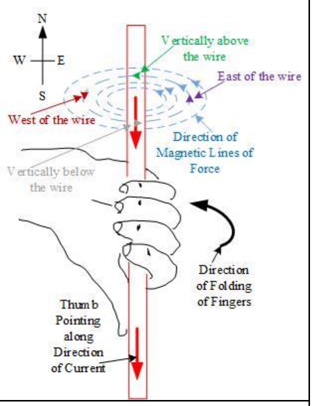
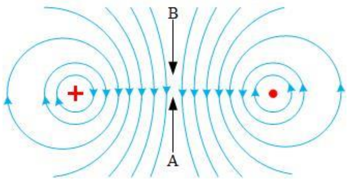
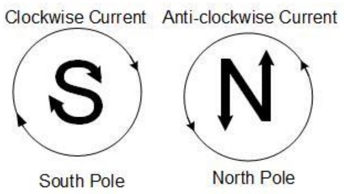
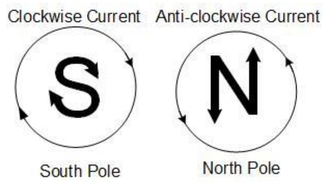
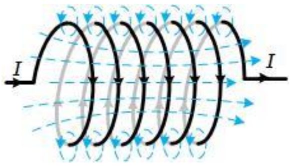
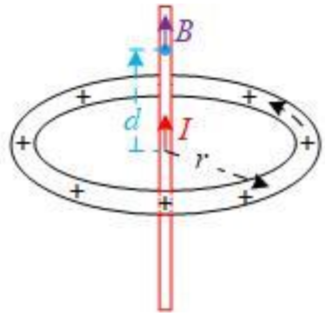
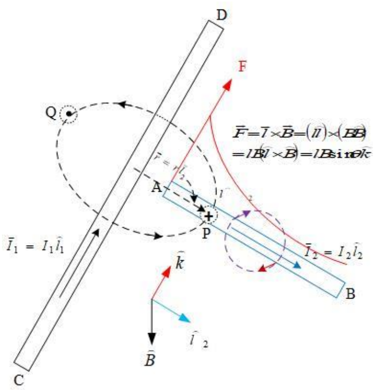
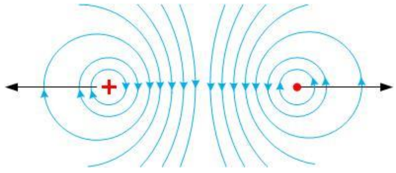
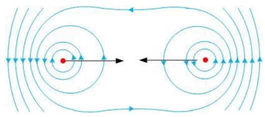
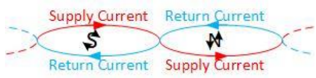
Code: Phy/ EM-4/MEC/001

Electromagnetism: Magnetic Effect of Electric Current

Illustrations of Typical Questions (Set-1)

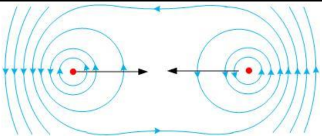
I-1	Applying Ampere's Right-Hand-Thumb rule where thumb points towards the direction of current. Then as per rule, fingers curling around the conductor indicate direction of the magnetic field shown by dotted lines. In this case for simplicity conductor is laid on the paper such that current flows from north to south, as given. Accordingly, in each of the case direction of magnetic field is – **(a) Coming out of paper** **(b) Entering paper** **(c) East to West** **(d) West to East** 
I-2	Applying Biot-Savart's Law magnitude of magnetic field at a point d from a long straight conductor carrying current i is $B=\frac{\mu_0 i}{2\pi d}$….(1). Here, μ_0 is absolute permeability in magnetic field. Further, speed of light in vacuum $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ …(2), where $\varepsilon_0$is absolute permittivity in electrostatic field. From (2), $c^2=\frac{1}{\mu_0\varepsilon_0}\Rightarrow\mu_0=\frac{1}{c^2\varepsilon_0}$…(3). Combing (1) and (3), $B=\frac{1}{\mu_0}\times\frac{i}{2\pi d}\Rightarrow B=\frac{1}{c^2\varepsilon_0}\times\frac{i}{2\pi d}\Rightarrow B=\boldsymbol{\frac{i}{2\pi c^2\varepsilon_0 d}}$, as desired.
I-3	A circular wire is placed vertically on a table such that its diameter lies on the table, piercing the table top through points diametrically opposite points, as shown in figure.  In the figure · indicates current coming out of the surface of the table and + indicates current entering the table. Thus, current in the circular wire when seen in the direction A is anticlockwise and when seen in the direction B is clockwise.  Further, as seen in the direction A magnetic lines of force, inside the circular wire, are coming towards observer and hence the circular wire acts like North pole. Likewise, when magnetic lines of force, inside the circular wire, seen in the direction B, it is going away from the observer. Accordingly, for an observer seeing current in clockwise direction, **magnetic field inside the circular wire, is going away from him,** and the surface of the coil is south pole.
I-4	Ampere's Law $\oint\vec{B}.d\vec{l}=\mu_0 i$ is an extension of Oersted's experiment and Biot-Savart's Law. Accordingly, current flowing through a wire in shape of a closed loop is being considered. For simplicity two loops are taken. In one loop marked S, current is in clockwise direction. And in other loop marked N, current is in anti-clockwise direction. As per Ampere's Right-Hand Thumb rule, magnetic field inside the closed is always on the left-hand side of the direction of current.  As regards, magnetic field outside the curve, i.e. on the right-hand side of the direction of current, it is outside the loop. Thus, net current is $i+(-i)=0$. It is explained by the fact in diametrically opposite ends of the loop currents are in opposite direction. Thus, **Ampere's Law, gives the contribution of only the currents crossing the area bound by the curve only,** and not outside it. Thus, **answer is Yes**.

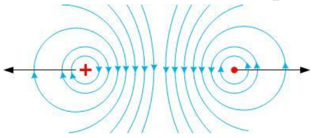
I-5	Magnetic field inside a loosely wound short solenoid is shown in the figure. We find there are small circular magnetic lines of force at every point of each and every turn. This makes magnetic lines of force curved, depending upon length of the solenoid and spacing between adjacent turns.  In this it given that – **a.** The solenoid is tightly wound - it implies that there is no space between adjacent turns, and hence it will not cause distortion in magnetic field produced by solenoid. **b.** The solenoid is long – it implies that end effect of solenoid will not distort magnetic lines of force as shown in the figure. Therefore, magnetic lines of force within the solenoid would remain parallel to its axis. Thus addition of extra loops at the end of the long solenoid would not considerably influence magnetic field inside the solenoid.
I-6	Ampere's Law is also known as Ampere's Circuital Law, where $\oint \vec{B}.d\vec{l} = \mu_0 i$, here $\oint \vec{B}.d\vec{l}$ is integral of magnetic field $\vec{B}$ along a closed loop and not open-ended line integral. While, μ_0 is absolute permeability and i is current intercepting the closed loop. Ampere's law valid for a loop that encloses wire carrying current.
I-7	Given system is shown in the figure where a straight conductor is carrying current I placed along axis of a ring, of radius r, having uniform charge q Coulomb per unit length. The ring is sent into rotation such that it linear speed is v m/s. We know that electric current at a point is rate of flow of charge $i = \frac{Q}{t}$, it can also be expressed as $i = qv$.  Thus, magnetic field B and its direction, at any point P on the conductor, placed along. axis of the ring as shown in the figure, applying Biot-Savart's Law is $\vec{B} = \frac{\mu_0 i r^2}{2(r^2+d^2)^{\frac{3}{2}}}\hat{d}$. Lorentz Force experienced by current carrying conductor placed in the magnetic field B produced by charged rotating ring is $\vec{F} = \vec{I} \times \vec{B} \Rightarrow \vec{F} = I\hat{d} \times \left(\frac{\mu_0 qvr^2}{2(r^2+d^2)^{\frac{3}{2}}}\hat{d}\right) \Rightarrow \vec{F} = \left(\frac{\mu_0 qvIr^2}{2(r^2+d^2)^{\frac{3}{2}}}\right)\hat{d} \times \hat{d}$. Since, cross product of a vector to itself is zero $\left(\hat{d} \times \hat{d} = 1 \sin 0 = 0\right)$, hence force experienced by the straight conductor is zero. It implies that the **straight current carrying conductor will not experience any force**.
I-8	Given are two wires AB and CD carrying currents $\vec{I_2} = I_2\hat{l_2}$ and $\vec{I_1} = I_1\hat{l_1}$ respectively. Here, I_1 and I_2 are magnitudes of the currents and $\hat{l_1}$ and $\hat{l_2}$ are direction vectors of the two currents and are along the length of the two conductors. Given that the two conductors are placed perpendicular to each other, it is visualized to be horizontally placed on the table top shown in the figure. Further, magnitudes of the two currents are equal i.e. $I_1 = I_2 = i$…(1); conductor carrying current I_1 is kept fixed while the conductor AB carrying current I_2 is free to move. Here, Two electromagnetic phenomenon come into play as under-  **Biot-Savart's Law:** Produces magnetic field $\vec{B} = B\hat{B} = \frac{\mu_0 i}{2\pi r}\hat{B}$…(2), around the conductor. Direction of the magnetic field, as per Ampere's Right-Hand Thumb Rule, is piercing the table top though the conductor current carrying current I_2 and emerging out of the table top from diametrically opposite point. **Lorentz's Force:** Magnetic flux $\vec{B}$ produced by conductor carrying CD current I_1 when interacts with current I_2 the conductor AB would experience a force $\vec{F} = I_2 B \sin 90\,\hat{k} \Rightarrow \vec{F} = \frac{\mu_0 i^2}{2\pi r}\hat{k}$ …(3), at each point.

	Here, $\hat{k}$ is perpendicular to the plane of unit vectors $\hat{B}$ and $\hat{l}_2$, as shown in the figure. Further, magnitude of $\vec{B} \propto \frac{1}{r}$ hence force at end A would be larger than that end B. Thus, non-uniform force on the conductor AB carrying would create two kinds of motion, D' Alembert's Principle, as per mechanics – **a.** Translational acceleration in the direction of force as per newton's Second Law of motion $a = \frac{F_R}{m}$, here $F_R = \int_a^b Fdr$ and is m mass of the rod. **b.** In this case non-uniform distribution of force F along the conductor, as shown through the graph in the figure, produces a torque. This torque produces a rotational motion in the conductor AB, about the center of mass. The direction of the motion is depicted through a circle around the center of mass of uniform conductor AB. As a result of the mechanics involved, end A will tend to move towards D and end B would tend to move towards end C, as shown in the figure. Eventually the conductor AB would tend align parallel to the conductor CD.  Once the two conductors are in parallel, currents through them would be in opposite directions leading to force of repulsion, causing the conductor AB to move away from the fixed conductor CD, as shown in the figure. **N.B.: 1.** The understanding of the figure (3D system on 2D) requires visualization; basic knowledge of drawings is helpful for this. **2.** This problems involves integration of multiple concepts, both in electromagnetism and mechanics, and thus it is of a high quality.
I-9	Current carrying conductors, two parallel but in opposite same direction experience force of attraction as shown in the figure. This is explained from the natural tendency of magnetic lines force, like elastic bend. In conductor charge carriers, generally electron, are free to move within the conductor but not independently.  But, in case of two proton beams in same directions, each proton is independent to move under influence of various forces acting on it and is defined as Lorentz's Force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ has two components $\vec{F} = \vec{F}_e + \vec{F}_{em}$, where, $\vec{F}_e = q\vec{E}$ and $\vec{F}_{em} = q\vec{v} \times \vec{B}$. Here, q is charge of the charge carrier in this case proton in the beam, $\vec{E}$ is the electric field established by the another beam, $\vec{v}$ is the velocity of charged particles i.e. protons, $\vec{B}$ is the magnetic field produced influencing protons of a beam by another beam. Further, electrostatically $\vec{F}_e = qE\hat{r}$ and electromagnetically $\vec{F}_{em} = qvB(-\hat{r})$, here $\hat{r}$ is separation vector between the two proton beams. Thus, relative magnitudes of F_e and F_{em} will determine whether proton beams would experience force of attraction or repulsion.
I-10	Ampere's Law is mathematically expressed as $\oint \vec{B}.d\vec{l} = \mu_0 i$. It is also known as Ampere's Circuital Law. The word Circuital defines that it is integral of a vector in a closed loop, also called circuit. In the instant case it is integral of magnetic field and the law defines its equivalence of current passing through the closed loop. If wire is straight and long, , as given, magnetic field around it is due to electric current flowing through the wire and it is defined by Biot-Savart's Law. Secondly, current in a loop requires to be closed and essentially it has a voltage source. Moreover, current in a circuit is defined by the principle of conservation of charge. Since the given statement is an open loop, there can be infinite loops of magnetic field, regular or irregular, and hence magnetic field at a point cannot be defined by it.
I-11	Twisting of electrical wires used in electrical appliances creates small loops as shown in the figure. Characteristically electric current produces magnetic field. Thus twists in connecting wires produce small loops as shown in the figure.  In case there are no twists the current loop formed by connecting wires will be large and magnetic field produced by it will cause stray effects on adjoining equipment.

<table>
<tr><td></td><td>But in case twists, small current loops formed by the connecting wires polarity of magnetic field in consecutive loops is opposite and therefore it closes in every consecutive loop.

N.B.: This problem is a good example of practical application. Students are advised to verify magnetic effect of current in connecting wires, using a magnetic compass, supplying electricity from battery or a AC/DC converter, both twisted and untwisted.</td></tr>
<tr><td>I-12</td><td>It is given that two current carrying wires may attract each other. The uncertainty about attraction or repulsion in the wires depends upon direction of current.



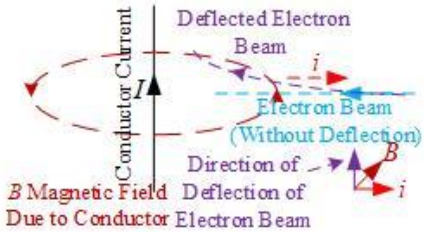


When both conductors carry currents in same direction there is force of attraction and when in opposite direction there is force of repulsion, as shown in the two figures.

Yet, taking given statement it is taken that directions of current in the two conductors corresponds to force of attraction.

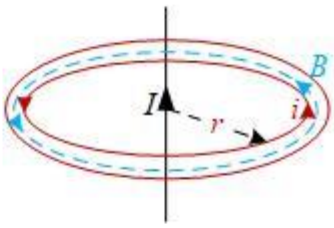
Now conductors have some mass, and when force acts upon them there is acceleration, as per Newton's Laws of Motion, and consequent displacement and increase in velocity of conductor in the direction of force as per equations of motion. The work done on the conductor $W = \vec{F}.d\vec{s}$ is converted into kinetic energy of the conductor, as per Law of Conservation of Energy.

Here, it is important to know that **cause of increase or change in kinetic energy of the conductor is current through it and not the magnetic field.**</td></tr>
<tr><td>I-13</td><td>System as given is shown in the figure. Magnetic field around the vertical wire carrying current say I is shown in a circular loop, such that plane of the loop is perpendicular to the conductor and direction of the magnetic field is shown in accordance with Ampere's Right-Hand Thumb Rule.



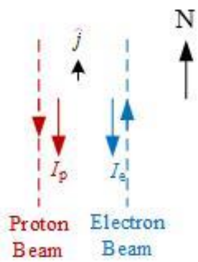
Electron beam is horizontally projected towards to conductor, The loop of the magnetic field and electron beam are taken to be coplanar.

Direction electric current i due to electron beam is opposite to the direction of the beam; it is attributed to the fact that electrons are (-Ve) charge carriers. Thus as per Fleming's Right-Hand Rule, as shown in the figure (3-D Vectors), force acting on the electron beam is upward. Therefore, as per Newton's Laws of Motion, the electron beam would experience upward deflection, as provided in **option (c), is the answer.**</td></tr>
<tr><td>I-14</td><td>Current I through a wire placed along axis of a circular loop carrying current i. As per Ampere's Right-Hand Thumb Rule, magnetic field $\vec{B}$ produced by the current I is in collinear to the current $\vec{\imath}$ through the circular loop. Thus, angle between the vectors $\vec{B}$ and $\vec{\imath}$ is $\theta = 0 \ldots(1)$



Interaction between $\vec{B}$ and $\vec{\imath}$ would produce a force $\vec{F}$ on the circular loop as per Lorentz's formula would be $\vec{F} = 2\pi r(\vec{\imath} \times \vec{B}) \Rightarrow F = 2\pi r i B \sin\theta \ldots(2)$.

