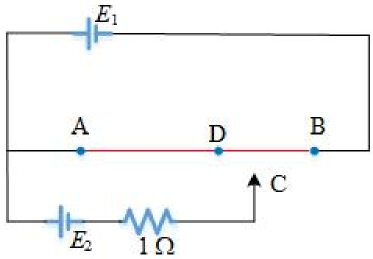
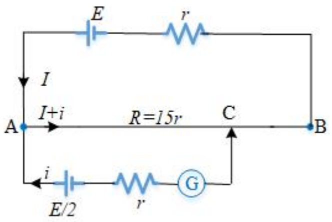
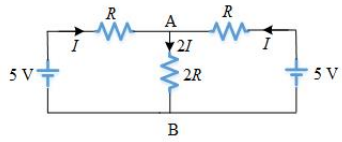
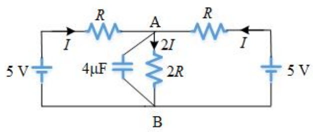
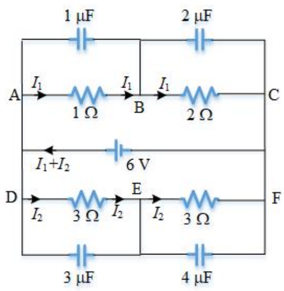
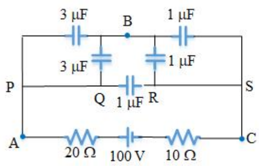
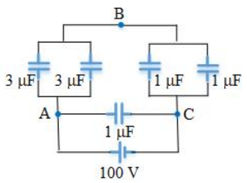
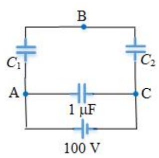
Code: Phy/ EM-2/CE/002

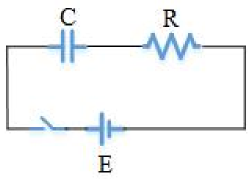
Electromagnetism: Effect of Current Electricity– Typical Questions (Set 2)

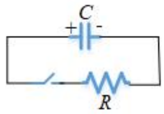
(Only Illustrations)

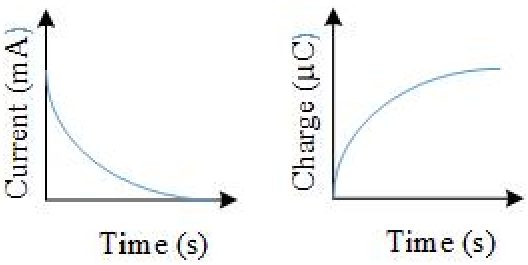
I-1	Wire of a potentiometer has uniform resistance per unit length say ρ Ω/cm. Given that length of wire is $L = 50$ cm, and length AD is $x = 30$ cm. Therefore, length DB is $l = L - x = 50 - 30 \Rightarrow l = 20$ cm. Accordingly, $R_{AD} = 30\rho\ \Omega$ and $R_{DB} = 20\rho\ \Omega$. The potentiometer is another form of Wheatstone bridge and in balanced condition where galvanometer has no deflection $\frac{R_1}{R_2} = \frac{R_3}{R_4}$…(1). Here, $R_1 = 6\ \Omega$, $R_2 = R_{AD} = 30\rho\ \Omega$, $R_3 = R\ \Omega$ and $R_4 = R_{DB} = 20\rho\ \Omega$. Substituting values of resistances in (1) we have $\frac{6}{30\rho} = \frac{R}{20\rho} \Rightarrow R = 6 \times \frac{2}{3} \Rightarrow R = \mathbf{4\ \Omega}$ **is the answer.**
I-2	This problem though looks similar to that on Wheatstone Bridge, it has some deviation from it. Let wire AB of length $L = 100$ cm has uniform resistance ρ Ω/cm. Let, distance AD is x cm corresponding to $R_1 = x\ \Omega$ the distance DB be $(L - x)$ cm corresponding to $R_2 = (L - x)\ \Omega$. Solving each part separately –  **Part (a):** Negative terminal is directly connected, without any resistance, to end B of the wire and the end is stated to be at potential Zero. Likewise, positive terminal of the battery with $E_1 = 6$ V is directly connected to end A of the wire. Hence, as per *Kirchhoff's Circuital Law* potential at A will be $V_A = V_B + E_1 \Rightarrow 0 + (+6) \Rightarrow \boldsymbol{V_A} = \mathbf{6}$ **V**. As regards potential at C, which is floating, it is connected to A through a resistance $r = 1\ \Omega$ and a battery having $E_2 = 4$ volts connected with reverse polarity.; the link carries no current $i = 0$. Therefore, $V_C = V_A - (E_2 + ir) \Rightarrow V_C = 6 - (4 + 0 \times 1) \Rightarrow \boldsymbol{V_C} = \mathbf{2}$ **V**. **Part (b):** The emf E_1 uniformly is spread out along wire AB such that potential gradient along the wire AB is $\delta = \frac{\Delta V}{\Delta l} = \frac{V_B - V_A}{L} = \frac{0-6}{100} \Rightarrow \delta = -0.06$ V/cm. For $V_D = V_c = 2$ V, distance x of point D on the wire say from end A is $V_D - V_A = x \times \delta \Rightarrow x = \frac{V_D - V_A}{\delta}$. Using the available data $x = \frac{2-6}{-0.06} \Rightarrow \boldsymbol{x} = \mathbf{66.7}$ **cm.** **Part (c):** Point C and point D determined at part (b) if connected by wire, resistance of which is say r' then current through wire would be D, $i = \frac{V_D - V_C}{r'}$. Using the available data, $i = \frac{2-2}{r'} = \frac{0}{r'} \Rightarrow \boldsymbol{i} = \mathbf{0}$ **Part (d):** Given that battery is changed such that $E_2 = 7.5$ V, with given set of connection resolving the three parts – **(i)** $V_A = V_B + E_1 \Rightarrow 0 + (+6) \Rightarrow \boldsymbol{V_A} = \mathbf{6}$ **V;** **(ii)** $V_C = V_A - (E_2 + ir) \Rightarrow V_C = 6 - (7.5 + 0 \times 1) \Rightarrow \boldsymbol{V_C} = \mathbf{-1.5}$**V** **(iii)** $x = \frac{-1.5-6}{-0.06} = 125$ cm it turns out to be outside the wire, while point D is shown to be between A and B. Thus, **no such point exits** Thus, **answers are (a) 6 V, 2 V (b) AD = 66.7 cm (c) Zero (d) 6 V, -1.5 V no such point D exists** **N.B.:** This problem can be structured into objection question on each part. But a full length questions needs to be solved in a stepwise manner.

I-3	 Resistance per unit length of a potentiometer wire AB of length $L = 600$ cm and resistance $R = 15r$ is $\rho = \frac{R}{L} = \frac{15r}{600} \Rightarrow \rho = \frac{r}{40}$. Each part is solved separately as under **Part (a):** Let jockey at a point C at distance x from A shows zero deflection of the galvanometer i.e. $i = 0$. Therefore, circuit current $I = \frac{E}{r+R} = \frac{E}{r+15r} \Rightarrow I = \frac{E}{16r}$. Therefore, potential gradient along the wire would be $\delta = -I\rho = -\frac{E}{16r} \times \frac{r}{40} \Rightarrow \delta = -\frac{E}{640}$ V/cm. Since, there is no current through the branch having galvanometer hence potential at C is $V_C = \frac{E}{2}$. As regards along the wire $V_C - V_A = x\delta \Rightarrow V_C = V_A + x\delta \Rightarrow \frac{E}{2} = E + x\left(-\frac{E}{640}\right) \Rightarrow x = \frac{1}{2} \times 640 \Rightarrow x = \mathbf{320\ cm}$. **Part (b):** Touch of the jockey on the wire at a point at distance $x = 560$ cm from A, other than null point determined in part (a), will create a current in galvanometer and change pattern of current and uniform voltage drop along the wire. Accordingly, voltage equations are formed for the two loops as under – *Upper loop*: $E - (I+i) \times \rho x - I\big((L-x)\rho + r\big) = 0 \Rightarrow I(\rho x + (L-x)\rho + r) = E - i\rho x$ $\Rightarrow I(L\rho + r) = E - i\rho x \Rightarrow I = \frac{E - i\rho x}{L\rho + r}$. *Lower Loop*: $\frac{E}{2} - (I+i) \times \rho x - ir = 0 \Rightarrow I\rho x = \frac{E}{2} - i(r + \rho x) \Rightarrow I = \frac{E - 2i(r+\rho x)}{2\rho x}$ Combing the two equations $\frac{E - i\rho x}{L\rho + r} = \frac{E - 2i(r+\rho x)}{2\rho x} \Rightarrow (E - i\rho x) \times 2\rho x = \big(E - 2i(r + \rho x)\big)(L\rho + r)$. Substituting, the available values we get – $\left(E - i \times \frac{r}{40} \times 560\right) \times 2 \times \frac{r}{40} \times 560 = \left(E - 2i\left(r + \frac{r}{40} \times 560\right)\right)\left(600 \times \frac{r}{40} + r\right)$. $\Rightarrow (E - 14ir) \times 28r = \big(E - 2i(r + 14r)\big)16r \Rightarrow 7(E - 14ir) = 4\big(E - 2i(r + 14r)\big)$ $\Rightarrow 7E - 98ir = 4E - 120ir \Rightarrow 3E = -22ir \Rightarrow i = -\frac{3E}{22r}$. Thus, current in the galvanometer is $\lvert i \rvert = \left\lvert -\frac{3E}{22r} \right\rvert = \boldsymbol{\frac{3E}{22r}}$ Thus, **answers are (a) 320 cm (b) $\boldsymbol{\frac{3E}{22r}}$** **N.B.: 1.** Here, loop equations have been framed differently by identifying branch currents instead of loop current. **2.** Galvanometer is an instruments which indicated bidirectional current through it, unlike current measuring instrument, Hence, it is the magnitude of current through the galvanometer is relevant, and that what has been asked.
I-4	In the circuit no switch is indicated, it implies that the capacitor is connected for a long time to be in steady state, i.e. fully charged. Thus charge $Q = CV$ on the capacitor is dependent on the voltage across 10 Ω resistance and it is, as per Ohm's Law $V = \frac{E}{R_1 + R_2} \times R_1$, here $E = 2$ V, $R_1 = 10\ \Omega$ and $R_2 = 20\ \Omega$. Using the available data $Q = (6 \times 10^{-6}) \times \left(\frac{2}{10+20} \times 10\right) = 6 \times 10^{-6}$ C or **4 μC is the answer.**

