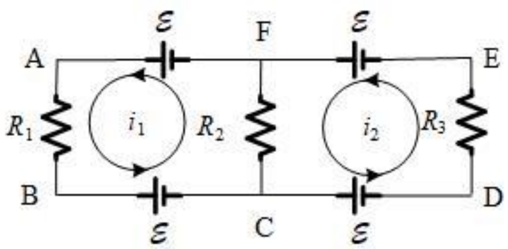
# Electromagnetism: Magnetic Effect of Electric Current

## Typical Questions (Set-2)

**I-01**

Given that emf of each of the battery is $E = 5$ V and resistances are marked in the circuit.



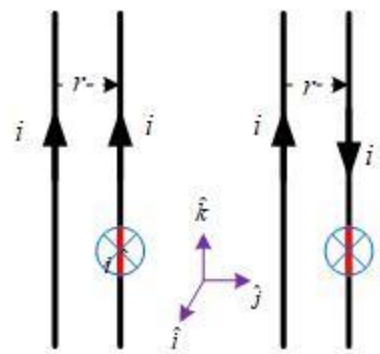

Applying Kirchhoff's Current law in loop ABCF, the emfs in braches Fa and BC are opposed to each other. Hence, current $i_1 = \frac{E-E}{R_1+R_2} = \frac{0}{R_1+R_2} \Rightarrow i_1 = 0\ldots.(1)$. This is independent of values of resistances.

Likewise, in loop FCDE, $i_2 = \frac{E-E}{R_2+R_3} = \frac{0}{R_2+R_3} \Rightarrow i_2 = 0\ldots(2)$.

Thus current in branch CF $i = i_1 - i_3$. Combining (1) and (2) current $i = 0$. Thus in entire circuit current is zero and hence there will be no magnetic field at any point either within or outside the circuit. **Hence proved**.

**I-02**

The problem states that current in both the conductors are equal, but is silent whether they are in the same direction or in the opposite directions. Taking current in upward direction $\vec{\imath} = i\hat{\jmath}$ therefore current in the opposite direction would be $\vec{\imath'} = i(-\hat{\jmath})$.




Magnetic field at appoint at distance $\vec{r} = r\hat{\jmath}$ from a long wire carrying current $i$ applying Biot-Savart's Law is $\vec{B} = \frac{\mu_0 i}{2\pi r}(-\hat{\imath})$ …(1), as shown in the figure. Three-dimensional unit vectors are also shown in the figure for convenience.

Lorentz's Force Law force stipulates force on a charge in presence of electric field as $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)\ldots(2)$. In the instant case there no electric field $\left(\vec{E} = 0\right)$, the entire effect of force is due to current. Accordingly (2) is modified as $\vec{F} = q\left(0 + \vec{v} \times \vec{B}\right) \Rightarrow \vec{F} = q\vec{v} \times \vec{B}\ldots(3)$.

Current is rate of flow of charge $q = \lambda l\ldots(4)$. Here, λ is charge per unit length and $l$ is length of the conductor. As per the Principle of Electrical an electrical system charge on conductor is being continuously replenished such that $\frac{\overrightarrow{dQ}}{dt} = q\vec{v} = (\lambda l)\vec{v} \Rightarrow q\vec{v} = l(\lambda\vec{v}) \Rightarrow q\vec{v} = l\vec{\imath}\ldots(5)$.

Figure shows both the cases when currents in both conductors is in the same direction and opposite. Accordingly, combing (3) and (5) with the stipulation of current in the beginning –

(a) **Currents in the same Direction:** Force on them would be $\vec{F} = l\vec{\imath} \times \vec{B} \Rightarrow \vec{F} = l\left(i\hat{k}\right) \times B(-\hat{\imath}) \Rightarrow \vec{F} = -Bil\hat{k} \times \hat{\imath}$. The vector product leads to $\vec{F} = Bil(-\hat{\jmath})$. The (-)ve sign depicts force is in direction opposite to the vector $\vec{r}$ i.e. attractive.

(b) **Currents in the same Direction:** Force on them would be $\vec{F} = l\vec{\imath'} \times \vec{B} \Rightarrow \vec{F} = l\left(i\left(-\hat{k}\right)\right) \times B(-\hat{\imath}) \Rightarrow \vec{F} = Bil\hat{k} \times \hat{\imath}$. The vector product leads to $\vec{F} = Bil\hat{\jmath}$. The (+)ve sign depicts force is in direction of the vector $\vec{r}$ i.e. repulsive.

Despite direction of the force in two cases discussed above magnitude of the force is $F = Bil\ldots(6)$. Thus magnitude of the force combining (1) and (6) is $F = \frac{\mu_0 i}{2\pi r} il \Rightarrow F = \frac{\mu_0 i^2 l}{2\pi r}$. Since, absolute permeability $\mu_0 = 4\pi \times 10^{-7}$, it leads to $F = \frac{\left(4\pi\times10^{-7}\right)\times i^2 l}{2\pi r} \Rightarrow F = \frac{\left(2\times10^{-7}\right)\times i^2 l}{r}\ldots(7)$

Using the given data in (7), $2.0 \times 10^{-5} = \frac{(2\times10^{-7})\times20^2\times0.1}{r} \Rightarrow r = 0.4$m or **40 cm is the answer.**

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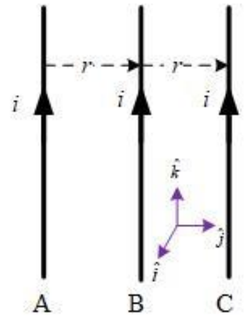
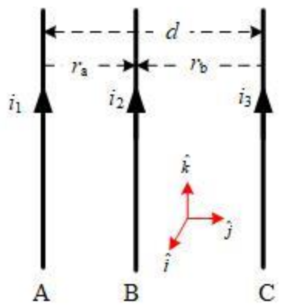
| I-03 | A per Biot-Savart's Magnetic field at point placed at a distance $d$ from a long conductor carrying current $i$ is $\vec{B}=\frac{\mu_0 i}{2\pi d}\hat{n}$ …(1). Here $\hat{n}$ is a unit vector perpendicular plane containing direction vector of $\hat{r}$ and $\hat{d}$. For the placement of current carrying conductors as shown in figure current in conductor A will produce magnetic field at B is $\vec{B}_{\mathrm{BA}}=\frac{\mu_0 i}{2\pi r}(-\hat{\imath})$…(2) and at C is $\vec{B}_{\mathrm{CA}}=\frac{\mu_0 i}{2\pi(2r)}(-\hat{\imath}) \Rightarrow \vec{B}_{\mathrm{CA}}=\frac{\mu_0 i}{4\pi r}(-\hat{\imath})$…(2).<br><br><br>Likewise., magnetic at B due to current in conductor at C is $\vec{B}_{\mathrm{BC}}=\frac{\mu_0 i}{2\pi r}(\hat{\imath})$…(3) and at C is $\vec{B}_{\mathrm{AC}}=\frac{\mu_0 i}{2\pi(2r)}(\hat{\imath}) \Rightarrow \vec{B}_{\mathrm{CA}}=\frac{\mu_0 i}{4\pi r}(\hat{\imath})$…(4).<br>Similarly, magnetic field A due to current in conductor B is $\vec{B}_{\mathrm{AB}}=\frac{\mu_0 i}{2\pi r}(\hat{\imath})$…(5) and at C is $\vec{B}_{\mathrm{CB}}=\frac{\mu_0 i}{2\pi(2r)}(-\hat{\imath}) \Rightarrow \vec{B}_{\mathrm{CA}}=\frac{\mu_0 i}{4\pi r}(-\hat{\imath})$…(6).<br>Thus, net magnetic field at B is $\vec{B}_B=\vec{B}_{\mathrm{BA}}+\vec{B}_{\mathrm{BC}}=\frac{\mu_0 i}{2\pi r}(-\hat{\imath})+\frac{\mu_0 i}{2\pi r}(\hat{\imath}) \Rightarrow \vec{B}_B=\frac{\mu_0 i}{2\pi r}(\hat{\imath}-\hat{\imath})=0$…(7).<br>While, net magnetic field at A is $\vec{B}_{\mathrm{A}}=\vec{B}_{\mathrm{AB}}+\vec{B}_{\mathrm{AC}}=\frac{\mu_0 i}{4\pi r}(\hat{\imath})+\frac{\mu_0 i}{2\pi r}(\hat{\imath}) \Rightarrow \vec{B}_{\mathrm{A}}=\frac{3\mu_0 i}{4\pi r}(\hat{\imath})$ …(8). And, net magnetic field at C is $\vec{B}_{\mathrm{C}}=\vec{B}_{\mathrm{CB}}+\vec{B}_{\mathrm{CA}}=\frac{\mu_0 i}{2\pi r}(-\hat{\imath})+\frac{\mu_0 i}{4\pi r}(-\hat{\imath}) \Rightarrow \vec{B}_{\mathrm{C}}=\frac{3\mu_0 i}{4\pi r}(\hat{\imath})$ …(9).<br>Force per unit length, experienced by each of the conductor as per Lorentz's Force Law is $\vec{F}=\vec{\imath}\times\vec{B}$ …(10). Here, $\vec{\imath}$ is the current in the conductor experiencing the force when placed in magnetic field $\vec{B}$.<br>Using (10) and magnetic field at $B_{\mathrm{B}}$ at B , $B_{\mathrm{A}}$ at A, and $B_{\mathrm{C}}$ at C as per (7), (8) and (9) respectively, while current in each of the conductor is given to be $i\hat{k}$ –<br>Force $F_{\mathrm{B}}$ experienced by conductor at B is Zero, since one of the multiplicand $B_{\mathrm{B}}=0.$, is one of the answer..<br>While, the force $F_{\mathrm{A}}$ experienced by conductor at A is $\vec{F}_A=(i\hat{k})\times\left(\frac{3\mu_0 i}{4\pi r}(\hat{\imath})\right) \Rightarrow$ s $\vec{F}_A=\frac{3\mu_0 i^2}{4\pi r}(\hat{k}\times\hat{\imath})$. It leads $\vec{F}_A=\frac{3\mu_0 i^2}{4\pi r}\hat{\jmath}$ …(11), i.e. attractive towards other two conductors.<br>Likewise, force $F_{\mathrm{C}}$ experienced by conductor at C is $\vec{F}_{\mathrm{C}}=(i\hat{k})\times\left(\frac{3\mu_0 i}{4\pi r}(-\hat{\imath})\right) \Rightarrow$ s $\vec{F}_A=-\frac{3\mu_0 i^2}{4\pi r}(\hat{k}\times\hat{\imath})$. It leads $\vec{F}_A=\frac{3\mu_0 i^2}{4\pi r}(-\hat{\jmath})$ …(12), i.e. attractive towards other two conductors.<br>Accordingly, using (11) and (12) magnitude of attractive force per unit length experienced by conductors A and C, with the available data $F_{\mathrm{A}}=F_{\mathrm{C}}=F=\frac{3\times(4\pi\times10^{-7})\times10^2}{4\pi\times0.05} \Rightarrow F=\frac{3}{5}\times10^{-3} \Rightarrow F=6.0\times10^{-4}$N is the answer.<br>**Thus, Force experienced by middle conductor is Zero, while forces experienced by extreme conductors are $\mathbf{6.0\times10^{-4}}$N and attractive is the answer.** |
| I-04 | Combining Biot-Savart's Law and Lorentz's Force law force experienced by conductor carrying current $i_{\mathrm{p}}$ when placed in magnetic field produced by another parallel conductor carrying current $i_q$ is $\vec{F}_P=\vec{\imath}_p\times\vec{B}_q$…(1).<br><br><br>It is required to find position of a third conductor w.r.t. two parallel conductors A and B carrying currents $i_1=10$ A and $i_3=40$ A, respectively, along $\hat{\jmath}$. The two conductors are separated by $d=r_a+r_b=0.10$ m…(2)<br>In respect of position of third conductor experiencing Zero Force, it is essential that direction of magnetic field produced two conductors A and C, carrying unequal currents, but in same direction $(\hat{k})$, as shown in the figure. This is possible when position vectors $\vec{r}_a$ and $\vec{r}_b$ are in opposite direc them as shown in the figure. Thus, $B_B=0$…(3) is possible |

