_{AB} = V_A - V_B$. Where, V_A and $V_B$are **electric potentials at points A and B and not the path**.
I-12	Electric field at any point is $\vec{p} = q\vec{d}$ ...(1), here d is the separation between two charges constituting dipole. Influence of a dipole at any point P as shown in the figure is $V = \frac{1}{4\pi\varepsilon_0} \times \frac{p\cos\theta}{r^2}$...(2). In (2) r is the distance of the point under consideration from midpoint of charge O. The mathematical expression (2) is valid for $\boldsymbol{d \ll r}$**, is the answer.** 
I-13	In solids where distance between atoms are small their electron orbits, also referred to as energy levels rearrange themselves to form energy bands, occupied by valance electrons. This concept is well explained by **Band Theory**, outside scope of the subject matter at this stage. Electron after impart of energy to them called excitation move into higher energy bands called conduction band where electron are free to perform **Brownian motion** Space in between valance band and conduction band is empty no electron stay there. Potential difference between valance band and conduction band is called **potential barrier**. Potential barrier in conductors is thin. Thus small amount of energy input to the conductor is good enough to excite electrons to jump from valance band to conduction band. This characteristic of conductor helps them to conduct electricity on application of small potential difference across it.

	But, in insulators the potential barrier is wide and require large amount of energy to cross it for electrons in valance band to enter into conduction band. Thus small potential difference across insulators is insufficient to cause conduction of current through it.
I-14	Piece of paper has distributed electrons on its surface, but its net charge remains zero. When a charges comb is brought near small pieces of paper based on the charge on the comb it will either attract electrons or repel them from the end of piece nearest to the comb, called electrostatic induction. This induction will either create efficiency or sufficiency of electrons (based on attraction or repulsion of electrons by the charged comb). In this process of induction there is no transfer of charge between comb and paper pieces. **N.B.:** Small pieces of paper, due to small weight, are important to make effect of electrostatic induction visible. This effect is not visible with large pieces of paper.
I-15	Strength of electric field at a point is concentration of electric lines of force represented by electric flux density per unit area. $E = \frac{\Delta\phi}{\Delta A}$. A close examination of the given figure shows that $E_A = E_C > E_B$ and it is provided **in option (c), is the answer.**
I-16	Change in electric potential $\Delta V = -\vec{E}.\Delta\vec{x} \Rightarrow \Delta V = -E\Delta x \cos\theta \ldots(1)$ here θ is the angle between vectors $\vec{E}$ (-and $\Delta\vec{x}$. Potential is a scalar quantity, yet quantitatively change of potential would be (+)ve when $\lvert\theta\rvert > 90^0$ i.e. an increase and would be (-)ve when $\lvert\theta\rvert > 90^0$ where $\cos\theta > 0$ i.e. a decrease. Change in potential energy is $\Delta W = q\Delta V \ldots(2)$. Combining (1) and (2), change of potential energy is $\Delta W = -qE\Delta x \cos\theta$. In the question magnitude of θ is not defined and hence it cannot be stated with certainty the potential energy would increase or decrease. This conclusion matches with **option (d) is the answer.**
I-17	When a charge is shifted from a lower electric potential V_1 to higher potential V_2 the change of electric potential is $\Delta V = V_2 - V_1 \Rightarrow \Delta V = V_2 > 0$ Where ΔV = (Final potential) – (Initial Potential)]. And change of potential energy $\Delta W = q\Delta V \ldots(1)$ The problem states that a (+)ve charge is shifted. Thus both the multiplicands in (1) are (+)ve and hence change of potential energy is (+)ve i.e. an increase. This conclusion matches with **option (a) is the answer.**
I-18	Potential at a point, at a distance R from a charge Q is $V = \frac{Q}{4\pi\varepsilon_0 R} \ldots(1)$. Let displacement between two charges be r and point between two equal (+q) charges placed at A and B. A point P between A and B under consideration, as shown in the figure, is at a distance x such that $0 < x < r$ from the charge A, then the distance of the point from B would be equal to $(r - x)$.  It is required to determine variation in potential for $0 < x < r$ Accordingly, as per (1) net potential at P due to charges at A and B is $V = V_A + V_B \Rightarrow V = \frac{q}{4\pi\varepsilon_0 x} + \frac{q}{4\pi\varepsilon_0 (r-x)}$. Thus, $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{1}{r-x}\right) \Rightarrow V = \frac{q}{4\pi\varepsilon_0} \times \left\{\frac{1}{x} + \frac{1}{(r-x)}\right\}$. Variation of potential V w.r.t. x is $\frac{dV}{dx} = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{x^2} - \frac{1}{(r-x)^2}(-1)\right] \Rightarrow \frac{dV}{dx} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-x)^2} - \frac{1}{x^2}\right] \Rightarrow \frac{dV}{dx} = \frac{q}{4\pi\varepsilon_0}\left[\frac{x^2-(r-x)^2}{\left(x(r-x)\right)^2}\right] \Rightarrow \frac{dV}{dx} = \frac{q}{4\pi\varepsilon_0} \times \frac{r(2x-r)}{\left(x(r-x)\right)^2} \ldots(2)$. In this expression $\frac{dV}{dx}$ all terms except $(r - 2x)$ are (+)ve, and this terms changes as analyzed below – **(a)** $\boldsymbol{0 < x < \frac{r}{2}}$: The factor $(2x - r) < 0$. Hence the slope is (-)ve. It implies that potential decreases with the increase of x. **(b)** $\boldsymbol{x = \frac{r}{2}}$**:** The factor $(2x - r) = 0$. Hence, slope becomes zero. It implies that potential is at its minimum, using concept of maxima-minima

	(c) $\frac{r}{2} < x < r$**:** The factor $(2x - r) > 0$. Hence the slope is (+)ve. It implies that potential increases with the increase of x. Thus analysis reveals that potential initially decreases and then increases as provided in **option (d), is the answer.**
I-19	Change of electric potential at a point $\Delta V = -\vec{E}.\Delta\vec{x} \Rightarrow \Delta V = -E\Delta x \cos\theta$. The given problem is expressed in the figure. For all displacements along X-axis potential with respect to $\vec{E}$,angle $\theta = 0 \Rightarrow \cos\theta = 1$is minimum. It implies that taking reference at origin O, for point A, $\Delta x = a \Rightarrow \Delta V_A = -Ea$, for point B, $\Delta x = 0 \Rightarrow \Delta V_B = 0$, for point C $\Delta x = -a \Rightarrow \Delta V_C = -E(-a) \Rightarrow \Delta V_C = Ea$ and for point D, $\Delta x = 0 \Rightarrow \Delta V_D = 0$. Thus it is seen that minimum value of potential is at A as provided in **option (a), is the answer.** 
I-20	During rubbing of bodies which are electrically neutral (basic property of a material in a free state) charging takes place due to transfer of charge from body to the other being rubbed. The body which gains electrons becomes negatively charged and the body which loses electrons becomes positively charged. Thus weight of negatively charged body would increase due to mass of electrons gained, and weight of positively charged body would decrease due to loss of electrons. Since it is not known that which body is positively or negatively charged hence increase or decrease in weight cannot be stated with certainty. Moreover, mass of electrons is quite small and hence increase or decrease is slight. This analysis is supported **by option (d), is the answer.**
I-21	Magnitude of force of $+q$ and $-q$ charges of the dipole due to uniform electric field is $F = qE$, for any angular position (θ) of the dipole w.r.t. electric field, but direction of the two forces as shown in the figure are opposite to each other. In an electric dipole the two charges coexist and hence net force on the dipole due to uniform electric field is $F = F_+ + F_- = qE - qE = 0$.. This analysis is supported by the **option (a), is the answer.** 
I-22	Electric potential is a scalar quantity and it is quantified for a charge Q at any point at a distance r from it as $V = \frac{Q}{4\pi\varepsilon_0 r}$. And work done moving a charge q from point A to B is $W = q(V_B - V_A)$. The given problem is elaborated with greater details in the figure. Accordingly, potential at point P is $V_P = \frac{Q}{4\pi\varepsilon_0 d}$, and points A,B,C are on a semicircular arc of radius R is $V_A = V_B = V_C = \frac{Q}{4\pi\varepsilon_0 R} = V$. Therefore, work done in moving a point charge q from P to any of the three points is $W = q(V - V_P)$. This conclusion since matches with **option (c), is the answer**. 
I-23	Electric potential is a scalar quantity and it is quantified for a charge Q at any point at a distance r from it as $V = \frac{Q}{4\pi\varepsilon_0 r}$. And work done moving a charge q from point A to B is $W = q(V_B - V_A)$. In rotating a charge around a circle of radius, to complete one evolution is $W = q(V_A - V_A) \Rightarrow W = 0$. This conclusion matches with **option (a) is the answer**. **N.B.:** An isolated charge creates circular equipotential surfaces centered at the charge. Hence, work done in moving a point charge from one point on an equipotential surface to any other point it is zero. It is not necessary to rotate the charge in one complete revolution. 

I-24	Total universe is electrically neutral. This makes it possible for electric lines of force created by (+)ve charge to terminate on complimentary charge. Thus total number of free or bound electrons/protrons carring equal and opposite chages are equal. Electrically neutral particles like neutran also have equal amount of bound (+)ve and (-) ve charges. Thus looking at universe wholistically total charge of universe is constant at Zer0. This conclusion is supported by **option (a), is the answer.**
I-25	Electric field at a point is vector sum if electric field at the point due charges around. Accordingly, net electric field is $\vec{E} = \sum_{i=1}^{n} \vec{E_i}$ where $\vec{E_i}$ is the electric field produced by is due to i[th] charge. Electric field is a vector quantity. Charges may be (+)ve or (-)ve and accordingly electric field contributed by the charge may have direction either summative or subtractive. Since question does not specify the nature of charge whether it is (+)ve or (-)ve, hence electric field due to a point charge in space nearby the point may increase if the point charge is (+)ve as provided in option (c), or decrease if the point charge is charge is (-)ve as provided in the option (d) Thus, **answer is option (c), and (d).**
I-26	Change of potential in a region is $\Delta V = -\vec{E}.\Delta\vec{x} \Rightarrow \frac{dV}{d\vec{x}} = -\vec{E}$. Thus mathematically if $V = \text{Constant}$ then $\vec{E}$ at that point is zero; this constant can be even zero. Based on this analysis each of the option is being analyzed – **Option (a):** If $\vec{E} = 0$ then V may be zero or any other constant. Hence, **option (a) is incorrect**. **Option (b):** Value of $\vec{E}$ depends upon slope $\frac{dV}{d\vec{x}}$ at the point and not the value of V at the point. Hence, if $V = 0$, it cannot be stated with certainty that E must be zero. Hence, **option (b) is incorrect**. **Option (c):** Value of $\vec{E}$ depends upon slope $\frac{dV}{d\vec{x}}$ at the point and not the value of V at the point. Hence, if $E \neq 0$, the slope $\frac{dV}{d\vec{x}}$ cannot be stated, while V may or may not be zero. Hence, it cannot be stated with certainty that V cannot be zero. Hence, **option (c) is incorrect.** **Option (d):** Value of $\vec{E}$ depends upon slope $\frac{dV}{d\vec{x}}$ at the point and not the value of V at the point. Hence, **option (d) is also incorrect.** Thus, **none of the options are correct.**
I-27	Change of potential in a region is $\Delta V = -\vec{E}.\Delta\vec{x} \Rightarrow \vec{E} = -\frac{\Delta V}{\Delta\vec{x}}$. It is also stated that electric potential decreases unformly such that $\Delta V = V_{+1} - V_{-1} \Rightarrow \Delta V = 80 - 120 = -40$ and $\Delta x = +1 - (-1) = 2$. This proposition is shown in the figure. Thus, electric field ay the origin is $\vec{E} = -\frac{(-40)}{2} \Rightarrow \vec{E} = 20$ V/cm 
I-28	Potential at a point is $V = -\int_{\infty}^{x} \vec{E}.d\vec{x}$ electric field at a point is $\vec{E} = \sum \vec{E_i}$ where due to an electric charge q_i electric field at a point is $\vec{E} = \frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\hat{r_i}$. Here, r_i is the displacement of the point from the charge and $\hat{r_i}$ is the direction vector of the point w.r.t. to the charge, zero potential is the situated at infinity. And potential energy of a charge q at a point is $W = qV$, in turn also depends upon zero potential energy which is the situated at infinity

Analysis, of options, for their dependence on choice of zero potential or potential energy, requires to examine each of them and is as under –

Option (a): Potential at a point depends the reference Zero potential, hence stipulation in this option that it does not depend upon the reference is wrong. Hence, **option (a) is incorrect.**

Option (b): Potential difference of point B w.r.t. point A is $\Delta V = V_B - V_A$. Thus choice of it depends upon choice of zero potential or zero potential energy, which is taken to be at infinity, influences potential difference between Two Points. Thus, **option (b) is correct**.

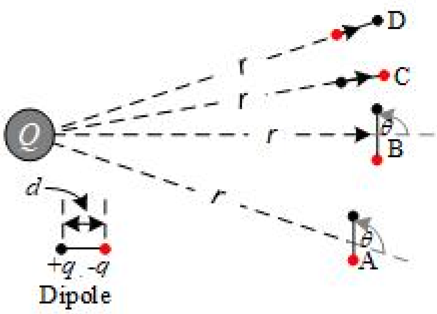
Option (c): Potential energy of a charge since depends upon reference zero potential energy it is not an absolute value. Further, potential energy between two charge system is $W = W_1 + W_2$. Therefore, it is incorrect to say that W does not depend choice of zero potential energy.Hence, **option (c) is incorrect**.

Option (d): Change in potential energy of a two charge system is $\Delta W = \Delta W_1 + \Delta W_2$. Since W_1 and WV_2 depends upon choice of zero potential energy. But, while changing potential energy of the two charges reference is not changing. Hence, ΔW does not depend upon the initial choice of potential energy. Thus, **option (d) is correct**

Thus, **answer is option (b) and (d)**

I-29

Electric field at a point due to an electric charge is $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$. Further, force on a charge q in the electric field is $\vec{F} = q\vec{E}$. And torque $(\vec{\tau})$ on a dipole is analyzed considering field to be uniform and is expressed as $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \tau = rF\sin\theta$, here is angle of $\vec{F}$ w.r.t. $\vec{r}$ as shown in the figure.



The problem states neither distance (r) of the charge Q from the dipole nor its orientation (θ) of the dipole w.r.t. electric field. Analysis of options provided is supported with the figure here.

Option (a): In case of dipole making and angle $\theta = 90^0$ with the field, as shown in orientation of dipole at B, both the charges of dipole will experience equal and opposite forces. It will lead to net force zero irrespective of value of r. But, in orientation at A when $\theta \neq 90^0$ and r is comparable to distance d between charges if dipole, then distance of (+)ve and (-)ve charges from Q are unequal $r_+ \neq r_-$ and field at the two charges would be unequal and hence resultant force ng $F = F_+ + F_- \neq 0$. Thus stipulation in **option (a) are incorrect**.

