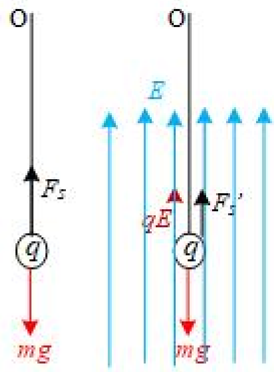
Electromagnetism: Electric Field and Gauss's Law – Typical Questions

(Set 2: Illustrations)

Important Note: Determination of Electric Field using Coulomb's Law basic concept based on which concept of electric potential has been developed. Yet, electric field at a point due to multiple charges or distributed charges becomes simple by determining electric potential. Extension of Coulomb's Law into Gauss's Law is a further simplification that is helpful in determining electric field at a point. It, however, require to define Gaussian surface with due consideration to geometrical symmetry of charge distribution. Illustrations here will lead to better understanding of the concepts and their applications.

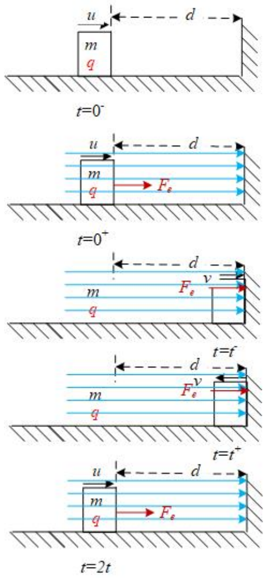
I-1	Given that a bob of mass $m = 40 \times 10^{-3}$ kg has a charge $Q = 4.0 \times 10^{-6}$ C. Time period of the oscillation of the bob is $T = 2\pi\sqrt{\frac{l}{a}}$. Here, $a = \frac{F_S}{m}$ is the acceleration of the bob at its mean position of suspension. This problem has two cases and each is analyzed as under -  **Case 1:** Here, $\vec{F_g} = m\vec{g} = mg(-\hat{\jmath})$. Thus force of the string is $\vec{F_S} = \vec{F_g}$. In this case net acceleration of the bob responsible for oscillation is $\vec{a} = \vec{g}$. Accordingly, $T = 2\pi\sqrt{\frac{l}{\frac{F_S}{m}}} == 2\pi\sqrt{\frac{l}{g}}$. In this it makes 20 oscillations in 45 s. It implies that $T = \frac{45}{20} = 2.25$ s. **Case 2:** When electric field $E = 2.5 \times 10^4$ is switched as stated, and shown in the figure, electric force on the bob in the direction of the electric field, is $\vec{F_E} = q\vec{E} = qE\hat{\jmath}$. Net force in the string is $\overrightarrow{F_s}' = \vec{F_g} + \vec{F_E} \Rightarrow \overrightarrow{F_s}' = mg(-\hat{\jmath}) + qE\hat{\jmath} \Rightarrow \overrightarrow{F_s}' = (qE - mg)\hat{\jmath}$. Thus, effective acceleration causing oscillation of the bob is $\overrightarrow{a}' = \frac{\overrightarrow{F_s}'}{m} \Rightarrow a' = \frac{qE-mg}{m} \Rightarrow \lvert a' \rvert = \left\lvert g - \frac{qE}{m} \right\rvert$. Therefore, time period of the oscillation is $T' = 2\pi\sqrt{\frac{l}{a'}} \Rightarrow T' = 2\pi\sqrt{\frac{l}{\left(g-\frac{qE}{m}\right)}}$. Combing result of the two cases $\frac{T\prime}{T} = \frac{2\pi\sqrt{\frac{l}{\left(g-\frac{qE}{m}\right)}}}{2\pi\sqrt{\frac{l}{g}}} \Rightarrow T' = T \times \sqrt{\frac{g}{\left(g-\frac{qE}{m}\right)}}$. Taking $g = 10$ m/s^2 and the given data $T' = 2.25 \times \sqrt{\frac{10}{\left(10-\frac{(4.0\times10^{-6})(2.5\times10^{4})}{(40\times10^{-3})}\right)}} \Rightarrow T' = 2.25 \times \sqrt{\frac{10}{(10-2.5)}} = 2.25 \times \sqrt{\frac{10}{7.5}} = 2.25 * 1.15$s. Therefore, time to complete 20 oscillation would be $= T' \times 20 = 2.25 \times 1.15 \times 20 = 51.75$s, Using principle of SDs **answer is 52 s.**
I-2	The mass m carrying a charge q in presence of electric field would experience a force $F_E = qE$….(1) This would cause an elongation l in the spring whose spring constant is k with one end fixed such that restraining force as per spring law $F_s = kl$…(2) .This is the position of equilibrium of the mass such that $F_s = F_E$…(3). Combining (1) and (2) in (3) we have $kl = qE \Rightarrow l = \frac{qE}{k}$. But, in the process, the mass will have acquired kinetic energy and this cause an overshoot of the mass by a distance x, where spring creates an additional restraining force $f = -kx$, as per spring law. This force f is proportional to displacement from mean position

and against direction of motion i.e. towards mean position. So also on achieving maximum displacement. This restraining force f will cause retardation on the mass until its velocity becomes zero. At this position, existence of f would accelerate the mass towards the mean position. But due to kinetic energy gained by the mass, as per Law of Conservation of Energy, it would again overshoot to compress the spring until it reaches its natural length. Thus, motion of the mass satisfies criteria of Simple Harmonic Motion (SHM) whose magnitude is $l = \frac{qE}{k}$. is the answer.

N.B.: In the problem natural length of spring has no role. And spring is assumed to be an ideal spring following law of linear restraining force.

I-3

The mass m carrying a charge q, placed on a smooth table would, in absence of electric field, would continue to be in state of rest as per Newton's Third and First Law of motion. As soon electric field is switched in would experience a force $F_E = qE$….(1) and thereby an acceleration a, as per Newton's Second Law of Motion, $F_E = ma \Rightarrow a = \frac{F_E}{m} = \frac{qE}{m}$…(2). This question has two parts – **Part 1:** time period of oscillation, **Part II:** Is the oscillation a simple harmonic motion (SHM). Analysis is facilitated with five instance diagrams, and each stage is being analyzed-



At $t = 0^-$: is initial position when electric field is not switched on.

At $t = 0^+$: under influence of F_E , the mass at rest $u = 0$, and it will move toward wall with acceleration a.

At $t = t^-$: while covering a distance d, before it collides with the wall, will acquire velocity, as per Third Equation of Motion $v^2 = u^2 + 2ad \Rightarrow v = \sqrt{2ad}$. Time t taken by the mass to reach the wall, as per First equation of motion, $v = u + at \Rightarrow t = \frac{v}{a} = \frac{\sqrt{2ad}}{a} = \sqrt{\frac{2d}{a}}$…(3). Combining (2) and (3), $t = \sqrt{\frac{2d}{\left(\frac{qE}{m}\right)}} = \sqrt{\frac{2md}{qE}}$.

At $t = t^+$: time: after the elastic collision , as stated in the problem, the mass will return with the same velocity v, but in a direction opposite to the electric field as shown in the figure.

At $t = 2t$: Thus electric force, due to charge on the mass, during $t = t^+$to $t = 2t$ will cause retardation a of magnitude, but in direction opposite to velocity of the mass. Thus, as per equations of motion in another time t it will reach the original position, with velocity Zero to complete one cycle of oscillation .

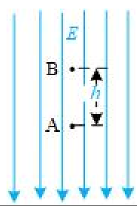
A Thus total period of oscillation would be $T = 2t = 2\sqrt{\frac{2md}{qE}} \Rightarrow T = \sqrt{\frac{8md}{qE}}$ **is the answer of part I.**

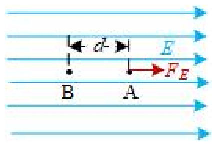
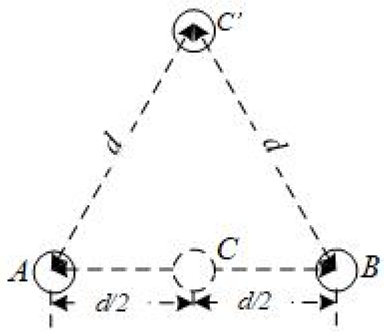
This kind of motion with time period T would continue, between the wall and initial position of the mass, as long as electric field exists. In this case force on the particle is constant and unidirectional irrespective of the fact whether mass is approaching the wall or moving away from the wall. This is against basic parameters of a simple harmonic motion (SHM), which requires force to be proportional to displacement a from mean position. Hence, **motion of the mass is not SHM, is the answer of part II.**

N.B.: Infact this question can be framed into option question. Yet framing such question in an illustrative problem requies that necessary considerations are brought out, rather than just straight answer in few steps.

I-4

Electric potential difference between two points work done in moving a unit (+) charge between the wo points expressed as $\Delta V = W = -\vec{E}.\vec{d}$. Let the two points be i.e. from A and B as shown in the figure. Given that electric field $\vec{E} = 10(-\hat{\jmath})$ is vertically downward, and displacement is upward i.e. $\vec{d} = 0.5\hat{\jmath}$. Thus, with the given data $\Delta V = W = 10(-\hat{\jmath}).(0.5\hat{\jmath}) =$ **5 V is the answer.**



I-5	A charge $q = 0.01\text{C}$ is moved from A to B by a displacement $\vec{d} = d(-\hat{\imath})$ in a direction against electric field $\vec{E} = E\hat{\imath}$. Therefore, workdone on the charge $W = -\vec{F}_E.\vec{d}$. Given that $W = 12$ J. Further, workdone is $W = \Delta V q$…(1) Here, given that $\Delta V = V_B - V_A$. Thus, using the available data in (1), $12 = (V_B - V_A) \times 0.01 \Rightarrow (V_B - V_A) = \frac{12}{0.01}$. It leads to $(V_B - V_A) = \mathbf{1200}$ **V is the answer.** 
I-6	Given that there are three charges of equal magnitude 2.0×10^{-7}C. Two of them are placed at A and B ad a distance $d = 0.2$m. The third equal charge is placed at midway at C. By principle of superimposition potential at C is $V_C = V_{AC} + V_{BC}$. Here, V_{AC} is potential at C due to charge at A and V_{BC} is potential at C due to charge at A. Potential V at a point at distance r due to a charge q is $V = \frac{q}{4\pi\varepsilon_0 r}$…(1)   The charge at C is required to be moved to a new position C' as shown in the figure. Thus potential at new position C' is $V_{C'} = V_{AC'} + V_{BC'}$. Accordingly, potential difference between C and C' is $V_{CC'} = V_{C'} - V_C$. Thus, $V_{CC'} = (V_{AC'} + V_{BC'}) - (V_{AC} + V_{BC})$…(2) here all the constituent quantities are scalar. Using (1) in (2) we have $V_{CC'} = \left(\frac{q}{4\pi\varepsilon_0 d} + \frac{q}{4\pi\varepsilon_0 d}\right) - \left(\frac{q}{4\pi\varepsilon_0\left(\frac{d}{2}\right)} + \frac{q}{4\pi\varepsilon_0\left(\frac{d}{2}\right)}\right) \Rightarrow V_{CC'} = \frac{q}{4\pi\varepsilon_0 d}(2-4) = -\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2q}{d}$. Using the given data and that $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2} we have $V_{CC'} = (-)9 \times 10^9 \times \frac{2\times(2.0\times10^{-7})}{0.2} = (-)1.8 \times 10^4$ V. Therefore work done I displacing charge from C to C' is $W_{CC'} = qV_{CC'} = (-1.8 \times 10^4) \times (2.0 \times 10^{-7}) = -3.6 \times 10^{-5}$J. Negative sign in in the final value indicates that work is done by the electric field. Hence, **answer is $\mathbf{3.6 \times 10^{-3}}$J.** **N.B.:** Solving this problem by determining net force on the third charge in any position along CC' an integrating to determine total work done is also possible and it would also lead to same answer. But, that makes the calculations more complex. Thus, effectiveness of the principle of potential difference is demonstrated in this problem.
I-7	Given that $\vec{E} = 20\hat{\imath}$ N/C …(1), is uniform and pair of coordinates of A and B in three sets. For each set $\Delta V = V_B - V_A$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_B - V_A = -(\vec{E}.\vec{r}_B - \vec{E}.\vec{r}_A) \Rightarrow V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_{AB})$…(2). Using the given data in (2), and taking each part separately – **Part (a):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 4\hat{\imath} + 2\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (4\hat{\imath} + 2\hat{\jmath})\right) = -20 \times 4 = \mathbf{-80}$**V** **Part (b):** $\vec{r}_A = 4\hat{\imath} + 2\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((4\hat{\imath} + 2\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right) = -20 \times 2 = \mathbf{-40}$**V** **Part (x):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right) = -20 \times 6 = \mathbf{-120}$**V** **Hence answers are (a) -80 V (b) -40 V (c) -120 V** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath} \cdot \hat{\imath} = 1$; $\hat{\jmath} \cdot \hat{\jmath} = 1$ and $\hat{\imath} \cdot \hat{\jmath} = 0$. This is done to encourage mathematical ability.

I-8	Given that $\vec{E} = 20\hat{\imath}$ N/C …(1), is uniform and pair of coordinates of A and B in three sets. For each set $\Delta V = V_B - V_A$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_B - V_A = -\left(\vec{E}.\vec{r}_B - \vec{E}.\vec{r}_A\right) \Rightarrow V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_{AB})$…(2). Further, change in potential energy is $U_B - U_A = q(V_B - V_A)$ and given that $q = -2.0 \times 10^{-4}$C. Using the given data in (2), and taking each part separately – **Part (a):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 4\hat{\imath} + 2\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (4\hat{\imath} + 2\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 4) = \mathbf{0.016}$ **J.** **Part (b):** $\vec{r}_A = 4\hat{\imath} + 2\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((4\hat{\imath} + 2\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 2) = \mathbf{0.008}$ **J.** **Part (x):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 6) = \mathbf{0.024}$ **J** **Hence answers are (a) 0.016 J, (b) 0.008 J, (c) 0.024 J** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath}\cdot\hat{\imath} = 1$; $\hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = 0$. This is done to encourage mathematical ability.
I-9	Given that electric field in pace is $\vec{E} = (20\hat{\imath} + 30\hat{\jmath})$ N/C …(1), and potential at origin $\vec{r}_O = 0\hat{\imath} + 0\hat{\jmath}$ is $V_O = 0$. It is required to find potential at a point P defined by $\vec{r}_P = 2\hat{\imath} + 2\hat{\jmath}$. Potential at point P ($\vec{r}_P$) w.r.t origin ($\vec{r}_O$) is $\Delta V = V_P - 0 \Rightarrow V_P = \Delta V$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_P = -\left(\vec{E}.\vec{r}_P - \vec{E}.\vec{r}_O\right) \Rightarrow V_P = \vec{E}.(\vec{r}_O - \vec{r}_P)$…(2). Using the given data in (2), $V_P = (20\hat{\imath} + 30\hat{\jmath}).\left((0\hat{\imath} + 0\hat{\jmath}) - (2\hat{\imath} + 2\hat{\jmath})\right) \Rightarrow V_P = -(20\hat{\imath} + 30\hat{\jmath})\cdot(2\hat{\imath} + 2\hat{\jmath})$. It leads to $V_P = -(20\hat{\imath}\cdot 2\hat{\imath} + 30\hat{\jmath}\cdot 2\hat{\jmath}) \Rightarrow V_P = -(40 + 60) = \mathbf{-100}$ **V is the answer.** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath}\cdot\hat{\imath} = 1$; $\hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{\imath} = 0$. This is done to encourage mathematical ability. At the same time it easier to solve the problem mathematically rather than graphically.
I-10	Given that electric field in space is $\vec{E} = Ax\hat{\imath}$ …(1), it is $\vec{E} = f(\vec{x})$ and potential at a point P, $\vec{r}_P = 10\hat{\imath} + 20\hat{\jmath}$ is $V_P = 0$. It is required to find potential at origin $\vec{r}_O = 0\hat{\imath} + 0\hat{\jmath}$. Potential at origin ($\vec{r}_O$) w.r.t. the point P ($\vec{r}_P$) is $\Delta V = V_0 - V_P \Rightarrow V_O = \Delta V$ and $\Delta V = -\vec{E}.\overrightarrow{\Delta r}$. Accordingly, $\Delta V = -\vec{E}_x\cdot\Delta\vec{r}_x \Rightarrow \Delta V = -Ax\hat{\imath}.(\Delta r_x\hat{\imath} + \Delta r_y\hat{\jmath}) \Rightarrow \Delta V = -Ax\Delta r_x$ …(2). Integrating (2) $V_0 = -10\int_{10}^{0} xdx = -10\left[\frac{x^2}{2}\right]_{10}^{0} = 5[100 - 0] = \mathbf{500}$ **V is the answer**.
I-11	Taking each part separately – **Part (a):** Potential at a point is defined as work done in moving a unit (+) charge from infinity to the point. Accordingly, as per dimensional analysis $[\text{Potential}] = \frac{[\text{Work}]}{[\text{Charge}]} \Rightarrow [\text{V}] = \frac{\text{ML}^2\text{T}^{-2}}{\text{IT}} = \text{ML}^2\text{T}^{-3}\text{I}^{-1}$…(1). Dimensional equivalent of the given expression is $[\text{V}] = [\text{A}]\text{L}^2$…(2). Combining (1) and (2), $[\text{A}]\text{L}^2 = \text{ML}^2\text{T}^{-3}\text{I}^{-1} \Rightarrow [\text{A}] = \frac{\text{ML}^2\text{T}^{-3}\text{I}^{-1}}{\text{L}^2} = \mathbf{MT^{-3}I^{-1}}$ **is the answer**. **Part (b):** Electric potential is $\Delta V = -\vec{E}\cdot\Delta\vec{r} \Rightarrow \vec{E} = -\frac{\partial V}{\partial r}$…(3). It is a partial derivative. Given that $V(x,y,x) = A(xy + yz + zx)$. Taking partial derivative $\vec{E} = \frac{\partial}{\partial r}A(xy + yz + zx)$. It is to be noted that $\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$. Therefore, $\vec{E} = A\left(\hat{\imath}\frac{d}{dx}(xy + zx) + \hat{\jmath}\frac{d}{dy}(xy + yz) + \hat{k}\frac{d}{dz}(yz + zx)\right)$. It

