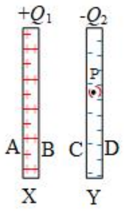
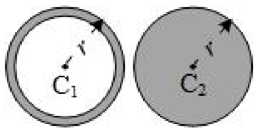
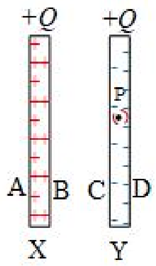
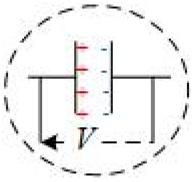
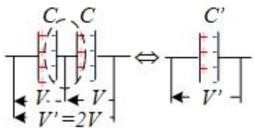
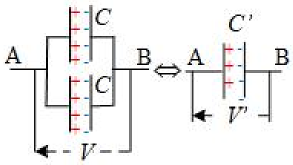
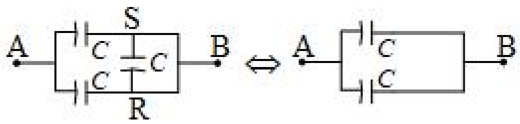
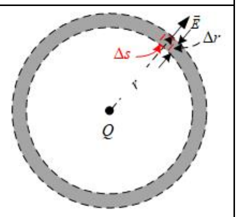
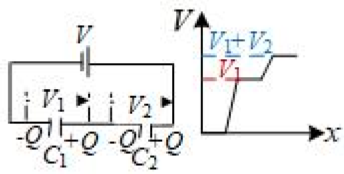
Electromagnetism: Capacitors – Typical Questions (Illustrations Only)

(Appendix contains derivation of star-delta conversion of capacitor connections)

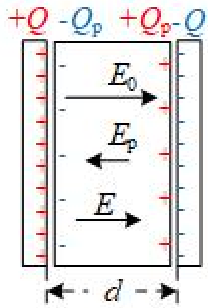
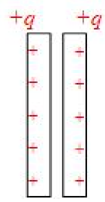
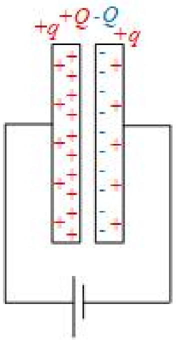
I-1	Given system is conceptualized in the figure. The plate X containing charge $+Q_1$ has two faces, outer surface is A and inner surface is B. Likewise, the plate Y containing charge $-Q_2$ has two faces, outer surface is C and inner surface is D. Charge on the capacitor is the charge on the two surfaces facing each other.  In the instant case these surfaces are B and C. Let, surface B has charge $Q_{\mathrm{B}} = +q \Rightarrow \rho_{\mathrm{B}} = \frac{Q_{\mathrm{B}}}{A} = \frac{q}{A}$ an accordingly corresponding complementary charge on surface C is $Q_{\mathrm{C}} = -q \Rightarrow \rho_{\mathrm{C}} = \frac{Q_{\mathrm{C}}}{A} = -\frac{q}{A}$. The system is standalone i.e. not connected to either a battery or any other material, therefore, after charge distribution on plates charge on the surfaces A and D is $Q_{\mathrm{A}} = +Q_1 - (+q) = Q_1 - q \Rightarrow \rho_{\mathrm{A}} = \frac{Q_{\mathrm{A}}}{A} = \frac{Q_1 - q}{A}$, and $Q_{\mathrm{D}} = -Q_2 - (-q) = -(Q_2 - q) \Rightarrow \rho_{\mathrm{D}} = \frac{Q_{\mathrm{D}}}{A} = \frac{-(Q_2 - q)}{A}$. Now, that capacitor plates are of conductive materials and hence charges reside on the surfaces of the plates and inside the plates electric potential is Zero. Taking a point P inside any of the plate say Y of the capacitor, between surfaces C and D. Therefore, primarily electric field at the point $\vec{E}_{\mathrm{P}} = 0$…(1). This is can be equated to electric field by charges on the four surfaces. As per Gauss's law electric field vector due to a charged surface is $\vec{E} = \frac{\rho}{\varepsilon_0}\hat{z}$, here $\hat{z}$ is direction vector perpendicular to the surface joining the point under consideration. Accordingly, $\vec{E}_{\mathrm{A}} = \frac{\rho_{\mathrm{A}}}{\varepsilon_0}\hat{z} = \frac{Q_1 - q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{B}} = \frac{\rho_{\mathrm{B}}}{\varepsilon_0}\hat{z} = \frac{q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{C}} = \frac{\rho_{\mathrm{C}}}{\varepsilon_0}\hat{z} = -\frac{q}{\varepsilon_0 A}\hat{z}$ and $\vec{E}_{\mathrm{D}} = \frac{\rho_{\mathrm{D}}}{\varepsilon_0}(-\hat{z}) = -\frac{Q_2 - q}{\varepsilon_0 A}(-\hat{z}) = \frac{Q_2 - q}{\varepsilon_0 A}\hat{z}$. Thus, analytically net electric field at P is $\vec{E}_{\mathrm{P}} = \vec{E}_{\mathrm{A}} + \vec{E}_{\mathrm{B}} + \vec{E}_{\mathrm{C}} + \vec{E}_{\mathrm{D}} \Rightarrow \vec{E}_{\mathrm{P}} = \frac{Q_1 - q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + -\frac{q}{\varepsilon_0 A}\hat{z} + \frac{Q_2 - q}{\varepsilon_0 A}\hat{z}$…(2). Combining (1) and (2), $\frac{Q_1 - q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + \left(-\frac{q}{\varepsilon_0 A}\hat{z}\right) + \frac{Q_2 - q}{\varepsilon_0 A}\hat{z} = 0 \Rightarrow (Q_1 - q) + (Q_2 - q) = 0 \Rightarrow q = \boldsymbol{\frac{Q_1 + Q_2}{2}}$, **is the answer.** **N.B.:** It is an interesting application of Gauss's law in capacitors.
I-2	Given expression is reframed out of the basic equation $Q = CV \Rightarrow Q \propto V$, here C is a proportionality constant based on geometry of the material and dielectric filling the space between plates of the capacitor. It is expressed as $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$, here A is the area of the capacitor plates, d is the separation between the plates, ε_0 is the absolute permittivity of vacuum and ε_r is the relative permittivity of the dielectric filling the space between plates of the capacitor. All these parameters are geometrical or physical property remain same and hence C is proportional to the charge Q. Thus, **answer is no.**
I-3	Given that two metal spheres A and B with centers C_1 and C_2, respectively, each of radius r carry equal charges Q. One of two is hollow and the other is solid, The charge given to it will be uniformly distributed on the surface of the sphere and is equivalent to point charge Q at the center of the respective spheres as shown in the figure.  The spheres being metallic will have equipotential surfaces. Accordingly, potential of sphere applying Coulomb's Law of sphere A would be $V_A = \frac{Q}{4\pi\varepsilon_0 r}$, and that of sphere B would be $V_B = \frac{Q}{4\pi\varepsilon_0 r}$. Thus, electric potential of both the spheres are **same, is the answer**.

I-4	Given system is conceptualized in the figure. The plate X and Y are given equal charge say $+Q$. The two faces of X, outer surface is A and inner surface is B. Let, B the inner surface has charge $Q_{\mathrm{B}} = q$ and therefore the outer surface A will have balance of the given charge $Q_{\mathrm{A}} = Q - q$. Thus, by electrostatic induction the inner surface C of the plate Y will have charge $Q_{\mathrm{C}} = -q$ and the surface D shall have balance charge $Q_{\mathrm{D}} = Q - (-q) = Q + q$.  The system is standalone i.e. not connected to either a battery or any other material, therefore,, surface charge densities platers of the capacitor, having each surface area *A,* which are essentially conductive will be $\rho_{\mathrm{B}} = \frac{Q_{\mathrm{B}}}{A} = \frac{q}{A}$, $\rho_{\mathrm{A}} = \frac{Q_{\mathrm{A}}}{A} = \frac{Q-q}{A}$, $\rho_{\mathrm{C}} = \frac{Q_{\mathrm{C}}}{A} = -\frac{q}{A}$ and $\rho_{\mathrm{D}} = \frac{Q_{\mathrm{D}}}{A} = \frac{Q+q}{A}$. Now, that capacitor plates are of conductive materials and hence charges reside on the surfaces of the plates and inside the plates electric potential is Zero. Taking a point P inside any of the plate say Y of the capacitor, between surfaces C and D. Therefore, primarily electric field at the point $\vec{E}_{\mathrm{P}} = 0 \ldots (1)$. This is can be equated to electric field by charges on the four surfaces. As per Gauss's law electric field vector due to a charged surface is $\vec{E} = \frac{\rho}{\varepsilon_0}\hat{z}$, here $\hat{z}$ is direction vector perpendicular to the surface joining the point under consideration. Accordingly, $\vec{E}_{\mathrm{A}} = \frac{\rho_{\mathrm{A}}}{\varepsilon_0}\hat{z} = \frac{Q-q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{B}} = \frac{\rho_{\mathrm{B}}}{\varepsilon_0}\hat{z} = \frac{q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{C}} = \frac{\rho_{\mathrm{C}}}{\varepsilon_0}\hat{z} = -\frac{q}{\varepsilon_0 A}\hat{z}$ and $\vec{E}_{\mathrm{D}} = \frac{\rho_{\mathrm{D}}}{\varepsilon_0}(-\hat{z}) = \frac{Q+q}{\varepsilon_0 A}(-\hat{z}) = -\frac{Q+q}{\varepsilon_0 A}\hat{z}$. Thus, analytically net electric field at P is $\vec{E}_{\mathrm{P}} = \vec{E}_{\mathrm{A}} + \vec{E}_{\mathrm{B}} + \vec{E}_{\mathrm{C}} + \vec{E}_{\mathrm{D}} \Rightarrow \vec{E}_{\mathrm{P}} = \frac{Q-q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + \left(-\frac{q}{\varepsilon_0 A}\hat{z}\right) + \frac{Q+q}{\varepsilon_0 A}(-\hat{z}) \ldots (2)$. Combining (1) and (2), $(Q - q) + q - q - (Q + q) = 0 \Rightarrow 2q = 0 \Rightarrow q = \mathbf{0}$**,** is the charge on the inner plates of the capacitor (i.e.) charge on the capacitor is **Zero is the answer.** Accordingly, the parts is charge on the outer surfaces of the capacitors is ***Q*****, is the answer.** **Hence, answers are Zero, *Q*.** **N.B.:** It is an interesting application of Gauss's law in capacitors. But, qualitatively the like charges would repel each other leading to net charge appearing on the outer surfaces. Thus, this can be framed as an object question also.
I-5	We know that charge on a capacitor is $Q = CV$, in the question capacitance C of the capacitor is given. Therefore, potential difference V is required to determine charge Q. Hence, **answers are No, and Potential difference across the capacitor.**
I-6	Dielectric constant of a material is dependent upon polarization, such that greater the polarization greater is the dielectric constant. Now, when temperature of dielectric material is increased greater is thermally induced vibration in molecules of the dielectric. This affects in reduction of the polarization of the moleculesof the dielectric and thus decease in dielectric constant.
I-7	Energy of a capacitor is $U = \frac{1}{2}CV^2 \ldots (1)$ and $Q = CV \Rightarrow V = \frac{Q}{C} \ldots (2)$. Combining the (1) and (2), $U = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 \Rightarrow U = \frac{Q^2}{C} \ldots (3)$. Now, capacitance of a capacitor is $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ and $\varepsilon_r > 1 \ldots (4)$ Hence, as the dielectric of the capacitor is inserted capacitance C of the capacitor decreases. And in turn energy of the capacitor increases. Now, using (2) in other form $C = \frac{Q}{V}$, (4) can be written as $U = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 \Rightarrow U = \frac{Q}{2V} \Rightarrow U \propto \frac{1}{V} \ldots (5)$. Since energy is decreases with insertion of dielectric, from (5) potential difference across the capacitor must increase. Further, for in a parallel plate capacitor there is uniform electric field hence $V = Ed \Rightarrow V \propto E \ldots (6)$. Here, d is separation between plates of a capacitor about which nothing is stated and hence taken to be constant. Hence, **electric field between the plates must increase with insertion of the dielectric**.

I-8	Given system is conceptualized in figure. The capacitor of capacitance C is charge to a potential V. There charge on the surfaces of the two plates facing each other would be $+Q$ and $-Q$ such that $Q = CV$…(1). As desired a Gaussian surface, enclosing the capacitor is shown in the figure. As per Gauss's Law electric field through the surface is $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$…(1), here q is the net charge inside the Gaussian surface.  In the instant case $q = (+Q) + (-Q) = 0$…(2). Thus combining (1) and (2) $\oint \vec{E}.d\vec{s} = \frac{0}{\varepsilon_0} = 0$….(3). For any real surface $\oint ds \neq 0$ and hence, essentially for (3), $\vec{E} = 0$, this is supported by option (d). Hence, **answer is option (d).**
I-9	Given that two capacitors, each of capacitance C and break down voltage V, are connected in series. The system is conceptualized in the figure. Since charge on a capacitor is $Q = CV$...(1), and hence each of the capacitor charged upto breakdown voltage V, will have charge $Q = CV$. But, in the given connection net voltage across the combination would be $V' = V + V = 2V$…(2). Yet, net charge on the capacitor  on effective capacitor C' remains Q. Thus, using (1) and (2), $C' = \frac{Q}{V'} = \frac{Q}{2V} \Rightarrow C' = \frac{1}{2} \times \frac{Q}{V} \Rightarrow C' = \frac{\boldsymbol{C}}{\boldsymbol{2}}$ and net maximum voltage across the combination is its breakdown voltage $V' = \boldsymbol{2V}$**.** Thus, values of C' and V' for the given combination is $C/2$ and $2V$ as provided in the option (d). Hence, **answer is option (d).**
I-10	The given system of two capacitors of capacitance C and breakdown voltage V, are joined in parallel. The system is sown in the figure. Thus, maximum voltage across each capacitor that can be applied across the parallel combination of capacitors is V. Thus, charge on each capacitor is $Q = CV$…(1), and total charge on plates connected to terminal A is $Q'_A = Q + Q = 2Q$ and to terminal B is $Q'_B = (-Q) + (-Q) = -2Q$…(2).  Thus, in accordance with (1) for the given combination of parallel capacitors, $Q' = C'V \Rightarrow 2Q = 2(CV)$. It leads to $C'V = 2CV \Rightarrow C' = \boldsymbol{2C}$, while breakdown voltage is least of the two capacitors connected in parallel. In the instant case breakdown voltage of both the capacitors is V. Hence, break down voltage of the parallel combination is $\boldsymbol{V}$. Both the values are provided in option (c). Thus, **answer is option (c).**
I-11	The given arrangement of three capacitors, each of capacitance C, is redrawn with charges on the capacitor. The capacitor between R and S is connected through a conductor and hence voltage across it will be zero. This is equivalent to a parallel connection of two capacitors, each of capacitance C, as shown in the figure. Thus equivalent capacitance is $C_{Eq} = C + C = 2C$, and it is provided in the option (b). Thus, **answer is option (b).** 
I-12	Charges on a parallel plate capacitors are uniformly distributed and the plates act as equipotential surface. Let an isolated capacitor of capacitance C, as given, carries a charge Q.Then potential difference across it would be $V = \frac{Q}{C}$…(1)Let separation between the plates is d then strength of uniform electric field would be $E = \frac{V}{d}$…(2). Therefore, as per Coulomb's Law force between the plates will be $F = EQ \Rightarrow F = \frac{V}{d}Q = \frac{\frac{Q}{C}Q}{d} \Rightarrow F = \frac{Q^2}{Cd}$…(3) Now when a dielectric with relative permittivity $\varepsilon_r$is inserted between the plates. Then capacitance would become $C' = \varepsilon_r C$…(4). Since the capacitor is isolated, charge on it would remain unchanged. Therefore, potential difference across it would change to $V' = \frac{Q}{C'} \Rightarrow V' = \frac{Q}{\varepsilon_r C} = \frac{V}{\varepsilon_r}$ …(5). Accordingly, on the lines of (2) electric field between the plates would change to $E' = \frac{V'}{d} \Rightarrow E' = \frac{\frac{V}{\varepsilon_r}}{d} = \frac{V}{\varepsilon_r d}$…(6). Therefore, on the lines of

	(3) using (6) force between the plates will be $F' = E'Q \Rightarrow F' = \frac{V'}{d}Q = \frac{\frac{Q}{C'}Q}{d} \Rightarrow F' = \frac{Q^2}{C'd}$ …(7). Combining (3), (4) an (7), force between the plates will be $F' = \frac{Q^2}{\varepsilon_r Cd} = \left(\frac{Q^2}{Cd}\right)\left(\frac{1}{\varepsilon_r}\right) \Rightarrow F' = \frac{F}{\varepsilon_r}$ …(8). In (8) all parameters in F expressed in (3) are same for the capacitor and hence force between the plates is inversely proportional to the relative permittivity $\varepsilon_r > 1$ of the dielectric that s inserted. Therefore, force between the plates of an isolated capacitor will decrease as provided in option (b). Hence, **answer is option (b).**
I-13	 Energy density $\rho_U = \frac{\Delta U}{\Delta V}$ …(1), at a point caused by work done $\Delta U = -\left(\vec{E} \times (\Delta s \rho_q)\right) \cdot \Delta\vec{r}$per unit volume ΔV. Here, ΔV volume swept by a unit charge in the given electric field. It is to be noted that here V stands for volume and not the potential generally used in electricity. Now as given electric field due to point charge Q at distance $\vec{r}$ from it, as per Coulomb's Law, is $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$…(2). This electric field is symmetrically radial around the point charge. Let unit charge is distributed on a hollow spherical surface of radius $r' = r + \Delta r$, it will lead to a charge density $\rho_q = \frac{1}{4\pi r^2}$…(3). This unit charge is moved by a $\Delta\vec{r} = \Delta r(-\hat{r})$. Taking a small area$\Delta\vec{s} = \Delta s\hat{s} \Rightarrow \Delta q = \Delta s \times \rho_q$…(4), as shown in the figure, work done in moving the distributed charge on the surface of a hypothetical sphere of radius r through a displacement $\Delta\vec{r}$ would be $\Delta U = -\left(\vec{E} \times (\Delta s \rho_q)\right) \cdot \Delta\vec{r} = -\left(\frac{Q\Delta s \rho_q}{4\pi\varepsilon_0 r^2}\hat{r}\right) \cdot \left(\Delta r(-\hat{r})\right) \Rightarrow \Delta U = \frac{Q\rho_q}{4\pi\varepsilon_0 r^2}(\Delta s \Delta r) \Rightarrow \Delta U = \frac{Q\rho_q}{4\pi\varepsilon_0 r^2}(\Delta V)$…(5). Here, $\Delta V = \Delta s \Delta r$…(6), is the volume swept by charge Δq. Combining (3), (5) and (6), we have $\frac{\Delta U}{\Delta V} = \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right) \times \frac{1}{4\pi r^2} \Rightarrow \frac{\Delta U}{\Delta V} = \left(\frac{Q}{(4\pi)^2\varepsilon_0}\right) \times \frac{1}{r^4} \Rightarrow \frac{\Delta U}{\Delta V} \propto \frac{1}{r^4}$…(7). Further, taking (7) with (1) we have $\rho_U \propto \frac{1}{r^4}$ as provided in **option (d), is the answer**.
I-14	Electric lines of force (though graphical representation of electric field at a point) are open loop and each of the inecreated at positive charge end at negative charge. When a parallel-plate capacitor with unequal areas is connected to a battery positive terminal of the battery attracts say $Q_- = -Q$…(1), free charges for the plate connected to it, he charge left on the plate, which is otherwise neutral is $Q_- + Q_+ = 0 \Rightarrow -Q + Q_+ = 0 \Rightarrow Q_+ = Q$…(2). The battery, does not retain the negative charge so collected, it passes on to negative plate which is otherwise neutral. Thus, magniude of charges on on two plates is $\lvert Q_- \rvert = Q = \lvert Q_+ \rvert$. Accordingly charges on the two plates are equal as provided in **option (b), is the answer.**
I-15	Capacitance of a capcitor is $Q = CV \Rightarrow C = \frac{Q}{V}$…(1) When plates of a capacitor seprated by a dielectric medium when connected across a potentital difference V it, a charge Q deverlops on the capacitor. But, when a metal plate P is placed between the plates of capacitor it bridges potential differece across the plates such that they become equipotential. It implies that $V = 0$…(2). Thus, combining (1) and (2) for any charge given the plate capacitance of the system becomes $C = \frac{Q}{0} \Rightarrow C = \infty$ is providee in the **option (d), is the answer**.
I-16	Given system is shown in the figure with charges induced on the plates of capacitors C_1 and C_2, with voltages across them V_1 and V_2 respectively. Magnitude of charges on each plate would be equal on the principle of electrical neutrality of each system, where each capacitor is a system in itself. Thus, $Q = C_1V_1 = C_2V_2 \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1}$…(1). In the given graph of voltage variation in the circuit $V_1 > V_2$, therefore, using (1), $C_2 > C_1$ or $C_1 < C_2$ as provided in **option (c), is the answer.** 