Option (b): In orientation at B net force is zero but not in A. But, in question orientation $\theta$and distance r are not defined, therefore stipulations is correct. Hence, **option (b) is correct.**

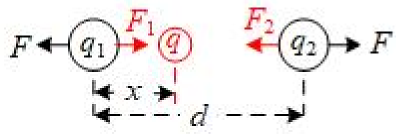
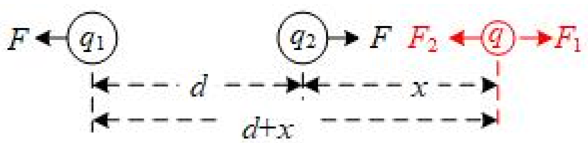
Option (c): For torque to b zero if $\theta = n\pi|_{n\in N} \Rightarrow \sin n\pi = 0$. Since, value of n is not defined and hence τ must be zero cannot be stated. Hence, **option (c) is incorrect**.

Option (d): Discussion at option (c) does not rule out possibility of taking an orientation. Hence, τ may be zero and thus **option (d) is correct**. .

Thus answer is option (b) and (d)

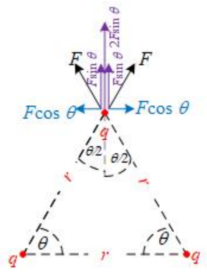
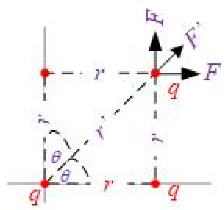
I-30	Electron and proton are charged particles with equal and opposite charges $-e$ and $+e$ respectively. A charged particle when placed in electric field experiences a force $\vec{F} = q\vec{E}$. Moreover, force is a vector quantity hence force on proton is $\vec{F_p} = +q\vec{E}$ and that on electron is $\vec{F_e} = -q\vec{E}$. In this context each of the option is being analyzed – **Option (a):** For two vectors to be equal their difference must be zero. In instant case $\vec{F} = \vec{F_p} + \vec{F_e}$. It leads to $\vec{F} = +q\vec{E} - (-q\vec{E}) = +2q\vec{E} \neq 0$. Hence, **this option is incorrect.** **Option (b):** Magnitude of a vector, in this case force, is absolute value without direction. Accordingly, on electron force is $F_e = \lvert\vec{F_e}\rvert = \lvert -q\vec{E}\rvert = qE$ and on proton it is $F_p = \lvert\vec{F_p}\rvert = \lvert q\vec{E}\rvert = qE$. Again, for two absolute quantities to be equal their difference must be zero. In instant case $F = F_p - F_e = qE - qE = 0$, Hence**, this option is correct**. **Option (c and d):** Acceleration of a particle, as per Newton's Second Law of motion is $\vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\vec{F}}{m}$. Thus, while mass is scalar, acceleration is a vector quantity. In instant case, since mass of electron and proton are unequal, therefore, while directions of their acceleration are opposite their magnitudes are also unequal. Therefore, for equality difference of acceleration of proton and electron must be zero. This criterial cannot be satisfied either for acceleration or their magnitudes and hence **both of these options are incorrect**. **Thus answer is option (b).**
I-31	Electric field at a point due to a system charges is $\vec{E} = \sum \vec{E_i}$ where due to a a particular charge q_i electric field at a point is $\vec{E} = \frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\hat{r_i} \Rightarrow E = \frac{q_i}{4\pi\varepsilon_0 {r_i}^2} \Rightarrow E \propto \frac{1}{r^2}$. But, it is stated in the problem that $E \propto r$. It is not a hypothetical case. Further, potential at a point is $V = -\int E dr \Rightarrow V \propto \int r dr \Rightarrow V \propto r^2$. This conclusion, matches with only option (c), is the answer. **N.B.:** In this question analysis leads to discrete answer matching with one of the options given. Hence, it is not necessary to analyze each of the option.
I-32	In dimensional analysis neither the magnitude nor the direction are considered. Therefore, expression in Coulomb's formula is being moderated $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \Rightarrow [F] = \left[\frac{q^2}{\varepsilon_0 r^2}\right] \Rightarrow [F] = \frac{[q^2]}{[\varepsilon_0][r^2]} \Rightarrow [\varepsilon_0] = \frac{[q^2]}{[F][r^2]}$....(1). Taking dimensions of the constituent parameters, for charge $q = \int I dt \Rightarrow [q] = \mathrm{IT} \Rightarrow [\mathrm{q}^2] = \mathrm{I}^2\mathrm{T}^2$. For force $[\mathrm{F}] = \mathrm{MLT}^{-2}$ and for distance $[r^2] = L^2$. Substituting, these dimensional expression in (1), $[\varepsilon_0] = \frac{\mathrm{I}^2\mathrm{T}^2}{(\mathrm{MLT}^{-2})(L^2)}$. Using theory of indices $[\varepsilon_0] =$ **$\mathbf{I^2M^{-1}L^{-3}T^4}$ is the answer.**
I-33	As per Coulomb's Law force between two charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^{2}C^{-2}, and given that $q_1 = q_2 = 1.0$ C and $r = 2.0 \times 10^3$ m. Using the available

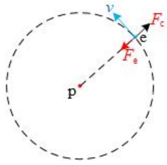
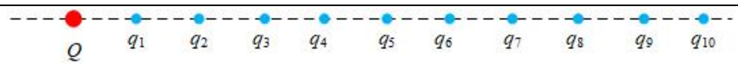
	data force between the two charges is $F = 8.99 \times 10^9 \times \frac{1.0\times1.0}{(2.0\times10^3)^2} \Rightarrow F = \frac{8.99}{4.0} \times 10^{(9-6)} \Rightarrow F =$ $\mathbf{2.25 \times 10^3}$ **N is the answer of first part** My weight is $W = mg = 50 \times 10 = 500$ N. Hence, $\frac{F}{W} = \frac{2.25\times10^3}{5\times10^2} = \mathbf{4.5}$**, is the answer of second part** **N.B:** Answer of second part is subjective and depends upon weight of person answering the question. In such questions it is advisable to use a value for it convenient for calculations.
I-34	As per Coulomb's Law force between two charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^{2}C^{-2}, and given that $q_1 = q_2 = 1.0$ C. Separation between the charges r is to be determined for a force equal to weight of a 50 kg person. It is to be noted that weight has a unit N, while given data is in kg, a unit of mass. Applying Newton's Second Law of motion $F = ma$ here $a = g = 10$ m/s^2 is used to convert given data in force. Accordingly, using the available data force between the two charges is $F = W \Rightarrow 50 \times 10 = 8.99 \times 10^9 \times \frac{1.0\times1.0}{r^2} \Rightarrow r^2 = 1.796 \times 10^3 \Rightarrow r^2 = 17.96 \times 10^2 \Rightarrow r = 4.238 \times 10^2$ m, say $r = \mathbf{424}$ **m is the answer of first part.** **N.B.:(a)** In this case precise answer would depend upon the value of *g* (not specified in the question) considered in the solution. Moreover, this value of *g* is in agreement with the SDs of the data given in the problem (b) Since data is given in conventional notation and hence, for convenience, the same is used in answer.
I-35	As per Coulomb's Law force between two charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^{2}C^{-2}, and given that separation between the charges $r = 1.0$ m but the two charges $q_1 = q_2 = q$ are to be determined such that force between them is equal to weight of a 50 kg person. It is to be noted that weight has a unit N, while given data is in kg, a unit of mass. Applying Newton's Second Law of motion $F = ma$ here $a = g = 10$ m/s^2 is used to convert given data in force. Accordingly, using the available data force between the two charges is $F = W \Rightarrow 50 \times 10 = 8.99 \times 10^9 \times \frac{q^2}{(1.0)^2} \Rightarrow q^2 = \frac{500}{8.99\times10^9} \Rightarrow q^2 = 5.56 \times 10^{-8} \Rightarrow q = \mathbf{2.4 \times 10^{-4}}$ **C is the answer**. **N.B.:(a)** In this case precise answer would depend upon the value of g (not specified in the question) considered in the solution. (b) Since data is given in conventional notation, yet, for convenience, scientific notation is used in answer. (c) Principle of SDs is used to answer.
I-36	As per Coulomb's Law force between two electrical charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^{2}C^{-2}, and charge of a proton is $= +1.602 \times 10^{-19}$ C and separation between them is $r = 1.0 \times 10^{-15}$m. Hence, using the available data the force between them is $F =$

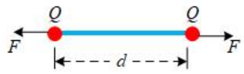
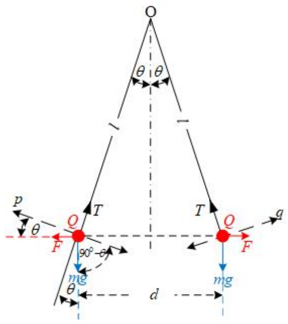
	$8.99\times10^9\times\frac{(1.602\times10^{-19})^2}{(1.0\times10^{-15})^2}\Rightarrow F=8.99\times10^9\times(1.602\times10^{-4})^2\Rightarrow F=89.9\times(1.602)^2=\mathbf{230}$ **N is the answer.**
I-37	The system given in the problem is shown in the figure. As per Coulomb' law of force between two electrical between charges $q_1=2.0\times10^{-6}$ and $q_2=1.0\times10^{-6}$is $F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0}=8.99\times10^9$ Nm^{2}C^{-2}, and separation between them $d=10^{-1}$m..  Both the charges q_1 and q_2 being positive would experience a single force of repulsion $F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{d^2}$…(1) Making net force on these A third charge q is introduced in the system of the two charges, such that net force on the two charges is zero. Thus, eventually both the charges would experience two forces of which second force on q_1 is $F_1=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q}{x^2}$ …(2), and the second force on q_2 is $F_2=\frac{1}{4\pi\varepsilon_0}\times\frac{q_2q}{(d-x)^2}$…(3) Thus, require of zero net force on both the charges q_1 is $F+F_1=0\Rightarrow F=-F_1$. In this combining (1) and (2) we have $\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{d^2}=-\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q}{x^2}\Rightarrow\frac{q_2}{d^2}=-\frac{q}{x^2}\Rightarrow q=q_2\left(\frac{x}{d}\right)^2$…(4). Likewise, net force q_2 is $F+F_2=0\Rightarrow F=-F_2$. In this combing combining (1) and (2) we have $\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{d^2}=-\frac{1}{4\pi\varepsilon_0}\times\frac{q_2q}{(d-x)^2}\Rightarrow\frac{q_1}{d^2}=-\frac{q}{x^2}\Rightarrow q=q_1\left(\frac{d-x}{d}\right)^2$…(5). Combining, (4) and (5), $q_2\left(\frac{x}{d}\right)^2=q_1\left(\frac{d-x}{d}\right)^2\Rightarrow\left(\frac{d-x}{x}\right)^2=\frac{q_2}{q_1}\Rightarrow\frac{d-x}{x}=\sqrt{\frac{q_2}{q_1}}$. Using the available data $\frac{0.1-x}{x}=\sqrt{\frac{1.0\times10^{-6}}{2.0\times10^{-6}}}\Rightarrow\frac{0.1-x}{x}=\frac{1}{\sqrt{2}}$. Applying componendo we have $\frac{0.1}{x}=\frac{1+\sqrt{2}}{\sqrt{2}}\Rightarrow x=\frac{0.1\times\sqrt{2}}{1+\sqrt{2}}\Rightarrow x=0.063$ m, or from q_1 **6.3 cm, the large charge is the answer**.. **N.B.:** Unless must, arithmetic calculations should be kept until the last; it leads to highly simplified calculations. This problem is a good example of it.
I-38	The system given in the problem is shown in the figure. As per Coulomb' law of force between two electrical between charges $q_1=2.0\times10^{-6}$ and $q_2=-1.0\times10^{-6}$is $F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0}=8.99\times10^9$ Nm^{2}C^{-2}, and separation between them $d=10^{-1}$m.  Both the charges q_1 and q_2 are opposite. A third charge q is introduced in the system of the two charges such that net force on it is zero. Since $q_2<q_1$ and are oppositely signed so also would be forces in opposite directions. Further, charge q must be placed along the line the two charges and closer to the weaker charge so that net force on it is zero.

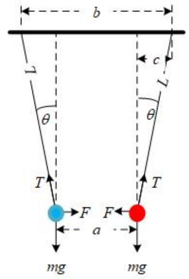
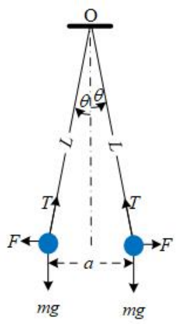
	Thus – (a) non-zero forces F_1 and F_2 on charge q, caused by q_1 and q_2, respectively, are collinear and (b) $\vec{F}_1 + \vec{F}_2 = 0$ …(1), as shown in the figure. Here, force due to q_1 is $F_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q}{(d+x)^2}$ …(2), and the force due to q_2 is $F_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q}{x^2}$…(3) Now, combining (1), (2) and (3) we have $\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q}{(d+x)^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q}{x^2} = 0 \Rightarrow \frac{q_1}{(d+x)^2} = -\frac{q_2}{x^2} \Rightarrow \frac{q_1}{q_2} = -\frac{(d+x)^2}{x^2}$ … (4). Using the available data $\frac{2.0\times10^{-6}}{-1.0\times10^{-6}} = -\left(\frac{0.1+x}{x}\right)^2 \Rightarrow \frac{0.1+x}{x} = \sqrt{2} \Rightarrow (\sqrt{2}-1)x = 0.1 \Rightarrow x = \frac{0.1}{\sqrt{2}-1} = 0.24\text{m}$ from q_2 or $(0.1 + 0.24) = 0.34$ m or **34 cm from q_1, the large charge is the answer**.
I-39	As per Coulomb's Law force between two electrical charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2C^{-2}, and given that $r = 1.0 \times 10^{-2}$. Therefore, for force to be minimum is the minimum magnitude of charges q_1 and q_2 irrespective of their signs. The smallest independent charge is of electron whose absolute magnitude 1.6×10^{-19} C. Thus taking $q_1 = q_2 = 1.6 \times 10^{-19}$ C. Hence, the required value is $F = (9 \times 10^9) \times \frac{(1.6\times10^{-19})^2}{(0.01)^2} = \mathbf{2.3 \times 10^{-24}}$ **N is the answer.**
I-40	One mol of an atom is equal to mass of substance in grams equal to atomic number $A = N + Z$, where N is number of electrons and Z is number of protons. Molecular mass of water (H_2O) is $\text{A}_\text{W} = 2 \times \text{A}_\text{H} + \text{A}_\text{O}$ and it is $\text{A}_\text{W} = 2 \times 1 + 16 = 18$ Further, number of molecules in A gram water is Avogadros' number (6.023×10^{23}). Therefore, number of atoms in 100 g water $N = \frac{100}{18} \times 6.023 \times 10^{23}$. Further, number of electrons in a water molecule $n_\text{W} = 2 \times n_\text{H} + n_\text{O} = 2 \times 1 + 8 = 10$. Therefore, total number of electrons in given quantity of water is $N_\text{e} = N \times n_\text{W} = \left(\frac{100}{18} \times 6.023 \times 10^{23}\right) \times 10 = 3.346 \times 10^{25}$ say $\mathbf{3.35 \times 10^{25}}$. It is known that charge of an electron is $e = (1.6 \times 10^{-19})$ C and hence total negative charge of electrons in the given quantity of water is $Q_\text{e} = N_\text{e} \times e = (3.346 \times 10^{25}) \times (1.6 \times 10^{-19}) \Rightarrow Q_\text{e} = 5.354 \times 10^6$ say $\mathbf{5.35 \times 10^6}$ **C.** **N.B.: (a)** Principle of SDs should be applied at last stage of reporting the answer, thus N_e is reported using SDs. **(b)** In calculating Q_e calculated value of $N_\text{e} = 3.346 \times 10^{25}$ is used but final value of Q_e is reported applying principle of SDs at the last stage. **(c)** This problem is a good example of application of principle of SDs.
I-41	Any matter in its natural state is electrically neutral, it implies that number of oppositely charged atomic particles is electrons and protons, electrically complementary to each other, are equal in number and quantity of charge. One mole of an atom is equal to mass of substance in grams equal to atomic number $A = N + Z$, where N is number of electrons and Z is number of protons. Molecular mass of water (H_2O) is $\text{A}_\text{W} = 2 \times \text{A}_\text{H} + \text{A}_\text{O}$ and it is $\text{A}_\text{W} = 2 \times 1 + 16 = 18$ Further, number of molecules in A gram water is Avogadros' number (6.023×10^{23}). Therefore, number of atoms in 100 g water $N = \frac{100}{18} \times 6.023 \times 10^{23}$. Further, number of electrons in a water molecule $n_\text{W} = 2 \times n_\text{H} + n_\text{O} = 2 \times 1 + 8 = 10$. Therefore, total number of electrons in given quantity of water is $N_\text{e} = N \times n_\text{W} = \left(\frac{100}{18} \times 6.023 \times 10^{23}\right) \times 10 = 3.346 \times 10^{25}$ say $\mathbf{3.35 \times 10^{25}}$. It is known that charge of an electron is $e = (1.6 \times 10^{-19})$ C and hence total negative charge of electrons in the given quantity of water is $Q_\text{e} = N_\text{e} \times e = (3.346 \times 10^{25}) \times (1.6 \times 10^{-19}) \Rightarrow Q_\text{e} = 5.354 \times 10^6$ say $\mathbf{5.35 \times 10^6}$ **C.**