Combining (1) and (2), $F = 0$, i.e. the wire will not exert any force on the loop, as provided in O**ption (c) the answer.**</td></tr>
<tr><td>I-15</td><td>The given system is shown in the figure. Since current is $I = \frac{d(nev)}{dt}$. Taking it vectorially, proton beam in in direction $(-\hat{\jmath})$ hence current in it is $\vec{I}_p = \frac{d(n_p e \vec{v}_p)}{dt} = n_p e \frac{dv_p}{dt}(-\hat{\jmath})$ as shown in the figure. Here, n_p is number of protons per unit length of the beam, e is charge of a proton, and $\vec{v}_p$ is the velocity of the proton beam. Since charge of proton is (+)ve and hence direction of current in proton beam is same as theis same as the direction of the beam i.e. in direction $(-\hat{\jmath})$. Accordingly, $\vec{I}_p = -n_p e \frac{dv_p}{dt}\hat{\jmath}$


</td></tr>
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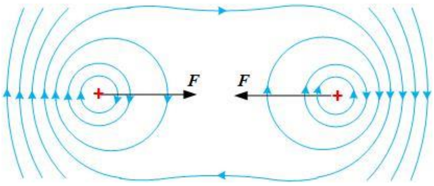
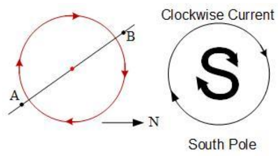
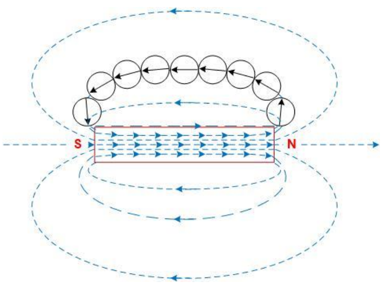
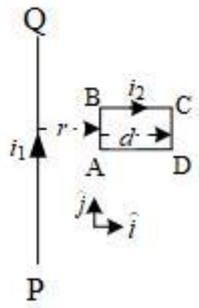
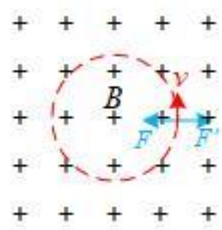
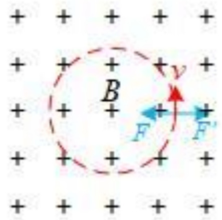
	But, in electron beam comprises of negative charge carrier and charges $(-e)$ are moving along $(\hat{j})$. Hence, electric current in the beam is $\vec{I}_e = \frac{d(n_e e \vec{v}_e)}{dt} = n_e(-e)\frac{dv_e}{dt}(\hat{j}) \Rightarrow \vec{I}_e = -n_e e \frac{dv_e}{dt}(\hat{j})$. Thus, electric current in both the beams is in the same direction.  Accordingly, forces acting on the two beams can be explained from nature of magnetic lines of force drawn using Ampere's Right-Hand Thumb Rule shown in the diagram. Alternatively using Biot-Savart's Law combined with Lorentz's Force. Taking simpler explaination in the diagram it is seen that both the beims being unidirectional would experience attractive forces of equal magnitude. Yet, ratio of mass of the electrons and protons $\frac{m_p}{m_e} = 1836$ and as per Newton's Second Law of Motion acceleration $a = \frac{F}{m}$. Therefore, acceleration of electrons and protons in the beam would be $\frac{a_e}{a_p} = \frac{\frac{F}{m_e}}{\frac{F}{m_p}} \Rightarrow \frac{a_e}{a_p} = \frac{m_p}{m_e} = 1836$. Since, acceleration of electron (a_e) in the beam is considerably larger than the acceleration of the protons (a_p) , Hence, electron beam will get deflected towards proton beam as provided in the **option (a), is the answer.**
I-16	Given system is shown in the figure. Deliberately points A and B on theaxis of the circular loop are taken far away from O the center of the loop to add clarity to the geometry. Direction of magnetic field and polarity of the face using Flemmig's Right-Hand Thumb Rule is shown in the figure.   Face of the loop seen from A is South Pole, as shown in the figure, and conversly, when seen from B it is north pole. The current carrying loop can be compared to a magnet. Accordingly, magnetic field inside a magnet and outside the magnet is examined. It is seen that inside the magnet magnetic field is from its south pole to north pole and outside the magnet it is from north to south pole. Therefore, magnetic field will be from A to B along the axis as provided in **option (d), is the answer.**
I-17	Wire PQ is carrying current i_1 and a loop ABCD is carrying current. A loop ABCD is carrying current i_2. The loop and the wire are is in vertical plane such that sides AD and CB of the loop are parallel to the wire PQ, while side DC and BA of the loop are perpendicular to the wire. Current in each branch of the loop is i_2. Magnetic field at any point at a distance r from PQ is $B_r = \frac{\mu_0 i}{2\pi r}$ …(1). The magnetic field as per Ampere's Right-Hand Thumb Rule would be entering the plane of paper, in geometry shown in the figure.  As per Fleming's Left Hand Rule branch AD of the loop would experience force of attraction towards the wire PQ i.e. along $(-\hat{\imath})$, while branch CB will experience force of repulsion i.e. along $(\hat{\imath})$. Both the arms carry current i_2, yet each of them experience different forces in accordance with (1). Accordingly, $F_{AD} \propto B \Rightarrow F_{AD} \propto \frac{1}{r}$ along $(-\hat{\imath})$ and $F_{AD} \propto B \Rightarrow F_{BC} \propto \frac{1}{r+d}$ along $(\hat{\imath})$. Thus, **net force on the loop** is $\vec{F}_i \propto \left(\frac{1}{r+d} - \frac{1}{r}\right)\hat{\imath}$ **is towards the wire PQ experienced by AD**. As regards branch BC and AD, they carry currents in opposite directions, yet distribution of magnetic field along the branches is identical. Thus, as per Fleming's Left Hand Rule force on branch BC is F_{BC} is along $\hat{j}$ while on branch AD is $F_{AD} = F_{BC}$…(2) but it is along$(-\hat{j})$. Thus, net force on the loop along $\hat{j}$ is $\vec{F}_j = (F_{BC} - F_{AD})\hat{j}$. Using (2), $\vec{F}_j = 0$, accordingly there will be **no displacement of loop along $\hat{j}$.** **Thus answer is option**

I-18	A charged particle, it could be (+)ve or (-)ve, is moving with a velocity $\vec{v}$ along line of a magnetic field $\vec{B}$. In case the charge on particles is positive then direction of current is same as that of the magnetic field. But, if charge is (-)ve the direction of current is opposite to the magnetic field. Lorentz' Force on a charged particles is $\vec{F} = q(\vec{v} \times \vec{B})$. Given that both the vectors $\vec{v}$ and $\vec{B}$ are collinear, and hence their cross product $\vec{v} \times \vec{B} = 0$. Therefore there will be no magnetic force on the particles, as per **option (d), is the answer.**
I-19	A charge as per Coulomb's Law produces electric field irrespective of the its dynamical state i.e. in rest or motion. But, when a charge is in motion it produces magnetic field as per Biot-Savart's Law. Thus a moving charge produces both electric and magnetic field as provided in **option (c), is the answer**.
I-20	When a charged particle is projected in a plane perpendicular to the magnetic field as shown in figure, it would experience a force in a direction expressed Fleming's Left-Hand Rule and quantified as Lorentz's Force $\vec{F} = q(\vec{v} \times \vec{B})$, here q is charge on the particle, $\vec{v}$ velocity-vector of the particle and $\vec{B}$ magnetic-field-vector. It is important to note that force $\vec{F}$ is perpendicular to the plane of vectors $\vec{v}$ and $\vec{B}$ and acts as centripetal force of magnitude $\vec{F} = qvB\hat{r}$ which will be compensated by centrifugal force $\overrightarrow{F'} = \frac{mv^2}{r}(-\hat{r})$ causing a circular motion of the charged particle inside the magnetic field. Here, m is the mass of the charged particle and r is the radial distance of the particle from the center of the circular path described by the charged particle.  After the particle is projected with velocity v inside the magnetic field its kinetic energy is $E = \frac{1}{2}mv^2$. It leads to $E \propto v^2$…(1) But, after that particle remains free to move in a state of equilibrium i.e. $\vec{F} + \overrightarrow{F'} = qvB\hat{r} + \frac{mv^2}{r}(-\hat{r}) = 0$, and there is no change in kinetic energy of the particle. It leads to $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{qvB} \Rightarrow r = \frac{mv}{qB}$…(2), the radius of the circular path described by the charged particle and it remains onstant since each of the term on R.H.S. has a predefined values. Area bound by the circular path described by the particle is $A = \pi r^2 = \pi\left(\frac{mv}{qB}\right)^2 \Rightarrow A = \left(\frac{\pi m^2}{q^2B^2}\right)v^2 \Rightarrow A \propto v^2$…(3). Comparing (1) and (3), both E and A are proportional to v^2. Therefore, as per Euclid's postulates $A \propto E$ as provided in **option (c), is the answer.**
I-21	Given that particles X and Y have equal charge say q and are accelerated through equal potential difference say V. Then potential energy of the particles after acceleration shall also be equal $E = qV$…(1) Let mass of the two particles be m_x and m_y. Then velocity attained by the particles will be $\frac{1}{2}mv^2 = E \Rightarrow \frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$….(2) Accordingly, velocity of particles X and Y is would be $v_x = \sqrt{\frac{2qV}{m_x}}$ and $v_y = \sqrt{\frac{2qV}{m_y}}$…(3), respectively. The forces on the particles inside magnetic field describe a circular motion in a plane perpendicular to the magnetic field, maintaining an equilibrium. Accordingly, kinetic energy remains at E.

It is important to note that electromagnetic force on particle X is $\vec{F_x} = qv_xB\hat{r}$ and on particle Y is $\vec{F_y} = qv_yB\hat{r}$. These forces are perpendicular to the plane of vectors $\vec{v}$ and $\vec{B}$ and act as centripetal force. Accordingly, centrifugal forces are $\overrightarrow{F_x'} = \frac{mv_x{}^2}{r_x}(-\hat{r})$ and $\overrightarrow{F_y'} = \frac{mv_y{}^2}{r_y}(-\hat{r})$, respectively.

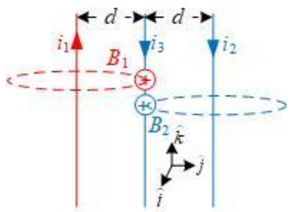
Necessary condition of circular motion of particles X is $\vec{F_x} + \overrightarrow{F_x'} = 0 \Rightarrow qv_xB = \frac{m_xv_x{}^2}{r_x} \Rightarrow m_x = \frac{qBr_x}{v_x}$. Likewise, for particle Y it is $\vec{F_y} + \overrightarrow{F_y'} = 0 \Rightarrow qv_yB = \frac{m_yv_y{}^2}{r_y} \Rightarrow m_y = \frac{qBr_y}{v_y}$…(4)



Combining set of equations (3) and (4) for the particle X is $m_x = \frac{qBr_x}{\sqrt{\frac{2qV}{m_x}}} \Rightarrow \sqrt{m_x} = \frac{\sqrt{q}\times Br_x}{\sqrt{2V}}$. Likewise for particle Y it is $m_y = \frac{qBr_y}{\sqrt{\frac{2qV}{m_y}}} \Rightarrow \sqrt{m_y} = \frac{\sqrt{q}\times Br_y}{\sqrt{2V}}$. Accordingly, $\frac{\sqrt{m_x}}{\sqrt{m_y}} = \frac{\frac{\sqrt{q}\times Br_x}{\sqrt{2V}}}{\frac{\sqrt{q}\times Br_y}{\sqrt{2V}}} \Rightarrow \sqrt{\frac{m_x}{m_y}} = \frac{r_x}{r_y} \Rightarrow \frac{m_x}{m_y} = \left(\frac{R_x}{R_y}\right)^2$, as provided in **option (c), is the answer**.

I-22

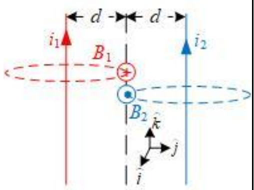
Given system is shown in the figure where $i_1 = 20$ A, $i_2 = 40$ A, and $i_3 = 20$ A. Using Ampere's Right-Hand Thumb Rule magnetic field due to conductor carrying current i_1 at the middle conductor is $\vec{B}_1 = \frac{\mu_0 i_1}{2\pi d}(-\hat{\imath})$…(1), and due to conductor carrying current i_2 at the middle conductor is $\vec{B}_2 = \frac{\mu_0 i_2}{2\pi d}(-\hat{\imath})$…(2) Therefore, combining (1) and (2) and using the available data, net magnetic field at the middle conductor is $\vec{B} = \vec{B}_1 + \vec{B}_2 \Rightarrow \vec{B} = \frac{20\mu_0}{2\pi d}(-\hat{\imath}) + \frac{40\mu_0}{2\pi d}(-\hat{\imath}) \Rightarrow \vec{B} = \frac{30\mu_0}{\pi d}(-\hat{\imath})$…(3).



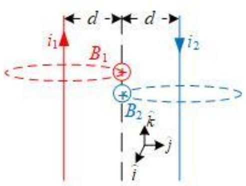
It is required to determine direction of force on the biddle conductor. As per Lorentz' Equation force perunit length of the conductor is $\vec{F} = q(\vec{v} \times \vec{B}) = \vec{\imath}_3 \times \vec{B}$. Using (3) and the given data, $\vec{F} = 20(-\hat{k}) \times \frac{30\mu_0}{\pi d}(-\hat{\imath})$. It leads to $\vec{F} = \frac{200\mu_0}{\pi d}\hat{k} \times \hat{\imath} \Rightarrow \vec{F} = \frac{200\mu_0}{\pi d}\hat{\jmath}$, we know from vector product that $\hat{k} \times \hat{\imath} = \hat{\jmath}$ i.e. towards conductor carrying , $i_2 = 40$, as provided in **option (b), the answer.**

I-23

Magnitude of the magnetic field at point at a distance d from a conductor carrying current i is $B = \frac{\mu_0 i}{2\pi d}$. In the problem there are two conductors carrying currents $i_1$and i_2 such that $i_1 > i_2$. Therefore, at a point between the two conductors, magnetic field due to currents $i_1$and i_2 as per Biot-Savart's Law is $B_1 = \frac{\mu_0 i_1}{2\pi d}$ and $B_2 = \frac{\mu_0 i_2}{2\pi d}$, respectively.

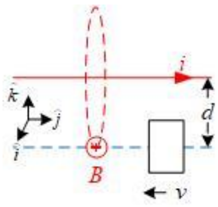


When both the currents $i_1$and i_2 are in same direction then direction of the magnetic fields $B_1$and B_2, as per Ampere's Right Hand Thumb Rule are opposed to each other. Therefore net magnetic field at midway is $B = B_1 - B_2 \Rightarrow B = \frac{\mu_0 i_1}{2\pi d} - \frac{\mu_0 i_2}{2\pi d} = 10$ T. It leads to $\frac{\mu_0}{2\pi d}(i_1 - i_2) = 10$ T…(1).



But, when currents in the two conductors in opposite direction, magnetic field at midway of the two conductors are in same direction. Hence, is $B' = B_1 - B_2$. It leads to $B' = \frac{\mu_0 i_1}{2\pi d} + \frac{\mu_0 i_2}{2\pi d} = 30 \Rightarrow \frac{\mu_0}{2\pi d}(i_1 + i_2) = 30$ T…(2).

Using (1) and (2), $\frac{\frac{\mu_0}{2\pi d}(i_1+i_2)}{\frac{\mu_0}{2\pi d}(i_1-i_2)} = \frac{30}{10} \Rightarrow \frac{i_1+i_2}{i_1-i_2} = \frac{3}{1}$…(3). Applying componendo-dividendo, of ratio & proportions to (3) we have $\frac{(i_1+i_2)+(i_1-i_2)}{(i_1+i_2)-(i_1-i_2)} = \frac{3+1}{3-1} \Rightarrow \frac{i_1}{i_2} = 2$ as provided in **option (2), is the answer**.