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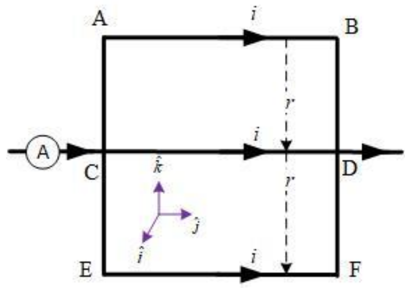
| | only when conductor B is placed parallel to conductors A and C and in-between the two, as shown in the figure. Thus, $\vec{r}_a = r_a\hat{\jmath}$ and $\vec{r}_b = r_b(-\hat{\jmath})$.<br>Magnetic field at B due to current in A, as per Biot-Savart's Law is $\vec{B}_{\mathrm{BA}} = \frac{\mu_0 i_1}{2\pi r_a}(-\hat{\imath})$…(4). Likewise, magnetic field at B due current in C is $\vec{B}_{\mathrm{BC}} = \frac{\mu_0 i_3}{2\pi r_b}(\hat{\imath})$…(5).<br>Combining (3) and (4), net magnetic field at B is $\vec{B}_{\mathrm{B}} = \vec{B}_{\mathrm{BA}} + \vec{B}_{\mathrm{BC}} \Rightarrow \vec{B}_{\mathrm{B}} = \frac{\mu_0 i_1}{2\pi r_a}(-\hat{\imath}) + \frac{\mu_0 i_3}{2\pi r_b}(\hat{\imath})$…(6).<br>Combining (3) and (6), $\frac{\mu_0 i_1}{2\pi r_a}(-\hat{\imath}) + \frac{\mu_0 i_3}{2\pi r_b}(\hat{\imath}) = 0 \Rightarrow \frac{\mu_0 i_1}{2\pi r_a} = \frac{\mu_0 i_3}{2\pi r_b} \Rightarrow \frac{i_1}{r_a} = \frac{i_3}{r_b} \Rightarrow \frac{i_1}{i_3} = \frac{r_a}{r_b}$. Applying invertendo-componendo, $\frac{i_1}{i_3+i_1} = \frac{r_a}{r_2+r_b} \Rightarrow r_a = (r_2 + r_b)\left(\frac{i_1}{i_3+i_1}\right)$. Using the available data, $r_a = 0.1 \times \left(\frac{10}{40+10}\right) = 0.02$ m or **2 cm from conductor A carrying current 10 A, is the answer.** |
| I-05 | <br>In the given system wires ACE and BDF have negligible resistances. Therefore, voltage drop across them are zero Thus nodes A,C and E are at same potential. Likewise, potentials of points B,D and F are at same potentials.<br>Further it is given that wires AB, CD and DE are long wires having identical resistances and thus form parallel circuit of long wires of equal resistances. Current through these wires is equal and it is $i$. Moreover separation between adjacent wires as shown in the figure is $r = 1.0 \times 10^{-2}$ m. As per Kirchhoff's law at node C current measured by ammeter 30 A = $3i \Rightarrow i = 10$ A. Magnetic field produced at apoint at a distance $d$ by a long current carrying wires as per Biot-Savart's Law is $\vec{B} = \frac{\mu_0 i}{2\pi d}\hat{n}$ …(1). Here, $\hat{n}$ is unit direction vector along $\hat{l} \times \hat{d}$ where $\hat{l}$ is the direction vector along which current is flowing and $\hat{d}$ is the direction vector of the point,w.r.t. current, where magnetic field is being considered. Further, as per Lorentz's Force Law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$…(2). In this case there is no static electric field i.e. $\vec{E} = 0$ therefore,(2) takes a form $\vec{F} = q\vec{v} \times \vec{B}$…(3). Here $q = \lambda l$ where is charge on conductor at any instant which is flowing in the conductor with a unform velocity $\vec{v}$, $\lambda$ is linear charge density on the wire of length $l$. Thus, (3) get further moderated to $\vec{F} = (\lambda l)(v\hat{\jmath}) \times \vec{B} \Rightarrow \vec{F} = (\lambda v)(l\hat{\jmath}) \times \vec{B} \Rightarrow \vec{F} = il\hat{\jmath} \times \vec{B}$…(4) Here, $\hat{\jmath}$ is unit vector along the length of the wire.<br>Thus, combining (1) and (4), force experienced by AB will have additive effect of currents in CD and EF in the same direction except that separation in case of CD is $r$ while for EF is $2r$. Thus net force on AB will be $\vec{F}_{AB} = \left(\frac{\mu_0 i \times i}{2\times r} + \frac{\mu_0 i \times i}{2\times 2r}\right)(-\hat{k}) \Rightarrow \vec{F}_{AB} = \frac{3\mu_0 i^2}{4r}(-\hat{k})$. Using the data, $\vec{F}_{AB} = \frac{3\mu_0 i^2}{4r}(-\hat{k}) = \frac{3(4\pi\times10^{-7})\times10^2}{4\times(1.0\times10^{-2})}$. This leads to a $\vec{F}_{AB} = 3\times10^{-3}(-\hat{k})$ N/m or a **downward force $3\times10^{-3}$ N/m is answer of one part.**<br>Likewise, force experienced by wire CD magnetic field produced by wire AB and EF equal in magnitude but in opposite direction to displacement vectors along $(\hat{\imath})$ and $(-\hat{\imath})$ which cancel each other. Thus, in absence of current carrying wire CD placed in Zero magnetic field force experienced by it will be **zero, is answer of second part.**<br>**Thus, answers are $3\times10^{-3}$ N/m downward force on wire AB and Zero force on wire CD.** |