I-17	Here, it is given that two metal plates having some separation, say d, are dipped in oil. Eventually, the given system is an isolated capacitor whose capacitance is $C=\frac{\varepsilon_0\varepsilon_r A}{d}$...(1). It is also given that charge on the capacitors is Q, while in a capacitor $Q=CV$...(2). Electric field between the parallel plates is $E=\frac{V}{d}$...(3). Combining (1), (2) and (3), $Q=\left(\frac{\varepsilon_0\varepsilon_r A}{d}\right)(Ed) \Rightarrow Q=\varepsilon_0\varepsilon_r AE$...(4). When oil is pumped out, $\varepsilon_r \to 1$, but charge on isolated capacitor remains the same. And thus (4) becomes $Q=\varepsilon_0 AE'$...(5). Combining (4) and (5), $\varepsilon_0 AE'=\varepsilon_0\varepsilon_r AE \Rightarrow E'=\varepsilon_r E$...(6). Oil is a dielectric for which $\varepsilon_r>1$ and hence $E'>E$, i.e. electric field increases as provided in the **option (a), is the answer**.
I-18	Two metal sphere of capacitance C_1 and C_2 carry say charges Q_1 and Q_2 respectively. When they are in contact, they will be at same potential and as per capacitance equation $Q=CV \Rightarrow V=\frac{Q}{C}$...(2). Thus for two capacitors at the same potential $\frac{Q_1}{Q_2}=\frac{C_1}{C_2}$ **as provided in option (b), is the answer.**
I-19	Given are three capacitor each of capacitance $C_1=C_2=C_3=6\ \mu$F. It is required to determine minimum an maximum capacitance using the three capacitors. **Minimum capacitance is** achieved by series combination of capacitors where $\frac{1}{C_{\min}}=\frac{1}{C}+\frac{1}{C_2}+\frac{1}{C_3} \Rightarrow \frac{1}{C_{\min}}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6} \Rightarrow \frac{1}{C_{\min}}=3\times\frac{1}{6} \Rightarrow \boldsymbol{C_{\min}=2\ \mu}$**F.** While maximum capacitance is achieved by making parallel combination of capacitors where $C_{\max}=C_1+C_1+C_1 \Rightarrow C_{\max}=3\times 6 \Rightarrow \boldsymbol{C_{\max}=18\ \mu}$**F.** These values match with those in **option (d), I the answer.**
I-20	Capacitance of a capacitor is its property defined by its geometry and the dielectric filled in the gap between the plates of the capacitor. Since question is silent either on geometry or the dielectric and hence air filled parallel pale capacitor is considered for analysis. Capacitance for such a capacitor is $C=\frac{\varepsilon_0 A}{d}$...(1). Here, A is area of the plates of the dielectric, d is separation between the plates and ε_0 is absolute permittivity of vacuum. Now each of the given option is analyzed – **Option (a):** Shape of the plates of the capacitor decides its area and hence capacitance depends upon shape. Thus, **this option is not true**. **Option (b):** Size of the plate of capacitor viz. length and breadth decides its area and capacitance. Thus, this **option is not true**. **Option (c):** Charge on capacitor has no mention in (1) and hence this. **option is true** **Option (d):** Separation between plates is an integral part of (1), hence this **option is not true**. Thus, **answer is option (c).**
I-21	Given is an isolated capacitor it implies that it cannot transfer charge of capacitor i.e. charge on its plates. Thus, **out right opion (b) is correct**. But, other options also needs to be analyzed. **Option (a):** When dielectric is inserted, no change in separation between plates of the capacitor is stated. Hence capacitance of the capacitor would change from $C=\frac{\varepsilon_0 A}{d}$ to $C'=\frac{\varepsilon_0\varepsilon_r A}{d}$, thus potential difference between plates would change from $V=\frac{Q}{C}$ to $V'=\frac{Q}{C'}$. Accordingly electric field $E=\frac{V}{d}=\frac{\frac{Q}{C}}{d}=\frac{Q}{\frac{\varepsilon_0 A}{d}\times d} \Rightarrow E=\frac{Q}{\varepsilon_0 A}$, would change to $E'=\frac{Q}{\varepsilon_0\varepsilon_r A}$, hence, **option (a) is not true.** **Option (c):** In analysis of option (a) it is seen that potential difference between plates changes from V to V'**.** Hence, **option (c) is not true.** **Option (d):** Energy stored in a capacitor is $U=\frac{1}{2}CV^2=\frac{1}{2}C\left(\frac{Q}{C}\right)^2 \Rightarrow U=\frac{Q^2}{2C}$. From analysis of option (a), $U=\frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} \Rightarrow U=\frac{Q^2 d}{2\varepsilon_0 A}$ would change to $U'=\frac{Q^2 d}{2\varepsilon_0\varepsilon_r A}$. Hence, **option (c) is not true.**

	Thus, **answer is option (d).**
I-22	Given is a capacitor of capacitance $C = \frac{\varepsilon_0 A}{d}$, having charge Q on it. Here, A is the area of plates of the capacitor and d is separation between the plates. Therefore, voltage across the capacitor will be $Q = CV \Rightarrow V = \frac{Q}{C} = \frac{Q}{\frac{\varepsilon_0 A}{d}} \Rightarrow V = \frac{Qd}{\varepsilon_0 A} \Rightarrow E_0 = \frac{V}{d} \Rightarrow E_0 = \frac{Q}{\varepsilon_0 A}$..(1).  Now a dielectric slab of dielectric constant $K = \varepsilon_r$…(2), here ε_r is the relative permittivity of the dielectric slab inserted between plates of the capacitor. Let this electric field would cause polarization in dielectric inducing a charge Q_p as shown in the figure. Accordingly, an electric field $E_p = \frac{Q_p}{\varepsilon_0 A}$ …(3), develops within the dielectric and it is in a direction opposite to E_0 as shown in the figure. Thus combined effect of Q charge on plates of the capacitor an induced charge in dielectric Q_p is the electric field within the dielectric $E = E_0 - E_p$ …(4). Combining (1) and (3) in (4), $E = \frac{Q}{\varepsilon_0 A} - \frac{Q_p}{\varepsilon_0 A} \Rightarrow E = \frac{Q-Q_p}{\varepsilon_0 A}$…(5). Net electric fields between the plates, in presence of dielectric filling the gap between the plates, is $E = \frac{Q}{\varepsilon_0 \varepsilon_r A}$…(6). Combining (5) and (6) $\frac{Q-Q_p}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 \varepsilon_r A} \Rightarrow \frac{Q_p}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 A} - \frac{Q}{\varepsilon_0 \varepsilon_r A} \Rightarrow Q_p = Q\left(1 - \frac{1}{\varepsilon_r}\right)$…(7). For dielectrics $\varepsilon_r > 1$ and hence $\left(1 - \frac{1}{\varepsilon_r}\right) < 1$. Accordingly, $Q_p < Q$ is must as provided in **option (d), is the answer**.
I-23	Given that each plate of a parallel-plate capacitor is given a charge q. The like charges would repel each other and would get uniformly distributed on outer surface of metallic plates of the capacitor.   Now the capacitor is connected to a battery as shown in the figure. Battery imparts say $+Q$ charge to one plate and $-Q$ on the other plate. These charges would appear on inner surfaces of the plates as shown in the figure in accordance with the principle of electrical neutraliy. It is observed that – • Inner surfaces of the plates of capacitor have equal and opposite charges as given in **option (a), is correct**. • Charge on left plate of the capacitor is $Q_L = Q + q$ while charge on the right plate is $Q_R = -Q + q$, thus $Q_L \neq Q_R$, thus, **option (b), is incorrect**. • Battery would supply equal and opposite charges $+Q$ and $-Q$ to the two plates of the capacitor, as per the principle of electrical neutrality. Thus, **option (c) is correct**. • Outer surfaces of the plates of the capacitor would continue to carry equal charge q while complying with the electrical neutrality of the battery. Thus, **option (d) is correct.** **Thus, answer is option (a), (c) and (d).**
I-24	The problem since does not indicate connection of the capacitor henceit is taken to be stand-alone capacitor having charge Q on it. Thus on increase of distance between the capacitors – **Option (a):** chargeon the stand alon capacitor would not change and ence, **option (b) is incorrect.** **Option (b):** Capacitance of a parallel plate ca[acitor is $C = \frac{\varepsilon_0 A}{d}$ …(1), and potential difference across the capacitor would be $V = \frac{Q}{C}$…(2). Combining (1) and (2), $V = \frac{Q}{\frac{\varepsilon_0 A}{d}} \Rightarrow V = \frac{Qd}{\varepsilon_0 A}$…(3). When

seperation between is increased to $d' > d$, potential difference across the capacitor would change to $V' = \frac{Qd'}{\varepsilon_0 A}$...(4). Hence, **option (b) is correct**.

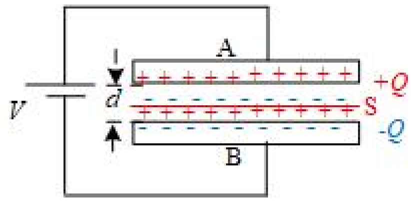
Option (c): Energy of the capacitor is $U = \frac{1}{2}QV$...(5). While, charge on the capacitor remains same voltage across it is increasing as per (3) and (4). Hence, energy on the capacitor changes. Thus, **option (c) is correct.**

Option (d): Energy density between the plates is $\rho_U = \frac{U}{Ad}$. Using (2) and (5), $\rho_U = \frac{\frac{1}{2}QV}{Ad} = \frac{Q\times\frac{Q}{C}}{2Ad} = \frac{Q^2}{2CAd}$. Using (1), $\rho_U = \frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)Ad} = \frac{Q^2}{2\varepsilon_0 A^2}$...(6). It iseen that (6) is indepenent of seperation between the plates. Thus, **option (d) is incorrect.**

Thus, answer is option (b) and (c).

I-25

Given system is conceptualized in figure where a capacitor comprising of parallel plates A and B with a separation d is connected to a battery. Capacitance of the will be $C = \frac{\varepsilon_0 A}{d}$ ...(1).



A thin metallic sheet S of negligible thickness $t \to 0$ ...(2), is inserted in gap between the plates A and B, such that d_1 is separation between A and upper surface of S and d_2 is separation between B and lower surface of S. Accordingly, using (2), $d = d_1 + t + d_2 \Rightarrow d = d_1 + d_2$...(3).

Further, upper surface of S will have induced charge $-Q$, while its lower surface will have $+Q$, marinating net charge of the plate S to be zero.

This becomes a series combination of two capacitors, One between A and upper surface of S where $C_1 = \frac{\varepsilon_0 A}{d_1}$...(4), and the other between B and lower surface of S where $C_2 = \frac{\varepsilon_0 A}{d_2}$....(5) Thus, net capacitance of the series combination is $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$...(6).

Combining, (4), (5) and (2), $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C'} = \frac{1}{\frac{\varepsilon_0 A}{d_1}} + \frac{1}{\frac{\varepsilon_0 A}{d_2}} \Rightarrow \frac{1}{C'} = \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 A} \Rightarrow \frac{1}{C'} = \frac{d_1 + d_2}{\varepsilon_0 A} \Rightarrow \frac{1}{C'} = \frac{d}{\varepsilon_0 A} \Rightarrow C' = \frac{\varepsilon_0 A}{d}$...(7).

It is seen from (1) and (7) that $C = C'$...(8).

In this context, each of the option is being analyzed –

Option (a): Capacitance of the modified system after insertion of thin metallic sheet remains unchanged and potential difference across the capacitor remains at V charge on capacitor would remain unchanged at $Q = CV$. Thus, **option (a) is incorrect.**

Option (b): From (8) it is seen that capacitance remains unchanged and hence **option (b) is incorrect.**

Option (c): Since battery connected across the capacitor is not changed and hence potential difference across the capacitor would remain unchanged. Thus, **(option (c) is incorrect.**

Option (d): Modified arrangement is a series combination of two capacitors and hence equal and opposite charges will develop on wo faces of thin metal sheet, and is shown in the figure. Hence, **option (d) is correct.**

Thus, **answer is option (d).**

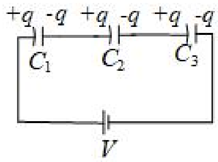
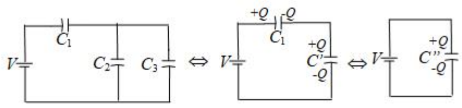
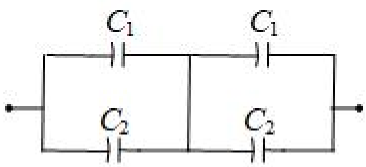
I-26

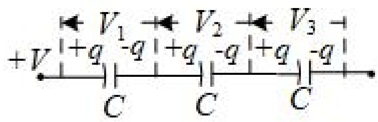
Each of the given option is being analyzed in context of sequence of operation –

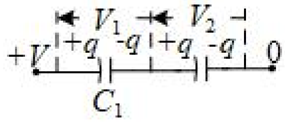
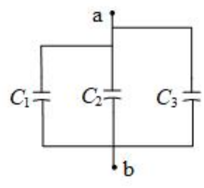
Option (a): A capacitor of capacitance say C prior to connection to a battery of emf E, has zero potential difference across it i.e. $V = 0$. Energy stored in the capacitor is $U = \frac{1}{2}CV^2$...(1). Thus initial

	energy stored in the capacitor is $U_0 = 0$.(2). Next in operation X, $V \rightarrow E$ and thus $U_1 = \frac{1}{2}CE^2$…(3). When operation Y is performed there no change in voltage across the capacitor and hence $U_3 = \frac{1}{2}CE^2$…(4). Now, when operation Z is performed, due to reversal of polarity of the battery, first capacitor will discharge and then will be recharged corresponding to reverse polarity at $U_4 = \frac{1}{2}C(-E)^2 \Rightarrow U_4 = \frac{1}{2}CE^2$…(5). Thus, in sequence of operations it is seen that U changes- **(i)** from 0 to $U_1 = \frac{1}{2}CE^2$; **(ii)** Remains at U_1 **(iii)** Changes from U_1 to zero and then recovers to U_1. Thus, changes in energy stored in capacitor makes **option (a) incorrect** **Option (b):** Charge on capacitor after operation X is $Q_X = CE$…(6). In the following operation W, battery remains connected and capacitance increases to $C' = \varepsilon_r C$ and thus charge becomes $Q_W = C'E = \varepsilon_r CE$…(7). On action Y following W increased charge Q_W continues to be held by the capacitor. While, operation Y after X, disconnection of battery, retains charge Q_X as per (6) on the capacitor. And operation W following it, changes capacitance $C \rightarrow C'$ which in turn affects only voltage across the capacitor. But, on account of disconnected battery, the charge on the capacitor remains at $Q_X = CE$. Comparing remnant charge $Q_W = \varepsilon_r CE$ in X-W-Y sequence of operations the remnant charge $Q_X = CE$ in X-Y-W sequence of operations and that $\varepsilon_r > 1$, in all certainty $Q_W > Q_X$. Thus, **option (b) is correct**. **Option (c):** In sequence of operations W-X-Y capacitor in presence of dielectric, in operation W, increases to $C' = \varepsilon_r C$. Then in action X capacitor is charged by battery and as a result it stores energy $U' = \frac{1}{2}C'E^2$. Following it action Y retains energy $U' = \frac{1}{2}(\varepsilon_r C)E^2 = \frac{1}{2}\varepsilon_r CE^2$. Whereas in sequence X-Y-W capacitor stores energy in operation X , $U = \frac{1}{2}CE^2$, and in following operation, battery is disconnected. Therefore, the last action W, does not lead to any change in energy. Comparing $U' = \frac{1}{2}\varepsilon_r CE^2$ with $U = \frac{1}{2}CE^2$,dielectric constant $\varepsilon_r > 1$, we have $U' > U$. Thus, **option (c) is correct.** **Option (d):** Electric field inside capacitor is $E = \frac{V}{d}$, here V is potential difference applied across the capacitor, in the instant case it is battery voltage, and d is the separation between plates of capacitor. In this expression dielectric constant does not appear as long as battery emf remains the same, this is ensured by action X. Thus, sequence of actions X and W does not change electric field in the capacitor. Thus, **option (d) is correct.** Accordingly, **answer is options (b), (c) and (d)** **N.B.:** This is a multiple choice objective question and can be solved with clarity of concepts. A detailed illustration has been made to bridge gaps, if any, in understanding of concepts.
I-27	We know that between two conducting parts capacitance is $Q = CV \Rightarrow C = \frac{Q}{V}$…(1). Given that $N = 1.0 \times 10^{12}$electrons are transferred from one conductor to the other while charge of a electron in $e = 1.6 \times 10^{-19}$C. Thus, charge on capacitor formed by the two conductors is $Q = N \times e = (1.0 \times 10^{12}) \times 1.6 \times 10^{-19} \Rightarrow Q = 1.6 \times 10^{-7}$C. Further it is given that the transfer of charge develops a potential difference between two conductors $V = 10$ V. Using the available data in (1) $C = \frac{1.6\times10^{-7}}{10} = \mathbf{1.6 \times 10^{-8}}$ **F, is he answer**.
I-28	Capacitance of a parallel plate capacitors is $C = \frac{\varepsilon_0 A}{d}$…(1) The given capacitor is formed by circular plates $r = 5.0 \times 10^{-2}$ m separated by a distance $d = 1.00 \times 10^{-3}$m. Thus, here $A = \pi r^2$…(2). Combining (1) and

	(2), $C=\frac{\varepsilon_0\pi r^2}{d}$. Using available data with $\varepsilon_0=8.85\times10^{-12}$ we have $C=\frac{(8.85\times10^{-12})\times\pi\times(5.0\times10^{-2})^2}{1.00\times10^{-3}}=7.0\times10^{-11}$F or $\mathbf{7.0\times10^{-5}}$ **μF is the answer**.
I-29	Capacitance of a parallel plate capacitors is $C=\frac{\varepsilon_0 A}{d}$…(1) Given that capacitance of the capacitor to be constructed is $C=1.0$ F using circular discs of radius say r, to be determined, separated by a distance $d=1.00\times10^{-3}$m. Thus, here $A=\pi r^2$…(2). Combining (1) and (2), $C=\frac{\varepsilon_0\pi r^2}{d}\Rightarrow r=\sqrt{\frac{Cd}{\varepsilon_0\pi}}$. Using available data with $\varepsilon_0=8.85\times10^{-12}$ we have $r=\sqrt{\frac{1.0\times(1.00\times10^{-3})}{(8.85\times10^{-12})\times\pi}}=6.0\times10^{3}$m or $\mathbf{6.0}$ **km is the answer**.
I-30	Capacitance of a parallel plate capacitors is $C=\frac{\varepsilon_0 A}{d}$…(1), where area of the plates is $A=25\times10^{-4}$ m^2, with a separation $d=1.00\times10^{-3}$m. It is also given that voltage across the capacitor is connected to a battery is $V=6.0$ V. Thus, as desired charge on a capacitor is $Q=CV$…(2). and work done by battery in charging the capacitor is analyzed a little later. In first part combining (1) and (2), $Q=\left(\frac{\varepsilon_0 A}{d}\right)V=\frac{\varepsilon_0 AV}{d}$. Using the available data with $\varepsilon_0=8.85\times10^{-12}$ we have $Q=\frac{(8.85\times10^{-12})(25\times10^{-4})6}{1.00\times10^{-3}}=1.33\times\mathbf{10^{-10}}$ C, say $\mathbf{1.3\times10^{-10}}$ **C is the answer.** And in second part energy of the capacitor result of part (1) is used. In a capacitor when Δq charge is transferred when potential difference across the capacitor is V' then work done transferring the charge is $\Delta W=\Delta q\times V'$. Using (2) this transforms into $\Delta W=\Delta q\times\frac{q}{C}$, this equation variable V' is substituted in terms of constant C and the variable charge q under consideration. Accordingly, $W=\int dw=\int_0^Q\left(\frac{q}{C}\right)dq\Rightarrow W=\frac{1}{C}\int_0^Q qdq\Rightarrow W=\frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q\Rightarrow W=\frac{Q^2}{2C}=\frac{1}{2}QV$…(3). It is a general expression of energy stored in a capacitor. But, the problem statement is that charge is flown through the battery of 6.0 V into a capacitor and charge in the instant case is $Q=1.33\times10^{-10}$ C. Therefore, as per principle of electrostatics work done in flowing a unit charge through a potential difference V is $W'=V$. Therefore, for flowing a charge Q through a potential difference V work done by the battery is $W=QV$…(4). This flow of charge is equivalent to time integral of flow of current $Q=\int_{t_1}^{t_2}Idt$. Thus, work done in the instant case is not as per general expression (3), instead it is as per case specific expression (4). Accordingly, W= $(1.33\times10^{-10})\times6=\mathbf{8.0\times10^{-10}}$ **J, is the answer**. Thus, **answers are** $\mathbf{1.33\times10^{-10}}$ **C,** $\mathbf{8.0\times10^{-10}}$ **J.** **N.B.: 1.** Precision of the problem statement is extremely important, and makes difference in solution. In high level competitions such linguistic twists are used to create a difference. It is excellently demonstrated in this problem. This made it essential to bring out a detailed illustration. **2.** As per principles of significant digits' intermediate results are retained until end and rounding to SDs is done at the last stage. This is becoming clear while using value of C in calculating U, while rounding to SDs is don for reporting both the results.
I-31	Capacitance of a parallel plate capacitors is $C=\frac{\varepsilon_0 A}{d}$…(1), where area of the plates is $A=25\times10^{-4}$ m^2, with a separation $d=2.00\times10^{-3}$m. It is also given that voltage across the capacitor is connected to a battery is $V=12.0$ V. Charge on a capacitor is $Q=CV$…(2). Thus combining (1) and (2) $Q=\left(\frac{\varepsilon_0 A}{d}\right)V$. Using the available data with $\varepsilon_0=8.85\times10^{-12}$ we have $Q=\frac{(8.85\times10^{-12})(25\times10^{-4})\times12}{2.00\times10^{-3}}=1.33\times\mathbf{10^{-10}}$ C, say $\mathbf{1.3\times10^{-10}}$ **C is the answer of part (a).**