I-24	 Given conductor of cross-sectional area A and n electrons per unit volume carries a current i, say in direction $\hat{j}$. Since current is $i = \frac{dQ}{dt} \Rightarrow i = qv'$...(1), here Q is charge past a point per sec, q is charge per unit length of the conductor, and v' is velocity of the charge along the conductor. Length of conductor (l) per unit volume is $Al = 1 \Rightarrow l = \frac{1}{A}$. With the given data number of free electrons per unit length of the conductor is $n' = \frac{n}{l} = \frac{n}{\frac{1}{A}} \Rightarrow n' = nA$. Therefore, charge of electrons per unit volume is $q = n'e = nAe$...(2). Combining (1) and (2), $i = (nAe)v' \Rightarrow v' = \frac{i}{nAe}$ ...(3). This velocity of electrons, which is opposite to the direction of current, is along $(-\hat{j})$. Given that an observer in a trolley moving with a velocity $v = \frac{i}{nAe}$ ...(4), along the direction opposite to the current i.e. along $(-\hat{j})$. Therefore, relative velocity of the charge carriers with respect to the observer, combining (3) and (4) is $\vec{V} = v(-\hat{j}) - v'(-\hat{j}) \Rightarrow \vec{V} = \frac{i}{nAe}(-\hat{j}) - \frac{i}{nAe}(-\hat{j}) = 0$...(5) Therefore, **apparently** current perceived by the observer would be $I = qV = 0$...(6). Since magnetic field as per Biot-Savart's Law is $B = \frac{\mu_0 i}{2\pi r}$...(7) in this case $i \to I$ and $B = \frac{\mu_0 I}{2\pi r}$...(8). Combining, (5) and (8) we should be getting $B = 0$. But it is not so as discussed in the following paragraph. But, **actually** the trolley at any position perceives current in the conductor an effect of the flow of the electrons inside it and not an electron beam. Therefore, motion trolley becomes insignificant. Thus, as per Biot-Savart's Law magnetic field seen by the observer is due to the current i inside the conductor and as per (7) it is $\boldsymbol{B} = \frac{\boldsymbol{\mu_0 i}}{\boldsymbol{2\pi r}}$ **as provided in the option (a), is the answer.** **N.B.:** These problems nicely discriminates effect of electric current inside the conductor and not the current due electron beam.
I-25	As per Biot-Savart's Law magnetic field $d\vec{B}$ at point with a position vector $\vec{r}$ due to current i through a wire of length $d\vec{l}$ is $d\vec{B} = \frac{\mu_0 i}{4\pi}\left(\frac{d\vec{l}\times\vec{r}}{r^3}\right)$ ...(1). The problem gives four expressions involving same variable. First expressions at option (a) is same as that (1), is correct. Further, the cross product, in (1), does not have commutative property i.e. $d\vec{l} \times \vec{r} \neq \vec{r} \times d\vec{l}$, yet $d\vec{l} \times \vec{r} = -\vec{r} \times d\vec{l}$ accordingly expression in option (b) is equivalent to that in (1), hence option is correct. But, expressions in Options (c) and (d) do not conform to (1) and hence they are incorrect. Thus, **answer is option (a) and (b).**
I-26	Lorentz Force equation is $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$, its dimensional equation is $[F] = [qE] + [qvB]$ such that $[qE] = [qvB] \Rightarrow [v] = \frac{[E]}{[B]} = LT^{-1}$...(1). Given that $x = \frac{E}{B} \Rightarrow [x] = \frac{[E]}{[B]}$...(2). Comparing (1) and (2), $[x] = LT^{-1}$...(4). We know that speed of light $c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$, its dimensional equation is $[c] = \left[\sqrt{\frac{1}{\mu_0 \varepsilon_0}}\right]$...(5). Using (1) and (2), $[c] = [v] \Rightarrow [c] = LT^{-1}$...(6). Given that $y = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$ ...(7). Comparing (5), (6) and (7), $[y] = LT^{-1}$ ...(8). Further, given that $z = \frac{l}{CR}$, here in RC circuit $[RC] = [T]$ being time constant and hence $[z] = \frac{L}{T} = LT^{-1}$...(9). It is seen from (4), (8) and (9), dimensions of given three variables x, y and z are same. Hence, **options (a), (b) and (c) are correct, are the answer**

I-27

Given that a long straight wire carries current i along Z-axis i.e. unit vector $(\hat{k})$ coming out of the plane of the figure. Current produced at point having position vector is $\vec{r}$ w.r.t. the wire is $\vec{B} = \frac{\mu_0 i}{2\pi r}(-\hat{\imath})$.

Taking points A, B and C with their position vectors $\vec{r}_A = r_1(-\hat{\jmath})$, $\vec{r}_B = r_1\hat{\jmath}$, and $\vec{r}_C = r_2\hat{\jmath}$, respectively. Here, $r_2 > r_1$. Accordingly, as per Biot-Savart's Law, simplified for use by Ampere's Right-Hand Thumb Rule, magnetic field at these points is $\vec{B}_A = \frac{\mu_0 i}{2\pi r_1}(\hat{\imath})$, $\vec{B}_B = \frac{\mu_0 i}{2\pi r_1}(-\hat{\imath})$ and $\vec{B}_C = \frac{\mu_0 i}{2\pi r_2}(\hat{\imath})$.

It is to be noted that magnetic field at point is vector, and for that both magnitude and directions matter while comparing magnetic field at two points. Now we are ready to evaluate each of the given options-

Option (a): Magnetic field at A and B are equal and magnitude but in opposite direction, hence $\vec{B}_A \neq \vec{B}_B$, hence this option is **incorrect**.

Option (b): Points B and C have same direction vectors i.e. $\hat{\jmath}$, but their distances$r_1 \neq r_2$. Therefore, though $\vec{B}_B \neq \vec{B}_C$, but their directions are same along $\hat{\imath}$. Hence this option is **correct**.

Option (c): As discussed at (a), though $\vec{B}_A \neq \vec{B}_B$, but magnitudes $B_A = \frac{\mu_0 i}{2\pi r_1}$ and $B_B = \frac{\mu_0 i}{2\pi r_1}$. Thus, $B_A = B_B$, Hence, this option is **correct**.

Option (d): As discussed at (c), despite equal magnitudes though $\vec{B}_A \neq \vec{B}_B$, but their directions are opposite to each other. Hence, this option is **correct**.

Thus, **answer is options (b), (c) and (d).**

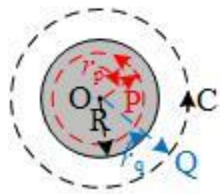
I-28

Magnetic field around a long straight wire of radius R carrying current I , as per Biot-Savart's Law is, $B = \frac{\mu_0 i}{2\pi r}$…(1), here r is the distance of the point from the axis of the wire, passing through O which is perpendicular to the plane of the figure, and i is the current enclosed inside the conductor.

It is given that the conductor is carrying current which is uniformly distributed over its cross section. Let the current density be ρ per unit area.

The possibilities for having different magnetic field can be two and are analyzed here under –

(i) Point P inside the conductor i.e. $0 \leq r_p < R$ in which case current $i = \pi {r_p}^2 \rho$….(2). In this case magnitude of the magnetic field at the point P, combining (1) and (2) is $B_P = \frac{\mu_0(\pi {r_p}^2 \rho)}{2\pi r_p} \Rightarrow B_P = \frac{\mu_0 r_p \rho}{2}$…(2).



(ii) Point Q is so placed that $R \leq r_q < \infty$ in which case current $i = I$. In this case magnitude of the magnetic field at the point P, combining (1) and (2) is $B_Q = \frac{\mu_0 I}{2\pi r_q}$…(3)

Now we are ready to analyze each of the given options, in respect of the magnitude of the magnetic field, as under –

Option (a): At the axis of wire $r = 0$, it is case (i) above, accordingly using (2), $B = B_P = 0 = B_{min}$, this is minimum and hence this option is **incorrect**.

Option (b): From the analysis at (a) above this is minimum and hence this option is **correct.**

Option (c): At the surface of the wire $r = R$ hence case (ii) is applicable and accordingly using Ampere's Circuital Law $\oint B.dl = \mu_0 I$ and hence $i_{max} = I$. Thus, using (3) $B = B_Q = \frac{\mu_0 I}{2\pi R} \Rightarrow B = \frac{\mu_0 i_{max}}{2\pi R}$ is maximum. Here, i in the numerator is constant and maximum value, while $r \to R$ in the denominator is minimum value of r. Accordingly, $B_{max} = \frac{\mu_0 i_{max}}{2\pi R}$ is the maximum value. Thus, this option is **correct**.

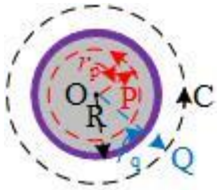
Option (d): From discussion at (c) above at the surface B has the maximum value $B_{max}=\frac{\mu_0 i_{max}}{2\pi R}$, while at (a) we found that minimum value $B_{max}=0$. Thus, $B_{max}\neq B_{min}$, hence this option is incorrect.

Thus, **answer is options (b) and (c).**

I-29

Magnetic field around a hollow tube of radius R carrying current say I along its length and is uniformly distributed over the length of the tube.

As per Biot-Savart's Law magnetic field at a distance r from axis of the tube, passing through O and perpendicular to the surface of the figure, is $B=\frac{\mu_0 i}{2\pi r}$…(1).



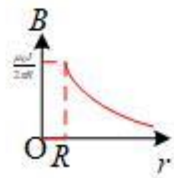
With the given geometry, as shown in the figure $i=I$.

The possibilities for having different magnetic field can be two and are analyzed here under –

(i) Magnetic field at any point P such that $0\leq r_P<R$, the current $i\to i_P=0$. Therefore, as per (1) magnetic field at point P is $B_P=\frac{\mu_0 i_P}{2\pi r}$…(2)

(ii) Magnetic field at a point Q such that $R\leq r_q<\infty$, the current $i\to i_q=I$. Therefore, as per (1) magnetic field at Q is $B_q=\frac{\mu_0 I}{2\pi r}$ ...(3)

Now we are ready to analyze each of the given options, in respect of the magnitude of the magnetic field, as under –



Option (a): Magnetic field from axis of the surface covers zones specified in two possibilities and can be shown with a graph. It contradicts the stipulation that B linearly increases with distance from axis O. Thus, this option is incorrect.

Option (b): The magnetic field inside tube is constant at Zero as per (2) and represented by line OR**.** Thus, this option is **correct**.

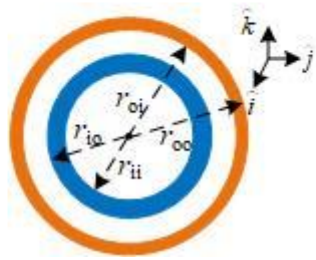
Option (c): As per discussions at (b), flux density is Zero at the axis of the tube O. Thus this option is **correct.**

Option (d): Magnetic field at a point at a distance r_q outside the tube proportional to its distance from axis decreases inversely with its distance, but $B_q\to 0$ as $r_q\to\infty$. Thus stipulations in this option is **incorrect**.

Thus, **answer is options (b) and (c).**

I-30

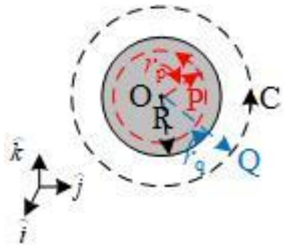
Given is coaxial cable as shown in the figure, comprises of two coaxial tubes having its cross-section on $\hat{\imath}-\hat{\jmath}$ plane. Inner tube has inner radius r_{ii} and outer radius r_{io}. The inner tube carries current say I along $\hat{\jmath}$ direction. The outer tube having its inner and outer radii r_{oi} and outer radius r_{oo}, respectively. Current though outer tube is given to be equal and opposite to that of the inner tube i.e. I along $(-\hat{\jmath})$ direction.

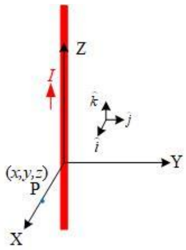
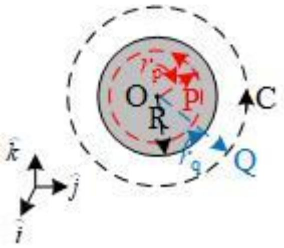
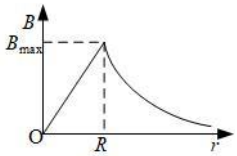
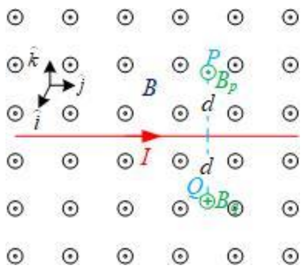


Magnetic field around the conductor is defined with Ampere's Circuital Law mathematically defined as $\oint\vec{B}.d\vec{l}=\mu_0 i$…(1). Here, i is the net current inside the closed path, in this case all paths are circular and concentric.

It is required to determine zone of Zero magnetic field w.r.t. the cable, out of four given options. Each of them is analyzed below.

Option (a): Outside the cable implies $r>r_{oo}$. In this case net current inside the closed circular path of the radius r is $i=I\hat{\jmath}+I(-\hat{\jmath})=0$. Though $d\vec{l}\neq 0$, according to (1), $B=0$. Thus, this option is **correct.**

	Option (b): Inside the inner conductor i.e. $r < r_{ii}$. Currents are flowing through inner and outer tubes only and are outside the point under consideration. Thus, in this case $i = 0$. Therefore, on the line of discussions at (a) above and according to (1), $B = 0$. Thus, this option is **correct.** **Option (c):** Inside the outer conductor in implies that $r_{oi} < r < r_{oo}$. Assuming current is uniformly distributed on the cross-section of the two tubes, $i = I - I' \neq 0$ since $I' < I$. Thus according to (1), $B \neq 0$. Thus, this option is **incorrect.** **Option (d):** Between the two conductors i.e. $r_{i0} < r < r_{oi}$ as per (1) only current I in the inner conductor will produce magnetic field and $B \neq 0$. Thus, this option is **incorrect.** Thus, **answer is options (a) and (b).**
I-31	Given that a steady current, say I, is flowing through a cylindrical conductor, say of radius R, shown in the figure in $\hat{\imath} - \hat{\jmath}$ plane The problem is silent about distribution of current through cross-section of the conductor. It is, therefore, considered that current density ρ in the conductor is uniform, $\rho = \frac{I}{\pi R^2}$.  Problem is in the form of possibilities given in the form of option, as under – **Option (a):** Uniform current density through the cross-section of the conductor implies that there is current through the axis of the conductor. As per Ohm's law flow of current through a conductor essential requires a potential difference V. Since, cylindrical conductor has a length L and therefore there is discrete electric field along the axis. Thus, stipulation given in this option is **incorrect.** **Option (b)**: Magnetic field around a long straight wire of radius R carrying current i , as per Biot-Savart's Law is, $B = \frac{\mu_0 i}{2\pi r}$…(1), here r is the distance of the point from the axis of the wire, passing through O which is perpendicular to the plane of the figure, and i is the current enclosed inside the conductor. **Option (c):** Any point Q in vicinity of the conductor in $\hat{\imath} - \hat{\jmath}$ plane is at a distance r_q from axis of the conductor such that $R < r_q < \infty$. Current is since flowing along $\vec{k}$ and there is no current along $\hat{\imath} - \hat{\jmath}$ plane, therefore, as per Ohm's Law discussed at (a) above, electric field in the vicinity of the conductor is zero. Thus stipulation in this option is **correct.** **Option (d):** As per Biot-Savart's Law the magnetic field in the vicinity of the conductor is non zero since current is flowing along $\hat{k}$ is nonzero i.e. $i = I \neq 0$. Hence, stipulation in this option is **incorrect.** Thus, **answer is option (b) and (c).**
I-32	Given are two expressions, the first one is vector equation, while the second one is scalar; it is required to ascertain the given units of magnetic field B and the permeability constant μ_0. Here it is to be remembered that algebra of units is scalar, yet direction creeps in with the vector quantities only. In the given units there is no content of direction. Accordingly, taking units of constituents in first vector expression $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{B} = \frac{\vec{F}}{q\vec{v}}$. Accordingly equations of units is $\mathrm{B} = \frac{\mathrm{N}}{\mathrm{Am}}$ or $B =$ **$\mathbf{Nm^{-1}A^{-1}}$ is ascertained.** Taking, constituents of second equation, $B = \frac{\mu_0 i}{2\pi r} \Rightarrow \mu_0 = \frac{B(2\pi r)}{i}$. Using unit of B ascertained above we have equation of units as $\mu_0 = \frac{(\mathbf{Nm^{-1}A^{-1}})(m)}{A} \Rightarrow \mu_0 =$ **$\mathbf{NA^{-2}}$ is ascertained.**