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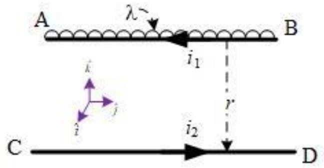
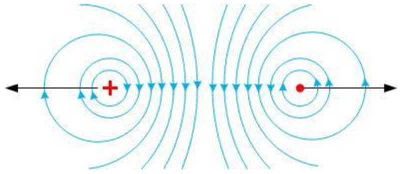
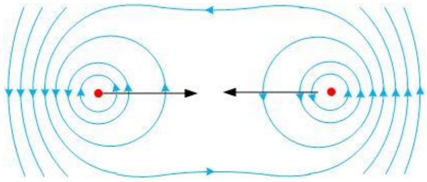
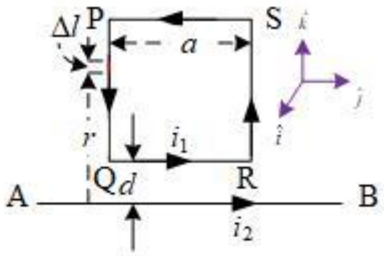
| I-06 | Given system of long wires is shown in the figure. The conductor CD carrying a current $i_2 = 50.0$ A is fixed. Another conductor AB having linear mass density $\lambda = 1.0 \times 10^{-4}$ kg-m$^{-1}$ is held directly above CD at a separation $r = 5.00 \times 10^{-3}$ m.   <br><br> It is required to find current in AB which by virtue of electromagnetic force balances its weight.  We know that force between two wires carrying current in same direction experience attractive electro-magnetic force. But, when currents are in opposite directions the force is repulsive. In the instant system weight of wire AB is $\vec{F}_g = \lambda\vec{g} \Rightarrow \vec{F}_g = \lambda g(-\hat{k})$…(1),  here acceleration due to gravity is taken to be $g = 10$ m/s$^{-2}$. <br><br> We know that force of repulsion between two wires carrying current in opposite directions, using Biot-Savart's Law and Lorentz Force Law, is $\vec{F} = \frac{\mu_0 i_1 i_2}{2\pi r}(\hat{k})$ N/m…(2). <br><br> Thus, combining (1) and (2) for equilibrium of forces on wire AB we have $\vec{F} + \vec{F}_g = 0$. It resolves into $\frac{\mu_0 i_1 i_2}{2\pi r}(\hat{k}) + \lambda g(-\hat{k}) = 0 \Rightarrow \frac{\mu_0 i_1 i_2}{2\pi r} = \lambda g \Rightarrow i_1 = \frac{2\pi r \lambda g}{\mu_0 i_2}$. Using the data $i_1 = \frac{2\pi \times (5.0\times10^{-3}) \times (1.0\times10^{-4}) \times 10}{(4\pi\times10^{-7})\times 50.0}$. It solves into $i_1 = \mathbf{0.50}$ **Amp in opposite direction is the answer.** <br><br> **N.B.:** The problem does not state value of $g$ and it has been taken as 10 m/s$^2$. Accordingly, numerical value of the answer would depend upon value of $g$along with the principle of Significant Digits. |
| I-07 | Magnetic field produced at apoint at a distance $d$ by a long current carrying wires as per Biot-Savart's Law is $\vec{B} = \frac{\mu_0 i}{2\pi d}\hat{n}$ …(1). Here, $\hat{n}$ is unit direction vector along $\hat{l} \times \hat{d}$ where $\hat{l}$ is the direction vector along which current is flowing and $\hat{d}$ is the direction vector of the point,w.r.t. current, where magnetic field is being considered. Further, as per Lorentz's Force Law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$…(2). In this case there is no static electric field i.e. $\vec{E} = 0$ therefore,(2) takes a form $\vec{F} = q\vec{v} \times \vec{B}$…(3). Here $q = \lambda l$ where is charge on conductor at any instant which is flowing in the conductor with a unform velocity $\vec{v}$, $\lambda$ is linear charge density on the wire of length $\Delta l$. Thus, (3) get further moderated to $\vec{F} = (\lambda l)(v\hat{j}) \times \vec{B} \Rightarrow \vec{F} = (\lambda v)(l\hat{j}) \times \vec{B} \Rightarrow \vec{F} = il\hat{j} \times \vec{B}$…(4) Here, $\hat{j}$ is unit vector along the length of the wire.   <br><br> The square wire loop consisists of broadly two parts – <br><br> **Part (a):** Sides PQ and RQ of length $a = 0.02$ m carrying current along length $(\hat{\imath})$ and $(-\hat{\imath})$. These currents are perpendicular to the magnetic field $B = \frac{\mu_0 i_2}{2\pi r}\hat{k}$ …(5) produced by wire AB carrying current $i_2(\hat{j})$, here $r$ is the distance of the portion of length $\Delta r\hat{\imath}$ of wiree PQ. This is in accordance with (1). This field when interacts with current $i_1$in portion $\Delta r\hat{\imath}$ produces a force as per (4), with appropriate variables and direction vectors in (5), is $\Delta\vec{F} = i_1\Delta r\hat{\imath} \times \vec{B}$. It further solves into $\Delta\vec{F} = i_1\Delta r\hat{\imath} \times \left(\frac{\mu_0 i_2}{2\pi r}\hat{k}\right) \Rightarrow \Delta\vec{F} = \frac{\mu_0 i_1 i_2}{2\pi r}\Delta r(-\hat{j})$…(6). Thus net force on side PQ is $\vec{F}_{\text{PQ}} = \left(\int_{a+d}^{d} \frac{\mu_0 i_1 i_2}{2\pi r} dr\right)(-\hat{j}) \Rightarrow \vec{F}_{\text{PQ}} = \left[\frac{\mu_0 i_1 i_2}{2\pi r^2}\right]_{a+d}^{d}(-\hat{j}) \Rightarrow \vec{F}_{\text{PQ}} = \left[\frac{\mu_0 i_1 i_2}{2\pi d^2} - \frac{\mu_0 i_1 i_2}{2\pi(a+d)^2}\right](-\hat{j})$. It solves to $\vec{F}_{\text{PQ}} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{1}{d^2} - \frac{1}{(a+d)^2}\right](-\hat{j}) \Rightarrow \vec{F}_{\text{PQ}} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{(a+d)^2 - d^2}{d^2(a+d)^2}\right](-\hat{j}) \Rightarrow \vec{F}_{\text{PQ}} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{(a+2d)a}{d^2(a+d)^2}\right](-\hat{j})$…(7). Where as force on side RS is $\vec{F}_{\text{RS}} = \left(\int_{d}^{a+d} \frac{\mu_0 i_1 i_2}{2\pi r} dr\right)(-\hat{j}) \Rightarrow \vec{F}_{\text{RS}} = \left[\frac{\mu_0 i_1 i_2}{2\pi r^2}\right]_{d}^{a+d}(-\hat{j}) \Rightarrow \vec{F}_{\text{RS}} = \left[\frac{\mu_0 i_1 i_2}{2\pi(a+d)^2} - \frac{\mu_0 i_1 i_2}{2\pi d^2}\right](-\hat{j}) \Rightarrow \vec{F}_{\text{RS}} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{d^2 - (a+d)^2}{a^2(a+d)^2}\right](-\hat{j})$. It solves into $\vec{F}_{\text{RS}} = (-)\frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{(a+2d)a}{a^2(a+d)^2}\right](-\hat{j}) \Rightarrow \vec{F}_{\text{RS}} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{(a+2d)a}{a^2(a+d)^2}\right](\hat{j})$…(8). From (7) and (8) **it is proved the forces on sides PQ and RQ are equal in magnitude and opposite in direction,** |