I-5	The problem requires application of Kirchhoff's Law which has been simplified considering symmetry of the circuit and accordingly currents and resistances in different branches have been indicated algebriacally. Accordingly, voltage equations for both the identical loops (left and right) will be $E - I \times R - 2I \times 2R = 0 \Rightarrow I = \frac{E}{5R}$, here $E = 5$ V and $R = 10$ Ω.  Therefore, $I = \frac{5}{5 \times 10} = 0.1$ A. Thus current through 10 Ω resistor is $2I = 2 \times 0.1 =$ **0.2 A is the answer of part (a).**  When a capacitor of $C = 4 \times 10^{-6}$ F is connected as shown in the steady state, voltage across it would be $V = 2I \times 2R = 0.2 \times 20 = 4$ V. Therefore energy stored in the capacitor is $U = \frac{1}{2}CV^2 = \frac{1}{2}(4 \times 10^{-6}) \times 4^2 = 32 \times 10^{-6}$ J or **32 μJ is the answer of part (b).** Thus, **answers are (a) 0.2 A (b) 32 μJ.**
I-6	In the circuit no switch is indicated, it implies that the capacitor is connected for a long time to be in steady state, i.e. fully charged and current through the capacitor is zero. Thus charge on the capacitor is dependent on the voltage across, $Q = CV$. Accordingly, it requires to determine current through resistors, as shown in the figure, to arrive at voltages across them, as per Ohm's Law. Thus current in branch A-B-C is $I_1 = \frac{6}{1+2} = 2$ A Therefore, voltage across 1 μF capacitor is $V_1 = I_1 \times 1 = 2 \times 1 = 2$ V and charge on it is $Q_1 = (1 \times 10^{-6}) \times V_1 \Rightarrow Q_1 = (1 \times 10^{-6}) \times 2 = 2 \times 10^{-6}$ C or **2 μC.** Likewise, voltage across 2 μF capacitor is $V_2 = I_1 \times 2 = 2 \times 2 = 4$ V and charge on it is $Q_2 = (2 \times 10^{-6}) \times V_2 \Rightarrow Q_1 = (2 \times 10^{-6}) \times 4 = 8 \times 10^{-6}$ C or **8 μC.**  Similarly, current in branch D-E-F is $I_1 = \frac{6}{3+3} = 1$ A Therefore, voltage across 1 μF capacitor is $V_3 = I_2 \times 3 = 1 \times 3 = 3$ V and charge on it is $Q_3 = (3 \times 10^{-6}) \times V_3 \Rightarrow Q_1 = (3 \times 10^{-6}) \times 3 = 9 \times 10^{-6}$ C or **9 μC.** Likewise, voltage across 42 μF capacitor, looking into circuit is $V_4 = V_3$; therefore, charge on it is $Q_4 = (4 \times 10^{-6}) \times V_4 \Rightarrow Q_1 = (4 \times 10^{-6}) \times 3 = 12 \times 10^{-6}$ C or **12 μC.** Thus, **answers are 2 μC, 8 μC, 9μC and 12 μC.**
I-7	Given circuit has been marked with additional nodes P,Q, R and S. Since the circuit   is stated to be in steady-state and hence therefore, there will be no current through the circuit. Accordingly, potentials $V_A = V_P = V_Q = 100$ V and $V_C = V_S = V_R = 0$ V. Accordingly merging equipotential points, electrical equivalent circuit is redrawn. Parallel combination of two capacitor between nodes A and B is $C_1 = 3 + 3 = 6$ μF, as show$V_1 = \frac{2}{2+6} \times 100 =$ 25n in the figure. Likewise, parallel combination of two capacitor between nodes A and B is $C_2 = 1 + 1 = 2$ μF. Thus, battery potential difference $V = 100$ is divided between A, B and C such that $V = V_1 + V_2$. The capacitors C_1 and C_2 connected in series between A and C acts as a potential divider. Accordingly, voltage between A and B is $V_1 = \frac{C_2}{C_1+C_2}V$ and between B and C in $V_2 = \frac{C_1}{C_1+C_2}V$. Using the available data $V_1 = \frac{2}{2+6} \times 100 =$ **25 V** and $V_2 = \frac{6}{2+6} \times 100 =$ **75 V**.  Thus, **answers are 25 V, 75 V.**

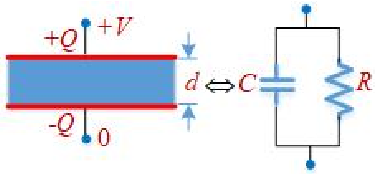
I-8	 Given circuit is shown in the figure. Determination of various electrical quantities, a time dependent circuit equation is as under $E - \frac{Q_t}{C} - i_t R = 0$. Here, at any instant t charge on capacitor is Q_t and circuit current is $i_t = \frac{dQ_t}{dt}$. Thus the circuit equation can be rewritten as $E - \frac{Q_t}{C} - R\frac{dQ_t}{dt} = 0 \Rightarrow \frac{dQ_t}{dt} = \frac{EC-Q_t}{RC} \Rightarrow \int \frac{dQ_t}{EC-Q_t} = \frac{1}{RC}\int dt$. Substituting $u = EC - Q_t \Rightarrow dQ_t = -du$. the equation transforms to $\int_0^t \frac{du}{u} = -\frac{t}{RC} \Rightarrow \ln u = -\frac{t}{RC} + K \Rightarrow u = K'e^{-\frac{t}{RC}} \Rightarrow EC - Q_t = K'e^{-\frac{t}{RC}}$. Here, K' is the integration constant. Imposing limit $t = 0$, charge on capacitor Q_0 =0, and therefore $EC - 0 = K'e^{-\frac{0}{RC}} \Rightarrow K' = KC$. Accordingly, solution of equation is $Q_t = EC - ECe^{-\frac{t}{RC}} \Rightarrow Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$ is the solution, which will be used to arrive maximum values in each part, as under : **Part (a):** Potential difference across resistor is $V_t = Ri_t = R\frac{d}{dt}\left(EC\left(1 - e^{-\frac{t}{RC}}\right)\right) = REC\frac{d}{dt}\left(1 - e^{-\frac{t}{RC}}\right)$. It leads to $V_t = REC\left(-\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}\right) = Ee^{-\frac{t}{RC}}$. Since, $\boldsymbol{V_t}$ is an exponentially decaying function and has **maximum value** at $t = 0$. Therefore, $V_0 = Ee^{-\frac{0}{RC}} = \boldsymbol{E}$**, is the answer.** **Part (b):** The current in the circuit, from fort (a), is $i_t = \frac{E}{R}e^{-\frac{t}{RC}}$, which is exponentially decaying and has **maximum value** at $t = 0$. Therefore, $i_0 = \frac{E}{R}e^{-\frac{0}{RC}} = \boldsymbol{\frac{E}{R}}$**, is the answer.** **Part (c):** The potential difference across the capacitor is $V_{ct} = E - V_t = E - Ee^{-\frac{t}{RC}}$. From this expression maximum value will be when decrement $D_t = Ee^{-\frac{t}{RC}}$ has minimum value. At $t = \infty \Rightarrow D_t = Ee^{-\frac{t}{RC}} = 0$. Accordingly, maximum potential difference across capacitor is $V_{C\infty} = E - 0 = \boldsymbol{E}$ **is the answer.** **Part (d):** Energy stored in the capacitor is $U_t = \frac{1}{2}CV_{ct}^2$ and maximum value of U_t is for maximum $V_{ct} = E$. Therefore, $U_{Max} = \boldsymbol{\frac{1}{2}CE^2}$ **is the answer.** **Part (e):** Power delivered by the battery is $P_t = Ei_t$. Therefore, maximum value of P_t is when i_t is maximum i.e. $i_0 = \frac{E}{R}$. Accordingly, $P_{Max} = Ei_0 = E \times \frac{E}{R} \Rightarrow P_{Max} = \boldsymbol{\frac{E^2}{R}}$ **is the answer.** **Part (f):** Power converted into heat is $P_{Ht} = i_t^2 R$. Therefore, maximum value of P_{Ht} is when i_t is maximum i.e. $i_0 = \frac{E}{R}$. Accordingly, $P_{H-Max} = (i_0)^2 \times R = \left(\frac{E}{R}\right)^2 \times R \Rightarrow P_{Max} = \boldsymbol{\frac{E^2}{R}}$ **is the answer.** Thus, **answer is (a)** $\boldsymbol{E}$ **(b)** $\boldsymbol{\frac{E}{R}}$ **(c)** $\boldsymbol{E}$ **(d)** $\boldsymbol{\frac{1}{2}CE^2}$ **(e)** $\boldsymbol{\frac{E^2}{R}}$ **(f)** $\boldsymbol{\frac{E^2}{R}}$
I-9	Charge on a capacitor in DC circuit, at any time t is $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$. Here, E is battery emf, R is resistance of circuit and C is the capacitance. At $t = \infty$ the capacitor gets fully charged to $Q_t = Q = EC$. In this equation at $t = \tau = RC$ charge on capacitor is $Q_\tau = EC(1 - e^{-1}) = 0.632Q$ or 63.2% of the maximum charge on the capacitor in the circuit. This time $\tau = RC$ is called time constant of an RCcircuit. In this problem with the given data $C = \frac{\varepsilon_a A}{d}$ where $\varepsilon_a = 8.85 \times 10^{-12}$ F/m. Accordingly, $C = \frac{(8.85\times10^{-12})\times(20\times10^{-4})}{1.0\times10^{-3}} = 17.7 \times 10^{-12}$ F. Therefore, $\tau = (10 \times 10^3) \times (17.7 \times 10^{-12}) = 0.177 \times 10^{-6}$s or say **0.18 μs is the answer**.
I-10	Charge on a capacitor in DC circuit, at any time t is $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$. Here, E is battery emf, R is resistance of circuit and C is the capacitance. With the given data $12.6 \times 10^{-6} = 2 \times (10 \times 10^{-6}) \times \left(1 - e^{\frac{50\times10^{-3}}{R\times(10\times10^{-3})}}\right)$. It leads to $0.63 = 1 - e^{-\frac{t}{RC}}$. This is indicative of $t \approx \tau = RC$, the time constant of the

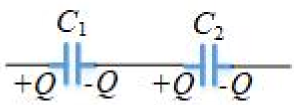
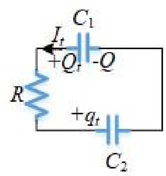
	circuit, which is not directly indicated in the problem. There, $R=\frac{\tau}{C}\approx\frac{t}{C}=\frac{50\times10^{-3}}{10\times10^{-6}}=5\times10^{3}\Omega$ or **5 kΩ is the answer.**
I-11	Charge on a capacitor in DC circuit, at any time t is $Q_t=EC\left(1-e^{-\frac{t}{RC}}\right)$. Here, E is battery emf, R is resistance of circuit and C is the capacitance. Time constant of the circuit $\tau=RC=100\times(20\times10^{-6})=2\times10^{-3}$. Since, it is required to find charge on the capacitor at $t=2$ ms i.e. at τ which is 63.2% of the maximum charge $Q_{\infty}=EC=6.0\times(20\times10^{-6})=120\times10^{-6}$C. Hence, $Q_{\tau}=0.632\times(120\times10^{-6})=75.84\times10^{-6}$ C say **76 μC is the answer.**
I-12	 The given problem is on discharge of a capacitor. Let at any time t , after the switch is closed, charge on the capacitor is Q_t. Therefore, potential difference across the capacitor would be $V_t=\frac{Q_t}{C}$. As per Kirchhoff's Law $V_t-i_tR=0\Rightarrow i_tR=V_t\Rightarrow -R\frac{d}{dt}Q_t=\frac{Q_t}{C}$. Here, negative sign is used since capacitor is discharging with the passage of time and hence gradient $\frac{d}{dt}Q_t$ has (-) value. On integration, $\int\frac{dQ_t}{Q_t}=-\frac{1}{RC}\int dt$. It leads to $\ln Q_t=-\frac{t}{RC}+K\Rightarrow Q_t=K'e^{-\frac{t}{RC}}$. Here, K' is integration constant whose value is determined with given limiting value at $t=0\Rightarrow Q_0=60\times10^{-6}$C. Accordingly, $K'=Q_0=60\times10^{-6}$ and with available data the capacitor discharge equation is $Q_t=(60\times10^{-6})\times e^{-\frac{t}{10\times(10\times10^{-6})}}$. Thus it leads to $Q_t=(60\times10^{-6})\times e^{-\frac{t}{10^{-4}}}$. With this charge on capacitor at given instances shall be as under – **(a) At $t=0$**, charge on the capacitor as determined above is $Q_0=60\times10^{-6}$or 60 μC. **(b) At $t=30\times10^{-6}$s**, charge on the capacitor is $Q_0=(60\times10^{-6})\times e^{-\frac{30\times10^{-6}}{10^{-4}}}\Rightarrow Q_0=(60\times10^{-6})\times e^{-0.3}\Rightarrow Q_0=(60\times10^{-6})\times0.74=\mathbf{44}$ **μC.** **(c) At $t=120\times10^{-6}$s**, charge on the capacitor is $Q_0=(60\times10^{-6})\times e^{-\frac{120\times10^{-6}}{10^{-4}}}\Rightarrow Q_0=(60\times10^{-6})\times e^{-1.2}\Rightarrow Q_0=(60\times10^{-6})\times0.87=\mathbf{18}$ **μC.** **(d) At $t=1\times10^{-3}$s**, it is a time constant of the circuit and hence charge on the capacitor is $Q_0=(60\times10^{-6})\times e^{-\frac{1\times10^{-3}}{10^{-4}}}\Rightarrow Q_0=(60\times10^{-6})\times e^{-10}\Rightarrow Q_0=(60\times10^{-6})\times4.5\times10^{-5}=\mathbf{2.7}$ **nC**. Thus, **answers are (a) 60 μC (b) 44 μC (c) 18 μC (d) 2.7 nC**
I-13	This a problem of charging of a capacitor for which charge on the capacitor at instant t after it connected to a battery is $Q_t=EC\left(1-e^{-\frac{t}{RC}}\right)$. Here, $E=6.0$ V emf of the battery, capacitance of the capacitor is $C=8.0\times10^{-6}$ F and resistance in the circuit $R=24\ \Omega$. Here, time constant of the circuit is $\tau=RC=24\times(8.0\times10^{-6})=0.192\times10^{-3}$s. Since, it is required to determine current and hence $i_t=\frac{d}{dt}\left(EC\left(1-e^{-\frac{t}{RC}}\right)\right)=\frac{E}{R}e^{-\frac{t}{\tau}}$. Accordingly, at $t=0^{+}$ when connections are just made the current is $i_{t0^{+}}=\frac{E}{R}e^{-\frac{0}{\tau}}=\frac{E}{R}=\frac{6.0}{24}=\mathbf{0.25}$ **A is the answer of part (a).** The current at one time constant i.e. $t=\tau$ is $i_{\tau}=\frac{E}{R}e^{-\frac{\tau}{\tau}}=\frac{E}{R}\times\frac{1}{e}=\frac{0.25}{2.72}=\mathbf{0.09}$ **A is the answer of part (b).** Thus, **answers are (a) 0.25 A (b) 0.09 A.**
I-14	It is basically a problem of charging of capacitor, taken forward to electric in capacitor $E_t=\frac{V_t}{d}$ at $t_{+}=10\times10^{-9}$s. Here, V_t is the potential difference across the plates of the capacitor and $d=1\times10^{-4}$ is separation between the plates of the parallel-plate capacitor. The potential difference is $V_t=\frac{Q_t}{C}$. And during charging of a capacitor $Q_t=EC\left(1-e^{-\frac{t}{RC}}\right)$. Here, E is the emf of the battery.