I-33	It is required to determine magnetic field at point P whose position vector, as per given data, shown in the diagram is $\vec{r} = 1\hat{\imath} + 0\hat{\jmath} + 0\hat{k} \Rightarrow \vec{r} = r\hat{\imath} = 1\hat{\imath} \Rightarrow r = 1 \ldots(1)$Current through a long straight wire is $\vec{I} = I\hat{k} = 10\hat{k} \Rightarrow I = 10 \ldots(2)$ It is required to find magnetic field vector at P. As per Biot-Savart's Law magnetic field $\vec{B} = \frac{\mu_0 \vec{I} \times \vec{r}}{2\pi r^2} \ldots(3)$.  Combining (1), (2) and (3) with that $\frac{\mu_0}{4\pi} = 10^{-7}$ we have, $\vec{B} = \frac{\mu_0(10\hat{k})\times(1\hat{\imath})}{2\pi 1^2}$T. It leads to $\vec{B} = 2\left(\frac{\mu_0}{4\pi}\right) \times 10 \times (\hat{k} \times \hat{\imath}) \Rightarrow \vec{B} = (2(10^{-7}) \times 10)\hat{\jmath} \Rightarrow \vec{B} = 2 \times 10^{-6}\hat{\jmath}$ T. Accordingly, flux density $\vec{B}$ at P is of magnitude **2×10^{-6}T along Y-axis, is the answer.** **N.B.:** It is to be noted that mathematical operator between two scalars is simple multiplication and it assumes vector product only when sandwiched between two vector quantities viz. $(\hat{k} \times \hat{\imath}) = \hat{\jmath}$, while $(\hat{\imath} \times \hat{k}) = (-\hat{\jmath})$.
I-34	A copper wire of diameter $d = 2R = 1.6$ mm or $R = 0.8 \times 10^{-3}$ m carries a current $I = 20$ A, but nothing is defined about current distribution in the conductor cross-section. Here, $r = \frac{d}{2}$ is the radius this problem current is considered to be uniformly distributed and has current density is $\rho = \frac{I}{\pi R^2}$.  As per Biot-Savart's Law magnetic field at a distance r from axis of the tube, passing through O and perpendicular to the surface of the figure, is $B = \frac{\mu_0 i}{2\pi r} \ldots(1)$. With the given geometry, as shown in the figure $i = I$. The possibilities for having different magnetic field can be two and are analyzed here under – (i) Magnetic field at any point P such that $0 \leq r_P < R$, the current $i \to i_P$ such that $i_p = \rho\pi r^2 = \left(\frac{I}{\pi R^2}\right)\pi r^2$. It leads to $i_p = \frac{Ir^2}{R^2} \ldots(2)$ Therefore, as per (1) and (2) magnetic field at point P is $B_P = \frac{\mu_0 I r}{2\pi R^2}$. It implies that $B_P \propto r \ldots(3)$  (ii) Magnetic field at a point Q such that $R \leq r_q < \infty$, the current $i \to i_q = I$. Therefore, as per (1) magnetic field at Q is $B_q = \frac{\mu_0 I}{2\pi r} \Rightarrow B_q \propto \frac{1}{r}$ ...(4). Plot of $r - B$ is shown in the figure reveals that B is maximum at $r \to R$ and $B_{max} = \frac{\mu_0 I}{2\pi R} = \left(2\frac{\mu_0}{4\pi}\right)\frac{I}{R}$. Using the available data $B_{max} = (2 \times 10^{-7})\frac{20}{0.8\times10^{-3}} \Rightarrow B_{max} =$ **5×10^{-3}T , is the answer.**
I-35	Given that a transmission wire is carrying a current $i = 100$A. It is required to determine magnetic field on road such that its distance from wire $r = 8$ m. As per Biot-Savart's Law $B = \frac{\mu_0 i}{2\pi r}$. We know that $\frac{\mu_0}{4\pi} = 10^{-7}$. Accordingly using the available data $B = 2\left(\frac{\mu_0}{4\pi}\right)\frac{i}{r} \Rightarrow B = 2 \times 10^{-7} \times \frac{100}{8} = 2.5 \times 10^{-6}$T or **2.5 μT is the answer.**
I-36	Given system is shown in figure in horizontal plane $\hat{\imath} - \hat{\jmath}$. A conductor is carrying current i of value $\vec{I} = 1.0\hat{\jmath}$ A. It is placed in a vertical magnetic field B of value $\vec{B} = 1.0 \times 10^{-5}\hat{k}$T…(1) It is required to determine resultant magnetic field at point P and Q at a distance $d = 2.0 \times 10^{-2}$m and their position vectors are defined as $\vec{r}_p = 2.0 \times 10^{-2}(-\hat{\jmath})$m and $\vec{r}_q = 2.0 \times 10^{-2}(\hat{\jmath})$m …(2), respectively.  Magnetic field vector at a point due to long strait current carrying wire, as per Biot-Savart's Law, is $\vec{B}' = \frac{\mu_0 \vec{I} \times \vec{r}}{2\pi r^2} \Rightarrow \vec{B}' = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r}$. With given orientation of conductor

and symmetrical position of points P and Q $\vec{B}_p{}' = \frac{\mu_0 I}{2\pi r}\hat{j} \times (-\hat{\imath}) \Rightarrow \vec{B}_p{}' = \frac{\mu_0 I}{2\pi r}\hat{k}$ and $\vec{B}_q{}' = \frac{\mu_0 I}{2\pi r}\hat{j} \times \hat{\imath} \Rightarrow \vec{B}_q{}' = \frac{\mu_0 I}{2\pi r}(-\hat{k})$….(3).

The final form in (3) is based on vector product, again using Right-Hand Thumb Rule in different context, $\hat{j} \times \hat{\imath} = -\hat{k}$ and $\hat{j} \times (-\hat{\imath}) = \hat{k}$.

Combining (1), (2) and (3), resultant magnetic field at P is $\vec{B}_p = \vec{B} + \vec{B}_p{}' \Rightarrow \vec{B}_p = B\hat{k} + \frac{\mu_0 I}{2\pi r}\hat{k}$. It leads to $\vec{B}_p = \left(B + \frac{\mu_0 I}{2\pi r}\right)\hat{k}$. Likewise, at Q it is $\vec{B}_q = \vec{B} + \vec{B}_q{}' \Rightarrow \vec{B}_q = B\hat{k} + \frac{\mu_0 I}{2\pi r}(-\hat{k})$.

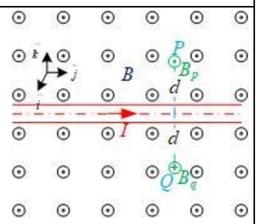
It leads to $\vec{B}_q = \left(B - \frac{\mu_0 I}{2\pi r}\right)\hat{k}$.

Using the available data, $\vec{B}_p = (1.0 \times 10^{-5})\hat{k} + 2 \times 10^{-7}\frac{1.0}{2.0\times10^{-2}}\hat{k} = (1.0 \times 10^{-5} + 1.0 \times 10^{-5})\hat{k} \Rightarrow \vec{B}_p = 2.0 \times 10^{-5}\hat{k}$, and $\vec{B}_q = (1.0 \times 10^{-5})\hat{k} + 2 \times 10^{-7}\frac{1.0}{2.0\times10^{-2}}\hat{k} = (1.0 \times 10^{-5} - 1.0 \times 10^{-5})\hat{k} \Rightarrow \vec{B}_q = 0$

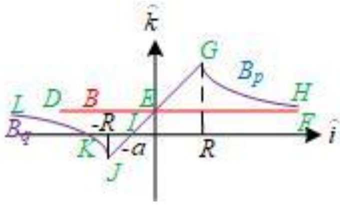
N.B.: 1. The problem can be easily solved applying Ampere's Right-Hand Thumb Rule. Yet, at first stage of practicing the problem-solving, it is illustrated using vector equation as per Biot-Savart's Law. Apparently, it is more tedious, yet it will help to develop visualization and conceptual clarity. As student's go into proficiency and revision stage they can comfortably and confidentially resort to short-cuts.

2. For zero quantity has neither direction nor unit.

I-37



Given system is shown in figure in horizontal plane $\hat{\imath} - \hat{j}$. A conductor is carrying current I of value $\hat{I} = I\hat{j}$ A. The current is uniformly distributed over the cross-section of the conductor, the current density is $\rho = \frac{I}{\pi R^2}$. It is placed in a vertical magnetic field B of value $\vec{B} = B\hat{k}$…(1) It is required to determine resultant magnetic field at point P and Q at a distance d and their position vectors are defined as $\vec{r}_p = d(-\hat{j})$m and $\vec{r}_q = d\hat{j}$ m …(2), respectively.



In this case magnetic field produced by the conductor interacts with magnetic field B around it. It has following aspects-

(i) Magnetic field above the axis of the conductor is along $(\hat{k})$ and is additive to the magnetic field B and is shown in the graph by curve sections EG and GH, already brought out in the figure above.

(ii) Magnetic field below the axis of the conductor is along $(-\hat{k})$ and is subtractive to the magnetic field B and is shown in the graph by curve sections EIJ and JKL, already brought out in the figure above.

(iii) Inside the conductor $0 \le r < R$ magnetic field due current in it, as per Biot-Savart's Law, combined with Ampere's Circuital Law, is $B_c = \frac{\mu_0 I r}{2\pi R^2}$ and is shown in the graph by curve sections EG, above the axis of the conductor and EJ, below the axis of the conductor.

(iv) Outside the conductor $R \le r < \infty$ magnetic field due current in it, as per Biot-Savart's Law, is $B_c = \frac{\mu_0 I}{2\pi r}$ and is shown in the graph by curve sections GH, above the axis of the conductor and JL, below the axis of the conductor

With this analysis we are ready find answer to the two parts of the question-

Part (a): It is seen from the graph of resultant magnetic field that maximum magnetic field occurs at the surface of the conductor where $r \to r_p \Rightarrow r = R$, above the axis of the conductor as discussed at (i) and (iii) above. Accordingly magnitude of the maximum magnetic field is $B_{max} = B + \frac{\mu_0 I}{2\pi R}$, is answer of part (a).

Part (a): Again observation of graph of resultant magnetic field, below the axis of the conductor inside the conductor it passes through Zero value at $r_q \to r \Rightarrow r = -a$. But, it attains minimum value at J on the graph when $r = -R$ and again starts increasing. Therefore, minimum flux density at occurs at $r = -R$ is $B_{min} = B - \frac{\mu_0 I}{2\pi R}$, is the answer of part (b).

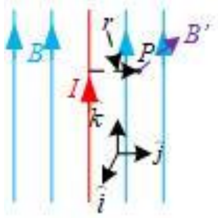
Thus, **answers are (a)** $\boldsymbol{B + \frac{\mu_0 I}{2\pi R}}$ **and (b)** $\boldsymbol{B - \frac{\mu_0 I}{2\pi R}}$**.**

N.B.: (a) This is an excellent problem on interaction of magnetic fields.

(b) Reference to the graph must be drawn from $\hat{\imath} - \hat{\jmath} - \hat{k}$ space shown in the figure.

(c) While determining minimum value concept of number like is important.

I-38



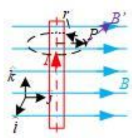
Given that a long straight wire is carrying current I shown in the figure and vectorially $\vec{I} = 30\hat{k}$ A. It placed in a uniform magnetic field B, parallel to the current, shown in the figure and is vectorially $\vec{B} = 4.0 \times 10^{-4}\hat{k}$ T.

It is required to find magnitude of the resultant magnetic field B_r at a point $r = 2.0 \times 10^{-2}$m away, as shown in the figure, and is vectorially $\vec{r} = 2.0 \times 10^{-2}\hat{\jmath}$m.

As per Biot-Savart's Law magnetic field at a point due to current in a long straight conductor, as given in the problem is $\vec{B}' = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r}$. Using the available data $\vec{B}' = \frac{(2\times10^{-7})\times30}{2.0\times10^{-2}}\hat{k} \times \hat{\jmath}$.

We know that vector product $\hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{\jmath} = (-\hat{\imath})$. Accordingly, $\vec{B}' = 3.0 \times 10^{-4}(-\hat{\imath})$. Therefore, resultant magnetic field at P is $\vec{B}_R = \vec{B} + \vec{B}' \Rightarrow 4.0 \times 10^{-4}\hat{k} + 3.0 \times 10^{-4}(-\hat{\imath})$ and magnitude of the resultant magnetic field is $B_R = \sqrt{B^2 + B'^2 + 2BB'\cos\theta}$, here θ is angle between associated vectors $\vec{B}$ and $\vec{B}'$. In the instant case angle between direction vectors $\hat{k}$ and $(-\hat{\imath})$, being orthogonal, is $\theta = 90^0$. Accordingly, $B_R = \sqrt{B^2 + B'^2} \Rightarrow B_R = \sqrt{(4.0 \times 10^{-4})^2 + (3.0 \times 10^{-4})^2} \Rightarrow B_R = \mathbf{5.0 \times 10^{-4}}$**T, is the answer**.

I-39



Given system is shown in the figure where a vertical long wire is carrying current $I = 10$ A upward and is vectorially $\vec{I} = 10\hat{k}$. The conductor is placed in horizontal magnetic field $B = 2.0 \times 10^{-3}$ T and is vectorially $\vec{B} = 2.0 \times 10^{-3}\hat{\jmath}$.

As per Biot-Savart's Law magnetic field at a point due to current in a long straight conductor, as given in the problem is $\vec{B}' = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r}$. Using the available data $\vec{B}' = \frac{(2\times10^{-7})\times10}{r}\hat{k} \times \hat{\jmath}$ or $B' = \frac{(2\times10^{-7})\times10}{r}$

We know that vector product $\hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{\jmath} = (-\hat{\imath})$. Accordingly, $\vec{B}' = 3.0 \times 10^{-4}(-\hat{\imath})$. Therefore, resultant magnetic field at P is $\vec{B}_R = \vec{B} + \vec{B}' \Rightarrow 4.0 \times 10^{-4}\hat{k} + 3.0 \times 10^{-4}(-\hat{\imath})$ and magnitude of the resultant magnetic field is $B_R = \sqrt{B^2 + B'^2 + 2BB'\cos\theta}$, here θ is angle between associated vectors $\vec{B}$ and $\vec{B}'$.

It is required to find position of point where resultant magnetic field is zero. It possible only when-

(a) Angle between $\vec{B}$ and $\vec{B}'$ $\theta = \pi$, since $\cos\pi = -1$ and
(b) $B = B'$

It leads to $B_R = 0 = B = B'$. Accordingly, using the available data $2.0 \times 10^{-3} = \frac{(2\times10^{-7})\times10}{r} \Rightarrow r = 1.0 \times 10^{-3}$m along $(-\hat{\imath})$ or 1mm behind the conductor.

I-40

Given that two parallel wires P and Q, separated by a distance $d = 4.0 \times 10^{-2}$m are carrying current $I = 10$ A in opposite directions. The wire in 3D vector space are laid along $\hat{\imath}$ on $\hat{\imath} - \hat{\jmath}$ plane as shown in the figure is currents in the wires would be $\vec{I}_p = 10\hat{k}$ and $\vec{I}_q = -10\hat{k}$.

It is required to determine magnetic field points A_1, A_2, A_3 and A_4, on $\hat{\imath} - \hat{\jmath}$ plane, which is combined effect of currents in wires P and Q. As per Biot-Savart's Law magnetic field at a point due to long straight wire is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r} = 2 \times 10^{-7}\left(\frac{I}{r}\right)(\hat{I} \times \hat{r})$…(1). It is, therefore, essential to first determine position vectors of each of the point w.r.t. wires P and Q.

Position vectors of these points w.r.t. wire P, as per given data and the figure $\vec{r}_{p1} = -2.0 \times 10^{-2}\hat{\jmath}$, $\vec{r}_{p2} = 1.0 \times 10^{-2}\hat{\jmath}$, $\vec{r}_{p3} = 2.0 \times 10^{-2}\hat{\jmath}$ and $\vec{r}_{p4} = 2.0 \times 10^{-2}\hat{\jmath} - 2.0 \times 10^{-2}\hat{\imath}$.

Likewise, relative position vectors of these points w.r.t. wire Q, whose position vector w.r.t. wire P is $\vec{r}_q = 4.0 \times 10^{-2}\hat{\jmath}$, are

(i) $\vec{r}_{q1} = \vec{r}_{p1} - \vec{r}_q = -2.0 \times 10^{-2}\hat{\jmath} - (4.0 \times 10^{-2}\hat{\jmath}) \Rightarrow \vec{r}_{q1} = -6.0 \times 10^{-2}\hat{\jmath}$.

(ii) $\vec{r}_{q2} = \vec{r}_{p2} - \vec{r}_q = 1.0 \times 10^{-2}\hat{\jmath} - (4.0 \times 10^{-2}\hat{\jmath}) \Rightarrow \vec{r}_{q1} = -3.0 \times 10^{-2}\hat{\jmath}$.

(iii) $\vec{r}_{q3} = \vec{r}_{p3} - \vec{r}_q = 2.0 \times 10^{-2}\hat{\jmath} - (4.0 \times 10^{-2}\hat{\jmath}) \Rightarrow \vec{r}_{q1} = -2.0 \times 10^{-2}\hat{\jmath}$.

(iv) $\vec{r}_{q4} = \vec{r}_{p4} - \vec{r}_q = (2.0 \times 10^{-2}\hat{\jmath} - 2.0 \times 10^{-2}\hat{\imath}) - (4.0 \times 10^{-2}\hat{\jmath}) \Rightarrow \vec{r}_{q1} = -2.0 \times 10^{-2}(\hat{\imath} + \hat{\jmath})$.

Using (1), we are now ready to determine magnitude of magnetic field at each of the given points. Here, it is important to recall product of unit vectors $\hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{\jmath} = -\hat{\imath}$. Likewise $\hat{k} \times \hat{\imath} = \hat{\jmath}$ and $\hat{\imath} \times \hat{k} = -\hat{\jmath}$.

Magnetic field at Point A₁: $\vec{B}_{A1} = \vec{B}_{P-A1} + \vec{B}_{P-A1}$

$$= 2 \times 10^{-7}\left[\left(\frac{10}{r_{p1}}\right)\left(\hat{k} \times (-\hat{\jmath})\right) + \left(\frac{10}{r_{q1}}\right)\left((-\hat{k}) \times (-\hat{\jmath})\right)\right]$$

$$= 2 \times 10^{-6}\left[-\left(\frac{1}{2.0\times10^{-2}}\right)(-\hat{\imath}) + \left(\frac{1}{6.0\times10^{-2}}\right)(-\hat{\imath})\right]$$

$$= 2 \times 10^{-4}\left[\frac{1}{2} - \frac{1}{6}\right]\hat{\imath} = \left(\frac{2}{3} \times 10^{-4}\right)\hat{\imath}.$$

Accordingly, magnitude of resultant magnetic field at A_1 is $B_{A1} = 0.67 \times 10^{-4}$T.

