Electromagnetism: Current Electricity – Typical Questions (Set 1)

(Only Illustrations)

<table>
<tr><td>I-1</td><td>Given is the three resistors each of value $R = 30\ \Omega$. Therefore, possible combination having different values of resistances are –
(a) All in series: $R_a = R + R \Rightarrow R_a = 60\ \Omega$
(b) Two in parallel and one in series $R_b = R_p + R \Rightarrow R_b = \frac{R\times R}{R+R} + R \Rightarrow R_b = \frac{3}{2}R = 45\ \Omega$
(c) All in parallel $\frac{1}{R_c} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \Rightarrow \frac{1}{R_c} = \frac{3}{R} \Rightarrow R_c = \frac{R}{3} = 10\ \Omega$
Hence, answers are (a) 60 Ω (b) 45 Ω (b) 10 Ω</td></tr>
<tr><td>I-2</td><td>Electric current is defined as $\vec{I} = \frac{dQ}{dt}\hat{r} \Rightarrow \vec{I} = \frac{(dN)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(dv)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(Adl)e_+}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)\frac{dl}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)v\hat{r}$.
In this current has a direction and hence its magnitude $\vec{I}$ is rate of flow of charge $\frac{dQ}{dt}$say in direction $\hat{r}$. Let number of protons dN drift in time dt and hence charge drifted is $dQ = (dN)e_+$, here e_+ is quantity of charge of proton. Let, n is density of protons in a unit volume, A is cross section of beam, and dl is the distance of drift of the beam in time dt.Hence, $dN = n(Adl)$. Accordingly, magnitude of drift velocity of proton beam is $v = \frac{dl}{dt}$., while direction of current remains $\hat{r}$. Thus in final form of expression $(nAe_+)v$ is magnitude of current and $\hat{r}$ is the direction of current, same as the direction of proton beam from east to west, is the answer.</td></tr>
<tr><td>I-3</td><td>Electric current is defined as $\vec{I} = \frac{dQ}{dt}\hat{r} \Rightarrow \vec{I} = \frac{(dN)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(dv)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(Adl)e_+}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)\frac{dl}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)v\hat{r}$.
In an electrolyte positive ions carrying charge q_+ this current has a direction left to right i.e. $\hat{r}$ and while magnitude of current due to positive ions $\vec{I}_+$ is rate of flow of charge $\frac{dQ}{dt}\hat{\imath}$. Let number of protons dN drift in time dt and hence charge drifted is $dQ_+ = (dN)q_+$. Let, n is density of positive ions a unit volume, A is cross section of beam, and dl is the distance of drift of the positive ions in time dt.Hence, $dN = n(Adl)$. Accordingly, magnitude of drift velocity of positive ions is $v = \frac{dl}{dt}$., while direction of current remains $\hat{r}$. Thus in final form of expression $(nAq_+)v$ is magnitude of current and $\hat{r}$ is the direction of current, same as the direction of positive ions from left . Accordingly, $\vec{I}_+ = (nAq_+)v\hat{r}$.
Further it is given that, which is obvious, that negative ions carrying charge $q_- = (-q_+)$ this current has a direction right to left i.e. $(-\hat{r})$ and while magnitude of current due to negative ions $\vec{I}_-$ is $\frac{dQ_-}{dt} = \big(nA(-q_+)\big)v(-\hat{r}) \Rightarrow \vec{I}_- = (nAq_+)v\hat{r}$.
Thus net current is $\vec{I} = \vec{I}_+ + \vec{I}_- \Rightarrow (nAq_+)v\hat{r} + (nAq_+)v\hat{r} \Rightarrow \vec{I} = 2(nAvq_+)\hat{r}$ from left to right, is the answer.
N.B.: Charge on positive and negative ions in an electrolyte are equal and opposite, while average density of positive and negative ions in an electrolyte are equal.</td></tr>
<tr><td>I-4</td><td>Electric current is defined as $\vec{I} = \frac{dQ}{dt}\hat{r} \Rightarrow \vec{I} = \frac{(dN)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(dv)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(Adl)e_+}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)\frac{dl}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)v\hat{r}$.
Charge of an electron is taken to be $e_- = (-e_+)$ and its direction from rear to front is taken to be. $\hat{r}$. Magnitude of current of electron beam $\vec{I}$ is rate of flow of charge $\frac{dQ}{dt}\hat{r}$. Let number of protons dN drift in time dt and hence charge drifted is $dQ_+ = (dN)e_-$. Let, n is density of positive ions a unit volume, A is cross section of</td></tr>
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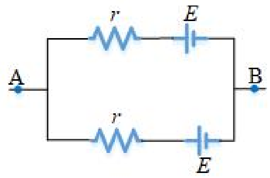
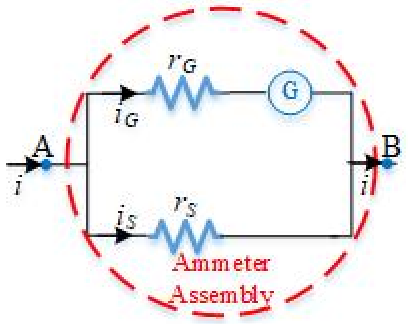
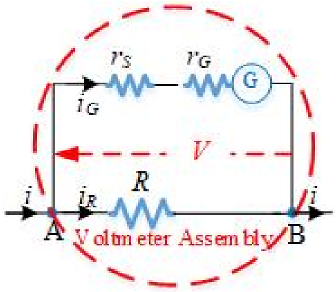
	beam, and dl is the distance of drift of the positive ions in time dt.Hence, $dN = n(Adl)$. Accordingly, magnitude of drift velocity of positive ions is $v = \frac{dl}{dt}$, while direction of current remains $\hat{r}$. Thus in final form $\vec{I} = (nAe_-)v\hat{r} \Rightarrow \vec{I} = (nAv)(-e_+)\hat{r} \Rightarrow \vec{I} = (nAve_+)(-\hat{r})$. Thus, direction of current is $(-\hat{r})$ opposite to the direction of electron tube in a TV tube. Thus, **answer is from front to rear**.
I-5	Electrons in a conductor, ions in gases and liquids perform random chaotic motion like that of molecules in gases or liquids. As per kinetic theory of gases, applied to Brownian motion, is based on assumption that atoms and/or molecules collide with other particles during Brownian motion. In every collision they impart kinetic energy and come to state of rest. Again, they get accelerated before next collision and average distance traversed by the particles between consecutive collisions is called mean free path, a statistical quantity. The statistical quantity is on a sample and is not valid on individual datum at any instance. Therefore, the **Brownian motion of the charged particle is the answer.**
I-6	In electrostatics, as the name suggests, charges are in state of rest. Further, conductor constitutes an equipotential surface i.e. $\Delta V = 0$ between two points on the surface. Thus, there is no flow of charges on surface. But, inside conductor electric field is $E = 0$and hence there cannot be flow of charges. In contradiction, in current electricity flow current between two points is due to drift of charges under influence of electric field between the two points. Thus, driving concept of electrostatics and current electricity are based on forces on charges $F = Eq \Rightarrow a = \frac{F}{m} \Rightarrow a = \frac{Eq}{m} \Rightarrow a = \frac{\frac{V}{\Delta l}q}{m} \Rightarrow a = \frac{Vq}{\Delta lm}$. Here, V is the potential difference between two points under consideration, and Δl is the distance between the points. Thus fallacy in argument is due to **difference in electrostatics and electrodynamics, is the answer.**
I-7	When current is established in a wire, potential difference between any two points is in steady state and remains constant. In such a state current is due to drift of cloud of charges such that in given space numbers of charges leaving is equal to number of charges entering. This is called ***principle of electrical neutrality***. It is this principle which ensures despite flow of charges, potential at each point remains constant. Thus, **number of free electrons in given space remain constant, is the answer.**
I-8	Electrical power supplied to a fan is $P = VI$, whereas power loss (Power consumed in winding) in fan is $p = I^2R$. Thus power used in blowing air is $P' = P - p \Rightarrow P' = P - I^2R$. Since supply voltage V is fixed and hence for a given power of fan current is $I = \frac{P}{V}$. Accordingly, current supplied for fan having copper and aluminum winding would be same, considering all other dimensions same. Further, in a fan magnetic field $\emptyset$ is produced by current through windings such that $\emptyset \propto NI \Rightarrow \emptyset = KNI$, here K is proportionality constant and N is number of turn in the winding. Assume that length l and area of cross-section A of each turn in fan having coller and aluminum winding is same. Now resistance of a wire is $R = \frac{\rho L}{A}$. Therefore, resistance of copper winding would be $R_{Cu} = \frac{\rho_{Cu}(Nl)}{A}$ and for aluminum winding $R_{Al} = \frac{\rho_{Al}(Nl)}{A}$. Therefore, power consumed in fan having copper winding is $p_{Cu} = \left(\frac{P}{V}\right)^2 \frac{\rho_{Cu}(Nl)}{A} \Rightarrow p_{Cu} = \left(\frac{P^2Nl}{P^2A}\right)\rho_{Cu} \Rightarrow p_{Cu} \propto \rho_{Cu}$. Likewise, power consumed in fan having aluminum winding is $p_{Al} \propto \rho_{Al}$. It leads to $\frac{p_{Cu}}{p_{Al}} = \frac{\rho_{Cu}}{\rho_{Al}}$. We know that copper is better conductor than aluminum and hence $\rho_{Cu} < \rho_{Al}$ and hence $\boldsymbol{p_{Cu} < p_{Al}}$ **is the reason.** **N.B.:** This question involves integration of electromagnetism applied to motor action and pre knowledge of comparison on resistivity of copper and aluminum.

I-9	Thermal energy in a resistor having resistance R is given to be $U = i^2Rt$…(1) Thus, $U \propto i^2$ taking time t to be constant while for a resistor R is constant. Here, change in resistance due heat developed in the resistor is ignored. Now, as per Ohm's Law $V = iR$…(2). Using (2), the equation (1) is transformed into $U = i(iR)t \Rightarrow U \propto iV$ …(3). The (3) is dependent on (2) where $V \propto i$…(4). Since, in (4) proportionality of U has an additional intrinsic proportionality as per Ohm's Law and hence it cannot be used for a resistor. Therefore, we should say $\boldsymbol{U \propto i^2}$**is the answer.**
I-10	Ideal battery whose emf is E. is that whose internal resistance is $r = 0$. Further, Power supplied by battery is $P_B = EI$…(1). It is given that a resistor is connected to the battery whose resistance is say R. Then as per Ohm's law current through resistance is $I = \frac{V}{R}$, ..(2)here V is the potential difference across the resistance. And thermal energy per second developed in resistance is equal to $P_R = VI$…(3). Again as per Ohm's Law, $E = V + Ir \Rightarrow V = E - Ir \Rightarrow V = E\|_{r=0}$…(4). Combining (1), (2), (3) and (4), $P_B = P_R = EI = E \times \frac{E}{R} = \frac{E^2}{R}$. Thus, **answer is Yes.**
I-11	There can be two situations as under- **Situation 1- Only resistor connected to the battery:** Let battery emf is E. and its internal resistance is r Further, Power supplied by battery is $P_B = EI$…(1). Let resistance of the resistor connected to the battery is R. Then as per Ohm's law current through resistance is $I = \frac{E}{R+r}$, ..(2). Accordingly, $P_B = E\left(\frac{E}{R+r}\right) \Rightarrow P_B = \frac{E^2}{R+r}$…(3). As regards thermal energy developed in electrical circuit is $P_T = I^2R_c$, here $R_c = R + r$ is total resistance of the circuit. Accordingly, $P_T = I^2(R + r)$…(4). Combining (2) and (4) we have $P_T = \left(\frac{E}{R+r}\right)^2(R + r) \Rightarrow P_T = \frac{E^2}{R+r}$…(5). It is to be noted that internal resistance of the battery is also a part of the circuit. Comparing (3) and (4) we find that $P_B = P_T = \frac{E^2}{R+r}$ i.e. work done by battery is equal to thermal energy developed in circuit, Hence answer is Yes $\Rightarrow P_R = \left(\frac{ER}{R+r}\right)\left(\frac{E}{R+r}\right) \Rightarrow P_R = \left(\frac{E}{R+r}\right)^2 R$ Potential difference across the resistance is $V = IR \Rightarrow V = \left(\frac{E}{R+r}\right)R = \frac{ER}{R+r}$…(3). Therefore thermal energy developed by the resistor is $P_R = VI \Rightarrow P_R = \left(\frac{ER}{R+r}\right)\left(\frac{E}{R+r}\right) \Rightarrow P_R = \left(\frac{E}{R+r}\right)^2 R$…(4). **Situation 2 – There is a capacitor in series connected to the resistance:** In this situation current in the circuit varies exponentially starting from instant of switching $t = 0^+$ at $I_{0^+} = \frac{E}{R+r}$ and is a function of time such that $i(t) = I_{0^+}\left(e^{-\frac{t}{RC}}\right)$ =progressively charging capacitor until it is fully charged at $t \to \infty$ and finally $I_\infty = 0$. Thus energy delivered by the battery is $E_B = \int_{0^+}^{\infty} i(t)dt$, out of the energy stored in the capacitor is $E_C = \frac{1}{2}CE^2$. Thus answer is that a **part of energy supplied by battery is stored in capacitor.**
I-12	In a non-ideal battery whose emf is E. is that whose internal resistance is $r \neq 0$. Further, Power supplied by battery is $P_B = EI$…(1). It is given that a resistor is connected to the battery whose resistance is say R. Then as per Ohm's law current through resistance is $I = \frac{E}{R+r}$, ..(2). Accordingly, $P_B = E\left(\frac{E}{R+r}\right) \Rightarrow P_B = \frac{E^2}{R+r}$ …(3) V is the potential difference across the resistance.

	And thermal energy per second developed in resistance is equal to $P_R = VI$…(4). Here, V is the potential difference across the resistance. Rewriting (2), $E = I(R + r) \Rightarrow V = E - Ir$…(5). Thus combing (4) and (5), $P_R = (E - Ir)\frac{E}{R+r} \Rightarrow P_R = \frac{E^2}{R+r} - \left(\frac{E}{R+r}\right)^2 r$...(6). It is seen from (3) and (6) that $P_B \neq P_R$. Hence a**nswer for non-ideal battery is No**. but discriminant of the two is $\Delta P = P_B - P_R = \left(\frac{E}{R+r}\right)^2 r$. Accordingly, $\Delta P = 0\|_{r=0}$for **an ideal battery** $P_B = P_R$, a**nswer is yes.**
I-13	Heat is developed in resister is transformation of electrical energy into heat energy in accordance with the principle of conservation of energy. While, under difference of temperature it is transfer of heat, both are different phenomenon. The **given statement is correct**
I-14	Current is manifestation of rate of flow of charges. Therefore, the given statement implies that **charges flow through wire.**
I-15	Voltmeter is a measuring device which undergoes deflection due to current flowing through it which is proportional to voltage. Thus, $V = E - Ir$, here $I = KV$ which is though quite small causes an error in measurement of emf. Whereas, potentiometer measures emf at null point of the galvanometer. This is a current sensing device which is used to decide null point when $E - V = 0 \Rightarrow E = V$. Hence, to measure emf **potentiometer is preferred.**
I-16	Conduction of current is based on drift of charges while marinating electrical neutrality i.e. quantity of charge ΔQ entering a conductor during an interval Δt is equal to quantity of charge leaving it during the interval. Thus change of charge in the conductor $\Delta Q' = \Delta Q - \Delta Q = 0$. Hence, **answer is No.**
I-17	Potential difference across a battery is $V = E - Ir$, here I is current supplied by the battery and r is the internal resistance of the battery. Thus, while battery supplies current $V < E$. But, when battery is charged current is pushed into the battery and therefore current becomes $I' = (-I)$. Accordingly, during charging of battery $V' = E - I'r \Rightarrow V' = E - (-I)r \Rightarrow V' = E + IR$. Thus, **during charging $V' > E$.**
I-18	Current in a metallic resistor is rate of flow of charges $I = \frac{dQ}{dt}$ in given space of the conductor. As per electro-mechanics (combining electricity with mechanics) $I = \frac{dQ}{dt} = Nqv$. Here N is number of electrons per unit volume and q is charge of electron and v is average velocity of electron Flow current is explained with Brownian Motion of free electrons, and is experimentally verifiable. As per this theory free electrons gain kinetic energy (i.e. velocity) from source of energy in the system; in the instant case it is electric field. During this acceleration they encounter another atom or molecule and collision occurs. During collision it imparts its kinetic energy and comes to state of rest, and the process of successive collision continues until it leaves the conducting medium, which is replaced by another free electron supplied by the source as per principle of electrical neutrality in current electricity. These discussion lead to that there is mean free path λ between two successive collisions over which it experiences force $F = Eq \Rightarrow F = \frac{V}{l}q$ and an acceleration as per Law of Motion $a = \frac{F}{m} \Rightarrow a = \frac{Vq}{ml}$. Thus mean velocity attained by free electron as per equation of motion $v^2 = 2as \Rightarrow v = \sqrt{2\left(\frac{Vq}{ml}\right)\lambda}$. Thus current is $I = Nq\sqrt{2\left(\frac{Vq}{ml}\right)\lambda}$…(1) It is given that by some process number of collisions are decreased from $n \rightarrow n'$ such that $n > n'$ and therefore mean free path $\lambda = \frac{l}{n}$ would increase to $\lambda' = \frac{l}{n'}$ i.e. $\lambda' > \lambda$…(2) Accordingly, in changed conditions average velocity would be $v' = \sqrt{2\left(\frac{Vq}{ml}\right)\lambda'}$. Accordingly, $I' = Nq\sqrt{2\left(\frac{Vq}{ml}\right)\lambda'}$…(3). Combing (1), (2) and (3) it is concluded that $I' > I$as provided in **option (a) is the answer.**