	Now, when separation is reduced to $d' = \frac{d}{2}$ then charge on capacitor would be $Q' = \frac{\varepsilon_0 AV}{d'} = \frac{2\varepsilon_0 AV}{d} \Rightarrow Q' = 2Q \Rightarrow Q' = 2(1.3 \times 10^{-10}) = 2.6 \times 10^{-10}$C. Accordingly, extra charge given b the battery to the positive plate is $\Delta Q = Q' - Q = 2.6 \times 10^{-10} - 1.3 \times 10^{-10} \Rightarrow \Delta Q = \mathbf{1.3 \times 10^{-10}}$ **C is the answer of part (b).** Thus, **answers are (a) $\mathbf{1.33 \times 10^{-10}}$ C (b) $\mathbf{1.33 \times 10^{-10}}$C**
I-32	Given figure shows that all the three capacitors are connected in parallel, to the battery. Therefore, voltage across each capacitor is same. Further, charge on capacitor is $Q = CV$. Therefore, using the given data - $Q_1 = C_1V \Rightarrow Q_1 = (2.0 \times 10^{-6}) \times 12 = 24 \times 10^{-6}$C or **24 μC,** $Q_2 = C_2V \Rightarrow Q_2 = (4.0 \times 10^{-6}) \times 12 = 48 \times 10^{-6}$C or **48 μC,** $Q_3 = C_3V \Rightarrow Q_3 = (6.0 \times 10^{-6}) \times 12 = 72 \times 10^{-6}$C or **72 μC.** Thus, **answer is 24 μC, 48 μC, 72 μC.**
I-33	When capacitors are connected in series, their equivalent capacitance C is such that $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Thus using the given data $\frac{1}{C} = \frac{1}{20\times10^{-6}} + \frac{1}{30\times10^{-6}} + \frac{1}{40\times10^{-6}} = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \times \frac{1}{10\times10^{-6}} = \frac{6+4+3}{12} \times \frac{1}{10\times10^{-6}} \Rightarrow C = \frac{12}{13}(10 \times 10^{-6})$. Accordingly, charge on capacitors connected in series combination would be $Q = C \times V \Rightarrow Q = \frac{12}{13}(10 \times 10^{-6}) \times 12 = 110 \times 10^{-6} \Rightarrow Q$= **110 μC**, it is equal in all capacitors being connected in series, as shown in the figure.  Further, work done by the battery is equal to energy stored in the combination and it is $U = QV$. Using the available data, $U = (110 \times 10^{-6})12 \Rightarrow U = 1.33 \times 10^{-3}$ J. Thus**, answers are 110 μC on each, $\mathbf{1.33 \times 10^{-3}}$ J**
I-34	The combination of capacitors can be reduced to that shown in figure where $C' = C_2 + C_3$…(1) i.e. a parallel combination of C_2 and C_3 and further to $C"$ such that $\frac{1}{C"} = \frac{1}{C_1} + \frac{1}{C'}$…(2). Combing (1) and (2), $\frac{1}{C"} = \frac{1}{C_1} + \frac{1}{C_2+C_3}$ …(3). Charge on a capacitor is $Q = CV$..(4).  Using the given data in (3), $\frac{1}{C"} = \frac{1}{8\times10^{-6}} + \frac{1}{4\times10^{-6}+4\times10^{-6}} \Rightarrow \frac{1}{C"} = \frac{1}{8\times10^{-6}} + \frac{1}{8\times10^{-6}} = \frac{1}{4\times10^{-6}} \Rightarrow C" = 4 \times 10^{-6}$ F. Therefore, charge on $C_1$is that on the combination $C"$ is equal to $Q_1 = C" \times V = (4 \times 10^{-6}) \times 12 = 48 \times 10^{-6}$ C or $\boldsymbol{Q_1} = \mathbf{48}$ **μC**…(4) But, the charge on C_2 and C_3 would divide such that $Q_1 = Q_2 + Q_3$…(5)as under – Let V' is capacitance across parallel combination then charge on C_2 is $Q_2 = C_2V' \Rightarrow V' = \frac{Q_2}{C_2}$ …(6), and charge on C_3 is $Q_3 = C_3V' \Rightarrow V' = \frac{Q_3}{C_3}$ …(7). Combining (6) and (7), $\frac{Q_2}{C_2} = \frac{Q_3}{C_3} \Rightarrow \frac{Q_2}{Q_3} = \frac{C_2}{C_3}$…(8). Applying componendo to (7), $\frac{Q_2+Q_3}{Q_3} = \frac{C_2+C_3}{C_3}$…(8). Combining (5) and (8), $Q_3 = \left(\frac{C_3}{C_2+C_3}\right)\left(\frac{1}{Q_2+Q_3}\right) \Rightarrow Q_3 = \left(\frac{C_3}{C_2+C_3}\right) \times \frac{1}{Q_1}$. Since given that $C_2 = C_3$ the charge Q_1 would equally divide between the two and thus, the available data $Q_2 = Q_3 = \mathbf{24}$ **μC.** Thus, **answer is 48 μC of 8 μF capacitor and 24 μC on each of the 4 μF capacitors.**  (a)

I-35	Given combination (a) is series combination of two C' while C' is a parallel combinations of C_1 and C_2 such that $C' = C_1 + C_2$. Using the available data $C' = 4.0 \times 10^{-6} + 6.0 \times 10^{-6} \Rightarrow C' = 10.0 \times 10^{-6}$F. Thus, $\frac{1}{C_a} = \frac{1}{C'} + \frac{1}{C'} = \frac{2}{C'}$. Accordingly, $C_a = \frac{C'}{2} = \frac{10.0\times10^{-6}}{2} \Rightarrow \boldsymbol{C_a = 5.0 \times 10^{-6}}$**F**. Further, combination in (b) is parallel combination of two C_a such that $C_b = C_a + C_a = 2C_a \Rightarrow C_b = 2 \times (5.0 \times 10^{-6}) \Rightarrow \boldsymbol{C_a = 10.0 \times 10^{-6}}$**F.** Thus, **answers are (a) 5 μF (b) 10 μF**
I-36	A close examination of the given circuit reveals that the battery voltage V is directly applied across each of C_1 and C_2. Thus, topologically both the capacitors are in parallel combination. Accordingly equivalent capacitance $C = C_1 + C_2$ and charge supplied by the battery is $Q = CV \Rightarrow Q = (C_1 + C_2)V$. Using the given data $Q = (5 \times 10^{-6} + 5 \times 10^{-6}) \times 10 \Rightarrow Q = 110 \times 10^{-6}$ C or **110 μC is the answer**,
I-37	A close examination of the given circuit reveals that the battery emf $V = 10$ V is directly applied across two cylindrical capacitors equal capacitance $C = 2.2 \times 10^{-6}$ F.. Thus, topologically both the capacitors are in parallel combination. Accordingly equivalent capacitance $C' = C + C = 2C$ and charge supplied by the battery is $Q = C'V \Rightarrow Q = 2CV$. Using the given data $Q = 2(2.2) \times 10 \Rightarrow Q = 44 \times 10^{-6}$ C or **44 μC is the answer**,
I-38	Electric potential due to a point charge Q at a distance R from it is $V = \frac{Q}{4\pi\varepsilon_0 R}$…(1). Given that two conducting sphere, which form equipotential surface, and has a geometrical symmetry. Thus expression (1) signifies electric potential at surface of a conducting sphere of radius R with charge Q on it. Since, $Q = CV \Rightarrow C = \frac{Q}{V}$ …(2). Accordingly, capacitance of conducting sphere using (1) and (2) is $C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 R}} \Rightarrow C = 4\pi\varepsilon_0 R$…(3). Using (3) individually capacitance of conducting spheres radii R_1 and R_2 are $C_1 = \boldsymbol{4\pi\varepsilon_0 R_1}$ and $C_2 = \boldsymbol{4\pi\varepsilon_0 R_2}$**.** Connecting the two sphere will create a parallel combination of C_1 and C_2 such that it will be a parallel of the two capacitors. Accordingly, capacitance of the arrangement would be combination would be $C' = C_1 + C_2 \Rightarrow C' = 4\pi\varepsilon_0 R_1 + 4\pi\varepsilon_0 R_2 \Rightarrow C' = \boldsymbol{4\pi\varepsilon_0(R_1 + R_2)}$**.** **Thus, answers are $\boldsymbol{4\pi\varepsilon_0 R_1}$, $\boldsymbol{4\pi\varepsilon_0 R_2}$, $\boldsymbol{4\pi\varepsilon_0(R_1 + R_2)}$**
I-39	With the given data that each of the capacitors in the figure have capacitance $C = 2 \times 10^{-6}$ F. that each branch of the combnation of capacitors three capacitors of capacitance C is a series combination. Thus, $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Rightarrow \frac{1}{C'} = \frac{3}{C} \Rightarrow C' = \frac{C}{3}$…(1) Further, the whole assembly of capacitors is a parallel combination of three cpacaitances C' where C' . Thus net capacitance of the combination is $C" = C' + C' + C' \Rightarrow C" = 3C' \Rightarrow C" = 3C'$ …(2). Combining (1) and (2), $C = 3 \times \frac{C}{3} \Rightarrow C" = C$…(3). Using the given data $C" = 2 \times 10^{-6}$ F or **2 μF.** Now, using identity symmetry across each banch of the capacitor $V_1 = V_2 = V_3 = QC$…(4) and total voltage across the cascaded capacitors is $V = V_1 + V_2 + V_3 \Rightarrow V = 3V_1 \Rightarrow V_1 = \frac{V}{3}$. Thus using the given data potential difference across each capacitor is $= \frac{60}{3} \Rightarrow V_1 = V_2 = V_3 = \mathbf{20}$ **V**.  Thus, **answers are 2 μF, 20 V/**
I-40	With the given data, number of capacitors of capacitance $C = 10 \times 10^{-6}$ and withstand voltage $V = 50$ V to be connected across a battery of emf E where $V < E$…(1), in series combination is $n_s = \frac{E}{V}$…(2). Here, n_s has to be essentially an integer, if not it is rounded to next higher integer. In the instant case $n_s = \frac{200}{50} = 4$, hence roundingis

	not required. Equivalent capacitance of a series combination of capacitors is $\frac{1}{C_s} = \sum_{i=1}^{n_s} \frac{1}{C_i}$…(3). Given that all capacitors have capacitance $C_i = C$, therefore, $\frac{1}{C_s} = \frac{n_s}{C} \Rightarrow C_s = \frac{C}{n_s} \Rightarrow C_s = \frac{C}{\frac{E}{V}} \Rightarrow C_s = \frac{CV}{E}$…(4). Now, to achieve a capacitance C the series combination having withstand capability E, as per (4), in the knowledge of inequality (1), a parallel combination of series combination is made such that $C_P = \sum_{k=1}^{n_p} C_{s_k}$ where capacitances of each parallel branch is equal. Accordingly, $C_P = kC_s \Rightarrow k = \frac{C_P}{C_s}$…(5). Combining (4) and (5), $k = \frac{C_P}{\frac{CV}{E}} \Rightarrow k = \frac{C_P E}{CV}$…(6). Since, objective is to achieve $C_P = C$, therefore, from (6) using the available data we have $k = \frac{CE}{CV} \Rightarrow k = \frac{200}{50} = 4$. Thus combination of capacitor will be four ($n_s = 4$) of the capacitor in series, and four ($k = 4$) parallel combination of the series combination, requiring total number of capacitor $N = n_s \times k = 4 \times 4 = 16$ number of the given capacitors. Thus, **answer is series-parallel combination of $4 \times 4 = 16$ capacitors as given.**
I-41	A close observation of the combination of capacitors reveal that the two parallel branches have two cascaded capacitors having ratio of their capacitances $\frac{C_1}{C_2} = \frac{4\times10^{-6}}{8\times10^{-6}} = \frac{3\times10^{-6}}{6\times10^{-6}} \Rightarrow k = \frac{C_1}{C_2} = \frac{1}{2}$…(1)  Voltage V_1 and V_2 across each of the capacitors C_1 and C_2, connected in series, in the branches is $V_1 = qC_1$…(2), $V_2 = qC_2$ …(3) and $V = V_1 + V_2 \Rightarrow V = q(C_1 + C_2)$…(4). Combining (2) and (4), $q = \frac{V_1}{C_1} = \frac{V}{C_1+C_2} \Rightarrow V_1 = \left(\frac{C_1}{C_1+C_2}\right)V \Rightarrow V_1 = \left(\frac{\frac{C_1}{C_2}}{1+\frac{C_1}{C_2}}\right)V$. . Using the available data we have $V_1 = \left(\frac{\frac{1}{2}}{1+\frac{1}{2}}\right)50 \Rightarrow V_1 = \frac{\mathbf{50}}{\mathbf{3}}$ **V is answer of part (a).** Further, in light of the given data, and (1), $V_C = V_D$ therefore, potential difference between points C and D is $V_{C-D} = V_C = V_D = 0$. Therefore, if a capacitor of any capacitance say C is connected between the two points charge on the capacitor would be $q = C \times V_{C-D} = C \times 0 \Rightarrow q =$ **0 is answer of part (b).** Thus, **answers are (a) $\frac{50}{3}$ μV at each point, (b) Zero.**
I-42	A close observation of the topology of the connection of the capacitors that upper three branches of capacitors are connected in parallel between points a and b. Out of this upper and lower branches have a series combination of capacitors C_1 and C_2 such that equivalent capacitance of the series combinations is $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1 C_2}{C_1 + C_2}$ …(1). Thus, eventually it is a parallel combination of two equivalent capacitors of capacitance C' and a capacitor of capacitance C_3. Thus equivalent capacitance between the points is $C = C' + C_3 + C' \Rightarrow C = C_3 + 2C'$…(2). Combining (2) and (3) we have $C = C_3 + 2\frac{C_1 C_2}{C_1 + C_2} \Rightarrow C = \boldsymbol{C_3 + \frac{2C_1C_2}{C_1 + C_2}}$ **is the answer.**
I-43	Given is the case of three capacitors C_1, C_2 and C_3 each separation between the plates d, $(d + b)$ and $(d + 2b)$ respectively. But, since depth and with of each horizontal plate (in the ladder forming capacitance with the bottom plate) will have equal area $A' = \frac{A}{3}$ …(1) Thus, capacitance of each of the capacitor is $C_1 = \frac{\varepsilon_0 A'}{d} \Rightarrow C_1 = \frac{\varepsilon_0 A}{3d}$…(2), $C_2 = \frac{\varepsilon_0 A}{3(d+b)}$ …(3), and $C_3 = \frac{\varepsilon_0 A}{3(d+2b)}$…(4).  The horizontal plate is continuous, while the vertical plates are connected with riser between the two steps. Thus, conceptually it is a parallel combination of three capacitors C_1, C_2 and C_3, between points a and b as shown in the figure, such that $C = C_1 + C_2 + C_3$ …(5).

	Combing (2), (3), (4) and (5), $C=\frac{\varepsilon_0 A}{3d}+\frac{\varepsilon_0 A}{3(d+b)}+\frac{\varepsilon_0 A}{3(d+2b)} \Rightarrow C=\frac{\varepsilon_0 A}{3}\left(\frac{1}{d}+\frac{1}{d+b}+\frac{1}{d+2b}\right) \Rightarrow C=\frac{\varepsilon_0 A}{3}\left(\frac{(d+b)(d+2b)+d(d+2b)+d(d+b)}{d(d+b)(d+2b)}\right)$. It solves into $C=\frac{\varepsilon_0 A}{3}\left[\frac{(d^2+3db+2b^2)+(d^2+2db)+(d^2+db)}{d(d+b)(d+2b)}\right]$. It, further, solves into $C=\frac{\boldsymbol{\varepsilon_0 A(3d^2+6bd+4b^2)}}{\boldsymbol{3d(d+b)(d+2b)}}$ **is the answer.**
I-44	Capacitance of a cylindrical capacitor, using Gauss's law is $C=\frac{2\pi\varepsilon_0 l}{\log_e\frac{R_O}{R_i}}$...(1). Here, l is length of the capacitor, r_i is the radius of the inner cylinder and r_O is radius of the outer cylinder, such that $\frac{R_O}{R_i}=\frac{4\times10^{-3}}{2\times10^{-3}}=2$ Thus, using the available data with $\varepsilon_0=8.85\times10^{-12}$, we have $C=\frac{2\pi(8.85\times10^{-12})\times(10\times10^{-2})}{\log_e\frac{4\times10^{-3}}{2\times10^{-3}}}=\frac{1.79\times\pi\times10^{-12}}{\log_{10}2\times\log_e 10}=\frac{1.79\times\pi\times10^{-12}}{\left(\frac{\log_{10}2}{\log_{10}e}\right)} \Rightarrow C=8\times10^{-6}$F or **8 μF is the answer of the part (a).** As regards another cylindrical capacitor of same length and ration of $\frac{R_O'}{R_i'}=\frac{8\times10^{-3}}{4\times10^{-3}}=2$, as per (1) **capacitance would be same as in part (a).** Thus, **answers are (a) 8 μF (b) same as in (a)**
I-45	
I-46	With the switch open in the circuit given, having each of the capacitor of capacitance $C=5\times10^{-6}$F, it is a series combination of C' and C, where $C'=C+C \Rightarrow C'=2C$...(1) equivalent capacitance of a parallel combination of two capacitor of capacitance C. Thus, equivalent capacitance C'' of a series combination of C' and C is $\frac{1}{C''}=\frac{1}{C'}+\frac{1}{C} \Rightarrow \frac{1}{C''}=\frac{1}{2C}+\frac{1}{C} \Rightarrow C''=\frac{2}{3}C$. Therefore, charge on the system of capacitors is $Q_1=C''V \Rightarrow Q_1=\left(\frac{2}{3}C\right)V$. Using the available data $Q_1=\left(\frac{2}{3}\times5.0\times10^{-6}\right)\times50 \Rightarrow Q_1=\frac{500\times10^{-6}}{3}$C...(1). When switch is closed capacitor C in the combination is shorted and thus effect capacitance across the battery becomes C' therefore charge on the system becomes $Q_2=C'V \Rightarrow Q_2=(2C)V \Rightarrow Q_2=(2\times5.0\times10^{-6})\times50 \Rightarrow Q_2=500\times10^{-6}$C...(2). It is seen that $Q_2>Q_1$ hence $Q_2-Q_1=500\times10^{-6}-\frac{500\times10^{-6}}{3}=3.3\times10^{-4}$ C charge supplied by the battery flows from A to B. Thus, **answer is $\boldsymbol{3.3\times10^{-4}}$ C.**
I-47	An observation of the figure shows that a potential difference V is applied across the combination of two capacitors, connected in series, each of capacitance $C=0.04\times10^{-6}=4.0\times10^{-8}$F having surface area of one side of the plates $A=100\times10^{-4}$m². The potential difference would divide equally across the two capacitors such that potential difference across each $V'=\frac{V}{2}$...(1) The particle of mass $m=10\times10^{-6}$kg having a charge $q=-0.01\times10^{-6}=-1.0\times10^{-8}$ C is to be kept in equilibrium in one of the capacitors as shown in the figure. It is possible only when $\vec{F_e}+\vec{F_g}=0$...(2). Here, electric force is $\vec{F_e}=\vec{E}q$...(3), and gravitational force $\vec{F_g}=m\vec{g} \Rightarrow \vec{F_g}=-mg\hat{j}$...(4). Here, $\hat{j}$ is unit vector in upward direction. Combining (2), (3) and (4), $\vec{E}q=mg\hat{j}$. Using the available data and taking $g=10$ m/s², $\vec{E}=\frac{mg\hat{j}}{q} \Rightarrow \vec{E}=-\frac{(10\times10^{-6})\times10}{-1.0\times10^{-8}}\hat{j} \Rightarrow E\hat{E}=1.0\times10^{4}(-\hat{j}) \Rightarrow E=1.0\times10^{4}$...(5) Here, negative sign signifies electric field in downward direction and is in agreement with the positive polarity of the battery connected to upper plate of the capacitor.

	Now, electric field across a capacitor is $E = \frac{V'}{d}$…(6), while capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d} \Rightarrow d = \frac{\varepsilon_0 A}{C}$…(7). Combining (1), (6) and (7), $E = \frac{\frac{V}{2}}{\frac{\varepsilon_0 A}{C}} \Rightarrow E = \frac{VC}{2\varepsilon_0 A}$ …(8). Using the available data with $\varepsilon_0 = 8.85 \times 10^{-12}$,we have $E = \frac{V(4.0\times10^{-8})}{2(8.85\times10^{-12})(100\times10^{-4})} \Rightarrow E = (2.26 \times 10^5)V$…(9). Comparing (5) and (9), $(2.26 \times 10^5)V = 1.0 \times 10^4 \Rightarrow V = 44 \times 10^{-3}$V or **44 mV is the answer**. **N.B.:** Choice of value of g slightly affects numerical value of answer.
I-48	The problem involves multiple concepts- (a) Division of potential difference V applied across the series combination into potential difference $V_1 = \left(\frac{C_2}{C_1+C_2}\right)V$ …(1) is applied across the upper capacitor; (b) Electric field within upper capacitor $E_1 = \frac{V_1}{d_1} \Rightarrow E_1 = \frac{\left(\frac{C_2}{C_1+C_2}\right)V}{d_1} \Rightarrow E_1 = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right)$ …(2); (c) Electric force on the electron when inside the capacitor $F_1 = E_1 q_e \Rightarrow F_1 = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right)q_e$…(3); (d) Acceleration of the electron perpendicular to its projected velocity $a' = \frac{F_1}{m_e} \Rightarrow a' = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right)\left(\frac{q_e}{m_e}\right)$…(4); it is given that only electric force is to be considered and hence gravitational force is ignored; (e) Time (t) taken by electron to sweep across the side of capacitor plate a , given in the problem, with its horizontal projected velocity v, along the central line of the capacitor shown in the figure, is $t = \frac{a}{v}$…(5); (f) Time taken by the projected electron to just reach the upper plate as per second equation of motion $\frac{d_1}{2} = 0 \times t + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{d_1}{a'}} \Rightarrow t = \sqrt{d_1\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{d_1}{V}\right)\left(\frac{m_e}{q_e}\right)} \Rightarrow t = d_1 \times \sqrt{\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{1}{V}\right)\left(\frac{m_e}{q_e}\right)}$… (6) (g) In order to satisfy the condition that electron projected along the central line of the capacitor does not collide with the upper plate, time in (5) and (6) must be equal, such that- $\frac{a}{v} = d_1 \times \sqrt{\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{1}{V}\right)\left(\frac{m_e}{q_e}\right)} \Rightarrow v = \sqrt{\left(\frac{a}{d_1}\right)^2\left(\frac{C_2}{C_1+C_2}\right)} \times \sqrt{V\left(\frac{q_e}{m_e}\right)}$…(7). (h) Here, $C_1 = \frac{\varepsilon_0 a^2}{d_1}$ and $C_2 = \frac{\varepsilon_0 a^2}{d_2}$ hence, $\frac{C_2}{C_1+C_2} = \frac{\frac{\varepsilon_0 a^2}{d_2}}{\frac{\varepsilon_0 a^2}{d_1}+\frac{\varepsilon_0 a^2}{d_2}} = \frac{d_1}{d_1+d_2}$…(8). Combining (7) and (8), $v = \sqrt{\left(\frac{a}{d_1}\right)^2\left(\frac{d_1}{d_1+d_2}\right)} \times \sqrt{V\left(\frac{q_e}{m_e}\right)} \Rightarrow v = \sqrt{\frac{\boldsymbol{Vq_e a^2}}{\boldsymbol{m_e d_1(d_1+d_2)}}}$ **is the answer**. **N.B.:** This is a good example of integration of multiple concepts.
I-49	When a pair of electron and proton are released from some point between the plates of a capacitor having separation $d = 2.00 \times 10^{-2}$m , at a distance say x from a negative plate. Therefore, distance of the pair from positive plates is $x' = d - x$…(1) Charge of electron and proton are equal and opposite say $(-e)$ and $(+e)$respectively. Say potential difference between the plates of the capacitor be V then electric field in the inter-spacing between plates of the capacitor would be $E = \frac{V}{d}$…(2) Therefore, both electron and proton will experience equal and opposite forces of magnitude $F = eE \Rightarrow F = \frac{eV}{d}$ …(3), but their accelerations would be $a_e = \frac{F}{m_e} = \frac{\frac{eV}{d}}{m_e} \Rightarrow a_e = \frac{eV}{dm_e}$…(4) and likewise $a_p = \frac{eV}{dm_p}$…(5), respectively. Due to opposite charges, proton would travel distance x towards negative plate in time t while, as per problem statement, in the same time electron would travel a distance x'reach positive plate.