leads to $\vec{E} = A\left((y+z)\hat{\imath}\frac{d}{dx}x + (x+z)\hat{\jmath}\frac{d}{dy}y + (x+y)\hat{k}\frac{d}{dz}z\right)$, and finally $\boldsymbol{\vec{E} = A\big((y+z)\hat{\imath} + (x+z)\hat{\jmath} + (x+y)\hat{k}\big)}$. **Is the answer.**

Part (c): Given $A = 10$ SI Unit and a point P is (1m, 1m, 1m). i.e. $x = 1$m, $y = 1$m and $z = 1$m. Therefore electric field at P, with the given data $\vec{E}_\mathrm{P} = 10\left((1+1)\hat{\imath} + (1+1)\hat{\jmath} + (1+1)\hat{k}\right) \Rightarrow \vec{E}_\mathrm{P} = 20(\hat{\imath}+\hat{\jmath}+\hat{k}) = 20 \times \sqrt{1^2+1^2+1^2} \times \hat{r} \Rightarrow \vec{E}_\mathrm{P} = 20\sqrt{3}\hat{r}$.Thus magnitude of electric field using principle of SDs is $20\sqrt{3} =$ **35 N/C is the answer.**

Thus, answers are (a) $\mathbf{MT^{-3}I^{-1}}$ (b) $\boldsymbol{-A\{\hat{\imath}(y+z) + \hat{\jmath}(z+x) + \hat{k}(x+y)\}}$ (c) 35 N/C.

N.B.: Part (b) of the illustration requires clarity of partial derivative of a vector, a part of Vector Calculus. This being an exclusive topic is being skipped in illustration

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I-12

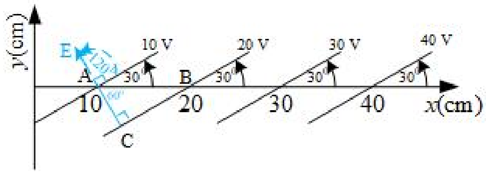
Two particles carrying charge $q = 2.0 \times 10^{-5}$ C are infinity an the are brought within a separation $r = 0.10$ m. The initial potential of the charges is zero. But when they are brought to a separation r, it is equivalent to either of the charge is moved is moved close to the other within the given separation. In the process amount of work done is the increase in potential energy. Force between the two charges as per Coulomb's Law is $\vec{F} = \frac{1}{4\pi\varepsilon_0} \times \frac{q\times q}{r^2}\hat{r} = (9 \times 10^9 \times q^2) \times \frac{1}{r^2}\hat{r}$. Therefore, potential energy of one of the charge moved from infinity is $W = -\int_\infty^r \vec{F}.d\vec{r} = -\int_\infty^r \left(\frac{1}{4\pi\varepsilon_0} \times \frac{q\times q}{r^2}\hat{r}\right).d\vec{r} \Rightarrow W = \frac{1}{4\pi\varepsilon_0} \times q^2\left[\frac{1}{r}\right]_\infty^r = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r}$. We know that $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ $\mathrm{Nm^2C^{-2}}$,using the given data $W = (9 \times 10^9) \times \frac{(2.0\times10^{-5})^2}{0.1} =$ **36J is the answer**.

I-13

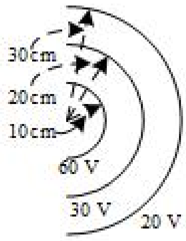
In the given figure two sets of equipotential surfaces are shown – A) Plane surfaces and (b) spherical surfaces.

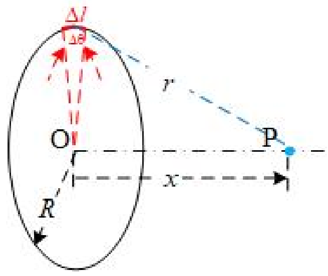
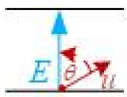
Using the basics two important considerations are (a) relation between electric potential and electric field is $\Delta V = -\vec{E}\cdot\Delta\vec{r}$ …(1), and (b) electric field is always perpendicular to the equipotential surface in a direction from higher potential to lower potential. Applying these two consideration to each of the two given sets of surfaces –

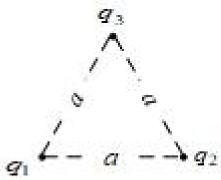
Set A: Equipotential surfaces are uniformly spaced. Therefore, taking points A and B on surfaces of potential 10 V and 20 V respectively $\Delta V = (20-10) = -E(r_b - r_a)\cos 120^0 \Rightarrow 10 = -E(0.2 - 0.1)(-0.5) \Rightarrow E = \frac{10}{0.1\times0.5} =$ **200V/m inclined at $\mathbf{120^0}$ to X-axis is the answer.**

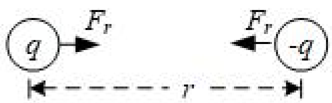


Set B: In this case surfaces are though parallel but they are spherical and therefore electric field will not be uniform in the space between two spherical surfaces. Further observation of the figure reveals that with the increase in the radius of the spherical surface potential is decreasing. Therefore, electric field is radially outward with charge causing the electric field at the center of the sphere. In such cases $V = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow Vr = \text{Cont}\ldots(2)$. Using (2), spacing of the electric field is to be verified. Three surfaces $V_1 = 60$ V, $V_2 = 30$ and $V_3 = 20$ V whose radius of curvature are $rV_1 = 0.10$ m, $r_2 = 0.20$ m and $r_3 = 0.30$ m. And we see that $V_1r_1 = V_2r_2 = V_3r_3 = 6$ Vm...(3) Hence, using (2) again, the premise that the equipotential spherical surfaces are caused by a charge $Q = Vr \times 4\pi\varepsilon_0$ ... (4). Accordingly, electric field at a distance r from O, the center of curvature of



	the concentric spherical surface, as per Coulomb's Law is $E=\frac{Q}{4\pi\varepsilon_0 r^2}$ …(5). Combining (4) and (5), $E=\frac{Vr\times 4\pi\varepsilon_0}{4\pi\varepsilon_0 r^2}=\frac{Vr}{r^2}$…(6). Combining (3) and (6), $E=\frac{6}{r}$ **V/m, is the answer, radially outward.** **Thus answers are (a) 200V/m inclined at 120^0 to X-axis is the answer (b) $\frac{6}{r}$ V/m, is the answer, radially outward.** **N.B.:** This questions has mix of two problems of same nature, but with different nature of equipotential surfaces. But, approach to solution requires identical basic concepts.
I-14	Electric potential V, a vector quantity, at point at a distance r, due to a charge q, is $V=\frac{q}{4\pi\varepsilon_0 r}$…(1) In the instant case a circular ring of radius R is uniformly charged with linear charge density λ C/m. The system is shown in the figure where point P is at a distance x from the center of the ring O. Considering an elemental charge $\Delta q=\lambda\Delta l=\lambda R\Delta\theta$ …(2) on length Δl of the ring as per (1) is $\Delta V=\frac{\Delta q}{4\pi\varepsilon_0 r}$…(3), here geometrically $r=\sqrt{R^2+x^2}$. Thus using (2), $\Delta V=\frac{\lambda R\Delta\theta}{4\pi\varepsilon_0\sqrt{R^2+x^2}}$…(4) Therefore, net potential at P due to charge distributed on the ring, by integrating (4), is $V=\int_0^{2\pi}\frac{\lambda R d\theta}{4\pi\varepsilon_0\sqrt{R^2+x^2}}\Rightarrow V=\frac{\lambda R}{4\pi\varepsilon_0\sqrt{R^2+x^2}}\int_0^{2\pi}d\theta\Rightarrow V=\frac{\lambda R}{4\pi\varepsilon_0\sqrt{R^2+x^2}}\times 2\pi$. It leads to $V=\frac{\lambda R}{2\varepsilon_0\sqrt{R^2+x^2}}$ **is answer of part (a).**  Further, electric field at point P due to radial symmetry of the ring will be along $\hat{\imath}$ and it is $\vec{E}=-\frac{dV}{dx}\hat{\imath}$. Using result of part (a), $\vec{E}=-\frac{d}{dx}\left(\frac{\lambda R}{2\varepsilon_0\sqrt{R^2+x^2}}\right)\hat{\imath}\Rightarrow\vec{E}=-\frac{\lambda R}{2\varepsilon_0}\frac{d}{dx}\left(\frac{1}{\sqrt{R^2+x^2}}\right)\hat{\imath}\Rightarrow\vec{E}=-\frac{\lambda R}{2\varepsilon_0}\left(\frac{-\frac{1}{2}}{\left(\sqrt{R^2+x^2}\right)^3}\right)\times\frac{d}{dx}x^2\hat{\imath}$. It resolves into $\vec{E}=-\frac{\lambda R}{2\varepsilon_0}\left(\frac{-\frac{1}{2}}{\left(\sqrt{R^2+x^2}\right)^3}\right)2x\hat{\imath}\Rightarrow\vec{E}=\frac{\lambda Rx}{2\varepsilon_0\left(\sqrt{R^2+x^2}\right)^3}\hat{\imath}$. Thus, magnitude of electric field at P is $\frac{\lambda Rx}{2\varepsilon_0\left(\sqrt{R^2+x^2}\right)^3}$ **is answer of part (b).**
I-15	Electric field between two parallel plates is perpendicular to the plates and uniform. Further, $\Delta V=-\vec{E}.\Delta\vec{x}$, here $\vec{E}$ and $\Delta\vec{x}$ are collinear therefore $\Delta V=-E\Delta x\cos 0\Rightarrow\Delta V=-E\Delta x$…(1). Taking each part separately – **Part (a):** Thus, on integration of (1), potential difference between plates, using the given data is $V=\left\|-\int_0^{0.02}1000dx\right\|\Rightarrow V=\left\|-1000[x]_0^{0.02}\right\|=$ **20 V is answer of part (a).** **Part (b):** Force on a charge placed in an electric field is $\vec{F}=\vec{E}q$. In instant case, for electron $q=-1.6\times 10^{-19}$C and its mass is $m=9.1\times 10^{-31}$kg. Therefore as per Newton's Second Law of acceleration of electron in the electric field will be $\vec{a}=\frac{\vec{F}}{m}\Rightarrow\vec{a}=\frac{\vec{E}q}{m}$…(2). The problem requires to determine velocity u of an the electron projected from lower plate which is just able to reach upper plate i.e. $v=0$, placed at a separation $s=0.02$ m. Since, electron has (-)ve charge and hence on electron projected from lower plate will be retardation to satisfy the given condition. As per Third Equation of Motion $v^2=u^2+2as$. ..(3). Thus using the available data $0^2=u^2+2\times\frac{1000\times(-1.6\times 10^{-19})}{9.1\times 10^{-31}}\times 0.02\Rightarrow u^2=\frac{6.4}{9.1}\times 10^{13}\Rightarrow u=$ **2.65×10^6 m/s is answer of part (b).** **Part (c):** Given that the electron is projected with velocity $u=2.65\times 10^6$ m/s, arrived at in part (b) at an angle $\theta=60^0$ with the field as shown in the figure. This is like a projectile motion where maximum height will be decided by the initial velocity of the electron along the direction of field $u'=u\cos\theta$ which is causing retardation 

	$\vec{a} = \frac{\vec{E}q}{m}$, since charge of electron is (-)ve. Further, at maximum height velocity of electron along the direction of the field will be zero. Accordingly, using (3) with the available data $0 = (2.65 \times 10^6 \times \cos 60^0)^2 + 2\left(\frac{1000\times(-1.6\times10^{-19})}{9.1\times10^{-31}}\right)s \Rightarrow s = \left((2.65 \times 10^6) \times 0.5\right)^2 \times \frac{9.1\times10^{-31}}{2(1000\times(-1.6\times10^{-19}))} = 5.0 \times 10^{-3}\text{m}$ or **0.50 cm is the answer** of part (c). **Thus answers are (a) 20 V (b) 2.65×10^6 m/s (c) 0.50 cm**
I-16	Given that electric field is in direction of X-axis $\vec{E} = 2.0\hat{\imath}$, it is required to find electric potential $V_P$at a point P represented by position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ when reference is origin O represented by position vector $\vec{O} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ where $V_O = 0$. Solving each part separately – **Part (a):** We know that $\Delta V = \vec{E}.\Delta\vec{r} \Rightarrow \Delta V = (2.0\hat{\imath}) \cdot (\Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}) \Rightarrow \Delta V = 2.0\Delta x$…(1). Therefore, potential at P is $V_P = -\int_0^x 2.0dx$. Thus, $V_P = -2.0[x]_0^x =$ **$-2.0x$ V is the answer of part (a).** **Part (b):** Using result of part (a) position of point where $V = 25$ is $25 = -2.0x \Rightarrow x = -12.5$ m. This equation of a perpendicular to X-axis and plane parallel to Y-Z plane. Hence answer is all points on plane **$x = -12.5$ m, is the answer of part (b).** **Part (c):** Given that $V_O = 100$ V in that case general expression for potential at any point using (1) is $\int_{V_O}^{V_P} dV = -\int_0^x 2.0dx \Rightarrow [V]_{100}^{V_P} = -2.0[x]_0^x \Rightarrow [V_P - 100] = -2.0[x - 0] \Rightarrow V_P =$ **$100 - 2.0x$V is answer of part (c).** **Part (d):** In this case, taking $V_\infty = 0$ using (1), $\int_{V_O}^{V_\infty} dV = -\int_0^\infty 2.0dx \Rightarrow [V]_{V_O}^0 = -2.0[x]_0^\infty \Rightarrow [0 - V_O] = -2.0[\infty - 0] \Rightarrow V_O = \infty$ **is the answer of part (d).** **Part (e):** In the situation defined in the problem, potential at infinity is determined by potential V_a at reference point on a plan $x = a$ such that Taking origin to be reference with potential V_O, using (1), $\int_{V_O}^{V_\infty} dV = -\int_0^\infty 2.0dx \Rightarrow [V]_{V_O}^{V_\infty} = -2.0[x]_a^\infty \Rightarrow [V_\infty - V_a] = -2.0[\infty - a]$. This further resolves into $V_\infty = V_a + 2.0a - \infty$, this in indeterminate expression for a determinate reference point $x = a$ and potential V_a. Hence, **answer to part (e) is No.** **Thus answers are (a) (–)$2.0x$ V (b) Points on the plane x = -12.5 m (c) $(100 - 2.0x)$V (d) Infinity (e) No**
I-17	 A charge in isolation is equivalent to a charge at infinity. But, when one q charge is brought near a charge Q, at a separation r, from infinity then work done is $W = Vq$, where $V = \frac{Q}{4\pi\varepsilon_0 r}$ is the potential due to charge Q at that point. Accordingly, $W = Vq \Rightarrow W = \left(\frac{Q}{4\pi\varepsilon_0 r}\right)q$ …(1). Potential and potential energy both are scalar quantities, therefore potential and potential energy of a system of charges arranged in a defined geometry is arithmetic sum of potential and potential energy of charges in the system is $V = \sum V_i$ and $W = \sum V_i q_i$…(2) Given that $q_1 = 4.0 \times 10^{-5}$C, $q_2 = 3.0 \times 10^{-5}$ and $q_3 = 2.0 \times 10^{-5}$ while displacement of charges from charge $q_1$is $r_{21} = 0.10$ m, $r_{32} = 0.10$ m and $r_{31} = 0.1$ m. Therefore, potential energy of the system is taken for arranging charges w.r.t. q_1 as per given geometry is $W = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q_1q_2}{r_{21}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right)$, here $\frac{1}{4\pi\varepsilon_0} = 9 \times$