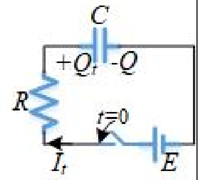
	Here, capacitance of the parallel plate capacitor with the given dimensions is $C=\frac{\varepsilon_0 A}{d}=\frac{(8.85\times10^{-12})\times(40\times10^{-4})}{1\times10^{-4}} \Rightarrow C=354\times10^{-12}$ F Accordingly, the electric field at an instance t is $E_t=\frac{\frac{Q_t}{C}}{d}=\frac{Q_t}{Cd}=\frac{EC\left(1-e^{-\frac{t}{RC}}\right)}{Cd}=\frac{E}{d}\left(1-e^{-\frac{t}{RC}}\right)$. Using the available data $E_t=\frac{2.0}{1\times10^{-4}}\left(1-e^{-\frac{10\times10^{-9}}{16\times(354\times10^{-12})}}\right)=2.0\times10^4\times\left(1-e^{-\frac{10\times10^{-9}}{5.664\times10^{-9}}}\right)=2.0\times10^4\times(1-e^{-1.77})$. It solves into $E_t=\mathbf{1.6\times10^4}$ **V/m is the answer**.
I-15	It is the problem of energy stored in a capacitor, after connection to a battery of emf $E=6$ V at $t=8.9\times10^{-6}$. Thje capacitor is connected through a resistor of resistance $R=100\times10^3=10^5\Omega$. Energy stored in a capacitor at any instant t is $U_t=\frac{1}{2}Q_tV_t=\frac{1}{2}Q_t\times\frac{Q_t}{C}=\frac{Q_t^2}{2C}$. Capacitance of the capacitor with the given data is $C=\frac{\epsilon_0 KA}{d}=\frac{(8.85\times10^{-12})\times5.0\times(20\times10^{-4})}{1.0\times10^{-3}}=88.5\times10^{-12}$F. . Charge on capacitor, during charging, is $Q_t=EC\left(1-e^{-\frac{t}{\tau}}\right)$, here time constant of the charging circuit $\tau=RC=10^5\times(88.5\times10^{-12})=8.85\times10^{-7}\approx8.9\times10^{-6}$s or 8.9 µs. Therefore, $Q_t=6.0\times(88.5\times10^{-12})\left(1-e^{-\frac{8.9}{8.9}}\right)=0.531\times10^{-9}\times(1-e^{-1})\approx0.336\times10^{-9}$C. Accordingly energy on the capacitor is $U_t=\frac{(0.336\times10^{-9})^2}{2\times(88.5\times10^{-12})}=\mathbf{6.3\times10^{-10}}$**J is the answer.**
I-16	Charging of a capacitor of capacitance $C=100\times10^{-6}$ F through a resistance $R=1.0\times10^6$ Ω when connected to a battery of emf $E=24$V is $Q_t=EC\left(1-e^{-\frac{t}{RC}}\right)$. Using the given data the equation can be rewritten as $Q_t=24\times(100\times10^{-6})\left(1-e^{-\frac{t}{(1.0\times10^6)(100\times10^{-6})}}\right)=24\times10^{-4}\times\left(1-e^{-t\times10^{-2}}\right)$. Accordingly, current in the circuit will be $I_t=\frac{d}{dt}Q_t=\frac{d}{dt}\left(24\times10^{-4}\times\left(1-e^{-t\times10^{-2}}\right)\right)=24\times10^{-6}\times e^{-t\times10^{-2}}$. **At $t=0$:** the charging current $I_0=24\times10^{-6}\times e^{-0\times10^{-2}}=24\times10^{-6}$ A or 26 mA. And charge on the capacitor is $Q_t=24\times10^{-4}\times(1-1)=0$. **At $t=600$ s:** the charging current $I_0=24\times10^{-6}\times e^{-600\times10^{-2}}=24\times10^{-6}\times e^{-6}=6\times10^{-8}$ A or 60 nA nearly zero.. And charge on the capacitor is $Q_t=24\times10^{-4}\times(1-0.002)\approx0.24$ µC. Using the above information, qualitative graphs are as under - 
I-17	**Charging of a capacitance** C through a resistance R when connected to a battery of emf E is $Q_t=EC\left(1-e^{-\frac{t}{RC}}\right)$. Accordingly, current in the circuit will be $I_t=\frac{d}{dt}Q_t=\frac{d}{dt}\left(EC\left(1-e^{-\frac{t}{RC}}\right)\right)=\frac{E}{R}e^{-\frac{t}{RC}}$. In terms of time constant $\tau=RC$ the current equation is $I_t=\frac{E}{R}e^{-\frac{t}{\tau}}$ and $I_0=\frac{E}{R}$

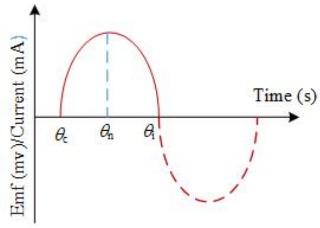
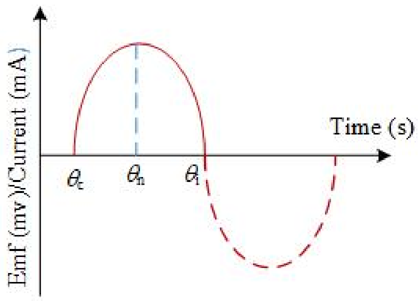
	Therefore, time $t = k\tau$ current from initial value I_0 to $0.5I_0$ can be expressed as $0.5I_0 = I_0 e^{-\frac{k\tau}{\tau}} \Rightarrow e^{-k} = 0.5$. It solves into $k = \ln 2 = \mathbf{0.69}$ **is the answer.** **While discharging of the capacitor** $Q_t = ECe^{-\frac{k\tau}{\tau}}$. Accordingly, $I_t = \frac{E}{R}e^{-\frac{k\tau}{\tau}} = I_0 e^{-k}$ .Therefore, for $0.5I_0 = I_0 e^{-k} \Rightarrow k = \ln 2 = \mathbf{0.69}$ **is the answer.** Thus, **answer is 0.69 time time constant in both the cases.**
I-18	While discharging of the capacitor $Q_t = ECe^{-\frac{k\tau}{\tau}} = Q_0 e^{-k}$ Here, $Q_0 = EC$ is the initial charge. Therefore, at time $t = k\tau$ for charge $Q_t = \frac{0.1}{100}Q_0$, the equation will be $\frac{0.1}{100}Q_0 = Q_0 e^{-k} \Rightarrow e^k = 1000 \Rightarrow k = \ln 1000 = 6.9$ **is the answer.**
I-19	Equilibrium value of energy in a capacitor is during its charging is when capacitor is fully charged. Thus its is the case of charging of a capacitor. Further, energy stored in a capacitor at any instant is $U_t = \frac{1}{2}Q_t V_t = \frac{Q_t^2}{2C}$. During charging charge on a capacitor is $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{k\tau}{\tau}}\right) \Rightarrow Q_t = Q\left(1 - e^{-k}\right)$, here $Q = EC$ is the charge on the capacitor when it is fully charged and $\tau = RC$ is the constant of the circuit. It is required to find k where $t = k\tau$. It is required to find time in number of time constants energy stored in capacitor reaches to half of its equilibrium value, which is expressed as $\frac{U_t}{U} = \frac{\frac{Q_t^2}{2C}}{\frac{Q^2}{2C}} = \frac{Q_t^2}{Q^2} = \frac{Q_t^2}{Q^2} = \frac{\left(Q\left(1-e^{-k}\right)\right)^2}{Q^2} \Rightarrow \frac{U_t}{U} = \left(1 - e^{-k}\right)^2$. It is given that $\frac{U_t}{U} = \frac{1}{2}$. Therefore, $\left(1 - e^{-k}\right)^2 = \frac{1}{2} \Rightarrow 1 - e^{-k} = \frac{1}{\sqrt{2}} \Rightarrow e^{-k} = \frac{\sqrt{2}-1}{\sqrt{2}} \Rightarrow e^k = \frac{\sqrt{2}}{\sqrt{2}-1} \Rightarrow k = \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right) = \mathbf{1.23}$ **is the answer**.
I-20	When a RC circuit is connected to battery having an emf, the capacitor acts initially as a short circuit allowing maximum current $I_0 = \frac{E}{R}$ to flow through it, and it exponentially decays, mathematically expressed as $I_t = \frac{d}{dt}Q_t = \frac{E}{R}e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{\tau}}$. Here, time-constant of circuit is $\tau = RC$ and $t = k\tau$. Thus, $I_t = \frac{E}{R}e^{-\frac{k\tau}{\tau}} \Rightarrow I_t = \frac{E}{R}e^{-k}$. Power delivered by battery at any instant is $P_t = EI_t = E \times \left(\frac{E}{R}e^{-k}\right) \Rightarrow P_t = \frac{E^2}{R}e^{-k}$. It is required to determine value of time in terms of k, when $P_t = \frac{1}{2}P_0 = \frac{1}{2}EI_0 = \frac{1}{2}E \times \frac{E}{R} = \frac{E^2}{2R}$. It reduces to $\frac{E^2}{R}e^{-k} = \frac{E^2}{2R} \Rightarrow e^k = 2 \Rightarrow k = \ln 2 \Rightarrow k = \mathbf{0.69}$ **is the answer.**
I-21	It is a case of capacitor charging in a RC circuit through a battery of emf E. Energy stored in a capacitor at any instant is $U_t = \frac{1}{2}Q_t V_t = \frac{1}{2}Q_t \times \frac{Q_t}{C} \Rightarrow U_t = \frac{{Q_t}^2}{2C}$. Therefore, rate at which energy is stored in the capacitor is $P_t = \frac{d}{dt}U_t = \frac{d}{dt}\frac{{Q_t}^2}{2C} = \frac{1}{2C} \times 2Q_t \times \frac{d}{dt}Q_t = \frac{Q_t}{C} \times \frac{d}{dt}Q_t$. Here, instantaneous charge on a capacitor is $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$ and $\frac{d}{dt}Q_t = EC \times \left(-\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}\right) \Rightarrow \frac{d}{dt}Q_t = \frac{E}{R}e^{-\frac{t}{RC}}$. Thus, $P_t = \frac{EC\left(1-e^{-\frac{t}{RC}}\right)}{C} \times \frac{E}{R}e^{-\frac{t}{RC}}$. It solves into $P_t = \frac{E^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)$. For maximum or minimum value of P_t the test is that value of t at which $\frac{d}{dt}P_t = 0$ and to judge it to be maximum is at that t determined in previous test $\frac{d^2}{dt^2}P_t < 0$ i.e. (-) and vice-versa for minimum. Accordingly, $\frac{d}{dt}P_t = \frac{d}{dt}\left[\frac{E^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)\right] = \frac{E^2}{R}\left[-\frac{e^{-\frac{t}{RC}}}{RC} - \left(\frac{-2e^{-\frac{2t}{RC}}}{RC}\right)\right] = 0 \Rightarrow 2e^{-\frac{2t}{RC}} = e^{-\frac{t}{RC}}$. It leads to $e^{\frac{t}{RC}} = 2 \Rightarrow \frac{t}{RC} = \ln 2 \Rightarrow t = \boldsymbol{RC}$ **ln 2, is the answer of second part One.**