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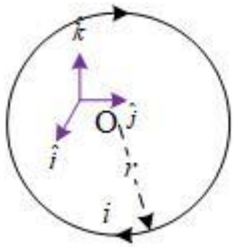
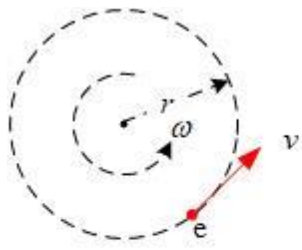
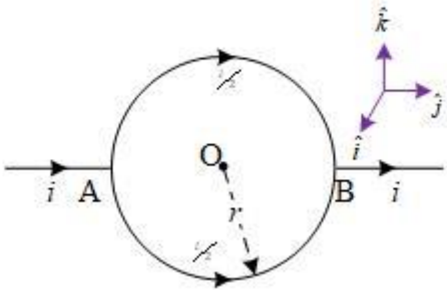
| | **Part (b):** We know that force of on repulsion between two parallel wires carrying current in, using Biot-Savart's Law and Lorentz Force Law, as per (6) would be $\vec{F} = i_1\vec{l} \times \left(\frac{\mu_0 i_2}{2\pi r}\hat{k}\right) \Rightarrow \vec{F} = \frac{\mu_0 i_1 i_2}{2\pi r}\left(\vec{l} \times \hat{k}\right)$…(9). Here, $\vec{l}$ is the vector length of wire along current $i_1$. Thus, for wire QR, $\vec{l}_{\text{QR}} = a\hat{j}$ and for wire SP, $\vec{l}_{\text{SP}} = a(-\hat{j})$.Further, the wire QR is at a distance $r_{\text{QR}} = d$ and $r_{\text{SP}} = a + d$. Accordingly, using (9) force on wire QR is $\vec{F}_{\text{QR}} = \frac{\mu_0 i_1 i_2}{2\pi r_{\text{QR}}}\left(\vec{l}_{\text{QR}} \times \hat{k}\right) \Rightarrow \vec{F}_{\text{QR}} = \frac{\mu_0 i_1 i_2}{2\pi d}\left(a(\hat{j}) \times \hat{k}\right) \Rightarrow \vec{F}_{\text{QR}} = \frac{\mu_0 i_1 i_2 a}{2\pi d}(\hat{\imath})$…(10). Likewise, force on wire SP is $\vec{F}_{\text{SP}} = \frac{\mu_0 i_1 i_2}{2\pi r_{\text{QR}}}\left(\vec{l}_{\text{SP}} \times \hat{k}\right) \Rightarrow \vec{F}_{\text{SP}} = \frac{\mu_0 i_1 i_2}{2\pi(a+d)}\left(a(-\hat{j}) \times \hat{k}\right) \Rightarrow \vec{F}_{\text{SP}} = \frac{\mu_0 i_1 i_2 a}{2\pi(a+d)}(-\hat{\imath})$…(11). Repulsive nature of force on the two sides is evident from (1) and (11), yet magnitudes of forces on the two sides are unequal.<br><br>Thus, net force on the loop PQRS is $\vec{F} = \left(\vec{F}_{\text{PQ}} + \vec{F}_{\text{QR}}\right) + \left(\vec{F}_{\text{RS}} + \vec{F}_{\text{SP}}\right)$. Using, derivation in part (a) and results in (10) and (11) we have $\vec{F} = \vec{F}_{\text{RS}} + \vec{F}_{\text{SP}} = \frac{\mu_0 i_1 i_2 a}{2\pi d}(\hat{\imath}) + \frac{\mu_0 i_1 i_2 a}{2\pi(a+d)}(-\hat{\imath}) \Rightarrow \vec{F} = \frac{\mu_0 i_1 i_2 a}{2\pi}\left(\frac{1}{d} - \frac{1}{a+d}\right)(\hat{\imath}) \Rightarrow \vec{F} = \frac{\mu_0 i_1 i_2 a}{2\pi} \times \frac{a}{d(a+d)}(\hat{\imath}) \Rightarrow \vec{F} = \frac{\mu_0 i_1 i_2 a^2}{2\pi d(a+d)}(\hat{\imath})$. Using the available data, $\vec{F} = \frac{(4\pi\times10^{-7})\times6\times10\times(2\times10^{-2})^2}{2\pi\times(1\times10^{-2})\times((2+1)\times10^{-2})}(\hat{\imath}) \Rightarrow \vec{F} = 160 \times 10^{-7}(\hat{\imath}) \Rightarrow \vec{F} = 1.6 \times 10^{-5}(\hat{\imath})$ N. Thus ,force on the loop is $1.6 \times 10^{-3}$ N towards wire AB.<br><br>Thus, **answers are (a) Proved, (b) $\mathbf{1.6 \times 10^{-5}}$ N towards wire AB** |
| I-08 | As per Biot-Savart's Law magnetic field at a point situated at a distance $r$ from a wire of length $\Delta\vec{l} = \Delta l\hat{l}$ carrying current $i$ along $\hat{l}$ as shown in the figure is $\Delta\vec{B} = \frac{\mu_0 i}{4\pi r^2}\Delta\vec{l} \times \hat{r} \Rightarrow \Delta\vec{B} = \frac{\mu_0 i}{4\pi r^2}\Delta l\hat{l} \times \hat{r} \Rightarrow \Delta\vec{B} = \frac{\mu_0 i\Delta l}{4\pi r^2}\left(-\hat{k}\right)$…(1). The elemental length $\Delta l$ is part of a circle of radius $r$ subtends an angle $\Delta\theta$ at the circle of the circle such that $\Delta l = r \times \Delta\theta$…(2). Combining (1) and (2), $\Delta\vec{B} = \frac{\mu_0 ir\Delta\theta}{4\pi r^2}\left(-\hat{k}\right)$. Thus, magnitude of the electric field is $\Delta B = \frac{\mu_0 i\Delta\theta}{4\pi r}$…(3).<br><br><br><br>Thus magnetic field at the center of the circular loop is $B = \int_0^{2\pi}\frac{\mu_0 i}{4\pi r}d\theta \Rightarrow B = \frac{\mu_0 i}{4\pi r}[\theta]_0^{2\pi} \Rightarrow B = \frac{\mu_0 i}{4\pi r} \times 2\pi$. It leads to $B = \frac{\mu_0 i}{2r}$…(4).<br><br>Using the available data in (4) $2.000 \times 10^{-3} = \frac{(4\pi\times10^{-7})\times5.00}{2r} \Rightarrow r = \frac{(4\pi\times10^{-7})\times5.00}{2\times(2.000\times10^{-3})} \Rightarrow r = 5.00 \times \pi \times 10^{-4} \Rightarrow r = 1.57 \times 10^{-2}$ m or **1.57 cm is the answer.** |
| I-09 | Magnetic field $B = 6.0 \times 10^{-5}$ T due to a current $i$ in a circular loop of radius $r = 5.0 \times 10^{-2}$ m at its center is $B = \frac{\mu_0 i}{2r}$. If it is a coil comprising of $n = 100$ turns then $B_n = \frac{\mu_0 in}{2r} \Rightarrow i = \frac{2rB_n}{\mu_0 n}$. With the given data current in the loop is $i = \frac{2\times(5.0\times10^{-2})\times(6.0\times10^{-5})}{(4\pi\times10^{-7})\times100} \Rightarrow i = \frac{15\times10^{-2}}{\pi}$. It solves into $i = 4.8 \times 10^{-2}$ or **48 mA is the answer.** |
| I-10 | Charge of an electron $q_e = -1.6 \times 10^{-19}$C is revolving in circle of radius $r = 0.5$ angstrom or $r = 0.5 \times 10^{-10}$ m making $n = 3 \times 10^5$ revolutions/sec. Then instantaneous velocity of the revolving electron $v = r\omega = r \times (2\pi n) \Rightarrow v = 2\pi rn$ m/s and the electron so revolving establishes a current similar to current in a circular loop $i = q_e v$ Amp. Therefore, magnetic field at the center of the circular path of the revolving electron is $B = \frac{\mu_0 i}{2r} = \frac{(4\pi\times10^{-7})\times q_e n}{2r} \Rightarrow B = \frac{(4\pi\times10^{-7})\times(1.6\times10^{-19})\times(3\times10^5)}{2\times(0.5\times10^{-10})} \Rightarrow B = 60 \times 10^{-11}$ T or $\mathbf{6 \times 10^{-10}}$ **T is the answer**.<br><br> |

**N.B.:** In case of charges moving in a straight wire, current is $i = qv$, here $q$ is the charge per unit length of the wire and $v$ is the velocity of displacement of the charge. But, in the instant case it a single electron is performing circular motion making $n$ revolutions per second. It is, therefore, equivalent to $q_e n$ coulomb charge passing through every point in every second, and hence current established in the circular loop is $\boldsymbol{i} = \boldsymbol{q_e} \times \boldsymbol{n}$, and not that applicable in case of a straight wire as shown earlier.

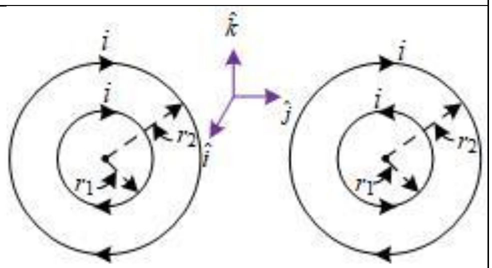
I-11 Applying Kirchhoff's Current Law at nodes incoming current $i$ will split along two identical semicircular arcs of the circle or radius $r$ each carrying current $\frac{i}{2}$ ., as shown in the figure. As per Ampere's Right Hand Thumb Rule (mathematically explained by Biot-Savart's Law) magnetic field produced by upper half of the circle at its center O $\vec{B}_U$ would be along $(-\hat{k})$. Likewise, magnetic field produced by the lower half at the center would be along $(\hat{k})$. Since, the magnetic field is produced beach of the complementary half of the circle carrying equal currents $\frac{i}{2}$ and hence magnitude as per Biot-Savart's Law would be equal and half of that produced by current in a circular loop $B_U = B_L = \frac{B}{2}$...(1)




Magnetic field produced by a circular current carrying loop, using Biot-Savart's Law is $B = \frac{\mu_0 i}{2r}$...(2). Thus combining (1), (2) and direction vectors of the fields discussed above is $\vec{B}_U = \frac{\mu_0 i}{4r}(-\hat{k})$ and $\vec{B}_L = \frac{\mu_0 i}{4r}(\hat{k})$. Thus net magnetic field at the center would be $\vec{B}_O = \vec{B}_U + \vec{B}_L \Rightarrow \vec{B}_O = \frac{\mu_0 i}{4r}(-\hat{k}) + \frac{\mu_0 i}{4r}(\hat{k}) = 0$. Thus, **Zero is the answer.**

**N.B.:** This problem unless asked as a part question or a full question would need to be elaborated accordingly. Else, it is worth an Objective question.

I-12 Magnetic field at the center of a coil in $\hat{\imath} - \hat{\jmath}$ plane current carrying in clockwise direction is along $(-\hat{k})$ and magnitude of the magnetic field is $B = \frac{\mu_0 n i}{2r}$...(1). Given are two concentric loops of radius $r_1 = 0.05$ m and $r_2 = 0.10$ m having turns $n_1 = 50$ and $r_2 = 100$ m , respectively. Each of the coil is carrying current $i = 2.0$ A in clockwise directions. Accordingly net magnetic field at the common center of the two coils is $\vec{B} = \vec{B}_1 + \vec{B}_2$. Here, as per figure

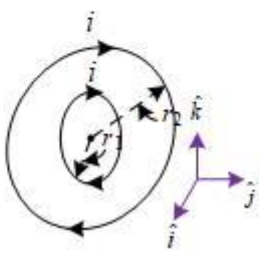



$\vec{B}_1 = \frac{\mu_0 n_1 i}{2r_1}(-\hat{k})$ and $\vec{B}_2 = \frac{\mu_0 n_2 i}{2r_2}(-\hat{k})$. Using the available data $\vec{B} = \left(\frac{\mu_0 n_1 i}{2r_1} + \frac{\mu_0 n_2 i}{2r_2}\right)(-\hat{k}) \Rightarrow \vec{B} = \left(\frac{\mu_0 i}{2}\right)\left(\frac{n_1}{r_1} + \frac{n_2}{r_2}\right)(-\hat{k}) \Rightarrow \vec{B} = \left(\frac{(4\pi\times10^{-7})\times2.0}{2}\right)\left(\frac{50}{0.05} + \frac{100}{0.10}\right)(-\hat{k}) \Rightarrow \vec{B} = 8\pi \times 10^{-4}(-\hat{k})$ T. Thus magnitude of the magnetic field is $\boldsymbol{8\pi \times 10^{-4}}$**T is answer of part (a).**