	Separation between the concentrated (+) ve and (-)ve charges is given to be 0.10 m. As per Coulomb's Law of Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, and with the given data $F = (9 \times 10^9) \times \frac{(5.35\times10^6)(-5.35\times10^6)}{(0.1)^2} \Rightarrow F = \mathbf{-2.56 \times 10^{16}}$ N attractive force is the answer. My weight is 50 kg, the unit used is of mass and hence quantitatively weight is $W = mg = 50 \times 10 =$ 500 N. Hence, the required ratio is $\frac{F}{W} = \frac{2.56\times10^{16}}{5\times10^2} = \mathbf{5.12 \times 10^{13}}$**times my weight.** **N.B: (a)** Answer of second part is subjective and depends upon weight of person answering the question. In such questions it is advisable to use a value for it convenient for calculations. **(b)** It is also to be noted that weight of persons is scaled in mass and accordingly unit used is kg. The reason being mass remain unchanged even in change of place. Therefore, it needs to be appropriately converted in Newton.
I-42	Radius of the given nucleus is 6.9 fermi $R = 6.9 \times 10^{-15}$m. Thus, maximum possible distance two protons in the nucleus is $d = 2R = 2 \times 6.9 \times 10^{-15}$m. Charge of a proton is $q_p = 1.6 \times 10^{-19}$. Further, as per Coulomb's Law of Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, $q_1 = q_2 = q_p = \left(1.6 \times 10^{-19}\right)$ and separation between charges is $r = d$. Therefore, using the available data $F = (9 \times 10^9) \times \frac{\left(1.6\times10^{-19}\right)^2}{(2\times6.9\times10^{-15})^2} \Rightarrow F = \mathbf{1.2}$ **N, is the answer**. The protons being like charges this force is repulsive and should make protons fly away. But, **short range strong nuclear forces keep them do no let it happen, is the answer.**
I-43	As per Coulomb's Law of Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}. The force is stated to be $F = 0.1$ N when two charged spheres is given to be $r = 0.01$m. It is required to determine number of electrons transferred from one sphere to another. Say n is the number of electrons transferred therefore charge on negatively charged sphere is $q_1 = nq_e$ where charge of an electron is $q_e = -1.6 \times 10^{-19}$C and charge positively charged sphere is $q_1 = nq_p$ where charge of an electron is $q_p = 1.6 \times 10^{-19}$C. Using the available data $0.1 = (9 \times 10^9) \times \frac{\left(n\times1.6\times10^{-19}\right)^2}{(0.01)^2} \Rightarrow n^2 = \frac{10^{-5}}{(4.8)^2\times10^{-28}} \Rightarrow n = \frac{10}{4.8} \times 10^{22} \Rightarrow$ 2.08×10^{11}. Using principle of SD $\mathbf{2 \times 10^{11}}$ **is the answer.**
I-44	Molecule of NaCl has electrovalent bond such that $\text{NaCl} \leftrightarrow \text{Na}^+ + \text{Cl}^-$. As per Coulomb's Law of Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} and separation between two oppositely charged ions is $r = 2.75 \times 10^{-8}$ cm or $r = 2.75 \times 10^{-10}$m. Magnitude o charge of the two ions is $q = -1.6 \times 10^{-19}$C. Thus using the available data $F = (9 \times 10^9) \times \frac{\left(1.6\times10^{-19}\right)^2}{(2.75\times10^{-10})^2} = \left(\frac{3\times1.6}{2.75}\right)^2 \times 10^{-9} \Rightarrow F = \mathbf{3.05 \times 10^{-9}}$ **N, is the answer**

I-45	As per Coulomb's Law of electrostatic Force $F_e = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. Here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, charge of a proton is $q_1 = q_2 = q = 1.6 \times 10^{-19}$C and r is the separation between two charges. Whereas, Newton's Law of gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$, here $G = 6.67 \times 10^{-11}$ and mass of a proton is $m_1 = m_2 = m = 1.67 \times 10^{-27}$. Thus the required ratio is $\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}}{G\frac{m^2}{r^2}} \Rightarrow \frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_0} \times \frac{1}{G} \times \left(\frac{q}{m}\right)^2$. Using the available data $\frac{F_e}{F_g} = (9 \times 10^9) \times \frac{1}{(6.67\times10^{-11})} \times \left(\frac{1.6\times10^{-19}}{1.67\times10^{-27}}\right)^2 \Rightarrow \frac{F_e}{F_g} = \frac{9}{6.67} \times \left(\frac{1.6}{1.67}\right)^2 \times 10^{36} \Rightarrow \frac{F_e}{F_g} = \mathbf{1.24 \times 10^{36}}$ **is the answer**
I-46	We know that $[e] = M^0L^0T^0$ is dimensionless. Hence, $[e^x] = M^0L^0T^0$ is a also dimensionless quantity. Here, $[x] = [Kr] = [K][r] = M^0L^0T^0$. Accordingly, $[K]L^1 = L^0 \Rightarrow [K] = \frac{L^0}{L^1} \Rightarrow \boldsymbol{[K] = L^{-1}}$ **is the answer** Further,$[F] = \frac{[C]}{[r^2]} \Rightarrow [C] = [F][r^2] \Rightarrow [C] = (MLT^{-2})(L^2) \Rightarrow \boldsymbol{[C] = ML^3T^{-2}}$ **is the answer.** **Thus answer is $\boldsymbol{ML^3T^{-2}}$ and $\boldsymbol{L^{-1}}$**
I-47	Given that three charges are equal $q = 2.0 \times 10^{-6}$ and placed symmetrically at the vertices of an equilateral triangle of side $r = 0.05$m. Therefore, analysis of quantity of force on any of the charge would be identical to other except the direction as shown in the figure. In this case $\theta = 60^0$ and hence $\cos 60^0 = \frac{1}{2}$ and $\sin 60^0 = \frac{\sqrt{3}}{2}$. It is seen from figure horizontal forces being in opposite direction would cancel each other and the resultant force on the charge at the vertex would be $F_R = 2F\sin\theta$, along bisector of the angle θ of the vertex.  As per Coulomb's Law of electrostatic Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. Here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, charge of a proton is $q_1 = q_2 = q = 2.0 \times 10^{-6}$. Using the available data $F = (9 \times 10^9) \times \frac{(2.0\times10^{-6})^2}{(0.05)^2}$ N. It reduces to $F = (3 \times 4 \times 10^{-5})^2$. Therefore, $F_R = 2 \times (3 \times 4 \times 10^{-5})^2 \frac{\sqrt{3}}{2} = \mathbf{24.9}$ **N. is the answer**.
I-48	Given that four equal charges $q = 2.0 \times 10^{-6}$ are placed at vertices of a square of side $r = 5$ cm. There is a geometrical symmetry in position of the charges and hence forces due to charges adjoining vertices having displacement r is F along the line joining corresponding charges and force due charge at the diametrical opposite charge having displacement r' is F'. Again geometrically $r' = \sqrt{r^2 + r^2} \Rightarrow r' = \sqrt{2}r$ and angle $\theta = 45^0$.  As per Coulomb's Law of electrostatic Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. Here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, charge of a proton is $q_1 = q_2 = q = 2.0 \times 10^{-6}$. Using the available data $F = (9 \times 10^9) \times \frac{(2.0\times10^{-6})^2}{(0.05)^2} \Rightarrow F = 14.4$ N. While, $F' = (9 \times 10^9) \times \frac{(2.0\times10^{-6})^2}{(\sqrt{2}\times0.05)^2} \Rightarrow F' = \frac{F}{2}$.

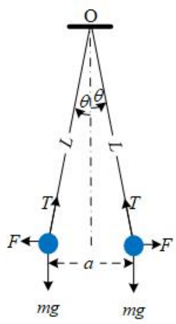
	Resultant of two forces of magnitude F is $F" = \sqrt{F^2 + F^2 + 2F \times F \times \cos 2\theta} \Rightarrow F" = \sqrt{2}F$ and this would be in line with F' and in the same direction. Thus resultant force on the charge would be $F_R = \frac{F}{2} + \sqrt{2}F \Rightarrow F_R = (0.500 + 1.414)F \Rightarrow F_R = 1.914F \Rightarrow F_R = 1.914 \times 14.4 = 27.56$ N say **27.6 N is the answer.**
I-49	Magnitude of charge of an electron and proton is $q = 1.6 \times 10^{-19}$ and radius of the orbit o electron is $r = 0.53 \times 10^{-10}$ m. As per Coulomb's Law of electrostatic Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. Here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} , charge of a proton is $q = 1.6 \times 10^{-19}$. Hence, using the available data $F = (9 \times 10^9) \times \frac{(1.6\times10^{-19})^2}{(0.53\times10^{-10})^2} \Rightarrow F = \mathbf{8.2 \times 10^{-8}}$ **N is the answer.**
I-50	Magnitude of charge of an electron and proton is $q = 1.6 \times 10^{-19}$ and radius of the orbit of electron is $r = 0.53 \times 10^{-10}$ m. As per Coulomb's Law of electrostatic force $\vec{F}_e = \frac{1}{4\pi\varepsilon_0} \times \frac{q(-q)}{r^2}\hat{r} = -\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}\hat{r}$. It is directed radially towards the center of the orbit; it acts as centripetal force. For electron of mass $m = 9.12 \times 10^{-31}$ kg orbiting around nucleus of hydrogen atom with a speed v experiences a centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$ directed radially away from the centre.  Since electron continues to orbit continuously with uniform speed and hence as per Newton's First Law of motion there be an equilibrium of forces such that $\vec{F}_e + \vec{F}_c = 0$ hence $\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{rm}}$. Using the available data, $v = \sqrt{(9 \times 10^9) \times \frac{(1.6\times10^{-19})^2}{(0.53\times10^{-10})(9.12\times10^{-31})}} \Rightarrow v = 2.18 \times \mathbf{10^6}$**m/s is the answer**.
I-51	 Arrangement of charges is as shown in the figure. Here, given that charge $Q = 1$ C is placed at origin. With an observation of distributed charges it can be said that $q_1 = 1.0^3 \times 10^{-8}$ C at a distance $r_1 = 0.1$m, $q_2 = 2.0^3 \times 10^{-8}$C at a distance $r_2 = 0.2$m, $q_3 = 3.0^3 \times 10^{-8}$ at a distance $r_3 = 0.1$m .....$q_{10} = 10.0^3 \times 10^{-8}$ at a distance $r_{10} = 1.0$m from Q. It is required to find net force of $F = \sum_{i=1}^{10} F_i \ldots(1)$ which is sum of forces caused by each of the distributed charges being like charges, placed collinear on the same side of Q . As per Coulomb's Law electrostatic force $F_i = \frac{1}{4\pi\varepsilon_0} \times \frac{Qq_i}{{r_i}^2}$ here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} · Using the available data $F = (9 \times 10^9)\left[\frac{1.0^3\times10^{-8}}{(1\times10^{-1})^2} + \frac{2.0^3\times10^{-8}}{(2\times10^{-1})^2} + \frac{3.0^3\times10^{-8}}{(3\times10^{-1})^2} \cdots \frac{10.0^3\times10^{-8}}{(1\times10^{-1})^2}\right]$. It, further, simplifies into $F = (9 \times 10^3)[1.0 + 2.0 + 3.0 \ldots 10.0] \Rightarrow F = (9 \times 10^3)S_n \ldots(1)$. Here, S_n is an arithmetic progression with $a = 1, d = 1$ and $n = 10$ and $S_n = \frac{n}{2}\{2a + (n-1)d\} \Rightarrow S_n = \frac{10}{2}\{2 \times 1 + (10-1)1\}$ It leads to $S_n = 5(2 + 9) \Rightarrow S_n = 55 \ldots(2)$. Combining (1) and (2) $F = (9 \times 10^3) \times 55 = \mathbf{4.95 \times 10^5}$**N is the answer**.