	Here solution second part is a pre-requisite for assertively arriving at answer to first part. Thus first derivative $\frac{d^2}{dt^2}P_t = \frac{d}{dt}\left(\frac{d}{dt}P_t\right) = \frac{d}{dt}\left(\frac{E^2}{R}\left[-\frac{e^{-\frac{t}{RC}}}{RC} + \frac{2e^{-\frac{2t}{RC}}}{RC}\right]\right) = \frac{E^2}{R^2C}\left[-\frac{4e^{-\frac{2t}{RC}}}{RC} - \left(-\frac{e^{-\frac{t}{RC}}}{RC}\right)\right] = \frac{E^2}{R^3C^2}\left[e^{-\frac{t}{RC}} - 4e^{-\frac{2t}{RC}}\right]$. Testing for maxima, with the value of $t = RC\ln 2$ of part 2 arrived at above value $e^{-\frac{t}{RC}} - 4e^{-\frac{2t}{RC}} < 0 \Rightarrow e^{\frac{t}{RC}} - 4 < 0$. To prove this inequality let $e^{\frac{t}{RC}} = x \Rightarrow e^{\frac{RC\ln 2}{RC}} = x \Rightarrow e^{\ln 2} = x \Rightarrow \ln 2 = \ln x \Rightarrow x = 2$. Accordingly, $e^{\frac{t}{RC}} - 4 < 0 \Rightarrow 2 - 4 = -2 < 0$. Thus it is established that $P_t = \frac{E^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)$ is maximum at $t = RC\ln 2$. Therefore, $P_{Max} = \frac{E^2}{R}\left(e^{-\frac{RC\ln 2}{RC}} - e^{-\frac{2RC\ln 2}{RC}}\right) = \frac{E^2}{R}\left(e^{-\ln 2} - e^{-2\ln 2}\right) = \frac{E^2}{R}(p - q)$. Going forward involves little of logarithmic algebra and accordingly let $e^{-\ln 2} = p \Rightarrow -\ln 2 = \ln p \Rightarrow \ln\frac{1}{2} = \ln p \Rightarrow p = \frac{1}{2}$.Likewise, $e^{-2\ln 2} = q \Rightarrow q = e^{-\ln 2^2} \Rightarrow q = e^{-\ln 4} \Rightarrow \ln q = -\ln 4 \Rightarrow \ln q = \ln\frac{1}{4} \Rightarrow q = \frac{1}{4}$. Using values of p and q we have $P_{Max} = \frac{E^2}{R}\left(\frac{1}{2} - \frac{1}{4}\right) = \boldsymbol{\frac{E^2}{2R}}$ **is the answer.** **N.B.: 1.** In this problem solution of part 2 is required for arriving at solution of part 1, as discussed above. Such problems are sometimes encountered. **2**. This problem involves simple use of calculus and logarithmic algebra. **3**. It is only comfort of a student to using associated mathematics promptly will help him to make a quick leap and minimize solution steps.
I-22	It is a problem of charging of a capacitor of capacitance $C = 12.0 \times 10^{-6}$ F, connected to a battery having $E = 6.00$ V and internal resistance $r = 1.00\ \Omega$, and external resistance $R = 0$. Basic equation of the circuit is of charging of a capacitor $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$ and $I_t = \frac{d}{dt}Q_t = EC \times \left(-\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}\right) \Rightarrow I_t = \frac{E}{R}e^{-\frac{t}{RC}}$. It is taken forward in solving each part, at $t = 12.0 \times 10^{-12}$s, separately – **Part (a):** The value of current is $I_t = \frac{E}{R}e^{-\frac{t}{RC}} = \frac{6.00}{1.00} \times e^{-\frac{12.0\times10^{-6}}{1.00\times(12.0\times10^{-6})}} = 6.00 \times e^{-1} = \mathbf{2.21A}$ **is the answer**. **Part (b):** Power delivered by the battery $P_t = EI_t = 6.00 \times 2.21 = 13.26$ A say **13.3 A, is the answer**. **Part (c):** Power dissipated in heat is in internal resistance r and hence $P_{Ht} = I_t^2r = (2.21)^2 \times 1.00 = \mathbf{4.88}$ **W is the answer.** **Part (d):** Energy stored in the battery is $U_t = \frac{1}{2}\frac{Q_t^2}{C}$, hence rate of energy stored $P_t = \frac{d}{dt}\left(\frac{1}{2}\frac{Q_t^2}{C}\right) = \frac{1}{2C}\frac{d}{dt}Q_t^2$. It leads to $P_t = \frac{2Q_t}{2C}\frac{d}{dt}Q_t = \frac{Q_tI_t}{C}$. Here, value of I_t is available from part (a), but Q_t has to be determined from charging equation. Using the available data $Q_t = 6.00 \times (12.0 \times 10^{-6}) \times \left(1 - e^{-\frac{12.0\times10^{-12}}{1.00\times(12.0\times10^{-6})}}\right) \Rightarrow Q_t = 72.0 \times 10^{-6} \times (1 - e^{-1}) = 72.0 \times 10^{-6} \times 0.632 = 45.5 \times 10^{-6}$. Using the available data $P_t = \frac{Q_tI_t}{C} = \frac{(45.5\times10^{-6})\times2.21}{12.0\times10^{-6}} = \mathbf{8.38}$ **W is the answer.** Thus, **answer is (a) 2.21 A (b) 13.3 W (c) 4.88 W (d) 8.37 W.**
I-23	This is a problem of discharge of a capacitor of capacitance C which is charged to a potential V is discharged through a resistance R. The discharge equation of the circuit is $Q_t = Q_0e^{-\frac{t}{RC}}$ here $Q_0 = VC$ is the initial charge on the capacitor. The discharge current is $I_t = \frac{d}{dt}Q_t = \frac{d}{dt}\left(VCe^{-\frac{t}{RC}}\right) = VC\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}} = -\frac{V}{R}e^{-\frac{t}{\tau}}$, here time constant $\tau = RC$. The power dissipated through a resistance in the form of heat is $P_t = I_t^2R$. Therefore, total heat dissipated during τ is $H = \int_0^\tau P_tdt = \int_0^\tau(I_t^2R)dt = R\int_0^\tau\left(-\frac{V}{R}e^{-\frac{t}{\tau}}\right)^2dt = \frac{V^2}{R}\int_0^\tau e^{-\left(\frac{2}{\tau}\right)t}dt \Rightarrow H =$

	$\frac{V^2}{R}\left[\left(-\frac{\tau}{2}\right)e^{-\left(\frac{2}{\tau}\right)t}\right]_0^{\tau} \Rightarrow H = \frac{V^2}{R}\times\left(-\frac{RC}{2}\right)\left[e^{-\left(\frac{2}{\tau}\right)\tau} - e^{-\left(\frac{2}{\tau}\right)0}\right] \Rightarrow H = \frac{CV^2}{2}\left[1-e^{-2}\right] \Rightarrow H = \boldsymbol{\frac{1}{2}\left(1-\frac{1}{e^2}\right)CV^2}$ **is the answer.**
I-24	This is a problem of charging of a capacitor of capacitance C, connected in series to a resistance R supplied by a battery of emf E. Equation of charge of a capacitor is $Q_t = EC\left(1-e^{-\frac{t}{RC}}\right)$. Therefore, instantaneous current through the resistor is $I_t = \frac{d}{dt}Q_t = \frac{d}{dt}\left(EC\left(1-e^{-\frac{t}{RC}}\right)\right) \Rightarrow I_t = EC\left(-\left(-\frac{1}{RC}\right)\right)e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{RC}}$. Power consumed by the resistor at any instant is $P_t = I_t^2R$. Therefore, heat dissipated by the resistor in time during charging is $H_t = \int_0^t I_t^2R\,dt = R\int_0^t\left(\frac{E}{R}e^{-\frac{t}{RC}}\right)^2dt = \frac{E^2}{R}\int_0^t e^{-\frac{2t}{RC}}dt = \left(\frac{E^2}{R}\right)\left(-\frac{RC}{2}\right)\left[e^{-\frac{2t}{RC}}\right]_0^t = \boldsymbol{\frac{E^2C}{2}\left(1-e^{-\frac{2t}{RC}}\right)}$. Now, during charging instantaneous power supplied to the capacitor is $P_t = I_tV_t = I_t\frac{Q_t}{C}$. Using the available values $P_t = \left(\frac{E}{R}e^{-\frac{t}{RC}}\right)\left(\frac{EC\left(1-e^{-\frac{t}{RC}}\right)}{C}\right) = \frac{E^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)$. Therefore, energy stored in the capacitor is $U = \int_0^t P_t dt$. It leads to $U_t = \int_0^t\left(\frac{E^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)\right)dt = \frac{E^2}{R}\left[\int_0^t e^{-\frac{t}{RC}}dt - \int_0^t e^{-\frac{2t}{RC}}dt\right] = \frac{E^2}{R}\left[RC\left(1-e^{-\frac{t}{RC}}\right) - \frac{RC}{2}\left(1-e^{-\frac{2t}{RC}}\right)\right]$. It solves into $U_t = E^2C\left[\left(1-e^{-\frac{t}{RC}}\right) - \frac{1}{2}\left(1-e^{-\frac{2t}{RC}}\right)\right] = \frac{E^2C}{2}\left(1-2e^{-\frac{t}{RC}}+e^{-\frac{2t}{RC}}\right)$. Time taken for the capacitor to be fully charged is $t\to\infty$ and at that heat developed $H = U_\infty = \frac{E^2C}{2}\left(1-e^{-\frac{2\infty}{RC}}\right) \Rightarrow \boldsymbol{H = \frac{E^2C}{2}}$ and energy stored in the capacitor is $U = U_\infty = \frac{E^2C}{2}\left(1-2e^{-\frac{\infty}{RC}}+e^{-\frac{2\infty}{RC}}\right) \Rightarrow \boldsymbol{U = \frac{E^2C}{2}}$. **It is thus proved that $\boldsymbol{H = U}$.** **N.B.: 1.** The problem states that capacitor is charged by connecting through a battery, without stating period of connection. It implies that capacitor is fully charged for which $t\to\infty$, and is used in final conclusion. **2.** From expression derived above the conclusion in this problem is not true for $0 < t < \infty$ and need not be taken as generic conclusion.
I-25	It is a case when dielectric filling the gap between plates of a parallel-plate capacitor is not pure dielectric, which is supposed to have infinite resistivity. Therefore, the capacitor when charged to a voltage say V and disconnected from the source would have a discharge circuit as shown in the figure. Here, capacitance of the parallel plate capacitor with dielectric having dielectric constant K is $C = \frac{\varepsilon_0 KA}{d}$, and resistance of the dielectric slab of resistivity is $R = \frac{\rho d}{A}$. Here, A is area of the plates and d is separation between the plates.  Time constant of a RC circuit is $\tau = RC = \left(\frac{\rho d}{A}\right)\times\left(\frac{\varepsilon_0 KA}{d}\right) = \rho\varepsilon_0 K \Rightarrow \tau = \boldsymbol{\varepsilon_0\rho K}$ is the answer. **N.B.:** Since $\boldsymbol{\varepsilon_0}$ is a universal constant it is given precedence of material dependent constant ρ and K, while reporting the answer,
I-26	Given circuit is a parallel combination of two equal capacitors of capacitance $C_1 = C_1 = 2\times10^{-6}$ F. Thus it equivalent to a case of charging RC circuit where $R = 25$ W and capacitance $C = C_1 + C_2 = 2(2\times10^{-6}) \Rightarrow C = 4\times10^{-6}$. Therefore time constant of the circuit is $\tau = RC = 25\times(4\times10^{-6}) = 0.1\times10^{-3}$. Charge on a capacitor in RC circuit at $t = 0.20\times10^{-3}$s is $Q_t = EC\left(1-e^{-\frac{t}{\tau}}\right) \Rightarrow Q_t = 6.0\times(4\times10^{-6})\left(1-e^{-\frac{0.2\times10^{-3}}{0.1\times10^{-3}}}\right)$. It leads to $Q_t = (24\times10^{-6})(1-e^{-2}) = (24\times10^{-6})\times0.86 = 20$ μC.