In part (b) direction of current in two coils is in opposite directions, as shown in the figure. Accordingly, $\vec{B} = \left(\frac{\mu_0 n_1 i}{2r_1}(-\hat{k}) + \frac{\mu_0 n_2 i}{2r_2}(\hat{k})\right) \Rightarrow \vec{B} = \left(\frac{\mu_0 i}{2}\right)\left(\frac{n_2}{r_2} - \frac{n_1}{r_1}\right)(\hat{k})$. Using the available data $\vec{B} = \left(\frac{\mu_0 i}{2}\right)(1000 - 1000)(\hat{k}) \Rightarrow \vec{B} = 0\left(\frac{\mu_0 i}{2}\right)\left(\frac{100}{0.10} - \frac{50}{0.05}\right)(\hat{k})$, i.e, **Zero is answer of the part (b)**

Thus. **answers are (a) $\boldsymbol{8\pi \times 10^{-4}}$ T (b) Zero.**

I-13 The two coils having radius $r_1 = 0.05$m and $r_2 = 0.10$m have turns $n_1 = 50$ $n_2 = 100$ recpectively. The axis of the inner coil having 50 turns along $\hat{\jmath}$ unit vector, while the coil having 100 turns is so rotated that its axis is along $\hat{k}$ unit vector. Cock-wise current $i = 2.0$A in outer coil produces magnetic field $\vec{B}_2 = \frac{\mu_0 n_2 i}{2r_2}(-\hat{k})$...(1), while clockwise current $i = 2.0$ A in inner coil produces magnetic field $\vec{B}_1 = \frac{\mu_0 n_1 i}{2r_1}(-\hat{\jmath})$...(2).



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Using the available data magnitudes are $B_1 = \frac{(4\pi\times10^{-7})\times50\times2.0}{2\times0.05} = (4\pi\times10^{-7})\times1000 \Rightarrow B_1 = 4\pi\times10^{-4}$T. On the similar lines the magnitude $B_2 = \frac{(4\pi\times10^{-7})\times100\times2.0}{2\times0.10} = (4\pi\times10^{-7})\times1000 \Rightarrow B_2 = 4\pi\times10^{-4}$T. It is seen that direction vectors of both the magnetic fields . of equal magnitudes $B_1 = B_2 = 4\pi\times10^{-4}$T are in perpendicular directions and hence net magnetic field is $B = (4\pi\times10^{-4})\times\sqrt{2} \Rightarrow 17.8\times10^{-4}$ T or **1.8 T is the answer**

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<td>I-14</td>
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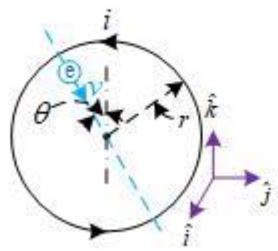
The problem involves application of Biot-Savart's Law for magnetic field produced by a circular loop in $\hat{\imath}-\hat{\jmath}$ plane of radius $r = 0.20$ m and carrying current $i = 10$ A, as shown in the figure. Accordingly, magnetic field at the center of the loop is $\vec{B} = \frac{\mu_0 i}{2r}\hat{k}$.




Next it is given that an electron having charge $q = -1.6\times10^{-19}$ C with moving a velocity $\vec{v} = 2.0\times10^{6}\hat{v}$ passes through center of the loop inclined at an angle $\theta = 30^0$ with the axis of the loop as shown in the figure. Therefore magnetic force on the electron as per Lorentz's Force law$\vec{F} = q\vec{v}\times\vec{B} \Rightarrow F = qvB\sin\theta$. Using the available data $F = (1.6\times10^{-19})\times(2.0\times10^{6})\times\frac{(4\pi\times10^{-7})\times10}{2\times0.20}\times\frac{1}{2} \Rightarrow B = \mathbf{16\pi\times10^{-19}}$**N is the answer.**

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<td>I-15</td>
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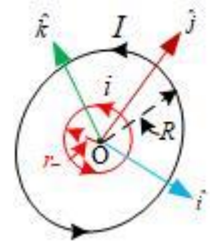
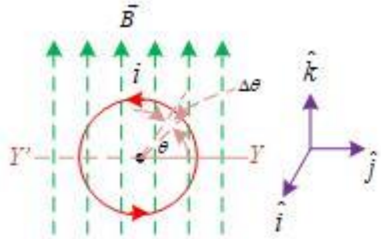
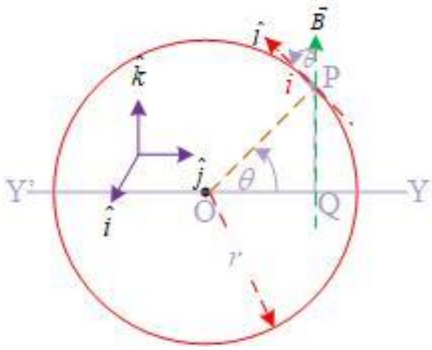
A circular loop of radius $R$ carrying a current $I$ is placed in $\hat{\imath}-\hat{\jmath}$ plane as shown in the figure. Axis of the loop is along $\hat{k}$. Applying Biot-Savart's law it will produce magnetic field $\vec{B} = \frac{\mu_0}{2R}\hat{k}$ ...(1), at the center of the loop. Further it is given that another circular loop of radius $r$ carrying current $i$ in anti-clockwise direction as seen against $\hat{\imath}$. The small loop, as shown in the figure, is in $\hat{\jmath}-\hat{k}$ plane.




Statement of the problem shown in the figure on the right side, it is observed that –

a) magnetic field at the center of the outer loop of radius $R$ is $\vec{B}$.
b) plane of the smaller loop of radius $r$ is along the magnetic field.
c) given that $r \ll R$, and geometrical symmetry of the loop, force experienced by inner coil as per will produce a torque about diameter of smaller coil. We take for convenience diameter of loop along Y-Y' i.e. $\hat{\jmath}$.




Taking forward analysis force on a small element of loop of length $\Delta\vec{l} = r\Delta\theta\hat{l}$...(2), carrying current $i$, is as per Lorentz's Force Law is $\Delta\vec{F} = i\vec{l}\times\vec{B} \Rightarrow \Delta\vec{F} = i\left(r\Delta\theta\hat{l}\,\right)\times\vec{B}$. The force as a result of cross-product is $\Delta\vec{F} = irB\sin\theta\,\Delta\theta(-\hat{\imath})$....(3).

Therefore, torque experienced by the element of loop about diameter Y-Y' would be $\Delta\vec{\Gamma} = \overrightarrow{QP}\times\Delta\vec{F} \Rightarrow \Delta\vec{\Gamma} = \left(r\sin\theta\,\hat{k}\right)\times\left(irB\sin\theta\,\Delta\theta(-\hat{\imath})\right)$....(4). This expression simplifies magnitude of the torque on small element of loop to $\Delta\Gamma = iBr^2\sin^2\theta\,\Delta\theta \Rightarrow \Delta\Gamma = \frac{iBr^2}{2}(1-\cos 2\theta)\Delta\theta$. Hence net torque on the inner loop would be $\Gamma = \int_0^{2\pi}\frac{iBr^2}{2}(1-\cos 2\theta)d\theta \Rightarrow \Gamma = \frac{iBr^2}{2}\left[\int_0^{2\pi}d\theta - \int_0^{2\pi}\cos 2\theta\;d\theta\right] \Rightarrow \Gamma = \frac{iBr^2}{2}\times 2\pi \Rightarrow \Gamma = iB\pi r^2$...(5).

Combining (1) and (5), $\Gamma = i\left(\frac{\mu_0 I}{2R}\right)\pi r^2 \Rightarrow \Gamma = \frac{\boldsymbol{\mu_0\pi i I r^2}}{\boldsymbol{2R}}$ **is the answer.**

**N.B.:** This is good example involving multiple vector operations, with the clarity of concepts. It is brought out in details, with basics, in Appendix-I.