I-19	Resistance of a resistor is a physical property and expressed as $R = \frac{\rho L}{A}$ …(1)where ρ is resistivity of material of the resistor, L and A are length and area of cross-section, respectively, of material forming resistor. In the instant case only resistivity of two materials used in forming resistors A and B is given and there is no information about their length and area of cross-section in (1) required to compare the two resistances R_A and R_B. Hence, **answer is option (d).**
I-20	Resistivity ρ is property of the material forming resistor and likewise conductivity is $\sigma = \frac{1}{\rho}$. Therefore, the required product $\rho \times \sigma = \rho \times \frac{1}{\rho}$=1 is unity, a constant. Hence it does not depend on any of the physical parameter. Hence, **answer is option (d).**
I-21	Resistivity ρ is property of the material forming resistor and likewise conductivity is $\sigma = \frac{1}{\rho}$. Therefore, the required product $\rho \times \sigma = \rho \times \frac{1}{\rho}$=1 is unity, a constant. Hence it does not depend on any of the physical parameter. It is given that temperature of the metallic resistor is increased. Since temperature is a physical quantity, it would change both ρ and σ such that t $\rho \times \sigma = 1$, reminds unchanged . Hence, **answer is option (c).**
I-22	Electric current in an electric circuit when power is supplied by the battery given positive charges flow from its positive terminal to negative terminal, and thus maintains principle of electrical neutrality. Thus inside the battery charges flow from negative terminal to its positive terminal. On the contrary, when battery is charged, current is pumped into the battery such that current enter battery from positive terminal and leaves it from negative terminal. Thus, inside the battery charges flow from positive terminal to negative terminal. Therefore, considering bidirectional flow of charges inside the battery **answer is option (b).**
I-23	n a non-ideal battery whose emf is E. is that whose internal resistance is $r \neq 0$. Further, Power supplied by battery is $P_B = EI$…(1). It is given that a resistor is connected to the battery whose resistance is say R. Then as per Ohm's law current through resistance is $I = \frac{E}{R+r}$, ..(2). Accordingly, $P_B = E\left(\frac{E}{R+r}\right) \Rightarrow P_B = \frac{E^2}{R+r}$ …(3) V is the potential difference across the resistance. And thermal energy per second developed in resistance is equal to $P_R = VI$…(4). Here, V is the potential difference across the resistance. Rewriting (2), $E = I(R + r) \Rightarrow V = E - Ir$…(5). Thus combing (4) and (5), $P_R = (E - Ir)\frac{E}{R+r} \Rightarrow P_R = \frac{E^2}{R+r} - \left(\frac{E}{R+r}\right)^2 r$...(6). It is seen from (3) and (6) that $P_B \neq P_R$. Hence a**nswer for non-ideal battery is No**. but discriminant of the two is $\Delta P = P_B - P_R = \left(\frac{E}{R+r}\right)^2 r$. Accordingly, $\Delta P = 0\|_{r=0}$for **an ideal battery** $P_B = P_R$, a**nswer is yes.**
I-24	Let charge of electron is $q_- =- e$ and charge of metal ions be s $q_+ = e$, likewise their masses be m_e and m_i, respectively. We know that mass of proton $m_p = 1836m_e$. Therefore, it can be conveniently inferred that $m_i \gg m_e$. Moreover, size of ion is also larged than that of electrom and therefore comparing mean free path λ of electron and ion it is safe to consider that $\lambda_e > \lambda_i$…(1) In the context of the above and electro-mechanics, flow of current in conductors is being analyzed as under – Current in a metallic resistor is rate of flow of charges $I = \frac{dQ}{dt}$ in given space of the conductor. As per electro-mechanics (combining electricity with mechanics) $I = \frac{dQ}{dt} = Nqv$. Here N is number of electrons per unit volume and q is charge of electron and v is average velocity of electron Flow current is explained with Brownian Motion of free electrons, and is experimentally verifiable. As per this theory free electrons gain kinetic energy (i.e. velocity) from source of energy in the system; in the instant case it is electric field. During this acceleration they encounter another atom or molecule and collision occurs. During collision it imparts its kinetic energy and comes to state of rest, and the process of successive collision

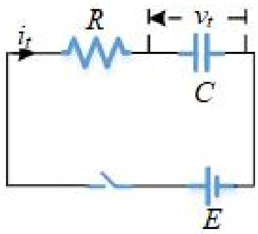
	continues until it leaves the conducting medium, which is replaced by another free electron supplied by the source as per principle of electrical neutrality in current electricity. These discussion lead to that there is mean free path λ between two successive collisions over which it experiences force $F = Eq \Rightarrow F = \frac{V}{l}q$ and an acceleration as per Law of Motion $a = \frac{F}{m} \Rightarrow a = \frac{Vq}{ml}$. Thus mean velocity attained by free electron as per equation of motion $v^2 = 2as \Rightarrow v = \sqrt{2\left(\frac{Vq}{ml}\right)\lambda}$. Thus, mean kinetic energy of a charged particle is $K = \frac{1}{2}mv^2 \Rightarrow K = \frac{1}{2}m \times 2\left(\frac{Vq}{ml}\right)\lambda \Rightarrow K = \frac{Vq}{l}\lambda$. ..(2) Accordingly, from (2) kinetic energy of electron is $K_1 = \frac{Vq}{l}\lambda_e$ and ion is $K_2 = \frac{Vq}{l}\lambda_i$. Thus comparing the kinetic energies $\frac{K_e}{K_i} = \frac{\lambda_e}{\lambda_i}$…(3). Combining (1) and (3), it is concluded that $K_1 > K_2$,. Thus, **option (c) is correct.**
I-25	Thermal energy$H$ developed in a resistor in time t $H = VIt \Rightarrow H = (IR)It \Rightarrow H = I^2Rt$. Given that the two resistors $R_1 = R$ and $R_2 = 2R$ are connected in series and hence current through them would be same. Accordingly, heat developed in the two resistors is $H_1 = I^2R_1t \Rightarrow H_1 = I^2Rt$ and $H_2 = I^2R_2t \Rightarrow H_1 = 2I^2Rt$. Thus comparing the heat developed in two resistors $\frac{H_1}{H_2} = \frac{I^2Rt}{2I^2Rt} \Rightarrow H_1{:}\,H_2 :: 1{:}\,2$. Thus, **answer is (a)**
I-26	Given that two resistors are connected in parallel in an electric circuit. Hence potential difference V across them would be equal such that $V = I_1R = I_2 \times 2R \Rightarrow \frac{I_1}{I_2} = 2$. Thermal energy$H$ developed in a resistor in time t is $H = VIt \Rightarrow H = (IR)It \Rightarrow H = I^2Rt$. Thus, heat developed in the two resistors is $H_1 = {I_1}^2Rt$ and $H_2 = {I_2}^2(2R)t$. Accordingly, $\frac{H_1}{H_2} = \frac{{I_1}^2Rt}{2{I_2}^2Rt} \Rightarrow \frac{H_1}{H_2} = \frac{1}{2}\left(\frac{I_1}{I_2}\right)^2 \Rightarrow \frac{H_1}{H_2} = \frac{1}{2}(2)^2 \Rightarrow H_1{:}H_2 :: 2{:}\,1$. Thus, **answer is (b).**
I-27	Given is a uniform wire of resistance $R = 50\ \Omega$. It is cut in $n = 5$ equal parts each having resistance $r = \frac{R}{n} = 10\ \Omega$...(1). These parts are connected in parallel and hence equivalent resistance $\frac{1}{R'} = \sum_{i=1}^{5}\frac{1}{r_i}$…(2). Further, from (1) we have $r_i = r = 10\ \Omega$. Accordingly, from (2) we have $\frac{1}{R'} = \frac{5}{10} \Rightarrow R' = 2\Omega$. Thus, **answer is (a).**
I-28	Analyzing each statement separately as under – **Statement 1**: As per Kirchhoff's junction law (KJL) $\sum_{k=i}^{n} I_k$ where I_k is current in k[th] branch of a junction having *n* branches. In KJL sign convention used is all currents entering node are positive and all currents leaving the node are positive. Currents is $I = \frac{dQ}{dt} \Rightarrow Idt = dq \Rightarrow \int_0^t Idt = \int_0^Q dq = Q$….(1) Accordingly, uniform time integral of each current would be $\int_0^t I_k dt = Q_k$ and it would lead to $\int_0^t (\sum_{k=i}^{n} I_k)dt = 0 = \sum_{k=1}^{n} Q_k$. Thus, **it follows conservative nature of charge.** **Statement 2**: As per Kirchhoff's loop law (KLL) $\sum_{k=i}^{n} V_k$ where V_k is current in k[th] branch of a junction having *n* branches. In KLL sign convention potential difference across a resistor (voltage drop) in the direction of loop current are negative and emf (voltage rise) of sources in the direction of loop current are positive. Further, potential difference $V = \int_a^b \vec{E}.d\vec{x}$ between two points say A and B is the amount of work done in moving a unit charge between them. Moreover, from (1), total work done is $U = VQ$. Considering conservative nature of charge, also called ***principle of electrical neutrality (PEN)*** of electric current, amount of work done in moving charge Q from A to B is $U_1 = (V_B - V_A)Q$, then in the circuit as per PEN $U_2 = (V_A - V_B)Q$. Thus, total work done in flow of charge Q around the circuit is $U = U_1 + U_2 \Rightarrow U = (V_B - V_A)Q + (V_A - V_B)Q \Rightarrow U = 0$. Thus, **it follows conservative nature of electric field.** Thus each of the statements A and B are correct. Hence, **answer is option (a).**

I-29	Given that two identical batteries each having emf E and internal resistance r are connected in series. It implies that their polarities are unidirectional and hence equivalent emf $E' = E + E = 2E$. Thus, $E' > E$. Accordingly, **statement A is True**. Since batteries are connected in series and hence their equivalent internal resistances will also be in series. Accordingly, $r' = r + r = 2r \Rightarrow r' > r$. Thus, **statement B is wrong.** Combined inferences of the two statements are provided in **option (b), is the answer.**
I-30	Given that When two identical batteries having emf say E and internal resistance say r are connected in parallel between nodes say A and B, as shown in the figure In this context the given statements are being analyzed.  **Statement 1:** The their emf of both the batteries are unidirectional and equivalent emf between nodes A and B is equal $E' = E \Rightarrow E' \ngtr E$. Hence, t**his statement is wrong** **Statement 2:** Internal resistances of the two batteries between the nodes A and B are in parallel and equivalent resistance $r' = \frac{r}{2} \Rightarrow r' < r$. **This statement is true.** These conclusion match with **option (c), is the answer.**
I-31	Ammeter is used to measure large current i. It has an inbuilt galvanometer which is sensitive to small current $i_G \ll i$…(1). Thus, use of galvanometer in ammeter is possible only when a small resistance called shunt is connected in parallel to the galvanometer, as per assembly of ammeter shown in the figure. This is based on principle that current i_G through galvanometer (having internal resistance r_G) and i_S through shunt having resistance r_S connected parallel to the galvanometer is such that $i_G r_G = i_S r_S \Rightarrow \frac{i_S}{i_G} = \frac{r_G}{r_S} \Rightarrow \frac{i_S + i_G}{i_G} = \frac{r_G + r_S}{r_S} \Rightarrow \frac{i}{i_G} = \frac{r_G + r_S}{r_S}$…(2). Comparing (1) and (2), it is evident that $r_S \ll r_G + r_S \Rightarrow r_S \ll r_G$…(3). It also implies that $r_G \approx r_G + r_S$…(4)  It is seen that both r_G and r_S are connected in parallel and therefore net resistance of ammeter $r = \frac{r_G \times r_S}{r_G + r_S}$. Using (3) and (4), $r \approx \frac{r_G \times r_S}{r_G} \Rightarrow r \approx r_S$ which small and leads to small current through galvanometer as given in (1). Thus, $i = i_S + i_G \Rightarrow i \approx i_S$ as per **option (d), is the answer.**
I-32	Voltmeter is used to measure large potential difference say $V = iR$ It has an inbuilt galvanometer which is sensitive to small current $i_G \ll i$…(1). Thus, use of galvanometer in voltmeter possible only when a large resistance is connected in series with the galvanometer, as per assembly of voltmeter shown in the figure. This is based on principle that current i_G through galvanometer (having internal resistance r_G and a large resistance $r_S \gg R \gg r_G$ and connected in series with the galvanometer) and i_R through resistance R across which potential difference is to be measured galvanometer is such that $i_G(r_G + r_S) = i_R R \Rightarrow \frac{i_G}{i_R} = \frac{R}{r_G + r_S} \Rightarrow \frac{i_G}{i_R} = \frac{R}{r_S} \Rightarrow \frac{i}{i_G} = \frac{r_G + r_S}{r_S}$…(2).  As per Kirchhoff's Junction Law $i_R = i - i_G$, and using $i_R \approx i \Rightarrow V = iR$ …(3). Thus, with $r_S \gg$ the voltage to be measured not appreciably change, as provided in **option (d), is the answer.**

I-33

A capacitor-charging circuit as stated in the problem is shown in the figure. The capacitor is initially discharged i.e. at $t = 0$ charge on capacitor is $Q_t = 0$, accordingly voltage across capacitor at the instant is $Q_t = Cv_t \Rightarrow v_0 = \frac{Q_O}{C} = 0$.

After closing the switch at any instant t, as per Kirchhoff's Loop Law in the circuit we have $E - i_tR - v_t = 0 \Rightarrow E - v_t = i_tR \Rightarrow E - \frac{Q_t}{C} = \frac{dQ_t}{dt}R \Rightarrow dQ_t = \left(\frac{EC-Q_t}{CR}\right)dt$ ...(1). Here, $i_t$is capacitor charging current at any instant t.



Equation (1) is a linear differential equation and its solution is $\int \frac{dQ_t}{EC-Q_t} = \frac{1}{CR}\int dt + K$...(2) here K is an integration constant whose value can be determined based on initial condition on solution of (2). Substituting $u = EC - Q_t \Rightarrow du = -dQ_t$ in (2) we have $-\int \frac{du}{u} = \frac{t}{CR} + K \Rightarrow -\ln u = \frac{t}{CR} + K$...(3). Reversing the substitution in (3) $\ln(EC - Q_t) = -\frac{t}{CR} - K \Rightarrow EC - Q_t = K'e^{-\left(\frac{t}{CR}\right)}$...(4) here $K' = e^{-K}$ is another form of the constant K.

Using the initial condition in (4), $EC - 0 = K'e^{-\left(\frac{0}{CR}\right)} \Rightarrow K' = EC$...(5). Combining (4) and (5), $Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)$...(6). Accordingly, charging current is $i_t = \frac{d}{dt}Q_t = \frac{d}{dt}EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}$ ...(7).

Results of (6) and (7) , where a term common to both is $f(t) = e^{-\left(\frac{t}{CR}\right)}$, with time stamps given in the problem are brought out in table below –

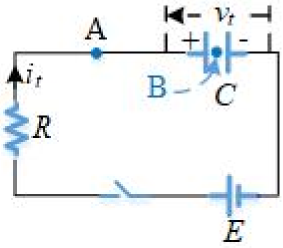
S No	Time (t)	$f(t) = e^{-\left(\frac{t}{CR}\right)}$	$\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$	**Conclusion**
1	0	1	$Q_0 = EC(1 - 1) = 0$	Initial condition
2	10 s	$\boldsymbol{e^{-\left(\frac{10}{CR}\right)}}$	$Q_{10} = EC\left(1 - e^{-\left(\frac{10}{CR}\right)}\right)$	$Q_1 = Q_{10} - Q_0 = Q_{10} = EC\left(1 - e^{-\left(\frac{10}{CR}\right)}\right)$ $Q_2 = Q_{20} - Q_0 = EC\left(e^{-\left(\frac{10}{CR}\right)} - e^{-\left(\frac{20}{CR}\right)}\right)$ Observation of factor $f(t) = e^{-\left(\frac{t}{CR}\right)}$ reveals that with increase of t it decreases exponentially. Accordingly, if $Q_1 - Q_2 > 0$ then $Q_1 > Q_2$. Thus, $Q_1 - Q_2 = EC\left(\left(1 + e^{-\left(\frac{20}{CR}\right)}\right) - 2e^{-\left(\frac{10}{CR}\right)}\right) > 0$, which would be more clear on using the numerical values of C and R, hence $\boldsymbol{Q_1 > Q_2}$.
3	10+10=20s	$\boldsymbol{e^{-\left(\frac{20}{CR}\right)}}$	$Q_{20} = EC\left(1 - e^{-\left(\frac{20}{CR}\right)}\right)$	
4	t_1 s	$\boldsymbol{e^{-\left(\frac{t_1}{CR}\right)}}$	$Q_{t_1} = EC\left(1 - e^{-\left(\frac{t_1}{CR}\right)}\right)$	$Q_{t_1} = EC\left(1 - e^{-\left(\frac{t_1}{CR}\right)}\right) = 10$ μC $Q_{t_1+t_2} - Q_{t_1} = EC\left(e^{-\left(\frac{t_1}{CR}\right)} - e^{-\left(\frac{t_1+t_2}{CR}\right)}\right) = 10$ μC. With the analysis at (2) and (3), it is concluded, which would become more clear on using the numerical values of C and R, that $\boldsymbol{t_1 < t_2}$
5	$t_1 + t_2$ s	$\boldsymbol{e^{-\left(\frac{t_1+t_2}{CR}\right)}}$	$Q_{t_1+t_2}$ $= EC\left(1 - e^{-\left(\frac{t_1+t_2}{CR}\right)}\right)$	

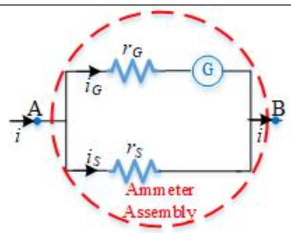
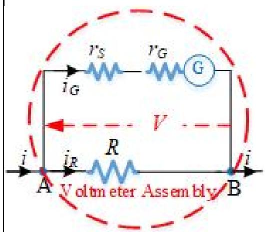
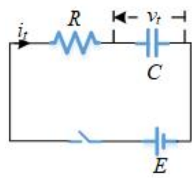
Thus summary of the conclusion in the table is $Q_1 > Q_2, t_1 < t_2$, provided in **option (b) is the answer**.