I-52	Given that $Q = 2.0 \times 10^{-8}$C and $d = 1$ m as shown in the figure. As per Coulomb's Law electrostatic force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q \times Q}{d^2} \Rightarrow F = \frac{1}{4\pi\varepsilon_0} \times \left(\frac{Q}{d}\right)^2$ here, constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}. Using the available data $F = (9 \times 10^9) \times \left(\frac{2.0\times10^{-8}}{1}\right)^2 \Rightarrow F = \mathbf{3.6 \times 10^{-6}}$ **N is the answer.** 
I-53	Given system is shown in the figure where charge on the two identical balls is $Q = 2.0 \times 10^{-7}$ C having mass $m = 0.1$ kg and length of strings $l = 0.5$ m. They are held in suspended state at a distance $d = 5.0 \times 10^{-2}$ m. The system being symmetrical about the vertical axis analysis of one ball would apply on the other. In state of free motion, ball is subjected to three forces – (a) gravitational force $F_g = mg$ N, here acceleration due to gravity is taken to be $g = 10$ m/s^2, (b) electrostatic force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{QQ}{d^2}$N, here $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, and (c)  Resultant of gravitational and electric forces $\vec{F}_R = \sqrt{F_g{}^2 + F^2}\hat{\imath}$ …(1) . Since, string, after the balls are released, remain taught therefore there is an equilibrium of forces. $\vec{T} + \vec{F}_R = 0$...(2). Geometrically, $\sin\theta = \frac{d/2}{l} = \frac{d}{2l} \Rightarrow \sin\theta = \frac{5.0\times10^{-2}}{2\times0.5} \Rightarrow \sin\theta = 5.0 \times 10^{-2}$…(3) and $\cos\theta = \sqrt{1 - \sin^2\theta} \Rightarrow \cos\theta = \sqrt{1 - (5.0 \times 10^{-2})^2} \Rightarrow \cos\theta = 0.999$ …(4). With this analysis, each of the part of the problem is being solved – **Part (a):** Electrostatic force on charge is $F = (9 \times 10^9) \times \frac{(2.0\times10^{-7})^2}{(5.0\times10^{-2})^2} = (9 \times 10^9) \times (4.0 \times 10^{-6})^2 = 1.44 \times 10^{-1}$N say **0.144 N, is the answer.** **Part (b):** Net force along the string as per (2) above as per (2) above is zero. But, net force perpendicular to the string is $F_p = mg\sin\theta - F\cos\theta$…(5). Combining (5) with (3) and (4) we have $F_p = (0.1 \times 10) \times (5.0 \times 10^{-2}) - (0.144) \times (0.999) = \mathbf{-0.094}$ **N, is the force perpendicular to the string, separating from the other charge.** **Part (c):** Tension in the string from (1), (2) and result in part (a) above, $\lvert\vec{\boldsymbol{T}}\rvert = \lvert\vec{\boldsymbol{F}}\rvert = \sqrt{F_g{}^2 + F^2} = \sqrt{(0.144)^2 + (0.1 \times 10)^2} = \mathbf{1.01}$ **N is the answer.** **Part (d):** Acceleration of the ball is determined from net of force F_p on the ball perpendicular to the string obtained in part (b) using Newton's Second Law of motion and $a_p = \frac{F_p}{m} \Rightarrow a_p = \frac{0.094}{0.1} = \mathbf{0.94}$ **m/s^2.** **Answers** are, (a) 0.144 N, (b) zero., 0.094 N away from other charge, (c) 1.01 N, (a) 0.94 m.s^{-2} perpendicular to the string and going away from the other charge **N.B.:** Value of g is not provided and hence value taken in solution would marginally influence the answer.
I-54	System given in the problem is shown in the figure, Ball are identical of mass charged by rubbing and have opposite charges of magnitude $q = 2.0 \times 10^{-8}$ C. The ball are suspended by strings of length $L = 0.20$ m with a separation $b = 0.05$ m at points of suspension. The balls are separated at a distance $a = 0.03$ m. It is required determine mass of the balls m.

	The balls are suspended under gravity and hence gravitational force decisive in solution depends upon value of $g = 10$ m/s^2 considered here. The system is symmetrical except direction of mutual electrostatic force between the two ball as per Coulomb's law $F = \frac{1}{4\pi\varepsilon_0} \times \frac{QQ}{d^2}$N…(1), where $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}. Thus analysis for one ball would be applicable to the other.  Angle of the string with vertical is θ, which geometrically leads to $\sin\theta = \frac{c}{L} = \frac{\frac{(b-a)}{2}}{L} = \frac{0.05-0.03}{2\times0.20} = 0.05$, accordingly $\cos\theta = \sqrt{1-\sin^2\theta} \Rightarrow \cos\theta = \sqrt{1-(0.05)^2} = 0.999$. Forces being vectors, from diagram $T\cos\theta = mg$ …(2) and $T\cos(90-\theta) = F \Rightarrow T\sin\theta = F$…(3). Combining (2) and (3) leads to $\frac{T\sin\theta}{T\cos\theta} = \frac{F}{mg} \Rightarrow m = F \times \frac{1}{g} \times \frac{\cos\theta}{\sin\theta}$. Using the available data and values derived above $F = (9\times10^9)\frac{(2.0\times10^{-8})^2}{(3.0\times10^{-2})^2} = 4.0\times10^{-3}$. Accordingly, $m = 4.0\times10^{-3}\times\frac{1}{10}\times\frac{0.999}{0.05} \Rightarrow m = 7.99\times10^{-3}$kg, say **8.0 g, is the one part of the answer.** Tension in the string $T = \sqrt{F^2+(mg)^2} \Rightarrow T = \sqrt{(4.0\times10^{-3})^2+(8\times10^{-3}\times10)^2}$. It simplifies into $T = 4.0\times10^{-3}\sqrt{1+400} \Rightarrow T = \mathbf{8\times10^{-2}}$**N is the answer of the second part**. **N.B.:** Value of g is not provided and hence value taken in solution would marginally influence the answer.
I-55	The system stated in the problem is shown in the figure. Let the two identical spheres of mass $m = 0.020$ kg carry a like charge Q. These balls are suspended from O with strings of length $L = 0.4\ m$. In the state of equilibrium separation between the two balls is $a = 0.04$ m. Parameters of the two balls being identical, forces on them shall also be identical but for directions.  In state of equilibrium with given separation angle of string with vertical axis will be θ such that $\sin\theta = \frac{\frac{a}{2}}{L} \Rightarrow \sin\theta = \frac{0.04}{2\times0.4} = 0.05$, accordingly $\cos\theta = \sqrt{1-\sin^2\theta} \Rightarrow \cos\theta = \sqrt{1-(0.05)^2} = 0.999$. As per Coulomb's law $F = \frac{1}{4\pi\varepsilon_0} \times \frac{QQ}{a^2}$N, where $\frac{1}{4\pi\varepsilon_0} = (9\times10^9)$ Nm^{2}C^{-2}. Thus, $F = (9\times10^9)\times\frac{Q^2}{(4\times10^{-2})^2} \Rightarrow F = \left(\frac{3Q}{4}\right)^2\times10^{13}$N. Taking $g = 10$ m/s^2, $F_g = (2.0\times10^{-2})\times10 \Rightarrow F_g = 2.0\times10^{-1}$N. In state of equilibrium horizontal and vertical components of tension in the string must be in equilibrium with the forces in these directions. Accordingly, $T\cos(90-\theta) = F \Rightarrow T\sin\theta = F$and $T\cos\theta = F_g$. It leads to $\frac{T\sin\theta}{T\cos\theta} = \frac{F}{F_g}$. Using the available data, $\frac{0.05}{0.099} = \frac{\left(\frac{3Q}{4}\right)^2\times10^{13}}{2.0\times10^{-1}} \Rightarrow Q = \sqrt{\left(\frac{1\times10^{-2}}{9.9\times10^{-2}}\right)\left(\frac{4}{3}\right)^2\times10^{-13}}$. It solves to $Q = \sqrt{\frac{160}{891}}\times10^{-7} \Rightarrow Q = 0.424\times10^{-7}$C or $\mathbf{4.24\times10^{-8}}$**C is the answer.** **N.B.: (a)** Value of g is not provided and hence value taken in solution would marginally influence the answer. **(b)** Management of power of 10 in arriving at solution and it's simplification is important.

I-56

The system stated in the problem is shown in the figure. Let the two identical spheres of mass m kg carry a like charge Q. These balls are suspended from O with strings of length L . In the state of equilibrium strings make an angle θ with the vertical. Separation between the two balls is this state of equilibrium is taken to be a . All quantities are taken to be n SI unit. Parameters of the two balls being identical, forces on them shall also be identical but for directions.

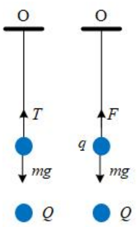


As per Coulomb's law $F = \frac{QQ}{4\pi\varepsilon_0 a^2} = \frac{Q^2}{4\pi\varepsilon_0 a^2}$...(1) and gravitational force $F_g = mg$...(2)

In state of equilibrium force F produces clockwise torque $\tau_C = (L\cos\theta)F$...(3) and F_g produces anti-clockwise torque $\tau_A = (L\sin\theta)F_g$ ...(4). In equilibrium using (3 & 4) $\tau_C = \tau_A \Rightarrow L\,F\cos\theta = LF_g\sin\theta$. Further, using (1&2), $\frac{Q^2}{4\pi\varepsilon_0 a^2}\cos\theta = mg\sin\theta$, here, $a = 2\times L\sin\theta$. Accordingly, $m = \frac{Q^2\cos\theta}{4\pi\varepsilon_0(2L\sin\theta)^2\times g\sin\theta}$, it simplifies into $m = \frac{Q^2\cot\theta}{16\pi\varepsilon_0 gL^2\sin^2\theta}$ **is the answer.**

I-57

Given system is shown in the figure. Initially the bob of mass $m = 0.1$ kg is suspended, it experience only gravitational force $F_g = mg$ and there is no electrostatic force since it is uncharged. Thus, to keep bob suspended a reaction in the form of tension $T + F_g = 0$ in the string exists, such that $T = -F_g$. Here, acceleration due to gravity is taken to be $g = 10$ m/s^2.



It is required to determine charge q to be given to the bob, when the string becomes loose. As soon as bob is given some charge it will experience an electric force F due to charge $Q = 2.0\times10^{-4}$ C directly below it. As per Coulomb's Law $F = \frac{Qq}{4\pi\varepsilon_0 r^2}$, where $\frac{1}{4\pi\varepsilon_0} = (9\times10^9)$ Nm^{2}C^{-2} and given that $r = 0.1$m. In this state of equilibrium $T + F = F_g$. Necessary requirement for the string to become loose is $T = 0$. Accordingly, $F = F_g \Rightarrow \frac{Qq}{4\pi\varepsilon_0 r^2} = mg \Rightarrow q = \frac{(mg)\times(4\pi\varepsilon_0 r^2)}{Q}$. Using the available data $q = \frac{(0.1\times10)\times r^2}{Q\times\frac{1}{4\pi\varepsilon_0}} \Rightarrow$ $q = \frac{(0.1)^2}{(2.0\times10^{-4})\times(9\times10^9)} \Rightarrow$ $\boldsymbol{q = 5.6\times10^{-9}}$**C is the answer.**

I-58

In the instant problem, as shown in the figure, all charges are placed on a smooth table they would experience electric force as per Coulomb's Law $F = \frac{q_1\times q_2}{4\pi\varepsilon_0 d^2}$ ...(1), where d is separation between the two charges under consideration.

It is given that a particle C which carries some charge say q is clamped on the table such that the particles A and B having charge Q and $2Q$ respectively remain at rest. It is seen that each of the three charge experiences Two forces due to other two charges. Thus for particle A and B to remain at rest the two forces on each charge must be in equilibrium. Taking forces on each particle separately -

Analysis of equilibrium of particle A having Charge $\boldsymbol{q_1 = Q}$:

It experiences two forces –

(a) mutual repulsive force between particles A due to particle B having charge $q_2 = 2Q$ such that $F = \frac{Q\times2Q}{4\pi\varepsilon_0 d^2}$ ...(1)

(b) force due to charge q on particle C at a distance x from A, it has to be opposite to the F such that $F_1 = \frac{Q\times q}{4\pi\varepsilon_0 x^2}$...(2)

Therefore, under equilibrium from (1 & 2), $F + F_1 = 0 \Rightarrow \frac{Q\times 2Q}{4\pi\varepsilon_0 d^2} = -\frac{Q\times q}{4\pi\varepsilon_0 x^2} \Rightarrow q = -2Q\left(\frac{x}{d}\right)^2$...(3)

Analysis of equilibrium of particle B having Charge $q_2 = 2Q$:

It experiences two forces –

(a) mutual repulsive force F between particles B due A is same as at (1).

(b) force due to charge q on particle C at a distance $(d-x)$ from B is $F_2 = \frac{2Q\times q}{4\pi\varepsilon_0 (d-x)^2}$....(4)

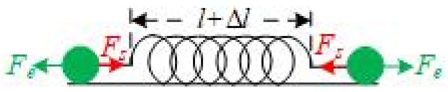
Therefore, under equilibrium from (1 & 4), $F + F_2 = 0 \Rightarrow \frac{Q\times 2Q}{4\pi\varepsilon_0 d^2} = -\frac{2Q\times q}{4\pi\varepsilon_0 (d-x)^2} \Rightarrow q = -Q\left(\frac{d-x}{d}\right)^2$...(5).

Combining (3 & 5) we have, $-2Q\left(\frac{x}{d}\right)^2 = -Q\left(\frac{d-x}{d}\right)^2 \Rightarrow \left(\frac{d-x}{x}\right)^2 = 2 \Rightarrow d - x = \sqrt{2}\,x$. It leads to $x = \frac{d}{\sqrt{2}+1}$. This is further resolved $x = \frac{d}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1} \Rightarrow x = (\sqrt{2}-1)d$ is the answer.

Using value of x in (3) $q = -2Q\left(\left(\sqrt{2}-1\right)d\right)^2 \Rightarrow q = -2Q\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 \Rightarrow q = -2\left(3-2\sqrt{2}\right)d^2 Q$. It leads to $q = -\boldsymbol{(6-4\sqrt{2})d^2 Q}$ **is the answer**.