Magnetic field at Point A_2: $\vec{B}_{A2} = \vec{B}_{P-A2} + \vec{B}_{P-A2}$

$$= 2 \times 10^{-7}\left[\left(\frac{10}{r_{p2}}\right)\left(\hat{k} \times (\hat{\jmath})\right) + \left(\frac{10}{r_{q2}}\right)\left((-\hat{k}) \times (-\hat{\jmath})\right)\right]$$

$$= 2 \times 10^{-6}\left[\left(\frac{1}{1.0\times10^{-2}}\right)(-\hat{\imath}) + \left(\frac{1}{3.0\times10^{-2}}\right)(-\hat{\imath})\right]$$

$$= 2 \times 10^{-4}\left[1 + \frac{1}{3}\right](-\hat{\imath}) = \left(\frac{8}{3} \times 10^{-4}\right)(-\hat{\imath}).$$

Accordingly, magnitude of resultant magnetic field at A_2 is $B_{A2} = 2.7 \times 10^{-4}$T.

Magnetic field at Point A_3: $\vec{B}_{A3} = \vec{B}_{P-A3} + \vec{B}_{P-A3}$

$$= 2 \times 10^{-7}\left[\left(\frac{10}{r_{p2}}\right)\left(\hat{k} \times (\hat{\jmath})\right) + \left(\frac{10}{r_{q2}}\right)\left((-\hat{k}) \times (-\hat{\jmath})\right)\right]$$

$$= 2 \times 10^{-6}\left[\left(\frac{1}{2.0\times10^{-2}}\right)(\hat{\imath}) - \left(\frac{1}{2.0\times10^{-2}}\right)(-\hat{\imath})\right] = 2 \times 10^{-6}\left[\left(\frac{2}{2.0\times10^{-2}}\right)(\hat{\imath})\right]$$

Accordingly, magnitude of resultant magnetic field at A_2 is $B_{A2} = 2.7 \times 10^{-4}$T.

Magnetic field at Point A_3: $\vec{B}_{A3} = \vec{B}_{P-A3} + \vec{B}_{P-A3}$

$$= 2 \times 10^{-7}\left[\left(\frac{10}{r_{p2}}\right)\left(\hat{k} \times (\hat{\jmath})\right) + \left(\frac{10}{r_{q2}}\right)\left((-\hat{k}) \times (-\hat{\jmath})\right)\right]$$

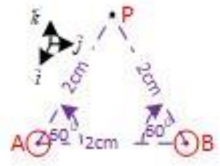
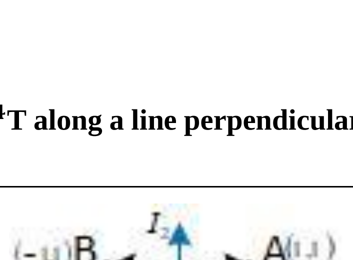
$$= 2 \times 10^{-6}\left[\left(\frac{1}{2.0\times10^{-2}}\right)(\hat{\imath}) - \left(\frac{1}{2.0\times10^{-2}}\right)(-\hat{\imath})\right] = 2 \times 10^{-6}\left[\left(\frac{2}{2.0\times10^{-2}}\right)(\hat{\imath})\right]$$

$$= 2.0 \times 10^{-4}\hat{\imath}$$

Accordingly, magnitude of resultant magnetic field at A_3 is $B_{A3} = 2.0 \times 10^{-4}$T.

Magnetic field at Point A_4: $\vec{B}_{A4} = \vec{B}_{P-A4} + \vec{B}_{P-A4}$

$$= 2 \times 10^{-7}\left[\left(\frac{10}{r_{p4}}\right)\left(\hat{k} \times (\hat{\jmath} - \hat{\imath})\right) + \left(\frac{10}{r_{q4}}\right)\left((-\hat{k}) \times (-\hat{\jmath} - \hat{\imath})\right)\right]$$

	Here, though vectorially $\vec{r}_{p4} \neq \vec{r}_{q4}$, yet geometrically, as shown in the figure, $r_{p4} = r_{q4} = \sqrt{2 \times (2.0 \times 10^{-2})^2} = \sqrt{2} \times (2.0 \times 10^{-2})$ $= 2 \times 10^{-6}\left[\left(\frac{1}{2\sqrt{2}\times 10^{-2}}\right)(\hat{k} \times \hat{j}) + \left(\frac{1}{2\sqrt{2}\times 10^{-2}}\right)(\hat{k} \times \hat{j})\right]$ $= 2 \times 10^{-4}\left[\frac{1}{\sqrt{2}}\right](-\hat{\imath}) = (\sqrt{2} \times 10^{-4})(-\hat{\imath})$. Accordingly, magnitude of resultant magnetic field at A_4 is $B_{A4} = 1.4 \times 10^{-4}$T. Thus, **answer is (a) 0.67×10^{-2} T, (b) 2.7×10^{-2}T (c) 2.0×10^{-2}T and (d) 1.4×10^{-2}T**
I-41	Given that two parallel wires A and B, separated by a distance $d = 2.0 \times 10^{-2}$m are carrying current $I = 10$ A in same direction. The wire in 3D vector space are laid along $\hat{\imath}$ on $\hat{\imath} - \hat{\jmath}$ plane as shown in the figure is currents in the wires would be $\vec{I}_p = 10\hat{k}$ and $\vec{I}_q = 10\hat{k}$.  It is required to determine magnetic field at point P, equidistant from A and B $d = 2.0 \times 10^{-2}$m, on $\hat{\imath} - \hat{\jmath}$ plane, which is combined effect of currents in wires A and B. As per Biot-Savart's Law magnetic field at a point due to long straight wire is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r} = 2 \times 10^{-7}\left(\frac{I}{r}\right)(\hat{I} \times \hat{r})$…(1). It is, therefore, essential to first determine position vectors of each of the point P w.r.t. wires A and B. Let us take wire A as reference, position vector of point P w.r.t. to A is $\vec{r}_{pa} = 2.0 \times 10^{-2}(\cos 60^0\hat{\jmath} - \sin 60^0\hat{\imath}) \Rightarrow \vec{r}_{pa} = 2.0 \times 10^{-2}\left(\frac{1}{2}\hat{\jmath} - \frac{\sqrt{3}}{2}\hat{\imath}\right)$. Position vectors of B w.r.t. wire A is $\vec{r}_{ba} = 2.0 \times 10^{-2}\hat{\jmath}$. Accordingly, position vector of point P w.r.t. wire B is $\vec{r}_{pb} = \vec{r}_{pa} - \vec{r}_{ba} \Rightarrow \vec{r}_{pb} = 2.0 \times 10^{-2}\left[\left(\frac{1}{2}\hat{\jmath} - \frac{\sqrt{3}}{2}\hat{\imath}\right) - \hat{\jmath}\right]$. It leads to $\vec{r}_{pb} = -2.0 \times 10^{-2}\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$. Here, magnitudes $r_{pa} = r_{pb} = 2.0 \times 10^{-2}$ Using (1), we are now ready to determine magnitude of magnetic field at point P. Here, it is important to recall product of unit vectors $\hat{\jmath} \times \hat{k} = \hat{\imath}$ and $\hat{k} \times \hat{\jmath} = -\hat{\imath}$. Likewise $\hat{k} \times \hat{\imath} = \hat{\jmath}$ and $\hat{\imath} \times \hat{k} = -\hat{\jmath}$. **Magnetic field at Point P:** $\vec{B}_P = \vec{B}_{Pa} + \vec{B}_{Pb}$ $= 2 \times 10^{-7}\left[\left(\frac{10}{r_{pa}}\right)\left(\hat{k} \times \left(\frac{1}{2}\hat{\jmath} - \frac{\sqrt{3}}{2}\hat{\imath}\right)\right) - \left(\frac{10}{r_{pb}}\right)\left(\hat{k} \times \left(\frac{1}{2}\hat{\jmath} + \frac{\sqrt{3}}{2}\hat{\imath}\right)\right)\right]$ $= 2 \times 10^{-6}\left[-\left(\frac{\sqrt{3}}{2.0\times 10^{-2}}\right)(\hat{k} \times \hat{\imath})\right]$ $= \sqrt{3} \times 10^{-4}(-\hat{\jmath})$. Accordingly, magnitude of resultant magnetic field at A_1 is $B_P =$ **1.7×10^{-4}T along a line perpendicular wires A-B, parallel to AB from in direction B to A, is the answer.**
I-42	Long straight wires along X and Y axes are carrying current $I = 5$A in positive directions, accordingly currents through them are $\vec{I}_1 = 5\hat{\imath}$ and $\vec{I}_2 = 5\hat{\jmath}$, respectively.  It is required to determine magnetic field at points A(1,1), B(-1,1), C(-1,-1) and D(1,-1). Magnetic field at point from a long straight wires is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r} = 2 \times 10^{-7}\left(\frac{I}{r}\right)(\hat{I} \times \hat{r})$. Here, $\vec{r} = r\hat{r}$ is position vector of the point from the current carrying wire, where r distance of the point from the wire and $\hat{r}$ is the unit direction vector of the point. Magnetic field at each of the given points shown in the figure is resultant of the magnetic fields produced by the two wires at the points. Accordingly,

Point A: $\vec{B}_A = \vec{B}_{A1} + \vec{B}_{A2} \Rightarrow \vec{B}_A = 2 \times 10^{-7}\left(\frac{5}{1}\right)[(\hat{\imath} \times \hat{\jmath}) + \hat{\jmath} \times \hat{\imath}] = 1.0 \times 10^{-6}(\hat{k} - \hat{k}) \Rightarrow \boldsymbol{\vec{B}_A = 0}$

Point B: $\vec{B}_B = \vec{B}_{B1} + \vec{B}_{B2} \Rightarrow \vec{B}_B = 2 \times 10^{-7}\left(\frac{5}{1}\right)[(\hat{\imath} \times \hat{\jmath}) + \hat{\jmath} \times (-\hat{\imath})] = 1.0 \times 10^{-6}\left(\hat{k} - (-\hat{k})\right) \Rightarrow \boldsymbol{\vec{B}_B = 2.0 \times 10^{-6}\hat{k}}$**.**

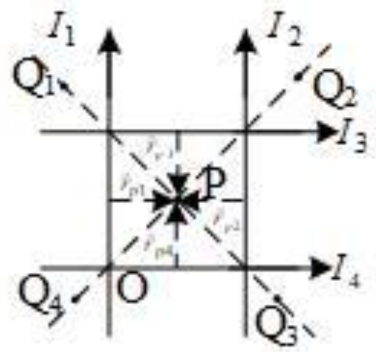
Point C: $\vec{B}_C = \vec{B}_{C1} + \vec{B}_{C2} \Rightarrow \vec{B}_C = 2 \times 10^{-7}\left(\frac{5}{1}\right)\left[\left(\hat{\imath} \times (-\hat{\jmath})\right) + \hat{\jmath} \times (-\hat{\imath})\right] = 1.0 \times 10^{-6}\left(-\hat{k} - (-\hat{k})\right) = 1.0 \times 10^{-6}(-\hat{k} + \hat{k}) \Rightarrow \boldsymbol{\vec{B}_C = 0}$**.**

Point D: $\vec{B}_D = \vec{B}_{D1} + \vec{B}_{D2} \Rightarrow \vec{B}_D = 2 \times 10^{-7}\left(\frac{5}{1}\right)\left[\left(\hat{\imath} \times (-\hat{\jmath})\right) + \hat{\jmath} \times (\hat{\imath})\right] = 1.0 \times 10^{-6}(\hat{k} + \hat{k}) = 2.0 \times 10^{-6}\hat{k} \Rightarrow \boldsymbol{\vec{B}_D = 2.0 \times 10^{-6}\hat{k}}$**.**

Thus, $\boldsymbol{B_A = 0}$**,** $\boldsymbol{B_B = 2.0 \times 10^{-6}}$**T coming out of the plane of the paper,** $\boldsymbol{B_C = 0}$**,** $\boldsymbol{B_D = 2.0 \times 10^{-6}}$**T entering the plane of the paper.**

I-43

Magnetic field at point from a long straight wires is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r} = 2 \times 10^{-7}\left(\frac{I}{r}\right)(\hat{I} \times \hat{r})$. Here, $\vec{r} = r\hat{r}$ is position vector of the point from the current carrying wire, where r distance of the point from the wire and $\hat{r}$ is the unit direction vector of the point.



Taking currents I_3 and I_4 along unit direction vector $\hat{\imath}$ and currents I_1 and I_2 along unit direction vector $\hat{\jmath}$, for the purpose of position vectors of Q_1, Q_2, Q_3, and Q_4 are situated on the diagonals of the square of side $d = 5.0 \times 10^{-2}$m formed by the four wires as defined point O as shown in the figure is taken as a reference.

Length of the diagonals of the square is $D = \sqrt{2}d$, accordingly geometrically position vectors of the points from the four wires point and corresponding magnetic fields are –

Point P: W.r.t. to wire 1, $\vec{r}_{p1} = 2.5\hat{\imath}$, and therefore $\vec{B}_{P1} = 2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{P1} = -4 \times 10^{-5}\hat{k}$;

W.r.t. to wire 2, $\vec{r}_{p2} = -2.5\hat{\imath}$, and therefore $\vec{B}_{P2} = -2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{P2} = 4 \times 10^{-5}\hat{k}$;

W.r.t. to wire 3, $\vec{r}_{p3} = -2.5\hat{\jmath}$, and therefore $\vec{B}_{P3} = -2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P3} = -4 \times 10^{-5}\hat{k}$;

W.r.t. to wire 4, $\vec{r}_{p4} = 2.5\hat{\jmath}$, and therefore $\vec{B}_{P4} = 2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P4} = 4 \times 10^{-5}\hat{k}$;

Therefore, magnetic field at point P is $\vec{B}_P = \vec{B}_{P1} + \vec{B}_{P2} + \vec{B}_{P3} + \vec{B}_{P4} = 4 \times 10^{-5}(-\hat{k} + \hat{k} - \hat{k} + \hat{k})$. It leads to $\boldsymbol{\vec{B}_P = 0}$ **is the answer**.

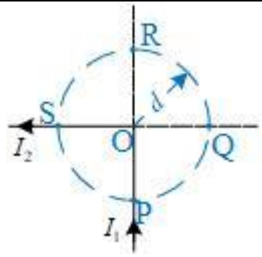
Point P is point of cross-section of the two diagonals. Yet, same set of equations can be used with the respective positions vectors of points Q_1, Q_2, Q_3, and Q_4, using geometrical symmetry. Accordingly,

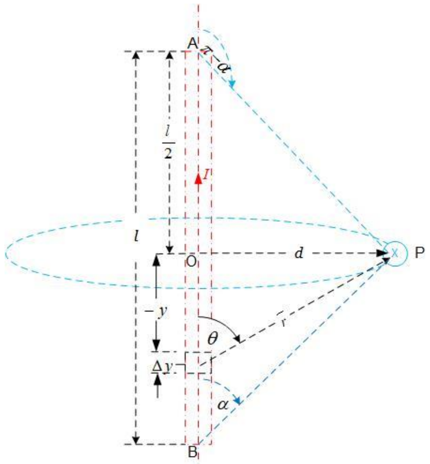
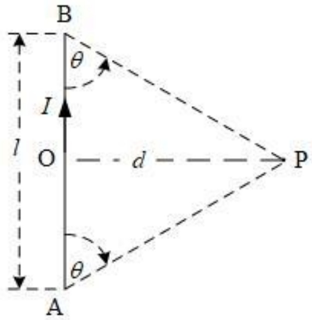
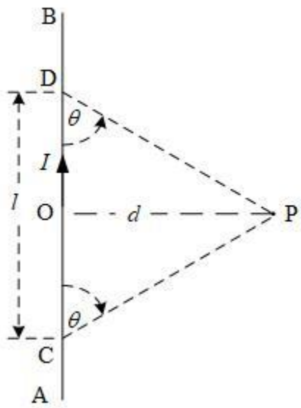
Point Q_1: W.r.t. to wire 1, $\vec{r}_{q1-1} = -2.5\hat{\imath}$, and therefore $\vec{B}_{q1-1} = -2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q1-1} = 4.0 \times 10^{-5}\hat{k}$;

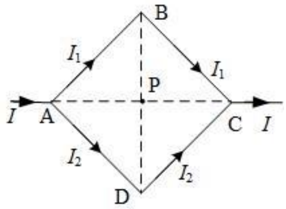
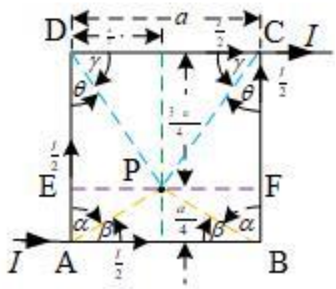
W.r.t. to wire 2, $\vec{r}_{q1-2} = -7.5\hat{\imath}$, and therefore $\vec{B}_{q1-2} = -2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q1-2} = 1.3 \times 10^{-5}\hat{k}$;

W.r.t. to wire 3, $\vec{r}_{q1-3} = 2.5\hat{\jmath}$, and therefore $\vec{B}_{q1-3} = 2 \times 10^{-7}\left(\frac{5}{2.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{q1-3} = 4.0 \times 10^{-5}\hat{k}$;