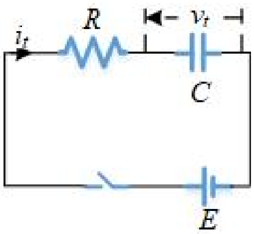
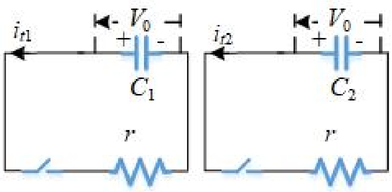
N.B.: This problem involves solution of linear differential equation of first order and its simplified version is brought out above for ready reference.

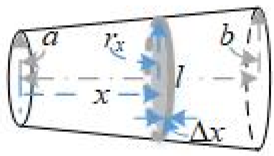
I-34

Thermo-emission phenomenon cause emission of electron by the filament. The emitted electron experience force $\vec{F} = q\vec{E}$. Given in the figure is $\vec{E} = E(-\hat{x})$ and charge of electron is $q = (-e)$, here (-)ve sign signifies nature of charge and e is quantity of charge of electron. Thus, $\vec{F} = (-e)E(-\hat{x}) \Rightarrow \vec{F} = eE\hat{x}$. Accordingly,

	acceleration of the electron as per Law of Motion $\vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{a} = \frac{eE\hat{x}}{m} \Rightarrow a = \frac{eE}{m}$…(1)It is seen that acceleration is in $\hat{x}$ and point B w.r.t. the filament, at which velocity of electron $u = 0$, is along $\hat{x}$ of point A or $x_B > x_A$…(2) Therefore, as per third equation of motion equation of motion $v_A^2 = 2ax_A \Rightarrow v_A^2 = 2\left(\frac{eE}{m}\right)x_A$. Likewise, $v_B^2 = 2\left(\frac{eE}{m}\right)x_B$. Therefore, $\frac{v_B^2}{v_A^2} = \frac{2\left(\frac{eE}{m}\right)x_B}{2\left(\frac{eE}{m}\right)x_A} \Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{x_B}{x_A}}$…(3). Combining (2) and (3) $v_B > v_A$as provided in **option (a), is the answer**.
I-35	At $t = 0$ when capacitor is connected to battery $Q_t = 0$, at $t > 0$ capacitor gets gradually charged and develops voltage across it as shown in the figure. voltage across capacitor at the instant is $Q_t = Cv_t \Rightarrow v_t = \frac{Q_t}{C}$…(1). When capacitor is fully charged to Q, voltage across it is $v_{t\to\infty} = E = \frac{Q}{C}$…(2). During charging flow of electrons constitute current in the circuit from (+)ve plate of battery to (-)ve plate of battery and it follows principle of conservation of charge. Thus there is no current through point B between the plates of the capacitor, as provided in **option (b), is the answer**.  When capacitor is fully charged at $t \to \infty$ it develops a potential difference $v_{t\to\infty} = E$, and then current in the circuit is $i_{t\to\infty} = \frac{E - v_{t\to\infty}}{R} \Rightarrow i_{t\to\infty} = \frac{E-E}{R} \Rightarrow i_{t\to\infty} = 0$. Therefore, current through circuit, point A is a part of it, as long as $0 < t < \infty$, that is while the capacitor is charging, as provided in option (c), is the answer. Thus, **answers is options (b) and (c).**
I-36	In a conductor electrons perform Brownian motion such that over a long time mean velocity of electron is zero and is expressed as $\sum \vec{v}_i = 0$…(1), but for an electron $\vec{v}_t = \int \vec{x}dt = 0$…(2). Further, over Δt since $\vec{v}_t = \frac{\Delta \vec{x}}{\Delta t} \neq 0$…(3). In this context each of the given options are being analyzed – **Option (a):** Since free electrons perform Brownian motion, and motion is associated with movement of electrons. Hence, **this option is wrong.** **Option (b):** As per discussion in option, electron changes place and thus the scalar $\Delta x \neq 0$ and hence average speed $= \frac{\Delta x}{t} \neq 0$. Hence, **this option is wrong.** **Option (c):** As per (2) **this option is correct.** **Option (d):** As per (1) average of velocities of all free electrons is zero and hence **this option is correct.** Thus, **answer is option (c) and (d).**
I-37	Analyzing each of the options- **Option (a):** Drift speed depends upon number of collisions per unit time which changes with temperature, as a result of heating. Hence, **this option is wrong.** **Option (b):** As per Brownian motion each electron accelerates from rest, while traversing mean free path, and imparts kinetic energy to colliding particles and comes to rest. And such collisions continue successively. The kinetic energy lost by free electrons contributes to heat developed in resistor. Thus with increase in number of collisions resistivity ρ increases. Hence, **this option is wrong.** **Option (c):** Resistance of resistor is $R = \rho\frac{L}{A}$. Thus in light of conclusion at (b) above resistance will also change on heating. Hence, **this option is wrong.** **Option (d):** When a resistor of resistance R is connected to a battery of potential difference Velectric current $I = \frac{V}{R}$ is established in it. The electric current follows principle of Conservation of Charges, Hence, number of free electrons in the resistor remains unchanged. Hence, **this option is correct.** Thus, **answer is option (d).**

I-38	Resistivity ρ of a conductor and its conductivity $\sigma = \frac{1}{\rho}$…(1). Therefore, ratio $\frac{\rho}{\sigma} = \frac{\rho}{\frac{1}{\rho}} \Rightarrow \frac{\rho}{\sigma} = \rho^2$…(2). Given that resistivity of a conductor changes. It is known that the change is increase. Therefore, the ratio $\frac{\rho}{\sigma}$ would also increase in square proportion as per (2). Hence, correct **option (a) is the answer.**
I-39	Given that current is passing through a wire of non-uniform cross-section. It is known that flow of current is rate drift of free-electron cloud while maintaining principle of conservation of charge. Thus, analyzing each of the given option – **Option (a):** The charge crossing in a given time interval will not change in accordance with the principle of conservation of charge, stated above. Hence, **this option is correct**. **Option (b):** Current through a conductor is $I = \frac{nAl}{t}$…(1), here n is number of electrons per unit volume, A is area of cross section of conductor, l is the length of conductor and t is time taken by electron cloud to traverse length l. Here, $s = \frac{l}{t}$ is the drift speed of free electron, accordingly $I = nAs$…(2). Thus for a given current I through a conductor having electron density n, which is characteristic to a conductor material, $s \propto \frac{1}{A}$…(3). Given that cross-section of conductor is non-uniform and hence drift speed s would change along the conductor. Hence, **this option is wrong.** **Option (c):** Using (1) current density $\delta = \frac{I}{A} = \frac{nl}{t} = ns \Rightarrow \delta \propto s$. In light conclusion at option (b) current density would change along the conductor. Hence, **this option is wrong.** **Option (d):** Free electron density n is characterstic to material which is uniform in the conductor. **Hence, this option is correct.** Thus, **answer is options (a) and (d).**
I-40	Ammeter is used to measure large current, while and voltmeter is used to measure high voltage. Both the instruments have an inbuilt galvanometer which is a highly sensitive to small current. Moreover, its resistance r_g is small. Thus, making possible to convert a galvanometer into an ammeter requires to split large current i into small current i_g within the range of galvanometer, and shunt current i_s as shown in the figure, is possible with the use of a shunt whose resistance $r_s \ll r_g$. Thus, for ammeter **option (a) is correct** while the **option (b) is wrong**.   As regards using galvanometer in voltmeter, small through it is achieved by connecting a high resistance r_s in series with the galvanometer such that $i_G = \frac{V}{r_s + r_G}$ is within the range of the galvanometer. Thus, for voltmeter **option (c) is wrong** while the **option (d) is correct**. Thus, **answer is options (a) and (d).**
I-41	A capacitor of capacitance $C = 500\ \mu\text{F}$ is connected to a battery through a $R = 10\ \Omega$ resistor as shown in the figure. Charge stored in capacitor in time $t_1 = 5$ s, from a generic formula, is $Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow Q_5 = EC\left(1 - e^{-\left(\frac{5}{CR}\right)}\right)$…(1) Using the available data $Q_5 = EC\left(1 - e^{-\left(\frac{5}{(500\times10^{-6})10}\right)}\right) \Rightarrow Q_5 \to EC(1-0) \Rightarrow Q_5 = EC$…(2)i.e. the capacitor in the circuit gets fully charged. Therefore, for any time $t > 5$s additional charge $\Delta Q = Q_t - Q_5 = 0$ …(3)stored in the capacitor is zero. Thus, using (2) and (3) for all options are correct, is the answer.  **N.B.:** This is a good problem on application of mathematical formulation of charging of capacitor.

I-42	It is a problem on discharge of capacitor. Given that two capacitors of capacitance $C_1 = 1\times10^{-6}$ F and $C_2 = 2\times10^{-6}$ F are separately connected for a long time $t \gg$ to a common battery of emf say E. Therefore, charge on the capacitors capacitance 1 μF and another capacitor C_2 of capacitance 2 μF are separately charged by a common battery for a long time as shown in the figure. Thus, charge on the capacitors would be $Q_t = EC\left(1-e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow Q_t = EC$ …(1). Accordingly, $Q_1 = EC_1 \Rightarrow Q_1 = (1\times 10^{-6})E$ and $Q_2 = (2\times10^{-6})E$.   These two charged capacitors are then separately discharged through equal resistors of resistance say r by connecting them $t = 0$, as shown in the figure. At this instance voltage across each is $V_0 = E = \frac{Q_1}{C_1}$. Applying Kirchhoff's Loop Law in the circuit we have $E + i_t r = 0$. It leads to $i_t r = -E \Rightarrow \frac{d}{dt}Q_t = -\frac{Q_t}{rC} \Rightarrow \frac{1}{Q_t}dQ_t = -\frac{dt}{rC}$. Integrating both sides we get $\int\frac{1}{Q_t}dQ_t = \frac{1}{rC}\int dt \Rightarrow \ln Q_t = -\frac{t}{rC} + K \Rightarrow Q_t = e^{-\left(\frac{t}{rC}+K\right)} \Rightarrow Q_t = K'e^{-\frac{t}{rC}}$…(2). Here, both K and K' are integrating constants whose value depends upon initial condition $t = 0$, such that $K' = Q_0 = EC$ ...(3). Accordingly, combining (1), (2) and (3), $Q_{1t} = EC_1e^{-\frac{t}{rC_1}}$ and $Q_{2t} = EC_2e^{-\frac{t}{rC_2}}$…(4). Therefore, discharge currents of two capacitors are $i_{1t} = \frac{d}{dt}Q_{1t} \Rightarrow i_{1t} = \frac{E}{r}e^{-\frac{t}{rC_1}}$.; likewise, $i_{2t} = \frac{E}{r}e^{-\frac{t}{rC_2}}$…(5). In this context, each of the option is being analyzed separately – **Option (a):** The current in each of the two discharging circuit at $t = 0$ are $i_{1\_0} = \frac{E}{r}e^{-\frac{0}{rC_1}} = \frac{E}{r}$ and likewise $i_{2\_0} = \frac{E}{r} \Rightarrow i_{1\_0} = i_{2\_0} = \frac{E}{r}$. Thus, this option suggesting $i_{1\_0} = i_{2\_0} = 0$ **is wrong.** **Option (b):** Taking forward analysis in option (a), $i_{1\_0} = i_{2\_0} = \frac{E}{r} \neq 0$. Thus, **this option is correct.** **Option (c):** Unequal currents in the two discharging circuits at $t = 0$ in light conclusion at (b), **this option is wrong.** **Option (d):** Let t_1 is the time taken by C_1 to lose loses 50% is $Q_{1t_1} = 0.5\times Q_{1_0}$is, as per (4), $0.5 = e^{-\frac{t_1}{rC_1}}$. …(6). Likewise, for capacitor C_2 in time t_1, $0.5 = e^{-\frac{t_2}{rC_2}}$…(7). Combining (6) and (7), $e^{-\frac{t_1}{rC_1}} = e^{-\frac{t_2}{rC_2}}$. Applying, theory of indices $\frac{t_1}{C_1} = \frac{t_2}{C_2} \Rightarrow t \propto C$. Accordingly, with the given $C_1 < C_2$ we have $t_1 < t_2$. Thus, **this option is correct.** **Thus, answer is options (b) and (d).** **N.B.:** This problem involves solution of linear differential equation of first order formulated for charge and discharge-current on a discharging capacitor, and is slightly different than charging of capacitor.
I-43	The problem is primarily of dimensional analysis with given that $Q(t) = At^2 + Bt + C$…(1). Further, $I(t) = \frac{d}{dt}Q(t)$…(2). In SI system dimension of current $[Current]$ = I, therefore, using (2), $[Charge]$ = IT…(3). Each part of the problem is analyzed separately. **Part (a):** In dimensional analysis all addend must have same dimension. Accordingly, $[At^2]$ = IT $\Rightarrow$ $[A]$T^2 = IT $\Rightarrow$ $[\boldsymbol{A}]$ = **IT^{-1}**. Further, $[Bt]$ = IT $\Rightarrow$ $[B]$T = IT $\Rightarrow$ $[\boldsymbol{B}]$ = **I .** Likewise, $[C]$ = IT $\Rightarrow$ $[\boldsymbol{C}]$ = **IT.** Thus, **answer of this part is IT^{-1}, I, IT.** **Part (b):** Given are the numerical values $A = 5$, $B = 3$ and $C = 1$ current at $t = 5$ s is obtained using (1) and (2). Accordingly, $I(t) = 2At + B \Rightarrow I(t) = 2\times5\times5 + 3 = 53$ A, is the answer.