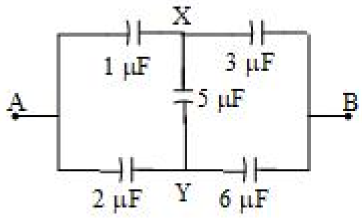
As per Second Equation of Motion, for electron, $x' = d - x = \frac{1}{2}a_e t^2 \Rightarrow t = \sqrt{\frac{2(d-x)}{a_e}} \Rightarrow t = \sqrt{\frac{2(d-x)}{\frac{eV}{dm_e}}} \Rightarrow t = \sqrt{\frac{2(d-x)dm_e}{eV}}$…(6). Likewise, for proton $t = \sqrt{\frac{2xdm_p}{eV}} \Rightarrow t = \sqrt{\frac{2xdm_p}{eV}}$…(7).

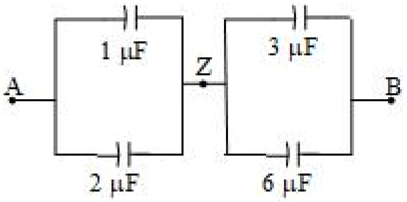
Comparing (6) and (7), $\sqrt{\frac{2xdm_p}{eV}} = \sqrt{\frac{2(d-x)dm_e}{eV}} \Rightarrow xm_p = (d-x)m_e \Rightarrow \frac{d-x}{x} = \frac{m_p}{m_e} \Rightarrow \frac{d}{x} = \frac{m_p}{m_e} + 1 \Rightarrow x = \frac{d}{\frac{m_p}{m_e}+1}$…(8). We know that $\frac{m_p}{m_e} = 1836$, accordingly using the available data $x = \frac{2.00\times10^{-2}}{1+1836} \Rightarrow x = \frac{2.00\times10^{-2}}{1837} =$ 1.09×10^{-5}m or **1.09×10^{-3} cm is the answer**.

N.B.: This a very good example of delaying numerical calculations till last stage where problem reduces to simple division. Mostly problems in physics are of this type and more require proficiency in handling algebraic formulations.

I-50

Topologically each of the three combinations are identical. And therefore any of the topology can be used for determine capacitance between points A and B. Taking (a), simpler to analyze, ratio of capacitance in upper and lower series combinations $\frac{1}{3} = \frac{2}{6} = k$. Therefore, if potential difference V is applied at A w.r.t. B, then potential at X and Y w.r.t. B would be $V_X = \frac{1}{1+k}V \Rightarrow V_X = \frac{1}{1+\frac{1}{3}}V \Rightarrow V_X = \frac{3}{4}V$. Likewise, potential at Y would be $V_Y = \frac{1}{1+\frac{1}{3}}V \Rightarrow V_Y = \frac{3}{4}V$. Thus, potential difference across points X and Y would be $\Delta V_{X-Y} = V_X - V_Y \Rightarrow \Delta V_{X-Y} = \frac{3}{4}V - \frac{3}{4}V = 0$. Thus, eventually capacitor is a shorted with its equivalent combination as shown in the figure.





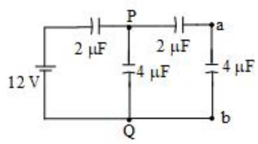
This is a pair of parallel combination between A-Z such that $C_1 = 1 \times 10^{-6} + 1 \times 10^{-6} \Rightarrow C_1 = 3 \times 10^{-6}$ and between Z and B such that $C_2 = 3 \times 10^{-6} + 6 \times 10^{-6} \Rightarrow C_2 = 9 \times 10^{-6}$.

This pair in turn is connected in series with an equivalent capacitance $C = \frac{C_1C_2}{C_1+C_2}$. Using the available data $C = \frac{(3\times10^{-6})(9\times10^{-6})}{3\times10^{-6}+9\times10^{-6}} \Rightarrow C =$ 2.25×10^{-6}F or 2.25 μF. Using the principle of significant digits $C =$ **2 μF is the answer.**

I-51

Solving each part separately to find $\Delta V = V_a - V_b$.

Part (a): The given connection of capacitors can be taken as a series combination of capacitance of 2 μF and the combination between P-Q.



The combination between P-Q is a parallel combination whose equivalent capacitance is $C' = 4 \times 10^{-6} + \frac{(2\times10^{-6})(4\times10^{-6})}{(2\times10^{-6})+(4\times10^{-6})}$

$\Rightarrow C' = 4 \times 10^{-6} + \frac{4}{3} \times 10^{-6} \Rightarrow C' = \frac{16}{3} \times 10^{-6} \Rightarrow C' = 5.33 \times 10^{-6}$F.

Thus voltage across P-Q would be $V' = \frac{2\times10^{-6}}{2\times10^{-6}+C'} \times 12 \Rightarrow V' = \frac{2\times10^{-6}}{2\times10^{-6}+5.33\times10^{-6}} \times 12 =$ $3.27\frac{9}{2}$Volts.

This voltage V' would further divide in series combination of capacitor containing points a-b. Therefore, voltage across a-b as desired would be $V_{ab} = \frac{2\times10^{-6}}{2\times10^{-6}+4\times10^{-6}} \times 3.27 \Rightarrow V_{ab} =$ **1.09 V.**

Part (b): In this circuit two batteries of equal emf are connected with their polarities reversed and thus they would cancel each other. Thus it is equivalent to both capacitors connected in parallel between points a and b, but without a battery. Therefore, the desired potential difference $V_{ab} =$ **0.**

Part (c): Symmetry of the circuit the circuit provides a clue that point **a** between two batteries of equal emf will be at mid potential of the potential difference across combination of batteries i.e. $V_a = 2$V.

Likewise, point **b** in the combination of the two equal capacitors connected in series across the combination of batteries and hence $V_b = \frac{2+2}{2} \Rightarrow V_b = 2$ V. Thus, potential difference between points a and b would be $V_{ab} = V_a - V_b = 2 - 2 \Rightarrow V_{ab} = \mathbf{0}$

Part (d): This combination of capacitors and batteries is a network having three branches connected in parallel between points a and b. Let potential difference between the points is V_{ab} and potential difference across a capacitor is $V = \frac{Q}{C}$. Accordingly, equations for three parallel branches are as under –

Upper Branch: $V_{ab} = 6 - \frac{Q_1}{4\times10^{-6}} \Rightarrow Q_1 = (6 - V_{ab})4 \times 10^{-6}$ …(1)

Middle Branch: $V_{ab} = 12 - \frac{Q_2}{2\times10^{-6}} \Rightarrow Q_2 = (12 - V_{ab})2 \times 10^{-6}$ …(2)

Lower Branch: $V_{ab} = 24 - \frac{Q_3}{1\times10^{-6}} \Rightarrow Q_3 = (24 - V_{ab})1 \times 10^{-6}$ …(3)

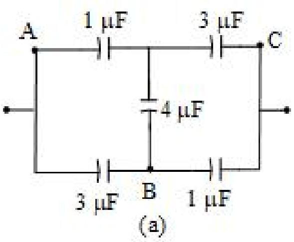
Further, it is an isolated system and hence, as per principle of electrical neutrality, net charge at on set of capacitors would remain zero such that $Q_1 + Q_2 + 4Q_3 = 0$…(4). Now we have four variables $V_{ab}, Q_1, Q_2,$ and Q_3 and a set of four independent linear equations.

Combining (1), (2), (3) and (4), $Q_1 + Q_2 + 4Q_3 = 0 = \big((6 - V_{ab})4 + (12 - V_{ab})2 + (24 - V_{ab})\big) \times 10^{-6}$

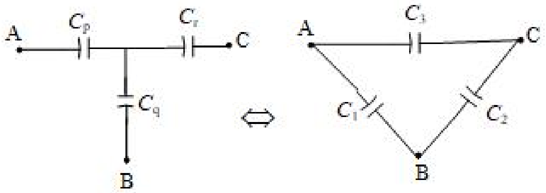
$\Rightarrow 72 - 7V_{ab} = 0 \Rightarrow V_{ab} = \frac{72}{7} = 10.3$V. Since all batteries have their positive polarity towards point b, hence, $V_a - V_b = -V_{ab} \Rightarrow V_a - V_b = \mathbf{-10.3V}$ **is the answer**.

Thus, **answers are (a) $\mathbf{1.09}$ V (b) Zero (c) Zero (d) -10.3 V**

I-52 This set of network involve star-delta connection of capacitances. Such questions can be easily solved with star-delta equivalent to be used interchangeably to simplify given network into series-parallel combination of capacitors. Taking each network separately –



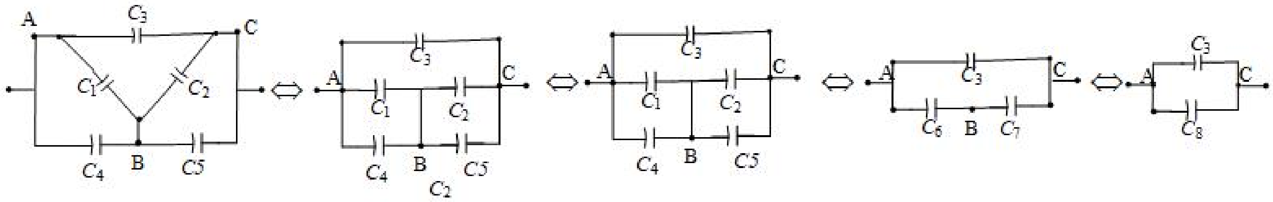
Network (a): In this case A, B, C nodes marked in the figure is seen as star connected capacitor where $C_p = 1 \times 10^{-6}$ F. $C_q = 4 \times 10^{-6}$ and $C_r = 3 \times 10^{-6}$.



This star connection is shown in the figure and is required to be converted into an equivalent delta connection of capacitors C_1, C_2, and C_3. As derived in Appendix $C_1 = \frac{C_p C_q}{C_p + C_q + C_r}$….(1), $C_2 = \frac{C_q C_r}{C_p + C_q + C_r}$ …(2), $C_3 = \frac{C_r C_p}{C_p + C_q + C_r}$ …(3).

Using the data, $C_1 = \frac{(1\times10^{-6})(4\times10^{-6})}{(1\times10^{-6})+(4\times10^{-6})+(3\times10^{-6})} \Rightarrow C_1 = \frac{(1\times10^{-6})(4\times10^{-6})}{(8\times10^{-6})} \Rightarrow, C_1 = 0.5 \times 10^{-6}$F….(4). Likewise, $C_2 = \frac{(4\times10^{-6})(3\times10^{-6})}{(8\times10^{-6})} \Rightarrow C_2 = 1.5 \times 10^{-6}$F…(5), and $C_3 = \frac{(3\times10^{-6})(1\times10^{-6})}{(8\times10^{-6})} \Rightarrow C_3 = 0.375 \times 10^{-6}$F…(6).

Using values in (4), (5) and (6), the network is reconstructed and reduced in stages to a simple parallel combination $C_2||C_3$, as shown in the figure. In this simplification algebraic values, from the given data are $C_4 = 3 \times 10^{-6}$ and $C_5 = 1 \times 10^{-6}$/



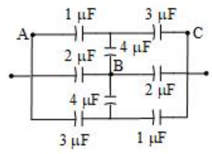
Accordingly, $C_6 = C_1||C_4 \Rightarrow C_6 = 0.5 \times 10^{-6} + 3 \times 10^{-6} \Rightarrow C_6 = 3.5 \times 10^{-6}$F.

Likewise, $C_7 = C_2||C_5 \Rightarrow C_7 = 1.5 \times 10^{-6} + 1 \times 10^{-6} \Rightarrow C_7 = 2.5 \times 10^{-6}$F.

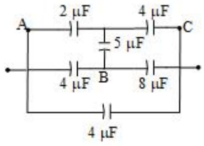
And series combination of C_6 and C_7 is $C_8 = \frac{C_6 \times C_7}{C_6 + C_7} \Rightarrow C_8 = \frac{(3.5\times10^{-6})\times(2.5\times10^{-6})}{(3.5\times10^{-6})+(2.5\times10^{-6})} \Rightarrow C_8 = 1.46 \times 10^{-6}$.

The last stage is , $C = C_3||C_8 \Rightarrow C = C_3 + C_8 \Rightarrow C = C_3 + C_8 \Rightarrow C = 0.375 \times 10^{-6} + 1.46 \times 10^{-6} \Rightarrow C = 1.83 \times 10^{-6}$ F or $1.83 = \frac{11}{6}$ **μF is the answer of part (a).**

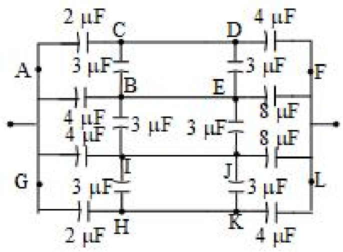
In a similar way networks in (b), (c) and (d) can be analyzed with clues as given below -



Network (b): In first stage determine $Y - \Delta$ equivalent of capacitors connected between A, B, C nodes. Then follow stage-wise simplification as done in network (a).



Network (c):): In first stage determine $Y - \Delta$ equivalent of capacitors connected between A, B, C nodes. Then follow stage-wise simplification as done in network (a).



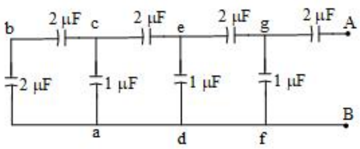
Network (d): In first stage split each of the 6 **μF** into a parallel combination of two capacitors of 3 **μF.** In next stage determine $\Delta - Y$ equivalent of capacitors connected across group of nodes (A, B, C), (D, E, F), (G, H, I) and (J, K, L), nodes. Then follow stage-wise simplification as done in network (a).

N.B.: 1. A generic conversion of a delta-star conversion of capacitor connections has been derived in Appendix, It may add clarity, and has been used in the problem as per need. These formula need not be derived in solution, instead. they can be used directly with care to designate nodes correctly. This simplifies the solution in a stage wise manner.

2. In such problems, determining equivalent capacitance at each stage, simplifies solving of long algebraic expressions, rather leaving them for last stage, as generally advised.

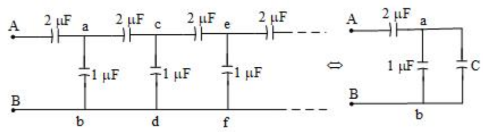
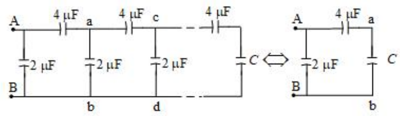
3. Proficiency in solving such networks grows patience, which increases with practice.

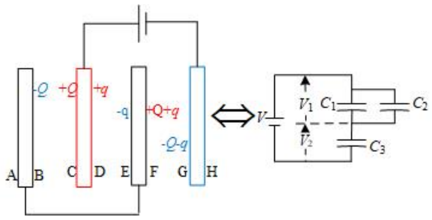
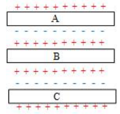
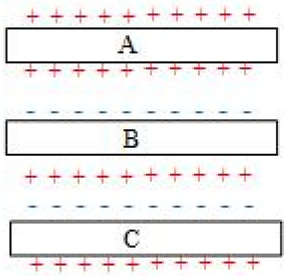
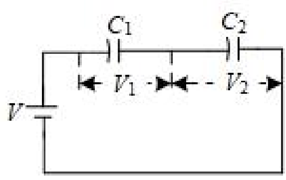
I-53



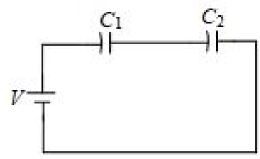
It is a case of capacitors connected in a formation of finite ladder where in first step two capacitor of 2 μF each between nodes a-b and b-c are connected in series. Thus its equivalent capacitance is $C = \frac{2\times2}{2+2} = 1$ μF. This C is connected in parallel to the capacitance of 1 μF between nodes a-c and thus net capacitance between nodes a-c is 2 μF. Likewise, progressively, net capacitance between nodes e-d and f-g is also 2 μF.

Finally, between nodes A-B two capacitors of 2 μF each are connected in series such that net capacitance is $= \frac{2\times2}{2+2} =$ **1 μF is the answer.**

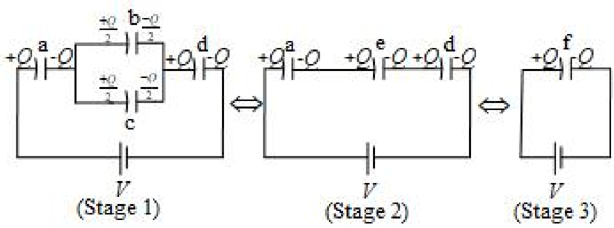
I-54	It is a case of infinite ladder where nodes have been marked in the figure. It is observed that each step is identical. Accordingly, effective capacitance following each step is taken to be C. Thus an equivalent capacitance of the ladder between nodes A-B would be $C_{AB} = C$. An equivalent network of capacitors' ladder is shown in the figure.  Capacitance between a-b is a parallel combination such that its equivalent capacitance is $C_{ab} = (1 + C)$ μF. This C_{ab} forms a series combination to give net capacitance $C_{AB} = \frac{2\times(1+C)}{2+(1+C)} \Rightarrow C \times (C + 3) = 2 + 2C \Rightarrow C^2 + C - 2 = 0$. Factorizing the equation $C^2 + (2 - 1)C - 2 = 0 \Rightarrow (C + 2)(C - 1) = 0$. This leads to two possible values, one of them is $C = -2$ μF which is not possible since capacitance is a scalar quantity and hence it is always positive, hence this value is ignored. The other value is $C = 1$ μF is realistic. Therefore, **answer is 1 μF.**
I-55	It is a case of infinite ladder of capacitors which is terminated at a capacitor of capacitance C. It is required to find value of C such that net capacitance of the ladder between nodes A and B is independent of C.  A close observation of capacitance of each element in the ladder is identical. Therefore, equivalent capacitance of elements connected in the ladder would remain unchanged. Accordingly, an equivalent network is drawn where net capacitance between A and B remains same at $C_{AB} = C$. From the equivalent circuit that ladder shown in figure $C = 2 + \frac{4\times C}{4+C} \Rightarrow (C + 4)(C - 2) = 4C$. It solves into $C^2 - 2C - 8 = 0 \Rightarrow (C - 4)(C + 2) = 0$. Thus possible values of $C = -2$μF which is not possible since capacitance is a scalar quantity and hence it is always positive, hence this value is ignored. The other value is $C = 4$ μF is realistic. Therefore, **answer is 4 μF.**
I-56	A capacitor having $Q_p = +Q$ on its positive plate and $Q_n = -Q$ on its negative plate then charge on capacitor is $Q_c = \frac{Q_p - Q_n}{2} \Rightarrow Q_c = \frac{(+Q)-(-Q)}{2} \Rightarrow Q_c = \frac{2Q}{2} \Rightarrow Q_c = Q$. It is given that charges on the two plates are unequal such that $Q_p = +2.0 \times 10^{-8}$ C and $Q_n = -1.0 \times 10^{-8}$C. Therefore, $Q_c = \frac{(+2.0\times10^{-8})-(-1.0\times10^{-8})}{2} \Rightarrow Q_c = 1.5 \times 10^{-8}$C. Given that capacitance of the capacitor is $C = 1.2 \times 10^{-9}$F. Therefore, potential difference between the plates with $Q_c = CV \Rightarrow V = \frac{Q_c}{C} = \frac{1.5\times10^{-8}}{1.2\times10^{-8}} \Rightarrow V = \mathbf{12.5}$ **V is the answer.**
I-57	A capacitor having $Q_p = +Q$ on its positive plate and $Q_n = -Q$ on its negative plate then charge on capacitor is $Q_c = \frac{\lvert Q_p - Q_n \rvert}{2}$…(1). Given that charge on positive plate of an isolated capacitor of capacitance $C = 10 \times 10^{-6}$ F is $Q_p = 20 \times 10^{-6}$C. There is no mention of charge given to the negative plate of the capacitor and hence implies that $Q_n = 0$. Thus using the available data $Q_c = \frac{20\times10^{-6}-0}{2} \Rightarrow Q_c = 10 \times 10^{-6}$. Further, potential difference between the plates with $Q_c = CV \Rightarrow V = \frac{Q_c}{C} = \frac{10\times10^{-6}}{10\times10^{-6}} \Rightarrow V =$ **1 V is the answer.**
I-58	A capacitor having $Q_p = +Q$ on its positive plate and $Q_n = -Q$ on its negative plate then charge on capacitor is $Q_c = \frac{\lvert Q_p - Q_n \rvert}{2}$…(1). Given that charge on positive plate of an isolated capacitor of capacitance $C = 0.1 \times 10^{-6}$ F is $Q_p = 1 \times 10^{-6}$C. There is no mention of charge given to the negative plate of the capacitor and hence implies that $Q_n = 2 \times 10^{-6}$C. Thus using the available data $Q_c = \frac{\lvert 1\times10^{-6}-2\times10^{-6} \rvert}{2} \Rightarrow Q_c = 0.5 \times 10^{-6}$. Further, potential difference between the plates with $Q_c = CV \Rightarrow V = \frac{Q_c}{C} = \frac{0.5\times10^{-6}}{0.1\times10^{-6}} \Rightarrow V =$ **5 V is the answer.**