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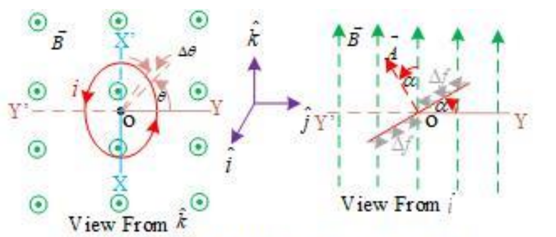
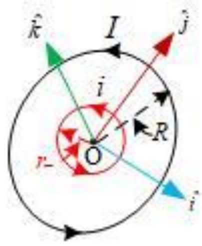
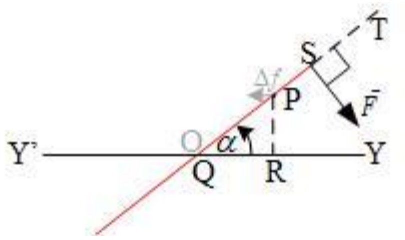
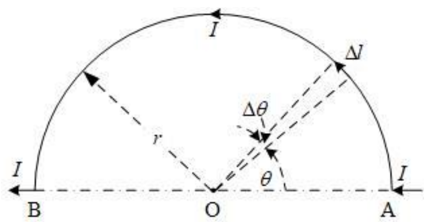
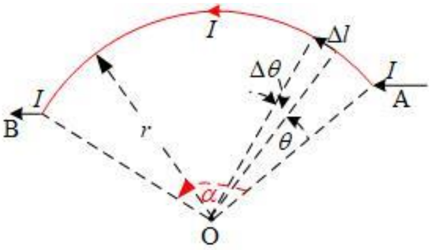
| I-16 | Given system of loop is shown in the figure. The outer loop of radius $R$ is carrying $I$ and produces magnetic field $B$ in accordance with the Biot and Savart's Law and in instant case  Conceptual drawing (Not to The scale)  when coil is along $\hat{\imath}-\hat{\jmath}$ and current in the loop in anticlockwise direction $\vec{B}=\frac{\mu_0 I}{2R}\hat{k}$…(1). Torque experienced by a loop of radius $r$ carrying current $i$ which in inclined to the plane of the out loop at an angle $\alpha=30^0$ as shown in the figure is $\vec{\Gamma}=\frac{\mu_0\pi iIr^2}{2R}\sin\alpha\,\hat{\imath}\Rightarrow\vec{\Gamma}=\frac{\mu_0\pi iIr^2}{4R}\hat{\imath}$…(2). The axis of rotation is X-X'.<br><br> This inner loop has to be held to held at an inclination $\alpha$ by application of an external single minimum force. Since, external torque $\vec{\Gamma}_e=\vec{r}\times\vec{F}\Rightarrow\vec{\Gamma}=rF\sin\emptyset(-\hat{\imath})$…(3). It implies that the direction of external torque is opposite to the torque due to internal forces in (2).<br><br>For external force $F$, applied on periphery of the loop, to be minimum as required, $r$ has to be maximum which is $r\to r$ radius of the inner loop and so also $\sin\emptyset=1$, i.e. $\emptyset=\frac{\pi}{2}$, as shown in the figure. It leads to $\vec{\Gamma}_e=\vec{\Gamma}=rF$.<br><br>Thus in state of equilibrium, $\vec{\Gamma}+\vec{\Gamma}_e=0\Rightarrow\frac{\mu_0\pi iIr^2}{4R}\hat{\imath}+rF(-\hat{\imath})=0\Rightarrow rF=\frac{\mu_0\pi iIr^2}{4R}\Rightarrow F=\boldsymbol{\frac{\mu_0\pi iI}{4R}}$ **is the answer.**<br><br>**N.B.:** This is good example involving multiple vector operations, with the clarity of concepts both in electromagnetism and mechanics. Concepts of electromagnetics are brought out in details, with basics, in Appendix-I. |
| I-17 |  Given system is shown in figure where a semicircular wire of radius $r=0.10$ m is carrying current $I=5.0$ A. It is required to determine magnetic field $B$ at the center of curvature O.<br><br>As per Biot-Savart's $\Delta\vec{B}=\frac{\mu_0 I}{4\pi r^2}\Delta\vec{l}\times\hat{r}\Rightarrow\Delta\vec{B}=\frac{\mu_0 I}{4\pi R^2}\Delta l\hat{l}\times\hat{r}$. Here, $\Delta l=r\Delta\theta$. Accordingly, $\Delta\vec{B}=\frac{\mu_0 Ir\Delta\theta}{4\pi r^2}\hat{k}\Rightarrow\Delta\vec{B}=\frac{\mu_0 I\Delta\theta}{4\pi r}\hat{k}$…(1). It is to be noted that in deriving this expression we encounter $\Delta\vec{l}\times\hat{r}$ and wires involving current entering the semicircular wire at A and leaving at B both the constituent vectors are collinear and hence $\Delta\vec{l}\times\hat{r}=\Delta lr\sin\theta\ \hat{n}=0$ for angle between collinear vectors $\theta=0\Rightarrow\sin 0=0$. Thus net magnetic field at the center O is obtained by integration on of (1) $B=\int_0^\pi\frac{\mu_0 Id\theta}{4\pi r}\Rightarrow B=\frac{\mu_0 I}{4\pi r}\int_0^\pi d\theta$. It solves into $B=\frac{\mu_0 I}{4\pi r}\times\pi\Rightarrow B=\frac{\mu_0 I}{4r}$…(2).<br><br>Using the given data in (2), $B=\frac{(4\pi\times10^{-7})\times5.0}{4\times0.10}=5\pi\times10^{-6}\Rightarrow B=1.6\times\times10^{-5}$ T, is the answer.<br><br>**N.B.:** This problem being full length question has been solved analytically. Otherwise it can be solved like an objective problem using formula of magnetic field prroduced by a circular current carrying loop $B=\frac{\mu_0 I}{2r}$. In this problem the wire is in semicircular shape and hence maggnetic field would be $B'=\frac{B}{2}=\frac{\mu_0 I}{4r}$, and using it directly as it is same as that in (2) |
| I-18 | Given system is shown in figure where a a wire shaped in arc of a circle forming angle $\alpha=120^0=\frac{2\pi}{3}$ rad of radius $r=0.200$ m is carrying current $I=6.00$ A. It is required to determine magnetic field $B$ at the center of curvature O. |

As per Biot-Savart's $\Delta\vec{B} = \frac{\mu_0 I}{4\pi r^2}\Delta\vec{l} \times \hat{r} \Rightarrow \Delta\vec{B} = \frac{\mu_0 I}{4\pi R^2}\Delta l\hat{l} \times \hat{r}$. Here, $\Delta l = r\Delta\theta$. Accordingly, $\Delta\vec{B} = \frac{\mu_0 I r\Delta\theta}{4\pi r^2}\hat{k} \Rightarrow \Delta\vec{B} = \frac{\mu_0 I\Delta\theta}{4\pi r}\hat{k}$…(1). Since, nothing is stated about either length or orientation of wires feeding and exiting the current, and hence they are ignored and analysis is limited to the given arc.




$\frac{\mu_0 I}{4\pi r} \times \pi$

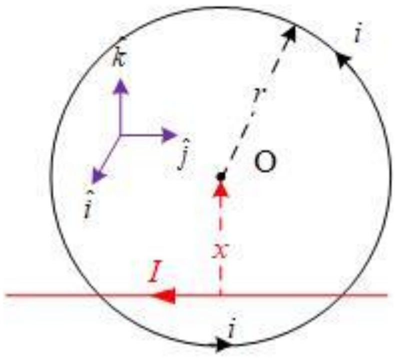
Thus net magnetic field at the center O is obtained by integration on of (1) $B = \int_0^{\frac{2\pi}{3}} \frac{\mu_0 I d\theta}{4\pi r} \Rightarrow B = \frac{\mu_0 I}{4\pi r}\int_0^{\frac{2\pi}{3}} d\theta$. It solves into $B = \frac{\mu_0 I}{4\pi r} \times \frac{2\pi}{3} \Rightarrow B = \frac{\mu_0 I}{6r}$…(2).

Using the given data in (2), $B = \frac{2\times(4\pi\times10^{-7})\times6.00}{6\times0.200} = 4.00 \times \pi \times 10^{-6} \Rightarrow B = \mathbf{1.26 \times 10^{-5}}$ **T, is the answer**.

**N.B.:** This problem being full length question has been solved analytically. Otherwise it can be solved like an objective problem using formula of magnetic field prroduced by a circular current carrying loop $B = \frac{\mu_0 I}{2r}$. In this problem the wire shaped in arc of a circle of angle $\alpha = \frac{2\pi}{3}$ and hence magnetic field would be $B' = \frac{2\pi}{3}B = \frac{\mu_0 I}{6r}$, and using it directly as it is same as that in (2).

I-19 As per Biot-Savart's Law, magnetic field at the center of loop of radius $r$ in $\hat{\imath} - \hat{\jmath}$ plane carrying current $i$ will produce magnetic field at O, center of the loop, $\vec{B}_1 = \frac{\mu_o i}{2r}\hat{k}$.

Whereas, a long wire in $\hat{\imath} - \hat{\jmath}$ plane carrying current $I$ along $\hat{\jmath}$ would produce magnetic field at a distance $x$ from it $\vec{B}_2 = \frac{\mu_o I}{x}\hat{k}$. If direction of current is revere i.e. along $(-\hat{\jmath})$ the magnetic field at same point would be $\vec{B}_3 = \frac{\mu_o I}{2\pi x}(-\hat{k})$.

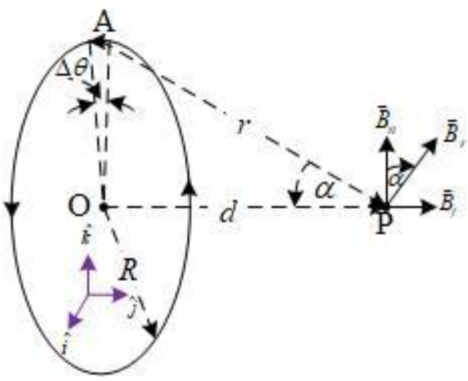



In the system as shown in the figure it is desired that magnetic field at the center of the loop is zero which with $\vec{B} = \vec{B}_1 + \vec{B}_2 \neq 0$ since both the constituent vectors are non-zero as well as unidirectional.

In the system as shown in the figure it is desired that magnetic field at the center of the loop is zero which with $\vec{B} = \vec{B}_1 + \vec{B}_2 \neq 0$ since both the constituent vectors are non-zero as well as unidirectional.