	Thus, **answers are (a) IT^{-1}, I, IT (b) 53 A.**
I-44	Current is $I = \frac{\Delta Q}{\Delta t}$. Given that in time $\Delta t = 1$s, $\Delta Q = n \times e$, where $n = 2.0 \times 10^{16}$ electron per second i.e. in Δt and charge of electron $e = -1.6 \times 10^{-19}$C. Since, in the question only magnitude of current is asked and not the direction and hence $I = \frac{(2.0\times10^{16})\times\lvert-1.6\times10^{-19}\rvert}{1} \Rightarrow I = \mathbf{3.2 \times 10^{-3}}$**A is the answer.**
I-45	Current is $I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta Q = I\Delta t$, here it is important to remember that Q is in Coulomb and time t is in seconds. Therefore, with the given data $\Delta Q = (2.0 \times 10^{-6}) \times (5 \times 60) \Rightarrow \Delta Q = \mathbf{6.0 \times 10^{-4}}$ **C is the answer**.
I-46	Current is $i = \frac{\Delta Q}{\Delta t} \Rightarrow Q = \int_0^t idt$. Given that $i = i_0 + \alpha t \Rightarrow i = 10 + 4t$. Therefore, $Q = \int_0^{10}(10 + 4t)dt$, it leads to $Q = \left[10t + \frac{4t^2}{2}\right]_0^{10} \Rightarrow Q = 10 \times 10 + 2 \times (10 \times 10) \Rightarrow Q =$ **300 C is the answer**.
I-47	
I-48	We know that $R = \rho\frac{L}{A}$ …(1). It given that resistance $R = 100\ \Omega$, length of wire $L = 1$ m and radius of wire $r = 0.1 \times 10^{-3}$m, hence area of wire $A = \pi r^2 \Rightarrow A = \pi(0.1 \times 10^{-3})^2 \Rightarrow A = \pi \times 10^{-8}$. Using the available data in (1), $\rho = \frac{RA}{L} \Rightarrow \rho = \frac{100\times(\pi\times10^{-8})}{1} \Rightarrow \rho = \boldsymbol{\pi \times 10^{-6}}$ **Ωm is the answer.**
I-49	We know that $R = \rho\frac{L}{A}$ …(1), given that $R = 100\ \Omega$. The wile is melted and recast into a wire of length $L' = 2L$…(2). In the process, area of cross-section of the wire would change to A', while the volume V of material of resistivity ρ remains unchanged. Accordingly, $V = LA = L'A' \Rightarrow A' = \frac{LA}{L'} \Rightarrow A' = \frac{LA}{2L} \Rightarrow A' = \frac{A}{2}$…(3) Using (1) with (2) and (3), resistance of the recast-wire is $R' = \rho\frac{L'}{A'} \Rightarrow R' = \frac{2L}{\frac{A}{2}} \Rightarrow R' = 4\left(\rho\frac{L}{A}\right) \Rightarrow R' = 4R$. Using the given value $R' = 4 \times 100 =$ **400 Ω is the answer.**
I-50	Given that length of wire is $L = 4$ m and area of cross-section of wire $A = 1 \times 10^{-6}$ m^2 is carrying current $I = 2$ A. We know that $I = \frac{dQ}{dt} \Rightarrow Q = It$ C …(1). With the given electron density $\alpha = 10^{29}$per meter cube, number of electrons in the wire would be $n = \alpha LA \Rightarrow n = 10^{29} \times 4 \times 1 \times 10^{-6} = 4 \times 10^{23}$. With the knowledge that charge of electron $e = -1.6 \times 10^{-19}$C, the charge of free electrons is $Q = n\lvert e\rvert \Rightarrow Q = (4 \times 10^{23}) \times (1.6 \times 10^{-19}) \Rightarrow Q = 6.4 \times 10^4$C…(2). Combining (1) and (2), $2t = 6.4 \times 10^4$ here t average time taken by a free electron in the wire to pass through its length. Accordingly, $t = \frac{6.4\times10^4}{2} = \mathbf{3.2 \times 10^4}$**s** it is equivalent to $\approx$ **8.9 hours is the answer.**
I-51	We know that $R = \rho\frac{L}{A}$ and given that $R = 10^3\ \Omega$, cross-sectional area $A = 0.01 \times 10^{-6}$m^2 and resistivity of copper used to create the resistance $\rho = 1.7 \times 10^{-8}\Omega$m. Therefore, desired is length of wire $L = \frac{RA}{\rho}$. Using the available data $L = \frac{10^3(0.01\times10^{-6})}{1.7\times10^{-8}} = 0.6 \times 10^3$m or **0.6 km is the answer**.
I-52	 We know that resistance of a conductor is $R = \rho\frac{L}{A}$…(1), here L is length of a wire of uniform cross-section A of a material of resistivity ρ. In the instant case given that circular cross-sectional area varies such that $a \le r \le b$ over a length l. This can be analyzed considering thin discs of thickness Δx at a distance x from the end of radius a stacked together to constitute the given shape. Therefore, $\Delta R = \rho\frac{\Delta x}{A_x}$…(2). Here, geometrically $A_x = \pi r_x^2$, here $r_x = a + \frac{b-a}{l}x$…(3). Accordingly, $R = \rho\int_0^l \frac{1}{\pi r_x^2}dx$…(4). Using (3) for substitution in the integral $dr_x = \frac{b-a}{l}dx \Rightarrow dx = \frac{l}{b-a}dr_x$ and modifying the limit, $x = 0 \Rightarrow r_0 = a$ and $x =$

$l \Rightarrow r_l = b$. Thus, rewriting (4), $R = \frac{\rho l}{\pi(b-a)} \int_a^b \frac{1}{r_x^2} dr_x \Rightarrow R = \frac{\rho l}{\pi(b-a)} \int_a^b \frac{1}{r_x^2} dr_x \Rightarrow R = \frac{\rho l}{\pi(b-a)} \left[-\frac{1}{r_x}\right]_a^b$. It solves into $R = \frac{\rho l}{\pi(b-a)} \left[\frac{1}{r_x}\right]_b^a \Rightarrow R = \frac{\rho l}{\pi(b-a)} \left[\frac{1}{a} - \frac{1}{b}\right] \Rightarrow R = \frac{\rho l}{\pi(b-a)} \left(\frac{b-a}{ab}\right) \Rightarrow R = \boldsymbol{\frac{\rho l}{\pi ab}}$ **is the answer.**

I-53 Given that resistance of a wire $R = 10^3\Omega$ is connected across a supply having potential difference $V = 20$ V. This problem has two parts. And second part uses information in first part,

Part (a): As per Ohm's Law current through the wire is $I = \frac{V}{R} = \frac{20}{10^3} = 20 \times 10^{-3}$A…(1). Further, $I = \frac{Q}{t}$ here, Q is the quantity of charge transferred in time t. Therefore, charge transferred in 1 sec is $Q = I$…(2). In the instant case, and specific to metal, current is due to flow of electrons, therefore, $I = n|e|$…(3). Here, n is number of electrons and magnitude of charge of electron $|e| = 1.6 \times 10^{-19}$C. Combining (1), (2) and (3), $n(1.6 \times 10^{-19}) = 20 \times 10^{-3} \Rightarrow n = \frac{20\times10^{-3}}{1.6\times10^{-19}} \Rightarrow n = \mathbf{1.25 \times 10^{17}}$**is answer of part (a).**

Part (b): Current density in a wire is $\delta = \frac{I}{A}$. Here, area of cross-section of wire is $A = \pi r^2$, where given that $r = 0.1 \times 10^{-3}$m, and current is available at (1). Accordingly, $\delta = \frac{20\times10^{-3}}{\pi(0.1\times10^{-3})^2} \Rightarrow \boldsymbol{\delta = 6.4 \times 10^6}$ **A/m²**.

I-54 Electric field along a uniform wire current carrying $I = 1$A is $E = \frac{V}{L}$…(1). Here, potential difference across a wire as per Ohm's Law is $V = IR$…(2), here resistance of wire is $R = \rho\frac{L}{A}$…(3). Here, resistivity of wire is $\rho = 1.7 \times 10^{-8}\ \Omega$m and area of cross-section of wire is $A = 2.0 \times 10^{-6}$.

Combining (1), (2) and (3), $E = \frac{I\left(\rho\frac{L}{A}\right)}{L} \Rightarrow E = \frac{I\rho}{A}$. Using the given data $E = \frac{1\times\left(1.7\times10^{-8}\right)}{2.0\times10^{-6}} = 0.85 \times 10^{-2}$ or **8.5 mV/m**

I-55 Electric field along a uniform wire current carrying $I = 1$A is $E = \frac{V}{L}$…(1). Here, potential difference across a wire as per Ohm's Law is $V = IR$…(2), here resistance of wire is $R = 5.0\ \Omega$…(3). Here, resistivity of wire is $\rho = 1.7 \times 10^{-8}\ \Omega$m and area of cross-section of wire is $A = 2.0 \times 10^{-6}$.

Combining (1), (2) and (3), $E = \frac{IR}{L} \Rightarrow E = \frac{10\times5.0}{2.0} = \mathbf{25}$ **V/m.**

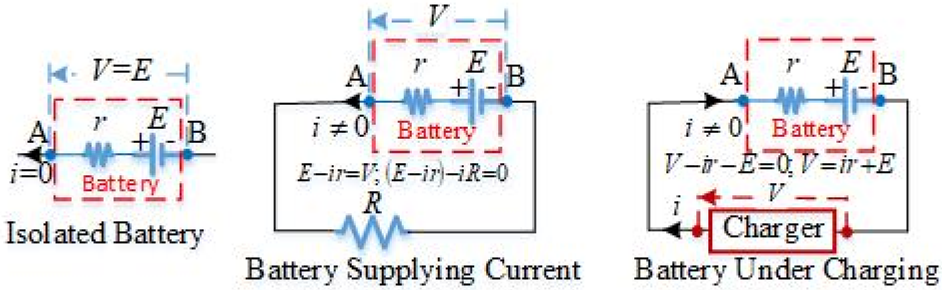
I-56 Resistance of a conductor varies with temperature such that $R_{t_2} = R_{t_1}\left(1 + \alpha(t_2 - t_1)\right)$…(1)

Given that at $t_1 = 20\ ^0$C, for iron wire $R_{t_1-Fe} = 3.9\ \Omega$ and for copper wire $R_{t_1-Cu} = 4.1\ \Omega$, thermal coefficient of resistivity of the two wires is $\alpha_{Fe} = 5.0 \times 10^{-3}$ K^{-1} and $\alpha_{Cu} = 4.0 \times 10^{-3}$ K^{-1}. Therefore, temperature t_2 at which the two resistances would be equal is obtained using (1).

Accordingly, $R_{t_2-Fe} = R_{t_2-Cu} \Rightarrow R_{t_1-Fe}(1 + \alpha_{Fe}\Delta t) = R_{t_1-Cu}(1 + \alpha_{Cu}\Delta t)$, here $\Delta t = t_2 - t_1$…(2). It is solves into $R_{t_1-Fe} \times \alpha_{Fe} \times \Delta t - R_{t_1-Cu} \times \alpha_{Cu} \times \Delta t = R_{t_1-Cu} - R_{t_1-Fe} \Rightarrow \Delta t = \frac{R_{t_1-Cu} - R_{t_1-Fe}}{R_{t_1-Fe}\times\alpha_{Fe} - R_{t_1-Cu}\times\alpha_{Cu}}$. Using the available data $\Delta t = \frac{4.1-3.9}{3.9\times(5.0\times10^{-3})-4.1\times(4.0\times10^{-3})} \Rightarrow \Delta t = \frac{0.2\times10^3}{19.5-16.4} = 64.5^0$C**.** Using (2) $t_2 = \Delta t + t_1 \Rightarrow t_2 = 64.5 + 20 = \mathbf{84.5^0}$**C have is the answer.**

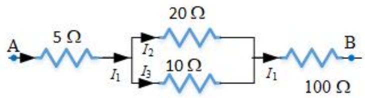
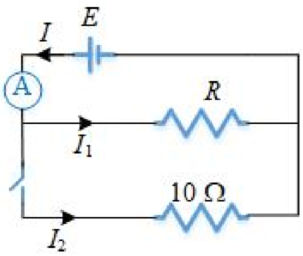
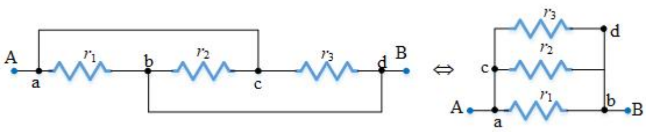
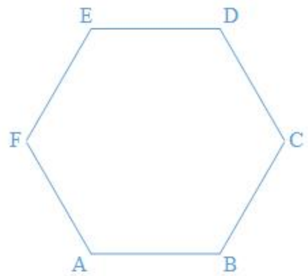
I-57 Given that ammeter is accurate while voltmeter has an error ϵ. With this two set of data are given as under –

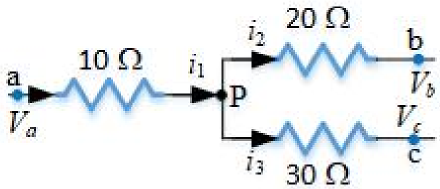
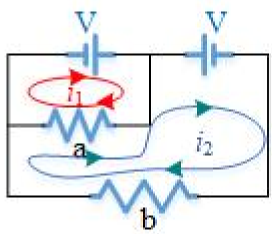
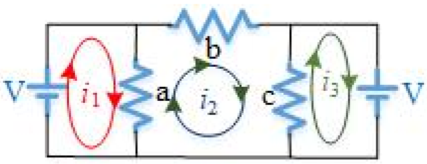
Set of Data	Ammeter Reading (A)	Voltmeter Reading (V)	Potential Difference (PD)	$R = \frac{PD}{I}$
Set 1	1.75	14.4	$14.4 - \epsilon$	$\frac{14.4 - \epsilon}{1.75}$
Set 2	2.75	22.4	$22.4 - \epsilon$	$\frac{22.4 - \epsilon}{2.75}$

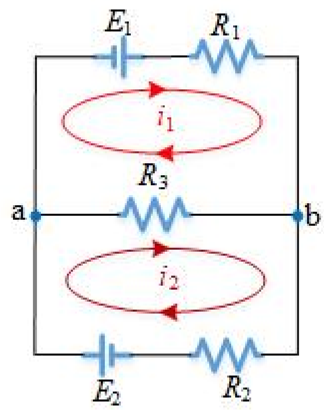
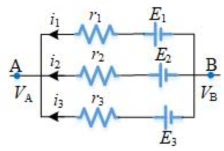
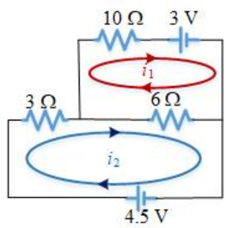
	Since measurements are on same resistor and hence equation results of the two sets is $\frac{14.4-\epsilon}{1.75}=\frac{22.4-\epsilon}{2.75}\Rightarrow(22.4-\epsilon)\times1.75=(14.4-\epsilon)\times2.75\Rightarrow(2.75-1.75)\epsilon=14.4\times2.75-24.4\times1.75\Rightarrow\epsilon=39.6-39.2=\mathbf{0.4}$ **V. is the answer.**
I-58	When switch is open, the voltmeter, having a high resistance, reads emf of the battery $E=\mathbf{1.52\ V}$. But, when switch is closed, current read by ammeter reads $I=1.0$ A. Therefore, as per Ohms law is $I=\frac{E}{r}$ and reading of the voltmeter is potential difference $V=1.45$ V across the battery $V=E-Ir\Rightarrow r=\frac{E-V}{I}$. Thus using the available data $r=\frac{1.52-1.45}{1.0}=\mathbf{0.07\ \Omega}$. **Thus answer is 1.52 V, 0.07 Ω.**
I-59	In a circuit a battery of emf $E=6.0$ V having internal resistance $r=1$ W, potential difference across the battery $V=E-Ir=5.8$ V across the battery when connected to an external resistance R. Thus circuit equation as per Kirchhoff's Circuit law is $E-I(r+R)=0\Rightarrow E-V=Ir$. Using the available data $I=\frac{E-V}{r}\Rightarrow I=\frac{6.0-5.8}{1}\Rightarrow I=0.2$ A. Further, using the loop equation, $R=\frac{E-Ir}{I}\Rightarrow R=\frac{V}{I}\Rightarrow R=\frac{5.8}{0.2}=\mathbf{29\ \Omega}$ **is the answer.**
I-60	When a battery of emf E is charged an external source of potential difference $V>E$ is connected with reverse polarity across the battery to pump current in the battery. Accordingly, the circuit equation is $V-Ir-E=0\Rightarrow r=\frac{V-E}{I}$. Using the available data $r=\frac{7.2-6.0}{2.0}\Rightarrow r=\mathbf{0.6\ \Omega}$ **is the answer.** **N.B.:** Circuit equation for a battery under charging needs to be noted carefully since in this case current is pumped into the battery causing a potential difference, higher than its emf, across it. Whereas when battery is supplying current potential difference across the battery is lower than its emf. The concept is shown in the figure. 
I-61	Given that an accumulator battery of emf $E=6$ V is fully discharged. In this state internal resistance of battery is $r_D=10\ \Omega$. But, when battery is fully charged its internal resistance reduces to $r_C=1\ \Omega$.. The discharged battery is connected to a charger which maintains a constant potential difference $V=9$ V. The look equation of a battery under charging is $V-ir-E=0\Rightarrow i=\frac{V-E}{r}$...(1) With this taking each part separately – **Part (a):** Just after connection of a discharged battery $r=r_D$, there current through the battery is circuit current as per (1). Accordingly using the available data $i=\frac{9-6}{10}=\mathbf{0.3}$ **A is the answer.** **Part (b): When battery is completely** charged battery $r=r_C$, there current through the battery is circuit current as per (1). Accordingly using the available data $i=\frac{9-6}{1}=$ **3 A is the answer.** Thus, **answer is (a) 0.3 A, (b) 3 A**
I-62	A close examination of circuits reveal that in first circuit the two batteries each of emf $E=6$ Vare connected in series with net emf in the circuit $E_1=E+E\Rightarrow E_1=2E$, and hence their internal resistances $r=5\ \Omega$ will also be in series $r_1=r+r=2r$. In turn supplying current to external resistance R. Thus, $i_1=\frac{E_1}{R+r_1}\Rightarrow i_1=\frac{2E}{R+2r}$...(1).