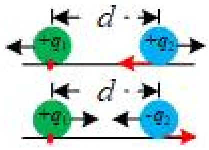
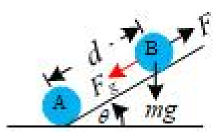
N.B.: The forces $F_1 = F_2 = F$ on particle A and B respectively, would produce equal and opposite forces on particle C having charge q. *Hence, it is not necessary to clamp charged particle C and in the final solution all the three charged particles charges would stay at rest, in equilibrium of forces.*

I-59

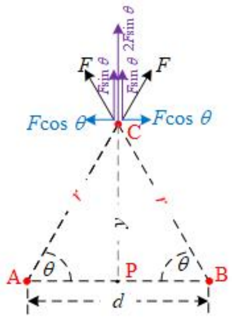


Given system is shown in the figure where a spring of natural length $l = 0.1$m with a spring constant $k = 100$ Nm^{-1}, has two charged particles having charge $q = 2.0\times 10^{-8}$C. The like charges would experience a electric force of repulsion as per Coulomb's Law $F = \frac{q_1\times q_2}{4\pi\varepsilon_0 d^2}$, where $\frac{1}{4\pi\varepsilon_0} = (9\times 10^9)$ Nm^{2}C^{-2}. In the instant case $F = F_e = \frac{1}{4\pi\varepsilon_0}\times\left(\frac{q}{l}\right)^2$ case Under the influence of repulsive electric force the spring will undergo alongation which will be restrained by spring constant k such that $F_s = -k\times\Delta l$ ...(1). This alongation would moderate to actual distance between charges $(l+\Delta l)$ under equilibrium and new value of electrostatic force would be $F_e = \frac{1}{4\pi\varepsilon_0}\times\left(\frac{q}{l+\Delta l}\right)^2$...(2). In state of equilibrium, using (1 & 2) $F_e + F_s = 0 \Rightarrow F_e = -F_s \Rightarrow \frac{1}{4\pi\varepsilon_0}\times\left(\frac{q}{l+\Delta l}\right)^2 = k\times\Delta l \Rightarrow (0.1+\Delta l)^2\times\Delta l = \frac{(9\times10^9)(2.0\times10^{-8})^2}{100}$. It is simplified into $10^{-4}(1+10\Delta l)^2\times\Delta l = 3.6\times10^{-8} \Rightarrow \Delta l + 20(\Delta l)^2 + 100(\Delta l)^3 = 3.6\times10^{-4}$....(3). A close examination of (3) reveals that despite significant coefficient of terms on LHS with powers (n) of Δl such that $n \geq 1$, the R.H.S. is quite small this can happen if-and-only-if $n \geq 1$, such that terms containing powers $n \geq 2$ are ignored. Accordingly, applying assumption $n \geq 1$ in (3) it simplifies to $\Delta l = \mathbf{3.6\times10^{-4}}$**m is the answer**.

N.B.: Value of g is not provided and hence value taken in solution would marginally influence the answer.

I-60	In the problem, as shown in the figure, it is required to determine charge $q_2 = q$ on free particle of mass $m = 0.08$ kg which remains in equilibrium with electrostatic force caused by charge $q_1 = 2.0 \times 10^{-6}$ C. Concept of physics in the problem is that – **(a)** Frictional force in a body in state of equilibrium is that $F + f = 0 \Rightarrow f = -F$…(1), here $F = \frac{q_1 \times q_2}{4\pi\varepsilon_0 d^2}$ …(2), is the electrostatic force as per Coulomb's law. Here, $d = 0.1$ m is given to be separation between two charges and $\frac{1}{4\pi\varepsilon_0} = (9 \times 10^9)$ Nm^{2}C^{-2}. **(b)** Frictional force $f \leq \mu mg$ …(3), here $\mu = 0.2$ is coefficient of friction between particle and the table, an $g = 10$ m/s^2 is acceleration due to gravity and value is approximated for convenience of calculations. **(c)** Frictional force is always opposite to force tending to cause motion with its limiting value as per (3).  **(d)** The limiting value of frictional force as per (1), (2) and (3) gives corresponding limiting value of charge which can be $\pm q$ in accordance with (c) above. Using the available data in (1) $\frac{(9\times10^9)(2.0\times10^{-6})\times q}{(0.1)^2} = 0.2 \times 0.08 \times 10 \Rightarrow q = \frac{1.6}{1.8} \times 10^{-7} = 8.9 \times 10^{-8}$C This value together with the concept at (d) above is $\pm \mathbf{8.9 \times 10^{-8}}$**C is the answer**.
I-61	Given that two particles A and B have same mass $m = 0.1$kg and carrying same charge $q = 2.0 \times 10^{-6}$ C are placed on incline at an angle $\theta = 30^0$. Particle A is on the bottom of the inline and particle B is up on the incline, as shown in the figure. It is required to find distance d when mass B stays in equilibrium.  Physics involved in the problem is as under – (a) Mass A is on the ground and offer equal and opposite reaction to forces acting on it. Thus it stays at rest (b) Electrostatic repulsive-force $F = \frac{q\times q}{4\pi\varepsilon_0 d^2}$ …(1), on B as per Coulomb's Law tends to move it up is the electrostatic force as per Coulomb's law. Here, $\frac{1}{4\pi\varepsilon_0} = (9 \times 10^9)$ Nm^{2}C^{-2}, and separation with charged particle $d = 0.1$ m. (c) The inclined plane being smooth cannot cause restraining force on B to maintain equilibrium. (d) Weight of the ball B is $W = mg$ has its component $F_g = W\cos(90-\theta) \Rightarrow F_g = mg\sin\theta$ …(2). It is equal acts in direction opposite to F to stay it in equilibrium. Thus combining (1 & 2) $\frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{d^2} = mg\sin\theta$. Using the available data $(9\times10^9) \times \frac{(2.0\times10^{-6})^2}{d^2} = 0.1 \times 10 \times \sin 30^0$. It simplifies to $d^2 = 7.2 \times 10^{-2} \Rightarrow d = 2.68 \times 10^{-1}$m say **27 cm is the answer**.
I-62	Given system is shown in figure and it is symmetrical about line PC with two charges Q at A and B separated at a distance d and another charge q at a distance on perpendicular bisector of line AB at a distance y from P. Geometrically C is equidistant from A and B. It is required to find y such that force acting on charge q is maximum.

Distance of charge q at point C from charges Q at points A and B is $r = \sqrt{y^2 + \left(\frac{d}{2}\right)^2} \Rightarrow r = \frac{\sqrt{d^2+4y^2}}{2}$ and $\alpha = 90^0 - \theta$ while $\sin\theta = \frac{y}{r} = \frac{d}{2r}$ and $\cos\theta = \frac{d}{2r}$.



As per Coulomb's Law magnitude of forces on charge q at C due to charges Q at A and B are of magnitude $F = \frac{Q\times q}{4\pi\varepsilon_0 r^2}$. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times 10^9)$ Nm^{2}C^{-2} . Resolution of the two force vectors parallel to AB are of magnitude $F\cos\theta$ but in opposite directions with zero resultant. But, the resolution of vectors along PC is equal and additive such that $F_R = 2F\sin\theta = 2\times\frac{Q\times q}{4\pi\varepsilon_0 r^2}\times\frac{y}{r}$. It simplifies to $F_R = 2F\sin\theta = \frac{qQy}{2\pi\varepsilon_0 r^3} \Rightarrow F_R = \frac{qQy}{2\pi\varepsilon_0\left(\frac{\sqrt{d^2+4y^2}}{2}\right)^3} = \frac{4qQ}{\pi\varepsilon_0}\times\frac{y}{(d^2+4y^2)^{\frac{3}{2}}}$...(1) It is seen that in F_R the only variable is y therefore for maximum value of F_R . As per differential calculus if $\frac{dF_R}{dy} = 0$ then F_R may be maximum or minimum, taking value of y so arrived at and using it in $\frac{d^2F_R}{dy^2} = \frac{d}{dy}\left(\frac{dF_R}{dy}\right)$ if it is (-)ve then F_R is maximum.

Observation figure and (1) reveals that minimum value of $F_R = 0$. And, when charge q is either at P, $y = 0 \Rightarrow F_R = 0$ or $y \to \infty \Rightarrow (d^2+4y^2)^{\frac{3}{2}} \gg y \Rightarrow F_R = 0$. Therefore, taking $\frac{dF_R}{dy} = 0$, and if either $y \neq 0$ or $y \neq \infty$, then the value of y arrived at would correspond to maximum value of F_R.

With little of differential calculus performing differentiation in two stages $\frac{d}{dy}\frac{y}{(d^2+4y^2)^{\frac{3}{2}}} = 0$, it leads to Substituting $u = d^2+4y^2 \Rightarrow \frac{du}{dy} = 8y$. Accordingly, $\left(\frac{d}{du}u^{-\frac{3}{2}}\times\frac{du}{dy}\right)y + u^{-\frac{3}{2}}\times 1 = 0$. It further simplifies to $\left(-\frac{3}{2}y\times u^{-\frac{5}{2}}\times 8y\right) + u^{-\frac{3}{2}} = 0 \Rightarrow 1 - \frac{12y^2}{d^2+4y^2} = 0 \Rightarrow y^2 = \left(\frac{d}{8}\right)^2 \Rightarrow y = \pm\frac{d}{2\sqrt{2}}$...(2)

Combining (1 & 2), $F_R = \frac{4qQ}{\pi\varepsilon_0}\times\frac{y}{(d^2+4y^2)^{\frac{3}{2}}} = \frac{4qQ\times\frac{d}{2\sqrt{2}}}{\pi\varepsilon_0\left(d^2+4\left(\frac{d}{2\sqrt{2}}\right)^2\right)^{\frac{3}{2}}} = \frac{\sqrt{2}qQ}{\pi\varepsilon_0 d^2}\times\frac{1}{\left(1+\frac{1}{2}\right)^{\frac{3}{2}}} \Rightarrow F_R = \frac{\mathbf{4qQ}}{\mathbf{3\sqrt{3}\pi\varepsilon_0 d^2}}$...(3). In electrostatics $\left(\frac{1}{4\pi\varepsilon_0}\right)$ occurs as a generic coefficient hence (3) is resolved as It solves into $F_R = \left(\frac{4\times 4}{3\sqrt{3}}\right)\frac{qQ}{4\pi\varepsilon_0 d^2} = \mathbf{3.08}\times\frac{\mathbf{qQ}}{\mathbf{4\pi\varepsilon_0 d^2}}$ **is more appropriate answer.**

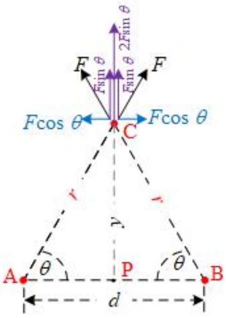
N.B.: (a) Presenting answer in more general form $3.08\times\frac{qQ}{4\pi\varepsilon_0 d^2}$ is more appropriate than as derived $\frac{4qQ}{3\sqrt{3}\pi\varepsilon_0 d^2}$.

(b) Units are not used in the answer being in algebraic form.

I-63 Given system is shown in figure and it is symmetrical about line PC with two charges Q at A and B separated at a distance d and another particle of mass m and charge q is placed at C at a distance y on the perpendicular bisector of line AB at point y from P. Geometrically C is equidistant from A and B. It is required to find y such that force acting on charge q is maximum.

Distance of charge q at point C from charges Q at points A and B is $r = \sqrt{y^2 + \left(\frac{d}{2}\right)^2} \Rightarrow r = \frac{\sqrt{d^2+4y^2}}{2}$ and $\alpha = 90^0 - \theta$ while $\sin\theta = \frac{y}{r} = \frac{d}{2r}$ and $\cos\theta = \frac{d}{2r}$.

In this context taking each part separately –



Part (a): As per Coulomb's Law magnitude of forces on charge q at C due to charges Q at A and B are of magnitude $F = \frac{Q\times q}{4\pi\varepsilon_0 r^2}$. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times 10^9)$ Nm^{2}C^{-2} . Resolution of the two force vectors parallel to AB are of magnitude $F\cos\theta$ but in opposite directions with zero resultant. But, the resolution of vectors along PC is equal and additive such that $F_R = 2F\sin\theta = 2\times\frac{Q\times q}{4\pi\varepsilon_0 r^2}\times\frac{y}{r}$. It simplifies to $F_R = 2F\sin\theta = \frac{qQy}{2\pi\varepsilon_0 r^3} \Rightarrow F_R = \frac{qQy}{2\pi\varepsilon_0\left(\sqrt{y^2+\left(\frac{d}{2}\right)^2}\right)^3} = \boldsymbol{\frac{qQ}{2\pi\varepsilon_0}\times\frac{y}{\left(y^2+\frac{d^2}{4}\right)^{\frac{3}{2}}}}$ **is answer of part (a).**

Part (b): When $y \ll d$, applying the limiting value in expression of F_R we have $\boldsymbol{y^2 \ll \frac{d^2}{4} \Rightarrow y^2 + \frac{d^2}{4} \approx \frac{d^2}{4}}$. Accordingly, the expression gets modified to $F_R = \frac{qQ}{2\pi\varepsilon_0}\times\frac{y}{\left(\frac{d^2}{4}\right)^{\frac{3}{2}}} = \frac{4qQ}{\pi\varepsilon_0 d^3}y \Rightarrow \boldsymbol{F_R \propto y}$**, hence proved**.

Part (c): Simple harmonic motion **(SHM)** requires that force on the oscillating particle is –

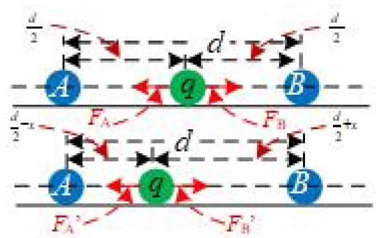
(1) proportional to displacement of particle from mean position, in this case it is point P, and
(2) force is directed towards the mean position i.e. point P.

Having proved condition (1) in part (b), the condition part (b) is possible when charge on particle C is (i.e. of sign opposite to that of the charge on particles A an B . Hence, **answer of part (c) is (-q).**

Part (d): Acceleration of a particle performing SHM at any displacement y is $F_y = -m\omega^2 y$, and comparing it results in Part(b) and (c), we have $m\omega^2 = \frac{4qQ}{\pi\varepsilon_0 d^3}$, here $\omega = 2\pi f = \frac{2\pi}{T}$. It leads to $\left(\frac{2\pi}{T}\right)^2 = \frac{4qQ}{m\pi\varepsilon_0 d^3} \Rightarrow T = \boldsymbol{\sqrt{\frac{m\pi^3\varepsilon_0 d^3}{qQ}}}$ **is the answer.**

N.B.: In part (d) answer is not written as $\left(\frac{2\pi}{T}\right)^2 = \frac{4qQ}{m\pi\varepsilon_0 d^3} \Rightarrow T = \left(\frac{m\pi^3\varepsilon_0 d^3}{qQ}\right)^{\frac{1}{2}} = \pm\sqrt{\frac{m\pi^3\varepsilon_0 d^3}{qQ}}$. Since, time period is always positive as much as surd $\sqrt{\frac{m\pi^3\varepsilon_0 d^3}{qQ}}$ is. In either case absolute value remain the same, but meaning of sign makes a difference.