	10^9. Using the given data we have $W = ((9 \times 10^9))\left(\frac{(4.0\times10^{-5})(3.0\times10^{-5})}{0.1} + \frac{(3.0\times10^{-5})(2.0\times10^{-5})}{0.1} + \frac{(2.0\times10^{-5})(4.0\times10^{-5})}{0.1}\right) = 9(12+6+8) =$ **234 J is the answer.**
I-18	Given that $\Delta KE = -10$ J…(1), here (-)sign is assigned due to decrease in kinetic energy stated in the problem. This decrease occurs when a charge moves in electric field from potential $V_1 = 100$ V to $V_2 = 200$ V. We know that $\Delta W = \Delta Vq \Rightarrow \Delta W = (V_2 - V_1)q$…(2) As per principle of conservation of energy, operating under internal forces change in internal energy $\Delta U = 0 = \Delta W + \Delta KE \Rightarrow \Delta W = -\Delta KE$…(3). Combining (1), (2) and (3), and using the given data $(V_2 - V_1)q = -\Delta KE \Rightarrow (200-100)q = -(-10)$. It leads to $q = \frac{10}{100} \Rightarrow q = \mathbf{0.10}$ **C is the answer**.
I-19	 Given that equal charges are of magnitude $q = 2.0 \times 10^{-4}$ C, having mass $m = 10 \times 10^{-3}$kg are separated by $r_i = 0.10$m. Potential energy of two charges at a separation r is $PE_r = V_r q \Rightarrow PE_r = \left(\frac{q}{4\pi\varepsilon_0 r}\right) q$. Therefore, initial potential energy would be $PE_{r_i} = \frac{q^2}{4\pi\varepsilon_0 r_i}$. When charges are separated at $r \gg$ then $PE_r = \frac{q^2}{4\pi\varepsilon_0 r}\Big\|_{r\gg} = 0$. Therefore, $\Delta PE = PE_r - PE_{r_i} \Rightarrow \Delta PE = 0 - \frac{q^2}{4\pi\varepsilon_0 r_i} = -\frac{q^2}{4\pi\varepsilon_0 r_i}$…(1) As per principle of conservation of energy, as system undergoing changes under internal forces $\Delta U = 0 = \Delta W + \Delta KE \Rightarrow \Delta KE = -\Delta W$. On its integration $KE = -W$…(2). Both the charges, having equal mass, are at initially at rest, and are experiencing equal and opposite forces. Therefore, will acquire equal velocities in direction of forces acting upon them, their total kinetic energy would be $KE = 2 \times \left(\frac{1}{2}mv^2\right)$…(3). Combing (1), (2) and (3) $2 \times \left(\frac{1}{2}mv^2\right) = -\left(-\left(\frac{q\times q}{4\pi\varepsilon_0}\right)\frac{1}{r_i}\right) \Rightarrow v = q\left(\sqrt{\frac{1}{4\pi\varepsilon_0} \times \frac{1}{mr_i}}\right)$ Using the data given in the problem $v = (2.0 \times 10^{-4})\left(\sqrt{(9 \times 10^9) \times \frac{1}{(10\times10^{-3})\times0.1}}\right) \Rightarrow v = (2.0 \times 10^{-4})(3 \times 10^6) \Rightarrow v = 6.0 \times 10^2$m/s **or 600 m/s is the answer.** **N.B.:** This problem can also be solved by using general expression of electric force, which is function of separation r, determining work done electric field, instead of direct use of general expression of potential energy of charges. But, that ignores concept of PE, gained and is less correct.
I-20	 Given that two charges are of magnitude $q_1 = 4.0 \times 10^{-5}$ C and $q_2 = -4.0 \times 10^{-5}$ C having mass $m = 5 \times 10^{-3}$kg are separated by $r_i = 1.0$m. Potential energy of two charges at a separation r is $PE_r = V_r q \Rightarrow PE_r = \left(\frac{q_1}{4\pi\varepsilon_0 r}\right) q_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$. Therefore, initial potential energy would be $PE_{r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_i}$. When charges are separated at $r_f = 0.50$ m, then $PE_{r_f} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_f}$. Therefore, $\Delta PE = PE_{r_f} - PE_{r_i} \Rightarrow \Delta PE = \frac{q_1 q_2}{4\pi\varepsilon_0 r_f} - \frac{q_1 q_2}{4\pi\varepsilon_0 r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$…(1) As per principle of conservation of energy, as system undergoing changes under internal forces $\Delta U = 0 = \Delta W + \Delta KE \Rightarrow \Delta KE = -\Delta PE$. On its integration $KE = -PE$…(2). Both the charges, having equal mass, are at initially at rest, and are experiencing equal and opposite forces. Therefore, will acquire equal velocities in direction of forces acting upon them, their total kinetic energy would be $KE = 2 \times \left(\frac{1}{2}mv^2\right)$…(3).

Combing (1), (2) and (3) $2\times\left(\frac{1}{2}mv^2\right)=-\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{m}\left(\frac{1}{r_f}-\frac{1}{r_i}\right)\right)\Rightarrow v=\left(\sqrt{\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{m}\times\left(\frac{1}{r_i}-\frac{1}{r_f}\right)}\right)$

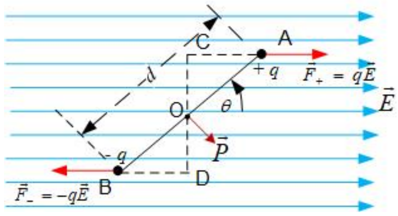
Using the data given in the problem $v=\left(\sqrt{(9\times10^9)\times\frac{(4.0\times10^{-5})(-4.0\times10^{-5})}{5\times10^{-3}}\times\left(\frac{1}{1}-\frac{1}{0.5}\right)}\right)$. It leads to

$v=\frac{(120)}{\sqrt{5}}\Rightarrow v=$ **54m/s is the answer.**

N.B.: This problem can also be solved by using general expression of electric force, which is function of separation r, determining work done electric field, instead of direct use of general expression of potential energy of charges. But, that ignores concept of PE, gained and is less correct.

I-21

Problem has been conceptualized in the figure, in which ion of the molecule of HCl is shown as a pair of charges $+q$ and $+q$ with a separation d are placed in an electric field $\vec{E}$. The line joining the pair of charges is inclined to electric field at an angle θ. The pair of charges $+q$ and $+q$ experience forces $\vec{F}_+=q\vec{E}$ and $\vec{F}_-=(-q)\vec{E}=-q\vec{E}$. These forces produce a moment $\vec{\tau}=\frac{1}{2}\vec{d}\times\vec{F}_++\frac{1}{2}(\overrightarrow{-d})\times\vec{F}_-$. It resolves into $\vec{\tau}=\frac{1}{2}\vec{d}\times(q\vec{E})+\frac{1}{2}(-\vec{d})\times(-q\vec{E})=q\vec{d}\times\vec{E}\Rightarrow\vec{\tau}=\vec{P}\times\vec{E}$, here dipole moment $\vec{P}=q\vec{d}$, as shown in the figure. Therefore, magnitude of torque is $\tau=qdE\cos(90-\theta)\Rightarrow\tau=PE\sin\theta\ldots(1)$. Here direction of rotation is determined as per rule of cross-multiplication of vectors.



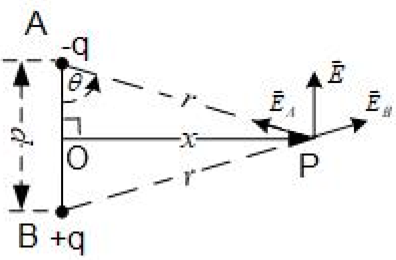
It is seen in (1) that $\tau=f(\theta)$, therefore for maximum torque, based on the principle of Maxima-Minima in differential calculus, $\frac{d}{d\theta}\tau=0$ will determine a values of θ for which $\vec{\tau}$ could be maximum or minimum. And for maximum torque $\frac{d^2}{d\theta}\tau<0$ or (-)ve. Accordingly, $\frac{d}{d\theta}\tau=\frac{d}{d\theta}qdE\sin\theta=dqE\cos\theta=0$. Thus possible values are $\theta=\frac{\pi}{2},-\frac{\pi}{2}$. Now, is verification of the magnitude of torque if it maximum, $\frac{d^2}{d\theta}\tau=\frac{d}{d\theta}\left(\frac{d}{d\theta}PE\sin\theta\right)$. It leads to $\frac{d^2}{d\theta}\tau=PE\frac{d}{d\theta}(\cos\theta)\Rightarrow\frac{d^2}{d\theta}\tau=-PE\sin\theta$. The (-) sign confirms that absolute magnitude of torque is maximum at $\theta=\frac{\pi}{2},-\frac{\pi}{2}$. Accordingly, using the available data in (1), $\tau=(3.4\times10^{-30})(2.5\times10^4)\sin 90^0=$ **8.5×10^{-26}nm is the answer.**

I-22

The two opposite charges of magnitude $q=2.0\times10^{-6}$ C placed at A and B are separated by $d=0.01$m as shown in the figure. Taking each part separately –

Part (a): Electric dipole moment is $\vec{P}=qd\vec{\imath}\ldots(1)$ Therefore, its magnitude using the available data in (1) we have $P=(2.0\times10^{-6})(1.0\times10^{-2})=2.0\times10^{-8}$ Cm is the answer.

Part (b): Electric field at a point P on the axis of the dipole, which passes from its midpoint O and is perpendicular to AB, as per Coulomb's Law is $\vec{E}=\frac{q}{4\pi\varepsilon_0r^2}\hat{r}$. Here, $\frac{1}{4\pi\varepsilon_0}=9\times10^9$. Accordingly, two charges of the dipole would produce $\vec{E}_A=\frac{+q}{4\pi\varepsilon_0r^2}\widehat{AP}$ and $\vec{E}_B=\frac{-q}{4\pi\varepsilon_0r^2}\widehat{BP}$ as shown in the figure and both the field are symmetrical about perpendicular to axis OP. Therefore, their components along $\hat{\jmath}$ would be equal and opposite cancelling each other. Whereas, their component along axis P would be $\vec{E}=\vec{E}_A+\vec{E}_B=\frac{2q}{4\pi\varepsilon_0r^2}\cos\theta\,\hat{\jmath}\Rightarrow E=\frac{2q}{4\pi\varepsilon_0r^2}\cos\theta\ldots(2)$ Here, from given data $r=\sqrt{x^2+\left(\frac{d}{2}\right)^2}$ and $\sin\theta=\frac{\frac{d}{2}}{r}=\frac{d}{2\times\sqrt{x^2+\left(\frac{d}{2}\right)^2}}$. Thus final form of (2) is $E=\frac{2q}{4\pi\varepsilon_0}\times$



$\frac{d}{2\left(\sqrt{x^2+\left(\frac{d}{2}\right)^2}\right)^3} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \times \frac{qd}{\left(\sqrt{x^2+\left(\frac{d}{2}\right)^2}\right)^3}$ …(3). Using the available data $E = (9\times10^9)\times \frac{(2.0\times10^{-6})(1.0\times10^{-2})}{\left(\sqrt{(1.0\times10^{-2})^2+\left(\frac{1.0\times10^{-2}}{2}\right)^2}\right)^3} = \frac{18}{\left(\frac{\sqrt{5}\times10^{-2}}{2}\right)^3} = \mathbf{1.3\times10^8}$ **N/C is answer of part (b).**

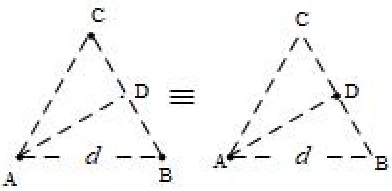
Part (c): In this part all other parameters remain the same as in part (b) except that $x = 1.0\ m$ or $x \gg d \Rightarrow x^2+\left(\frac{d}{2}\right)^2 \approx x^2 \Rightarrow \left(\sqrt{x^2+\left(\frac{d}{2}\right)^2}\right)^3 = x^3$. Accordingly, (3) approximates to $E = \frac{1}{4\pi\varepsilon_0}\times\frac{qd}{x^3}$. Thus using the available data $E = (9\times10^9)\times\frac{(2.0\times10^{-6})(1.0\times10^{-2})}{1^3} = 180$ **N/C is answer of part (c).**

Thus answers are (a) $\mathbf{2.0\times10^{-8}}$Cm (b) $\mathbf{1.3\times10^7}$ N/C (c) 180 N/C.

N.B.: Part (b) and (c) are identical except value of x, in part (b) $x = d$ and hence approximation is not done. But, in part (c) $x \gg d$ and leads to approximation. It is a very explicit case which emphasizes that conceptual approach is more error proof, while use of direct formula may incorporate approximation which may not be valid in specific problem, and thus lead to wrong answer.

I-23

This problem can be considered in two ways



(a) Taking D as center of the two (-)ve charges B and C as center of equivalent charge $(-2q)$, and then considering it's separation from charge $2q$ to determine dipole moment, or
(b) Forming two pair of charges one of $+q$ at A with $-q$ at B and other pair of remaining charge $+q$ at A with $-q$ at C. Determining dipole moments of both the pair and their vector addition to determine net dipole moment.