	In second circuit both the batteries are connected in parallel and hence emf in the circuit is $E_2 = E$, while potential difference across external resistance is $V_2 = i_2R$. The internal resistances of the two batteries shall also be in parallel such that $r_2 = \frac{r\times r}{r+r} \Rightarrow r_2 = \frac{r}{2}$. Hence, current in the circuit $i_2 = \frac{E}{R+\frac{r}{2}} \Rightarrow i_2 = \frac{2E}{2R+r}$…(2). *Expression in this case can be verified by applying Kirchhoff's Loop Law also.* Accordingly, the desired ratio using (1) and (2) is $\frac{i_1}{i_2} = \frac{\frac{2E}{R+2r}}{\frac{2E}{2R+r}} \Rightarrow \frac{i_1}{i_2} = \frac{2R+r}{R+2r}$…(3). Using this final form with the available data for each of the value of R – **Case (a):** $R = 0.1\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 0.1+5}{0.1+2\times 5} = 0.51$ **Case (b):** $R = 1\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 1+5}{1+2\times 5} = 1$ **Case (c):** $R = 10\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 10+5}{10+2\times 5} = 1.25$. **N.B.:** Solving problem algebraically and substituting numerical values at last stage greatly simplifies calculation. All that it needs is proficiency and patience to handle algebraic expressions.
I-63	All cells in the combination are of emf E and internal resistance r. The combination comprises of n_2 lines each $n_1$cells in series. Therefore, emf of each line would be$E' = n_1 \times E$ …(1), and series resistance of each line would be $r' = n_1 \times r$…(2) Further, each line is identical and hence emf of the circuit will be $E" = E'$ but internal resistance of batteries would be equivalent resistance of parallel combination of r' with n_2 branches such that $N = n_1n_2$ …(3), and thus will be $r" = \frac{r'}{n_2}$…(4). With this analysis taking each part separately – **Part (a):** Current through external resistance $I" = \frac{E"}{R+r"}$. Using (1), (2) and (4), $I" = \frac{E'}{R+\frac{r'}{n_2}} \Rightarrow I" = \frac{n_1E}{R+\frac{n_1r}{n_2}}$ **is the answer.** **Part (b):** For maximum current through R, using (3), result of part (a) can be written as $I" = \frac{n_1E}{R+\frac{n_1r}{\frac{N}{n_1}}}$, it reduces to $I" = \frac{Nn_1E}{NR+(n_1)^2r}$. Therefore, applying maxima $\frac{d}{dn_1}I" = \frac{d}{dn_1}\left(\frac{Nn_1E}{NR+(n_1)^2r}\right) = 0$. Here, a bit of differential calculus is applied, $\frac{d}{dn_1}\left(\frac{Nn_1E}{NR+(n_1)^2r}\right) = \frac{\frac{d}{dn_1}(Nn_1E)\times(NR+(n_1)^2r)-(Nn_1E)\times\frac{d}{dn_1}(NR+(n_1)^2r)}{(NR+(n_1)^2r)^2} = 0$. It solves into $NE \times (NR + (n_1)^2r) - (Nn_1E) \times 2n_1r = 0 \Rightarrow NR + (n_1)^2r - 2(n_1)^2r = 0 \Rightarrow NR = (n_1)^2r \Rightarrow n_1n_2R = (n_1)^2r \Rightarrow \boldsymbol{n_2R = n_1r}$ **is the answer.** Thus, **answers are (a)** $\boldsymbol{\frac{n_1E}{R+\frac{n_1r}{n_2}}}$ **(b)** $\boldsymbol{rn_1 = Rn_2}$
I-64	The system can be considered as a battery of emf $E = 100$ V and internal resistance $r = 10\times 10^3$ W supplying current to an external resistance $R \leq 100\ \Omega$.. Thus current supplied by the battery is $I = \frac{E}{R+r}$. In this since is in the denominator and applying the specified constraint $I = \frac{100}{100+10\times 10^3} \Rightarrow I = \frac{1}{101} \approx 10$ mA and if $R = 0$ the current is $I = \frac{100}{0+10\times 10^3} \Rightarrow I = \frac{1}{100} \approx 10$ mA. Thus constant current upto two SDs is **10 mA is the answer.** **N.B.: (a)** This is a good example to numerically understand how constant current source of small current can be achieved. **(b)** In this a large r in series with battery is considered as an internal resistance of battery is only a circuit analogy for analysis. In fact internal resistance is of small value.

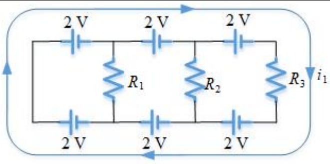
I-65	Resistances 20 Ω and 30 Ω carrying current $I_1 = 2.4$ A measured by A₁ and I_2 measured by A₂ are connected in parallel and hence current distribution between them would be $\frac{I_1}{I_2} = \frac{R_2}{R_1} \Rightarrow I_2 = I_1\left(\frac{R_1}{R_2}\right)$. Using the available data $I_2 = 2.4\left(\frac{20}{30}\right) = \mathbf{1.6}$ **A is current measured by ammeter A₂**. As regards current measured by ammeter A₃ as per Kirchhoff's Node law $I_3 = I_1 + I_2 \Rightarrow I_3 = 2.4 + 1.6 = \mathbf{4.0}$ **A is the answer.** Thus, **answers are 1.6 A, 4.0 A.**
I-66	Current through the circuit will be $I = \frac{V}{R}$…(1) here $V = 5.5$ V and equivalent resistance of the circuit $R = \frac{10\times20}{10+20} + r \Rightarrow R = \frac{20}{3}$…(2), here r is the value of variable resistant in the range $0 \le r \le 30$…(3) Thus, combining (1), (2) and (3), minimum current in circuit will be for maximum value of $r = 30$ Ω in (1) and it is $I_{min} = \frac{5.5}{\frac{20}{3}+30} \Rightarrow I_{min} = \frac{5.5\times3}{110} = \mathbf{0.15}$ **A.** For maximum current will be for maximum value of $r = 0$ in (1) and it is $I_{min} = \frac{5.5}{\frac{20}{3}+0} \Rightarrow I_{min} = \frac{5.5\times3}{20} =$ $\mathbf{0.83}$ **A.** Thus, **answers are 0.15 A, 0.83 A**
I-67	Three bulbs each of resistance $R = 180$ W are connected in parallel across a battery of $V = 60$ V and can be switched on selectively, in three ways are as under – **Way (a):** All bulbs are switched on hence equivalent resistance would be $R_a = \frac{R}{3}$, therefore current $I_a = \frac{V}{R_a} \Rightarrow I_a = \frac{60}{\frac{180}{3}} = \mathbf{1.0}$ **A, is the answer**. **Way (b):** Two bulbs are switched on hence equivalent resistance would be $R_b = \frac{R}{2}$, therefore current $I_b = \frac{V}{R_b} \Rightarrow I_b = \frac{60}{\frac{180}{2}} = \mathbf{0.67}$ **A, is the answer**. **Way (c):** Only one bulbs is switched on hence equivalent resistance would be $R_c = R$, therefore current $I_c = \frac{V}{R_c} \Rightarrow I_b = \frac{60}{180} = \mathbf{0.33}$ **A, is the answer**. Thus, **answers are (a) 1.0 A, (b) 0.67 A, (c) 0.33 A** **N.B.:** Principle of SDs is used while reporting the answer.
I-68	Given that three resistances $R_1 = 20$ Ω, $R_2 = 50$ Ω, and $R_3 = 100$ Ω. When resistances are connected in parallel the equivalent resistance $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ …(1), is less than the minimum resistance (R_1) among given resistors. But, when resistances are connected in series then equivalent resistance $R_S = R_1 + R_{21} + R_3$ is more than the maximum resistance (R_3). Accordingly, minimum resistance $R_{min} = R_P$ and from (1), $\frac{1}{R_P} = \frac{1}{20} + \frac{1}{50} + \frac{1}{100} = \frac{8}{100} \Rightarrow R_P = \frac{100}{8} \Rightarrow R_{min} =$ $\mathbf{12.5}$ **Ω**. And maximum resistance $R_{max} = R_S$ and from (2), $R_S = 10 + 50 + 100 \Rightarrow R_S = 180 \Rightarrow R_{max} =$ **180 Ω**. Thus, **answers are 12.5 Ω, 170 Ω.**
I-69	The two filaments of a bulb of 15 V are of $P_1 = 5$ W and $P_2 = 10$ W, when they operate in parallel in parallel $P_3 = P_1 + P_2 \Rightarrow P_3 = 5 + 10 = 15$ W. Power of a bulb is $P = VI = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$. Accordingly, resistance of filament $P_1 = 5$ W is $R_1 = \frac{V^2}{P_1} \Rightarrow R_1 = \frac{15^2}{5} = \frac{225}{5} = 45$ Ω. Likewise, resistance of filament $P_2 = 10$ W is $R_2 = \frac{V^2}{P_2} \Rightarrow R_2 = \frac{225}{10} = 22.5$ Ω. Thus, **answers are 45 Ω, 22.5 Ω.**

I-70	Given that current through resistance of 5 kΩ is $I_1 = 12 \times 10^{-3}$A. Therefore, as per Kirchhoff's Node Law, as shown in the figure $I_1 = I_2 + I_3$...(1), here I_2 and I_3 are currents through resistances of 20 Ω and 10 Ω, respectively, connected in parallel. Therefore, the distribution of I_1 in I_2 and I_3 would be $\frac{I_2}{I_3} = \frac{10}{20}$, Applying componendo and using (1) $\frac{I_2+I_3}{I_3} = \frac{10+20}{20} \Rightarrow \frac{I_1}{I_3} = \frac{30}{20} \Rightarrow I_3 = \frac{2}{3} \times (12 \times 10^{-3}) = 8 \times 10^{-3}$A or **8 mA through 10 Ω resistor.** Accordingly, using (1), $I_2 = I_1 - I_3 \Rightarrow I_2 = 12 \times 10^{-3} - 8 \times 10^{-3} \Rightarrow I_2 = 4 \times 10^{-3}$A or **4 mA through 20 Ω resistor**. And current **through 100 Ω resistor** as per (1) is $I_1 = I_2 + I_3 = 12 \times 10^{-3}$A or **12 mA.**  The equivalent resistance of the series-parallel-series combination of resistors is $R = 5 + \frac{10\times20}{10+20} + 100 = 111.7\ \Omega$. Therefore, voltage drop across the combination of resistor is $V = I_1 R = (12 \times 10^{-3}) \times 111.7 = \mathbf{1.34\ V.}$ Thus, **answers are 4 mA in 20 Ω resistor, 8 mA in 10 Ω resistor, and 12 mA in 100 Ω resistor, 1.34 V.**
I-71	Given that an ideal battery of emf say E send current $I = I_1 = 5$A...(1), through a resistor R. When another resistor of resistance $R_2 = 10\ \Omega$ the battery current becomes $I = I_1 + I_2 = 6$ A...(2).  The battery being ideal, as shown in the figure, the current I_1 remains unchanged on connection of resistor of R_2. Therefore, current of $I = 6$ A would distribute, as per Kirchhoff's Node Law and Ohm's Law such that $\frac{I_1}{I-I_1} = \frac{10}{R} \Rightarrow \frac{5}{6-5} = \frac{10}{R} \Rightarrow R = \frac{10}{2} =$ **2 Ω is the answer.**
I-72	Given connection of resistors is redrawn in the figure by identifying each resistor as r_1 , r_2, and r_3, though each of them equal to r, with nodes at ends of each.  Connecting wires are taken to be of zero resistances, and accordingly the equivalent connection of resistors is reorganized along the given connection. It is seen that the three resistances $r_1 = r_2 = r_3 = r$ are connected in parallel between given nodes A and B. Therefore, equivalent resistance is $R = \frac{r}{3}$ **is the answer.**
I-73	Given that a wire of resistance 15.0 Ω is bent into a regular hexagon ABCDEFA. Thus each side of the hexagon would be of resistance $r = \frac{15.0}{6} = 2.5\Omega$. Resistance between pair of any two point (vertices) of the polygon (a closed shape) would form a parallel circuit. Accordingly, resistance between pair of points specified in each part is as under –  **Part (a):** Between points A and B is a parallel combination of r (corresponding to side AB) and 5r (corresponding to resistances five sides BC, CD, DE, EF and FA in series). Thus equivalent resistance would be $R = \frac{r\times5r}{r+5r} \Rightarrow R = \frac{5}{6} \times 2.5 \Rightarrow R = \mathbf{2.08\ \Omega.}$ **Part (b):** Between points A and C is a parallel combination of $2r$ (corresponding to resistances of two sides AB and BC in series) and 4r (corresponding to resistances four sides CD, DE, EF and FA in series). Thus equivalent resistance would be $R = \frac{2r\times4r}{2r+4r} \Rightarrow R = \frac{8}{6} \times 2.5 \Rightarrow R = \mathbf{3.33\ \Omega.}$ **Part (c):** Between points A and D is a parallel combination of $3r$ (corresponding to resistances of three sides AB, BC and CD in series) and $3r$ (corresponding to resistances three sides, DE, EF and FA in series). Thus equivalent resistance would be $R = \frac{3r\times3r}{3r+3r} \Rightarrow R = \frac{3}{2} \times 2.5 \Rightarrow R = \mathbf{3.75\ \Omega.}$ Thus, **answers are (a) 2.08 Ω, (b) 3.33 Ω, (c) 3.75 Ω**
I-74	When switch S is open as per part (a) the two resistances are in series combination with $R_a = 10 + 20\ \Omega$. Therefore, current in the circuit would be $I_a = \frac{V}{R_a} \Rightarrow I_a = \frac{3}{30} = \mathbf{0.1A}$.

	But, when switch S is closed, resistance of 20 Ω is bypassed and circuit resistance is $R_b = 20\ \Omega$. Therefore, current in the circuit would be $I_b = \frac{V}{R_b} \Rightarrow I_a = \frac{3}{10} = \mathbf{0.3A}$. Thus, **answers are (a) 0.1 A, (b) 0.3 A**
I-75	The two batteries are connected with opposite polarities. Hence, circuit emf is $E = 4 - 2 = 2$ V. The resistor of resistance **4 Ω in the upper arm** of the circuit is short-circuited and hence current through it **is zero**. Thus, resistance of 4 Ω and 6 Ω in left and right arms, respectively, form a series combination of the with circuit resistance $R = 4 + 6 = 10\ \Omega$. Therefore, current through these two resistances would be $I = \frac{E}{R}$. Using the available data $I = \frac{2}{10} = \mathbf{0.2}$ **A.** Thus, **answer is Zero in upper 4 Ω resistor and 0.2 A in the rest two.**
I-76	Given network is redrawn with currents i_1, i_2 and i_3 in each branch and voltages $V_a = 30$ V, $V_b = 12$ V and $V_c = 2$V at the specified nodes a, b and c. A new node mark P is junction of three branches.  Applying Kirchhoff's Junction Law at node P $i_1 - i_2 - i_3 = 0$…(1). Further forming voltage equations for branches a-P-b we have $V_a - 10 \times i_1 - 20 \times i_2 - V_b = 0 \Rightarrow 18 - 20 \times i_1 - 20 \times i_2 = 0$ …(2). Likewise, for branch a-p-c, $V_a - 10 \times i_1 - 30 \times i_3 - V_c = 0 \Rightarrow 28 - 10 \times i_1 - 30 \times i_3 = 0$…(3). Thus, there are three equation and three unknown currents i_1, i_2 and i_3. Eliminating i_1 using (1) and (2) $[(1) \times 10 + (2)]$ we have $18 - 30 \times i_2 - 10 \times i_3 = 0$…(4). Likewise, Eliminating i_1 using (1) and (3) $[(1) \times 10 + (2)]$ we have $28 - 10 \times i_2 - 40 \times i_3 = 0$…(5). Now, eliminating i_2 using (4) and (5) $[(5) - (4) \times 4]$ we have $28 - 72\ - 10 \times i_2 + 120 \times i_2 = 0$. It solves into $110 \times i_2 = 112 \Rightarrow i_2 = \frac{44}{110} \Rightarrow \boldsymbol{i_2} = \mathbf{0.4}$ **A.** Using value of i_2 in, any of (4) or (5), say (4), $18 - 30 \times 0.4 - 10 \times i_3 = 0 \Rightarrow 10 \times i_3 = 6 \Rightarrow \boldsymbol{i_3} = \mathbf{0.6}$ **A.** Using values of i_2 and i_3 in (1), $i_1 - 0.4 - 0.6 = 0 \Rightarrow \boldsymbol{i_1} = \mathbf{1}$ **A.** Thus, **answer is 1 Amp through 4 Ω , 0.4 A through 20 Ω and 0.6 A through 30 Ω** **N.B.:** This is a simple case of application of Kirchhoff's Laws.
I-77	Given circuits can be easily analyzed using Kirchhoff's Laws, for which loops with their respective loop currents are marked in the circuit to facilitate the analysis. Taking each circuit separately –  **Circuit (A):** Loop equation in Loop-1 is $V - (i_1 - i_2)R_a = 0 \Rightarrow (i_1 - i_2)10 = 10 \Rightarrow i_1 - i_2 = 1$ …(1). Likewise, in Loop-2 is $-V - i_2R_b - (i_2 - i_1)R_a = 0 \Rightarrow i_1 - 2i_2 = 1$…(2).  Solving (1) and (2) we have, current **through resistance b** is $i_2 = \mathbf{0}$ and **through resistance a** is $i_1 = \mathbf{1}$ A. **Circuit (B):** This circuit is analyzed as per foot-note and leads to current through resistance b, $i_2 = \frac{V-V}{R_b} \Rightarrow i_2 = \frac{10-10}{10} \Rightarrow \boldsymbol{i_2} = \mathbf{0}$. Accordingly, current through resistance a is $i_i - i_2 = \frac{V}{R}$. Using the available data $i_i - 0 = \frac{10}{10} \Rightarrow \boldsymbol{i_i} = \mathbf{1}$A. Thus, in both the circuits**, current through a is 1A and through b Zero.** **N.B.:** This can be an objective question, which can be solved by observation that across b two equal batteries are connected in series with opposite polarities. Hence potential difference across resistance b, is zero and hence current is zero. While, resistance a is connected directly across a battery and current thought it is, as per Ohm's Law is $I = \frac{E}{R}$