Hence, for magnetic $\vec{B} = 0$ it essential that fields at O are in opposite direction which is there when current in straight wire is along $(-\hat{\jmath})$. Thus, for $\vec{B} = 0$ condition is $\vec{B} = \vec{B}_1 + \vec{B}_3 = 0 \Rightarrow \frac{\mu_o i}{2r}\hat{k} + \frac{\mu_o i}{2\pi x}(-\hat{k}) = 0 \Rightarrow \frac{\mu_o i}{2r} = \frac{\mu_o i}{2\pi x} \Rightarrow x = \frac{rI}{i\pi}$. Using the available data $x = \frac{r(4i)}{i\pi} \Rightarrow x = \frac{4r}{\pi}$. In this case wire is placed in that half of the circle where current in minor arc is in direction opposite to the that in straight wire.

I-20 Given is a circular coil containing $n = 200$ turns of radius $R = 0.10$ m in $\hat{\imath} - \hat{k}$ plane carrying current $I = 2.0$ A. As per Biot-Savart's Law, magnetic field at point P on the axis of the loop at a distance $d$ from the center of the loop O; a distance $r$ from a small element of wire of length $\Delta\vec{l} = r\Delta\theta\hat{l}$ will be $\Delta\vec{B}_P = \frac{\mu_o nIR\Delta\theta}{4\pi r^2}\widehat{B_r}$, here $\Delta B_P = \frac{\mu_o nIR\Delta\theta}{4\pi r^2}$…(1)




This magnetic field has two components $\Delta\vec{B}_P = \Delta\vec{B}_j + \Delta\vec{B}_N \Rightarrow \Delta\vec{B}_P = \Delta B_P \cos\left(\frac{\pi}{2} - \alpha\right)\hat{\jmath} + \Delta B_P \sin\left(\frac{\pi}{2} - \alpha\right)\hat{N}$. It simplifies into $\vec{B}_P = \Delta B_P \sin\alpha\,\hat{\jmath} + \Delta B_P \cos\alpha\,\hat{N}$…(2), as shown in the figure. With

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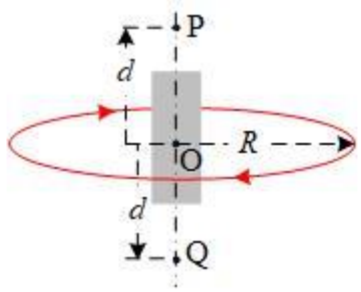
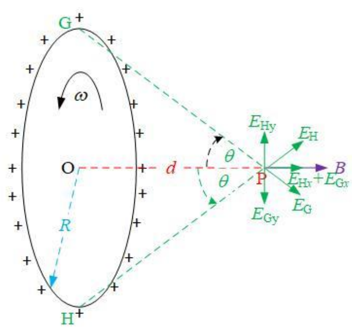
| | the symmetry of the loop about its axis O, the component of magnetic field along the loop $\hat{N}$ will cancel out leaving the component along $\hat{j}$ to be only effective.<br><br>Thus, combining (1) and (2), net magnetic field at point P, due to the loop is $B_P = \int_0^{2\pi} \frac{\mu_o nIR\Delta\theta}{4\pi r^2} \sin\alpha \Rightarrow B_P = \frac{\mu_o nIR \sin\alpha}{4\pi r^2} \int_0^{2\pi} d\theta \Rightarrow B_P = \frac{\mu_o nIR \sin\alpha}{4\pi r^2} \times 2\pi \Rightarrow B_P = \frac{\mu_o nIR \sin\alpha}{2r^2}$…(3). It is seen from the figure that $\sin\alpha = \frac{R}{r}$, therefore, $B_P = \frac{\mu_o nIR^2}{2r^3}$. Further, $r = \sqrt{d^2 + R^2}$ therefore, $B_P = \frac{\mu_o nIR^2}{2(d^2+R^2)^{\frac{3}{2}}}$ …(4).<br><br>With this generic analysis, the problem in two parts is being solved as under –<br>**Part (a):** At the center of the coil point P → O, $r \to R$ and $\alpha \to \frac{\pi}{2}$ and, therefore, using (3) $B_O = \frac{\mu_o nIR \sin\frac{\pi}{2}}{2R^2}$. It solves into $B_O = \frac{\mu_o nI}{2R}$, Using the available data, $B_O = \frac{(4\pi\times10^{-7})\times200\times2.0}{2\times0.10} = 0.8\pi \times 10^{-3} \Rightarrow B_O =$ **$2.51$ mT, is the answer.**<br><br>**Part (b):** It is required to find distance $d$ of point P at which magnetic field intensity drops to half of the value at center of the coil O, determined in part (a) i.e. $B_P = \frac{B_O}{2}$. Thus, using (4) with the given data, $\frac{\mu_o nIR^2}{2(d^2+R^2)^{\frac{3}{2}}} = \frac{1}{2} \times \frac{\mu_o nI}{2R} \Rightarrow 2R^3 = (d^2 + R^2)^{\frac{3}{2}} \Rightarrow 4 = \left(\frac{d^2+R^2}{R^2}\right)^3 \Rightarrow \left(\frac{d}{R}\right)^2 + 1 = \sqrt[3]{4} \Rightarrow \left(\frac{d}{R}\right)^2 + 1 = 1.59 \Rightarrow \left(\frac{d}{R}\right)^2 = 0.59$. It leads to $\frac{d}{R} = \sqrt{0.59} \Rightarrow d = 0.77 \times R \Rightarrow d = 0.77 \times 0.10 = 0.077$m or **7.7 cm is the answer.**<br>Thus, **answers are (a) $2.51$ mT and 7.7 cm.**<br><br>**N.B.:** This analysis is brought out in Annexure II |
| I-21 | Figure shows a gray block at the center of the loop is just to conceptualize clockwise direction of current in the loop. Magnetic field due to a coil at a distance $d$, along the axis of a circular loop, from its center is $B = \frac{\mu_o IR^2}{2(d^2+R^2)^{\frac{3}{2}}}$…(1), as brought out in Appendix-II. As per Ampere's Right-Hand-Thumb-Rule, upper face of the coil will act as South Pole and accordingly. direction of magnetic field shall be **downward at point P**. Likewise, lower face of the coil will act as north pole maintaining **downward field at Q**, and also satisfying continuity of magnetic field.<br><br><br>Both points P and Q are placed symmetrically on the opposite sides of the loop and hence magnitude of the magnetic field as per (1) with the given data would be $B = \frac{(4\pi\times10^{-7})\times5.0\times(4\times10^{-2})^2}{2((3\times10^{-2})^2+(4\times10^{-2})^2)^{\frac{3}{2}}} = \frac{(2\pi\times10^{-6})\times(4\times10^{-2})^2}{2\times(5\times10^{-2})^3} \Rightarrow B = \frac{(\pi\times10^{-6})\times16\times10^{-4}}{125\times10^{-6}} \Rightarrow B = 4.02 \times 10^{-5}$T.<br>Thus, **answer is $B = 4.02 \times 10^{-5}$T at both the points P and Q downward at point P.** |
| I-22 | Given system is shown in figure where ring of radius $R = 0.20$ m carrying a charge $q = 3.14 \times 10^{-6}$C rotates with an angular velocity $\omega = 60.0$ rad/s. It is required to find ratio $\frac{E_x}{B}$ at a point along axis of the loop displaced from the center O by $d = 0.05$ m.<br><br><br>**Determination of $E_x$:** Consider a small length of ring $\Delta l$ at two diametrically opposite points G and H as shown in the figure. Charge on the small lengths at the two points carry charge $\Delta q = \frac{Q}{2\pi R}\Delta l$…(1). Magnitude electric field at |

point due to two charges as per Coulomb's Law is $\left|\vec{E}_G\right| = \left|\vec{E}_H\right| = \frac{\Delta q}{4\pi\varepsilon_0(d^2+R^2)}$...(2). Each of these two fields due to diametrically opposite points have two components.

(i) Components of these two fields perpendicular X-axis are equal in magnitude and opposite in direction, as shown in figure, and hence would cancel out.

(ii) Components of along X-axis of magnitude $E_{Gx} = E_{Hx} = \left|\vec{E}_H\right| \cos\theta$ ...(3), are additive. Here, $\cos\theta = \frac{d}{\sqrt{d^2+R^2}}$...(4). Combining (1)...(4) we have $\Delta E_x = 2E_{Gx} = 2 \times \frac{\frac{q}{2\pi R}\Delta l}{4\pi\varepsilon_0(d^2+R^2)} \times \frac{d}{\sqrt{d^2+R^2}} \Rightarrow \Delta E_x = \frac{q}{\pi R} \times \frac{d\Delta l}{4\pi(d^2+R^2)^{\frac{3}{2}}}$...(5). This is magnitude of electric field at P due to two diametrically opposite small elements as discussed above. Thus net electric field at P is integral of (5) over semicircular ring. Accordingly, we have $E_x = \int_0^{\frac{L}{2}} \frac{q}{\pi R} \times \frac{d}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}}\Delta l \Rightarrow E_x = \frac{q}{\pi R} \times \frac{d}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}}\int_0^{\frac{L}{2}}\Delta l \Rightarrow E_x = \frac{q}{\pi R} \times \frac{d}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}} \times \frac{L}{2}$...(6).
Here, perimeter of the ring $L = 2\pi R$, therefore using (6), $E_x = \frac{q}{\pi R} \times \frac{d}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}} \times \frac{2\pi R}{2} \Rightarrow E_x = \frac{qd}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}}$...(7).