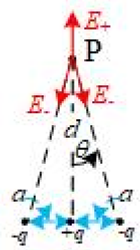
Here, method (a) is used where charge at A is $2q$ and equivalent charge $(-2q)$ at D. Geometrically, separation between AD is $d' = d\sin 60^0$. Therefore, dipole moment of the system of charges would be $P = 2q\times d\frac{\sqrt{3}}{2} \Rightarrow P = \boldsymbol{qd\sqrt{3}}$ Cm. Further, direction of the electric dipole moment is from (-)ve charge to positive charge i.e, along $\widehat{DA}$ away from the triangle and DA is bisector of the angle A of the equilateral triangle of the geometry of charges where charge $2q$ is placed. Therefore, **answer is $\boldsymbol{qd\sqrt{3}}$** Cm along the bisector of the angle at $2q$, away from the triangle

I-24

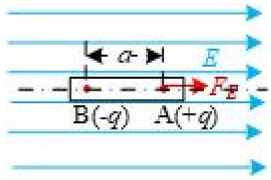
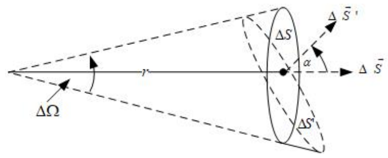
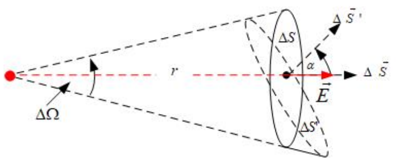
Taking each case separately-

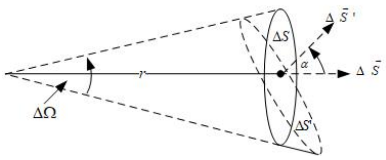
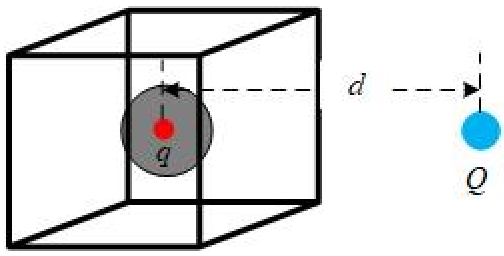
Case (a): It has a single charge and, therefore, as per Coulomb's Law, magnitude of the electric field is $E = \frac{q}{4\pi\varepsilon_0 d^2}$ **is answer of part (a).**

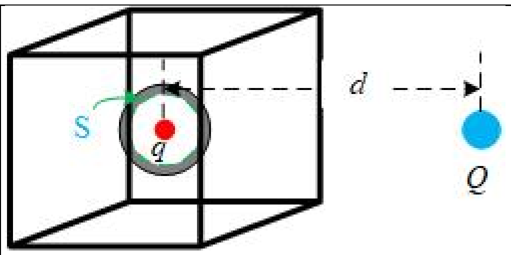
Case (b): It is case of a dipole where $d \gg a$, and distance of point P from charges is $r = \sqrt{d^2+a^2} \Rightarrow r \approx d$. Here, as shown in the figure, as per geometrical symmetry resultant of fields E_+ and E_- along OP is zero, however, electric field at P parallel to line q_+ and q_- is $E = E_+\sin\theta + E_+\sin\theta = \frac{q}{4\pi\varepsilon_0 d^2}\times 2\sin\theta$. It leads to $E = \frac{q}{4\pi\varepsilon_0 d^2}\times\frac{2a}{d} \Rightarrow E = \frac{2aq}{4\pi\varepsilon_0 d^3}$. It is given that $2qa = p$, therefore. $E = \frac{p}{4\pi\varepsilon_0 d^3}$ **is answer of part (b).**

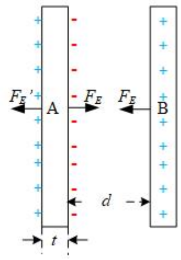
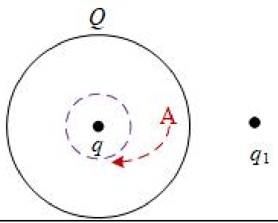
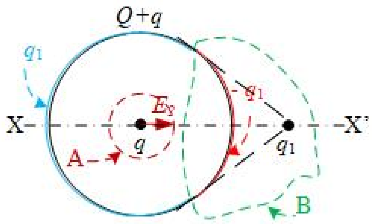
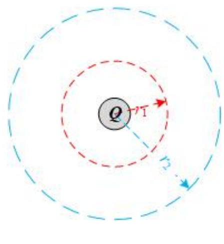


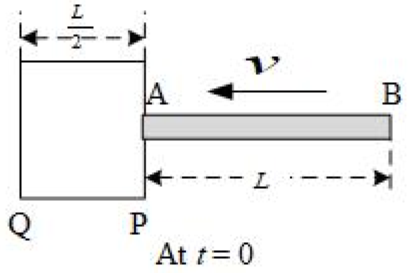
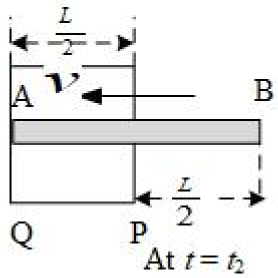
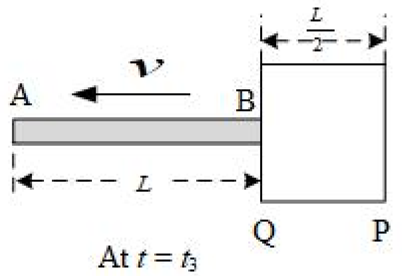
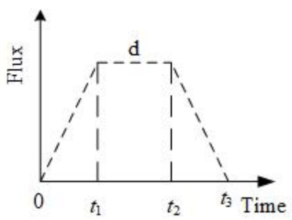
Case (c): Net electric field at P due system of charges as shown in the figure $E = E_+ - 2E_-\cos\theta$. Here, $E_- = \frac{q}{4\pi\varepsilon_0(d^2+a^2)} = \frac{q}{4\pi\varepsilon_0 d^2}\Big|_{d\gg a}$, $E_+ = \frac{q}{4\pi\varepsilon_0 d^2}$ and $\cos\theta = \frac{d}{\sqrt{d^2+a^2}} \Rightarrow$

	$\cos\theta = 1\|_{d\gg a}$. Accordingly, $E = \frac{q}{4\pi\varepsilon_0 d^2} - 2\times\frac{q}{4\pi\varepsilon_0 d^2}\times 1 = -\frac{q}{4\pi\varepsilon_0 d^2}$ **is the answer of part (c).**
I-25	The problem states a dipole created by fixing two charges $+q$ and $-q$, each of mass m, at the ends of a light rod of length a i.e. mass of rod is to be neglected. The rod is fixed at one end say B. The rod is placed in electric field Electric field along the axis of the dipole as shown in the figure. Further it states that neglect effect of acceleration due to gravity, time period of the oscillation is to be determined.  The concepts of physics that goes into the problem are- (a) This situation can be visualized when the system is placed on a smooth horizontal field, where only electric forces affect oscillation of the system. (b) Taking end B having charge $-q$ is fixed, as provided in the problem, and hence force on it due to electric field will have no effect on the oscillation of the system. (c) The force $F_E = Eq$ on charge $+q$ at free end A will act as restraining force when the system is slightly tilted. (d) This system can be compared with a pendulum where time period of oscillation is $T = 2\pi\sqrt{\frac{l}{g}}$...(1), where $l \leftrightarrow a$ length of pendulum and $g \leftrightarrow a'$ restraining acceleration on the mass due to force acting on it. In this case mass m of charge $+q$ and electric force $F_E = qE$, will decide restraining acceleration as per Newton's Second Law of motion. Accordingly, $a' = \frac{qE}{m}$. Thus using quantities determined at (d) in (1), time period of oscillation if the system will be $T = 2\pi\sqrt{\frac{a}{a\prime}} \Rightarrow$ $T = 2\pi\sqrt{\frac{a}{\frac{qE}{m}}} \Rightarrow T = 2\pi\sqrt{\frac{ma}{qE}}$ **is the answer**.
I-26	The concept involved in the problem is that of Avogadro's Number $N = 6\times 10^{23}$ is number of atoms in atomic weight of any substance. In this case substance is copper wire whose atomic mass is $A = 64$ gm/mol. Mass of wire is given to be $m = 6.4$ g, and that each atom contributes one free electron. Thus, combining these data, number of free electrons in the given sample of wire is $n = \frac{Nm}{A} \Rightarrow n = \frac{(6\times10^{23})\times6.4}{64} \Rightarrow n = 6\times 10^{22}$ **is the answer.**
I-27	As per Gauss's Law flux out of a surface area s per Gauss's Law flux out of a surface area $\Delta\phi = \vec{E}.\Delta\vec{S}$. Here, $\Delta\vec{S}$ is small surface-element, it is vector quantity of magnitude ΔS and direction perpendicular to the the elemental surface area as shown in the figure and $\vec{E}$ is the electric field at ΔS, while total flux $\Delta\phi$ is a scalar quantity of magnitude $\Delta\phi = E\Delta S\cos\alpha$, where $\alpha$is the angle between vectors $\vec{E}$ and $\Delta\vec{S}$.   In the instant case a small plane of the area ΔS is being rotated in an electric field $\vec{E}$, which are constant, and it is required orientation α when flux through $\Delta\phi$ is maximum and minimum. **Maximum value of $\Delta\phi$ is when $\cos\alpha\|_{\alpha=0} = 1$ and minimum value of $\Delta\phi$ is when $\cos\alpha\|_{\alpha=\pm\frac{\pi}{2}} = 0$ or $\Delta\phi = 0$** **Thus answers are (a) $\Delta\phi\|_{Max}$ at $\alpha = 0$ and (b) $\Delta\phi = 0$ at $\alpha = \pm\frac{\pi}{2}$**

I-28	Area is a vector whose direction is to its surface as shown in the figure. Given that circular ring of radius r has area $\vec{A} = A\hat{n} = \pi r^2\hat{n}$ and it is placed parallel to electric field $\vec{E} = E\hat{e}$. It implies that $\hat{n}\cdot\hat{e} = 1$. Therefore, in this position flux through the ring is $\phi = \vec{E}\cdot\vec{A}$. It leads to $\phi = (E\hat{e})\cdot(\pi r^2\hat{n}) \Rightarrow \phi = E\pi r^2(\hat{n}\cdot\hat{e}) \Rightarrow \phi = E\pi r^2$.  When is rotated though 180^0 ring $\overrightarrow{A'} = -\vec{A} = -A\hat{n} = -\pi r^2\hat{n}$. Therefor flux through the ring in its new position is $\phi' = \vec{E}\cdot\vec{A'} \Rightarrow \phi' = (E\hat{e})\cdot(-\pi r^2\hat{n}) \Rightarrow \phi' = -E\pi r^2(\hat{n}\cdot\hat{e}) \Rightarrow \phi' = -E\pi r^2$. Therefore, change of flux is $\phi' - \phi = (-E\pi r^2) - (E\pi r^2) = -2E\pi r^2 \neq 0$, therefore, **change of flux is there**. Further, the change of flux $\phi' - \phi$ being negative implies that there is **decrease in flux**. **Thus answers are Yes, decrease.**
I-29	As per Gauss's Law on a closed surface $\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$, where Q is the charge inside closed surface. Given that Q is charge distributed on a thin spherical shell. Accordingly taking each question separately – (a) Centre of the sphere is inside the surface on which charge is distributed, therefore, no charge is there at the center and hence electric field at the center is **zero is the answer to this question.** (b) When a point charge say $+q$ is brought close to the shell, then due to electrostatic induction opposite charge $-q$ will appear on the surface of the shell facing the charge $+q$. But, on the surface of the shell opposite to the point charge will have an addition charge $+q$ such that net charge on the sphere remains $(-q) + Q + (+q) = Q$. Thus, uniform charge distribution becomes non-uniform, creating a virtual dipole of charges $-q$ and $+q$; as a result electric field will appear at the center of the shell causing a change. **Hence, yes is answer to this part of the question**. (c) In case the spherical shell is non-conduction, the charge distribution would be more non-uniform depending upon dielectric property of the material of the shell. Thus there would be change of electric field at the center of the shell but this change would be different than that at (b). **Thus answer to this part of question is Yes.** **Thus answers are Zero, Yes, Yes**
I-30	As per Gauss's Law on a closed surface $\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$, where Q is the charge inside closed surface. Given that Q is charge distributed on a thin spherical shell. Accordingly taking each question separately – (a) Centre of the sphere is inside the surface on which charge is distributed, therefore, no charge is there at the center and hence electric field at the center is **zero is the answer to this question.** **(b)** When the shell is deshaped, without altering the charge Q on its surface, yet the charge inside the shell continues to be zero and hence there is no change in the electric field inside the shell. Thus, **answer to this part of question is No.** **Thus answers are Zero, No**
I-31	As shown in the figure, given that a charge q is placed in metal box within a small cavity and charge Q is outside the metal box. It is required to determine qualitatively whether charge q will experience a force.  Taking forward the analysis, the charge Q is shielded by metal box. Therefore, as per Gauss's Law electric field caused by Q at q is $E_Q = 0$ …(1) Now inside the box, the charge q , by electrostatic induction, would induce an equal and opposite charge $(-q)$ on the inner surface of the cavity.

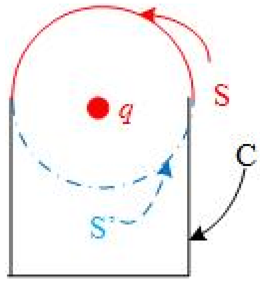
	 Take a Gaussian surface S, inside the cavity but enclosing the charge q as shown in the next figure. Thus, it would electrostatically induce a charge $(-q)$ on inner surface of the cavity, which is outside the Gaussian surface S. Therefore, as per Gauss's Law electric field E_{-q} by the charge $(-q)$ inside the S would be $E_{-q} = 0$…(2) Thus, using principle of superimposition, combining (1) and (2) electric field at q is $E = E_Q + E_{-q} = 0$ …(3). Accordingly, using (3) force experienced by charge q is $F = Eq = 0$. **Thus, answer is No**
I-32	Let a rubber balloon is spherical of radius r charge Q is uniformly distributed over its surface. It is equivalent to charge Q placed at center with uniform electric field, as per Coulomb's law $\vec{E} = \frac{Q}{4\pi\varepsilon_0 d r^2}\hat{r}$. Electric flux out of any surface area as per Gauss's Law would be $\Delta\phi = \vec{E}_S \cdot \Delta\vec{S} = \frac{\Delta q}{\varepsilon_0}$ and the elemental area is equivalent to charge Δq and center of the balloon placed at $(-\vec{r})$. Taking center sphere for geometrical simplicity, each elemental charge uniformly distributed would cancel electric field produced by an elemental area placed diametrically opposite to it. Thus net field at the center is zero. Same is mathematically true for any point inside the sphere. But, if balloon is not sphere yet mathematical analysis would reveal that net electric field inside the balloon would remain at Zero. In any case of transitory change in shape of balloon, which causes in-equilibrium of field the reactionary forces, as per Newton's Third Law of Motion would readjust the shape of the balloon to create an equilibrium of electric field inside the balloon. **Hence, answer is Yes.**
I-33	The question comprises of three sub-questions and are illustrated separately – In all electrical process protons are held in their position due to strong nuclear forces. Therefore giving charge to a conductor be it (-)ve or (+)ve protons do not come to surface and hence **answer to this part is No.** Giving charge to a conductor, as current electricity per Kirchhoff's Law is a $\sum I = 0$ and $I = \frac{\Delta Q}{\Delta t} \Rightarrow (\sum I)\Delta t = \Delta Q = 0$. Therefore, giving charge to a conductor has to be a process of electrostatic induction where, giving (-)ve charge implies transfer of electrons to the conductor. Eventually it is addition to free electrons which would appear on the surface, while all electrons of the conductor remain unaffected. Likewise, giving positive charge to a conductor is eventually taking away free electron in the conductor, and not all the electrons. **Hence, answer to this part is No.** All free electron, in an uncharged conductor, are in a state of Brownian motion. But when giving charge to a conductor is adding free electrons that would appear on the surface or cause depletion of free electrons which are equivalent to the charge given to the conductor. Therefore, all free electrons are not affected. Hence, **answer to this part is No.** **Answer to three parts of the question are No, No, No.**
I-34	As per Gauss's Law $\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$…(1),here $\vec{E}$ is electric field on an elemental area $\Delta\vec{S}$ caused by a charge Q. It is given that plastic plate carrying a uniformly distributed charge Q. The $\vec{E}$ on every elemental area $\Delta\vec{S}$ i.e. L.H.S of (1), but its R.H.S. remains unaffected. electric field at a point P close to the field is combined effect of Q which is same and is given to be 10 V/m. It is given that plastic plate is replaced by a copper plate. Thus, change of property of material of the plate would cause change of $\vec{E}$ on every elemental area $\Delta\vec{S}$ i.e. L.H.S of (1), but its R.H.S. remains unaffected. electric field at a point P close to the field is combined effect of Q which is same and is given to be 10 V/m, as given in the option (c). Hence, **answer is option (c).**