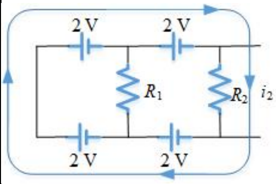
I-78	Pictorially circuits (A) and (B)look different but graphically or electrically (considering interconnection of elements of circuit) both of them are identical. Therefore, $V_a - V_b$ would be same for both the circuits. Therefore, circuit (a) is being analyzed. This circuit is redrawn identifying two loops with their respective currents $i_1$and i_2.  The required potential difference R_3as per Ohms law is $V_a - V_b = \Delta V = (i_2 - i_1)R_3 \Rightarrow (i_2 - i_1) = \frac{\Delta V}{R_3}$…(1). Applying Kirchhoff's Loop Law in Loop-1, $E_1 - i_1R_1 - (i_1 - i_2)R_3 = 0$. Using (1) it transforms into $E_1 - i_1R_1 - \left(-\frac{\Delta V}{R_3}\right)R_3 = 0 \Rightarrow i_1 = \frac{E_1+\Delta V}{R_1}$…(2) Likewise, in Loop-2 $E_2 - i_2R_2 - (i_2 - i_1)R_3 = 0$. Using (1) it transforms into $E_2 - i_2R_2 - \left(\frac{\Delta V}{R_3}\right)R_3 = 0 \Rightarrow i_2 = \frac{E_2-\Delta V}{R_2}$…(3). Combining (1), (2) and (3), $\frac{E_2-\Delta V}{R_2} - \frac{E_1+\Delta V}{R_1} = \frac{\Delta V}{R_3} \Rightarrow \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\Delta V = \frac{E_2}{R_2} - \frac{E_1}{R_1} \Rightarrow \Delta V = \frac{\frac{E_2}{R_2}-\frac{E_1}{R_1}}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$. Using value of ΔV in (1), $V_a - V_b = \frac{\frac{E_2}{R_2}-\frac{E_1}{R_1}}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$ **is the answer.** **N.B.:** Simple application of Kirchhoff's Law, it requires care in using sign convention.
I-79	Let potential at A be V_A and at B be V_B and potential difference between the two points be $\Delta V = V_A - V_B$…(1) Here, $V_A > V_B$ since positive terminals of each of the battery in the parallel branches is towards A. Now writing voltage equation for each of the parallel branch having batteries of emf E_1, E_2 and E_3, carrying currents i_1, i_2 and i_3 as shown in the figure, and using the available data –  **Top Branch:** $E_1 - i_1r - \Delta V = 0 \Rightarrow 3 - i_1 = \Delta V \Rightarrow i_1 = 3 - \Delta V$…(2). **Middle Branch:** $E_2 - i_2r - \Delta V = 0 \Rightarrow 2 - i_2 = \Delta V \Rightarrow i_2 = 2 - \Delta V$…(3). **Bottom Branch:** $E_3 - i_3r - \Delta V = 0 \Rightarrow 1 - i_3 = \Delta V \Rightarrow i_3 = 1 - \Delta V$ …(4). Further, the network is isolated and hence applying Kirchhoff's Node Law, $i_1 + i_2 + i_3 = 0$…(5). Combining (1)…(4), we have $(3 - \Delta V) + (2 - \Delta V) + (1 - \Delta V) = 0 \Rightarrow 3\Delta V = 6 \Rightarrow V_A - V_B =$ **2 V.** Using value of ΔV in (2)…(4), $\boldsymbol{i_1} = 3 - 2 =$ **1 A**, $\boldsymbol{i_2} = 2 - 2 = \mathbf{0}$ and $\boldsymbol{i_3} = 1 - 2 = \mathbf{-1}$ **A.** **Thus, answer is 2 V, $\boldsymbol{i_1} = \mathbf{1}$ A, $\boldsymbol{i_2} = \mathbf{0}$, $\boldsymbol{i_3} = \mathbf{-1}$ A.**
I-80	The given circuit is redrawn by identifying two loops with respective currents i_1 and i_2 as shown in the figure. Writing loop equations as per Kirchhoff's Circuit Law as under –  **Loop 1:** Loop current is i_1; $-3 - (i_1 - i_2)6 - i_1 \times 10 = 0 \Rightarrow -16i_1 + 6i_2 = 3$…(1). **Loop 2:** Loop current is i_2; $4.5 - i_2 \times 3 - (i_2 - i_1)6 = 0 \Rightarrow 6i_1 - 9i_2 = -4.5$…(2). To solve these equations $(1) \times 3 + (2) \times 2$ leads to $(-48 + 12)i_1 = 0$. This $i_1 = 0$ A is the answer. **N.B.:** Simple application of Kirchhoff's Law, it requires care in using sign convention as under- **1)** Generally each loop is identified with a loop current. Emf of all batteries (or voltage sources) with polarities aligned in the direction of the loop current is taken to be (+)ve. Thus, emf of batteries with polarities against direction of loop current is assigned (-)ve. **2)** In a resistor, loop current of the loop for which KLL equation being written is taken (+)ve and any other current in the resistor is assigned sign relative to the loop current. **3)** Potential difference across a resistor, with the consideration of current through it as per (2) above is assigned (-)ve.
I-81	The given circuit, with each resistance $R_1 = R_2 = R_3 = 1\ \Omega$, as shown in the figure, is analyzed in three stages using Kirchhoff's Loop Law as under –

Loop current i_1: It has only one resistance with six batteries each of 2 V, three in the upper line are in direction opposite to the loop current, while the three in bottom in direction of the loop current. Accordingly, the loop equation is:



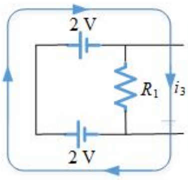
$$(-2)+(-2)+(-2)-i_1R_3+2+2+2=0 \Rightarrow$$

$\boldsymbol{i_1=0}$.



Loop Current i_2: It has Four batteries, two in the upper line are in direction opposite to the loop current, while the two in bottom in direction of the loop current. Accordingly, the loop equation is:

$(-2)+(-2)-i_2R_2+2+2=0 \Rightarrow \boldsymbol{i_2=0}$.



Loop Current i_3: It has tour batteries, one in the upper line are in direction opposite to the loop current, while the other in bottom in direction of the loop current. Accordingly, the loop equation is:

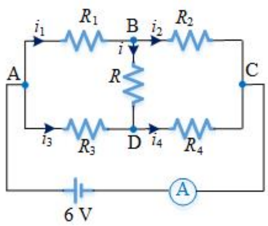
$(-2)-i_3R_1+2=0 \Rightarrow \boldsymbol{i_3=0}$.

Thus, answer is Zero.

N.B.: A close observation of the given circuit has been simplified with simple equation of external loops, instead of considering internal loops. It is specific to the problem and cannot be taken as a general rule.

I-82

The given circuit is redrawn, in a generic sense, with resistances of 10 Ω shown algebraically such that $R_1=R_2=R_3=R_4=10\,\Omega$, with branch currents i_1, i_2, i_3, i_4 and i, and nodes A,B,C and D identified, as shown in the figure. It is also given that current $i=0\ldots(1)$, through resistance R. With this this value of R is to be determined.



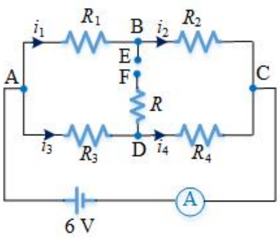
The current i through R, as per Ohm's Law, is $i=\frac{V_B-V_D}{R}\ldots(2)$, considering assigned direction to the current from node B to D. Combining (1) and (2), $\frac{V_B-V_D}{R}=0$ is possible when - **(a)** Either $V_B-V_D=0\ \ldots(3)$, or **(b)** $R\to\infty$.

Taking possibility (a) and applying voltage equation to branches AB, BC, AD and DC we have –

$V_A-V_B=i_1R_1\ldots(4)$, $V_A-V_D=i_3R_3\ldots(5)$, $V_B-V_C=i_2R_2\ldots(6)$, and $V_D-V_C=i_4R_4\ldots(7)$.

And as per Kirchhoff's current law at node B $i_1=i+i_2$; using (1) $i_1=i_2\ldots(8)$., and at node D $i_3+i=i_4$, again using (1) $i_3=i_4\ldots(9)$.

Using (3), (4) and (5) , $V_B-V_D=i_1R_1-i_3R_3=0 \Rightarrow \frac{i_1}{i_3}=\frac{R_3}{R_1}\ldots(10)$. Likewise, using (3), (6) and (7), we have $V_B-V_D=i_2R_2-i_4R_4=0 \Rightarrow \frac{i_2}{i_4}=\frac{R_4}{R_2}\ldots(11)$.



Now combining (11) with (8) and (9), $\frac{i_1}{i_3}=\frac{R_4}{R_2}\ldots(12)$. Comparing (10) and (12) $\frac{R_3}{R_1}=\frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2}=\frac{R_3}{R_4}.\ \ldots(13)$.

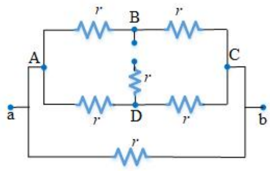
Using the given values of resistances $\frac{R_1}{R_2}=\frac{R_3}{R_4}=\frac{10}{10}=1$ for $i=0$ as given at (1). This is similar to absence of connection between B and D, i.e. open circuit with nodes E and F, as shown in the figure. Therefore, **any value of R will do** is the answer.

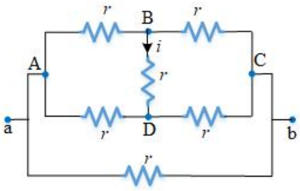
N.B.: The given network is different graphical representation of Wheatstone Bridge, Appendix-II. It is a ***very important circuit*** extensively used for different application in different names. It was developed by Samuel Hunter Christie (year 1833) and modified by Sir Charles Wheatstone year 1943) and known after his name.

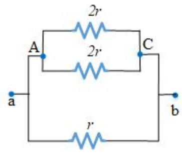
I-83

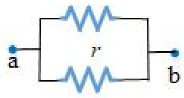
Each of the circuit is being analyzed separately,

Circuit A: Electrical equivalent of the circuit is shown in the figure. It is seen that network between nodes A-B-C-D-A is a Wheatstone bride where $\frac{R_{AB}}{R_{AD}} = \frac{R_{BC}}{R_{DC}} = 1$, elaborated in Appendix-I. Thus it satisfies condition current though branch B-D is $i = 0$. it is analogues to branch an open circuit as shown in the figure. Thus it simplifies into a circuit as shown in the figure taking series resistances of branches A-B-C and A-D-C. Next step is equivalent resistance of two parallel resistors of resistances $2r$. It further simplifies into $\frac{2r \times 2r}{2r+2r} = r$. In its last form it is parallel combination of two resistors of resistance r as shown in the circuit. Thus, the network reduces to an equivalent resistance $R = \frac{r \times r}{r+r} = \frac{r}{2}$ **is the answer.**

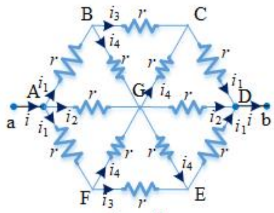




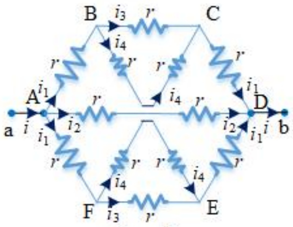




Circuit B: The network of resistances at a first glance is combination of delta-connection. But, all resistance being equal to r adds symmetry to the network both about a-b and central line perpendicular to a-b and passing through G. This symmetry simplifies problem instead of using Δ-Y transformations as per Appendix-III. Accordingly, currents through different branches have been indicated in figure of stage 1.

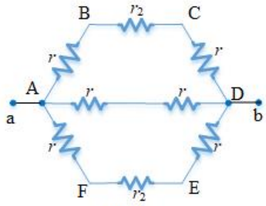


Stage 1



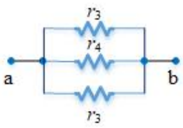
Stage 2

Considering mirror symmetry of the currents as shown in the figure which is – (i) current i_2 entering node G in branch AG is equal to current leaving the node in branch GD, (ii) Current i_4 entering node G through branches BG and FG would be equal to currents leaving the node through branches GC and GE. These observations have been utilized to isolate upper delta and lower delta connections having vertex at G as shown ion the figure of stage 2. Thus results into parallel connection between nodes BC and FE such that $r_2 = \frac{r \times 2r}{r+2r} \Rightarrow r_2 = \frac{2r}{3}$ as shown in the network of stage 3.



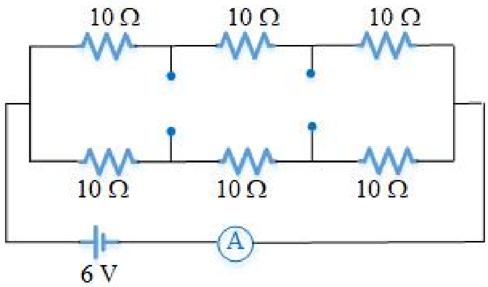
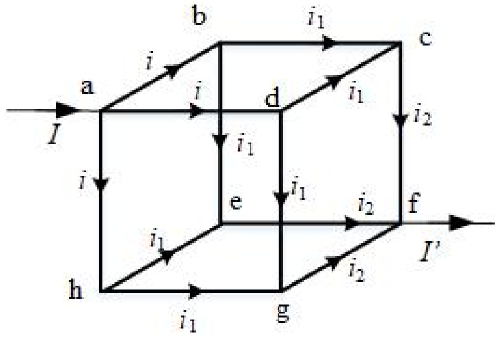
Stage 3

Thus network reduces to parallel combination of three branches ABCD and AFED, each having equivalent resistances $r_3 = r + \frac{2r}{3} + r \Rightarrow r_3 = \frac{8r}{3}$. And AD having equivalent resistance $r_4 = r + r \Rightarrow r_4 = 2r$. Thus, further reduced network is shown in figure of stage 4 having three resistances r_3, r_4 and r_3 in parallel. Thus equivalent resistance R of the network between nodes a and b is $\frac{1}{R} = \frac{1}{r_3} + \frac{1}{r_4} + \frac{1}{r_3} = \frac{1}{\frac{8r}{3}} + \frac{1}{2r} + \frac{1}{\frac{8r}{3}} \Rightarrow \frac{1}{R} = \frac{3}{8r} + \frac{1}{2r} + \frac{3}{8r} = \frac{10}{8r} \Rightarrow R = \frac{4r}{5}$



Stage 4

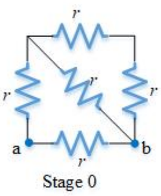
N.B.: Such, network problems require systematic use of series-parallel equivalent resistances, Wheatstone Bridge wherever applicable and Star-Delta equivalent networks, discussed in Appendix II and III. This approach is useful to solve any complex network without forming long node and loop equation as per Kirchhoff's Law and Ohm's Law. These two basic laws are the foundation of network analysis, all that it needs is a patience to analyze the network.

I-84	This problem a symmetry in numerical values of the resistances, which is graphically also symmetrical. Therefore, based on these observation of the network, currents through resistors of 50 Ω would be zero and accordingly, equivalent network is as shown in the figure. Here, $r_1 = r + r + r = 3r$, here given that $r = 10\ \Omega$. Thus, $R = \frac{r_1 \times r_1}{r_1 + r_1} \Rightarrow R = \frac{3r \times 3r}{3r + 3r} \Rightarrow R = \frac{3r}{2}$. Accordingly as per Ohm's Law current through circuit is $I = \frac{6}{\frac{3\times10}{2}} \Rightarrow I = \mathbf{0.4}$ **A is measured by ammeter, is the answer.** 
I-85	Analyzing each part of figure (a) separately- **Figure 1:** **Part (a):** Current I through circuit in clock-wise direction, applying Kirchhoff's Loop Law the loop equation, as per sign convention, is $-I \times 10 - 12 - I \times 5 - 6 = 0 \Rightarrow 15I = -18 \Rightarrow I = -\frac{18}{15} = -1.2$ A. Negative sign signifies the current is in anticlockwise direction of magnitude **1.2 A, is the answer.** **Part (b):** Potential drop (magnitude) across resistance of 5 Ω is $\lvert V_5 \rvert = \lvert I \rvert \times 5 \Rightarrow \lvert V_5 \rvert = 1.2 \times 5 = \mathbf{6}$ **V, is the answer.** **Part (c):** Potential drop (magnitude) across resistance of 10 Ω is $\lvert V_{10} \rvert = \lvert I \rvert \times 10 \Rightarrow \lvert V_{10} \rvert = 1.2 \times 10 \Rightarrow \lvert V_{10} \rvert = \mathbf{12}$ **V, is the answer.** **Figure 2:** This circuit graphically different due to interchange in placement of 5 V battery and resistance of 5 Ω, but electrically it is identical to figure (A). Hence, three the three electrical quantities would be s**ame as in part (a), (b) and (c).** Thus, **answer is (a) 1.2 A, (b) 6 V, (c) 12 V, (d) Same as the parts (a), (b) and (c).**
I-86	Equivalent resistance of the given network is being analyzed considering its electrical symmetry and accordingly distribution of current through various branches of the network as shown in the figure. Thus, writing Kirchhoff's Node Equation at different nodes – **Node a:** $I - i - i - i = 0 \Rightarrow 3i = I \Rightarrow i = \frac{I}{3}$ …(1). **Nodes b, d and h:** $i - i_1 - i_1 = 0 \Rightarrow 2i_1 = i \Rightarrow i_1 = \frac{i}{2} \Rightarrow i_1 = \frac{\frac{I}{3}}{2} \Rightarrow i_1 = \frac{I}{6}$…(2). **Nodes c, g and e:** $i_1 + i_1 - i_2 = 0 \Rightarrow i_2 = 2i_1 \Rightarrow i_2 = 2 \times \frac{I}{6} \Rightarrow i_2 = \frac{I}{3}$…(3) **Node f:** $i_2 + i_2 + i_2 - I' = 0 \Rightarrow I' = 3 \times \frac{I}{3} = I' \Rightarrow I$…(4)  The shortest path from node a to f having unidirectional currents are (i) a-d-g-f, (ii) a-d-c-f, (iii) a-b-c-f, (iv) a-b-e-f, (v) a-h-g-f and (vi) a-h-e-f. Given Each branch of the network is of a wire of has resistance r. Utilizing the network symmetry as observed voltage equation is $V_a - ir - i_1 r - i_2 r - V_f = 0$. Using equations (1), (2) and (3), $\Delta V = V_a - V_f = (i + i_1 + i_2)r \Rightarrow \Delta V = \left(\frac{I}{3} + \frac{I}{6} + \frac{I}{3}\right) r \Rightarrow \Delta V = \frac{5}{6} Ir$…(5) Using (4), considering equivalent resistance between points a and f to be R we have $\Delta V = IR$…(6). Combining (5) and (6), $IR = \frac{5}{6} Ir \Rightarrow R = \frac{5}{6}\boldsymbol{r}$ **is the answer**. **N.B.:** Combing geometrical symmetry and electrical symmetry, with careful observation of the given network greatly simplifies the solution.