I-59	Given that each capacitor has surface area on one side $A = \frac{96}{\varepsilon_0} \times 10^{-12}$ Fm and separation $d = 4.00 \times 10^{-3}$m. Capacitance of a capacitor is $C = \frac{\varepsilon_o A}{d}$...(1). A battery of emf 10 Volts is connected in the manner shown in the figure. The plates having surfaces A-B and E-F are connected and hence at same potential. Surface B and C form a capacitor C_1 likewise surface D and E form a capacitor C_2 and surface F and G forms capacitor C_2. Each of the capacitor is of capacitance C as given. Distribution of charges based on the principle of electrical neutrality on surfaces B, C, D, E, F and G is shown in the figure while surfaces A and are charge free. Accordingly, topologically equivalent network is shown in the figure. Net capacitance of the system is $C' = \frac{2C \times C}{2C + C} = \frac{2}{3}C$...(2).  Combining (1) and (2), with the available data $C' = \frac{2}{3}\left(\frac{\varepsilon_o A}{d}\right) \Rightarrow C' = \frac{2}{3}\left(\frac{\varepsilon_o \times \left(\frac{96}{\varepsilon_0} \times 10^{-12}\right)}{4.00 \times 10^{-3}}\right) \Rightarrow C' = 16 \times 10^{-6}$ F...(3). Therefore, charge supplied by the battery is $Q + q = Q' = C'V \Rightarrow Q' = (16 \times 10^{-9}) \times 10 \Rightarrow Q' = 160 \times 10^{-9}$C or **0.16 μC is the answer**.
I-60	Given that capacitance between plates A and B is $C = 50 \times 10^{-9}$ F and same is between B and C. Given system of capacitors is isolated as shown in the figure and symmetrical to the middle plate B. Further, it is given that a charge $Q = 1.0 \times 10^{-6}$C is placed on the middle plate B. This due to symmetry would divide on both the surfaces of plate B such that $Q' = \frac{Q}{2} \Rightarrow Q' = \frac{1.0 \times 10^{-6}}{2} = 0.50 \times 10^{-6}$. Therefore, inner surface of A will have induced charge $Q'' = -Q' \Rightarrow Q'' = -0.50 \times 10^{-6}$C.  Based on principle of electrical neutrality of an isolated system complementary charge on outer surface of the upper plate A will be $= -Q'' = -(-0.50 \times 10^{-6}) = 0.50 \times 10^{-6}$ C or **0.50 μC is answer of part (a)** Further, as per capacitor equation $Q = CV \Rightarrow V = \frac{Q''}{C}$. Thus using the available data $V = \frac{0.50 \times 10^{-6}}{50 \times 10^{-9}} =$ **10 V is answer of part (b)** **Thus, answers are (a) 0.50 μC (b) 10 V**
I-61	The three plates of the given system are named A, B and C. As given a charge $q = 1.0 \times 10^{-6}$C is placed on the upper plate A which will equally distribute on its two faces of plate A each having a charge $q' = 0.5 \times 10^{-6}$ C. As a consequence induced charges equal to q' on each face of the plates B and C are shown in the figure.  Thus charges of capacitor formed by plates A and B and that by plates B and C are $q' = 0.5 \times 10^{-6}$. Given that capacitance of these two capacitors are $c = 50 \times 10^{-9}$F. Voltage across capacitor is $Q = CV \Rightarrow V = \frac{Q}{C}$, Accordingly with the given data $V = \frac{q'}{C} = \frac{0.5 \times 10^{-6}}{50 \times 10^{-9}} \Rightarrow V = 10$V. Thus**, answer of both the parts is (a) 10 V, (b) 10 V.**
I-62	Given that the two capacitors of capacitance $C_1 = 20.0 \times 10^{-12}$F and $C_2 = 50.0 \times 10^{-12}$F are connected in series. Therefore, equivalent capacitance of the series combination is $C = \frac{C_1 C_2}{C_1 + C_2}$...(1). Using the given data, $C = \frac{(20.0 \times 10^{-12})(50.0 \times 10^{-12})}{20.0 \times 10^{-12} + 50.0 \times 10^{-12}} \Rightarrow C = 14.3 \times 10^{-12}$F.  Further, on each capacitor, based on the principle of electrical neutrality would be $Q = CV$...(2), as shown in the figure. Accordingly, in this case $V_1 = \frac{Q}{C_1}$...(3). Combining, (1), (2) and (3) with the given $V = 6.00$ V, voltage across capacitor $C_1$is $V_1 = \frac{\left(\frac{C_1 C_2}{C_1 + C_2}\right)V}{C_1} \Rightarrow V_1 = \frac{C_2 V}{C_1 + C_2} \Rightarrow V_1 = \frac{(50.0 \times 10^{-12}) \times 6.00}{20.0 \times 10^{-12} + 50.0 \times 10^{-12}} = \frac{50.0 \times 6.00}{70.0} = \mathbf{4.29}$

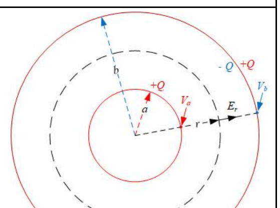
	V. Likewise, voltage across capacitor $C_2$is $V_2=\frac{\left(\frac{C_1C_2}{C_1+C_2}\right)V}{C_2}\Rightarrow V_2=\frac{C_1V}{C_1+C_2}\Rightarrow V_2=\frac{(20.0\times10^{-12})\times6.00}{20.0\times10^{-12}+50.0\times10^{-12}}=\frac{50.0\times6.00}{70.0}=\mathbf{1.71V.}$ Further, energy supplied by the battery in charging capacitors is $E'=CV^2$ out of which energy supplied stored to the capacitor is $E"=\frac{1}{2}CV^2$ accordingly energy supplied to the capacitor C_1 is $E_1^"=\frac{1}{2}C_1V_1^2$. Using the available data $E_1^"=\frac{1}{2}(20.0\times10^{-12})(4.29)^2\Rightarrow E_1^"=$ **184 pJ**. Likewise, to the capacitor C_2 is $E_2^"=\frac{1}{2}C_2V_2^2$. Accordingly,$E_2^"=\frac{1}{2}(50.0\times10^{-12})(1.71)^2\Rightarrow E_1^"=\mathbf{73.1}$ **pJ**. Thus, **answers are (a) 4.29 V, 1.71 V (b) 184 pJ, 73.1 pJ**
I-63	Given that the two capacitors of capacitance $C_1=4.0\times10^{-6}$F and $C_2=6.0\times10^{-6}$F are connected in series. Therefore, equivalent capacitance of the series combination is $C=\frac{C_1C_2}{C_1+C_2}$…(1). Using the given data, $C=\frac{(4.0\times10^{-6})(6.0\times10^{-6})}{4.0\times10^{-6}+6.0\times10^{-6}}\Rightarrow$ $C=2.4\times10^{-6}$F.

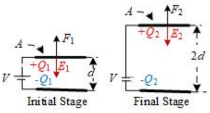


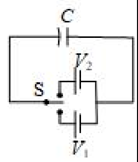
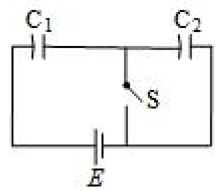
	Further, energy supplied by the battery in charging capacitors is $E'=CV^2$ out of which energy supplied stored to the capacitor is $E''=\frac{1}{2}CV^2$. Accordingly, using the available data energy supplied by the series combination of capacitors is $E'=CV^2\Rightarrow E'=CV^2\Rightarrow(2.4\times10^{-6})\times20^2=960\times10^{-6}$J or **960 μJ is the answer.**
I-64	The given circuit considering capacitors each of capacitance $C=10\times10^{-6}$F, the charge on each capacitor plate, based on the principle of electrical neutrality is shown in figure (Stage 1). Further, this circuit has been simplified for analysis in its equivalents in Stage 2 and Stage 3.

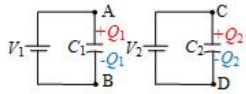
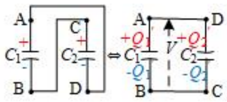
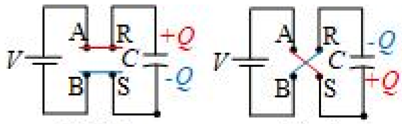


	Capacitor e in Stage 2 is equivalent of a parallel combination of capacitors b and c has capacitance $C_2=2C$. Stage 3 is equivalent of series combination of capacitors a, e and d in Stage 2 with an equivalent capacitor of capacitance C_3 such that $\frac{1}{C_3}=\frac{1}{C}+\frac{1}{2C}+\frac{1}{C}\Rightarrow\frac{1}{C_3}=\frac{5}{2C}\Rightarrow C_3=\frac{2C}{5}$. Thus charge Q on the capacitors, as shown in the figure and with the available data is $Q=C_3V\Rightarrow Q=\frac{2(10\times10^{-6})}{5}\times100=\frac{2}{5}\times10^{-3}$ C. Energy stored in a capacitor is $E'=\frac{1}{2}CV^2=\frac{Q^2}{2C}$. Accordingly, **energy stored in capacitors a and d is** $E_1'=\frac{\left(\frac{2}{5}\times10^{-3}\right)^2}{2(10\times10^{-6})}=\frac{1}{125}=8\times10^{-3}$ J or **8 mJ**. Likewise, **energy stored in equal capacitors b and c**, connected in parallel, is $E_2'=\frac{\left(\frac{Q}{2}\right)^2}{2C}=\frac{\left(\frac{1}{2}\left(\frac{2}{5}\times10^{-3}\right)\right)^2}{2(10\times10^{-6})}=\frac{1}{500}\Rightarrow2\times10^{-3}$J or **2 mJ**, Thus, answers are 8 mJ in (a) and (d), 2 mJ in (b) and (c)..
I-65	Let two equal capacitors are of capacitance C. One of them has energy stored $E'=4.0$J. Energy stored in capacitor is $E'=\frac{1}{2}QV$. Since $Q=CV\Rightarrow V=\frac{Q}{C}\Rightarrow E'=\frac{Q^2}{2C}$…(1) When this charged capacitor is connected to another uncharged capacitor makes sense when both are in parallel. Thus charge on each capacitor in new formation would be equally shared such that $Q'=\frac{Q}{2}$…(2). Thus, combining (1) and (2), energy stored in each capacitor would be $E=\frac{\left(\frac{Q}{2}\right)^2}{2C}=\frac{1}{4}\left(\frac{Q^2}{2C}\right)=\frac{E'}{4}=\frac{4.0}{4}\Rightarrow E=1.0$J.

	Accordingly energy stored in the two capacitors is $= 2E" = 2 \times 1.0 = \mathbf{2.0}$ **J is the answer**.
I-66	Charge on capacitor of capacitance $C_1 = 2.0 \times 10^{-6}$ F when charge to a potential difference $V = 12$ V is $Q_1 = C_1 V$…(1) $\Rightarrow Q_1 = (2.0 \times 10^{-6}) \times 12 = 24 \times 10^{-6}$ F. When it is connected to an uncharged capacitor of capacitance $C_2 = 4.0 \times 10^{-6}$, there will be charge sharing across the two capacitor until both the capacitors are at a common potential difference V' such that charges on the two capacitors C_1 and C_2 would be $Q_1' = C_1 V'$ and $Q_2' = C_2 V'$, respectively. As shown in the figure, in the problem, it is an isolated system, and therefore it would conserve charge such that $Q_1 = Q_1' + Q_1'$…(2). Accordingly, combining (1) and (2) with the available data $(2.0 \times 10^{-6}) \times 12 = (2.0 \times 10^{-6}) \times V' + (4.0 \times 10^{-6}) \times V'$, we have $V' = \frac{(2.0\times10^{-6})\times12}{2.0\times10^{-6}+4.0\times10^{-6}} = 4$V. With this basics taking each part separately – **Part (a):** Charge on capacitor $C_1$is $Q_1' = C_1 V' \Rightarrow Q_1' = (2.0 \times 10^{-6}) \times 4 = 8.0 \times 10^{-6}$ or **8.0 μC.** Likewise, charge on capacitor $C_1$is $Q_1' = (4.0 \times 10^{-6}) \times 4 = 16.0 \times 10^{-6}$ or **16.0 μC**. **Part (b):** Energy stored in capacitor in capacitor $C_1$is $E_1' = \frac{1}{2} C_1 V'^2 \Rightarrow E_1' = \frac{1}{2} \times (2.0 \times 10^{-6}) \times 4^2 = 16.0 \times 10^{-6}$ or 16 **μJ.** Likewise, charge on capacitor C_2 is $E_1' = \frac{1}{2} C_2 V'^2 \Rightarrow E_1' = \frac{1}{2} \times (4.0 \times 10^{-6}) \times 4^2 = 32.0 \times 10^{-6}$or **32 μJ**. **Part (c):** Energy initially stored in the capacitor $C_1$is $E_1" = \frac{1}{2} C_1 V^2 \Rightarrow E_1' = \frac{1}{2} \times (2.0 \times 10^{-6}) \times 12^2 = 144.0 \times 10^{-6}$ or 144 **μJ.** While energy stored in the combination of the two capacitors, using analysis in part (b) is $E' = E_1' + E_2' \Rightarrow E' = 16 + 32 = 48$ **μJ**. Therefore, The heat produced during the charge transfer from one capacitor to the other is the difference of energy $E_H = E_1' - E' \Rightarrow 144 - 48 = 96$ **μJ.** **Thus, answers are (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ.**
I-67	When a charge Q is placed at the origin of a sphere an outside sphere of radius *R*, potential of the sphere is $V = \frac{Q}{4\pi\varepsilon_0 R}$. Thus energy stored in the sphere is $E' = \frac{1}{2} QV \Rightarrow E' = \frac{1}{2} Q \left(\frac{Q}{4\pi\varepsilon_0 R}\right) = \boldsymbol{\frac{Q^2}{8\pi\varepsilon_0 R}}$**, is the answer**.
I-68	 When sphere of radius $a = R$ is charged to a potential $V_a = V$ relation between charge $+Q$ on it and its potential is given by $V_a = \frac{Q}{4\pi\varepsilon_0 a} \Rightarrow Q = 4\pi\varepsilon_0 a V_a \Rightarrow Q = 4\pi\varepsilon_0 RV$…(1). With another concentric sphere of radius $b = 2R$ its inner surface will have an induced charge $-Q$, while charge on the outer sphere would be $+Q$. Thus potential of outer surface would be $V_b = \frac{Q}{4\pi\varepsilon_0 b} \Rightarrow Q = 4\pi\varepsilon_0 b V_b$…(2) These two sphere form a capacitor having a charge Q and a potential difference $V = V_a - V_b \Rightarrow V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b} \Rightarrow V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 2R} \Rightarrow V = \frac{Q}{8\pi\varepsilon_0 R}$…(3) Energy charged in a capacitor is $E' = \frac{1}{2} QV'$. Accordingly combining (1) and (3), we have $E' = \frac{1}{2}(4\pi\varepsilon_0 RV)\frac{(4\pi\varepsilon_0 RV)}{8\pi\varepsilon_0 R} \Rightarrow E' = \boldsymbol{\pi\varepsilon_0 RV^2}$ **is the answer of part (a).** As regards part (b), energy stored in electrostatic field outside sphere of radius $b = 2R$ is $E" = \frac{1}{2} QV_b$, while work done on the system is $E"' = QV_b$ Combining (1) and (2) $E" = \frac{1}{2} QV_b \Rightarrow E" = \frac{1}{2}(4\pi\varepsilon_0 RV)\left(\frac{Q}{4\pi\varepsilon_0 2R}\right) = \frac{VQ}{4} \Rightarrow E" = \frac{V}{4}(4\pi\varepsilon_0 RV) = \boldsymbol{\pi\varepsilon_0 RV^2}$, it is the same as that inside the two spheres at part (a). **Thus, contention in part (b) is proved.**
I-69	Given that a large conducting plane has uniform charge density $\rho = 1.0 \times 10^{-4}$ C/m². This will create a uniform electric field perpendicular to the surface of the sphere. Accordingly as per Gauss's Law $\oint \vec{E}.d\vec{s} =$

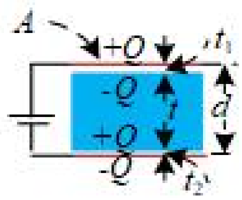
	$\frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{\varepsilon_0 A} \Rightarrow E = \frac{\rho}{\varepsilon_0}$. Here, $q = \rho A$ and $A = a^2$, where $a = 1 \times 10^{-2}$ is the edge of the cubical volume of side a. Therefore, energy stored in the cubical volume of side is work done in moving q against $\vec{E}$ through a distance a is $E' = \frac{1}{2} \times E \times q \times a \Rightarrow$ Accordingly, $E' = \frac{\rho}{\varepsilon_0} \times (\rho A) \times a \Rightarrow E' = \frac{\rho^2}{\varepsilon_0} \times (aA) \Rightarrow E' = \frac{\rho^2}{\varepsilon_0} \times (aA) = \frac{\rho^2}{\varepsilon_0} a^3$, here a^3 is the volume of the cubical space. Using the available data with $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $E' = \frac{1}{2} \times \frac{(1.0\times10^{-4})^2}{8.85\times10^{-12}} \times (1 \times 10^{-2})^3 \Rightarrow E' = \frac{10^{-2}}{2\times8.85} = \mathbf{5.6 \times 10^{-4}}$**J is the answer**.
I-70	Given system of parallel-plate capacitors having plate area $A = 20 \times 10^{-4}\text{m}^2$ having a separation $d = 1.00 \times 10^{-3}$ m. Therefore, initial capacitance of the capacitor is$C_i = \frac{\varepsilon_0 A}{d}$ …(1). Electric field between the plates is $E_i = \frac{V}{d}$…(2) Holding the capacitor in this initial positon requires a restraining force $F_i = E_i \times Q_i$ …(3), as shown in the figure, here $Q_i = C_i \times V$…(4). Combing (1), (2), (3) and (4) initially energy stored in the capacitor is $E_i' = \frac{1}{2} \times C_i V_i^2 = \frac{1}{2} C_i V^2$, here $V_i = V$…(5) Likewise, energy stored in the capacitor to take it to final stage, is $E_f' = \frac{1}{2} \times C_f V_f^2 = \frac{1}{2} C_f V^2$, here $V_f = V$….(6). And Using these basic, taking each part separately. **Part (a):** Charge flown though the process of initial to final stage is $\Delta Q = Q_i - Q_f \Rightarrow \Delta Q = C_i \times V - C_f \times V \Rightarrow \Delta Q = (C_i - C_f)V \Rightarrow \Delta Q = \left(\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{2d}\right)V \Rightarrow \Delta Q = \frac{\varepsilon_0 AV}{2d}$. Using the available data and $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $\Delta Q = \frac{(8.85\times10^{-12})(20\times10^{-4})12.0}{2\times(1.00\times10^{-3})} \Rightarrow \Delta Q = \mathbf{1.06 \times 10^{-10}}$**C.** **Part (b):** How much energy is absorbed by the battery during the process is $E' = \frac{Q}{2}V$, here $\Delta Q = \frac{Q}{2}$ is the charge returned by the capacitor at a constant voltage V of the battery. Accordingly, using result of part (a) $E' = (1.06 \times 10^{-10}) \times 12 = \mathbf{12.7 \times 10^{-10}}$J. **Part (c):** Energy stored in the capacitor is initially is $E_i' = \frac{1}{2} \times C_i V_i^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2 = \mathbf{12.7 \times 10^{-10}}$ J, is same as that in part (b). But energy stored in the capacitor after the process is $E_f' = \frac{1}{2} \times C_f V_f^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{2d}\right)V^2 = \frac{12.7\times10^{-10}}{2} = \mathbf{6.35 \times 10^{-10}}$**J.** **Part (d):** It is desired to use the expression for the force between the plates to find the work done by the person pulling the plates apart. Accordingly, work done by the person in increasing the separation from d to $2d$ such that for every change in separation $\Delta W = F_x \Delta x = (E_x Q_x)\Delta x = \left(\left(\frac{V}{x}\right)(C_x V)\right)\Delta x \Rightarrow \Delta W = V^2\left(\frac{C_x}{x}\right)\Delta x \Rightarrow \Delta W = V^2\left(\frac{\frac{\varepsilon_0 A}{x}}{x}\right)\Delta x \Rightarrow \Delta W = \varepsilon_0 A V^2 \frac{1}{x^2}\Delta x$. It is to be noted charge on the capacitor and potential difference both are function of separation between the plates and eventually $\Delta W = f(x)\Delta x$. Accordingly, it would require integration of the function such that $W = \varepsilon_0 A V^2 \int_d^{2d} \frac{1}{x^2} dx \Rightarrow W = -\varepsilon_0 A V^2 \left[\frac{1}{x}\right]_d^{2d} \Rightarrow W = -\varepsilon_0 A V^2 \left[\frac{1}{2d} - \frac{1}{d}\right] \Rightarrow W = \frac{\varepsilon_0 A V^2}{2d}$. Using the available data $W = \frac{(8.85\times10^{-12})(20\times10^{-4})12^2}{2\times(1.00\times10^{-3})} = \mathbf{12.7 \times 10^{-10}}$ J **Part (e):** The amount of work done in the process [Answer of part (d)] is equal to amount of energy absorbed by the battery in the process [Answer of part (b)]. It is therefore concluded that no heat is produced in the process. **Answer is True.** **Thus, answers are (a)** $\mathbf{1.06 \times 10^{-10}}$ **C (b)** $\mathbf{12.7 \times 10^{-10}}$ **J (c)** $\mathbf{12.7 \times 10^{-10}}$ **J,** $\mathbf{6.35 \times 10^{-10}}$**J,** **(d)** $\mathbf{6.35 \times 10^{-10}}$ **J (e) True**