I-35	A metallic particle shown as a plate A in the figure, for clarity, initially has no net charge on it, which implies that negative number of electrons and protons in it is zero. There is another plate B which has say $+q$ charge on it. The plate A is brought near the plate B. As a result of electrostatic induction surface of the plate A facing B will induce an equal and opposite charge $-q$ while the other surface of plate A will induce $+q$ charge maintaining net charge on A to be Zero. But, pair of surfaces of B having charge $+q$ and surface of B having charge $-q$ will experience a force of attraction $F_E = \frac{q\times(-q)}{4\pi\varepsilon_0 d^2}$.  Likewise, the other face of A having charge $+q$ will experience will experience a force of repulsion $F_E' = \frac{q\times q}{4\pi\varepsilon_0(d+t)^2}$. Since, $t > 0 \Rightarrow d+t > d \Rightarrow F_E > F_E'$. Therefore, net force on A is $F = F_E - F_E'$ is along plate B and is in accordance with the **option (a), is the answer**.
I-36	Given system is shown in the figure. In this an additional Gaussian surface A is taken to be enclosing charge q of the thin metallic shell. Since charges Q and q are outside A, and hence electric field caused by these charges at the center of the sphere as per Gauss's Law is $E = 0$. Therefore force experience by the charge $q$is $F = qE = q \times 0 = 0$. This conclusion is provided only **in option (d) is the answer.** 
I-37	It is required to determine force experienced on charge q placed at the center of thin spherical shell. All the charges are given to be positive. The problem is solved in two phases an then direction of the net force on q is determined using superimposition principle as under- **Phase I - Electric field at q due to charge Q:** Consider a Gaussian surface A around the charge q. Since the charge The Q is outside A hence electric field at A would be $E_1 = 0$. Therefore, force on charge q would be $\boldsymbol{F_1 = E_1 q}$**...(1)** **Phase II – Electric field due to spherical shell due to charge q_1:** The charge $q_1$would electrostatically induce charge $(-q_1)$ on the part of surface of the shell facing the charge q_1, this area would be enclosed in conical cap tangential to the spherical shell whose boundary is depicted by tangents on sphere drawn from q_1. While the remaining part of the spherical shell will have $(+q_1)$ charge. Thus, based on induced charge on the spherical shell a Gaussian surface B is y=taken enclosing the charge q_1 and induced charge $(-q_1)$. Net charge inside B is $q_1 + (-q_1) = 0$, hence electric field at B caused by charges inside B is zero. But, the induced charge $(+q_1)$ on the remaining part of the shell is on the left of charge q and is symmetrical about the line X-X' passing through charge q. Therefore, by geometrical symmetry it will produce electric field E_2 at q directed towards right. Therefore, force on charge q due to shell, eventually charges on the shell will be $\boldsymbol{F_2 = E_2 q}$ **...(2), will be directed towards right.**  Superimposing analysis in phase I and phase II ne force on charge q due to charge on shell would be obtained by combining (1) and (2) $F = F_1 + F_2 = 0 + E_2 q = \boldsymbol{E_2 q}$ in **option (b) is the answer.** **N.B.:** It is a good case of superimposition of electric field.
I-38	A small volume, taken to be spherical as shown in the figure, has charge Q distributed in it. Given that electric flux through a spherical surface of radius $r_1 = 0.10$ m is $\phi_1 = 25$ Vm. Here, it is to be ensured that shape of the volume containing charge, not necessarily spherical, is within the given Gaussian surface, and is there in the figure. Accordingly, as per Gauss's Law surface integral of electric field E_1, which is geometrically uniform, will have a total flux emerging out of it as $\phi_1 = \oint \vec{E}_1 \cdot d\vec{s} = \frac{Q}{\varepsilon_0} \Rightarrow E_1 \times 4\pi r_1{}^2 = \phi_1 = \frac{Q}{\varepsilon_0}$ ...(1) 

	The other Gaussian surface is also a concentric sphere of radius $r_2 = 0.20$ m such that $r_2 > r_1$. Therefore, this surface will also act as Gaussian surface within which the same charge is Q. Therefore, as per Gauss's Law $\phi_2 = \oint \vec{E}_2 \cdot d\vec{s} = \frac{Q}{\varepsilon_0} \Rightarrow E_2 \times 4\pi r_2^2 = \phi_2 = \frac{Q}{\varepsilon_0}$ …(2). R.H.S of (1) and (2) being same the flux through would be same $\phi_2 =$ **25 Vm, as given in option (a), the answer.**
I-39	In the given figure the charged rtod is passing through an imaginary cube withat a constant speed.. The cube in this case is like a Gaussian surface. As per Gauss's Law $\phi = \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$…(1), here Q is the amount of charge inside the Gaussian surface. It is given that charge on the rod is uniformly distributed and therefore charge per unit length of the rod is $q = \frac{Q}{L}$…(2). The problem is analyzed in Five states as under –  **State 1 at *t* = 0:** End A of the rod, while being outside the cube, is touching the center of its surface P as shown in the figure. Since there is no charge inside the Gaussian Surface i.e. cube, hence the flox through surface of the cube is $\phi_0 = \frac{\Delta q}{\varepsilon_0} = 0$. **Stage 2 during $0 < t < t_2$:** Rod is moving through the cube (i.e. entering it) and is short of touching surface Q such that end A of the rod is ion-between face P and Q. Since, size of the cubical box is $\frac{L}{2}$, for the rod moving at a constant speed $t_2 = \frac{\frac{L}{2}}{v} \Rightarrow t_2 = \frac{L}{2v}$…(3)Further, charge at any time inside the Gaussian surface is $q_t = (vt)q$…(4). Combining (1), (2) and (4), flux through the box will be $\phi_t = \frac{q_t}{\varepsilon_0} \Rightarrow \phi_t = \frac{1\times(vt)q}{\varepsilon_0} \Rightarrow \phi_t = \frac{Qv}{\varepsilon_0 L}t \Rightarrow \phi_t \propto t$, since, $Q.v, L$ and ε_0 are constant. It implies that during this period flox throgh the cube is increasing linearly.  **Stage 3 during $t = t_2$:** At his instant End A of the rod touches face Q of the cube and at P is the mid point of the rod. Thus half of the length of the rod is inside the cube. Therefore, combining (1), (2) and (3) flux through the cube would be $\phi_t = \frac{q_2}{\varepsilon_0} \Rightarrow \phi_t = \frac{1\times(vt_2)q}{\varepsilon_0} \Rightarrow \phi_t = \frac{1}{\varepsilon_0} \times v \times \frac{L}{2v} \times \frac{Q}{L} \Rightarrow \phi_t = \frac{Q}{2}$. **Stage 4 during $t_2 < t < t_3$:** Rod is moving through the cube (i.e. coming out of it). During this period end B of the rod moves from face P towards face Q and is short of leaving the face Q. Using the discussions in stage 2, length of the rod inside the cube reduces linearly. Therefore, $\phi_t = -\frac{Qv}{\varepsilon_0 L}t \Rightarrow \phi_t \propto -t$.  **Stage 5 during $t = t_3$:** At his instant End B of the rod will be touching face Q of the cube charge inside the cube $q_3 = 0$. Hence at this instant electric flux throgh the cube as per (1) would be $\phi_3 = \frac{q_3}{\varepsilon_0} \Rightarrow \phi_3 = \frac{0}{\varepsilon_0} = 0$.  This kind of variation of flux through the cube is characterized only in **graph (d) the answer.**

I-40

Charge q is placed at the center o the opening of a cylinder C as shown in the figure. A Gaussian surface combining S and S' encloses charge q and is symmetrical about it. Therefore, electric field at the two surfaces. would be radial and equal in magnitude at every point on them $|\vec{E}_S| = |\vec{E}_{S'}|$. Moreover their areas $A_S = A_{S'}$



Thus, as per Gauss's Law $\phi = \oint \vec{E} \cdot d\vec{s} = \int \vec{E}_S \cdot d\vec{s} + \int \vec{E}_{S'} \cdot d\vec{s'} = \frac{q}{\varepsilon_0}$. According to the discussions above, $\phi = \int \vec{E}_S \cdot d\vec{s} + \int \vec{E}_{S'} \cdot d\vec{s'} \Rightarrow \phi = \phi_S + \phi_{S'} \Rightarrow 2\phi_{S'} = \frac{q}{\varepsilon_0} \Rightarrow \phi_{S'} = \frac{q}{2\varepsilon_0}$.

The surface S' together with the surface C form a closed Gaussian surface within which no charge. Therefore as per Gauss's Law $\phi_{S'} + \phi_C = \frac{0}{2\varepsilon_0} \Rightarrow \phi_C = -\phi_{S'}$For this combined new Gaussian surface (-)ve sign is assigned to flux entering it and therefore conversely flux leaving the surface would be (+)ve. Thus taking $\phi_{S'}$, as per sign convention $\phi_C = -\left(-\frac{q}{2\varepsilon_0}\right) \Rightarrow \phi_C = \frac{q}{2\varepsilon_0}$**, is the answer as per option (c).**

I-41

As per Gauss's Law $\phi = \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$. Here, Q is total charge within a closed surface, which need not be either symmetrical or concentric. In light of this each of the option I being analyzed-

Option (a): It requires symmetrical charge distribution within Gaussian surface for validity of Gauss's Law. Hence, **this is not correct.**

Option (b): Gauss's law does not impose a pre-condition on medium housing the charge. Hence, **this is not correct.**

Option (c): Electric field in the surface integral is at the element of the Gaussian surface and not within the Gaussian surface. Hence, **this is not correct.**

Option (d): This statement is in accordance with the Gauss's law. Hence, **option (d) is correct.**

Thus answer is option (d).

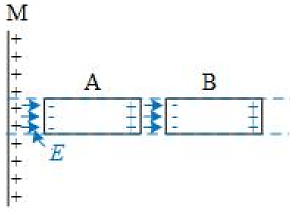
I-42

Given that a positive charge Q is brought near an isolated metal cube. It implies that there is no charge inside the cube. Therefore, ϕ_i part of electric flux $\phi = \frac{Q}{\varepsilon_0}$ due to charge Q, is intercepted by the metal cube. Since, there is no charge, therefore, total flux emerging from the cube $\phi_c = 0 \Rightarrow \phi_i + \phi_o = 0$, here $\phi_o$is the flux emanating out of the cube.

In absence of the metal cube flux would exit. Therefore, there must be a flux equal in magnitude and opposite in direction of the flux produced by Q in the space occupied by the cube. This is possible only if there is a non-uniform distribution of charge on the surface of the cube as provided in option (d). This non-uniform distribution of charge is caused by electrostatic induction of charge $(-Q)$ charge on the surface of the cube facing the external charge Q and $(+Q)$ on the surface of the cube away from the cube. The non-uniform distribution of charge is provided in **option (d), the answer.**

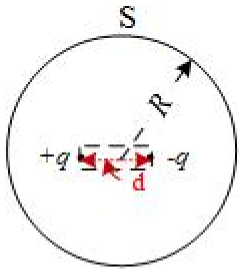
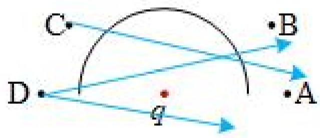
I-43

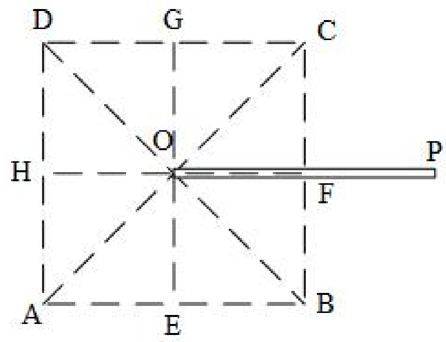
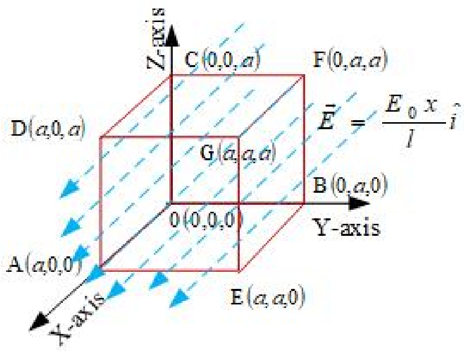
Given that a large non-conducting sheet has a uniform charge density say q C/m^2. Thus, as per Gauss's law, in accordance with geometry of the sheet, it will create a electric field $\vec{E}$ perpendicular to it will b such that $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$, here A is area of cross section of the sheet enclosing charge q and is aligned to the metal rod A and B. Thus charges induced in the rod will create an electric field inside the rods to maintain zero electric field inside it. As shown in the figure.

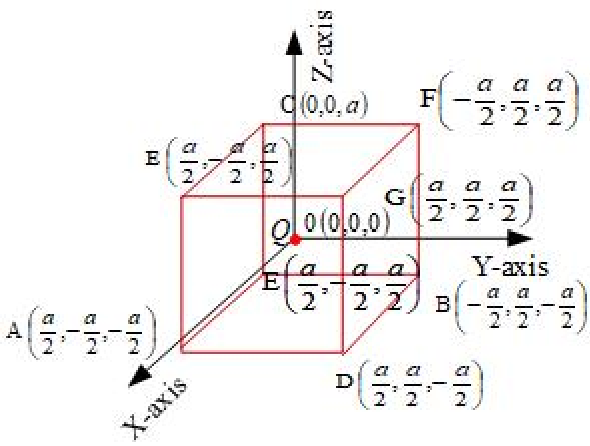
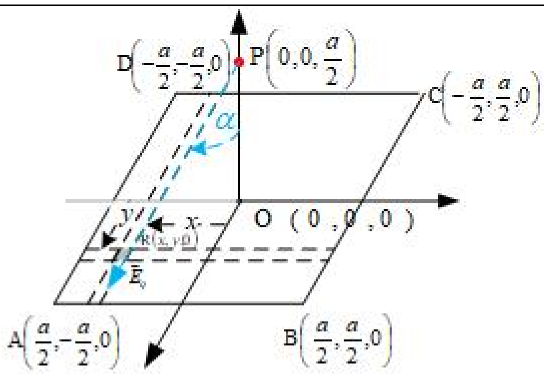


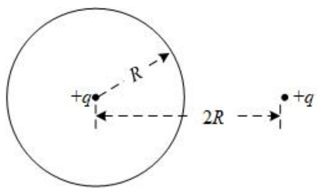
Thus, as seen in the figure, opposite charges are faced by M and A causing attraction, M and B, through A to cause attraction, A and B and vice versa would experience forces of attraction.