I-87

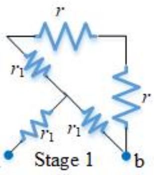
It is required to equivalent resistance R between nodes a and b, in the given five different combinations of resistors of resistances r shown in figure A, B, C, D and E. Each of the combination is being solve, in a stage-wise manner, independently using series, parallel and star-delta equivalent as per need. Simplification of network in each stage is shown in respective figure –

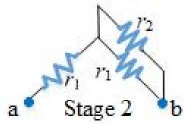
Figure A:



Stage 0: It is the given network in which lower lower electrically similar Δ is selected for its Y equivalent. This choice is based on consideration that each of the equivalent resistance remains part of the network. Whereas upper delta will leave one of the star limb open in the network. Accordingly, symmetrical star equivalent resistance, as per Appendix-III is $r_1 = \frac{r}{3}$. It is taken forward in stage 1 resolution of network, as under.

Stage 1: In this upper triangular connection is not delta rather it is a parallel combination of resistance $r_1$with a series combination of resistances $r_1 + r + r = \frac{r}{3} + 2r \Rightarrow r_2 = \frac{7r}{3}$. This equivalent is used in stage 2 resolution of network, as under.



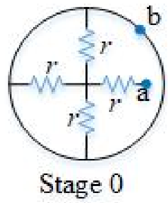


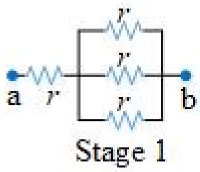
Stage 2: Parallel combination of resistances r_1 and r_2 in this stage is $r_3 = \frac{r_1 \times r_2}{r_1 + r_2} = \frac{\left(\frac{r}{3}\right) \times \left(\frac{7r}{3}\right)}{\frac{r}{3} + \frac{7r}{3}} \Rightarrow r_3 = \frac{7r}{24}$. This equivalent is used in stage 2 resolution of network, as under.

Stage3: In final stage it is a series combination of resistances r_1 and r_2. Thus equivalent resistance of the given combination is is $R = r_1 + r_3 \Rightarrow R = \frac{r}{3} + \frac{7r}{24} = \frac{15r}{24} \Rightarrow R = \frac{5r}{8}$ **is the answer.**

Figure B:

Stage 0: A close observation of given connection of resistors reveal that **e**lectrically it contains a parallel combination of three resistors of resistance r as shown in stage 1.





Stage 1: Equivalent resistance of the parallel combination is $r_1 = \frac{r}{3}$. It is taken forward in stage 2 resolution of network, as under.

Stage 2: As shown in the figure it is a series combination of two resistances r and r_1. Thus equivalent resistance of the given combination is of the parallel combination is $R = r + r_1 \Rightarrow R = r + \frac{r}{3} \Rightarrow R = \frac{4r}{3}$ **is the answer.**

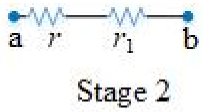
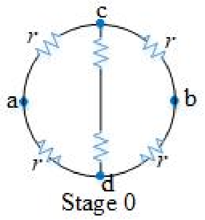
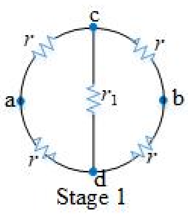


Figure C:

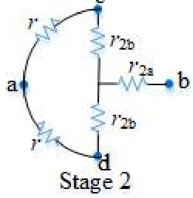
Stage 0: Given network has a series combination of two resistors of resistance r connected along diameter between nodes marked c and d. Thus, $r_1 = r + r = 2r$, as shown in the figure in stage 1.



Stage 1: In the reduced network of resistances any of the two identical geometrical half can be taken as Δ connection of resistances. Accordingly, choice is made of right-half for leading to equivalent resistance. It is to be seen that it is not an equilateral delta as resistances between nodes b-c and b-d are r while resistance between nodes c-d is r_1. Thus, equivalent resistances of star-equivalent as per Appendix-III is $r_{2a} = \frac{r \times r}{r + r + r_1} = \frac{r \times r}{r + r + 2r} \Rightarrow r_{2a} = \frac{r}{4}$, while $r_{2b} = \frac{r \times r_1}{r + r + r_1} = \frac{r \times 2r}{r + r + 2r} \Rightarrow r_{2b} = \frac{r}{2}$. It is taken forward in stage 2 for resolution of network, as under.



Stage 2: It has two identical series combination of resistances r and r_{2b} such that $r_3 = r + r_{2b} = r + \frac{r}{2} \Rightarrow r_3 = \frac{3r}{2}$. It is used in stage 2 for solving network, as under.

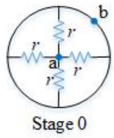


Stage 3: It has a parallel combination of two identical resistors r_3 such that $r_4 = \frac{r_3}{2} = \frac{\frac{3r}{2}}{2} \Rightarrow r_4 = \frac{3r}{4}$. It is used in stage 2 for solving network, as under.

Stage 4: It is the last stage having series combination of two identical resistors r_4 and r_{2a}. Thus equivalent resistance is $R = r_4 + r_{2a} = \frac{3r}{4} + \frac{r}{4} \Rightarrow R = \boldsymbol{r}$ **is the answer**.

Figure D:

Stage 0: A close observation of the geometry of the network reveals that one end of all four resistors is connected to node a. Whereas, the other end of all the four resistances is electrically connected to node b. This is shown in the figure in stage 1.



Stage 1: Thus, parallel combination of four resistors of resistance $r_1 = \frac{r}{4}$. There is no other resistance between nodes a and b. Hence equivalent resistance is $R = \frac{\boldsymbol{r}}{\boldsymbol{4}}$ **is the answer.**

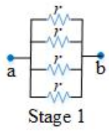
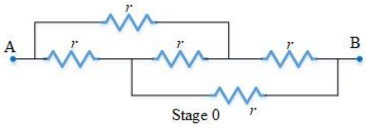
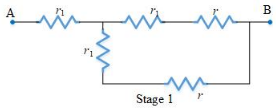


Figure E:

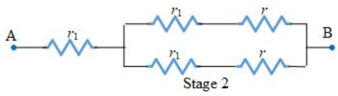
Stage 0: The given network has two symmetrical Δ-connection of resistors one with resistance r above the three identical resistors of resistance r directly between A and B and the other with identical resistance below the line A-B. Any of Δ-connection can be converted into Y-connection, as per Appendix-III. Choice is made of Δ-connection with the resistance r above the line AB, $r_1 = \frac{r}{3}$. This is used in stage 1.

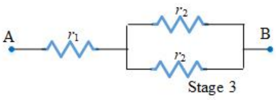


Stage 1: As shown in the figure which has a parallel combination of two branches of identical resistances connected in series. This connection for clarity is redrawn in stage 2.



Stage 2: Equivalent resistance of each of the branch of parallel combination is $r_2 = r_1 + r = \frac{r}{3} + r = \frac{4r}{3}$. It is used in stage 3 for further reducing the network.





Stage 3: The parallel combination of two resistors of resistance r_2 is $r_3 = \frac{r_2}{2} \Rightarrow r_3 = \frac{\frac{4r}{3}}{2} \Rightarrow r_3 = \frac{2r}{3}$. This is taken forward in last stage of the simplification in stage 4.

Stage 4: As shown in the figure it is a series combination of resistances r_1 and r_3, and finally $R = r_1 + r_3 = \frac{r}{3} + \frac{2r}{3} \Rightarrow R = \boldsymbol{r}$ **is the answer.**

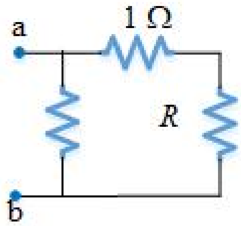
Thus, **answers are** (a) $\frac{5}{8}r$, (b) $\frac{4}{3}r$, (c) r, (d) $\frac{r}{4}$, (e) r

N.B.: 1. Geometrical connection of resistors is at times to test conceptual clarity in respect of electrical connection.

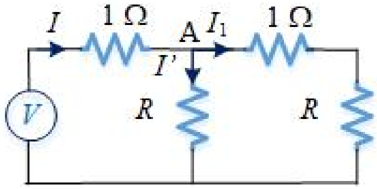
2. Given set of Five connections tries to combine different types of connections and solving each of them will help to develop confidence in determining equivalent resistance of a network of resistances.

3. ***Patience is the key*** *in successfully handling problems of electricity* during studies. It is an opportunity to develop patience required in practice in real life, specially involving electricity, which otherwise could lead to dangerous experiences.

I-88



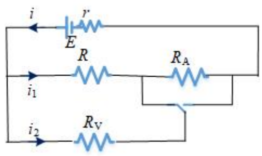
An infinite ladder is given where all steps are identical. Let at any intermediate whose equivalent circuit is shown in the figure. Equivalent resistance R seen by each step is identical. Given that resistances of 1Ω and 2 Ω in each step as shown in the figure. Accordingly, $R_{ab} = R = \frac{(1+R)\times 2}{(1+R)+2} \Rightarrow R(R+3) = 2R+2 \Rightarrow R^2 + R - 2 = 0$. It is a quadratic equation solves into $(R+2)(R-1) = 0$. Thus, the possible $R = -2\ \Omega$ it is not feasible as resistance being a scalar has always (+)ve magnitude. Thus another value is $R = 1\ \Omega$ is acceptable. Therefore, equivalent resistance of the ladder is $R_{eq} = 1 + R = 1 + 1 \Rightarrow$ R_{eq} **= 2Ω is the answer of part (a)**



In part (b) current through resistance of 2Ω , is obtained, Accordingly, the infinite ladder is replaced by R_{eq} determined in step 1, as shown in the figure. Thus, potential difference across the resistor 2Ω, based on loop equation of first step would be $V' = V - I \times 1 \Rightarrow V' = V - I$…(1). Accordingly, current in 1Ω in the second step of the ladder would be $I_1 = \frac{V'}{1+R}$ …(2). Combining (1) and (2), $I_1 = \frac{V-I}{1+R}$. Using the available data, we have $I_1 = \frac{6-I}{1+1} \Rightarrow I_1 = \frac{6-I}{2}$ A…(3). Thus applying Node equation at A, $I = I' + I_1 \Rightarrow I' = I - I_1$…(4). And $I = \frac{V}{R_{eq}} \Rightarrow I = \frac{6}{2} = 3$ A…(5). Combining (3), (4) and (5), $I' = 3 - \frac{6-3}{2} =$ **1.5 A, is the answer of part (b).**

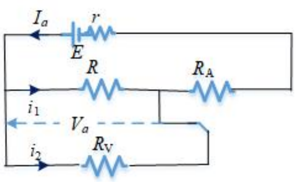
Thus, answers are (a) 2 Ω, (b) 1.5 A.

I-89

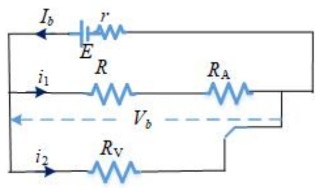


Given is a circuit with significant resistances of ammeter $R_A = 2.0\ \Omega$ and voltmeter $R_V = 200\ \Omega$, respectively. These resistances are inherent resistances of the two meters having a current sensing device. Therefore, reading of ammeter and voltmeter is proportional to current flowing through it and in turn it is dependent on voltage across the meter. The circuit having battery of emf $E = 4.8$ V and internal resistance $r = 1.0$ Ω is connected with the two meters to measure current and voltages about an external resistance $R = 50\ \Omega$. The circuit is redrawn, as shown in the figure, with the meters replaced by respective resistances.

Accordingly, the given circuit to measure voltage and current about resistor R in part (a) and about resistor $R + R_A$ in part (b). These voltages and currents are calculated using circuit equations of each part separately.



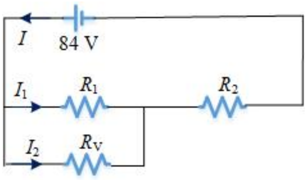
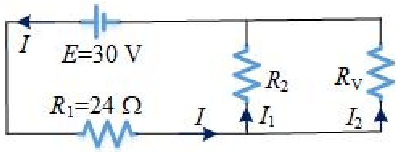
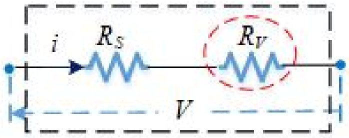
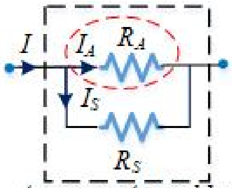
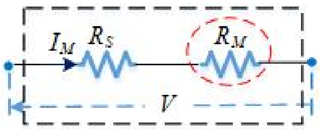
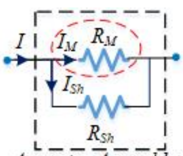
Part (a): In this R and R_V form parallel combination which is in series with r and R_A. Thus equivalent resistance of the circuit is $R_a = \frac{R \times R_V}{R+R_V} + R_A + r \Rightarrow R_a = \frac{50\times 200}{50+200} + 2.0 + 1.0 \Rightarrow R_a = 43\ \Omega$. Therefore, circuit current is $I_a = \frac{E}{R_a} = \frac{4.3}{43} =$ **0.10 A**. Though the circuit current branches into $i_1$and i_2 in resistors R and R_V, yet current through ammeter remains I_a and therefore the ammeter would read $I_a = 0.11$ A. As regards voltmeter, it will read the voltage across parallel combination of R and R_V equal to $V_a = E - I_a \times (r + R_A) = 4.3 - 0.1(1.0 + 2.0) \Rightarrow V_a = 4.0$ V, say **4.0 V.**

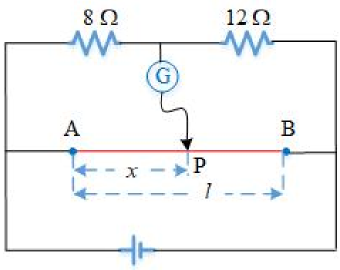


Part (b): This places voltmeter across resistance R and ammeter resistance R_A. Thus, equivalent resistance of the circuit changes to $R_b = \frac{(R+R_A)\times R_V}{(R+R_A)+R_V} + r \Rightarrow R_b = \frac{(50+2.0)\times 200}{(50+2.0)+200} + 1.0 \Rightarrow R_b = 41.3\ \Omega$. Therefore, circuit current will be also $I_b = \frac{4.3}{41.3} = 0.10$ A. Accordingly, voltage across the voltmeter would be $V_b = E - I_b r = 4.3 - 0.12 \times 1.0 \Rightarrow V_b = 4.18$ V, say **4.2 V** As regards current $I_b = i_1 + i_2$, hence $i_1$and i_2, in parallel branches are in inverse proportion of their resistances, $\frac{i_2}{i_1} = \frac{R+R_A}{R_V}$. It leads to $\frac{i_2+i_1}{i_1} = \frac{R+R_A+R_V}{R_V} \Rightarrow i_1 = \frac{I_b R_V}{R+R_A+R_V} = \frac{0.10\times 200}{50+2.0+200} \Rightarrow i_1 = 0.079$ A say **0.08 A.**

Thus, **answers are (a) 0.10 A, 4.0 V (b) 0.08 A, 4.2 V.**

N.B.: This example nicely highlights impact of meter resistance and connected resistance on meter reading.