	W.r.t. to wire 4, $\vec{r}_{q1-4} = 7.5\hat{\jmath}$, and therefore $\vec{B}_{q1-4} = 2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P1-4} = 1.3 \times 10^{-5}\hat{k}$; Therefore, magnetic field at point P is $\vec{B}_{q1} = \vec{B}_{q1-1} + \vec{B}_{q1-2} + \vec{B}_{q1-3} + \vec{B}_{q1-4} = (4.0 + 1.3 + 4.0 + 1.3) \times 10^{-5}\hat{k}$. It leads to $\vec{B}_{q1} = 1.1 \times 10^{-4}\ \hat{k}$ or $\boldsymbol{B_{q1} = 1.1 \times 10^{-4}}$ **T coming out of the plane of the figure, is the answer**. **Point Q₂:** W.r.t. to wire 1, $\vec{r}_{q2-1} = 7.5\hat{\imath}$, and therefore $\vec{B}_{q2-1} = 2 \times 10^{-7}\left(\frac{5}{7.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q2-1} = -1.3 \times 10^{-5}\hat{k}$; W.r.t. to wire 2, $\vec{r}_{q2-2} = 2.5\hat{\imath}$, and therefore $\vec{B}_{q2-2} = 2 \times 10^{-7}\left(\frac{5}{2.5}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q2-2} = -4.0 \times 10^{-5}\hat{k}$; W.r.t. to wire 3,$\vec{r}_{q2-3} = 2.5\hat{\jmath}$, and therefore $\vec{B}_{q2-3} = 2 \times 10^{-7}\left(\frac{5}{2.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{q2-3} = 4 \times 10^{-5}\hat{k}$; W.r.t. to wire 4, $\vec{r}_{q1-4} = 7.5\hat{\jmath}$, and therefore $\vec{B}_{q2-4} = 2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P2-4} = 1.3 \times 10^{-5}\hat{k}$; Therefore, magnetic field at point P is $\vec{B}_{q2} = \vec{B}_{q2-1} + \vec{B}_{q2-2} + \vec{B}_{q2-3} + \vec{B}_{q2-4} = (-4 - 1.3 + 4 + 1.3) \times 10^{-5}\hat{k}$. It leads to $\boldsymbol{\vec{B}_{q2} = 0}$ **is the answer**. **Point Q₃:** W.r.t. to wire 1, $\vec{r}_{q3-1} = 7.5\hat{\imath}$, and therefore $\vec{B}_{q3-1} = 2 \times 10^{-7}\left(\frac{5}{7.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q3-1} = -1.3 \times 10^{-5}\hat{k}$; W.r.t. to wire 2, $\vec{r}_{q3-2} = 2.5\hat{\imath}$, and therefore $\vec{B}_{q3-2} = 2 \times 10^{-7}\left(\frac{5}{2.5}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q3-2} = -4.0 \times 10^{-5}\hat{k}$; W.r.t. to wire 3, $\vec{r}_{q3-3} = -2.5\hat{\jmath}$, and therefore $\vec{B}_{q3-3} = -2 \times 10^{-7}\left(\frac{5}{2.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{q3-3} = -4 \times 10^{-5}\hat{k}$; W.r.t. to wire 4, $\vec{r}_{q3-4} = -7.5\hat{\jmath}$, and therefore $\vec{B}_{q3-4} = -2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P3-4} = -1.3 \times 10^{-5}\hat{k}$; Therefore, magnetic field at point P is $\vec{B}_{q3} = \vec{B}_{q3-1} + \vec{B}_{q3-2} + \vec{B}_{q3-3} + \vec{B}_{q3-4} = (-4 - 1.3 - 4 - .3) \times 10^{-5}\hat{k} \Rightarrow \vec{B}_{q2} = -1.1 \times 10^{-4}\ \hat{k}$. It leads to $\boldsymbol{\vec{B}_{q3}} = 1.1 \times 10^{-4}$ **T entering the plane of the figure, is the answer**. **Point Q₄:** W.r.t. to wire 1, $\vec{r}_{q4-1} = -2.5\hat{\imath}$, and therefore $\vec{B}_{q4-1} = -2 \times 10^{-7}\left(\frac{5}{2.5\times10^{-2}}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q4-1} = 4.0 \times 10^{-5}\hat{k}$; W.r.t. to wire 2, $\vec{r}_{q4-2} = -7.5\hat{\imath}$, and therefore $\vec{B}_{q4-2} = -2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\jmath} \times \hat{\imath}) \Rightarrow \vec{B}_{q4-2} = 1.3 \times 10^{-5}\hat{k}$; W.r.t. to wire 3,$\vec{r}_{q4-3} = -7.5\hat{\jmath}$, and therefore $\vec{B}_{q4-3} = -2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{q4-3} = -1.3 \times 10^{-5}\hat{k}$; W.r.t. to wire 4, $\vec{r}_{q4-4} = -2.5\hat{\jmath}$, and therefore $\vec{B}_{q4-4} = 2 \times 10^{-7}\left(\frac{5}{7.5}\right)(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{B}_{P4-4} = -4.0 \times 10^{-5}\hat{k}$; Therefore, magnetic field at point P is $\vec{B}_{q4} = \vec{B}_{q4-1} + \vec{B}_{q4-2} + \vec{B}_{q4-3} + \vec{B}_{q4-4} = (4 + 1.3 - 4 - 1.3) \times 10^{-5}\hat{k}$. It leads to $\boldsymbol{\vec{B}_{q4} = 0}$ **is the answer**.

	Thus, magnetic fields are $\vec{\boldsymbol{B}}_{\boldsymbol{P}}=\mathbf{0}$, $\boldsymbol{B}_{\boldsymbol{q1}}=\mathbf{1.1}\times\mathbf{10^{-4}}$ **T coming out of the plane of the figure, is the answer**, $\vec{\boldsymbol{B}}_{\boldsymbol{q2}}=\mathbf{0}$ **is the answer**, $\vec{\boldsymbol{B}}_{\boldsymbol{q3}}=1.1\times10^{-4}$ **T entering the plane of the figure, is the answer**, $\vec{\boldsymbol{B}}_{\boldsymbol{q4}}=\mathbf{0}$ **is the answer**.
I-44	 Given system is shown in the figure on the $\hat{\imath}-\hat{\jmath}$ plane, in which $\vec{I_1}=I_1\hat{\jmath}$ and $\vec{I_2}=-I_2\hat{\jmath}$, yet $I_1=I_2=I$. Since, the wire is stated to be bent at right angle at O, the dotted portions of the wires are nonexistent, and the given wire is represented by only solid lines. As per Biot-Savart's Law $\vec{B}=\frac{\mu_0 I}{2\pi r}\hat{I}\times\hat{r}$…(1). Since wires are only on left and below point O, as shown in the figure, i.e. half-length of long straight wire, hence magnetic field so caused would be $\vec{B'}=\frac{\vec{B}}{2}\Rightarrow\vec{B'}=\frac{\mu_0 I}{4\pi r}\hat{I}\times\hat{r}=10^{-7}\left(\frac{I}{r}\right)\left(\hat{I}\times\hat{r}\right)$….(2). Any current carrying wire has finite radius R and when nothing is stated about current distribution then uniform distribution of current is assumed over the cross-section of the wire such that current density is $\rho=\frac{I}{\pi R^2}$. If a point is considered along axis of the wire then combining Biot-Savart's Law (2) and Ampere's Circuital Law$\oint\vec{B}.d\vec{l}=\mu_0 I$, at the axis of the conductor where $r\to0$ the Zero magnetic field is $B_A=\frac{\mu_0}{4\pi}\times(\rho\times\pi r^2)\times\frac{1}{r}\Big\vert_{r\to0}\Rightarrow B_A=\frac{\mu_0}{4\pi}\times\left(\frac{I}{\pi R^2}\times\pi r^2\right)\times\frac{1}{r}\Big\vert_{r\to0}=\left(\frac{\mu_0 I}{4\pi R^2}\right)r\Big\vert_{r\to0}\Rightarrow B_A=0$ Accordingly, magnitude of magnetic field at points P, Q R and S are determined using (2), here under – **Point P:** $\vec{B}_P=\vec{B}_{P1}+\vec{B}_{P2}=0+10^{-7}\left(\frac{I}{d}\right)\left((-\hat{\imath})\times(-\hat{\jmath})\right)$. Since, point P is along axis of wire carrying current I_1. Accordingly, $\vec{B}_P=10^{-7}\left(\frac{I}{d}\right)(\hat{\imath}\times\hat{\jmath})\Rightarrow\vec{B}_P=10^{-7}\left(\frac{I}{d}\right)\hat{k}$. Thus magnitude $B_P=10^{-7}\left(\frac{I}{d}\right)$, is the answer. **Point Q:** $\vec{B}_Q=\vec{B}_{Q1}+\vec{B}_{Q2}=10^{-7}\left(\frac{I}{d}\right)\left((-\hat{\jmath})\times(\hat{\imath})\right)+0$. Since, point Q is along axis of wire carrying current I_2. Accordingly, $\vec{B}_Q=-10^{-7}\left(\frac{I}{d}\right)(\hat{\jmath}\times\hat{\imath})\Rightarrow\vec{B}_Q=10^{-7}\left(\frac{I}{d}\right)\hat{k}$. Thus magnitude $B_Q=10^{-7}\left(\frac{I}{d}\right)$, is the answer. **Point R:** $\vec{B}_R=\vec{B}_{R1}+\vec{B}_{R2}=0+10^{-7}\left(\frac{I}{d}\right)\left((-\hat{\imath})\times(\hat{\jmath})\right)$. Since, point R is along axis of wire carrying current I_1. Accordingly, $\vec{B}_R=-10^{-7}\left(\frac{I}{d}\right)(\hat{\imath}\times\hat{\jmath})\Rightarrow\vec{B}_R=-10^{-7}\left(\frac{I}{d}\right)\hat{k}$. Thus magnitude $B_R=10^{-7}\left(\frac{I}{d}\right)$, is the answer. **Point S:** $\vec{B}_S=\vec{B}_{S1}+\vec{B}_{S2}=10^{-7}\left(\frac{I}{d}\right)\left((-\hat{\imath})\times(\hat{\jmath})\right)+0$. Since, point S is along axis of wire carrying current I_2. Accordingly, $\vec{B}_S=-10^{-7}\left(\frac{I}{d}\right)(\hat{\imath}\times\hat{\jmath})\Rightarrow\vec{B}_S=-10^{-7}\left(\frac{I}{d}\right)\hat{k}$. Thus magnitude $B_S=10^{-7}\left(\frac{I}{d}\right)$, is the answer. Thus, **magnitudes of magnetic field is equal at all points P, Q, R and S and it is $\mathbf{10^{-7}}\left(\frac{I}{d}\right)$T, is the answer.**
I-45	Given system is shown in the figure in $\hat{\imath}-\hat{\jmath}$ plane, where a wire AB of length l laid along $\hat{\jmath}$ is carrying a current I from B towards A. It is required to find magnetic field at a point P at a distance d from midpoint O of the wire. Consider a small element of wire of length $\Delta y\to0$ at a distance $-y$ from its midpoint. As per Biot-Savart's Law magnetic field at point P due to current I in the element of wire $d\vec{B}=\frac{\mu_0 I}{4\pi}\left(\frac{d\vec{y}\times\hat{r}}{r^2}\right)$.It leads to $d\vec{B}=\frac{\mu_0 I}{4\pi}\frac{1}{r^2}\sin\theta\,dy\hat{k}\Rightarrow dB=\frac{\mu_0 I}{4\pi}\frac{\sin\theta}{r^2}dy$, in direction entering the plane of the figure as shown therein.

	Therefore, magnitude of magnetic field at B due to wire AB is $B=\frac{\mu_0 I}{4\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{\sin\theta}{d^2+y^2}dy$. Trigonometrically, $y=d\cot\theta\Rightarrow dy=-d\ \csc^2\theta\, d\theta$. Likewise, limits would change to $-\frac{l}{2}\to\theta$ and $\frac{l}{2}\to(\pi-\theta)$. Using this substitution $B=\frac{\mu_0 I}{4\pi}\int_{\theta}^{\pi-\theta}\frac{\sin\theta}{d^2(1+\cot^2\theta)}(-d\ \csc^2\theta\, d\theta)$. Again using trigonometry $(1+\cot^2\theta=\csc^2\theta)$, we have $B=\frac{\mu_0 I}{4\pi}\left[\int\frac{\sin\theta}{d^2(\csc^2\theta)}(-d\ \csc^2\theta\, d\theta)\right]_{\theta}^{\pi-\theta}$. This expression solves into $B=\frac{\mu_0 I}{4\pi d}[\int\sin\theta\, d\theta]_{\theta}^{\pi-\theta}\Rightarrow B=-\frac{\mu_0 I}{4\pi d}[\cos\theta]_{\theta}^{\pi-\theta}\Rightarrow B=\frac{\mu_0 I}{4\pi d}[\cos\theta-\cos(\pi-\theta)]\Rightarrow B=\frac{\mu_0 I}{2\pi d}\cos\theta$. Here, $\cos\theta=\frac{\frac{l}{2}}{r}\Rightarrow\cos\theta=\frac{\frac{l}{2}}{\sqrt{d^2+\left(\frac{l}{2}\right)^2}}\Rightarrow\cos\theta=\frac{l}{\sqrt{4d^2+l^2}}$.  It is asked to determine B in two cases – **Case 1:** When $d\gg l$. From the figure $\cos\theta\to\frac{l}{2d}$, accordingly $B=\frac{\mu_0 I}{2\pi d}\times\frac{l}{2d}\Rightarrow B=\frac{\mu_0 Il}{4\pi d^2}\Rightarrow \boldsymbol{B\propto\frac{1}{d^2}}$ **proved.** **Case 2:** When $d\ll l$. From the figure $\cos\theta\to 1$, accordingly $B=\frac{\mu_0 I}{2\pi d}\Rightarrow\boldsymbol{B\propto\frac{1}{d}}$ **proved.**
I-46	Given system is shown in the figure where wire AB of length $l=0.10$ m is carrying a current $I=10$ A. A point P forming an equilateral triangle with the ends A and B of the wire is so positioned that $\theta=60^0$ and distance of point P from the wire. Angle is dependent upon length of wire and $d=l\sin 60^0$.  Magnetic at appoint P in the given system as per Biot-Savart's law is $B=\frac{\mu_0 I}{2\pi d}\cos\theta$. Using the available data, $B=2\times10^{-7}\times\frac{10}{0.10\times\sin 60^0}\cos 60^0$. It solves into $B=2\times10^{-5}\times\cot 60^0\Rightarrow B=\frac{2}{\sqrt{3}}\times10^{-5}\Rightarrow B=11.54\times10^{-6}$T say **12 T, is the answer.**
I-47	Given long straight wire AB carrying current I is shown along with a point P at a distance d from the wire. Magnetic field at the point as per Biot-Savart's Law is $B_1=\frac{\mu_0 I}{2\pi d}$…(1) A section of wire CD of length l is so located, as given, that angle between position vector and the wire is θ. In this case magnetic field with the application of Biot-Savart's Law is $B_2=\frac{\mu_0 I}{2\pi d}\cos\theta$…(2), here trigonometrically $\cos\theta=\frac{\frac{l}{2}}{r}$ where distance CP is $r=\sqrt{d^2+\left(\frac{l}{2}\right)^2}$. Accordingly, $\cos\theta=\frac{\frac{l}{2}}{\sqrt{d^2+\left(\frac{l}{2}\right)^2}}\Rightarrow\cos\theta=\frac{l}{\sqrt{4d^2+l^2}}$...(3)  Given that $\frac{B_1-B_2}{B_1}=\frac{1}{100}$. Combining (1), (2) and (3) $\frac{\frac{\mu_0 I}{2\pi d}-\frac{\mu_0 I}{2\pi d}\times\frac{l}{\sqrt{4d^2+l^2}}}{\frac{\mu_0 I}{2\pi d}}=\frac{1}{100}$. It simplifies into $\frac{\sqrt{4d^2+l^2}-l}{\sqrt{4d^2+l^2}}=\frac{1}{100}$. Applying dividendo $\frac{l}{\sqrt{4d^2+l^2}}=\frac{99}{100}\Rightarrow\frac{\sqrt{4\left(\frac{d}{l}\right)^2+1}}{1}=\frac{100}{99}\Rightarrow\left(2\frac{d}{l}\right)^2+1=\left(\frac{1}{1-0.01}\right)^2\Rightarrow\left(2\frac{d}{l}\right)^2+1=(1-0.01)^{-2}$…(4). As per binomial expansion $(1-x)^n=1-nx\vert_{x\ll}$.