I-71	Given system is shown in the figure where ca capacitor of capacitance $C = 100 \times 10^{-6}$F is initially connected to a battery of $V_1 = 24$ V. Later, the battery V_1 is disconnected and capacitor is connected to another battery of $V_2 = 12$V such that capacitor is joined to positive terminal of the battery. It implies that polarity of the capacitor remains unchanged. With this each part of the problem is being illustrated.  **Part (a):** Charge on capacitor before connection $Q_1 = CV_1 \Rightarrow Q_1 = (100 \times 10^{-6}) \times 24 = 2400 \times 10^{-6}$C or **2400 μC**. And charge on it after reconnection is $Q_2 = CV_{21} \Rightarrow Q_2 = (100 \times 10^{-6}) \times 12 = 1200 \times 10^{-6}$C or **1200 μC**. **Part (b):** In process of reconnection reduction in charge on capacitor from $Q_1$to Q_2 has only one way is that through battery V_2. Thus charge flown through the battery is $Q = Q_1 - Q_2 = (2400 - 1200) \times 10^{-6} = 1200 \times 10^{-6}$ C or **1200 μC.** **Part (c):** Since charge is being returned to the battery V_2 and hence work is done on the battery of magnitude $E' = Q \times V_2 = (1200 \times 10^{-6}) \times 12 = 14.4 \times 10^{-3}$ or **14.4 mJ** **Part (d):** Electrostatic field energy is stored in the capacitor before disconnection is $E_1' = \frac{1}{2}CV_1^2$. Using the available data $E_1' = \frac{1}{2}(100 \times 10^{-6})24^2$J. Likewise, the energy after reconnection is $E_2' = \frac{1}{2}CV_2^2 \Rightarrow E_2' = \frac{1}{2}(100 \times 10^{-6})12^2$. Thus decrease in energy $\Delta E = E_1' - E_2' = \frac{1}{2}(100 \times 10^{-6})24^2 - \frac{1}{2}(100 \times 10^{-6})12^2 \Rightarrow \Delta E = (50 \times 10^{-6})(24^2 - 12^2) \Rightarrow \Delta E = 21.6 \times 10^{-3}$J or **21.6 mJ.** **Part (e):** Heat energy developed during the process is $H = \Delta E - E'$, where ΔE is the energy released by the capacitor determined in part (d) and E' , is the energy returned to the reconnected battery determined in part (c). Accordingly, and $H = 21.6 - 16.4 = \mathbf{7.2}$ **mJ is the answer**. **Thus, (a) 2400 μC, 1200 μC, (b) 1200 μC (c) 14.4 mJ, (d) 21.6 mJ, (e) 7.2 mJ**
I-72	Each of the part is being taken up progressively – **Part (a):** When switch open for a longtime the capacitors C₁ and C₂ of capacitance C connected in series with equivalent capacitance $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} \Rightarrow C' = \frac{C}{2}$ will be fully charged such that $Q' = C'E \Rightarrow Q' = \frac{C}{2}E = \frac{CE}{2}$**.** When the switch S is closed, the capacitor C_2 is short circuited and therefore charge on its two plates is mutually neutralized. But, the potential difference appearing exclusively on the capacitor C_1 develop charge $Q'' = CE$. Thus, the difference of the charge supplied by the battery is $\Delta Q = Q" - Q' = CE - \frac{CE}{2} \Rightarrow \Delta Q = \boldsymbol{\frac{CE}{2}}$  **Part (b):** Work done by the battery of emf E to supply the charge ΔQ is $W = \Delta QE$, using the result of part (a) $W = \left(\frac{CE}{2}\right)E = \boldsymbol{\frac{CE^2}{2}}$**.** **Part (c):** Energy stored in the capacitors, when switch S is open, is $E_i' = \frac{1}{2}C'E^2 = \frac{1}{2}\left(\frac{C}{2}\right)E^2 \Rightarrow E_i' = \frac{CE^2}{4}$. But, when switch S is closed energy stored in C_1 becomes $E_{1f}' = \frac{1}{2}CE^2$ and in capacitor C2, being short-circuited is $E_{2f}' = 0$. Thus, total energy on the capacitors in switched closed condition is $E_f' = E_{1f}' + E_{2f}' \Rightarrow E_f' = \frac{1}{2}CE^2 + 0 \Rightarrow E_f' = \frac{1}{2}CE^2$. Accordingly, change in energy stored in capacitors is $\Delta E' = E_f' - E_i' = \frac{1}{2}CE^2 - \frac{1}{4}CE^2 \Rightarrow \Delta E' = \boldsymbol{\frac{1}{4}CE^2}$. **Part (d):** Heat developed in the process is the energy dissipated and is arrived at in part (c) is equal to $\Delta E' = \boldsymbol{\frac{1}{4}CE^2}$**.** **Thus, answers are (a)** $\boldsymbol{\frac{CE}{2}}$ **(b)** $\boldsymbol{\frac{CE^2}{2}}$ **(c)** $\boldsymbol{\frac{CE^2}{4}}$ **(d)** $\boldsymbol{\frac{CE^2}{4}}$.
I-73	Given are two capacitors of capacitance $C_1 = 5.00 \times 10^{-6}$F and $C_2 = 6.00 \times 10^{-6}$ F charged to $V_1 = 24.0$ V and $V_2 = 12.0$ V are initially independent. With this each of the part is being taken up progressively –

	Part (a): Initially, energy stored in capacitor C₁ is $E_1' = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times(5.00\times10^{-6})(24.0)^2 \Rightarrow E_1' = 1.44\times10^{-3}$J or **1.44 mJ**. Likewise, $E_2' = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times(6.00\times10^{-6})(12.0)^2 \Rightarrow E_1' = 0.432\times10^{-3}$ or **0.432 mJ.**  **Part (b):** Capacitors charged independently at $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$ as shown in figure part (a) are now connected as shown in the figure here. It becomes a closed loop electrically a parallel combination of the capacitors. Thus charge Q on the combination is $Q = Q_1 - Q_2 = (5.00\times10^{-6})\times24 - (6.00\times10^{-6})\times12 = 48.0\times10^{-6}$ C. Let voltage across the parallel combination $C' = C_1 + C_2 = 11\times10^{-6}$ F is $V' = Q\times C' \Rightarrow V' = \frac{Q}{C'} = \frac{48.0\times10^{-6}}{11\times10^{-6}} = \frac{48}{11}$V.  Accordingly, charge on capacitor C_1 is $Q_1' = C_1V' = (5.00\times10^{-6})\times\frac{48}{11} = 21.8\times10^{-6}$C or $\mathbf{21.8}$ **μC.** Likewise, charge on capacitor C_2 is $Q_2' = C_2V' = (6.00\times10^{-6})\times\frac{48}{11} = 26.2\times10^{-6}$C or $\mathbf{26.2}$ **μC.** **Part (c):** Initial energy stored in both the capacitors, based on part (a) is $E' = E_1' + E_2' = 1.44 + 0.432 = 1.87$mJ. But, in new configuration $E'' = E_1'' + E_2'' = \frac{1}{2}\times Q_1'V' + \frac{1}{2}\times Q_2'V' = \frac{1}{2}\times QV$. Thus using the quantities arrived at above $E'' = \frac{1}{2}\times(48.0\times10^{-6})\times\frac{48}{11} = 0.105$ mJ. Accordingly, , loss of electrostatic energy is $\Delta E = E' - E'' = 1.87 - 0.105 = \mathbf{1.76}$ **mJ.** **Part (d):** The loss of energy in part (c) is **dissipated as heat**. **Thus, answers are (a) 1.44 mJ, 0.432 mJ (b) 21.8 μC, 26.2 μC (c) 1.76 mJ (d) Dissipated as heat** N.B.: Decimal points in the given data needs to watched carefully and used in reporting answers on the principle of SDs.
I-74	The given system with its initial and final connections is shown in the figure. Initial charge on the capacitor is $Q = CV = (5.0\times10^{-6})\times12 = 60\times10^{-6}$ C. But, when connections are reversed as given in the problem charge on plates in final connection is $Q' = C(-V) = (5.0\times10^{-6})\times(-12) = -60\times10^{-6}$C.  Initial Final The only source of charge is battery and thus charge that flows through battery is $\Delta Q = Q - Q' = 60\times10^{-6} - (-60\times10^{-6}) = 120\times10^{-6}$C. Accordingly, energy supplied by the battery, in the process, is $E' = \Delta Q\times V$. Using the available data $E' = (120\times10^{-6})\times12 = 1440\times10^{-6}$ or 1.44 mJ. Using the principle of the significant digits $E' = \mathbf{1.4}$ **mJ and it is dissipated as heat, is the answer.**
I-75	Given is a dielectric slab having two square faces having area $A = (0.2\times10^{-1})^2 = 4.0\times10^{-2}\text{m}^2$. The square faces of the dielectric having a separation $d = 1.0\times10^{-3}$m, The square faces of the slab having dielectric constant $K = 4.0$and are metal coated which makes it a capacitor of capacitance $C = \frac{\varepsilon_0KA}{d}$. Thus, using the available data $C = \frac{(8.85\times10^{-12})\times4.0\times(4.0\times10^{-2})}{1.0\times10^{-3}} = 1.416\times10^{-9}$F or **1.4 nF is the answer**, as per principle of SDs.
I-76	Given is a dielectric slab having two square faces having area $A = (0.2\times10^{-1})^2 = 4.0\times10^{-2}\text{m}^2$. The square faces of the dielectric having a separation $d = 1.0\times10^{-3}$m, The square faces of the slab having dielectric constant $K = 4.0$and are metal coated which makes it a capacitor of capacitance $C = \frac{\varepsilon_0KA}{d}$. Thus, using the available data $C = \frac{(8.85\times10^{-12})\times4.0\times(4.0\times10^{-2})}{1.0\times10^{-3}} \Rightarrow C = 1.416\times10^{-9}$F . The coated surfaces are connected to a battery $V = 6.0$ V. With this each part of the problem is solved as under – **Part (a):** Charge supplied by the battery in charging the capacitor to a potential difference V is $Q = CV$. Using the available data $Q = (1.416\times10^{-9})\times6 = 8.5\times10^{-9}$ or **8.5 nC**. **Part (b):** Charge induced on dielectric is $Q' = Q\left(1 - \frac{1}{K}\right) \Rightarrow Q' = (8.5\times10^{-9})\left(1 - \frac{1}{4}\right) \Rightarrow \mathbf{6.4}\times10^{-9}$ or **6.4 nC.** **Part (c):** Net charge appearing on one face of the coated surface is $Q'' = Q - Q' = 8.5 - 6.4 = \mathbf{2.1}$ **nC.** **Thus, answers are (a) 8.5 nC, (b) 6.4 nC, (c) 2.1 nC**

I-77

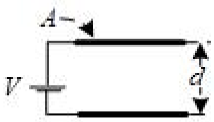
The given is shown in the figure having area $A = 100 \times 10^{-4}$ m^2 of the parallel- plates having a separation $d = 5.00 \times 10^{-3}$ m. The gap is filled with a metal sheet of thickness $t = 4.00 \times 10^{-2}$ m. The sheet is so placed that separation between its upper surface with the upper plate is t_1 and that between similar lower pair is t_2.



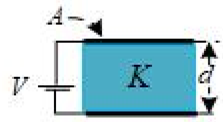
When the system is connected to a battery charges induced on plates of the capacitor and surface of the metal sheet facing it will form two capacitors such that $C_1 = \frac{\varepsilon_0 A}{t_1}$ and $C_2 = \frac{\varepsilon_0 A}{t_2}$. Since, charges in metal, being conductor resides on its surface and hence the metal sheet acts like a connection between series combination of the two capacitors such that its equivalent capacitance is $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C = \frac{\left(\frac{\varepsilon_0 A}{t_1}\right)\left(\frac{\varepsilon_0 A}{t_2}\right)}{\frac{\varepsilon_0 A}{t_1} + \frac{\varepsilon_0 A}{t_2}} \Rightarrow C = \frac{\varepsilon_0 A}{t_1 + t_2}$. Geometrically, $t_1 + t_2 = t - d$, accordingly $\boldsymbol{C = \frac{\varepsilon_0 A}{t - d}}$**, this is independents of $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$** i.e. where the metal sheet is placed within the gap between plates of the parallel-plate capacitor. Using the available data with $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $C = \frac{(8.85\times10^{-12})\times(100\times10^{-4})}{5.00\times10^{-2} - 4.00\times10^{-2}} = 885 \times 10^{-14} =$ **88 pF** in accordance with the principle of SDs.

I-78

Given that a capacitor of capacitance $C = 50 \times 10^{-6}$ F is connected to a batter with a potential difference V. When space between the plates of the capacitor is with a dielectric of dielectric constant K, a charge $\Delta Q = 100 \times 10^{-6}$C flows through the battery.



Initial Stage



Final Stage

Initially charge on the battery is $Q = CV$…(1). This is followed by filling the space between the plates of the capacitor $Q' = C'V$…(2). The final capacitance $C' = KC$…(3). Therefore, combining (1), (2) and (3), the change in charge on the battery is $\Delta Q = Q' - Q \Rightarrow \Delta Q = KCV - CV \Rightarrow \Delta Q = (K-1)CV \Rightarrow Q' - Q = (K-1)Q$…(4). Using the given data, $100 \times 10^{-6} = (K-1)50 \times 10^{-6} \Rightarrow 2 = K - 1 \Rightarrow$ $\boldsymbol{K = 3}$ **is the answer**.

I-79

Given is a parallel plate capacitor of capacitance $C = 5 \times 10^{-6}$F is connected to a battery with $V = 6$ V. Separation between the plates is $d = 2 \times 10^{-3}$m. With this each part of the problem is solved progressively-

Part (a): Charge on positive plate of the capacitor is $Q = CV \Rightarrow Q = (5 \times 10^{-6}) \times 6 = 30 \times 10^{-6}$ C=**30 μC.**

Part (b): Electric field between the plates is $E = \frac{V}{d} \Rightarrow E = \frac{6}{2\times10^{-3}} = \boldsymbol{3 \times 10^3}$ **V/m.**

Part (c): Gap between plates of the capacitor is filled with a dielectric of thickness $t = 1 \times 10^{-3}$m the capacitor $C = \frac{\varepsilon_0 A}{d} \Rightarrow \varepsilon_0 A = Cd$ is equivalent to series combination of two capacitor one with air occupying free space of thickness $t' = d - t$ having capacitance $C_1 = \frac{\varepsilon_0 A}{t'}$ and the other with dielectric of dielectric constant $k = 5$ having thickness t such that $C_2 = \frac{\varepsilon_0 KA}{t}$. Thus equivalent capacitance of the new configuration is $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C' = \frac{\left(\frac{\varepsilon_0 A}{d-t}\right)\left(\frac{\varepsilon_0 KA}{t}\right)}{\frac{\varepsilon_0 A}{d-t} + \frac{\varepsilon_0 KA}{t}}$. It solves into $C' = \frac{\varepsilon_0 AK}{t(1-K) + dK} \Rightarrow C' = \frac{CdK}{t(1-K) + dK}$. Using the available data $C' = \frac{(5\times10^{-6})(2\times10^{-3})5}{(1\times10^{-3})(1-5) + (2\times10^{-3})5} = \frac{50\times10^{-6}}{6} \Rightarrow C' = 8.3 \times 10^{-6}$F or **8.3 μF.**

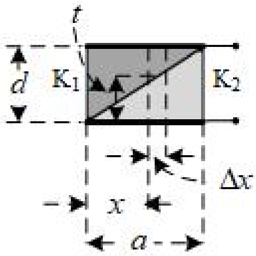
Part (d): Charge held by the capacitor in this new configuration is $Q' = C'V \Rightarrow Q' = (8.3 \times 10^{-6}) \times 6 = 49.8 \times 10^{-6}$F,or 49.8 μC . Thus additional charge supplied by the battery is $\Delta Q = Q' - Q = 49.8 - 30 = 19.8$ μC. Using principle of SDs answer is **20 μC.**

Thus answers are (a) 30 μC, (b) 3 × $\boldsymbol{10^3}$ V/m (c) 8.3 μF, (d)20 μC.

I-80

Given is a parallel plate capacitor of capacitance $A = 100 \times 10^{-4}$ m^2 with a separation between the plates $d = 1 \times 10^{-2}$m. The complete gap between the plates is filled with two dielectric materials having dielectric constants $K_1 = 6.0$ and $K_2 = 4.0$. This system of two capacitor C_1 and C_2 is equivalent to series combination of two capacitor of thickness $t_1 = 6.0 \times 10^{-3}$ filled with dielectric K_1 and the other capacitor of thickness $t_2 = d - t_1 = 4.0 \times 10^{-3}$ filled with dielectric K_2. Capacitance $C_1 = \frac{\varepsilon_0 K_1 A}{t_1} = \frac{(5.85\times10^{-12})\times(100\times10^{-4})\times6.0}{(6.0\times10^{-3})} \Rightarrow$

	$C_1 = 5.85 \times 10^{-13}$. Likewise, $C_2 = \frac{\varepsilon_0 K_2 A}{t_2} = \frac{(8.85\times10^{-12})\times(100\times10^{-4})\times4.0}{(4.0\times10^{-3})} \Rightarrow C_2 = 8.85 \times 10^{-11}$. It is seen that $C_1 = C_2$. Further, equivalent capacitance of the new configuration is $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1}{2}$. Accordingly, $C' = \frac{8.85\times10^{-11}}{2} = 4.4 \times 10^{-11}$ or **44 pF.**
I-81	Given is a parallel-plate capacitor of area $A = 400 \times 10^{-4}\text{m}^2$ and separation between the plates $d = 1.0 \times 10^{-3}\text{m}$ is connected to a supply of $V = 100$. A dielectric slab of thickness $t = 0.5 \times 10^{-3}\text{m}$ of dielectric constant $K = 5.0$ is inserted in the gap between the plates. Taking this information each part is solved progressively – **Part (a):** Initially energy stored in the capacitor is $E_1' = \frac{1}{2}CV^2$, here $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})\times(400\times10^{-4})}{1.0\times10^{-3}} = 35.4 \times 10^{-11}\text{F}$. When dielectric slab is inserted capacitance changes to C' which is series combination of two capacitors, one is C_1 having the dielectric slab of thickness t and dielectric constant K has capacitance $C_1 = \frac{\varepsilon_0 KA}{t}$ and other capacitor is C_2 having the dielectric slab of thickness $t' = d - t$ and dielectric constant K has capacitance $C_2 = \frac{\varepsilon_0 A}{d-t}$. Accordingly, $6.0 \times 0.5 \times 10^{-3}$equivalent capacitance of the combination is $C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\varepsilon_0 KA}{t}\right)\left(\frac{\varepsilon_0 A}{d-t}\right)}{\frac{\varepsilon_0 KA}{t} + \frac{\varepsilon_0 A}{d-t}} \Rightarrow C' = \frac{\varepsilon_0 KA}{K(d-t)+t}$. Using the available data $C' = \frac{(8.85\times10^{-12})\times5.0\times(400\times10^{-4})}{5.0(1.0\times10^{-3}-0.5\times10^{-3})+0.5\times10^{-3}} \Rightarrow C' = \frac{8.85\times20\times10^{-11}}{3.0} = 59.0 \times 10^{-11}\text{F}$. Energy stored in the capacitor after inserting the dielectric is $E_2' = \frac{1}{2}C'V^2$. Accordingly, increase in electrostatic energy is $\Delta E_{1-2} = E_2' - E_1' = \frac{1}{2}C'V^2 - \frac{1}{2}CV^2$. It solves into $\Delta E_{1-2} = \frac{1}{2}C'V^2 - \frac{1}{2}CV^2 = \frac{100^2}{2}(59.0 \times 10^{-11} - 35.4 \times 10^{-11}) = 1.18 \times 10^{-6}$ J or **1.18 μJ.** **Part (b):** Charge on capacitor when dielectric slab is inserted is $Q' = C'V = (59.0 \times 10^{-11}) \times 100 \Rightarrow Q' = = 59.0 \times 10^{-9}\text{C}$. Now battery is disconnected and then dielectric slab is taken out. In this situation charge remains trapped on capacitor, and capacitance of the capacitor changes to its initial value $C$and therefore electrostatic energy would be $E" = \frac{1}{2}Q'V = \frac{1}{2}Q'\left(\frac{Q'}{C}\right) = \frac{1}{2} \times \frac{Q'^2}{C} = \frac{(59.0\times10^{-9})^2}{2\times35.4\times10^{-11}} = 4.9 \times 10^{-6}$ J. While, $E_2' = \frac{100^2}{2}(59.0 \times 10^{-11}) = 2.95 \times 10^{-6}$ J. Thus, in this part removal of dielectric slab creates further increase in electrostatic energy in the capacitor by $\Delta E"=E"- E_2' = 4.9 \times 10^{-6} - 2.95 \times 10^{-6} \Rightarrow \Delta E"=1.95 \times 10^{-6}$J or **1.95 μJ.** **Part (c):** The increase in energy is due to **work done by external force** in removing the dielectric slab, this external force is against the electrostatic pull exerted on the dielectric. **Thus, answers are (a) 1.18 μJ (b) 1.95 μJ (c) Work done by external force in removing the dielectric.**
I-82	Filling of a gap between plates of a capacitor with dielectric parallel to the surface of the plates converts the capacitor into a combination of series capacitors with respective dielectric constant and thickness of the medium filling the gap. This is applicable to configuration (a) and (b). But, if dielectric is filled laterally then it becomes parallel combination of capacitors. With this capacitance each configuration is being derived – **Figure (a):** Here the capacitor is considered to be series combination of C_1 and C_2 filled with dielectrics of equal thickness $t = \frac{d}{2}$ of dielectric constant K_1 and K_2. Therefore, $C_1 = \frac{\varepsilon_0 K_1 A}{\frac{d}{2}} = \frac{2\varepsilon_0 K_1 A}{d}$ and $C_2 = \frac{\varepsilon_0 K_2 A}{\frac{d}{2}} = \frac{2\varepsilon_0 K_2 A}{d}$. These two capacitors forming series combination its capacitance is $C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{2\varepsilon_0 K_1 A}{d}\right)\left(\frac{2\varepsilon_0 K_2 A}{d}\right)}{\frac{2\varepsilon_0 K_1 A}{d} + \frac{2\varepsilon_0 K_2 A}{d}}$. It solves into $C = \frac{\left(\frac{2\varepsilon_0 K_1 A}{d}\right)\left(\frac{2\varepsilon_0 K_2 A}{d}\right)}{\frac{2\varepsilon_0 K_1 A}{d} + \frac{2\varepsilon_0 K_2 A}{d}} = \frac{2\varepsilon_0 K_1 K_2 A}{d(K_1 + K_2)}$. **Figure (b):** Here the capacitor is considered to be series combination of C_1 and C_2 filled with three different dielectrics of equal thickness $t = \frac{d}{3}$ of dielectric constant K_1, K_2 and K_3. Therefore, $C_1 = \frac{\varepsilon_0 K_1 A}{\frac{d}{3}} = \frac{3\varepsilon_0 K_1 A}{d}$. Likewise, $C_2 = \frac{3K_2 A}{d}$. And $C_3 = \frac{3K_3 A}{d}$These two capacitors forming series combination its