I-90	Reading of the voltmeter will be potential difference across it. Accordingly, circuit is being modified by replacing voltmeter with its resistance and then calculate potential difference across it. Given that resistance of voltmeter $R_V = 400\ \Omega$, potential difference is required to be measured resistance $R_1 = 100\ \Omega$ which is connected in series with another resistance $R_2 = 100\ \Omega$. The circuit has a battery of $E = 84$ V as shown in the circuit. Thus, circuit resistance is $R = \frac{R_1 \times R_V}{R_1 + R_V} + R_2$, it works out to $R = \frac{100\times400}{100+400} + 200 \Rightarrow R = 280\ \Omega$. Therefore, the circuit current $I = \frac{E}{R}$. Thus taking each part separately -  **Part (a):** The voltmeter reading is $V = E - IR_2$. Using the available data, $V = 84 - \left(\frac{84}{280}\right)200 \Rightarrow V = \mathbf{24V}$ **Part (b):** When voltmeter is removed circuit resistance becomes $R' = 100 + 200 = 300\ \Omega$. And circuit current becomes $I' = \frac{E}{R'}$. Therefore, potential difference across $R_1 = 100\ \Omega$ will become $V' = I'\ R_1 \Rightarrow V' = \left(\frac{84}{300}\right)100 = \mathbf{28\ V.}$ Thus, **answers are (a) 24 V (b) 28 V**
I-91	The given circuit is redrawn with voltmeter replaced by its resistance R_V to be determined. The voltmeter reads $V = 18$ V across resistance $R_2 = 50\ \Omega$. With this data in the given circuit current will be $I = \frac{E-V}{R_1} = \frac{30-18}{24} \Rightarrow I = 0.50$.  Accordingly, in the circuit, $V = I\left(\frac{R_2 \times R_V}{R_2 + R_V}\right) \Rightarrow V \times R_2 + V \times R_V = I \times R_2 \times R_V \Rightarrow R_V(I \times R_2 - V) = V \times R_2$. This leads to $R_V = \frac{V \times R_2}{I \times R_2 - V}$. Using the available data $R_V = \frac{18\times50}{0.50\times50-18} \Rightarrow R_V = \frac{18\times50}{7} = 128.6\ \Omega$ say **130Ω is the answer.** **N.B.:** The answer is reported using principle of SDs.
I-92	Given that resistance of coil of a voltmeter is $R_V = 25\ \Omega$ and it is connected in series with a resistance $R_S = 575\ \Omega$. A current $i = 10 \times 10^{-3}$ A causes full scale deflection in the voltmeter. Thus maximum potential difference that can be measure by the combination, as shown in the figure is $V = i(R_S + R_V)$. Thus using the available data $V = 10 \times 10^{-3} \times (25 + 575) = \mathbf{6\ V}$ **is the answer.**  *Voltmeter Assembly*
I-93	Given that resistance of coil of an ammeter coil is $R_A = 25\ \Omega$ gives full scale deflection for a current $i = 1 \times 10^{-3}$ A. A shunt of resistance R_S is chosen of value that the ammeter, as shown in the assembly can read upto maximum current $I = 2.0$ A. It is clear from the circuit that line current I being measured branches into ammeter and shunt such that $I = I_A + I_S$ into parallel combination of R_A and R_S. This branching follows $\frac{I_A}{I_S} = \frac{R_S}{R_A} \Rightarrow \frac{I_A}{I_A + I_S} = \frac{R_S}{R_A + R_S} \Rightarrow \frac{I_A}{I} = \frac{R_S}{R_A + R_S}$. Using the given data $\frac{1\times10^{-3}}{2.0} = \frac{R_S}{25 + R_S}$.  *Ammeter Assembly* It solves into $(25 + R_S) \times (1 \times 10^{-3}) = 2.0 \times R_S \Rightarrow 25 + R_S = 2.0 \times 10^3 \times R_S \Rightarrow 2005R_S = 25$. It leads to $R_S = \frac{25}{2005} = \mathbf{1.25 \times 10^{-2}\Omega}$ **is the answer.**
I-94	Voltmeter is an assembly of a current sensing device having a resistance defined as that of its coil given to be $R_M = 50.0\ \Omega$ with a series resistance R_S. In the instant case given that with $R_S = 1.15 \times 10^3\Omega$ to read maximum voltage $V = 12$ V. Thus, maximum current through meter is $I_M = \frac{V}{R_M + R_S} = \frac{12}{50.0 + 1.15\times10^3} \Rightarrow I_M = 0.01\text{A}\ldots(1)$.  *Voltmeter Assembly*  *Ammeter Assembly* The same coil, meaning the same current sensing device, is used to measure maximum current $I = 2.0$ A. This is achieved by using a shunt of resistance R_{Sh} in parallel to the device as shown in the figure. This causes branching of current into I_M and I_{Sh} through R_M and R_{Sh}

	such that $I = I_M + I_{Sh}$…(2). In parallel combination $\frac{I_M}{I_{Sh}} = \frac{R_{Sh}}{R_M} \Rightarrow \frac{I_M}{I_M+I_{Sh}} = \frac{R_{Sh}}{R_{Sh}+R_M} \Rightarrow I_M(R_{Sh}+R_M) = IR_{Sh} \Rightarrow (I-I_M)R_{Sh} = I_M R_M \Rightarrow R_{Sh} = \frac{I_M R_M}{I-I_M}$. Using the available data $R_{Sh} = \frac{0.01\times 50.0}{2.0-0.01} = \mathbf{0.251\Omega}$ **is the answer.**
I-95	Let a point P is a distance x from end A of the potentiometer wire, then distance of wire between P and B is $l-x$. Potentiometer works on principle of Wheatstone bridge at its null point of zero-deflection.  Let resistance per unit length of the potentiometer wire is ρ Ω/cm. Therefore, resistance between A-P is $R_1 = \rho x$ and resistance between P-B is $R_2 = \rho(l-x)$. Likewise, $R_3 = 8\ \Omega$ and $R_4 = 12\ \Omega$. Thus at null point $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \frac{\rho x}{\rho(l-x)} = \frac{8}{12} \Rightarrow \frac{x}{x+(40-x)} = \frac{8}{8+12} \Rightarrow \frac{x}{40} = \frac{8}{20}$. It solves into $x = 40 \times \frac{8}{20} \Rightarrow x =$ **16 cm is the answer.**

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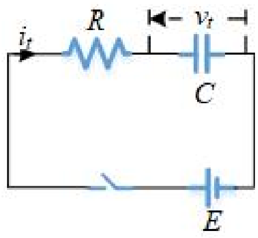
Appendix-I

Charging and Discharging of a Capacitor Through Resistance

Charging and discharging of a capacitor-charging in current electricity is dynamic application of electrostatics. While basics concept remains same initial condition of capacitor gives different results for both the processes. Accordingly, they are being discussed separately –

Charging of a Capacitor: A capacitor charging circuit is shown in the figure. The capacitor is initially discharged i.e. at $t = 0$ charge on capacitor is $Q_t = 0$, accordingly voltage across capacitor at the instant is $Q_t = Cv_t \Rightarrow v_0 = \frac{Q_o}{C} = 0$.



After closing the switch at any instant t, as per Kirchhoff's Loop Law in the circuit we have $E - i_tR - v_t = 0 \Rightarrow E - v_t = i_tR \Rightarrow E - \frac{Q_t}{C} = \frac{dQ_t}{dt}R \Rightarrow dQ_t = \left(\frac{EC-Q_t}{CR}\right)dt$ …(1). Here, $i_t$is capacitor charging current at any instant t.

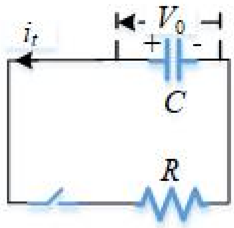
Equation (1) is a linear differential equation and its solution is $\int\frac{dQ_t}{EC-Q_t} = \frac{1}{CR}\int dt + K$…(2) here K is an integration constant whose value can be determined based on initial condition on solution of (2). Substituting $u = EC - Q_t \Rightarrow du = -dQ_t$ in (2) we have $-\int\frac{du}{u} = \frac{t}{CR} + K \Rightarrow -\ln u = \frac{t}{CR} + K$…(3). Reversing the substitution in (3) $\ln(EC - Q_t) = -\frac{t}{CR} - K \Rightarrow EC - Q_t = K'e^{-\left(\frac{t}{CR}\right)}$…(4) here $K' = e^{-K}$ is another form of the constant K.

Using the initial condition in (4), $EC - 0 = K'e^{-\left(\frac{0}{CR}\right)} \Rightarrow K' = EC$…(5). Combining (4) and (5), $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$…(6). Accordingly, charging current is $i_t = \frac{d}{dt}Q_t = \frac{d}{dt}EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow \boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$ …(7).

Finally, during charging of a capacitor, at any instant t , charge $\boldsymbol{Q_t}$ on it and current through the resistor $\boldsymbol{i_t}$ is-

(A) $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$ (B) $\boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$

Discharging of a Capacitor: It is with an initial state of capacitor when it carries some charge. But, in this simplified case no voltage source is considered in the circuit, as shown in the figure.



Let, initial charge on capacitor of capacitance C, when switch is closed, be Q_0, therefore, initial potential difference across it will be $V_0 = \frac{Q_0}{C}$.

Let, at any instant t charge on the capacitor is Q_t and therefore potential difference across it is $V_t = \frac{Q_t}{C}$. Accordingly, at that instant current through discharge is resistance R applying Kirchhoff's Loop Law in the circuit we have $V_t - i_t R = 0 \Rightarrow i_t = \frac{V_t}{R}$. During discharge charge on capacitor decreases, unlike charging, leads to $i_t = -\frac{d}{dt}Q_t \Rightarrow \frac{d}{dt}Q_t = -\frac{\frac{Q_t}{R}}{C} \Rightarrow \frac{dQ_t}{Q_t} = -\frac{dt}{RC}$. Integrating both sides we get $\int \frac{1}{Q_t} dQ_t = -\frac{1}{rC}\int dt \Rightarrow \ln Q_t = -\frac{t}{rC} + K \Rightarrow Q_t = e^{-\left(\frac{t}{rC}+K\right)} \Rightarrow Q_t = K'e^{-\frac{t}{rC}}$...(8). Here, both K and K' are integrating constants whose value depends upon initial condition $t = 0$, such that $K' = Q_0 = V_0 C$ ...(9).

Accordingly, combining (8) and (9), $Q_t = V_0 C e^{-\frac{t}{RC}}$ ...(10). Therefore, discharge currents through the resistor is $i_t = \frac{d}{dt}Q_t \Rightarrow i_t = \frac{E}{r}e^{-\frac{t}{rC_1}}$...(11).

Finally, during discharging of a capacitor, at any instant t , charge $\boldsymbol{Q_t}$ on it and current through the resistor $\boldsymbol{i_t}$ is-

(C) $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$ (D) $\boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$

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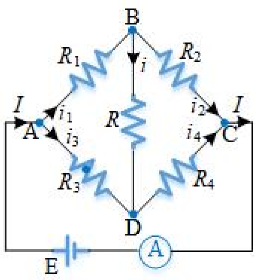
Appendix-II

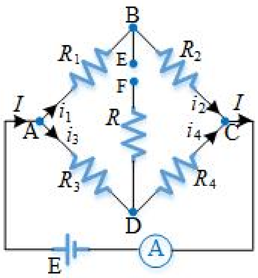
Wheatstone Bridge

Wheatstone Bridge is a typical connection of resistors which bridges nodes B and D through a resistor, yet the branch B-D is an open circuit. It is a ***very important circuit*** *extensively used for different application in different names. It was developed by Samuel Hunter Christie (year 1833) and modified by Sir Charles Wheatstone year 1843) and known after the name of the latter.*



Typical connection of resistors is shown in the figure. The given circuit is redrawn, in a generic sense, with resistances that R_1, R_2, R_3, R_4 and R with branch currents i_1, i_2, i_3, i_4 and i, and nodes A,B,C and D are identified for analysis. As per initial premise current $i = 0$...(1), through resistance R. With this this value of R is to be determined.

As per Ohm's Law, is $i = \frac{V_B - V_D}{R}$...(2), considering assigned direction to the current from node B to D. Combining (1) and (2), $\frac{V_B - V_D}{R} = 0$ is possible when - **(a)** Either $V_B - V_D = 0$ ...(3), or **(b)** $R \to \infty$**.** Taking possibility (a) in (3), $V_B = V_D$ ...(4). Accordingly, voltage equation for branches AB, BC, AD and DC we have – $V_A - V_B = i_1 R_1$...(5), $V_A - V_D = i_3 R_3$...(6), $V_B - V_C = i_2 R_2$...(7), and $V_D - V_C = i_4 R_4$...(8).



And as per Kirchhoff's current law at node B $i_1 = i + i_2$; using (1) $i_1 = i_2$…(9), likewise, at node D $i_3 + i = i_4 \Rightarrow i_3 = i_4$…(10).

Using (5) and (6) , $V_B - V_D = i_3R_3 - i_1R_1 = 0 \Rightarrow \frac{i_1}{i_3} = \frac{R_3}{R_1}$…(11). Likewise, using (7) and (8), we have $V_B - V_D = i_2R_2 - i_4R_4 = 0 \Rightarrow \frac{i_2}{i_4} = \frac{R_4}{R_2}$ …(12).

Now combining (12) with (9) and (10), $\frac{i_1}{i_3} = \frac{R_4}{R_2}$…(13). Comparing (11) and (13) $\frac{R_3}{R_1} = \frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$. …(13).

It concludes that $\boldsymbol{\frac{R_3}{R_1} = \frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}}$ is a result of the basic premise $\boldsymbol{i = 0}$, or vice-versa. Thus, any value of resistance $\boldsymbol{0 \le R, \infty}$ will do. However, to verify that the circuit satisfies the premise a current sensitive device viz. galvanometer is used in series with resistor $\boldsymbol{R}$.

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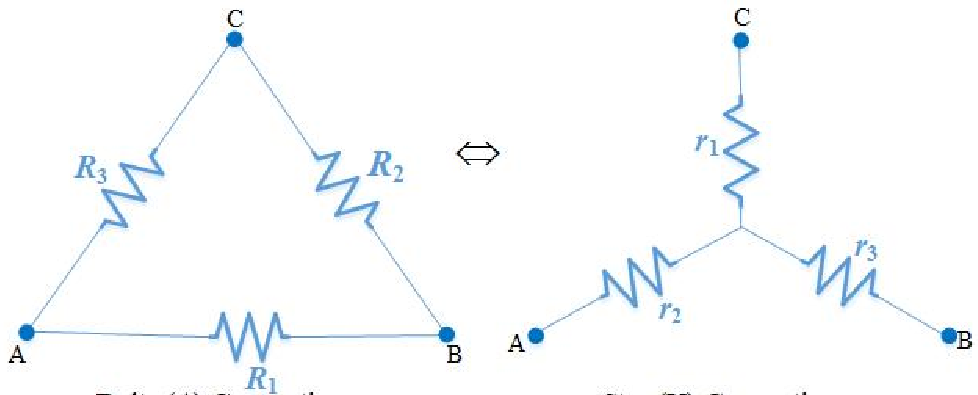
Appendix-III

Conversion of Delta-Star (Δ-Y) Equivalent of Resistance Network



Delta (Δ) Connection   Star (Y) Connection

Operation	Delta Connection	Star Connection	Eqn.
Equivalent resistance between nodes A-B	$R_{AB} = R_1 \| \| (R_2 + R_3)$ $\Rightarrow \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$	$R_{AB} = r_2 + r_3$	(1)
Equivalent resistance between nodes B-C	$R_{BC} = R_2 \| \| (R_1 + R_3)$ $\Rightarrow \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$	$R_{BC} = r_3 + r_1$	(2)
Equivalent resistance between nodes C-A	$R_{CA} = R_3 \| \| (R_2 + R_1)$ $\Rightarrow \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$	$R_{CA} = r_3 + r_2$	(3)

Delta to Star Conversion			
(2)-(1): $R_{AB} - R_{BC}$	$\frac{R_1(R_2+R_3)}{R_1+R_2+R_3} - \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$ $\Rightarrow \frac{R_3(R_1-R_2)}{R_1+R_2+R_3}$	$r_1 - r_3$	(4)
(3)+(4): $R_{CA} + (R_{AB} - R_{BC})$	$\frac{R_3(R_1+R_2)}{R_1+R_2+R_3} + \frac{R_3(R_2-R_1)}{R_1+R_2+R_3}$ $\Rightarrow \frac{2R_2R_3}{R_1+R_2+R_3}$	$2r_1$	(5)
Final Form of Conversion Formula	$\boldsymbol{r_1} = \frac{\boldsymbol{R_2R_3}}{\boldsymbol{R_1+R_2+R_3}}$; Likewise, $\boldsymbol{r_2} = \frac{\boldsymbol{R_1R_3}}{\boldsymbol{R_1+R_2+R_3}}$; $\boldsymbol{r_3} = \frac{\boldsymbol{R_1R_2}}{\boldsymbol{R_1+R_2+R_3}}$		(6)
Star to Delta Conversion			
Using (6)	$r_1r_2 + r_2r_3 + r_3r_1 = \left(\frac{R_2R_3}{R_1+R_2+R_3}\right)\left(\frac{R_1R_3}{R_1+R_2+R_3}\right) + \left(\frac{R_1R_3}{R_1+R_2+R_3}\right)\left(\frac{R_1R_2}{R_1+R_2+R_3}\right) + \left(\frac{R_1R_2}{R_1+R_2+R_3}\right)\left(\frac{R_2R_3}{R_1+R_2+R_3}\right)$ $\Rightarrow \frac{R_1R_2R_3}{R_1+R_2+R_3}$		(7)
(7)/(6)	$\frac{r_1r_2+r_2r_3+r_3r_1}{r_1} = \frac{\frac{R_1R_2R_3}{R_1+R_2+R_3}}{\frac{R_2R_3}{R_1+R_2+R_3}}$		(8)
Final Form of Conversion Formula	$\boldsymbol{R_1} = \frac{\boldsymbol{r_1r_2+r_2r_3+r_3r_1}}{\boldsymbol{r_1}}$, likewise, $\boldsymbol{R_2} = \frac{\boldsymbol{r_1r_2+r_2r_3+r_3r_1}}{\boldsymbol{r_2}}$ and $R_3 = \frac{r_1r_2+r_2r_3+r_3r_1}{r_3}$		(9)

N.B.: 1. Derivation of $(Y - \Delta)$ is based on series-parallel equivalents resistances with algebraic manipulations.

2. A close observation of formulae derived, in relation to the figure, will help to develop an easy way to remember it.

3. Care in naming resistances in star connection and relating it to delta connection is important for ease of application.

4. A similar, but with a difference formulation for star-delta network of capacitances has been developed and is available in question bank on capacitors.

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