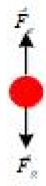
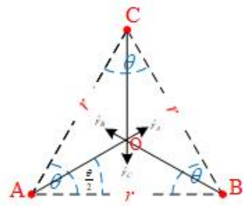
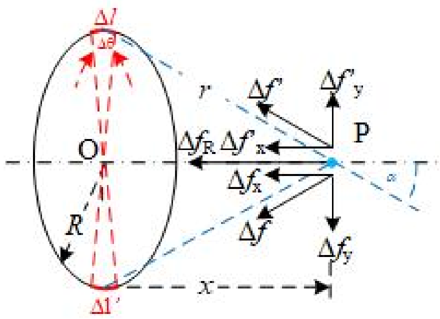
I-64



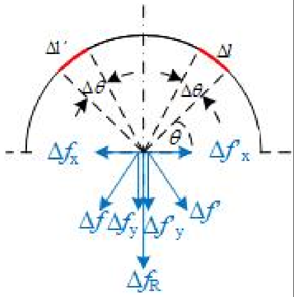
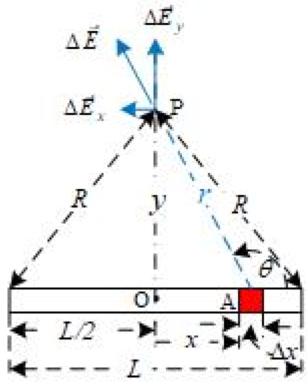
Given system is shown in figure and it is symmetrical about line PC with two charges Q at A and B separated at a distance d and another particle of mass m and charge q is placed at C, a midpoint of A and B. Particle C is a displaced by x from the midpoint, thus its distance from particle A becomes $\left(\frac{d}{2} - x\right)$ and from particle B is $\left(\frac{d}{2} - x\right)$. The

	problem is analysis of forces and motion of the particle C when it is displaced. As per Coulomb's Law magnitude of forces on charge q at C due to charges Q at A and B are of magnitude $F=\frac{Q\times q}{4\pi\varepsilon_0 r^2}$ …(1) Here, $\frac{1}{4\pi\varepsilon_0}=(9\times10^9)$ Nm^{2}C^{-2} . All charges expressed algebraically are taken to be (+)ve and hence forces exerted on particles C by charges on particles A and B are repulsive, but in opposite directions. Therefore, only forces due to charged particles A and B are considered in the analysis of each part as under - **Party (a):** When particle is displaced by a distance x towards from the midpoint, net force is $F_x=F_A'-F_B'$. Accordingly, as per (1) $F_x=\frac{Q\times q}{4\pi\varepsilon_0\left(\frac{d}{2}-x\right)^2}-\frac{Q\times q}{4\pi\varepsilon_0\left(\frac{d}{2}+x\right)^2}=\frac{qQ}{4\pi\varepsilon_0}\times\frac{2dx}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}\Rightarrow F_x=\frac{\boldsymbol{qQd}}{\boldsymbol{2\pi\varepsilon_0}}\times\frac{x}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}$ **is the answer of part (a).** **Part (b):** When $x\ll d\Rightarrow x\ll\frac{d}{2}\Rightarrow x^2\ll\left(\frac{d}{2}\right)^2\Rightarrow\left(\left(\frac{d}{2}\right)^2-x^2\right)\rightarrow\frac{d^4}{16}$. Applying the limiting value in expression of F_R arrived at part (a) we have$F_x=\frac{\boldsymbol{qQd}}{\boldsymbol{2\pi\varepsilon_0}}\times\frac{x}{\frac{d^4}{16}}\Rightarrow F_x=\left(\frac{\boldsymbol{8qQ}}{\boldsymbol{\pi\varepsilon_0 d^3}}\right)\boldsymbol{x}$. In this expression except x all other values are constant. Therefore, $\boldsymbol{F_R\propto x}$ **is proved.** **Part (c):** When particle C is moved towards particle A net force on particle C is towards particle B, i.e. it in direction opposite to the displacement. Thus this together with derivation in part (b) satisfies necessary conditions for Simple Harmonic Motion. Acceleration of a particle performing SHM at any displacement y is $F_x=m\omega^2 x$,. Combining results in Part(a) and (b), we have $\omega^2=\frac{\boldsymbol{8qQ}}{\boldsymbol{\pi\varepsilon_0 d^3 m}}$, here $\omega=2\pi f=\frac{2\pi}{T}$. It leads to $\left(\frac{2\pi}{T}\right)^2=\frac{\boldsymbol{8qQ}}{\boldsymbol{\pi\varepsilon_0 d^3 m}}\Rightarrow T=\sqrt{\frac{\boldsymbol{\pi^3\varepsilon_0 d^3 m}}{\boldsymbol{2qQ}}}$ **is the answer.** **N.B.:** In part (d) answer is not written as $\left(\frac{2\pi}{T}\right)^2=\frac{\boldsymbol{8qQ}}{\boldsymbol{\pi\varepsilon_0 d^3 m}}\Rightarrow T=\left(\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}\right)^{\frac{1}{2}}=\pm\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$. Since, time period is always positive as much as surd $\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$ is. In either case absolute value remain the same, but meaning of sign makes a difference.
I-65	Force $\left(\vec{F}=1.5\times10^{-3}\hat{\imath}\right)$experienced by a charge $q=1.0\times10^{-6}$C in an electric field $\left(\vec{E}\right)$ is $\vec{F}=q\vec{E}$., here $\hat{\imath}$ is the unit vector along the direction of force. Using the available data $\vec{E}=\frac{\vec{F}}{q}\Rightarrow\vec{E}=\frac{1.5\times10^{-3}\hat{\imath}}{1.0\times10^{-6}}\Rightarrow$ $\vec{E}=\boldsymbol{1.5\times10^3\hat{\imath}}$ **N/C i.e.** $\boldsymbol{1.5\times10^3}$ **N/C along the direction of the force is the answer.**

I-66	Given that particles A and B are fixed in position separated by a distance $d = 0.2$ and A carries charge $q_1 = +2.00 \times 10^{-6}$C and B it is $q_2 = -4.00 \times 10^{-6}$C. It is required to locate a point where (a) electric field is zero and (b)potential is zero. Electric field at a point is defined net as electrostatic force experienced by a unit positive charge $q = +1$ C at the point which as per Coulomb's Law is magnitude $\vec{E} = \sum \frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\widehat{r_i}$ …(1), here, $\frac{1}{4\pi\varepsilon_0} = (9 \times 10^9)$ Nm^{2}C^{-2} and electric potential, which is a scalar quantity, at a point is sum of electric potential due to charges surrounding the point and is expressed as $V = \sum V_i$ where $V_i = \frac{q_i}{4\pi\varepsilon_0 r_i}$ …(2). The given is shown in the figure where +1 C charge is placed at P a distance $\vec{r}_1 = x\hat{\imath}$ from particle A and farther from particle B $\vec{r}_2 = (d + x)\hat{\imath}$.  Therefore, net electric field at the point P due to charges on A and B is $\vec{E}_p = \vec{E}_1 + \vec{E}_2$. Thus as per (1) $\vec{E}_p = \frac{q_1}{4\pi\varepsilon_0 {r_1}^2}\hat{\imath} + \frac{q_2}{4\pi\varepsilon_0 {r_2}^2}\hat{\imath} \Rightarrow \vec{E}_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{{r_1}^2} + \frac{q_2}{{r_2}^2}\right)\hat{\imath}$. Thus using the given data and the condition at (a) we have $\left(\frac{+2.00\times10^{-6}}{x^2} + \frac{-4.00\times10^{-6}}{(0.200+x)^2}\right) = 0 \Rightarrow \frac{2.00}{x^2} = \frac{4.00}{(0.200+x)^2} \Rightarrow \left(\frac{0.200+x}{x}\right)^2 = 2.00 \Rightarrow \frac{0.200+x}{x} = \pm1.41$. It leads to $\frac{0.200}{x} = \pm1.41 - 1 \Rightarrow x = \frac{0.20}{\pm1.41-1}$. In view of the fact that particles A and B are oppositely charged and hence direction of the forces on P would be opposite. Further, $q_1 < q_2$, therefore, to achieve condition of equilibrium the +1 C charge must be closer to weaker charge q_1 and farther from the stronger charge q_2 such that $x < (d + x)$ and $x > 0$ i.e. x is (+). Accordingly, $= \frac{0.20}{1.41-1} = \frac{0.20}{0.41} = 0.49$ m or **49 cm from A along AB is the answer for the point of zero electric field.** As regards point of zero potential $V_p = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} \Rightarrow \vec{E}_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$. Thus using the given data and the condition $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 0 \Rightarrow \frac{q_1}{r_1} = -\frac{q_2}{r_2} \Rightarrow \frac{+2.00\times10^{-6}}{x} = -\frac{-4.00\times10^{-6}}{0.20+x} \Rightarrow \frac{0.20+x}{x} = \frac{4.00}{2.00}$. It solves into $x = 0.20$ m or **20 cm from A along AB is the answer for the point of zero electric potential.** **N.B.:** Since distance between A and B is given in cm and hence it is more appropriate to answer value x of in cm.
I-67	It is required to find magnitude of electric charge which produces electric field electric $\vec{E} = 50$ field $\vec{E} = 50$ NC^{-1} at a distance of $\vec{r} = 0.40\hat{r}$ m cm from it. Electric field at a point is defined due to a charge q is the electrostatic force experienced by a unit positive charge $q = +1$ C at the point which as per Coulomb's Law is magnitude $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\widehat{r_i}$ …(1), here, $\frac{1}{4\pi\varepsilon_0} = (9 \times 10^9)$ Nm^{2}C^{-2} . Using the available data $50\widehat{r_i} = (9 \times 10^9) \times \frac{q}{(0.40)^2}\widehat{r_i} \Rightarrow q = \frac{50\times0.16}{9\times10^9} = \frac{80}{9} \times 10^{-11}$or $\mathbf{8.9 \times 10^{-11}}$**C is the answer**.
I-68	Water particle of mass $m = 10.0 \times 10^{-6}$kg carries a charge $q = 1.50 \times 10^{-6}$ C. It is stays suspended in an electric field $\vec{E}$.

	The particle will experience two forces as shown in the figure, (a) Gravitational force $\vec{F_g} = m\vec{g} = mg(-\hat{k}) = -mg\hat{k}$, here acceleration due to gravity which acts vertically downward i.e. along $(-\hat{k})$, and (b) electric force due to electric field $\vec{E}$ such that $\vec{F_e} = q\vec{E}$. For the state of suspension of the particle $\vec{F_e} + \vec{F_e} = 0 \Rightarrow q\vec{E} = -(-mg\hat{k}) \Rightarrow \vec{E} = \frac{mg}{q}\hat{k}$. Using the available data and $g =$ 10m/s^2, we have $\vec{E} = \frac{(10.0\times10^{-6})\times10g}{1.50\times10^{-6}}\hat{k} = 66.7$ NC^{-1}, upward, is the answer.  **N.B.:** Value of g is not provided and hence value taken in solution would marginally influence the answer. Hence, it must be clearly mentioned.
I-69	Given the geometry of the system where distance between the vertices is $r = 0.20$m form sides of an equilateral triangle ABC. And each of the vertices carry a charge $q = 1.0 \times 10^{-8}$ C , distance of the center O of the equilateral from vertices is $OA = OB = OC = d = \frac{r}{2} \times \sec\frac{\theta}{2}$, Here, for the equilateral triangle, as given, $\theta = 60^0 \Rightarrow \frac{\theta}{2} = 30^0 \Rightarrow \sec 30^0 = \frac{2}{\sqrt{3}}$.  Electric field at a point is defined net as electrostatic force experienced by a unit positive charge $q = +1$ C at the point which as per Coulomb's Law is magnitude $\vec{E} = \sum\frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\hat{r_i}$ …(1), here, $\frac{1}{4\pi\varepsilon_0} = (9 \times 10^9)$ Nm^{2}C^{-2} and electric potential, which is a scalar quantity, at a point is sum of electric potential due to charges surrounding the point and is expressed as $V = \sum V_i$ where $V_i = \frac{q_i}{4\pi\varepsilon_0 r_i}$ …(2). Since center of the triangle is equidistant from its vertices and hence it will experience equal electric field at an angular displacement of 120^0. It forms and equilateral triangle of forces in equilibrium. Hence net electric field at the center of the triangle is **zero, is the answer.** As regards electric potential at O, the center of the triangle, $V = V_A + V_B + V_C$, where $V_A = V_B = V_C$. Therefore, $V = 3V_A$. Using the available data $V = 3 \times (9 \times 10^9) \times \frac{1.0\times10^{-8}}{\left(\frac{0.2}{2}\times\frac{2}{\sqrt{3}}\right)} = 27\sqrt{3} \times 50$=2165.1V or $\mathbf{2.2 \times 10^3}$ **V is the answer.**
I-70	Given is a circular ring of radius R carrying a uniformly distribute positive charge Q. A particle of mass m having a negative charge q is placed ata distance x from the center of the circle O. As per Coulomb's Law force between two charges Q and q is $\vec{F} = \frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$ …(1). Once resultant force F_R due to the ring is determined acceleration of the charged particle at P can be determined.  The system, as given, is shown in the figure with detailing for the purpose of analysis. Let a small elemental arc of length $\Delta l = R\Delta\theta$ is considered. Charge on the element would be $\Delta Q = \frac{Q}{2\pi R} \times R\Delta\theta = \frac{Q}{2\pi}\Delta\theta$. Thus force $\Delta\vec{f} = \Delta\vec{f_x} + \Delta\vec{f_y}$…(2), due to this element on charge q at P at a distance $\vec{r} = \left(\sqrt{x^2 + R^2}\right)\hat{r}$. ..(3)This force attractive since charge Q and q are oppositely signed. Component of force $\vec{F_x}$ is along axis $-\hat{\imath}$ while the component $\vec{F_y}$ is perpendicular to it. Likewise, another complementary element $\Delta l'$ causes force on the particle $\Delta\vec{f'} = \Delta\vec{f'}_x + \Delta\vec{f'}_y$…(4) The perpendicular