	Since equal capacitor are in parallel and hence 20 μC charge will be distribute equally on each of the capacitor such $Q_{1t} = Q_{2t} = \frac{Q_t}{2} = \frac{20}{2} =$ **10 μC is the answer.**
I-27	In the given circuit switch S is kept closed for a long time during which capacitor would be charged drawing no current and thus the two resistors would act as a series combination. Hence, voltage across the capacitor is the voltage across the middle resistor such that $V = \frac{E}{2R} \times R = \frac{E}{2} = \frac{12}{2} = 6.0$ V. Therefore, charge on the capacitor would be $Q = CV = (25 \times 10^{-6}) \times 6.0 = 0.15 \times 10^{-3}$C. Now when switch is opened it form a simple series RC circuit with only 10 Ω middle resistor having a time constant $\tau = RC = 10 \times (25 \times 10^{-6}) = 0.25 \times 10^{-3}$s. Equation of discharge-current of a capacitor is $I_t = \frac{E}{R} e^{-\frac{t}{\tau}}$. And therefore at $t = 1.0 \times 10^{-3}$s the current would be $I_t = \frac{12}{10} e^{-\frac{1.0 \times 10^{-3}}{0.25 \times 10^{-3}}} = 1.2 \times e^{-4} = 0.022$ A or **22 mA is the answer**.
I-28	The problem has two phases as under – **Phase 1:** Capacitor of capacitance $C = 100 \times 10^{-6}$ is charged by a battery of emf $E = 6.0$ V through a resistor of $R = 20 \times 10^3$ for $t_1 = 4.0$ s. Thus time-constant of the circuit is $\tau = RC = (20 \times 10^3) \times (100 \times 10^{-6}) = 2.0$s. **Phase 2:** The battery is disconnected and replaced with thick wire and allowed to discharge for a period $t_1 < t \leq t_2$ here $t_2 - t_1 = 4.0$ s. During this period, the thick wire is of negligible resistance and discharge resistance remains at R. Therefore, time-constant of the discharge circuit would remain at $\tau = 2.0$ s. Equation of charging of capacitor is $Q_t = EC\left(1 - e^{-\frac{t}{\tau}}\right) \Rightarrow Q_{t_1} = 6.0 \times (100 \times 10^{-6}) \times \left(1 - e^{-\frac{4.0}{2.0}}\right)$. It leads to $Q_{t_1} = (0.6 \times 10^{-3})(1 - e^{-2.0}) = 0.519 \times 10^{-3}$ CC In phase 2 the capacitor discharges as per equation $Q_{t2} = Q_{t_1} \times e^{-\frac{t}{\tau}} \Rightarrow Q_{t2} = (0.519 \times 10^{-3}) \times e^{-2} = 70 \times 10^{-6}$ C or **70 μC is the answer.**
I-29	It is problem of charging of a series capacitor whose equivalent resistance is $C = \frac{C_1 C_2}{C_1 + C_2}$. Equation of charging capacitor is $Q_t = EC\left(1 - e^{-\frac{t}{RC}}\right)$. Here, value of resistance $R = r$ and capacitance is $C = \frac{C_1 C_2}{C_1 + C_2}$ and therefore charge on the combination is $Q_t = EC\left(1 - e^{-\frac{t}{rC}}\right)\Big\|_{C = \frac{C_1 C_2}{C_1 + C_2}}$. It is required to find charge on capacitor C_1 as a function of t. It is to be noted that each cascaded capacitor has same charge as that on the equivalent capacitor, as shown in the figure. Therefore, charge on the capacitor C_1 is $Q_t = \boldsymbol{EC\left(1 - e^{-\frac{t}{rC}}\right)\Big\|_{C = \frac{C_1 C_2}{C_1 + C_2}}}$ **is the answer**. 
I-30	Circuit of the given problem is shown in the figure. Charge on capacitor $C_1 = C$ is given a charge $Q_1 = Q$. This capacitor is connected to another identical capacitor C_2 through a resistance R in series.. It is a case of simultaneous charging-discharging of capacitors. While charging of a capacitor C_2 by a capacitor C_1 which is at a potential difference $V_t = \frac{Q_t}{C}$. Accordingly, the discharge equation of the circuit for $t > 0$ i$V_t - V_t - \frac{q_t}{C} - RI_t = 0 \Rightarrow V_t - \frac{q_t}{C} - R\frac{d}{dt}q_t = 0 \Rightarrow \frac{d}{dt}q_t = \frac{Q_t}{RC} - \frac{q_t}{RC}$. As per principle of conservation of charge discharge on isolated capacitors-circuit remains constant at $Q = Q_t + q_t \Rightarrow Q_t = Q - q_t$. Accordingly, the charging equation can be rewritten as $\frac{d}{dt}q_t = \frac{Q - q_t}{RC} - \frac{q_t}{RC} \Rightarrow \frac{d}{dt}q_t = \frac{Q - 2q_t}{RC} \Rightarrow \int \frac{dq_t}{Q - 2q_t} = \frac{1}{RC}\int dt$. This solves into $-\frac{1}{2}\ln(Q - 2q_t) = \frac{t}{Rc} + K \Rightarrow Q -$ 

	$2q_t = K'e^{-\frac{2t}{RC}}$. At $t = 0$ the charge on te capacitor C_2 is $q_t = 0$, therefore, $Q - 0 = K'^{e^0} \Rightarrow K' = Q$. It leads to $Q - 2q_t = Qe^{-\frac{2t}{RC}} \Rightarrow 2q_t = Q\left(1 - e^{-\frac{2t}{RC}}\right)$. In its final form charge on capacitor C_2 is $q_t = \frac{Q}{2}\left(\mathbf{1} - e^{-\frac{2t}{RC}}\right)$ **is the answer.** **N.B.:** This problems prompts to visualize situations close to that occurring in practical circuits
I-31	Given that the capacitor of capacitance C is charged at Q. At $t = 0$ it is connected to an ideal batter of emf E whose internal resistance $r = 0$. As soon as switch is closed for $t > 0$, a current I_t is established giving additional charge to the capacitor until $V_t = \frac{Q_t}{C} < E$. Thus circuit equation is $E - RI_t - V_t = 0 \Rightarrow E - R\frac{d}{dt}Q_t - \frac{Q_t}{C} = 0$. Here, current $I_t = \frac{d}{dt}Q_t$ in the circuit is rate of change of charge on the capacitor. This linear differential equation is rewritten as $\frac{d}{dt}Q_t = \frac{EC - Q_t}{CR} \Rightarrow \int\frac{dQ_t}{EC - Q_t} = \frac{1}{CR}\int dt \Rightarrow -\ln(EC - Q_t) = \frac{t}{CR} + K$. It leads to $EC - Q_t = K'e^{-\frac{t}{CR}}$. Here, K is an integration constant and K' is its mathematically transformed value.  At $t = 0$, charge on capacitor $Q_0 = Q$. Accordingly, the circuit equation becomes $EC - Q = K'e^{-\frac{0}{CR}}$, it leads to $K' = EC - Q$, with this $EC - Q_t = (EC - Q)e^{-\frac{t}{CR}} \Rightarrow Q_t = \boldsymbol{EC}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{t}{CR}}\right) + \boldsymbol{Qe}^{-\frac{t}{CR}}$ **is the answer.**
I-32	Increase of temperature T of the order of 3000^0C, of filament of an incandescent bulb, leads state of incandescence where it starts radiating luminance is the principle. Moreover, the filament of the bulb has some resistance R_a at ambient temperature T_a. When it is connected to a constant potential difference V, as per Joule's Law it initially produces heat $H_a = VI_a = V \times \frac{V}{R_a}$. A part of the heat is utilized in raising temperature of the filament and part in radiating heat and light. At the equilibrium temperature T the resistance of the filament undergoes change as per $R_T = R_a\big(1 + \alpha(T - T_a)\big)$. Here, α is the thermal coefficient of resistance which is of the order of 4×10^{-3} C^{-1} and is quite small. Therefore, $R_T = R_a + \epsilon$. Thus, current through filament changes from $I_a = \frac{V}{R_a}$ to $I_T = \frac{V}{R_T} = \frac{V}{R_a + \epsilon}$. Accordingly, $\frac{I_T}{I_a} = \frac{\frac{V}{R_a + \epsilon}}{\frac{V}{R_a}} \Rightarrow I_T = I_a\left(1 + \frac{\epsilon}{R_a}\right)^{-1}$. Here, $\frac{\epsilon}{R_a} \ll$, therefore as per Binomial expansion $\left(1 + \frac{\epsilon}{R_a}\right)^{-1} \approx 1 - \frac{\epsilon}{R_a}$. Thus, $I_T = I_a\left(1 - \frac{\epsilon}{R_a}\right)$, or $\boldsymbol{I_T}$ **is slightly less than** $\boldsymbol{I_a}$**is explained.**
I-33	Current through R_1 is $I_1 = \frac{E}{R_1 + r}$, and as per Joule's Law thermal energy developed by the resistor in time t is $H_1 = I_1^2R_1t = \left(\frac{E}{R_1 + r}\right)^2 R_1t$. Likewise, for resistor R_1, heat developed is $H_2 = \left(\frac{E}{R_2 + r}\right)^2 R_2t$. Therefore, we have $\frac{H_1}{H_2} = \frac{\left(\frac{E}{R_1 + r}\right)^2 R_1t}{\left(\frac{E}{R_2 + r}\right)^2 R_2t} \Rightarrow \frac{H_1}{H_2} = \frac{R_1}{R_2} \times \left(\frac{R_2 + r}{R_1 + r}\right)^2$. Given that $R_1 \neq R_2$ and hence $\boldsymbol{H_1 \neq H_2}$ **is part One of the answer**. For, $\boldsymbol{H_1 = H_2}$ necessary condition is $\frac{R_1}{R_2} \times \left(\frac{R_2 + r}{R_1 + r}\right)^2 = 1 \Rightarrow \frac{(R_2 + r)^2}{R_2} = \frac{(R_1 + r)^2}{R_1} \Rightarrow \frac{R_2^2 + 2rR_2 + r^2}{R_2} = \frac{R_1^2 + 2rR_1 + r^2}{R_1}$. It leads to $R_2 + 2r + \frac{r^2}{R_2} = R_1 + 2r + \frac{r^2}{R_1} \Rightarrow R_2 - R_1 = r^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow R_2 - R_1 = r^2\left(\frac{R_2 - R_1}{R_1R_2}\right) \Rightarrow r^2 = R_1R_2$. It summarizes to $\boldsymbol{r = \sqrt{R_1R_2}}$ **is answer of part two.** Thus, **answers are No,** $\boldsymbol{r = \sqrt{R_1R_2}}$
I-34	In adiabatic process heat in a system is not exchanged with the environment. Whereas, heat produced by resistance as per Joules' Law is exchanged with provided and it is used as an electoral means of producing heat. Hence, **it is not an adiabatic process.**
I-35	When a charge q is moved through a potential difference ΔV in time t then as per definition of potential difference and Joules' Law $\Delta Q = q\Delta V = \frac{q}{t} \times \Delta V \times t = \Delta Vit$. This heat is utilized to raise temperature of the