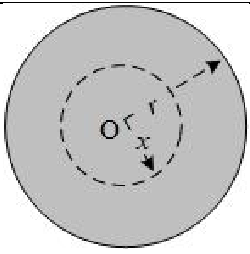
Hence, **all options are correct, is the answer.**

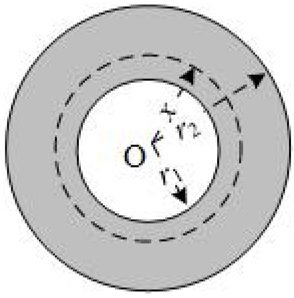
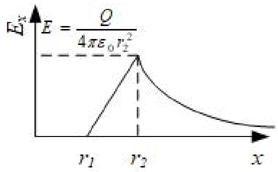
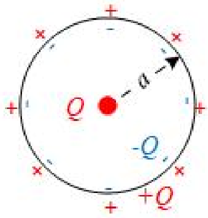
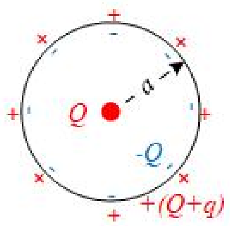
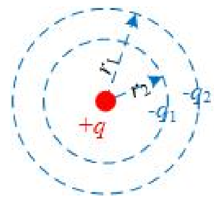
I-44	As per Gauss's law, $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$ where is the flux through a closed surface, $\oint \vec{E}.d\vec{s}$ is the surface integral of electric field $\vec{E}$ along the surface vector $d\vec{s}$, and q is the net electric charge enclosed inside the surface. Therefore, for $\phi = 0$, necessary requirements are either $\vec{E}$ is zero everywhere on the surface leading to $\oint \vec{E}.d\vec{s} = 0$, as provided in option (b), or the net charge inside the surface $q = 0$ as povied in the option (c). Hence, **answer is options (b) and (c).**
I-45	The given problem is conceptualized in the figure shown. As per Gauss's law, $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$ where is the flux through a closed surface, $\oint \vec{E}.d\vec{s}$ is the surface integral of electric field $\vec{E}$ along the surface vector $d\vec{s}$, and q is the net electric charge enclosed inside the surface. Here, sphere acts like a Gaussian surface, and it is enclosing a dipole. whose net charge is $Q = +q + (-q) = 0$, while surface area of any sphere is non-zero. Therefore, $\phi = 0$ as provided in **option (a), and it is correct.**  Further, assuming any other Gaussian surface inside the sphere, enclosing the dipole, electric field at any point on it would be zero as per Gauss's Law. **It makes option (c) as correct.** Hence, **answer is options (a) and (c).** **N.B.:** In the figure distance d between charges of dipole is small as compared to the geometry under consideration. But, it could not be shown in the figure due to visual clarity.
I-46	Electric flux through the hemisphere ϕ_q due to charge q is half of the flux, if it were sphere as per Gauss's Law thus $\phi_q = \frac{q}{2\varepsilon_0}$.  Flux through charge Q would influence flux ϕ_q of the electric field through the sphere if it intercepts sphere at only one point and is true or points C as shown in the figure. The same is true for point B also and given in **option (b) and (c); hence, these are not the answer**. But, for points D which is on the line at the brim of the hemisphere. Thus, flux through this point either flux does not pass through the hemisphere or intercepts for in and out making ne flux through it Zero. Same is true for point A. Thus, these points are provided in **options (a) and (d), are the answer**.
I-47	A portion of circuit is enclosed by a surface S, which can betterated as a Guassian Surface. When switch is open current in the circuit is zero, Hence net charge on the portion remains $Q = 0$…(1), and hence as per Gauss's law flux due to electric field through the suface **is zero as provided in option (d).** But when switch is closed, given that number of electrons entering the surface $(+n)$ is equal to the numberof electrons leaving it $(-n)$. Let charge of electron is e. Therefore, charge on the portion of circuit $Q' = Q + ne - ne = Q = 0$. Thus even after closing switch charge on the portion of circuit does not change and hence fluxthrough the surface S remains unchanged, **as provided in opion (c).** Thus, **answers are option (c) and (d).**
I-48	A positive charge on point P would induce $(-)$ ve charge on the surface of conducting sphere facing P and on the complementary surface of the sphere, which was otherwise neutral and is unearthed, it will induce $(+)$ ve . The closed surface is since enclosing that part of the sphere which carries $(+)$ ve induced charg9oe, it will radiate flux and which is assigned $(+)$ ve, **as provided in option (b), the answer/**
I-49	Equation of a surface area $A = 0.2$ m^2 parallel to Y-Z plane is $\vec{A} = A\hat{\imath}$. Given that $\vec{E} = \frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}$ where $E_0 = 2.0 \times 10^3$. Then as per Gauss's Law flux through the area is $\phi = \oint \vec{E}.d\vec{s} = \left(\frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}\right) \cdot A\hat{\imath} \Rightarrow \phi = \frac{3}{5}E_0A(\hat{\imath}\cdot\hat{\imath}) + \frac{3}{5}E_0A(\hat{\imath}\cdot\hat{\jmath}) = \frac{3}{5}E_0A$. Thus using the given data $\phi = \frac{3}{5}(2.0 \times 10^3)(0.2) =$ **240 Nm^{2}C^{-1} is the answer.**

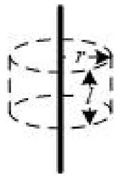
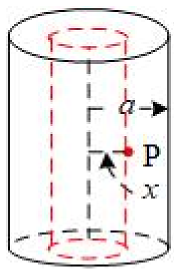
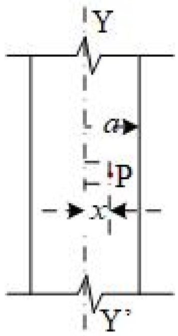
	N.B.: Illustration has been developed in a pure mathematical manner without a supporting figure to catalyze visualization of concept among reader.
I-50	Given that a charge Q is uniformly distributed over a rod of length l leading to a charge per unit length $\rho = \frac{Q}{l}$.The rod is placed inside a hypothetical cube of side l such that one end of the rod is at the center of the cube. It is required to find minimum possible flux of electric field of charge on the rod through the surface of the cube.  The cube serves as a Gaussian surface, and charge inside the cube is $q = \rho x \Rightarrow q = \frac{Q}{l}x \Rightarrow q \propto x$. And maximum length of the rod inside the cube is $x_{max} = \sqrt{2}l$ when rod with its center at O, as given, is aligned to the diagonals i.e. OA, OB, OC and OD. And it is minimum when rod is aligned to OE, OF, OG and OH such that $x_{min} = \frac{l}{2}$. Therefore, for minimum possible flux $x = x_{min} = \frac{l}{2} \Rightarrow q = \frac{Q}{l} \times \frac{l}{2} \Rightarrow q = \frac{Q}{2}$ And as per Gauss's Law $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0} \Rightarrow \phi = \frac{1}{\varepsilon_0} \times \frac{Q}{2} \Rightarrow \phi = \boldsymbol{\frac{Q}{2\varepsilon_0}}$ **is the answer**.
I-51	Given that electric field is uniform at all points in a region. Further a region has an enclosing surface. Therefore as per Gauss's Law $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$…(1). Since surface is enclosing the region electric field remains uniform $\vec{E}$ but the vector $d\vec{s}$,which is perpendicular to the surface and outward in direction will be $(+)d\vec{s}$ and $(-)d\vec{s}$ for elemental surfaces opposite to each other. This will lead to $\oint \vec{E}.d\vec{s} = 0$…(2). Thus combining (1) and (2), $\frac{q}{\varepsilon_0} = 0 \Rightarrow \boldsymbol{q = 0}$**, i.e. there is no charge inside the region, hence proved.**
I-52	Charge inside a region as per Gauss's Law is $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$…(1) The region has been specified by a cubical surface and as shown in the figure. Area of the surfaces enclosing the region are $A_1 = (\text{AEGD})_{ar} = a^2$, $A_1 = (\text{AEGF})_{ar} = a^2$, $A_2 = (\text{COBF})_{ar} = a^2$, $A_3 = (\text{AOCD})_{ar} = a^2$, $A_4 = (\text{EBFG})_{ar} = a^2$, $A_5 = (\text{DGFC})_{ar} = a^2$ and $A_6 = (\text{AEB0F})_{ar} = a^2$.  It is given that $\vec{E} = \frac{E_0 x}{l}\hat{\imath}$. Thus, using (1) $\frac{q}{\varepsilon_0} = \left[\frac{E_0 x}{l}\hat{\imath}.\left((A_1 + A_2)\hat{\imath} + (A_3 + A_4)\hat{\jmath} + (A_5 + A_2)\hat{k}\right)\right]$….(2) Using the magnitudes of each surface area of the cubical volume in (2) with the corresponding DOT products $\frac{q}{\varepsilon_0} = \left[\frac{E_0 x}{l}(A_1 + A_2)\right]$…(3). Using values of x corresponding to A_1 and A_2 as a and 0 respectively. Thus, (3) gets transformed into $\frac{q}{\varepsilon_0} = \left[\frac{E_0}{l}(a \times a^2 + 0 \times a^2)\right] \Rightarrow q = \frac{\varepsilon_0 E_0 a^3}{l}$. Using the available data $q = \frac{(8.85\times10^{-12})\times(5\times10^{3})\times(1\times10^{-2})^3}{(2\times10^{-2})} \Rightarrow q = \boldsymbol{2.2 \times 10^{-12}}$ **C is the answer.**

I-53	Given system is shown in figure with charge Q at the center of a cube of side say a. As per Gauss's law electric flux due to the charge Q is $\phi = \oint \vec{E}.d\vec{s} = \frac{Q}{\varepsilon_0}$. ..(1). Electric field at the each surface can be taken to be at the center of the surface at a distance $\frac{a}{2}$ from the venter, i.e. the origin where the charge is placed. Therefore, electric field at the surface ADGE $\vec{A}_1 = a^2\hat{\imath}$...(2), considering geometrical symmetry is equated to $\vec{E}_1 = \frac{\frac{Q}{6}}{4\varepsilon_0\left(\frac{a}{2}\right)^2}\hat{\imath} \Rightarrow \vec{E}_1 = \frac{Q}{6\varepsilon_0 a^2}i$ ...(3). Therefore, using (2) and (3) in (1), flux emerging out of the surface is $\phi_1 = \vec{E}_1 \cdot \vec{A}_1$. . The vector product resolves into scalar $\phi_1 = \left(\frac{Q}{\varepsilon_0 a^2}\right)a^2 \Rightarrow \phi_1 = \frac{Q}{\pi 6}$...(4).  Again based on geometrical symmetry of the cube as a Gaussian surface, total flux through the six surfaces is $\phi = 6 \times \frac{Q}{6\varepsilon_0} \Rightarrow \phi = \frac{Q}{\varepsilon_0}$, is the answer. **N.B.:** Equation (3) can be arrived at analytically, and it is, required to be derived in question no 54.
I-54	Given system is shown in the figure. Where, charge Q is placed at $P\left(0,0,\frac{a}{2}\right)$ and the given surface ABCD is along $\hat{\jmath}$. Consider an elemental area at $R(x,y,0)$ of size $\Delta\vec{A} = (\Delta x \times \Delta y)(-\hat{\jmath})$. ..(1). Electric field at the point shall be along PQ such that $\vec{E}_R = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$...(2) Here, $\vec{r} = r\hat{r} \Rightarrow r = \sqrt{x^2 + y^2 + \left(\frac{a}{2}\right)^2}$ ..(3). Therefore, flux through the surface ABCD as per Gauss's Law is $\phi = \int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}} \vec{E}_R \cdot \vec{A}dx\right)dy$...(4).  Using (1), (2) and (3) in (4) $\phi = \frac{Q}{4\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{\cos\alpha}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)}dx\right)dy$...(5). Here, $\alpha = \angle OPR$ and $\cos\alpha = \frac{OP}{OR} = \frac{\frac{a}{2}}{\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\alpha = \frac{a}{2\times\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}}$...(6). Combining (5) and (6) $\phi = \frac{Q}{4\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)} \times \frac{a}{2\times\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}}dx\right)dy \Rightarrow \phi = \frac{aQ}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx\right)dy \Rightarrow \frac{aQ}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}(I)\,dy$...(7). Here, $I = \int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx$ is the inner integral. For solving $I = \int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx$ a substitution is made $p^2 = y^2 + \left(\frac{a}{2}\right)^2$ and $x = p\tan\theta \Rightarrow x = p\sec^2\theta\, d\theta$ We have $I = \int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{(p^2\tan^2\theta + p^2)^{\frac{3}{2}}}p\sec^2\theta\, d\theta = \frac{1}{p^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\sec^3\theta}\sec^2\theta\, d\theta = \frac{1}{p^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\theta\, d\theta = \frac{1}{p^2}[\sin\theta]_{-\frac{a}{2}}^{\frac{a}{2}}$. We know

	that $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{x}{(\sqrt{x^2+p^2})}$. This leads to $I = \frac{1}{p^2}\left[\frac{x}{\sqrt{x^2+p^2}}\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{1}{p^2}\left[\frac{\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2+p^2}} - \frac{\left(-\frac{a}{2}\right)}{\sqrt{\left(-\frac{a}{2}\right)+p^2}}\right] \Rightarrow I = \frac{2a}{p^2\times\sqrt{a^2+4p^2}}$. Substituting p we have $I = \frac{2a}{\frac{(4y^2+a^2)}{4}\times\sqrt{4y^2+2a^2}} \Rightarrow I = \frac{8a}{(4y^2+a^2)\times\sqrt{4y^2+2a^2}} \Rightarrow I = \frac{4\sqrt{2}a}{(4y^2+a^2)\times\sqrt{2y^2+a^2}}$. Using expression of I in (7) we have $\phi = \frac{4\sqrt{2}a^2Q}{8\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{1}{(4y^2+a^2)\times\sqrt{2y^2+a^2}}\right)dy$. Instead of taking this expression as an integral of product of two functions a substitution is made $\sqrt{2}y = a\tan\varphi \Rightarrow \sqrt{2}dy = a\sec^2\varphi\, d\varphi$, we have $\phi = \frac{4\sqrt{2}a^2Q}{8\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{1}{a^2(2\tan^2\varphi+1)\times a\sec\varphi}\right)\frac{a\sec^2\varphi}{\sqrt{2}}d\varphi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\sec\varphi}{2\,sec^2\varphi-1}\right)d\varphi$. We know that $sec^2\varphi = 1 + tan^2\varphi \Rightarrow 2\tan^2\varphi + 1 = 2\,sec^2\varphi - 1$. Thus, $\phi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\sec\varphi}{2\,sec^2\varphi-1}\right)d\varphi \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\frac{1}{\cos\varphi}}{\frac{2}{\cos^2\varphi}-1}\right)d\varphi$. It solves to $\phi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\cos\varphi}{2-\cos^2\varphi}\right)d\varphi \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\cos\varphi}{1+\sin^2\varphi}\right)d\varphi$. Substituting, $\sin\varphi = t \Rightarrow \cos\varphi\, d\varphi = dt$. Accordingly, $\phi = \frac{Q}{2\pi\varepsilon_0}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{1+t^2}dt$. At this stage standard integration $\int\frac{1}{1+t^2}dt = \tan^{-1}t$ is used. It leads to , $\phi = \frac{Q}{2\pi\varepsilon_0}[\tan^{-1}t]_{-\frac{a}{2}}^{\frac{a}{2}}$. Here, limits are for initial variable y in (5). Hence, doing reverse substitution $\tan^{-1}t = \tan^{-1}(\sin\varphi) = \tan^{-1}\left(\frac{\tan\varphi}{\sqrt{1+\tan^2\varphi}}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{2}y}{a}}{\sqrt{1+\frac{2y^2}{a^2}}}\right) = \tan^{-1}\left(\frac{\sqrt{2}y}{\sqrt{a^2+2y^2}}\right)$. Accordingly, $\phi = \frac{Q}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{\sqrt{2}y}{\sqrt{a^2+2y^2}}\right)\right]_{-\frac{a}{2}}^{\frac{a}{2}} \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{\sqrt{2}\frac{a}{2}}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right) - \tan^{-1}\left(\frac{\sqrt{2}\left(-\frac{a}{2}\right)}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)\right]$. It leads to $\phi = \frac{Q}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{\sqrt{2}\frac{a}{2}}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{2}\left(-\frac{a}{2}\right)}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)\right] \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \phi = \frac{Q}{2\pi\varepsilon_0}\times\frac{\pi}{3} \Rightarrow \phi = \boldsymbol{\frac{Q}{6\varepsilon_0}}$, **is the answer.** **N.B.:** This has become an intensely mathematical solution. It could have been solved using Gauss's Law and make it worth an objective problem. Therefore, for the problem being subjective full length analytical illustration has been made. Thus, indispensability of mathematical proficiency in handling problems on physics I unique problems has been emphasized.
I-55	In the given system both the charges at the center of the spherical surface of radius R and at a distance $2R$ from the center are $q = 10^{-7}$C. As per Gauss's law $\phi = \oint \vec{E}\cdot d\vec{s}$ …(1),net flux through the surface of sphere is due to charge inside it, while the charge outside the sphere makes no contribution to the flux through the surface. Further, as per Coulomb's Law electric field at every elemental area is radial and $\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\vec{r}$ and elemental area is $d\vec{s} = ds\hat{r}$ is uniformly at a constant distance R.  Therefore, $\vec{E}\cdot d\vec{s} = \left(\frac{q}{4\pi\varepsilon_0 R^2}\vec{r}\right)\cdot(ds\hat{r}) \Rightarrow \vec{E}\cdot d\vec{s} = \frac{q}{4\pi\varepsilon_0 R^2}ds$…(2). Using (2) in (1), $\phi = \oint\frac{q}{4\pi\varepsilon_0 R^2}ds = \frac{q}{4\pi\varepsilon_0 R^2}\oint ds \Rightarrow \phi = \frac{q}{4\pi\varepsilon_0 R^2}\times 4\pi R^2 \Rightarrow \phi = \frac{q}{\varepsilon_0}$, here $\varepsilon_0 = 8.85\times10^{-12}$. Thus using available data $\phi = \frac{10^{-7}}{8.85\times10^{-12}} = \mathbf{1.1\times10^4}$ **Nm^{-2}C^{-1}is the answer.**
I-56	A per Gauss's Law electric flux ϕ due to charge q through an imaginary surface enclosing the charge is $\phi = \oint\vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}$…(1) In the system given is a Gaussian surface in hemispherical shape with charge placed at its center. Considering the imaginary hemspherical surface with its remaining half of the spherical surface, as