	Accordingly, (4) reduces takes a form $\left(2\frac{d}{l}\right)^2+1=1+0.02 \Rightarrow \left(2\frac{d}{l}\right)^2=2\times10^{-2} \Rightarrow \frac{d}{l}=\frac{10^{-1}}{\sqrt{2}} \Rightarrow \frac{d}{l}=0.07$ is the answer. **N.B.:** It involves simple application of algebra.
I-48	Given system is shown in figure, on $\hat{\imath}-\hat{\jmath}$ plane, in which resistance of path ABC of current I_1 is r and path ADC of current I_2 is $2r$. Applying Kirchhoff's Current Law at node A and C, $I=I_1+I_2$…(1) Let potential difference between nodes A and C is V, then as per Ohm's Law $I_1=\frac{V}{r}$ and $I_2=\frac{V}{2r}$. It leads to $\frac{I_1}{I_2}=2$…(2).  Combining (1) and (2), $I_2=\frac{I}{3}$ and $I_1=\frac{2I}{3}$. Geometrically, distance of point P from current carrying branches AB, BC, AD and DC, each of length a, from their mid points is $d=\frac{a}{2}$. As per Biot-Savart's Laws magnetic field, at a point, due to a wire of length l at a distance d from the midpoint is $B=\frac{\mu_0 I}{2\pi d}\cos\theta$. Here, θ is the angle formed by line joining ends of the wire and the point under consideration, and current carrying wire. In the given geometry ABCD is a square and the angle $\theta=45^0$. Thus, $\cos 45^0=\frac{1}{\sqrt{2}}$ and $B=\frac{\mu_0 I}{2\sqrt{2}\pi d}$…(3) Direction of magnetic field at point P is determined using Ampere's Right-Hand Thumb Rule; as per direction convention magnetic field entering paper is assigned $(-\hat{k})$ direction and coming out of paper is assigned direction $(\hat{k})$. Accordingly, magnetic field at point P due to each branch of wire, using the available data, is as under – **Branch AB:** $\vec{\boldsymbol{B}}_{\boldsymbol{AB}}=\frac{\mu_0 I_1}{2\sqrt{2}\pi d}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{AB}}=\frac{\mu_0\left(\frac{2I}{3}\right)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{AB}}=\frac{2\mu_0 I}{3\sqrt{2}\pi a}(-\hat{k})$ **Branch BC:** $\vec{\boldsymbol{B}}_{\boldsymbol{BC}}=\frac{\mu_0 I_1}{2\sqrt{2}\pi d}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{BC}}=\frac{\mu_0\left(\frac{2I}{3}\right)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{BC}}=\frac{2\mu_0 I}{3\sqrt{2}\pi a}(-\hat{k})$ **Branch AD:** $\vec{\boldsymbol{B}}_{\boldsymbol{AD}}=\frac{\mu_0 I_2}{2\sqrt{2}\pi d}(\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{AD}}=\frac{\mu_0\left(\frac{I}{3}\right)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{AD}}=\frac{\mu_0 I}{3\sqrt{2}\pi a}(\hat{k})$ **Branch DC:** $\vec{\boldsymbol{B}}_{\boldsymbol{DC}}=\frac{\mu_0 I_2}{2\sqrt{2}\pi d}(\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{DC}}=\frac{\mu_0\left(\frac{I}{3}\right)}{2\sqrt{2}\pi\left(\frac{a}{2}\right)}(-\hat{k}) \Rightarrow \vec{\boldsymbol{B}}_{\boldsymbol{DC}}=\frac{\mu_0 I}{3\sqrt{2}\pi a}(\hat{k})$. Resultant magnetic field at point P is $\vec{B}_R=\vec{B}_{AB}+\vec{B}_{Bc}+\vec{B}_{Ad}+\vec{B}_{AC} \Rightarrow \vec{B}_R=\frac{\mu_0 I}{3\sqrt{2}\pi a}(-2\hat{k}-2\hat{k}+\hat{k}+\hat{k})$. It leads to $\vec{B}_R=-\frac{2\mu_0 I}{3\sqrt{2}\pi a}\hat{k} \Rightarrow \vec{B}_R=-\frac{\sqrt{2}\mu_0 I}{3\pi a}\hat{k}$ or $B_R=\frac{\sqrt{2}\boldsymbol{\mu_0 I}}{\boldsymbol{3\pi a}}$ **entering into the paper, is the answer**.
I-49	Given geometry is detailed in the figure, on $\hat{\imath}-\hat{\jmath}$ plane. Electrically square loop is made of uniform wire. The current I entering node A routes to exit node C through two paths ABC and ACD of equal lengths $2a$ and hence will have equal resistances. Therefore, the current I will get split into two paths equally at $\frac{I}{2}$.  As per Biot-Savart's Laws magnetic field, at a point, due to a wire of length l at a distance d from the midpoint is $B=\frac{\mu_0 I}{2\pi d}\cos\theta$ ...(1). Here, θ is the angle formed by line joining ends of the wire and the point under consideration, and current carrying wire. Accordingly, magnetic field due to half-length is $B=\frac{\mu_0 I}{4\pi d}\cos\theta$ ...(2). Equation (2) is useful to determine magnetic field due to wire-lengths AD and BC, while equation (1) is useful to determine magnetic field due to wire-lengths DC and AB.

In the figure $\cos\theta = \frac{\frac{3}{4}a}{\sqrt{\left(\frac{3}{4}a\right)^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\theta = \frac{3}{\sqrt{13}}$; $\cos\gamma = \frac{\frac{1}{2}a}{\sqrt{\left(\frac{3}{4}a\right)^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\gamma = \frac{2}{\sqrt{13}}$;

$$\cos\alpha = \frac{\frac{1}{4}a}{\sqrt{\left(\frac{1}{4}a\right)^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{5}}\ ;\ \cos\beta = \frac{\frac{1}{2}a}{\sqrt{\left(\frac{1}{4}a\right)^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\beta = \frac{2}{\sqrt{5}}$$

Direction of magnetic field at point P is determined using Ampere's Right-Hand Thumb Rule; as per direction convention magnetic field due to wire ABC, coming out of paper, is assigned $(\hat{k})$ direction and magnetic field due to wire ADC, entering into the plane of paper is assigned direction $(-\hat{k})$.

Accordingly, magnetic field at point P due to each branch of wire, using the available data, is as under –

Wire AB: Using (1); $\vec{B}_{AB} = \frac{\mu_0\frac{I}{2}}{2\pi\frac{a}{4}}\left(\frac{2}{\sqrt{5}}\right)(\hat{k}) \Rightarrow \vec{B}_{AB} = \frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k})$

Wire BC: Using (2) for sections BF and FC; $\vec{B}_{BC} = \vec{B}_{BF} + \vec{B}_{FC} \Rightarrow \vec{B}_{BC} = \frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{1}{\sqrt{5}}\right)(\hat{k}) + \frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{3}{\sqrt{13}}\right)(\hat{k})$. It leads to $\vec{B}_{BC} = \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](\hat{k})$

Wire AD: Using (2) for sections AE and ED; $\vec{B}_{AD} = \vec{B}_{AE} + \vec{B}_{ED} \Rightarrow \vec{B}_{AD} = \frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{1}{\sqrt{5}}\right)(-\hat{k}) + \frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{3}{\sqrt{13}}\right)(-\hat{k})$. It leads to $\vec{B}_{AD} = \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](-\hat{k})$

Wire DC: Using (1); $\vec{B}_{DC} = \frac{\mu_0\frac{I}{2}}{2\pi\frac{3a}{4}}\left(\frac{2}{\sqrt{13}}\right)(-\hat{k}) \Rightarrow \vec{B}_{DC} = \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k})$.

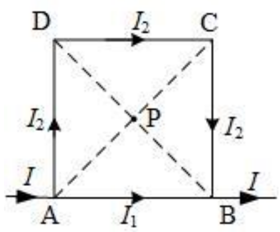
Thus, resultant magnetic field at P is $\vec{B} = \vec{B}_{AB} + \vec{B}_{BC} + \vec{B}_{AD} + \vec{B}_{DC} = \frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k}) + \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](\hat{k}) + \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](-\hat{k}) + \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k}) \Rightarrow \vec{B} = \frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k}) + \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k})$.

It leads to $\vec{B} = \frac{2\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right)\hat{k}$**, or** $\boldsymbol{B} = \frac{2\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right)$ **coming out of the paper.**

N.B.: Solution of this problem could be simplified using geometrical symmetry of sides AD and BC, as well as asymmetry of sides DC and AB w.r.t. point P. Yet, for clarity of concepts the problem.

I-50

Given system is shown in figure, on $\hat{\imath} - \hat{\jmath}$ plane. The square loop is made of uniform wire and therefore resistances R_1 and R_2 of the path AB and ADC would be proportional to their lengths, accordingly, $\frac{R_1}{R_2} = \frac{1}{3}$. Further, both the paths are parallel between AB and currents in the two paths shall be $\frac{I_1}{I_2} = \frac{3}{1}$, such that $I_1 + I_2 = I$. Applying componendo to the ratio of currents $\frac{I}{I_2} = 4 \Rightarrow I_2 = \frac{I}{4}$, and $I_1 = \frac{3I}{4}$.



Geometrically, distance of point P, being midpoint, from current carrying branches AB, BC, AD and DC, each of length a, is $d = \frac{a}{2}$.

As per Biot-Savart's Laws magnitude of magnetic field, at a point, due to a wire of length l at a distance d from the midpoint is $B = \frac{\mu_0 i}{2\pi d}\cos\theta$. Here, i is the current through the wire θ is the angle formed by line joining ends of the wire and the point under consideration, and current carrying wire. Geometrically, $\theta = 45^0 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$. Accordingly, magnitude of magnetic field due to half-length is $B = \frac{\mu_0 i}{2\sqrt{2}\pi d}$ ...(1).

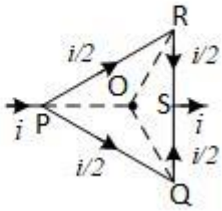
As per Ampere's Right-Hand Thumb Rule magnetic field at point P due to wire AD, DC and CB carrying equal current $I_2 = \frac{I}{4}$ would be equal and entering plane of the figure i.e. along $(-\hat{k})$. Therefore, magnetic field at P due to these three branches would be $\vec{B}_2 = 3\left(\frac{\mu_0 \frac{I}{4}}{2\sqrt{2}\pi d}\right)(-\hat{k}) \Rightarrow \vec{B}_2 = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(-\hat{k})$. Likewise, due to wire AB carrying current $I_2 = \frac{3I}{4}$ would be leaving plane of the figure i.e. along $(\hat{k})$. Thus magnetic field due to AB would be $\vec{B}_1 = \frac{\mu_0 \frac{3I}{4}}{2\sqrt{2}\pi d}(\hat{k}) \Rightarrow \vec{B}_1 = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(\hat{k})$.

Thus net magnetic field at P is $\vec{B} = \vec{B}_1 + \vec{B}_2 \Rightarrow \vec{B} = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(\hat{k}) + \frac{3\mu_0 I}{8\sqrt{2}\pi d}(-\hat{k}) \Rightarrow \boldsymbol{\vec{B} = 0}$ **is the answer**.

N.B.: This can be made an objective problem also.

I-51

In the given system magnetic field at O, center of the triangle PQR, as per Biot-Savart's Law, resultant of the magnetic fields as under –



a) Due to current $\frac{i}{2}$ through wire section PR, to which point O is symmetrically placed along its length is $\vec{B}_{\mathrm{PR}} = \frac{\mu_0 \frac{i}{2}}{2\pi d}\cos\theta\,(-\hat{n})$, here geometrically $\theta = \angle\mathrm{RPO} = -\frac{\pi}{6}$, $d = \frac{a}{2}\sin\theta$ and $\hat{n}$ is unit vector perpendicular to the plane of the figure. In this length of side of the equilateral triangle PQR is a. Accordingly, $\vec{B}_{\mathrm{PR}} = \frac{\mu_0 \frac{i}{2}}{2\pi\left(\frac{a}{2}\sin\theta\right)}\cos\theta\,(-\hat{n}) \Rightarrow \vec{B}_{\mathrm{PR}} = \frac{\mu_0 i}{2\pi a}\cot\theta(-\hat{n})$,,,(1).

b) Due to current $\frac{i}{2}$ through wire section PQ, to which point O is symmetrically placed along its length is $\vec{B}_{\mathrm{PQ}} = \frac{\mu_0 \frac{i}{2}}{2\pi d}\cos\theta\,(\hat{n})$, here geometrically $\theta = \angle Q\mathrm{PO} = \frac{\pi}{6}$ and $d = \frac{a}{2}\sin\theta$, where length of side of the equilateral triangle ABC is a. Accordingly, $\vec{B}_{\mathrm{PQ}} = \frac{\mu_0 \frac{i}{2}}{2\pi\left(\frac{a}{2}\sin\theta\right)}\cos\theta\,(\hat{n}) \Rightarrow \vec{B}_{\mathrm{PQ}} = \frac{\mu_0 i}{2\pi a}\cot\theta(\hat{n})$…(2).

c) Due to current $\frac{i}{2}$ through wire section RS, half the length RQ., magnetic field at O which is symmetrically placed to RQ, is half of that it would produce, were the current flowing through RQ. Accordingly, in line with (1), $\vec{B}_{\mathrm{RS}} = \frac{1}{2}\vec{B}_{\mathrm{RQ}} = \frac{1}{2}\times\frac{\mu_0 i}{2\pi a}\cot\theta(-\hat{n}) \Rightarrow \vec{B}_{\mathrm{RS}} = \frac{\mu_0 i}{4\pi a}\cot\theta(-\hat{n})$…(3).

d) Similar to that at (c) above, magnetic field at O due to current $\frac{i}{2}$ through wire section QS, half the length QR., magnetic field at O which is symmetrically placed to RQ, is half of that it would produce, were the current flowing through RQ. Accordingly, in line with (2), $\vec{B}_{\mathrm{QS}} = \frac{1}{2}\vec{B}_{\mathrm{QR}} = \frac{1}{2}\times\frac{\mu_0 i}{2\pi a}\cot\theta(\hat{n})$. It leads to $\vec{B}_{\mathrm{QS}} = \frac{\mu_0 i}{4\pi a}\cot\theta(\hat{n})$…(4).

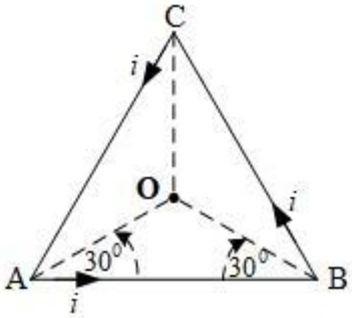
Thus, net magnetic field at O, $B = \vec{B}_{\mathrm{PR}} + \vec{B}_{\mathrm{PQ}} + \vec{B}_{\mathrm{RS}} + \vec{B}_{\mathrm{QS}}$…(5). Combining (1) to (5), we have magnetic field at O $B = \frac{\mu_0 i}{2\pi a}\cot\theta(-\hat{n}) + \frac{\mu_0 i}{2\pi a}\cot\theta(\hat{n}) + \frac{\mu_0 i}{4\pi a}\cot\theta(-\hat{n}) + \frac{\mu_0 i}{4\pi a}\cot\theta(\hat{n})$. All terms on RHS cancel with each other and accordingly $\boldsymbol{B = 0}$ **is the answer.**

N.B.: This problem has been solved in full length being a subjective problem. Otherwise, based on observation of geometrical symmetry and currents through it, the problem can reduced to an objective solution.

I-52

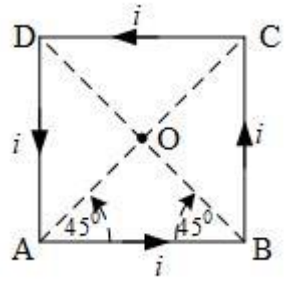
Given is a wire of length l, carrying a current i. The problem is in two parts. Given is a wire of length l, carrying a current i. The problem is in two parts.

Part (a): The wire is bent to form an equilateral triangle ABC of side length $a = \frac{l}{3}$, as shown in the figure. The point O, center of the triangle, is symmetrical to its three sides with vertices making an angle $\theta = 30^0$. As discussed in Appendix-I, magnetic field at O due to current in the side AB is $\vec{B}_{\text{AB}} = \frac{\mu_0 i}{2\pi d}\cos\theta\,(\hat{n})$…(1), here $d = \frac{a}{2}\tan\theta$ is distance of point O from the side AB and $\theta = 30^0$ both determined geometrically, and $\hat{n}$ is direction of vector perpendicular to the plane of the triangle ABC. Using (1) and the available data, it leads $\vec{B}_{\text{AB}} = \frac{\mu_0 i}{2\pi\left(\frac{a}{2}\tan\theta\right)}\cos\theta\,(\hat{n}) \Rightarrow \vec{B}_{\text{AB}} = \frac{\mu_0 i\frac{\sqrt{3}}{2}}{\pi\frac{l}{3}\frac{1}{\sqrt{3}}} \Rightarrow \vec{B}_{\text{AB}} = \frac{9\mu_0 i}{2\pi l}(\hat{n})$…(2).