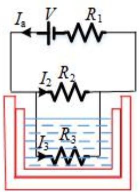
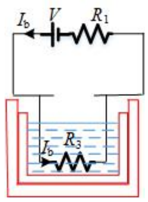
	resistance $\Delta U = ms\Delta t$ and thermal expansion of material, leading to compression of the surrounding expressed as $\Delta W = p\Delta v$. Since, Δt is positive ΔU is positive. Likewise, ΔV is positive and the resulting pressure is also positive making ΔW also positive. Thus, **all the three quantities are non-zero and positive is the answer**
I-36	Emf of a thermocouple is $E = a\theta + \frac{1}{2}b\theta^2$ here, $\theta' = \theta_c + \theta$ where $\theta' > \theta_c$C is the temperature of hot junction while cold junction is maintained at 0^0C while a and b are constants specific to a pair of metals forming the thermocouple. This is an empirical equation approximating characteristic of a thermos couple as shown in the figure -  Thus for thermocouple-emf to be zero, $0 = a\theta + \frac{1}{2}b\theta^2 \Rightarrow \theta_i = \theta = -\frac{2a}{b}$. Whereas for maximum emf $\frac{d}{d\theta}E = a + \frac{1}{2} \times 2b\theta = 0 \Rightarrow \theta_n = \theta = -\frac{a}{b}$. If $\theta_c = 0^0$ C and temperatures are indicated in Celsius then $\theta_i = 2\theta_n$. It is to be noted that θ_n for a thermocouple is fixed and depends on variation of electron density with temperature forming the thermocouple. Occurrence of this point of inflexion called inversion or neutral temperature requires that values of a and b are signed oppositely. Therefore, neutral temperature would not occur when pair of metals having values of a and b are of opposite signs, this is not true e.g. Pb-Ag, Pb-Cu… Hence, **answer is No for all thermocouples.**
I-37	Emf of a thermocouple is $E = a\theta + \frac{1}{2}b\theta^2$ here, $\theta' = \theta_c + \theta$ where $\theta' > \theta_c$ is the temperature of hot junction while cold junction is maintained at 0^0C while a and b are constants specific to a pair of metals forming the thermocouple. This is an empirical equation approximating characteristic of a thermos couple as shown in the figure -  Thus for thermocouple-emf to be zero, $0 = a\theta + \frac{1}{2}b\theta^2 \Rightarrow \theta_i = \theta = -\frac{2a}{b}$. Whereas for maximum emf $\frac{d}{d\theta}E = a + b\theta = 0 \Rightarrow \theta_n = \theta = -\frac{a}{b}$. If $\theta_c = 0^0$ C and temperatures are indicated in Celsius then $\theta_i = 2\theta_n$. It is to be noted that θ_n for a thermocouple is fixed and depends on variation of electron density with temperature forming the thermocouple. Hence, **answer is Yes and temperatures are indicated in Celsius and $\boldsymbol{\theta_c} = \mathbf{0}\,^0$C.**
I-38	Analysis of characteristic of thermocouple $E = a\theta + \frac{1}{2}b\theta^2$ here, $\theta' = \theta_c + \theta$ where $\theta' > \theta_c$ is the temperature of hot junction while cold junction is maintained at 0^0C while a and b are constants specific to a pair of metals forming the thermocouple. It leads to If $\theta_c = 0^0$ C and temperatures are indicated in Celsius then $\theta_i = 2\theta_n$. Hence, **answer is Yes if $\boldsymbol{\theta_c = 0^0}$ C and temperatures are indicated in Celsius.**
I-39	Polarity of an electrode used in electrolysis is the polarity of the battery to which it is connected. Accordingly an electrode connected to (+)ve terminal of battery will become anode and during electrolysis it will cause deposition of (-)ve ions of the electrolyte and vice-versa. Hence, **answer is No.**
I-40	Increase of temperature of electrolyte has two affects – (a) viscosity of electrolyte decreases, causing more mobility of ions, (b) dissociation of molecules into ions increases with increases giving rise to more charges free to move. Thus on account of the above two reasons conductivity σ of electrolyte increases and resistivity of electrolyte $\rho = \frac{1}{\sigma}$ decreases. Hence, **answer is Yes.**
I-41	As per Joule's Law thermal energy produced by a resistance R due to flow of current I in a given time t is $H = I^2Rt \Rightarrow H \propto I^2$a parabolic function of current. In the given figure curves (a) and (d) are parabolic [$x^2 =$

	$4A_a y]$; in graph a whose focal length A_a is less than slope $m > 4\,A_a$ of graph (b) $[x = my + C\|_{C=0}]$. Equations are written in this form for comparison of curves. Whereas for graph (c) $[x^2 = 4A_b y]$; in graph a whose focal length A_a is less than slope $m < 4\,A_b$. With the given graphs qualitative analysis to this extent is possiblymore quant It therefore requires to make best choice among the two. Curve (a) lies below the linear curve (b), and is an appropriate choice is **curve (a), the answer**
I-42	Rate of heat produced by resistance R when current I it is $P = \frac{d}{dt}U = \frac{d}{dt}(i^2Rt) = i^2R$. Since, it is stated that a constant is flowing through the resistor hence $\frac{d}{dt}U \propto R$. As current start flowing through a resistor, conversion of electrical energy into heat occurs. This heat is partially utilized in radiation and partially to raise temperature of the resistance. Variation of resistance with temperature is $R_T = R(1 + \alpha \times \Delta T) \Rightarrow R_T = (R\alpha)\Delta T + R \equiv m\Delta T + C$ can be compared with an equation of a line. Here coefficient of thermal variation of resistance α is positive and resistance is also positive hence slope of the line $m = R\alpha > 0$ and at $\Delta T = 0 \Rightarrow R_T = R$. Both these conditions are satisfied by **line (d), is the answer**
I-43	Neutral temperature θ_n depends upon density of free electrons of the metals forming the thermocouple. Thus **statement (A) is correct**. Whereas, the temperature of cold junction θ_c decides $\Delta\theta = \theta_n - \theta_c$. Accordingly, inversion temperature is $\theta_i = \theta_n + \Delta\theta$. Thus, **statement B is wrong.** Accordingly, option (b) is correct.
I-44	Let us analyze each of the given option – **Option (a):** In Thomson effect heat produced in conductor is proportional to the current passing through it. Hence, **this option is wrong.** **Option (b):** In Peltier effect heat produced in conductor is proportional to the current passing through it. Hence, t**his option is wrong.** **Option (c):** As per Joules' Law heat produced in conductor is proportional to the square of the current passing through it. Hence, t**his option is Correct.** **Option (d):** Since option (c) out of the three above is correct, hence **this option is wrong.** Thus, **answer is option (c).**
I-45	Peltier effect is caused due to concentration of electrons in the two metals forming thermocouple for which both the statements (A) and (B) are true. It is provided only in **option (a)., is the answer.**
I-46	In conductors when its two ends are held at different temperatures, electron density varies along its length. Electrons tend to diffuse across cross-sections from higher density to lower density and thus release energy to equalize the temperature along the length. Accordingly, when current is flown across a thermocouple junction from it causes release of energy acquired by electron in metal having higher concentration and thus cooling is produced. This difference in concentration of free electron reverses at colder junction causing pumping causing electrical work and thus producing heat tending to rise of temperature. But, maintaining temperatures of hot and cold junctions helps to sustain emf in the thermocouple and consequent current. Thus, flow current due different temperatures of the junctions is stipulation of Seebeck effect as per statement A and not due to B. Thus, **option (b) is the answer.**
I-47	In metal conductor different density of free electrons exists at different temperature. Thus across every cross-section of conductor the differential free-electron-densities produce an emf. This is the Thmoson effect. Temperature along conductor has a gradient between hot junction and cold junction. Thus emf along each of the metal strip of thermocouple is due **free-electron density dependent upon temperature provided in statement A** and is independent of electron densities in the two metal strips forming thermos couple. Hence, **answer is option (c)**.

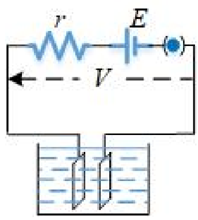
I-48	Faraday's constant is defined as magnitude of charge per mole of electron. Avogadro's Number $N_A = 6.022 \times 10^{23}$ is number of molecules in One mol (*M*) of substance. Here, 1 mol of substance is substance equal to molecular mass of substance. For example molar mass of hydrogen H_2 is 2 $(= 2 \times 1)$ while that of carbon is 12. Thus molar mass of methane CH4 is $16 = (12 + 2 \times 2)$. Thus molar mass of substance is based on atomic structure of atoms forming molecules of substance. Number of atoms per gram-mol of substance is $N = N_A \times M$ is a constant. Magnitude of charge of an electron is $e = 1.602 \times 10^{-19}$. Accordingly, charge of once gram of electrons is defined as Faraday's Constant is $F = N_A \times e = (6.022 \times 10^{23}) \times (1.602 \times 10^{-19}) = \mathbf{96.4 \times 10^3}$ **C/mol^{-1} is universal constant.** Here, *1 faraday of charge* 1*f* = 96.4×10^3C. **N.B.:** Electrochemical equivalent Z of substance is amount of substance deposited when 1 C charge (equivalent to 1A current for 1 s) passes through an electrolyte. Let a substance of molecular mass m and valency v is deposited through electrolysis will exchange charge $q = it$ which is equal to $q = v(N_A \times e) = vF$. Therefore, electrochemical equivalent of the substance, as per definition is, $\boldsymbol{Z} = \frac{m}{v(N_A \times e)} = \frac{\boldsymbol{m}}{\boldsymbol{vF}} = \frac{\boldsymbol{m}}{\boldsymbol{it}} \Rightarrow$ $\boldsymbol{m = Zit}$.
I-49	Given are two resistors such that $R_1 = R_2 = R$ are connected in series. A current I is passed through the combination of resistances and therefore in a time t heat produced in R_1 is $H_1 = I^2R_1 = I^2R$. Likewise, $H_2 = = I^2R$. Thus, regarding heat developed out of the two options (a) and (b), **option (a) is correct and is the answer.** As regards temperature rise it depends upon physical parameters of the two resistances, and not the electrical resistances. Accordingly, out of heat developed $H = H_a + H_d$, here H_a is heat retained by the resistor which goes into buildup of temperature $H_a = S\Delta T$ here S is the heat capacity and ΔT is the temperature rise. Whereas, H_d is the heat dissipated by the two resistors. Therefore, for equal temperature rise $\Delta T = \frac{H - H_a}{S}$ proportion of heat dissipated by the two resistors and specific heat capacities must be equal. Since given data is insufficient to decide and hence among option (c) and (d), option (c) is ruled out. Hence, **answer is option (d).** Thus, **answers are option (a) and (d).**
I-50	Given system is an experimental set up of two different metal strips joined at one end and kept at 0^0C and other free ends of the two strips are maintained at 100^0C. Accordingly, - • There will be potential difference at the two free ends of copper strip, as per Thomson's Effect. Thus **option (a) is correct.** • There will be potential difference between ends of cooper and iron strips at the junction as per Seebeck effect. Thus **option (b) is correct.** • There will be potential difference at the two free ends of iron strip, as per Thomson's Effect. Thus **option (c) is correct.** • There will be potential difference between free ends ends of cooper and iron strips as per Seebeck effect. Thus **option (d) is correct.** Thus, **answer is all options are correct.**
I-51	Equation of thermocouple emf is $E = a\theta + \frac{1}{2}b\theta^2$. Thus requirement of neutral temperature is when slope $\frac{dE}{d\theta} = 0$. It necessitates $a + b\theta = 0 \Rightarrow \theta = -\frac{a}{b}$. Since temperature of cold junction is $\theta_c = 0$, hence neutral temperature $\theta_n > \theta_c \Rightarrow \theta_n > 0$ implies that θ is positive. In the instant case with positive values of a and b θ is negative. Hence neutral temperature would not exit. Thus, **option (a) is correct.** For inversion temperature to exist, it has to be preceded by neutral temperature. Since there is no neutral temperature as concluded above, there will be no inversion temperature. Thus, **option (b) is correct.** Since both $a \neq 0$ and $b \neq 0$ in case of the two junctions kept at different temperatures therefore will be thermo-emf. Thus, **option (c) is incorrect.**