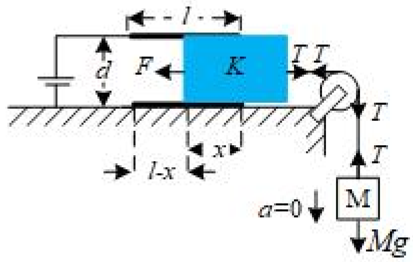
	capacitance is $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_1C_2+C_2C_3+C_3C_1}{C_1C_2C_3}$. Accordingly, it solves into $C=\frac{\left(\frac{3\varepsilon_0K_1A}{d}\right)\left(\frac{3\varepsilon_0K_2A}{d}\right)\left(\frac{3\varepsilon_0K_3A}{d}\right)}{\left(\frac{3\varepsilon_0K_1A}{d}\right)\left(\frac{3\varepsilon_0K_2A}{d}\right)+\left(\frac{3\varepsilon_0K_1A}{d}\right)\left(\frac{3\varepsilon_0K_2A}{d}\right)+\left(\frac{3\varepsilon_0K_1A}{d}\right)\left(\frac{3\varepsilon_0K_2A}{d}\right)} \Rightarrow C=\boldsymbol{\frac{3\varepsilon_0K_1K_2K_3A}{d(K_1K_2+K_2K_3+K_1K_3)}}$. **Figure (a):** Here the capacitor is considered to be series combination of C_1 and C_2 filled with dielectrics of equal thickness d and equal area $A_1=A_2=\frac{A}{2}$ dielectric constant K_1 and K_2. Therefore, $C_1=\frac{\varepsilon_0K_1\left(\frac{A}{2}\right)}{d}=\frac{\varepsilon_0K_1A}{2d}$, likewise, $C_2=\frac{\varepsilon_0K_2A}{2d}$. These two capacitors forming parallel combination and hence $C=C_1+C_2=\frac{\varepsilon_0K_1A}{2d}+\frac{\varepsilon_0K_2A}{2d}\Rightarrow C=\boldsymbol{\frac{\varepsilon_0A}{2d}(K_1+K_2)}$. **Thus, answers are (a)** $\boldsymbol{\frac{2K_1K_2\varepsilon_0A}{d(K_1+K_2)}}$ **(b)** $\boldsymbol{\frac{3AK_1K_2K_3\varepsilon_0}{d(K_1K_2+K_2K_3+K_3K_1)}}$ **(c)** $\boldsymbol{\frac{\varepsilon_0A}{2d}(K_1+K_2)}$
I-83	 The given system having linear variation in thickness of the two dielectrics filling the gap between the square plates, having edge a, of the capacitor can be considered as a parallel combination of thin slits of dielectric at distance x from left edge. The slit has width a and a thickness Δx. In this slit, geometrically, separation between plates filled by the dielectric K_2 is $t_2=\left(\frac{d}{a}\right)x=\frac{dx}{a}$ and that filled by dielectric K_2 is $t_1=d-t_2\Rightarrow t_1=d-\frac{dx}{a}\Rightarrow t_1=\frac{d(a-x)}{a}$. Thus capacitance of these two elemental capacitors is $\Delta C_{1x}=\frac{\varepsilon_0K_1A}{t_1}=\frac{\varepsilon_0K_1(a\Delta x)}{\frac{d(a-x)}{a}}\Rightarrow \Delta C_{1x}=\left(\frac{\varepsilon_0K_1a^2}{d}\right)\frac{\Delta x}{a-x}$. Likewise, $\Delta C_{2x}=\frac{\varepsilon_0K_2A}{t_2}=\frac{\varepsilon_0K_2(a\Delta x)}{\frac{dx}{a}}\Rightarrow \Delta C_{2x}=\left(\frac{\varepsilon_0K_2a^2}{d}\right)\frac{\Delta x}{x}$. These two capacitors forming series combination its capacitance2is $\Delta C_x=\frac{\Delta C_{1x}\Delta C_{2x}}{\Delta C_{1x}+\Delta C_{2x}}=\frac{\left(\left(\frac{\varepsilon_0K_1a^2}{d}\right)\frac{\Delta x}{a-x}\right)\left(\left(\frac{\varepsilon_0K_2a^2}{d}\right)\frac{\Delta x}{x}\right)}{\left(\frac{\varepsilon_0K_1a^2}{d}\right)\frac{\Delta x}{a-x}+\frac{2\varepsilon_0K_2a^2}{d}\frac{\Delta x}{x}}\Rightarrow \Delta C_x=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1x+K_2(a-x))}\right)\Delta x\Rightarrow \Delta C_x=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_2a+(K_1-K_2)x)}\right)\Delta x$. Thus, equivalent capacitance of the system is $C=\sum\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_2a+(K_1-K_2)x)}\right)\Delta x=\int_{x=0}^{a}\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_2a+(K_1-K_2)x)}\right)dx$. Here, it involves little integral calculus wherein a substitution $u=K_2a+(K_1-K_2)x\Rightarrow du=(K_1-K_2)dx\Rightarrow dx=\frac{du}{(K_1-K_2)}$ is made. Using these substitutions, $C=\int_{x=0}^{a}\left(\frac{\varepsilon_0K_1K_2a^2}{u}\right)\frac{du}{(K_1-K_2)}=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)\int_{x=0}^{a}\frac{1}{u}du\Rightarrow C=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)[\ln u]_{x=0}^{a}$. Making reverse substation to arrive results $C=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)[\ln(K_2a+(K_1-K_2)x)]_{x=0}^{a}$. This expression using the limits becomes $C=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)[\ln(K_2a+(K_1-K_2)a)-\ln(K_2a)]\Rightarrow C=\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)[\ln(K_1a)-\ln(K_2a)]$. In its final form $C=\boldsymbol{\left(\frac{\varepsilon_0K_1K_2a^2}{d(K_1-K_2)}\right)\ln\left(\frac{K_1}{K_2}\right)}$**.** **N.B.:** This problem is an excellent example of integration of concepts and higher mathematics. It needs a systematic mathematical formulation of the problem and solving it with patience.
I-84	Capacitance C of a parallel-plate capacitor changes to $C'=KC$ when the gap between the capacitor is filled with a dielectric of dielectric constant $K=3$. In the given system two identical capacitors A and B of capacitance say C are initially connected in parallel to a battery filled with the capacitors and therefore initially energy stored in each of the capacitors is $E_A'=E_B'=\frac{1}{2}QV=\frac{1}{2}(CV)V\Rightarrow E_A'=\frac{CV^2}{2}$. Total energy in the system is $E'=2E_A'=CV^2$. Next action is the switch is opened such that capacitor A remains connected to the battery while capacitor B is isolated. Thus, each of the capacitor will have a stored charge $Q=CV$. In this state a dielectric of dielectric constant $K=3$ fills the space between the plates, thus $C'=3C$. To this both the capacitors will perform differently as under –

	Capacitor A: It continues to be connected to the battery and thus with the insertion of dielectric energy stored in the capacitor would change to $E_A{}'' = \frac{C'V^2}{2} = \frac{KCV^2}{2}$. **Capacitor B:** It becomes isolated and charge on its stays at $Q = CV$. Therefore, with the insertion of dielectric, energy stored in the capacitor would be $E_B{}'' = \frac{1}{2}QV' = \frac{1}{2}Q\frac{Q}{C'} = \frac{Q^2}{2C'} \Rightarrow \frac{(CV)^2}{2KC} = \frac{CV^2}{2K}$. In final form total energy stored in the capacitor system in terms of known variables *V, C* and *K* is $K'' = E_A{}'' + E_B{}'' \Rightarrow K'' = \frac{KCV^2}{2} + \frac{CV^2}{2K} \Rightarrow K'' = \frac{CV^2}{2}\left(K + \frac{1}{K}\right) \Rightarrow K'' = \frac{CV^2}{2}\left(\frac{K^2+1}{K}\right)$. Accordingly the desired ratio of initial and final energy stored in capacitors is $R = \frac{K'}{K''} = \frac{CV^2.}{\frac{CV^2}{2}\left(\frac{K^2+1}{K}\right)} = \frac{2K}{K^2+1}$. Using the given value of K we arrive at $R = \frac{2\times3}{3^2+1} \Rightarrow R = \frac{6}{10} \Rightarrow R :: \mathbf{3:5}$ **is the answer**.
I-85	Given is a capacitor having plate area *A* and separation *d* is charged to a potential difference *V*. In this state energy stored in the capacitor is $E' = \frac{QV}{2} = \frac{(CV)V}{2} \Rightarrow E' = \frac{CV^2}{2}$. Next battery is disconnected and a dielectric slab of dielectric constant *K* is then inserted to fill space between the plates of the capacitor. In this state charge on the capacitor stays at $Q = CV$, but capacitance changes to $C' = CK$. Accordingly, in this new state energy stored in the system is $E'' = \frac{Q}{2}\left(\frac{Q}{C'}\right) \Rightarrow E'' = \frac{Q^2}{2C'} \Rightarrow E'' = \frac{(CV)^2}{2CK}$. It leads to $E'' = \frac{CV^2}{2K}$. Thus change in energy stored in the capacitor is $\Delta E' = E'' - E' \Rightarrow \Delta E' = \frac{CV^2}{2K} - \frac{CV^2}{2}$. Using $C = \frac{\varepsilon_0 A}{d}$, we get $\Delta E' = \left(\frac{\varepsilon_0 A}{d}\right)\frac{V^2}{2}\left(\frac{1}{K} - 1\right) \Rightarrow \Delta E' = \boldsymbol{\frac{\varepsilon_0 A V^2}{2d}\left(\frac{1}{K} - 1\right)}$ **is the answer**.
I-86	Given is a capacitor of capacitance of 100 μF is charged to a potential difference of 50 V. Taking part-wise solution of the problem is as under – **Part (a):** Charge on the capacitor is $Q = CV \Rightarrow Q = (100 \times 10^{-6}) \times 50 \Rightarrow Q = 5.0 \times 10^{-3}$C or **5.0 mC.** **Part (b):** Battery is now disconnected and therefore on this isolated capacitor retains charge 5.0×10^{-3}C. Now when dielectric of dielectric constant $K = 2.5$ is inserted to fill the space between plates of capacitor, its capacitance changes to $C' = CK$. In the process the only variable that can chage is potential difference across the capacitor such that $V' = \frac{Q}{C'} = \frac{Q}{CK} \Rightarrow V' = \frac{5.0\times10^{-3}}{(100\times10^{-6})\times2.5} \Rightarrow V' = \mathbf{20}$ **V**. **Part (c):** The charge on the capacitor, in absence of the dielectric, that would produce potential difference $V' = 20$ V is $Q' = CV' \Rightarrow Q' = (100 \times 10^{-6}) \times 20 \Rightarrow Q' = 2.0 \times 10^{-3}$C or **2.0 mC**. **Part (d):** Charge induced on the surface of the dielectric is $\Delta Q = Q - Q' \Rightarrow \Delta Q = 5.0 - 2.0 \Rightarrow \Delta Q = \mathbf{3.0}$**mC.** **Thus, answers are (a) 5.0 mC (b) 20 V (c) 2.0 mC (d) 3.0 mC.**
I-87	The given assembly is equivalent to series combination of two spherical capacitors – One of capacitance C_1 is of inner radius $r_{i1} = a$ and outer radius $r_{o1} = c$ filled with a dielectric of dielectric constant K. And the other of capacitance C_2 is of inner radius $r_{i2} = c$ and outer radius $r_{o2} = b$ filled with air. Capacitance of a spherical capacitor is $C = \frac{4\pi\varepsilon_0 K r_i r_o}{r_o - r_i}$. In the given problem inner capacitor with the dielectric $C_1 = \frac{4\pi\varepsilon_0 K r_{i1} r_{o1}}{r_{o1} - r_{i1}} \Rightarrow C_1 = \frac{4\pi\varepsilon_0 acK}{c-a}$ and for the outer without dielectric $C_2 = \frac{4\pi\varepsilon_0 r_{i2} r_{o2}}{r_{o2} - r_{i2}} \Rightarrow C_2 = \frac{4\pi\varepsilon_0 bc}{b-c}$. Therefore net capacitance of the series combination is $C = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C = \frac{\left(\frac{4\pi\varepsilon_0 acK}{c-a}\right)\left(\frac{4\pi\varepsilon_0 bc}{b-c}\right)}{\frac{4\pi\varepsilon_0 acK}{c-a} + \frac{4\pi\varepsilon_0 bc}{b-c}}$. It further solves to $C = \frac{4\pi\varepsilon_0 abc^2 K}{c\left(Ka(b-c) + b(c-a)\right)} \Rightarrow C = \boldsymbol{\frac{4\pi\varepsilon_0 abcK}{Ka(b-c) + b(c-a)}}$ **is the answer.**
I-88	The given configuration can be seen as series combination of two spherical capacitors –One of capacitance C_1 is of inner radius $r_{i1} = a$ and outer radius $r_{o1} = b$, and the other of cacitance C_2 is of inner radius $r_{i2} = b$ and outer radius $r_{o2} = c$ filled with air. Capacitance of a spherical capacitor is $C = \frac{4\pi\varepsilon_0 r_i r_o}{r_o - r_i}$, here r_i is the inner radius and r_o is the outer radius.

	In the given problem inner capacitor with the dielectric $C_1 = \frac{4\pi\varepsilon_0 r_{i1} r_{o1}}{r_{o1}-r_{i1}} \Rightarrow C_1 = \frac{4\pi\varepsilon_0 ab}{b-a}$ and for the outer without dielectric $C_2 = \frac{4\pi\varepsilon_0 r_{i2} r_{o2}}{r_{o2}-r_{i2}} \Rightarrow C_2 = \frac{4\pi\varepsilon_0 bc}{c-b}$. Therefore net capacitance of the series combination is $C = \frac{C_1 C_2}{C_1+C_2} \Rightarrow C = \frac{\left(\frac{4\pi\varepsilon_0 ab}{b-a}\right)\left(\frac{4\pi\varepsilon_0 bc}{c-b}\right)}{\frac{4\pi\varepsilon_0 Kab}{b-a}+\frac{4\pi\varepsilon_0 bc}{c-b}}$. It further solves to $C = \frac{4\pi\varepsilon_0 ab^2 c}{b(a(c-b)+c(b-a))} \Rightarrow C = \frac{4\pi\varepsilon_0 abc}{bc-ab} \Rightarrow C = \frac{4\pi\varepsilon_0 ac}{c-a}$ **is the answer.**
I-89	The given configuration can be seen as series combination of two spherical capacitors –One of capacitance C_1 is of inner radius $r_{i1} = a$ and outer radius $r_{o1} = b$ filled with a dielectric of dielectric constant K. And the other of capacitance C_2 is of inner radius $r_{i2} = b$ and outer radius $r_{o2} = c$ filled with air. Capacitance of a spherical capacitor is $C = \frac{4\pi\varepsilon_0 K r_i r_o}{r_o - r_i}$, here r_i is the inner radius and r_o is the outer radius. In the given problem inner capacitor with the dielectric $C_1 = \frac{4\pi\varepsilon_0 K r_{i1} r_{o1}}{r_{o1}-r_{i1}} \Rightarrow C_1 = \frac{4\pi\varepsilon_0 Kab}{b-a}$ and for the outer without dielectric $C_2 = \frac{4\pi\varepsilon_0 r_{i2} r_{o2}}{r_{o2}-r_{i2}} \Rightarrow C_2 = \frac{4\pi\varepsilon_0 bc}{c-b}$. Therefore net capacitance of the series combination is $C = \frac{C_1 C_2}{C_1+C_2} \Rightarrow C = \frac{\left(\frac{4\pi\varepsilon_0 Kab}{b-a}\right)\left(\frac{4\pi\varepsilon_0 bc}{c-b}\right)}{\frac{4\pi\varepsilon_0 Kab}{b-a}+\frac{4\pi\varepsilon_0 bc}{c-b}}$. It further solves to $C = \frac{4\pi\varepsilon_0 Kab^2 c}{b(Ka(c-b)+c(b-a))} \Rightarrow C = \frac{4\pi\varepsilon_0 Kabc}{Ka(c-b)+c(b-a)}$ **is the answer.**
I-90	Given that charge $Q = 12 \times 10^{-6}$ C on a capacitor of capacitance C when connected to a potential difference $V = 1200$ V. Since, $Q = CV \Rightarrow C = \frac{Q}{V}$. Using the given data $C = \frac{12\times10^{-6}}{1200} = 10 \times 10^{-9}$F. Capacitance of a capacitor is $C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \left(\frac{C}{\varepsilon_0}\right) d \Rightarrow A \propto d$ i.e. when d is minimum, area of the plate of capacitor A would also be minimum. Further, given that dielectric strength i.e. maximum electric field that air can withstand is $E_{max} = \frac{V}{d_{min}}$. Accordingly with the given data, $d_{min} = \frac{1200}{3\times10^6} = 0.4 \times 10^{-3}$. Accordingly, $A_{min} = \left(\frac{10\times10^{-9}}{8.85\times10^{-12}}\right)(0.4 \times 10^{-3}) \Rightarrow A_{min} = \mathbf{0.45\ m^2}$
I-91	Given is a parallel-plate capacitor having plate area $A = 100 \times 10^{-4}$m^2 and separation between plates $d = 1.0 \times 10^{-2}$ m. Capacitance of the capacitor is $C = \frac{\varepsilon_0 A}{d} \Rightarrow C = \frac{(8.85\times10^{-12})(100\times10^{-4})}{1.0\times10^{-2}} \Rightarrow C = 8.85 \times 10^{-12}$ F. The capacitor is connected across battery $V = 24$ V. Thus, charge on the capacitor, using the available data is $Q = CV \Rightarrow Q = (8.85 \times 10^{-12}) \times 24 \Rightarrow Q = 0.21 \times 10^{-9}$C. A close examination of Coulomb's Law reveals that force experienced by a charge q is due to electric field E produced by charge(s) other than itself and is expressed as $F = Eq$. Accordingly, force experienced by charge on a plate of the capacitor is due to field produced by charge on the other plate, and it is symmetrical to both the plates. In parallel plate capacitor uniform electric field $E = \frac{\rho}{2\epsilon_0}$is produced by charge of $(+Q)$ on the positive plate of the capacitor. Thus, force experienced by negative plate carrying charge $(-Q)$ is $F = \frac{\rho}{2\epsilon_0}(-Q) \Rightarrow F = \frac{\rho A}{2\epsilon_0 A}(-Q) \Rightarrow F = \frac{(+Q)}{2\epsilon_0 A}(-Q) \Rightarrow F = -\frac{Q^2}{2\epsilon_0 A}$. The (-) ve sign signifies that the force is attractive having magnitude $\lvert F\rvert = \frac{Q^2}{2\epsilon_0 A}$. Accordingly attractive force between the two plates, using the available data is $\lvert F\rvert = \frac{(0.21\times10^{-9})^2}{2\times(8.85\times10^{-12})\times(100\times10^{-4})} \Rightarrow \lvert F\rvert = 0.25 \times 10^{-6}$ N. **N.B.: (1)** The problem involves clarity of concept of electric field in capacitor and electrostatic force between the plates of the capacitors. (2) Mutual force on plates of a charged capacitor is different from that on a charge q in between the plates of a charged capacitor. In this case force on the charge is combined effect of external field due to positively charged plate $\vec{E}_+ = \frac{\rho}{2\epsilon_0}\hat{r}$ and that due to negatively charged plate$\vec{E}_- = \frac{-\rho}{2\epsilon_0}(-\hat{r}) = \frac{\rho}{2\epsilon_0}\hat{r}$. Thus net electric

field influencing force on the charge q is $\vec{E}=\vec{E}_{+}+\vec{E}_{-}=\frac{\rho}{2\epsilon_0}\hat{r}+\frac{\rho}{2\epsilon_0}\hat{r}\Rightarrow\vec{E}=\frac{\rho}{\epsilon_0}\hat{r}$. This is double of that considered in illustration of the solution. Moreover, magnitude of the electric field at charge q is $E=\frac{\rho}{\epsilon_0}=\frac{\rho A}{\epsilon_0 A}\Rightarrow E=\frac{Q}{\frac{\epsilon_0 Ad}{d}}\Rightarrow E=\frac{Q}{\left(\frac{\epsilon_0 A}{d}\right)d}\Rightarrow E=\frac{Q}{Cd}\Rightarrow E=\left(\frac{Q}{C}\right)\times\frac{1}{d}\Rightarrow E=V\times\frac{1}{d}\Rightarrow E=\frac{V}{d}$.

I-92



Capacitance C of a capacitor change in absence of dielectric $C'=\frac{\epsilon_0 A}{d}$ changes to $C"=\frac{\epsilon_0 KA}{d}$ and hence energy stored in capacitor without dielectric $E'=\frac{1}{2}QV\Rightarrow E'=\frac{C\prime V^2}{2}\Rightarrow E'=\frac{\epsilon_0 AV^2}{2d}$ changes to $E"=\frac{\epsilon_0 KAV^2}{2d}$.