	components $\Delta\vec{f_y} + \Delta\vec{f'}_y = 0$ …(5) being equal and opposite cancel out each other. While axial components are equal and unidirectional $\Delta\vec{f_x} = \Delta\vec{f'}_x$…(6). Thus on combining (2,4, 5 & 6), $\Delta\vec{f_R} = 2\Delta\vec{f_x} = 2\Delta f \cos\alpha\,\hat{\imath}$…(7). Here, using (1) $\Delta\vec{f} = \frac{\Delta Q\times q}{4\pi\varepsilon_0 r^2}(-\hat{\imath}) = \frac{\left(\frac{Q}{2\pi}\Delta\theta\right)\times q}{4\pi\varepsilon_0\left(\sqrt{x^2+R^2}\right)^2}(-\hat{\imath}) = \frac{Qq}{8\pi^2\varepsilon_0(x^2+R^2)}\Delta\theta(-\hat{\imath})$ and geometrically $\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+R^2}}$. Accordingly, using these values in (7) $\Delta f_R = 2\times\frac{Qq}{8\pi^2\varepsilon_0(x^2+R^2)}\Delta\theta\times\frac{x}{\sqrt{x^2+R^2}}(-\hat{\imath}) = \frac{Qqx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\Delta\theta(-\hat{\imath})$ Therefore, net resultant force $\vec{F_R} = \int d\vec{f_R} = (-\hat{\imath})\int_0^{\pi}\frac{Qqx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}d\theta \Rightarrow \vec{F_R} = (-\hat{\imath})\frac{Qqx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\int_0^{\pi}d\theta$. It is to be noted that since each element Δl is being considered with complementary element Δl ‘ and therefore limit of integration is taken from 0 to π. It simplifies to using $\vec{F_R} = (-\hat{\imath})\frac{Qqx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\times\pi = \frac{Qq}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}(-\vec{x}) \Rightarrow \vec{F} = k(-\vec{x})$ where $k = \frac{Qq}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}$. Given that $x \ll R \Rightarrow x^2 \ll R^2 \Rightarrow (x^2+R^2)^{\frac{3}{2}} \approx R^3$, therefore, $\vec{F_R} = \frac{Qq}{4\pi\varepsilon_0 R^3}(-\vec{x}) \Rightarrow \vec{F_R} \propto (-\vec{x})$…(9) This satisfies conditions of SHM which require (a) Force is proportional to displacement from mean position, and (b) is always directed towards to mean position as signified with (-)ve sign . When particle C is moved towards particle A net force on particle C is towards particle B, i.e. it in direction opposite to the displacement. Thus this together with derivation in part (b) satisfies necessary conditions for Simple Harmonic Motion. Acceleration of a particle performing SHM at any displacement x is $F_R = \frac{Qq}{4\pi\varepsilon_0 R^3}x = m\omega^2 x$,. Combining results in Part(a) and (b), we have $\omega^2 = \frac{Qq}{4\pi\varepsilon_0 R^3 m}$, here $\omega = 2\pi f = \frac{2\pi}{T}$. It leads to $\left(\frac{2\pi}{T}\right)^2 = \frac{Qq}{4\pi\varepsilon_0 R^3 m} \Rightarrow T = \sqrt{\frac{16\pi^3\varepsilon_0 R^3 m}{qQ}}$ **is the answer.** **N.B.:** (a) Answer is not written as $\left(\frac{2\pi}{T}\right)^2 = \frac{Qq}{4\pi\varepsilon_0 R^3 m} \Rightarrow T = \left(\frac{16\pi^3\varepsilon_0 R^3 m}{qQ}\right)^{\frac{1}{2}} = \pm\sqrt{\frac{16\pi^3\varepsilon_0 R^3 m}{qQ}}$. Since, time period is always positive as much as surd $\sqrt{\frac{16\pi^3\varepsilon_0 R^3 m}{qQ}}$ is. In either case absolute value remain the same, but meaning of sign makes a difference. (b) Discussions on limit of integration are important in light of the fact that $\Delta\vec{f_R}$ is taking into account complementary element also.
I-71	Given is a semi-circular ring made of a rod of length L is carrying a uniformly distributed positive charge Q. It is required to find electric field at the center of curvature O of the semicircle. As per Coulomb’s Law force between two charges Q and q is $\vec{F} = \frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$. And electric field due to charge is force caused by it on unit positive charge. Accordingly, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$…(1)

	The system, as given, is shown in the figure with detailing for the purpose of analysis. Let a small elemental arc of length $\Delta l = R\Delta\theta$ is considered in first quadrant, here radius of curvature of the semicircle is $R = \frac{L}{\pi}$. Charge on the element would be $\Delta Q = \frac{Q}{\pi R} \times R\Delta\theta = \frac{Q}{\pi}\Delta\theta$. Thus field $\Delta\vec{E} = \Delta\vec{E}_x + \Delta\vec{E}_y$…(2). Component of force $\vec{E}_x$ is along axis $-\hat{\imath}$ while the component $\vec{E}_y$ is perpendicular to it. Likewise, another complementary element $\Delta l'$, in second quadrant, causes force on the particle $\Delta\vec{E'} = \Delta\vec{E'}_x + \Delta\vec{E'}_y$…(3). Components along X-axis $\Delta\vec{E}_x + \Delta\vec{E'}_x = 0$ …(4) being equal and opposite cancel out each other. While components along Y-axis are equal and unidirectional $\Delta\vec{E}_y = \Delta\vec{E'}_y$…(5). Thus on combining (2,4, 5 & 6), $\Delta\vec{E}_R = 2\Delta\vec{E}_y = 2\Delta E\cos\theta\,(-\hat{\jmath})$…(6).  Here, using (1) $\Delta\vec{E} = \frac{\Delta Q}{4\pi\varepsilon_0 R^2}(-\hat{\jmath}) = \frac{\left(\frac{Q}{\pi}\Delta\theta\right)}{4\pi\varepsilon_0 R^2}(-\hat{\jmath}) = \frac{Q}{4\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath})$. Accordingly, using these values in (6) $\Delta f_R = 2 \times \frac{Q\cos\theta}{4\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath}) = \frac{Q\cos\theta}{2\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath})$ Therefore, net electric field $\vec{E}_R = \int d\vec{f}_R = (-\hat{\jmath})\int_0^{\frac{\pi}{2}}\frac{Q\cos\theta}{2\pi^2\varepsilon_0 R^2}d\theta \Rightarrow \vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta$. It is to be noted that since each element Δl is being considered with complementary element Δl ‘ and therefore limit of integration is taken from 0 to $\frac{\pi}{2}$. It simplifies to using $\vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}[\sin\theta]_0^{\frac{\pi}{2}} = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}[1-0] = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}$. Substituting the value of $R = \frac{L}{\pi}$, we have $\vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0\left(\frac{L}{\pi}\right)^2} \Rightarrow \vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0\left(\frac{L}{\pi}\right)^2} = (-\hat{\jmath})\frac{Q}{2\varepsilon_0 L^2}$. . Thus magnitude of electric field at the center is $E_R = \boldsymbol{\frac{Q}{2\varepsilon_0 L^2}}$ **is the answer**.
I-72	Given is rod of length $L = 0.10$ carries a charge $Q = 50\ \times 10^{-6}$C. It is required to find electric field which is at a distance $R = 0.10$ m from both the ends of the rod. As per Coulomb’s Law force between two charges Q and q is $\vec{F} = \frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$ …(1). And electric field due to charge is force caused by it on unit positive charge. Accordingly, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times 10^9)$ Nm^{2}C^{-2}.  The system, as given, is shown in the figure with detailing for the purpose of analysis. Charge on the element of length $\Delta x \to 0$ is $\Delta Q = \frac{Q}{L}\times\Delta x$…(2). Thus electric field at point P is force $\Delta\vec{E} = \Delta\vec{E}_x + \Delta\vec{E}_y = E\cos\theta\,\hat{\imath} + E\sin\theta\,\hat{\jmath}$…(3). Geometrically, distance of point P, equidistant from both ends of the rod. from the midpoint of the rod O is $y = \sqrt{R^2 - \left(\frac{L}{2}\right)^2} = \frac{\sqrt{4R^2 - L^2}}{2}$…(4) And field at point P due an element at A is $\Delta\vec{E} = \frac{\Delta Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\Delta Q}{4\pi\varepsilon_0(x^2+y^2)}\hat{r}$, since $r = \sqrt{x^2+y^2}$. And as per (3) it is $\Delta\vec{E} = \frac{\left(\frac{Q}{L}\times\Delta x\right)\cos\theta}{4\pi\varepsilon_0(x^2+y^2)}\hat{\imath} + \frac{\left(\frac{Q}{L}\times\Delta x\right)\sin\theta}{4\pi\varepsilon_0(x^2+y^2)}\hat{\jmath}$…(5). Net field at P requires integration of (4), but here $\Delta E = f(x,\theta)$ where $\theta < 90^0$ appears in the form of trigonometric ratios such that $\cos\theta =$

$\frac{-x}{r} = -\frac{x}{\sqrt{x^2+y^2}}$ and $\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$. Accordingly, $\Delta\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left(-\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{\imath} + \frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{\jmath}\right)\Delta x$ is in one variable x. Therefore, $\vec{E} = \int d\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left[\left\{\int_{-\frac{x}{2}}^{\frac{x}{2}}\frac{y}{(x^2+y^2)^{\frac{3}{2}}}dx\right\}\hat{\jmath} - \left\{\int_{-\frac{x}{2}}^{\frac{x}{2}}\frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx\right\}\hat{\imath}\right] = \frac{Q}{4\pi\varepsilon_0 L}[I_1\hat{\jmath} + I_2\hat{\imath}]_{-\frac{L}{2}}^{\frac{L}{2}}$. …(6)

Here, *it involves a bit of calculus*, which is first solved as indefinite integral and limits of definite integration shall be used in its final form.

(a) $I_1 = y\int\frac{1}{(x^2+y^2)^{\frac{3}{2}}}dx$. Substitute $x = y\cot(\pi-\theta) = -\cot\theta \Rightarrow dx = -(-y\,\mathrm{cosec}^2\,\theta\, d\theta) = y\,\mathrm{cosec}^2\,\theta\, d\theta$. It leads to $I_1 = y\int\frac{y\,\mathrm{cosec}^2\,\theta}{(y^2\cot^2\theta+y^2)^{\frac{3}{2}}}d\theta = \frac{1}{y}\int\frac{\mathrm{cosec}^2\,\theta}{\mathrm{cosec}^3\,\theta}d\theta = \frac{1}{y}\int\sin\theta\, d\theta = -\frac{1}{y}\cos\theta = -\frac{1}{y}\times\frac{\cot\theta}{\sqrt{1+\cot^2\theta}} = -\frac{1}{y}\times\frac{\frac{x}{y}}{\sqrt{1+\left(\frac{x}{y}\right)^2}} = -\frac{x}{yr}$. Accordingly, $I_1' = [I_1]_{\frac{L}{2}}^{-\frac{L}{2}} = \frac{1}{yr}[-x]_{\frac{L}{2}}^{-\frac{L}{2}} = \frac{1}{yr}[x]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{1}{yr}\left[\frac{L}{2} - \left(-\frac{L}{2}\right)\right] = \frac{L}{yr}$…(7)

(b) $I_2 = \int\frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx$. This an integration of an even function and its definite integral within limit $-\frac{L}{2} \le x \le \frac{L}{2}$ is zero. On its definite integral within limit $-\frac{L}{2} \le x \le \frac{L}{2}$ is zero. Accordingly, $I_2' = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx = 0$…(8)

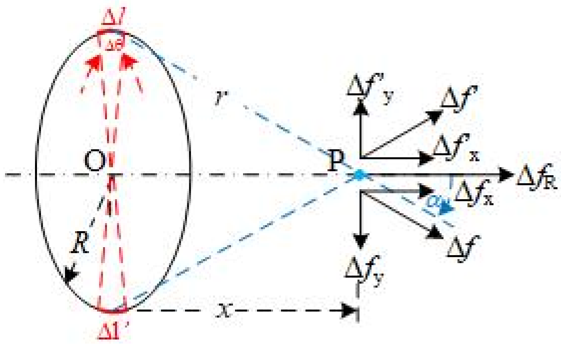
Combining (4),(6), (7) and (8) we have, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left(\frac{L}{yr}\right)\hat{\jmath} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{\left(\sqrt{r^2-\left(\frac{L}{2}\right)^2}\right)r}$…(9). Using (4)On substituting the available data, magnitude of electric field at point P is $E = \left(9\times10^9\right)\left(\frac{50\times10^{-6}}{0.1\times\sqrt{0.75}}\right) = 4.5\times10^5$N/C is the answer. 5.2×10^7 N/C

N.B.: This problem being a little simple, electric field is determined without simplification of analysis using symmetry of the geometry. It is good case study for students to practice for gaining analytical proficiency. However, using geometrical symmetry for simplification of illustration and ease of understanding is generally used.

I-73

Given is a circular ring of radius R carrying a uniformly distribute positive charge Q. A particle of mass m having a negative charge q is placed ata distance x from the center of the circle O.

As per Coulomb's Law force between two charges Q and q is $\vec{F} = \frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$ and therefore electric field being net force on a unit (+)ve charge is $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$…(1). Once resultant force E_R due to the ring at point P at distance x from O, the center of ring, using differential calculus (principle of Maxima), position of maximum field can be determined.



The system having a unit (+)ve charge at P, as given, is shown in the figure with details for the purpose of analysis. Let a small elemental arc of length $\Delta l = R\Delta\theta$ is considered. Charge on the element would be $\Delta Q = \frac{Q}{2\pi R}\times R\Delta\theta = \frac{Q}{2\pi}\Delta\theta$. Thus force $\Delta\vec{E} = \Delta\vec{E}_x + \Delta\vec{E}_y$…(2), due to this element on unit charge at P at a distance $\vec{r} = \left(\sqrt{x^2+R^2}\right)\hat{r}$. ..(3). Taking charge Q

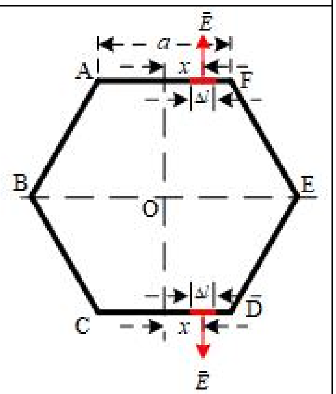
to be (+)ve electric field caused by it is along $\hat{r}$. As a matter of fact magnitude of field is independent of the nature of charge Q, which would be otherwise in the reverse direction. Component of field $\Delta\vec{E}$ due to element Δl along $\hat{\imath}$ is $\Delta\vec{E}_x$while the component $\Delta\vec{E}_y$ is perpendicular to it along $-\hat{\jmath}$. Likewise, another complementary element $\Delta l'$ causes force on the particle $\Delta\vec{E'} = \Delta\vec{E'}_x + \Delta\vec{E'}_y$…(4) The perpendicular components being equal in magnitude, but in opposite directions, lead to $\Delta\vec{E}_y + \Delta\,\vec{E'}_y = 0$ …(5). While axial components are equal and unidirectional $\Delta\vec{E}_x = \Delta\vec{E'}_x$…(6). Thus on combining (2,4, 5 & 6), $\Delta\vec{E}_R = 2\Delta\vec{E}_x = 2\Delta E\cos\alpha\,\hat{\imath}$…(7).