	In view of the existence of thermo-emf as per conclusion in option (c) there will be current in the thermocouple, **making option (d) incorrect.** Thus, **answers are option (a) and (b).**
I-52	In the given situation each of the options are analyzed as under – **Option (a):** When battery terminals are reversed, polarity of anode and cathode in a voltameter are reversed electrolysis continues with reversal of deposition of anions and cations. Thus, **statement at option (a) is wrong.** **Option (b):** In light of discussions in option (a), rate of liberation would remain same as all other parameters are same except, change of polarity. Thus, **option (b) is wrong.** **Option (c):** As per discussions at (b) rate of liberation of material would remain same. Thus, **option (c) is correct.** **Option (d):** In light of discussions at option above, and equilibrium of ionization of electrolyte, resistivity of the electrolyte would remain same and so also heat produced. Thus, **option (d) is incorrect.** Thus, **answer is option (c).**
I-53	Electrochemical equivalent Z of substance is amount of substance deposited when 1 C charge (equivalent to 1A current for 1 s) passes through an electrolyte. Let mass of substance M and valency v is deposited through electrolysis will exchange charge $q = it$ which is equal to $q = v(N_A \times e) = vF$. Therefore, electrochemical equivalent of the substance, as per definition is, $Z = \frac{M}{v(N_A \times e)} = \frac{M}{vF} = \frac{M}{it}$. Mass of substance $M = nm$, here is m molar of substance say 16 for carbon and is specific to each material. And n is number on moles . If $it =$ 1C then $Z = m$. Thus, **option (a) is correct.** Discussions on option (a) above, electrochemical equivalent being characteristic to material does not depend upon current through electrolyte, option (b), amount of charge $Q = it$ in option (c), and amount of material present in electrolyte option (d). Thus, **answer is option (a).**
I-54	As per Joule's law heat developed $H = I^2Rt$. Given that $I = 2.0$ A, $R = 2\ \Omega$ and $t = 60$ s, $H = 2.0^2 \times 25 \times 60 = 6.0 \times 10^3$ = **60 kJ is the answer**..
I-55	As per Joule's law $H = I^2Rt$, since given is voltage and resistance and therefore Combining Ohm's law we have $H = \left(\frac{V}{R}\right)^2 Rt = \frac{V^2}{R}t$…(1). It is also given that heat capacity of the coil is $S = 4.0$ JK-1 and temperature of the coil is raised is $\Delta T = 10\ ^0$C =15K. Thus, $H = S \times \Delta T$…(2). Equating (1) and (2), $\frac{V^2}{R}t = S \times \Delta T \Rightarrow t = \frac{S \times \Delta T \times R}{V^2}$. Using the available data $t = \frac{4.0 \times 15 \times 100}{6^2} = \frac{1000}{s}$ s or **2.8 min is the answer.**
I-56	As per Joules' and Ohms Law $P = \left(\frac{V}{R}\right)^2 \Rightarrow R = \frac{V^2}{P} \Rightarrow R = \frac{250 \times 250}{500} = \mathbf{125\ \Omega}$ **is the answer of first part.** Now keeping voltage to be same power of the heater coils is $P' = 1000$ W, hence resistance of the coil would be $R' = \frac{V^2}{P'} \Rightarrow R' = \frac{250 \times 250}{1000} = \mathbf{62.5\ \Omega}$ **is the answer of first part.** Thus, **answers are 125 Ω, 62.5 Ω**
I-57	Starting with Joules' Law taking each part progressively with data given – **Part (a):** $P = \left(\frac{V}{R}\right)^2 \Rightarrow R = \frac{V^2}{P} \Rightarrow R = \frac{250 \times 250}{500} = \mathbf{125\ \Omega}$ **is the answer.** **Part (b):** Resistance of wire based on its physical parameters $R = \rho\frac{L}{A} \Rightarrow L = \frac{RA}{\rho}$. Given that $\rho = 1.0 \times 10^{-6}\ \Omega$m and cross-sectional area of wire $A = 0.5$ mm² =0.5×10^{-6}m², $L = \frac{125 \times (0.5 \times 10^6)}{1.0 \times 10^6} =$ $\mathbf{62.5}$ **m is the answer.**

	Part (c): The resistance wire is compacted in the form of a coil having turns or radius $r = 4.0 \times 10^{-3}$m, the number turns would be $N = \frac{L}{2\pi r} = \frac{62.5}{2\pi \times (4.0\times10^{-3})} = 2487$turns, using principle of significant digits **2500 turns is the answer**. **Thus, answers are (a) 125 Ω (b) 62.5 m (c) 2500 turns**
I-58	Resistance of bulb of rating 250 V and 100 W as per Joules' Law is $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} \Rightarrow R = \frac{250\times250}{100} = 625$ Ω. Resistance of the wire to and fro connecting the bulb situated at a distance $L = 10$ m is $r = \rho\frac{2L}{A} = (1.7 \times 10^{-8})\frac{2\times10}{5\times10^{-6}} = 6.8 \times 10^{-2}$. Thus current drawn by the bulb connected to a supply $V' = 220$ V is $I' = \frac{V'}{R+r}$. Hence, power consumed by the connecting wires is $P' = I'^2 r = \left(\frac{V'}{R+r}\right)^2 r$. Using the available data $P' = \left(\frac{220}{625+0.068}\right)^2 \times 0.068 =$ **5.3 mW is the answer.**
I-59	As per Joule's law for the given data $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} \Rightarrow R = \frac{220\times220}{60} = 800$ Ω. When supply voltage drops to $V_1 = 180$ V, power consumed by the bulb will be $P_1 = \frac{V_1{}^2}{R} = \frac{180^2}{800} =$ **40 W is the answer of the first part.** Now voltage is raised to $V_2 = 240$ V, power consumed by the bulb will be $P_1 = \frac{V_1{}^2}{R} = \frac{240^2}{800} =$ **71 W is the answer of the second part.** Thus, **answers are 40 W, 71 W**
I-60	As per Joule's law for the given data $P = \frac{V^2}{R}$. It is given that voltage fluctuates by $\pm$ 1% and therefore power consumed by the bulb would be $P' = \frac{V'^2}{R} = \frac{(V(1\pm0.01))^2}{R} = \frac{V^2}{R}(1 \pm 0.01)^2 \approx \frac{V^2}{R}(1 \pm 2 \times 0.01) = 100(1 \pm 0.02)$. Therefore, minimum and maximum power consumed by the bulb are $\boldsymbol{P_{Min}} = 100(1 - 0.02) =$ **98 W** and $\boldsymbol{P_{Max}} = 100(1 + 0.02) =$ **102 W**. Thus, **answers are 98 W, 102 W.**
I-61	As per Joules' Law $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$…(1). Bulb with specifications $V = 220$ V and power $P = 100$ W gets fused at $P' \geq 100$ W. Therefore, maximum operating voltage of the bulb is $P' = \frac{V'^2}{R} \Rightarrow R = \frac{V'^2}{P'}$…(2). Equating (1) and (2), $\frac{V'^2}{P'} = \frac{V^2}{P} \Rightarrow V' = V\sqrt{\frac{P'}{P}}$. Using the available data $V' = 220\sqrt{\frac{150}{100}} = 269$ V. Using the principle of significant digits $V \leq$ **270 V is the answer.**
I-62	Amount of heat required to raise temperature of water $V = 0.01$ m³ whose mass (density of water being $\rho = 1000$ kg /m³) is $M = \rho V = 1000 \times 0.01 = 10$ kg. Its temperature is to be raised by $\Delta T = 40 - 15 = 25$ ^{0}C. Therefore heat required is $H = M \times s \times \Delta T$, here specific heat of water is $s = 4200$ J/kg/ ^{0}C. Accordingly, $H = 10 \times 4200 \times 25 = 1050 \times 10^3$J. It is given that 60% of input energy is utilized to heat water, therefore amount of electrical energy consumed is $E = (1050 \times 10^3) \times \frac{100}{60} =\approx 1750 \times 10^3$J. With the given specification of heater $P = 1000$ W, 220 V, required electrical energy is $E = P \times t \Rightarrow t = \frac{E}{P}$, here t is time in s. Therefore, time in minutes will be $t' = \frac{E}{P\times60} = \frac{1750\times10^3}{1000\times60} = 29$ minutes is the answer. **N.B.:** Here it is given that immersion heater is used at its rated voltage and hence voltage of the heater is notional and is not required to be used in calculations
I-63	Mass of water of 4 cups of water is $m = n \times V \times \rho$, here $n = 4$ cups of water, $V = 200$ cc/cup and $\rho = 1$ g/cc for water. Room temperature $T_1 = 25$ ^{0}C. Boiling point of water at normal atmospheric pressure is $T_2 =$

	100 ^{0}C. Therefore, heat required to boil the water is $H = ms(T_2 - T_2) \times 4.2 \Rightarrow H = (n \times V \times \rho)s(T_2 - T_2) \times 4.2$ J. Here, 4.2 is mechanical equivalent of heat. Accordingly, with the available data $H = (4 \times 200 \times 1) \times 1 \times (100 - 25) \times 4.2 = 252$ kJ. Given is rate of energy $R = 7.00$ per unit. And 1 unit is $= 1000$ Watt hour $= 1000 \times 3600$ W-s. Thus cost of energy required for boiling the water is $C = \frac{H}{1000\times3600} \times R \Rightarrow C = \frac{252\times1000}{1000\times3600} \times 7.00 \Rightarrow C = 0.49$ Rupee or **49 paise is answer of part (a).** When room temperature is $T_1 = 5$ 0C then $= (4 \times 200 \times 1) \times 1 \times (100 - 5) \times 4.2 = 319.2$ kJ. Hence cost of electricity will be $C = \frac{319.2\times1000}{1000\times3600} \times 7.00 \Rightarrow C = 0.62$ Rupee or **62 paise is answer of part (b).** Thus, **answers are (a) 49 paise (b) 62 paise**
I-64	An electric bulb of $P = 100$ W, $V = 220$ V starts glowing at 40 W. And for power input $P' > 40$ W it is given that power used in luminance is $P_L{}' = (P' - 40) \times 0.6$ W and into heat is $P_H{}' = (P' - 40) \times 0.4$ W. Further, it is required to find drop in luminance when supply voltage changes to $V' = 200$ V. Resistance of a bulb of given power does not change with change of voltage and for the given bulb it is $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$. Therefore, the bulb when operated at V' its power is $P' = \frac{V'^2}{R} \Rightarrow P' = \frac{V'^2}{\frac{V^2}{P}} \Rightarrow P' = \left(\frac{V'}{V}\right)^2 P$. With the given data $P' = \left(\frac{200}{220}\right)^2 \times 100 = 82.6$ W. At rated operation of $V = 220$ V the illuminating power is $P_L = (100 - 40) \times 0.6 = 36$ W and on operation at $V' = 200$ V the illuminating power is $P_L = (82.6 - 40) \times 0.6 = 25.56$ W. Accordingly, light intensities $\frac{I_L{}'}{I_L} = \frac{P_L{}'}{P_L} \Rightarrow \frac{I_L{}'}{I_L} = \frac{25.56}{36} = 0.71$ or 71%. Therefore, drop in light intensity is $= 100 - 71 =$ **29% is the answer.**
I-65	Heating of water in the given problem is dependent on heat $H = I^2Rt$ developed in $R_3 = 2\ \Omega$, here $t = 15 \times 60 = 900$ s is the time for which the circuit is active. Accordingly, rise of temperature of water is $H = S \times \Delta T$, here $S = 2000$ JK^{-1} is the heat capacity of heat system consisting of calorimeter and water. Here, units of electrical energy and heat capacity in joules and therefore, use of mechanical equivalent of heat J is not required. The problem has two parts having change in the electrical circuit where battery voltage is $V = 6$ V, resistances $R_1 = 1\ \Omega$, $R_2 = 6\ \Omega$, and $R_1 = 2\ \Omega$. Whereas, the thermal system remains unchanged. Each part is solved separately.  **Part (a):** Considering series parallel combination of the circuit, equivalent resistance of the circuit is $R_{Eq} = R_1 + \frac{R_2R_3}{R_2+R_3} = 1 + \frac{2\times6}{2+6} = 2.5\ \Omega$. Therefore, current in the circuit $I_a = \frac{V}{R_{Eq}} = \frac{6}{2.5} = 2.4$ A. This current would distribute $I_a = I_2 + I_3$ among R_2 and R_3 in inverse proportion of their resistances such that $\frac{I_3}{I_2} = \frac{R_2}{R_3} \Rightarrow \frac{I_3}{I_2+I_3} = \frac{R_2}{R_2+R_3} \Rightarrow I_3 = \left(\frac{R_2}{R_2+R_3}\right) \times I_a$. Using the available data $I_3 = \left(\frac{6}{6+2}\right) \times 2.4 = 1.8$ A. Accordingly, $\Delta T_a = \frac{I_3{}^2R_3t}{S} = \frac{1.8^2\times2\times900}{2000} =$ **2.9 ^{0}C is answer of the apart (a).** **Part (b):** In this part it is given that R_2 gets burnt. Thus the electrical circuit becomes a series combination of R_1 and R_3. Thus current in the circuit $I_b = \frac{V}{R_1+R_3} = \frac{6}{1+2} = 2$ Amp. Therefore, $\Delta T_b = \frac{I_b{}^2R_3t}{S} = \frac{2^2\times2\times900}{2000} = 3.$**6 ^{0}C is answer of the apart (b).**  Hence, **answers are (a) 2.9^{0}C (b) 3.6^{0}C** **N.B.:** Incremental rise of temperature in each part is to be determined and not the cumulative rise in temperature. It needs to be noted carefully to be right at the answer.