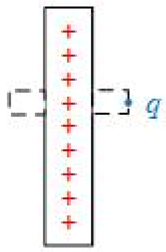
	complete Gaussian surface enclosing the charge $\phi = \frac{q}{\varepsilon_0}$ ...(2). The two hemispherical syrfaces have geometrical symmetry w.r.t. the charge an hence flux emanating out of the two hemispheres ϕ_1 and ϕ_2 will be equal i.e. $\phi_1 = \phi_2$ and $\phi = \phi_1 + \phi_2 \Rightarrow \phi = 2\phi_1 = \frac{q}{\varepsilon_0} \Rightarrow \phi_1 = \frac{q}{2\varepsilon_0}$ **is the answer**.
I-57	 Say the given spherical volume of radius r having a uniform charge density $\rho = 2.0 \times 10^{-4}$ C/m^3. It is required to determine electric field at a distance $x = 4.0 \times 10^{-2}$m. As per Gauss' Law electric flux at a distance x in the given system is due to charge inside the volume enclosed by Gaussian surface of radius x, and the charge is $q = \left(\frac{4}{3}\pi x^3\right)\rho$. While charge outside the region of radius x a does not contribute to flux at the Gaussian surface. Further, per Coulomb's Law the electric field would be $E = \frac{q}{4\pi\varepsilon_0 x^2} \Rightarrow E = \frac{\left(\frac{4}{3}\pi x^3\right)\rho}{4\pi\varepsilon_0 x^2} = \frac{x\rho}{3\varepsilon_0}$. Here, $\varepsilon_0 = 8.85 \times 10^{-12}$. Using the available data, $E = \frac{\left(\frac{4}{3}\pi x^3\right)\rho}{4\pi\varepsilon_0 x^2} = \frac{(4.0\times10^{-2})(2.0\times10^{-4})}{3(8.85\times10^{-12})} = \mathbf{3.0 \times 10^5}$**N/C is the answer.**
I-58	In the given problem spherical surface of nucleus of radius $r = 7.0 \times 10^{-15}$ is considered to be Gaussian surface. Charged particles inside the nucleus are protons each have charge $q = 1.6 \times 10^{-19}$C and Z number is the number of protons. Hence charge inside the nucleus is $Q = Zq$. As per Gauss's Law electric flux at the surface is due to charge Q enclosed by the Gaussian Surface. And as per Coulomb's Law electric field at the surface is $E = \frac{Q}{4\pi\varepsilon_0 r^2}$. Thus each part is being solved separately. **Part (a):** Using the available data with $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}. Electric field at the surface of the nucleus is $E = (9 \times 10^9) \times \frac{(79\times(1.6\times10^{-19}))}{(7.0\times10^{-15})^2} = \mathbf{2.3 \times 10^{21}}$ **N/C is answer of part (a).** **Part (b):** Electric field at mid of radius will involve two considerations. First is Gaussian surface has radius $r' = \frac{r}{2}$ and Second that charge inside Gaussian surface, be based on uniform distribution of charge, is $q' = \frac{Q}{\left(\frac{4}{3}\pi r^3\right)}\left(\frac{4}{3}\pi r'^3\right) = Q\left(\frac{r'}{r}\right)^3 = \frac{Q}{8}$. Therefore, electric field is $E' = \frac{Q'}{4\pi\varepsilon_0 r'^2}$. Using the available data $E' = (9 \times 10^9) \times \frac{\frac{79\times(1.6\times10^{-19})}{8}}{\left(\frac{7.0\times10^{-15}}{2}\right)^2} = \frac{E}{2} \Rightarrow E' = \frac{2.3\times10^{21}}{2} = \mathbf{1.15 \times 10^{21}}$ **N/C is answer of part (b).** **Part (c):** Nucleus is an integral part of atoms of each material be it conductor, insulator or semiconductor. It is the distribution of electrons that changes in the three kinds of materials. As regards they are held in position in the nucleus due to nuclear forces caused by nucleons. This causes charge of protons uniformly distributed in the nucleus. Hence, **answer to part (c) is Yes.** **Hence, answers are (a)** 2.32×10^{21} N/C (b) 1.16×10^{21} N/C (c) Yes

I-59	Electric field is required to be determined at any point at distance x from the center of the system of charges in the system shown in the figure. Uniform charge density in the given shell is $q=\frac{Q}{4\pi(r_2^3-r_1^3)}$..(1) As per Gauss's law the spherical cell of radius x forms the Gaussian surface within which charge is $Q_x=q\times 4\pi(x^3-r_1^3)$…(2) Further, as per Coulomb's Law the charge within the Gaussian surface causes electric field as if the charge is at its center. Accordingly, $E_x=\frac{Q_x}{4\pi\varepsilon_0 x^2}$…(3). Combining (1) and (2) in (3), $E_x=\frac{\left(\frac{Q}{4\pi(r_2^3-r_1^3)}\right)\times 4\pi(x^3-r_1^3)}{4\pi\varepsilon_0 x^2}\Rightarrow E_x=\frac{\frac{Q(x^3-r_1^3)}{(r_2^3-r_1^3)}}{4\pi\varepsilon_0 x^2}\Rightarrow E_x=\frac{Q(x^3-r_1^3)}{4\pi\varepsilon_0 x^2(r_2^3-r_1^3)}$ **is the answer**.   The approximate graph of electric field vs distance from the center of the shall is as shown in the figure. Electric field is zero for $x<r_1$, for region $r_1<x<r_2$ it increases nearly linear and for $x>r_2$ electric field reduces as per Coulomb's Law.
I-60	Taking each part separately. **Part (a):** Charge Q is placed at the center of an uncharged thin hollow metallic sphere. It would induce a charge $-Q$ on the inner surface and its complementary charge $+Q$ would be induced on the outer surface. Since t a metallic sphere the charge distribution would be uniform such that charge density on the inner surface $\rho_i=-\frac{Q}{4\pi a^2}$ and on the outer surface the charge density would be $\rho_o=\frac{Q}{4\pi a^2}$. Thus, **answer of part (a) is** $-\frac{Q}{4\pi a^2}, \frac{Q}{4\pi a^2}$.  **Part (b):** When charge q is placed on the sphere, it would move to outer surface and get uniformly distributed. But, charge distribution on the inner surface of hollow sphere would remain unchanged at $\rho_i=-\frac{Q}{4\pi a^2}$. But on the outer surface of the sphere would be $\rho_o=\frac{Q+q}{4\pi a^2}$. Thus, **answer of the part (b) is** $-\frac{Q}{4\pi a^2}, \frac{Q+q}{4\pi a^2}$  **Part (c):** In either of the two cases electric field inside the hollow sphere at a distance x from its center, as per Gauss's law would be due to charge Q at the center of the sphere. This field as per Coulomb's Law would be $E=\frac{Q}{4\pi\varepsilon_0 x^2}$ **in both the situations, is the answer of part (c).** **Thus, answers are** $-\frac{Q}{4\pi a^2}, \frac{Q}{4\pi a^2}$ (b) $-\frac{Q}{4\pi a^2}, \frac{Q+q}{4\pi a^2}$ (c) $\frac{Q}{4\pi\varepsilon_0 x^2}$ in both the situation
I-61	Given that there are four protons in beryllium atom and hence charge at nucleus is $q=4\times(+1.6\times10^{-19})=+6.4\times10^{-19}$C. Electric field at just inside 1s orbit of radius as per Coulomb's Law is $r_1=1.3\times10^{-11}$ will be influenced by only q and accordingly $E_1=\frac{q}{4\pi\varepsilon_0 r_1^2}$. Using the available data $E_1=(9\times10^9)\frac{6.4\times10^{-19}}{(1.3\times10^{-11})^2}=$ $\mathbf{3.4\times10^{13}}$ **N/C is answer of part (a).**  But, for electric field just inside 2s orbit, the 1s Orbit acts as Gaussian surface and net charge inside it is $q'=q+2\times(-1.6\times10^{-19})=3.2\times10^{-19}$ would play its role. Accordingly, $E_2=\frac{q'}{4\pi\varepsilon_0 r_2^2}=(9\times10^9)\frac{3.2\times10^{-19}}{(5.2\times10^{-11})^2}=\mathbf{1.1\times10^{12}}$ **N/C is answer of part (b).** **Thus answers are (a)** $\mathbf{3.4\times10^{13}}$ **N/C (b)** $\mathbf{1.1\times10^{12}}$**N/C**

I-62	Given that line charge density is $\rho = 2 \times 10^{-6}$ C/m and therefore to determine electric field at a point $r = 4.0 \times 10^{-2}$m. As per Gauss's Law $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$. Taking a cylindrical Gaussian surface of radius r with the line charge at its axis. Considering the geometrical symmetry, as per Coulomb's law, electric field is $\vec{E} = E\hat{r}$ and $\Delta\vec{s} = \Delta s\hat{r}$. Accordingly, $\oint \vec{E} \cdot d\vec{s} = \oint(E\hat{r}) \cdot (\Delta s\hat{r}) = \frac{\Delta q}{\varepsilon_0} \Rightarrow E\oint ds = \frac{\Delta q}{\varepsilon_0}$...(1) Here, lateral surface area of the Gaussian surface is $\oint ds = 2\pi r\Delta l$ ...(2) and $\Delta q = \rho\Delta l$ ...(3). Combining (2) and (3) in (1), $E(2\pi r\Delta l) = \frac{\rho\Delta l}{\varepsilon_0} \Rightarrow E = \frac{\rho}{2\pi r\varepsilon_0}$. Using the given data and $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $E = \frac{2\times10^{-6}}{2\pi(4.0\times10^{-2})(8.85\times10^{-12})} \Rightarrow E = \mathbf{9 \times 10^5}$ **N/C is the answer.**
I-63	Given that a log cylindrical wire carries a linear charge density is $\rho = 2 \times 10^{-8}$ C/m and an electron revolves in a circular path. In this (+)ve linear charge would cause an inward force of attraction to act as centripetal force $\vec{F_i} = Ee(-\hat{r})$...(1), here E is the electric field on the electron having a change – e. If the electron is revolving in a circular path of radius r, electric field using standard formulation is a $E = \frac{\rho}{2\pi r\varepsilon_0}$ N/C...(2). But, the outward centrifugal force on the electron due to circular motion would be $\vec{F_o} = \frac{mv^2}{r}\hat{r}$...(3). And finally an equilibrium $\vec{F_i} + \vec{F_o} = 0$ ...(4), exit that keep electron revolving around .  Combining (1), (2), (3) and (4), $\frac{\rho}{2\pi r\varepsilon_0}e(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow mv^2 = \frac{\rho e}{2\pi\varepsilon_0}$. Therefore kinetic energy of electron is $KE = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \times \rho e$. We know that $e = 1.6 \times 10^{-19}$C and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$. Thus, using the available data $KE = \frac{1}{2}mv^2 = (9 \times 10^9) \times (2 \times 10^{-8})(1.6 \times 10^{-19}) = \mathbf{2.9 \times 10^{-17}}$**J is the answer**.
I-64	Given that the long cylindrical volume has uniform charge density ρ per unit volume. Therefore, considering the geometrical symmetry, this can be considered like a uniformly distributed linear charge at the axis of the cylinder for the volume inside coaxial Gaussian surface passing through the point P under consideration, inside the cylinder such that $x < a$ as shown in the figure. This point P is at a distance x from the axis of the cylinder. Therefore, the linear charge per unit length would be $q = \frac{\pi x^2 l\rho}{l} \Rightarrow q = \pi x^2\rho$...(1). Electric field due to a linear charge at a distance x from as per Gauss's Law is $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$ ...(2).Again by geometrical symmetry  $\vec{E} = E\hat{r}$ and $d\vec{s} = ds\vec{r}$, here magnitude E is constant at every point on the Gaussian surface. Thus, $\vec{E} \cdot d\vec{s} = E\hat{r} \cdot ds\vec{r} \Rightarrow \vec{E} \cdot d\vec{s} = Eds$. Therefore, LHS of (2) resolves into $E\oint ds = E(2\pi x)$...(3). Combining (1), (2) and (3) $E(2\pi x) = \frac{\pi x^2\rho}{\varepsilon_0} \Rightarrow E = \frac{x\rho}{2\varepsilon_0}$ **is the answer**.
I-65	Given system of a large non-conducting sheet of thickness a is shown in the figure as side view having projection in its length and width. The point P in consideration is at a distance $0 < \|x\| < \|a\|$ it implies that it can be on either side of central plane shown by YY'.  Therefore, as per Gauss's Law Taking a Gaussian surface, encompassing point P, of are $\Delta\vec{s} = \Delta s\hat{s}$...(1), the charge inside it would be $q = (\Delta sx)\rho$...(2) Electric field due to a linear charge at a distance x from as per Gauss's Law is $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$ ...(3). Again by geometrical symmetry $\vec{E} = E\hat{s}$, here magnitude E is constant at every point on the Gaussian surface. Thus, $\vec{E} \cdot \Delta\vec{s} = E\hat{s} \cdot \Delta s\hat{s} \Rightarrow \vec{E} \cdot \Delta\vec{s} = E\Delta s$ ...(4). Therefore, LHS of (3) resolves into $E\oint ds = E\Delta s$...(5). Combining (2), (3) and (5) $E\Delta s = \frac{\Delta sx\rho}{\varepsilon_0} \Rightarrow E = \frac{x\rho}{\varepsilon_0}$ **is the answer**.

I-66

Given a non-conducting plate having a charge density $\rho = 4.0 \times 10^{-6}$ C/m^2. Considering a Gaussian surface with point P, close to the plate, at it right cross-section n the right of the plate as shown in the figure. Electric flux emanating from cross-section of the plate, as per Gauss's Law is $\phi = \oint \vec{E} \cdot d\vec{s} = \frac{\Delta q}{\varepsilon_0}$,…(1). Here $\Delta q = \rho \Delta s$…(2), considering proximity of P to the plate and geometrical symmetry, $\vec{E} = E\hat{s}$, here magnitude E is constant at every point on the Gaussian surface and $\Delta\vec{s} = \Delta s\hat{s}$. Thus, $\vec{E} \cdot \Delta\vec{s} = E\hat{s} \cdot \Delta s\hat{s} \Rightarrow \vec{E} \cdot \Delta\vec{s} = E\Delta s$ …(3). Therefore, in (1) $E \oint ds = 2E\Delta s$…(4). Here, $\oint ds = 2\Delta s$ considering the cross-sections of the Gaussian surface on both sides of the charged plate.



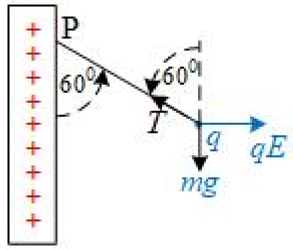
Combining (1), (2) and (4) $2E\Delta s = \frac{\Delta s \rho}{\varepsilon_0} \Rightarrow E = \frac{\rho}{2\varepsilon_0}$…(5).

Further, as per Coulomb's Law attractive force experience by charge q is $F = Eq$…(6). Combining (5) and (6) using available data $\varepsilon_0 = 8.85 \times 10^{-12}$, we have $F = \frac{\rho q}{2\varepsilon_0} \Rightarrow F = \frac{(4.0\times10^{-6})(2.0\times10^{-6})}{2\times(8.85\times10^{-12})} = \mathbf{0.45}$ **N is the answer.**

I-67

Electric field at point near a non-conducting vertical plate having a charge density ρ C/m^2 is $E = \frac{\boldsymbol{\rho}}{\mathbf{2\varepsilon_0}}$ …(1).

Horizontal force on ball of mass $m = 10 \times 10^{-3}$ kg carrying a charge $q = 4.0 \times 10^{-6}$ tied at free end of string of length $l = 0.10$ m is $F_E = Eq$. The ball will experience a vertical gravitational force $F_g = mg$. The sting is tied to the vertical plate at P, as shown in the figure. In state of equilibrium the string carrying a tension T makes an angle $\theta = 60^0$ with the vertical. This geometry necessitates that charge on vertical plate is positive.