Here, using (1) $\Delta\vec{E} = \frac{\Delta Q}{4\pi\varepsilon_0 r^2}(\hat{\imath}) = \frac{\left(\frac{Q}{2\pi}\Delta\theta\right)}{4\pi\varepsilon_0\left(\sqrt{x^2+R^2}\right)^2}(-\hat{\imath}) = \frac{Q}{8\pi^2\varepsilon_0(x^2+R^2)}\theta(\hat{\imath})$ and geometrically $\cos\alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2+R^2}}$. Accordingly, using these values in (7) $\Delta\vec{E}_R = 2\times\frac{Q}{8\pi^2\varepsilon_0(x^2+R^2)}\Delta\theta\times\frac{x}{\sqrt{x^2+R^2}}(\hat{\imath}) = \frac{Qx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\Delta\theta(\hat{\imath})$

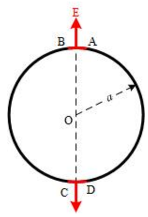
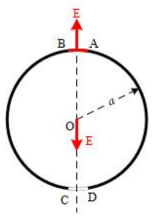
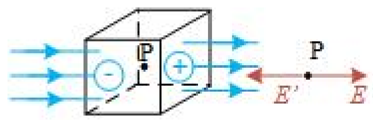
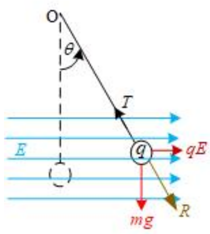
Therefore, net resultant force $\vec{E}_R = \int d\vec{E}_R = (\hat{\imath})\int_0^{\pi}\frac{Qx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}d\theta \Rightarrow \vec{E}_R = (\hat{\imath})\frac{Qx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\int_0^{\pi}d\theta$. It is to be noted that since each element Δl is being considered with complementary element $\Delta l'$ and therefore limit of integration is taken from 0 to π and not 0 to 2π. It simplifies to using $\vec{E}_R = (\hat{\imath})\frac{Qx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\times\pi = \frac{Q}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}(\vec{x})$. Since, it is required to find distance where electric field is maximum hence its magnitude is of relevance. Accordingly, $E_R = \frac{Qx}{4\pi^2\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}\times\pi = \frac{Qx}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}}$…(8)

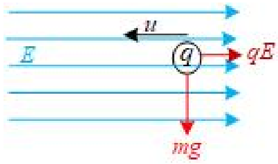
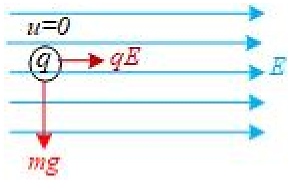
The distance x for maximum of E_R in (8), based on the principle of maxima, is solution of $\frac{d}{dx}\frac{Qx}{4\pi\varepsilon_0(x^2+R^2)^{\frac{3}{2}}} = 0 \Rightarrow \frac{Q}{4\pi\varepsilon_0}\frac{d}{dx}\frac{x}{(x^2+R^2)^{\frac{3}{2}}} = 0 \Rightarrow \frac{d}{dx}\frac{x}{(x^2+R^2)^{\frac{3}{2}}} = 0$. Solving L.H.S as derivative of product of two functions $\frac{1}{(x^2+R^2)^{\frac{3}{2}}}\times\frac{d}{dx}x + x\times\frac{d}{dx}\frac{1}{(x^2+R^2)^{\frac{3}{2}}} = 0 \Rightarrow \frac{1}{(x^2+R^2)^{\frac{3}{2}}} + x\times\left(\frac{d}{du}\left(\frac{1}{u^3}\right)\times\frac{du}{dx}\right) = 0$…(9). Here $u^2 = x^2+R^2 \Rightarrow 2xdx = 2udu \Rightarrow \frac{du}{dx} = \frac{x}{u}$ S. Solving second term on LHS $x\times\left(\frac{d}{du}\left(\frac{1}{u^3}\right)\times\frac{du}{dx}\right) = x\times\left(\frac{-3}{u^4}\times\frac{x}{u}\right)$. Using this in (9) $\frac{1}{(x^2+R^2)^{\frac{3}{2}}} - \frac{-3x^2}{(x^2+R^2)^{\frac{5}{2}}} = 0 \Rightarrow \frac{x^2+R^2-3x^2}{(x^2+R^2)^{\frac{5}{2}}} = 0 \Rightarrow R^2 - 2x^2 = 0 \Rightarrow x = \frac{R}{\sqrt{2}}$. Thus, distance of point P from O is $\boldsymbol{\frac{R}{\sqrt{2}}}$ **is the answer.**

I-74 Given that a charge q is distributed uniformly on a regular hexagon of side a. This is the case of an equipotential perimeter, where charges stay at heir place and creates a uniform electric field perpendicular at every point on the perimeter, and directed outward. Tis is conceptualized in the figure. Thus there is no electric field, mathematically **Zero**, inside the perimeter and is valid for center of the hexagon O **is the answer**.



N.B.: This can be mathematically verified by calculating electric field at O due to two small elements, as shown in the figure, of length Δl each containing on sides AF and CD charge $\Delta q = \frac{q\Delta l}{6a}$. Net electric field at point O can be calculated in accordance with Coulomb's Law, due to the two sides which will work out to be zero. So also it will be true for pair of sides AB and DE and the other pair of sides BC and EF.

I-75	Given that a charge Q is distributed uniformly on a circular wire-loop of radius a. This is the case of an equipotential perimeter, where charges stay at their place, and there is a uniform electric field perpendicular at every point on the perimeter, and directed outward as per Gauss's Law. This is conceptualized in the figure. Strength of the electric field as per Coulomb's Law is $E = \frac{Q\times1}{4\pi\varepsilon_0 a^2} = \frac{Q}{4\pi\varepsilon_0 a^2}$…(1) Thus there is no electric field, at every point inside the perimeter i.e. mathematically it is **Zero**; and it applies to center O also.   Now, a small length dl of wire CD is cut carrying a charge $dQ = \frac{Qdl}{2\pi a}$ …(2) This changes circular configuration from a closed loop to an open loop where portion AB, complementary to CD, creates electric field around it in 360^0, and this field extends until infinity as per (2). Accordingly, combining (1) and (2), electric field at O, the center of the wire, combining (1) an (2) is $E = \frac{\left(\frac{Qdl}{2\pi a}\right)}{4\pi\varepsilon_0 a^2} \Rightarrow E = \frac{\boldsymbol{Q}}{\boldsymbol{8\pi^2\varepsilon_0 a^3}}$ **is the answer.**
I-76	A conducting cube in an electric field will act like a equipotential surface and electric field at every point on it is perpendicular to the conducting surface. Yet, surface of the cube will develop charge such that surface of the cube facing incoming electric field would develop a (-)ve charge on it and surface meaning electric field would develop a (+)ve charge on it. This charge distribution on the conducting surface of the cube should create an electric field E' at point P, center of the cube.  In absence of the cube charge q as per Coulomb's Law would produce at point P, at a distance d from the charge would produce an electric field $E = \frac{q}{4\pi\varepsilon_0 d^2}$. Any conducting closed surface has Zero electric field inside it. As per Principle of Superimposition , this is possible only if $\vec{E} + \vec{E'} = 0$ or $\vec{E} = -\vec{E'}$, i.e. the two field are equal in magnitude and opposite in directions. Therefore, electric field at P, the center of the cube due to charge produced on the cube is of magnitude $\frac{\boldsymbol{q}}{\boldsymbol{4\pi\varepsilon_0 d^2}}$ **is the answer.**
I-77	Given is a bob of mass $m = 8\times10^{-6}$kg carries a charge $q = 2\times10^{-8}$C. It is at rest ion a uniform electric field $E = 20\times10^3$ V/m. The system of the bob suspended by a thread rom O, in rest, under influence of electric field and earth's gravitational field is shown the figure. Thus, the bob is in a state of equilibrium such that $\vec{T} + \vec{F_g} + \vec{F_E} = 0 \Rightarrow \vec{T} + mg(-\hat{\jmath}) + qE\hat{\imath} \Rightarrow \vec{T} + \vec{R} = 0 \Rightarrow \vec{T} = -\vec{R}$ …(1) i.e. $\vec{\mathrm{T}}$ and $\vec{\mathrm{R}}$ are equal in magnitude and opposite in direction.  Here, $\vec{T}$ is the tension in the string while $\vec{F_g}$ and $\vec{F_E}$ are forces due to gravitational and electric field on the bob such that they are orthogonal to each other. Magnitude of the resultant of the two forces $\vec{F_g}$ and $\vec{F_E}$ is $R = \sqrt{(mg)^2 + (qE)^2}$. Using the given data and taking $g = 10\text{m/s}^2$, $R = \sqrt{\left((80\times10^{-6})\times(10)\right)^2 + \left((2\times10^{-8})\times(20\times10^3)\right)^2}$. It

	leads to $R=\sqrt{(8.0\times10^{-4})^2+(4.0\times10^{-4})^2}=4\times10^{-4}\sqrt{2.0+1.0}\Rightarrow R=4\times10^{-4}\times2.2$N or $R=8.8\times10^{-4}$ N. Thus in accordance with inference of (1), tension in the string is $\mathbf{8.8\times10^{-4}}$ **N, is the answer**.
I-78	Given system is shown in the figure particle has a mass m and carries a charge q, is thrown with a horizontal speed u in an electric field of strength $\vec{E}$. Thus particle is subjected to two forces $\vec{F}_g=mg(-\hat{j})$ and $\vec{F}_E=qE\hat{\imath}$. The particle is since thrown against the direction of electric field i.e. $\vec{u}=u(-\hat{\imath})$.  This is the case when particle will move along the resultant force. The electric field $\vec{E}$ being in direction opposite to the initial velocity $\vec{u}=u(-\hat{\imath})$ it will create a retardation until the particle comes to a state of rest along $\hat{\imath}$. Thereafter, it will start its motion along $\vec{E}$ under the influence of $\vec{F}_E$. But, $\vec{F}_g$ along $(-\hat{j})$ will not have its resolution along $\hat{\imath}$ being perpendicular to each other $[\cos 90^0=0]$. Therefore, motion of the particle in this case has to be analyzed only under influence of $\vec{E}$ such that retardation of the particle would be $\vec{a}=\frac{\vec{F}_e}{m}=\frac{qE\hat{\imath}}{m}=\frac{qE}{m}\hat{\imath}$. Accordingly, using magnitudes third equation of motion, which is in scalar form, $v^2=u^2+2as$, where $v=0$. Thus, when momentarily the particle comes to state of rest $u^2+2as=0\Rightarrow s=-\frac{u^2}{2a}$. Using the available data $s=-\frac{u^2}{2\left(\frac{qE}{m}\right)}=-\frac{mu^2}{2qE}$. Thus, magnitude of distance covered in a direction opposite to the electric field is $\boldsymbol{\frac{mu^2}{2qE}}$ **is the answer.**
I-79	Given is a particle of mass $m=1.0\times10^{-3}$ kg and it carries a charge $q=2.5\times10^{-4}$C. The particle is released from a state of rest $u=0$ in an electric field $E=1.2\times10^{4}$ NC^{-1} The particle, since has a mass it will be subjected to gravitational force $\vec{F}_g=mg(-\hat{j})$ … (1)and electric force $\vec{F}_E=qE\hat{\imath}$ …(2).  In this particle will move along the resultant force. The electric field $\vec{E}$ being along $\hat{\imath}$ it will create an acceleration, as per Newton's Second Law of Motion, in that direction $\vec{a}=\frac{\vec{F}_E}{m}=\frac{qE}{m}\hat{\imath}$…(3) In this context, each part of the question is being analyzed independently. **Part (a):** Magnitude of the Electric force as per (2) is $F_E=qE=(2.5\times10^{-4})\times(1.2\times10^{4})\Rightarrow F_E=3.0$ N. And gravitational force as per (1) is $F_g=mg=(1.0\times10^{-3})\times10\Rightarrow F_g=1.0\times10^{-2}$ N. here we take $g=10$ m/s^2. Thus, 3.0 N and 1.0×10^{-2} are the answers. **Part (b):** Comparing the two forces in Part (a), $\frac{F_g}{F_E}=\frac{1.0\times10^{-2}}{3.0}\Rightarrow F_g=3.\dot{3}\times10^{-3}$. Thus, F_g is 0.3% of F_E and can be ignored for approximate analysis. Hence answer is Yes. **Part (c):** It is required to find distance travelled by the particle along electric field and hence only acceleration a as per (3) would be considered. Thus, $a=\frac{F_E}{m}$. The particle is initially at $u=0$, hence time taken to travel a distance $s=0.40$ m as per Second Equation of

motion will be $s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{\frac{qE}{m}}} \Rightarrow t = \sqrt{\frac{2ms}{qE}}$. Using the available data $t = \sqrt{\frac{2(1.0\times10^{-3})\times0.40}{(2.5\times10^{-4})\times(1.2\times10^{4})}} \Rightarrow t = \sqrt{\frac{8.0\times10^{-2}}{3}} = \mathbf{1.6}$ **s, is the answer**.

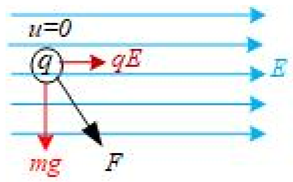
Part (d): Speed of the particle, in this case, after travelling a distance $s = 0.40$ m as per Third equation of motion $v^2 = u^2 + 2as \Rightarrow v = \sqrt{2as}$. Using the available data, $v = \sqrt{2\left(\frac{(2.5\times10^{-4})\times(1.2\times10^{4})}{(1.0\times10^{-3})}\right)(0.40)} = \sqrt{2.4 \times 10^3} = 10 \times \sqrt{24} \Rightarrow v = 10 \times 4.9 = \mathbf{49}$ **m/s is the answer.**

Part (e): Work done by electric field in moving a charge by a distance $s = 0.40$ m is $W = \vec{F}_E.\vec{s}$. Using the available data $W = \left((2.5 \times 10^{-4}) \times (1.2 \times 10^{4})\hat{\imath}\right).(0.4\hat{\imath}) = \mathbf{1.2}$ **J is the answer.**

Thus, answers are (a) 3.0 N, $\mathbf{1.0 \times 10^{-2}}$N (b) Yes (c) $\mathbf{1.6 \times 10^{-2}}$ s (d) 49 m/s (e) 1.2 J.

I-80

Given is a particle of mass $m = 100 \times 10^{-3}$ kg and it carries a charge $q = 4.9 \times 10^{-5}$C. The particle is released from a state of rest $u = 0$ in an electric field $E = 2.0 \times 10^4$ NC^{-1} The particle, since has a mass it will be subjected to gravitational force $\vec{F}_g = mg(-\hat{\jmath})$ … (1)and electric force $\vec{F}_E = qE\hat{\imath}$ …(2). Illustration of each part of the solution is as under-



Part (a): Resultant force on the particle $F = \sqrt{{F_g}^2 + {F_E}^2}$. Using the available data $F = \sqrt{\left((100 \times 10^{-3}) \times 9.8\right)^2 + \left((4.9 \times 10^{-5}) \times (2.0 \times 10^4)\right)^2} \Rightarrow F = \sqrt{(9.8 \times 10^{-1})^2 + (9.8 \times 10^{-1})^2}$. It leads to $F = (9.8 \times 10^{-1}) \times \sqrt{2} = 1.4$ N. Since, both the components F_g and $F_E$are equal and perpendicular to each other, the F would be equally inclined to both the forces. Thus, $\boldsymbol{F} = \mathbf{1.4}$ **N at (-)45^0 is the answer.**

Part (b): As per Newton's Second Law of motion, the particle will move along **the direction of resultant force F, is the answer.**

Part (c): To determine the distance travelled by the particle in $t = 2$s, when $u = 0$, it ill be required to use Second equation of motion $s = ut + \frac{1}{2}at^2$, with an acceleration as per Newton's Second Law of Motion, based on the force determined in part (a). Accordingly, $a = \frac{F}{m} = \frac{1.4}{0.1} = 14$ m/s^2. Using the available data, $s = \frac{1}{2} \times 14 \times 2^2 =$ **28m from the initial position is the answer.**

Thus answers are – (a) 1.4 N at an angle $\mathbf{-45^0}$, (b) Along the direction of the resultant force, (c) 28m from the initial position is the answer.

	N.B: In this case value of g is taken to be 9.8 m/s^2, instead of 10 m/s^2. This is based on the observation of the given data and the value which would lead to simplest answer with minimum calculations.

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