In the given system say with a width x of the dielectric inside the gap, it is equivalent to two capacitors as under –

(a) With Dielectric Filled: $C_1=\frac{\epsilon_0 KA\left(\frac{x}{l}\right)}{d}\Rightarrow C_1=\frac{\epsilon_0 KAx}{ld}$

(b) Without dielectric: $C_2=\frac{\epsilon_0 A\left(\frac{l-x}{l}\right)}{d}\Rightarrow C_1=\frac{\epsilon_0 A(l-x)}{ld}$.

This system is equivalent to a parallel combination of capacitors C_1 and C_2 having an equivalent capacitance $C=C_1+C_2\Rightarrow C=\frac{\epsilon_0 KAx}{ld}+\frac{\epsilon_0 A(l-x)}{ld}\Rightarrow C=\frac{\epsilon_0 A}{ld}(Kx+l-x)\Rightarrow C=\frac{\epsilon_0 A}{ld}(l+(K-1)x)$. Accordingly, energy stored in the system is $E'=\frac{\left(\frac{\epsilon_0 A}{ld}(l+(K-1)x)\right)V^2}{2}\Rightarrow E'=\frac{\epsilon_0 AV^2}{2ld}(l+(K-1)x)$. Now, when dielectric is moved $\Delta E'=\frac{d}{dx}\left(\frac{\epsilon_0 AV^2}{2ld}(l+(K-1)x)\right)\Delta x\Rightarrow\Delta E'=-\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. If Δx decreases it becomes negative and hence $\Delta E'=-\frac{\epsilon_0 AV^2(K-1)}{2ld}(-\Delta x)\Rightarrow\Delta E'=\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. Further, $\Delta E'=\vec{F}.\Delta\vec{x}$ and hence magnitude of inward force on dielectric being pulled out is $F=\frac{\epsilon_0 AV^2(K-1)}{2ld}$.

Here, an element of concepts mechanics is involved. Natural tendency of the mass M is to lower down under gravity. But, the system is stated to be in equilibrium and therefore $a=0$ such that $M\vec{g}+\vec{T}=0\Rightarrow M\vec{g}=-\vec{T}$. With the sequence of forces in the string are shown in the figure eventually for dielectric slab to be in equilibrium $\vec{F}=-\vec{T}$. Ignoring intermediate forces in the string connecting its final form $F=Mg$. It implies that both the forces are equal in magnitude and opposite in direction. Accordingly, $Mg=\frac{\epsilon_0 AV^2(K-1)}{2ld}$, it leads to $M=\frac{\epsilon_0 AV^2(K-1)}{2ldg}$.

It is given in the problem that width of each plates of capacitors is b and their length as per given figure is l and, therefore, $A=bl$. It leads to $M=\frac{\epsilon_0(bl)V^2(K-1)}{2ldg}\Rightarrow$ $\boldsymbol{M=\frac{\epsilon_0 bV^2(K-1)}{2dg}}$ **is the answer**.

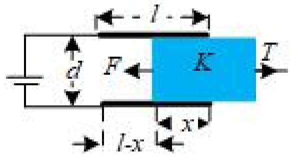
N.B.: (1) This problem is a good example of integration of concepts as one either moves forward in physics or application of concepts of physics.

(2) Work or energy being scalar, what is important is change in energy as per relevant formulae and not the direction of mechanical force or electric field.

I-93

Capacitance C of a capacitor change in absence of dielectric $C' = \frac{\epsilon_0 A}{d}$ changes to $C" = \frac{\epsilon_0 KA}{d}$ and hence energy stored in capacitor without dielectric $E' = \frac{1}{2}QV \Rightarrow\ E' = \frac{C\prime V^2}{2} \Rightarrow\ E' = \frac{\epsilon_0 AV^2}{2d}$ changes to $E" = \frac{\epsilon_0 KAV^2}{2d}$.

In the given system say with a width x of the dielectric inside the gap, it is equivalent to two capacitors as under –

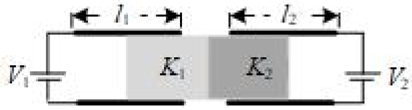


(a) With Dielectric Filled: $C_1 = \frac{\epsilon_0 KA\left(\frac{x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 KAx}{ld}$

(b) Without dielectric: $C_2 = \frac{\epsilon_0 A\left(\frac{l-x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 A(l-x)}{ld}$.

This system is equivalent to a parallel combination of capacitors C_1 and C_2 having an equivalent capacitance $C = C_1 + C_2 \Rightarrow C = \frac{\epsilon_0 KAx}{ld} + \frac{\epsilon_0 A(l-x)}{ld} \Rightarrow C = \frac{\epsilon_0 A}{ld}(Kx + l - x) \Rightarrow C = \frac{\epsilon_0 A}{ld}(l + (K-1)x)$. Accordingly, energy stored in the system is $E' = \frac{\left(\frac{\epsilon_0 A}{ld}(l+(K-1)x)\right)V^2}{2} \Rightarrow E' = \frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)$. Now, when dielectric is moved $\Delta E' = \frac{d}{dx}\left(\frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)\right)\Delta x \Rightarrow \Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. If Δx decreases it becomes negative and hence $\Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}(-\Delta x) \Rightarrow \Delta E' = \frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. Further, $\Delta E' = \vec{F}.\Delta\vec{x}$ and hence magnitude of inward force on dielectric being pulled out is $F = \frac{\epsilon_0 AV^2(K-1)}{2ld} = \frac{\epsilon_0 b(lb)V^2(K-1)}{2ld} \Rightarrow F = \frac{\epsilon_0 bV^2(K-1)}{2d}$. It is to be noted that inward electrostatic force on dielectric caused is independent of depth of the slab inside the capacitor plates and length of the plates of the capacitors.

It is given that two parallel plate capacitors have plates of equal width b but with lengths l_1 is connected to a battery of emf V_1 and the other with length l_2 is connected to a battery of emf V_2. Therefore, area of plates of capacitor First capacitor is $A_1 = l_1 b$ and $A_2 = l_b b$. Let, dielectric constant of the dielectric slab partially inserted in first capacitor is K_1 and that in the second capacitor is K_2. Thus inward force caused on the dielectric slab in first capacitor is $F_1 = \frac{\epsilon_0 bV_1^2(K_1-1)}{2d}$. Likewise, on inward force on the second dielectric is $F_2 = \frac{\epsilon_0 bV_2^2(K_2-1)}{2d}$.



As per given figure the two dielectric slabs are rigidly connected and geometrically they are so placed that the inward force on the two dielectric slabs would be in opposite directions. Therefore, in state of equilibrium as given $F_1 = F_2 \Rightarrow \frac{\epsilon_0 bV_1^2(K_1-1)}{2d} = \frac{\epsilon_0 bV_2^2(K_2-1)}{2d} \Rightarrow V_1^2(K_1 - 1) = V_2^2(K_2 - 1) \Rightarrow \left(\frac{V_1}{V_2}\right)^2 = \frac{K_2-1}{K_1-1} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{\boldsymbol{K_2-1}}{\boldsymbol{K_1-1}}}$ **is the answer.**

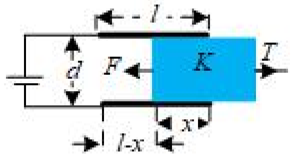
N.B.: (1) This problem is a good example of integration of concepts as one either moves forward in physics or application of concepts of physics.
(2) Work or energy being scalar, what is important is change in energy as per relevant formulae and not the direction of mechanical force or electric field.

I-94

Any object which experiences force and executes periodic motion it must have a mass and let mass of the dielectric slab in the system is m. It is given that a dielectric slab of dielectric constant K is held in a state of rest such that it partially fills the gap to a length a. Length and width of the slab and dielectric plates are l and b respectively.
The dielectric slab is released is released from a state of rest and it is required to demonstrate that it performs a periodic motion and to find periodic motion.
In the first go it is required to determine force on a dielectric slab when it is partially filling the gap and it is analyzed as under.

Capacitance C of a capacitor change in absence of dielectric $C' = \frac{\epsilon_0 A}{d}$ changes to $C" = \frac{\epsilon_0 KA}{d}$ and hence energy stored in capacitor without dielectric $E' = \frac{1}{2}QV \Rightarrow\ E' = \frac{C\prime V^2}{2} \Rightarrow\ E' = \frac{\epsilon_0 AV^2}{2d}$ changes to $E" = \frac{\epsilon_0 KAV^2}{2d}$.

In the given system say with a width x of the dielectric inside the gap, it is equivalent to two capacitors as under –

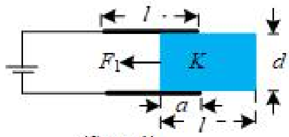


(a) With Dielectric Filled: $C_1 = \frac{\epsilon_0 KA\left(\frac{x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 KAx}{ld}$

(b) Without dielectric: $C_2 = \frac{\epsilon_0 A\left(\frac{l-x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 A(l-x)}{ld}$.

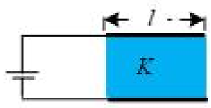
This system is equivalent to a parallel combination of capacitors C_1 and C_2 having an equivalent capacitance $C = C_1 + C_2 \Rightarrow C = \frac{\epsilon_0 KAx}{ld} + \frac{\epsilon_0 A(l-x)}{ld} \Rightarrow C = \frac{\epsilon_0 A}{ld}(Kx + l - x) \Rightarrow C = \frac{\epsilon_0 A}{ld}(l + (K-1)x)$. Accordingly, energy stored in the system is $E' = \frac{\left(\frac{\epsilon_0 A}{ld}(l+(K-1)x)\right)V^2}{2} \Rightarrow E' = \frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)$. Now, when dielectric is moved $\Delta E' = \frac{d}{dx}\left(\frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)\right)\Delta x \Rightarrow \Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. If Δx decreases it becomes negative and hence $\Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}(-\Delta x) \Rightarrow \Delta E' = \frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. Further, $\Delta E' = \vec{F}.\Delta\vec{x}$ and hence magnitude of inward force on dielectric being pulled out is $F = \frac{\epsilon_0 AV^2(K-1)}{2ld} = \frac{\epsilon_0 b(lb)V^2(K-1)}{2ld} \Rightarrow F = \frac{\epsilon_0 bV^2(K-1)}{2d}$. It is to be noted that inward electrostatic force on dielectric caused is independent of depth of the slab inside the capacitor plates and length of the plates of the capacitors.

Now an element of mechanics is involved and there are three states of the block as under –



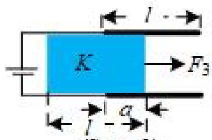
(State 1)

State 1: In the state of rest initial velocity of the block $u = 0$. But, due to the uniform dielectric force $F_1 = \frac{\epsilon_0 bV^2(K-1)}{2d}$ the dielectric slab would experience uniform acceleration, as per Newton's Second Law of Motion $k_1 = \frac{F_1}{m} \Rightarrow k_1 = \frac{\epsilon_0 bV^2(K-1)}{2dm}$, as shown in the figure (i).



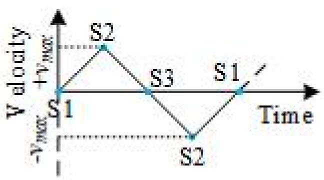
(State 2)

State 2: The slab remains accelerated until it fill gap completely as shown if figure (ii), and by then acquires velocity as per Third Equation of Motion $v^2 = 2ks \Rightarrow v = \sqrt{2ks} \Rightarrow v_{max} = \sqrt{2\left(\frac{\epsilon_0 bV^2(K-1)}{2dm}\right)(l-a)} = \sqrt{\left(\frac{\epsilon_0 bV^2(K-1)}{dm}\right)(l-a)}$. At this state when gap is completely filled force on the dielectric becomes zero.



(State 3)

State 3: In state 2, under inertia as per Newton's First Law of Motion would continue with the instantaneous velocity to create gap on right side and experience electrostatic force $F_3 = F_1$ i.e. of same magnitude but in opposite direction which would retard the slab until it reaches state 3.



Considering the three states the slab would continue perform a periodic motion in the system where effect of friction and gravity is ignored. Thus, transition of slab from one state to the other is shown in the figure where at $t = 0$ the slab is in state 1 (S1), at $t = \frac{T}{4}$ the slab is in state 2 (S2). at $t = \frac{T}{2}$ the slab is in state 3 (S3). at $t = \frac{3T}{4}$ the slab is in state 2 (S2) and at $t = T$ the slab is back in state 1 (S1). Like this periodic motion continues. Accordingly from First Equation of motion $v_{max} = 0 + k_1 \times \frac{T}{4} \Rightarrow T = \frac{4v_{max}}{k_1}$. Using results in stage 1 and 2, $T = \frac{4\sqrt{\left(\frac{\epsilon_0 bV^2(K-1)}{2dm}\right)(l-a)}}{\frac{\epsilon_0 bV^2(K-1)}{2dm}} \Rightarrow T = 8 \times \sqrt{\frac{(l-a)md}{\epsilon_0 bV^2(K-1)}} \Rightarrow T = 8 \times \sqrt{\frac{(l-a)mdl}{\epsilon_0 (bl)V^2(K-1)}} = \mathbf{8} \times \sqrt{\frac{(l-a)mdl}{\epsilon_0 AV^2(K-1)}}$ **is the answer.**

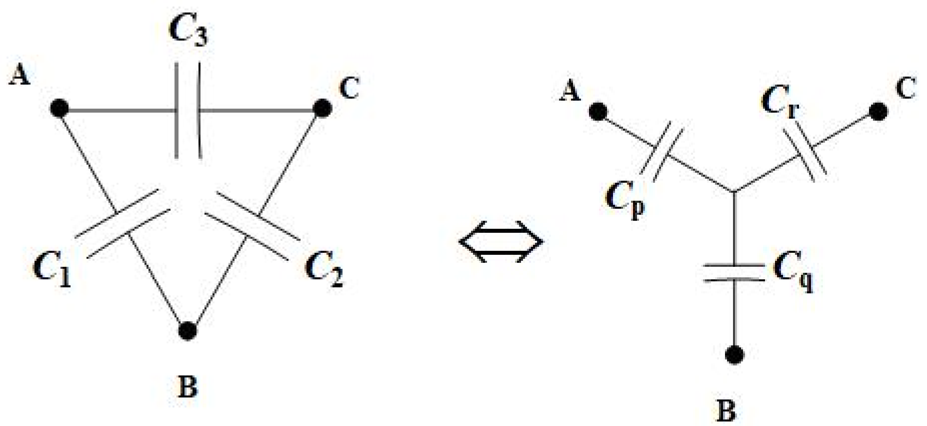
N.B.: (1) This problem is a good example of integration of concepts as one either moves forward in physics or application of concepts of physics.

(2) Work or energy being scalar, what is important is change in energy as per relevant formulae and not the direction of mechanical force or electric field.

	(3) Step by step application of concepts makes solution of a problem, apparently difficult, very simple and adds to clarity of concepts.

APPENDIX

Star-Delta $(\mathrm{Y}-\Delta)$ Equivalent of Capacitor Connection



Delta Connected Capacitors

Star Connected Capacitors

Operation	Delta Connection	Star Connection	Eqn.
Equivalent capacitance across nodes a-b	$C_{ab}=C_1+\frac{C_2C_3}{C_2+C_3}$ $\Rightarrow \frac{C_1C_2+C_2C_3+C_3C_1}{C_2+C_3}$ $\frac{1}{C_{ab}}=\frac{C_2+C_3}{C_1C_2+C_2C_3+C_3C_1}$	$C_{ab}=\frac{C_pC_q}{C_p+C_q}$ $\frac{1}{C_{ab}}=\frac{C_p+C_q}{C_pC_q}$	(1)
Equivalent capacitance across nodes b-c	$C_{bc}=C_2+\frac{C_3C_1}{C_3+C_1}$ $\Rightarrow \frac{C_1C_2+C_2C_3+C_3C_1}{C_3+C_1}$ $\frac{1}{C_{bc}}=\frac{C_3+C_1}{C_1C_2+C_2C_3+C_3C_1}$	$C_{bc}=\frac{C_qC_r}{C_q+C_r}$ $\frac{1}{C_{bc}}=\frac{C_q+C_r}{C_qC_r}$	(2)
Equivalent capacitance across nodes c-a	$C_{ca}=C_3+\frac{C_1C_2}{C_1+C_2}$	$C_{ca}=\frac{C_rC_p}{C_r+C_p}$	

<table>
<tr><td></td><td>$$\Rightarrow \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2}$$
$$\frac{1}{C_{\mathrm{ca}}} = \frac{C_1 + C_2}{C_1C_2 + C_2C_3 + C_3C_1}$$</td><td>$$\frac{1}{C_{\mathrm{bc}}} = \frac{C_{\mathrm{r}} + C_{\mathrm{p}}}{C_{\mathrm{r}}C_{\mathrm{p}}}$$</td><td>(3)</td></tr>
<tr><td>(2)-(1):
$$\frac{1}{C_{\mathrm{bc}}} - \frac{1}{C_{\mathrm{ab}}}$$</td><td>$$\frac{C_1 - C_2}{C_1C_2 + C_2C_3 + C_3C_1}$$</td><td>$$\frac{C_{\mathrm{q}} + C_{\mathrm{r}}}{C_{\mathrm{q}}C_{\mathrm{r}}} - \frac{C_{\mathrm{p}} + C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}}$$
$$\Rightarrow \frac{C_{\mathrm{p}}C_{\mathrm{q}} + C_{\mathrm{p}}C_{\mathrm{r}} - C_{\mathrm{r}}C_{\mathrm{p}} - C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$$
$$\Rightarrow \frac{C_{\mathrm{p}}C_{\mathrm{q}} - C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$$</td><td>(4)</td></tr>
<tr><td colspan="4">Delta to Star Conversion</td></tr>
<tr><td rowspan="2">(3)+(4)</td><td>$$\frac{2C_1}{C_1C_2 + C_2C_3 + C_3C_1}$$</td><td>$$\left(\frac{C_{\mathrm{r}} + C_{\mathrm{p}}}{C_{\mathrm{r}}C_{\mathrm{p}}}\right) \times \frac{C_{\mathrm{q}}}{C_{\mathrm{q}}} + \frac{C_{\mathrm{p}}C_{\mathrm{q}} - C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$$
$$\Rightarrow \frac{2C_{\mathrm{p}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}} = \frac{2}{C_{\mathrm{r}}}$$</td><td rowspan="2">(5)</td></tr>
<tr><td colspan="2">$$\frac{C_1}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{1}{C_{\mathrm{r}}}$$
$$\boldsymbol{C_{\mathrm{r}} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1}}$$</td></tr>
<tr><td rowspan="2">Applying analogy of (5)</td><td colspan="2">$$\boldsymbol{C_{\mathrm{p}} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_2}}$$</td><td>(6)</td></tr>
<tr><td colspan="2">$$\boldsymbol{C_{\mathrm{q}} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1}}$$</td><td>(7)</td></tr>
<tr><td colspan="4">Star to Delta Conversion</td></tr>
<tr><td>(5) × (6)</td><td colspan="2">$$C_{\mathrm{r}}C_{\mathrm{p}} = \frac{(C_1C_2 + C_2C_3 + C_3C_1)^2}{C_1C_2}$$
$$\Rightarrow \frac{1}{C_{\mathrm{r}}C_{\mathrm{p}}} = \frac{C_1C_2}{(C_1C_2 + C_2C_3 + C_3C_1)^2}$$</td><td>(8)</td></tr>
</table>

(6) × (7)	$$C_pC_q = \frac{(C_1C_2 + C_2C_3+C_3C_1)^2}{C_2C_3}$$ $$\Rightarrow \frac{1}{C_pC_q} = \frac{C_2C_3}{(C_1C_2 + C_2C_3+C_3C_1)^2}$$	(9)
(5) × (7)	$$C_qC_r = \frac{(C_1C_2 + C_2C_3+C_3C_1)^2}{C_3C_1}$$ $$\Rightarrow \frac{1}{C_qC_r} = \frac{C_3C_1}{(C_1C_2 + C_2C_3+C_3C_1)^2}$$	(10)
(8)+(9)+(10)	$$\frac{1}{C_rC_p} + \frac{1}{C_pC_q} + \frac{1}{C_qC_r} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{(C_1C_2 + C_2C_3+C_3C_1)^2}$$ $$\Rightarrow \frac{1}{C_1C_2 + C_2C_3+C_3C_1} = \frac{1}{C_rC_p} + \frac{1}{C_pC_q} + \frac{1}{C_qC_r}$$ $$\Rightarrow \frac{1}{C_1C_2 + C_2C_3+C_3C_1} = \frac{C_p + C_q+C_r}{C_pC_qC_r}$$	(11)
(5) × (11)	$$\left(\frac{C_1C_2 + C_2C_3+C_3C_1}{C_1}\right) \times \left(\frac{1}{C_1C_2 + C_2C_3+C_3C_1}\right) = C_r \times \left(\frac{C_p + C_q+C_r}{C_pC_qC_r}\right)$$ $$\Rightarrow \frac{1}{C_1} = \frac{C_p + C_q+C_r}{C_pC_q}$$ $$\boldsymbol{C_1} = \frac{\boldsymbol{C_pC_q}}{\boldsymbol{C_p + C_q+C_r}}$$	(12)
Applying analogy of (125)	$$C_2 = \frac{\boldsymbol{C_qC_r}}{\boldsymbol{C_p + C_q+C_r}}$$	(13)
	$$C_3 = \frac{\boldsymbol{C_rC_p}}{\boldsymbol{C_p + C_q+C_r}}$$	(14)

N.B.: 1. Derivation of $(\mathrm{Y} - \Delta)$ is based on series-parallel equivalents capacitances with algebraic manipulations.

2.Derivation of equivalent capacitance in A close observation of formulae in relation to the figure will help to develop easy way to remember it.

2. A similar, but with a difference formulation for star-delta network of resistance in DC circuit analysis and impedances in AC circuit analysis will be supplemented at a later stage.

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