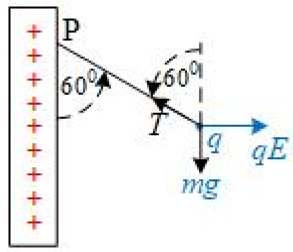


In this position of equilibrium $T\sin\theta = qE$…(2) and $T\cos\theta\, mg$…(3). Combining (1), (2) and (3) $\frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg} \Rightarrow \tan\theta = \left(\frac{q}{mg}\right)\left(\frac{\boldsymbol{\rho}}{\mathbf{2\varepsilon_0}}\right)$. Taking $g = 10$ m/s^2 and $\varepsilon_0 = 8.85 \times 10^{-12}$ and the available data $\rho = \frac{2mg\varepsilon_0\tan\theta}{q} \Rightarrow \rho = \frac{2\times(10\times10^{-3})\times10\times(8.85\times10^{-12})\times\sqrt{3}}{4.0\times10^{-6}} \Rightarrow \rho = \mathbf{7.7 \times 10^{-7}}$ **C/m^2 is the answer.**

I-68

Electric field at point near a non-conducting vertical plate having a charge density ρ C/m^2 is $E = \frac{\boldsymbol{\rho}}{\mathbf{2\varepsilon_0}}$ …(1). Horizontal force on ball of mass $m = 10 \times 10^{-3}$ kg carrying a charge $q = 4.0 \times 10^{-6}$ tied at free end of string of length $l = 0.10$ m is $F_E = Eq$. The ball will experience a vertical gravitational force $F_g = mg$. The sting is tied to the vertical plate at P, as shown in the figure. In state of equilibrium the string carrying a tension T makes an angle $\theta = 60^0$ with the vertical. This geometry necessitates that charge on vertical plate is positive.

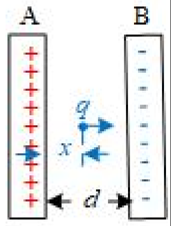
In this position of equilibrium $T\sin\theta = qE$…(2) and $T\cos\theta\, mg$…(3). Combining (1), (2) and (3) $\frac{T\sin\theta}{T\cos\theta} = \frac{qE}{mg} \Rightarrow \tan\theta = \left(\frac{q}{mg}\right)\left(\frac{\boldsymbol{\rho}}{\mathbf{2\varepsilon_0}}\right)$. Taking $g = 10$ m/s^2 and $\varepsilon_0 = 8.85 \times 10^{-12}$ and the available data $\rho = \frac{2mg\varepsilon_0\tan\theta}{q} \Rightarrow \rho = \frac{2\times(10\times10^{-3})\times10\times(8.85\times10^{-12})\times\sqrt{3}}{(4.0\times10^{-6})} \Rightarrow \rho = 7.7 \times 10^{-7}$.



Using (1), (2) and (3), $T = \sqrt{(mg)^2 + (qE)^2} \Rightarrow T = \sqrt{(mg)^2 + \left(\frac{q\rho}{2\varepsilon_0}\right)^2}$. Using the available data we have $T = \sqrt{\left((10 \times 10^{-3}) \times 10\right)^2 + \left(\frac{(4.0\times10^{-6})(7.7\times10^{-7})}{2(8.85\times10^{-12})}\right)^2} \Rightarrow T = \sqrt{10^{-2} + 3 \times 10^{-2}} =$ $\mathbf{0.20}$ **N.is answer of part (a).**

When the all is pushed aside slightly, either way, the restraining force is T combined effect of mg and not qE. Thus it will act like **a** simple pendulum with time period of oscillation $= 2\pi\sqrt{\frac{l}{g}}$. In the instant case acceleration would be $g \to a = \frac{T}{m}$. Accordingly, time period would be $= 2\pi\sqrt{\frac{l}{\frac{T}{m}}} = 2\pi\sqrt{\frac{ml}{T}} = 2\pi\sqrt{\frac{(10\times10^{-3})\times0.1}{0.2}}$ $=$ **0.44s is answer of part (b)**

I-69



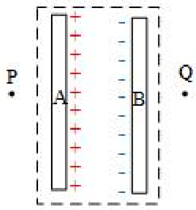
Given the large conducting parallel plates plate having charge on their inner surface is shown in the figure. Thus electric field at a point between the two plates due to plate A having positive charge, as per Gauss's Law, is $\vec{E}_A = \frac{\rho}{2\varepsilon_0}\hat{z}$, here ρ C/m^2 is the charge density on the plate. Charge on A would induce negative charge density $(-\rho)$ C/m^2 and as a result electric field at the point $\vec{E}_B = \frac{(-\rho)}{2\varepsilon_0}(-\hat{z}) = \frac{\rho}{2\varepsilon_0}\hat{z}$. Therefore, force on a electron carrying charge $q = -1.6 \times 10^{-19}$ is $\vec{F} = q \times (\vec{E}_A + \vec{E}_B) = \frac{q\rho}{\varepsilon_0}\hat{z}$. Accordingly, as per Newton's Second law of motion the particle will experience acceleration $\vec{a} = \frac{\vec{F}}{m} = \frac{q\rho}{m\varepsilon_0}\hat{z}$ …(1), here $m = 9.1 \times 10^{-31}$kg and $\varepsilon_0 = 8.85 \times 10^{-12}$.

Further, given that an electron is projected from a plate towards another plate at a separation $d = 2.00 \times 10^{-2}$ m, with a velocity u m/s such that it reaches the another plate i.e. $v = 0$ in $t = 2.00 \times 10^{-6}$. With this system shown in the figure, it can be only when electron is project with velocity $\vec{u} = u\hat{z}$ and $\vec{a}$ acts as retardation. Thus, as per Third equation of motion $v^2 = u^2 + 2ad \Rightarrow u = \sqrt{(-2ad)}$. Since, $\vec{u} = u\hat{z}$ and $\vec{a} = a(-\vec{z})$, algebraically, $u = \sqrt{2ad}$. Further, as per First Equation of Motion, $v = u + at \Rightarrow 0 = u + at \Rightarrow \sqrt{2ad} = -at \Rightarrow a = \frac{2d}{t^2}$…(2).

Combining (1) and (2), $\frac{q\rho}{m\varepsilon_0} = \frac{2d}{t^2} \Rightarrow \rho = \frac{2dm\varepsilon_0}{qt^2}$. Using the available data $\rho = \frac{2\times(2.00\times10^{-2})(9.1\times10^{-31})(8.85\times10^{-12})}{(1.6\times10^{-19})(2.00\times10^{-6})^2}$ $=$ **5.03×10^{-13} C/m^2 is the answer.**

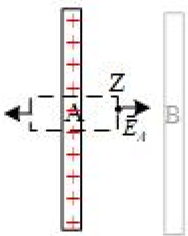
N.B.: This problem is a good example of integration of multiple concepts.

I-70



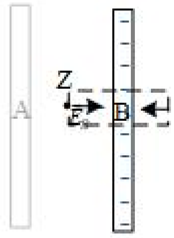
Given system of two large conducting plates A and B of surface area S having charge densities $(+\sigma)$ C/m^2 and $(-\sigma)$ C/m^2 respectively. The charges would reside and bound on the inner surface of the two plates. A Gaussian surface enclosing the two plates is taken. Further, in respect part (a) and (c) of the problem, points X and Y, respectively are taken outside the Gaussian surface.

Therefore. as per Gauss's Law, $\oint E.ds = \frac{Q}{\varepsilon_0}$, here $Q = Q_A + Q_B \Rightarrow Q = S(+\rho) + S(-\rho)$. It leads to $Q = 0$ and, therefore, $\oint E.ds = 0$…(1). Since, surface area of a Gaussian surface is never zero and hence for (1) to be valid only necessity is $E = 0$ at points X and Y, which are outside the Gaussian surface.



Hence, **answer for part (a) and (c) is Zero**.

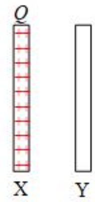
Now analysis of part (b) is done for electric field at a point Z in space between the two plates. Taking an elemental area Δs of late to which point Z is placed perpendicularly as shown in the figure. Charge enclosed inside the cylindrical Gaussian surface is $\Delta q = \rho\Delta s$. As per Gauss's Law electric field by geometrical symmetry is perpendicular to the cross-sectional area of the Gaussian surface $\oint \vec{E}_A.d\vec{s} = 2(E_A\hat{z})(\Delta s\hat{z}) = \frac{\rho\Delta s}{\varepsilon_0} \Rightarrow \vec{E}_A = \frac{\rho}{2\varepsilon_0}\hat{z}$, it is directed towards the plate B. Likewise, electric field at Z due to plate B is $\vec{E}_B = \frac{\rho}{2\varepsilon_0}\hat{z}$, directed towards B, as shown in the figure. Thus net electric field at Z is $\vec{E}_Z = \vec{E}_A + \vec{E}_B = \frac{\rho}{2\varepsilon_0}\hat{z} + \frac{\rho}{2\varepsilon_0}\hat{z} = \frac{\rho}{\varepsilon_0}\hat{z}$, it is perpendicularly directed towards plate B. $\vec{E}_Z = \vec{E}_A + \vec{E}_B = \frac{\rho}{2\varepsilon_0}\hat{z} + \frac{\rho}{2\varepsilon_0}\hat{z} = \frac{\rho}{\varepsilon_0}\hat{z}$. Hence, **answer to part (c) is $\frac{\rho}{\varepsilon_0}$ perpendicularly directed towards plate B.**

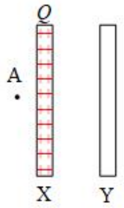


Thus, answers are (a) Zero (b) $\frac{\sigma}{\varepsilon_0}$ perpendicularly directed towards negatively charged plate (c) Zero

I-71

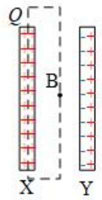
Given that both the plates X and Y are conducting plates, each of them have surface area A each side. Plate X is given a charge Q. Characteristically, charge would appear on the surface of the plate Therefore, charge distributed on each face of X would be $Q_X' = \frac{Q}{2}$. Accordingly, charge density at inner surface of X as desired in part (a) is $\rho_a = \frac{\frac{Q}{2}}{A} = \boldsymbol{\frac{Q}{2A}}$**, is answer of part (a).**

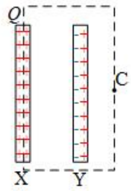




The plate Y is since neutral, as per Gauss's law electric field at a point left A as shown in the figure is $E = \frac{\rho_a}{\varepsilon_0}$, thus using the answer of part (a), $E = \frac{\frac{Q}{2A}}{\varepsilon_0} \Rightarrow E = \boldsymbol{\frac{Q}{2A\varepsilon_0}}$ **is answer of part (b).**

Inner face of plate Y will have an induced charge $Q_Y' = -\frac{Q}{2}$ and outer surface shall have $Q_Y'' = \frac{Q}{2}$ to maintain its neutrality. Thus as per Gauss's law electric field at B due to effective charge $Q_X' = \frac{Q}{2}$ A within the Gaussian surface as shown in the figure is $E = \frac{\frac{Q}{2A}}{\varepsilon_0} \Rightarrow$ $E = \boldsymbol{\frac{Q}{2A\varepsilon_0}}$**, thus using the answer of part (c).**



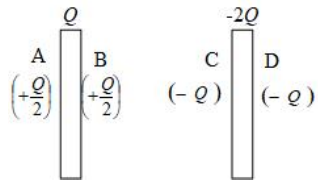


Electric field at a point C to the right of plates, as shown in the figure, is due to effective charge $Q_X' = \frac{Q}{2}$ and hence as per Gauss's Law electric field at C is $E = \frac{\frac{Q}{2A}}{\varepsilon_0} \Rightarrow E = \boldsymbol{\frac{Q}{2A\varepsilon_0}}$**, thus using the answer of part (d).**

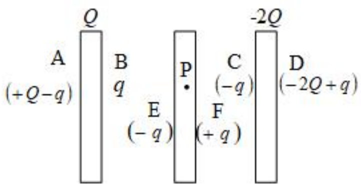
Thus, **answers are (a)** $\boldsymbol{\frac{Q}{2A}}$ **(b)** $\boldsymbol{\frac{Q}{2A\varepsilon_0}}$ **towards left (c)** $\boldsymbol{\frac{Q}{2A\varepsilon_0}}$ **towards right (d)** $\boldsymbol{\frac{Q}{2A\varepsilon_0}}$ **towards right**

I-72

The given system of identical metal plates is shown in figure two stages. Taking first stage with the two charged plates and excluding the middle plates which is electrically neutral. The plates being metallic charge on it will get distributed on their surfaces. The two surfaces A and B of the left plates carrying charge will have charge densities $\rho_A = \frac{+Q}{2A}$ and $\rho_B = \frac{+Q}{2A}$ and the two surface C and D of right plate will have charge densities $\rho_C = \frac{-2Q}{2A} = \frac{-Q}{A}$ and $\rho_D = \frac{-2Q}{2A} = \frac{-Q}{A}$.



Now, when middle, electrically neutral plate is introduced as shown in the figure. It will have induced charges say $-q$ and $+q$ on its each of the two surfaces E and F, to maintain its electrically neutrality, which are electrically opposite to the charge on the inner surfaces B and C such that charge densities are $\rho_E = \frac{-q}{A}$ and $\rho_F = \frac{+q}{A}$ respectively. This will result in redistribution of charges on surfaces A leading to surface charge densities $\rho_A' = \frac{+(Q-q)}{A}$ and $\rho_B' = \frac{+q}{A}$ such that net charge on the plate remains at $+Q$. Likewise, surfaces C and D will be densities $\rho_C' = \frac{-q}{A}$ and $\rho_D' = \frac{(-2Q+q)}{2A}$ leading to net charge on it to $-2Q$.



Next step is to determine electric field at point P, due to each of the surface, which as per general equation of Gauss's law is $E = \frac{\rho}{\varepsilon_0}$, where ρ is the charge density of the corresponding surface. It is independent of its distance from the surface. Accordingly, $\vec{E}_A = \frac{+(Q-q)}{A\varepsilon_0}\hat{z}$, $\vec{E}_B = \frac{+q}{A\varepsilon_0}\hat{z}$, $\vec{E}_C = \frac{-q}{A\varepsilon_0}(-\hat{z}) = \frac{q}{A\varepsilon_0}\hat{z}$, $E_D = \frac{(-2Q+q)}{A\varepsilon_0}(-\hat{z}) = \frac{(2Q-q)}{A\varepsilon_0}\hat{z}$, $E_E = \frac{-q}{A\varepsilon_0}\hat{z}$ and $E_F = \frac{+q}{A\varepsilon_0}(-\hat{z}) = \frac{-q}{A\varepsilon_0}\hat{z}$, a set of equations (2). In this negative sign with unit vector $\hat{z}$ in E_C, E_D and E_F is indicative of direction of field at point P, due to respective charge distribution. Likewise, E_A, E_B and E_E are towards right i.e. along $+\hat{z}$.

Now a point P is taken inside the middle metal plate electrical field $E_P = 0$ …(3) it is characteristic to the metals. Thus using superimposition of electric fields due to each charged resultant electric field at point P is Combing (2) and (3) we have $E_A + E_B + E_C + E_D + E_E + E_F = E_P$…(4). Combining (2) and (3) in (4) we have $\frac{(Q-q)}{A\varepsilon_0} + \frac{q}{A\varepsilon_0} + \frac{q}{A\varepsilon_0} + \frac{(2Q-q)}{A\varepsilon_0} + \frac{-q}{A\varepsilon_0} - \frac{q}{A\varepsilon_0} = 0 \Rightarrow Q + 2Q - -2q = 0 \Rightarrow q = \frac{3}{2}Q$…(5)

	Accordingly, the charge appearing on the outer surface of the rightmost plate, as required to be determined, using (5) is $Q_{\mathrm{D}} = (-2Q + q) = \left(-2Q + \frac{3}{2}Q\right) = -\frac{\boldsymbol{Q}}{\mathbf{2}}$ **is the answer**. **N.B.:** Surfaces B and E, and F and C are acting like pair of capacitor plates having equal and opposite charges remaining charge on the two charged plates appear on surfaces A and D.

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