I-66	The emf equation of the thermocouple is $E = a\theta + \frac{1}{2}b\theta^2$. Using the given data where temperature of the junction $\theta = \theta_C + 0.001^0$C is given while the other junction is maintained at $\theta_C = 0$ ^{0}C. Accordingly, the thermo-emf that is developed in the thermo-couple is $E = (-46 \times 10^{-6}) \times 0.001 + \frac{1}{2}(-0.48 \times 10^{-6} \times (0.001)^2) \Rightarrow E = \mathbf{-0.46 \times 10^{-8}}$ **V is the answer.**
I-67	With the given values of a and b for metals forming thermo-couple $a_{Cu-Ag} = a_{Cu-Pb} - a_{Ag-Pb}$, it leads to $a_{Cu-Ag} = 2.76 \times 10^{-6} - 2.5 \times 10^{-6} = 0.26 \times 10^{-6}$ V^{0}C^{-1} and $b_{Cu-Ag} = b_{Cu-Pb} - b_{Ag-Pb} = 0$, since both values on R.H.S are equal. Further given that temperature of cold junction is $\theta_C = 0^0$C and that of the hot junction is $\theta_H = 40$ ^{0}C, accordingly $\theta = \theta_H - \theta_C = 40^0$C. Therefore, as per equation $E = a\theta + b\theta^2 \Rightarrow E = a_{Cu-Ag} \times 40 + 0 = (0.26 \times 10^{-6}) \times 40 = \mathbf{1.04 \times 10^{-5}}$ **V is the answer.**
I-68	Equation of thermo-emf is $E = a\theta + \frac{1}{2}b\theta^2$. In this neutral temperature is at $\frac{d}{d\theta}E = \frac{d}{d\theta}(a\theta + b\theta^2) = a + 2b\theta = 0$ or with the given $\theta_C = 0$, at $\theta = -\frac{a}{b} = \theta_n$. For the given copper-iron thermocouple $a_{Cu-Fe} = a_{Cu} - a_{Fe} \Rightarrow a_{Cu-Fe} = 2.76 \times 10^{-6} - 16.6 \times 10^{-6} = -13.84 \times 10^{-6}$ V^{0}C^{-1}and $b_{Cu-Fe} = b_{Cu} - b_{Fe} \Rightarrow b_{Cu-Fe} = 0.12 \times 10^{-6} - (-0.030) \times 10^{-6} = 0.042 \times 10^{-6}$ V^{0}C^{-1}. Therefore, $\theta_n = -\frac{(-13.84\times10^{-6})}{(0.042\times10^{-6})} = \mathbf{330}$ **^{0}C is one part of the answer.** For inversion temperature θ_i we have $E = a\theta_i + \frac{1}{2}b\theta_i^2 = 0 \Rightarrow \theta_i = -2 \times \frac{a}{b} = -2 \times \left(\frac{-13.84\times10^{-6}}{0.042\times10^{-6}}\right)$. It leads to $\theta_i = \mathbf{659}$ **^{0}C is one part of the answer.** Thus, **answers are 330^{0}C, 659^{0}C**
I-69	Charge of a monovalent ion is $q_m = e = 1.6 \times 10^{-19}$C. While for a divalent atom is $q_d = 2e = 2 \times (1.6 \times 10^{-19}) = 3.2 \times 10^{-19}$. Therefore, charge required to flow through an electrolyte to liberate one atoms of monovalent material and a divalent material, respectively is $\mathbf{1.6 \times 10^{-19}}$ **C and $\mathbf{3.2 \times 10^{-19}}$ C, are the answers.**
I-70	Amount of charge flown during $I = 0.500$ for time $t = 1 \times (60 \times 60) = 3600$ s is $Q = It = 0.500 \times 3600 = 1800$ C. Given that atomic mass of silver is 107.9 g.mol^{-1} Further, 1 gm-mol of substance contains $N_A = 6.022 \times 10^{23}$ atoms and that silver being mono-valance atom charge in 1 gm-mol of silver its electro-chemical equivalent is $Z = \frac{m}{N_A e} = \frac{107.9}{(6.022\times10^{23})\times(1.602\times10^{-19})} = 11.18 \times 10^{-4}$. Therefore, mass of silver deposited is $M = Z \times Q = (11.18 \times 10^{-4}) \times 1800 = \mathbf{2.01}$ **g is the answer.**
I-71	Given the electrochemical equivalent of silver $Z = 1.12 \times 10^{-6}$ kg.C^{-1} or 1.12×10^{-3} g.C^{-1}. Amount of silver deposited is $M = 3.0$ then charge required to pass through the electrolyte is $M = ZQ = ZIt \Rightarrow I = \frac{M}{Zt}$, here I is the current passed for given $t = 3 \times 60 = 180$ s. Therefore, $I = \frac{3.0}{(1.12\times10^{-3})\times180} = 14.9$ A. Using the principle of Significant Digits, **the answer is 15 A.**
I-72	Molecular mass of hydrogen is $m = 2.0$ g which at STP occupies 22.4 litr, therefore mass of the hydrogen liberated is $M = \frac{m}{22.4} = \frac{2.0}{22.4}$ gm. Further, $M = ZIt$ for hydrogen being monovalent electrochemical equivalent is $Z = \frac{1}{(6.022\times10^{23})\times(1.602\times10^{-19})} = \frac{1}{96500}$ therefore, $t = \frac{M}{ZI} = \frac{\frac{2}{22.4}}{\frac{1}{96500}\times5.0} = 1723$ s $= \frac{1723}{60} = 28.7$ min or, using principle of significant digits, **29 minutes is the answer.**
I-73	Amount of substance deposited is $M = ZIt$ ...(1), here electrochemical equivalent of the substance is $Z = \frac{m}{vA_v e}$ g.C^{-1} ...(2), here valency $v = 3$, and $A_v e = 96500$. Further, given that current $I = 2$ and which passed through the voltameters for $t = 1.50 \times 60 \times 60 = 5400$ s mass of the metal deposited is $M_3 = 1.00$ g.

	Combining (1) and (2) $M_3 = \frac{m_3}{v_3(A_v e)} \times I \times t \Rightarrow m_3 = \frac{M_3 v_3 (A_v e)}{It} = \frac{1.00 \times 3 \times 96500}{2 \times 5400} = \mathbf{26.8}$ **g/mol^{-1} is the answer of part (a).** From the part (a) $Z_3 = \frac{m_3}{v_3(A_v e)} = \frac{26.8}{3 \times 96500}$. For silver being monoatomic having $m_1 = 107.9$ g.mol^{-1} its electrochemical equivalent is $Z_1 = \frac{m_1}{v_1(A_v e)} = \frac{107.9}{1 \times 96500}$. Since the two voltameters are connected in series, therefore, charge passed through them $Q = It$ is equal. Accordingly, using (1) & (2), $\frac{M_1}{M_3} = \frac{Z_1 Q}{Z_3 Q} \Rightarrow M_1 = M_3 \left(\frac{m_1 v_3}{m_3 v_1}\right) = 1.00 \times \frac{107.9 \times 3}{26.8 \times 1} = \mathbf{12.1}$ **g is the answer of part (b).** Thus, **answers are (a) $\mathbf{26.8}$ g/mol^{-1} (b) $\mathbf{12.1}$ g.**
I-74	Mass of silver deposited is $M = V\rho = (2Ad)\rho$, given that area of one side of the plate is $A = 200$ cm^2 and thickness of silver coat on each side is $d = 1 \times 10^{-2}$ cm and specific density of silver translates into its density $\rho = 10.5$ gm/cm^3. Therefore, mass of silver deposit is $M = (2 \times 200 \times 0.01) \times 10.5 = 42$ g. Further, from basics of electrolysis $M = \frac{m}{v(A_v e)} \times I \times t$. Given that atomic mass of silver is 107.9 g.mol^{-1} and current $I = 15$ A is passed for a certain time t. Therefore, $t = \frac{M \times v \times (A_v e)}{m \times I} = \frac{42 \times 1 \times 96500}{107.9 \times 15}$ s or $t = \frac{42 \times 1 \times 96500}{107.9 \times 60} = 41.7$ min. Using principle of significant digits **42 minutes is the answer.**
I-75	Mass of silver (M) deposited during electrolysis is $M = \frac{m}{v(A_v e)} \times I \times t \Rightarrow I = \frac{96500 \times M}{m \times t}$ …(1), here given that mass of silver deposited $M = 2.68$ g, atomic mass of sliver $m = 107.9$ g.mol^{-1}, valency of silver $v = 1$, Avogadro's number A_v and charge of an electron e such that $A_v e = 96500$ a constant, $t = 10 \times 60 = 600$ s for which a current I is passed through the electrolyte. Therefore, as desired, heat developed in resistor $R = 20\ \Omega$ is to be determined, which as per Joules' Law is $H = I^2 Rt$ J….(2). Combining (1) and (2) $H = \left(\frac{96500 \times M}{m \times t}\right)^2 Rt = \left(\frac{96500 \times M}{m}\right)^2 \times \frac{R}{t}$…(3). Using the available data $H = \left(\frac{96500 \times 2.68}{107.9}\right)^2 \times \frac{20}{600} = 191496$ J. Using principle of significant digits $H =$ **190 kJ is the answer.**
I-76	It is a problem combining concepts of electrical circuit and electrolysis. It is required to determine mass of silver M deposited in time $t = 30 \times 60 = 1800$ s. Atomic mass of silver is $m = 107.9$ g.mol^{-1}. In electrolysis $M = \frac{m}{v(A_v e)} \times I \times t$…(1), Here, we know that valency of silver $v = 1$, Avogadro's number A_v and charge of an electron e such that $A_v e = 96500$ a constant. In (1) the only unknown is current I  From electrical circuit, the voltage equation is $E - Ir - V = 0 \Rightarrow I = \frac{E-V}{r} = \frac{12-10}{2} = 1$ A. Thus with the available data, $M = \frac{107.9}{96500} \times 1 \times 1800 = 2.01$ g. Thus using principle of significant digits **answer is 2g.**
I-77	This problem has four parts, connected progressively, as under – **Part (a):** electrolytic deposition copper of thickness $d = 10 \times 10^{-6}$m on both sides of plate having area $A = 10 \times 10^{-4}$m^2, density of copper is $\rho = 9000$ kg.m^{-3}. This will lead to geometrical determination of mass M of copper deposited on the given plate. Accordingly, using the available data, $\boldsymbol{M} = V\rho = (2A \times d)\rho = (2 \times (10 \times 10^{-4}) \times 10 \times 10^{-6}) \times (9000) = \mathbf{1.8} \times \mathbf{10^{-4}}$ **kg.** **Part (b):** Amount of charge $Q = It$ passed during electrolysis for deposition of mass M of copper is $M = ZQ$ given that electrochemical equivalent of copper is $Z = 3 \times 10^{-7}$ kg.C^{-1}. Accordingly, $\boldsymbol{Q} = \frac{M}{Z} = \frac{M}{Z} = \frac{1.8 \times 10^{-4}}{3 \times 10^{-7}} = \mathbf{6.0 \times 10^2}$**C** **Part (c):** Amount of energy spent by cell $U = VIt = VQ$, here cell has $V = 12$ V.Therefore, $\boldsymbol{U} = 12 \times (6.0 \times 10^2) = 7.2 \times 10^3$ J or **7.2 kJ**

	Part (d): If energy calculated in part (c) is spent on heating water of mass $m_w = 0.1$ kg the rise of temperature of the water whose specific heat is $s = 4200$ J.kg^{-1}.K^{-1}is $U = H = m_w s\Delta T \Rightarrow \Delta T = \frac{U}{m_w s} = \frac{\mathbf{7.2\times10^3}}{0.1\times4200} = 17$ K **Thus, answers are $\mathbf{7.2}$ kJ, 17 K.** N.B.: Decomposition of problems in step-wise solution makes it easier. This is called algorithmic approach of solving any problem howsoever complex it may appear, at a first glance. Such an approach is extremely useful in solving an unknown problem, as one moves forward.

Important Note: You may encounter need of clarification on contents and analysis or an inadvertent typographical error. We would gratefully welcome your prompt feedback on mail ID: subhashjoshi2107@gmail.com. If not inconvenient, please identify yourself to help us reciprocate you suitably.

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