Current in all sides of the triangle is identical and unidirectional, i.e. anti-clockwise, hence each side w.r.t. which point O is symmetrical would produce equal and unidirectional magnetic field and hence at O, $\vec{B} = 3\vec{B}_{\text{AB}}$. Accordingly, magnitude of magnetic field at O is $B = 3 \times \frac{9\mu_0 i}{2\pi l} \Rightarrow B = \boldsymbol{\frac{27\mu_0 i}{2\pi l}}$ **is the answer of part (a).**

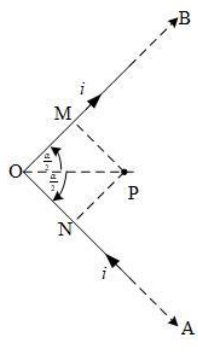
Part (b): In this case wire is bent in the form of a square ABCD of side length $a = \frac{l}{4}$, as shown in the figure. The point O, center of the square, is symmetrical to its four sides with vertices making an angle $\theta = 45^0$. As discussed in Appendix-I, magnetic field at O due to current in the side AB is $\vec{B}_{\text{AB}} = \frac{\mu_0 i}{2\pi d}\cos\theta\,(\hat{n})$…(1), here $d = \frac{a}{\sqrt{2}}$ is distance of point O from the side AB and $\theta = 45^0$ both determined geometrically, and $\hat{n}$ is direction of vector perpendicular to the plane of the triangle ABC. Using (1) and the available data, it leads $\vec{B}_{\text{AB}} = \frac{\mu_0 i}{2\pi\left(\frac{a}{2}\tan\theta\right)}\cos\theta\,(\hat{n}) \Rightarrow \vec{B}_{\text{AB}} = \frac{\mu_0 i\frac{1}{\sqrt{2}}}{\pi\frac{l}{4}\times 1} \Rightarrow \vec{B}_{\text{AB}} = \frac{2\sqrt{2}\mu_0 i}{\pi l}(\hat{n})$…(2).



Current in all sides of the square is identical and unidirectional, i.e. anti-clockwise, hence each side w.r.t. which point O is symmetrical would produce equal and unidirectional magnetic field and hence at O, $\vec{B} = 4\vec{B}_{\text{AB}}$. Accordingly, magnitude of magnetic field at O is $B = 4 \times \frac{2\sqrt{2}\mu_0 i}{\pi l} \Rightarrow B = \boldsymbol{\frac{8\sqrt{2}\mu_0 i}{\pi l}}$ **is the answer of part (b).**

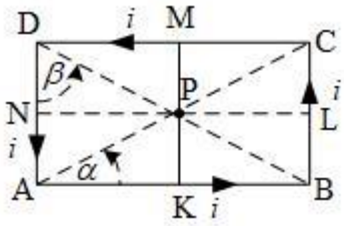
Thus answers are (a) $\boldsymbol{\frac{27\mu_0 i}{2\pi l}}$ **and (b)** $\boldsymbol{\frac{8\sqrt{2}\mu_0 i}{\pi l}}$**.**

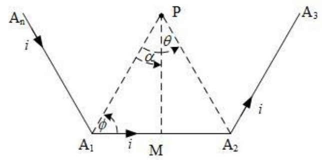
I-53 This problem combines asymmetry of point P w.r.t. a long wire AB bent at O at an angle α such that the point P at a distance x from the vertex O, is symmetrical to portions of wire AO and OB as shown in the figure.



Applying Biot-Savart's Law magnetic field due current i in wire , as illustrated in Appendix-I, is $\overrightarrow{dB} = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dl(\hat{n})$…(1). Here, dl is length of the element of wire carrying current i, r is the distance of the element from the point P at which magnetic field is to be determined, θ is the angle of the vector $\vec{r}$ w.r.t. $\overrightarrow{dl}$ in which current i is flowing and $\hat{n}$ is the unit direction vector of the magnetic field.

Applying (1) to the portion of wire OA wire the angle θ is clockwise direction and hence it's value is (-)ve and at A $\theta_i \to 0$ while at O $\theta_f = -\left(\pi - \frac{\alpha}{2}\right)$. Therefore, net magnetic

	field due to wire is $\vec{B}_{AB} = \left[\int_0^{-\left(\pi-\frac{\alpha}{2}\right)} \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2} dl\right]\hat{n}$...(2). This calculation can be simplified decomposing asymmetry of point P with the wire length AO as AO=AN+NO. Accordingly, field due to wire AN is $\vec{B}_{\mathrm{AN}} = \frac{1}{2}\left(\frac{\mu_0 i}{2\pi b}\right)(-\hat{n})$, here $b = \mathrm{NP} = \mathrm{OP}\sin\frac{\alpha}{2} \Rightarrow b = x\sin\frac{\alpha}{2}$, is half of the field due to a long wire. Thus using the available data $\vec{B}_{\mathrm{AN}} = \frac{\mu_0 i}{4\pi x \sin\frac{\alpha}{2}}(-\hat{n})\ldots(3)$. But, for portion NO it is half of the length of wire symmetrical w.r.t. P along AO. Thus, $\vec{B}_{\mathrm{NO}} = \frac{1}{2}\left(\frac{\mu_0 i}{2\pi b}\cos\frac{\alpha}{2}\right)(-\hat{n})$. Using the available data $\vec{B}_{\mathrm{NO}} = \frac{1}{2}\left(\frac{\mu_0 i}{2\pi\left(x\sin\frac{\alpha}{2}\right)}\cos\frac{\alpha}{2}\right)(-\hat{n}) \Rightarrow \vec{B}_{\mathrm{NO}} = \frac{\mu_0 i}{4\pi x}\cot\frac{\alpha}{2}(-\hat{n})\ldots(4)$. Thus, net field at P due to wire AO, combining (3) and (4) is $\vec{B}_{\mathrm{AO}} = \vec{B}_{\mathrm{AN}} + \vec{B}_{\mathrm{NO}} = \frac{\mu_0 i}{4\pi x\sin\frac{\alpha}{2}}(-\hat{n}) + \frac{\mu_0 i}{4\pi x}\cot\frac{\alpha}{2}(-\hat{n})$. It leads to $\vec{B}_{\mathrm{AO}} = \frac{\mu_0 i}{4\pi x}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}}(-\hat{n})\ldots(5)$. Here, a little trigonometric manipulation is needed such that $\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\frac{\alpha}{2}} = \frac{\left(1+\left(2\cos^2\frac{\alpha}{4}-1\right)\right)}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}} = \frac{2\cos^2\frac{\alpha}{4}}{2\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}} = \frac{\cos\frac{\alpha}{4}}{\sin\frac{\alpha}{4}} = \cot\frac{\alpha}{4}\ldots(6)$. Combining, (5) and (6), $\vec{B}_{\mathrm{AO}} = \frac{\mu_0 i}{4\pi x}\cot\frac{\alpha}{4}(-\hat{n})\ldots(7)$. It is seen from the figure that magnetic field at P due to the same current i in portion OB is symmetrical and additive to the magnetic field due to the current in AO. Thus, magnitude of the net magnetic field at P due to wire AOB is $B = 2B_{AO} = 2\times\frac{\mu_0 i}{4\pi x}\cot\frac{\alpha}{4} \Rightarrow$ $\boldsymbol{B = \frac{\mu_0 i}{2\pi x}\cot\frac{\alpha}{4}}$ **is the answer.** **N.B.:** Complexity in the solution of the problem due to integration involved in application of Biot-Sevart's Law can be avoided by using standard formulation of magnetic field due to a long wire and a section of wire symmetrically placed about a point. It involves systematic observation of the geometry of the problem combining asymmetry into symmetries.
I-54	Given system is shown in the figure in which a wire in shape of rectangular loop of length l and width b is carrying a current i. It is required to determine magnetic field B at point P.  Symmetry in the geometry of the problem reveals that sides AB and CD each of length l are carrying current in anti-clockwise direction with the only difference point P is on the left of the current in the sides AB and CD such that it is in the middle of the both the wires and at a distance $d_1 = \frac{b}{2}$ from them. Thus magnetic field at P due to both the wires, as per Biot-Savart's Law illustrated in Appendix I, is $\vec{B}_{\mathrm{AB}} = \vec{B}_{\mathrm{CD}} = \frac{\mu_0 i}{2\pi}\frac{\cos\alpha}{d_1}(\hat{n})\ldots(1)$, here $\sin\alpha = \frac{\mathrm{AK}}{\mathrm{AP}} = \frac{\frac{l}{2}}{\frac{\sqrt{l^2+b^2}}{2}} \Rightarrow \sin\alpha = \frac{l}{\sqrt{l^2+b^2}}$ Similar geometry in respect of sides BC and CD leads to magnetic field at P is $\vec{B}_{\mathrm{BC}} = \vec{B}_{\mathrm{DA}} = \frac{\mu_0 i}{2\pi}\frac{\sin\beta}{d_2}(\hat{n})\ldots(1)$,here $\sin\beta = \frac{\mathrm{PN}}{\mathrm{DP}} = \frac{\frac{b}{2}}{\frac{\sqrt{l^2+b^2}}{2}} \Rightarrow \sin\beta = \frac{b}{\sqrt{l^2+b^2}}$. Here, $d_2 = \frac{l}{2}$ Thus net magnetic field at the center P is $\vec{B} = \vec{B}_{\mathrm{AB}} + \vec{B}_{\mathrm{BC}} + \vec{B}_{\mathrm{CD}} + \vec{B}_{\mathrm{DA}} = 2\frac{\mu_0 i}{2\pi}\frac{\sin\alpha}{d_1} + 2\frac{\mu_0 i}{2\pi}\frac{\sin\beta}{d_2}$. Using the available data $\vec{B} = \left(2\frac{\mu_0 i}{2\pi}\frac{\frac{l}{\sqrt{l^2+b^2}}}{\frac{b}{2}} + 2\frac{\mu_0 i}{2\pi}\frac{\frac{b}{\sqrt{l^2+b^2}}}{\frac{l}{2}}\right)(\hat{n}) = \frac{2\mu_0 i}{\pi\sqrt{l^2+b^2}}\left(\frac{l}{b}+\frac{b}{l}\right)(\hat{n}) \Rightarrow B = \frac{2\mu_0 i}{\pi\sqrt{l^2+b^2}}\times\frac{l^2+b^2}{bl} \Rightarrow$ $\boldsymbol{B = \frac{2\mu_0 i}{\pi l b}\sqrt{l^2+b^2}}$ **is the answer.**

I-55	Given system of a regular polygon having sides n made out by bending a wire of total length $l = 2\pi r$ is shown in the figure. Length of each side is $a = \frac{2\pi r}{n}$ and is carrying current i. Point P, at the center of the polygon is symmetrically placed w.r.t. each side of the polygon at a distance $d = \mathrm{AM}\cos\alpha$. Here, $\mathrm{AM} = \frac{a}{2}$, $\alpha = \frac{\theta}{2}$, where $\theta = \frac{2\pi}{n} \Rightarrow \alpha = \frac{\pi}{n}$. Therefore, $d = \frac{a}{2}\tan\alpha \Rightarrow d = \frac{a}{2}\tan\frac{\pi}{n}$…(1).  Using Biot-Savart's Law, as per Appendix-I, amgnetic field at P due current i in first side A_1A_2 of the polygon is $\vec{B}_1 = \frac{\mu_0 i}{2\pi}\frac{\cos\phi}{d}(\hat{n})$ …(2). Here, again from geometry $\phi = \frac{\pi}{2} - \alpha \Rightarrow \phi = \frac{\pi}{2} - \frac{\pi}{n} \Rightarrow \tan\phi = \tan\left(\frac{\pi}{2} - \frac{\pi}{n}\right) \Rightarrow \tan\phi = \cot\frac{\pi}{n}$ …(3). Each branch of the polygon would produce equal and unidirectional magnetic field at P such that magnitude of the magnetic field would be $B = nB_1 \Rightarrow B = n \times \frac{\mu_0 i}{2\pi}\frac{\cos\phi}{d}$. Using the available data $B = \frac{n\mu_0 i}{2\pi} \times \frac{\sin\frac{\pi}{n}}{\frac{a}{2}\cot\frac{\pi}{n}} \Rightarrow B = \frac{n\mu_0 i}{\pi\frac{2\pi r}{n}}\sin\frac{\pi}{n}\tan\frac{\pi}{n} \Rightarrow B = \boldsymbol{\frac{\mu_0 i n^2}{2\pi^2 r}\sin\frac{\pi}{n}\tan\frac{\pi}{n}}$ **is the answer of part (a).** Further, when geometry of wire is circular as $n \to \infty \Rightarrow \frac{\pi}{n} \to 0$, then $\sin\frac{\pi}{n} \to \frac{\pi}{n}$ and $\tan\frac{\pi}{n} \to \frac{\pi}{n}$. Accordingly, $B = \frac{\mu_0 i n^2}{2\pi^2 r} \times \frac{\pi}{n} \times \frac{\pi}{n}$. It simplifies to $B = \boldsymbol{\frac{\mu_0 i}{2r}}$ **is answer of part (b).** Thus, **answers are (a)** $\boldsymbol{\frac{\mu_0 i n^2}{2\pi^2 r}\sin\frac{\pi}{n}\tan\frac{\pi}{n}}$ **and (b)** $\boldsymbol{\frac{\mu_0 i}{2r}}$**.**

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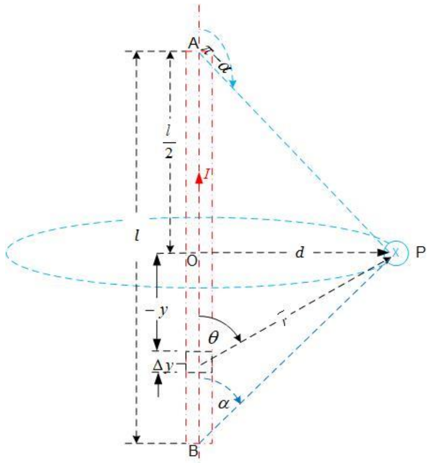
Appendix-I

Magnetic Field due to a Long Current Carrying Wire at a Point P

(Application of Biot-Savart's Law)



Given system is shown in the figure in $\hat{\imath} - \hat{\jmath}$ plane, where a wire AB of length l laid along $\hat{\jmath}$ is carrying a current I from B towards A. It is required to find magnetic field at a point P at a distance d from midpoint O of the wire.

Consider a small element of wire of length $\Delta y \to 0$ at a distance $-y$ from its midpoint. As per Biot-Savart's Law magnetic field at point P due to current I in the element of wire $d\vec{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\vec{y}\times\hat{r}}{r^2}\right)$.It leads to $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{1}{r^2}\sin\theta\, dy\hat{k} \Rightarrow dB = \frac{\mu_0 I}{4\pi}\frac{\sin\theta}{r^2}dy$, in direction entering the plane of the figure as shown therein.

Therefore, magnitude of magnetic field at B due to wire AB is $B = \frac{\mu_0 I}{4\pi}\int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{\sin\theta}{d^2+y^2}dy$. Trigonometrically, $y = d\cot\theta \Rightarrow dy = -d\ \csc^2\theta\, d\theta$. Likewise, limits would change to $-\frac{l}{2} \to \theta$ and $\frac{l}{2} \to (\pi - \theta)$. Using this substitution $B = \frac{\mu_0 I}{4\pi}\int_{\theta}^{\pi-\theta}\frac{\sin\theta}{d^2(1+\cot^2\theta)}(-d\ \csc^2\theta\, d\theta)$. Again using trigonometry $(1 + \cot^2\theta = \csc^2\theta)$, we have $B = \frac{\mu_0 I}{4\pi}\left[\int\frac{\sin\theta}{d^2(\csc^2\theta)}(-d\ \csc^2\theta\, d\theta)\right]_{\theta}^{\pi-\theta}$. This expression solves into $B =$

$\frac{\mu_0 I}{4\pi d}\left[\int \sin\theta\, d\theta\right]_{\theta}^{\pi-\theta} \Rightarrow B = -\frac{\mu_0 I}{4\pi d}[\cos\theta]_{\theta}^{\pi-\theta} \Rightarrow B = \frac{\mu_0 I}{4\pi d}[\cos\theta - \cos(\pi-\theta)] \Rightarrow B = \frac{\mu_0 I}{2\pi d}\cos\theta$…(I) Here, $\cos\theta = \frac{\frac{l}{2}}{r} \Rightarrow \cos\theta = \frac{\frac{l}{2}}{\sqrt{d^2+\left(\frac{l}{2}\right)^2}} \Rightarrow \cos\theta = \frac{l}{\sqrt{4d^2+l^2}}$.

The generic expression of magnetic field (I), can be resolved in two specific cases as under -

Case 1: When $d \gg l$. From the figure $\cos\theta \to \frac{l}{2d}$, accordingly $B = \frac{\mu_0 I}{2\pi d} \times \frac{l}{2d} \Rightarrow B = \frac{\mu_0 Il}{4\pi d^2} \Rightarrow \boldsymbol{B \propto \frac{1}{d^2}}$ **proved.**

Case 2: When $d \ll l$. From the figure $\cos\theta \to 1$, accordingly $B = \frac{\mu_0 I}{2\pi d} \Rightarrow \boldsymbol{B \propto \frac{1}{d}}$ **proved.**

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Important Note: You may encounter need of clarification on contents and analysis or an inadvertent typographical error. We would gratefully welcome your prompt feedback on mail ID: subhashjoshi2107@gmail.com. If not inconvenient, please identify yourself to help us reciprocate you suitably.

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