**Determination of $B$ :** The ring carrying charge $q$ is rotating with an angular velocity $\omega = 2\pi f \Rightarrow \omega = \frac{2\pi}{T} \Rightarrow \frac{1}{T} = \frac{\omega}{2\pi}$...(8). Here $f$ is the number of revolutions per second and $T$ is time taken to complete one revolution i.e. for the charge to go around the ring once. Moreover, current is $I = \frac{\Delta Q}{\Delta t} \Rightarrow I = \frac{\Delta Q}{\Delta t}$...(9). In the instant case $\Delta Q = q$ and $\Delta t = T$. Therefore, combining (8) and (9), current caused by the rotating ring is $I = q \times \frac{\omega}{2\pi} \Rightarrow I = \frac{q\omega}{2\pi}$ ...(10).

Magnetic field along axis of a current carrying loop as discussed in Appendix-II is $B = \frac{\mu_o I R^2}{2(d^2+R^2)^{\frac{3}{2}}}$...(11).

Thus, combining (10) and (11), $B = \frac{\mu_o\left(\frac{q\omega}{2\pi}\right)R^2}{2(d^2+R^2)^{\frac{3}{2}}} \Rightarrow \frac{\mu_o q\omega R^2}{4\pi(d^2+R^2)^{\frac{3}{2}}}$...(12).

Using (7) and (12) in the required ratio, $\frac{E_x}{B} = \frac{\frac{qd}{4\pi\varepsilon_0(d^2+R^2)^{\frac{3}{2}}}}{\frac{\mu_o q\omega R^2}{4\pi(d^2+R^2)^{\frac{3}{2}}}} \Rightarrow \frac{E_x}{B} = \frac{d}{\mu_o\varepsilon_0\omega R^2}$...(13). We know that $\mu_o\varepsilon_0 = \frac{1}{c^2}$, here $c = 3 \times 10^8$ m/s. Using it in (13) with he available data $\frac{E_x}{B} = \frac{dc^2}{\omega R^2} \Rightarrow \frac{E_x}{B} = \frac{0.05\times(3\times10^8)^2}{60\times(0.20)^2} \Rightarrow \frac{E_x}{B} = 1.9 \times 10^{15}$ m/s is the answer.

**N.B.:** Unit of the answer, in a simple way, is derived from units of quantities in its final form as under –

$$\frac{E_x}{B} = \frac{m\left(\frac{m}{s}\right)^2}{\left(\frac{1}{s}\right)m^2} \Rightarrow \frac{E_x}{B} = \frac{\frac{m^3}{s^2}}{\frac{m^2}{s}} \Rightarrow \frac{E_x}{B} = \frac{m}{s} \text{ or } \frac{m}{s}.$$

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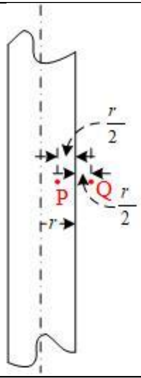


Given system of a thin but long hollow cylindrical tube of radius $r$ is a shown in the figure. The tube is carrying a current $i$. It is required to find magnetic field at points P and Q situated at a distance $\frac{r}{2}$.

**Magnetic field at Point P:** Since electric current is flowing through the hollow tube and therefore electric current within periphery of radius $r_p = \frac{r}{2}$ enclosed by point P is $i_P = 0$. Therefore, as per Ampere's Circuital law magnetic field is $B_P$ **= 0, is answer of part (a).**

**Magnetic field at Point Q:** Again applying Ampere's Circuital Law for point due to current $i$ in the tube, which is inside the periphery of a circle of radius $r_q = r + \frac{r}{2} \Rightarrow r_q = \frac{3}{2}r$ ...(1), at which

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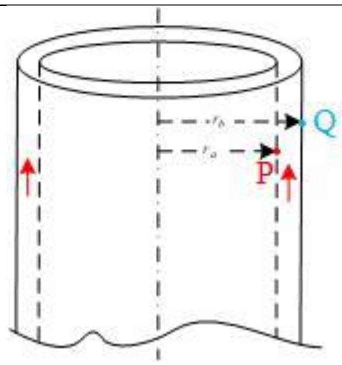
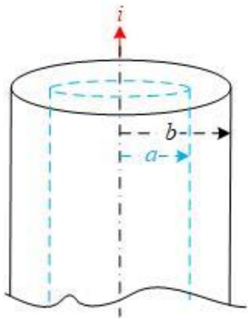
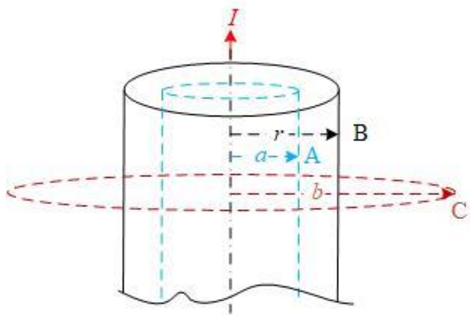
| | point Q is located. Therefore, as per Ampere's Circuital Law $\mu_0 i = \oint B_q\, dl \Rightarrow \mu_0 i = B_q \oint dl$….(2). Using (1), the value of $\oint dl = 2\pi r_q = 2\pi \times \left(\frac{3}{2} r\right) \Rightarrow \oint dl = 3\pi r$…(3).<br>Combining (2) and (3), $\mu_0 i = B_q \times (3\pi r) \Rightarrow B_q = \frac{\mu_0 i}{3\pi r}$ **is the answer of part (b)**<br>Thus, **answers are (a) Zero (b)** $\boldsymbol{\frac{\mu_0 i}{3\pi r}}$ |
| I-24 | Given system of a tube of inner radius $r_a = a$ and outer radius $r_b = b$ is carrying current $i$. It is required to find magnetic field at points P at Q.<br><br>**Magnetic Field at P:** It is given that point P is at inner surface of the tube. Since current is flowing through tube, outside the periphery of point P, and inside this periphery electric current is Zero. Hence, as per Ampere's Circuital law, magnetic field at P is **Zero is the answer of part (a).**<br>**Magnetic Field at Q:** Again applying Ampere's Circuital Law for this point due to current $i$ in the tube, which is inside the periphery of a circle of radius $r_b = b$, at which point Q is located. Therefore, as per Ampere's Circuital Law we have $\mu_0 i = \oint B_q\, dl \Rightarrow \mu_0 i = B_q \oint dl$….(1). The value of $\oint dl = 2\pi b$…(2)<br>Combining (1) and (2), $\mu_0 i = B_q \times (2\pi b) \Rightarrow B_q = \frac{\mu_0 i}{2\pi b}$ **is the answer of part (b)**<br>Thus, **answers are (a) Zero and (b)** $\boldsymbol{\frac{\mu_0 i}{2\pi b}}$ |
| I-25 | Given system of a long cylindrical wire of radius $b$ is carrying electric current $i$ uniformly distributed over its cross-section. It is required to find magnetic field at any point P at a distance $a$ from the loop, as shown in the figure.<br><br>Effective current, responsible for producing magnetic field at the point P, as per Ampere's Circuital Law is $i_P = \frac{i}{\pi b^2} \times \pi a^2 \Rightarrow i_P = \frac{a^2}{b^2} i$…(1).<br>As per Ampere's Law we have $\mu_0 i_p = \oint B_p\, dl \Rightarrow \mu_0 i_p = B_p \oint dl$….(2). The value of $\oint dl = 2\pi a$…(3) Combining (1), (2) and (3), $\mu_0 \left(\frac{a^2}{b^2} i\right) = B_p \times (2\pi a) \Rightarrow B_p = \frac{\mu_0 i a}{2\pi b^2}$ **is the answer.** |
| I-26 | Given system of a long solid wire of radius $r = 0.10$m is carrying electric current $I = 5.0$ A. The current is uniformly distributed over its cross-section. It is required to find magnetic field at any points A, B and C, as shown in the figure.<br><br>**Magnetic Field at A:** Effective current, responsible for producing magnetic field at the point A at a distance $a = 0.02$m from axis of the wire, as per Ampere's Circuital Law is $i_P = \frac{I}{\pi r^2} \times \pi a^2 \Rightarrow i_A = \frac{a^2}{r^2} I$…(1).<br>As per Ampere's Law we have $\mu_0 i_A = \oint B_A\, dl \Rightarrow \mu_0 i_p = B_A \oint dl$….(2). The value of $\oint dl = 2\pi a$…(3) Combining (1), (2) and (3), $\mu_0 \left(\frac{a^2}{r^2} I\right) = B_A \times (2\pi a) \Rightarrow B_A = \frac{\mu_0 I a}{2\pi r^2}$ …(4)<br>Using the available data, $B_A = \frac{(4\pi \times 10^{-7}) \times 5.0 \times 0.02}{2\pi \times (0.10)^2} \Rightarrow B_A = 2.0 \times 10^{-6}$ T or **2.0 μT is the answer of part (a)**.<br>**Magnetic Field at B:** Taking forward analysis for point A above, here $r = 0.10$ m, i.e. surface of the wire entire current $I$ is responsible for producing flux and therefore, using (4), $B_B = \frac{\mu_0 I r}{2\pi r^2} = \frac{\mu_0 I}{2\pi r}$. Using the available data, $B_A = \frac{(4\pi \times 10^{-7}) \times 5.0}{2\pi \times 0.10} \Rightarrow B_A = 10 \times 10^{-6}$ or **10 μT is the answer of part (b)**. |

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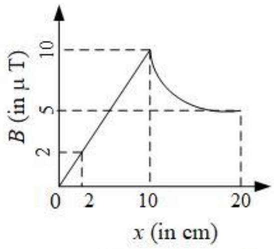
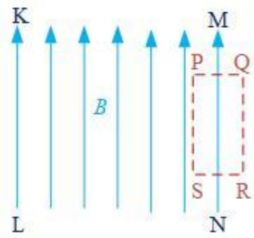
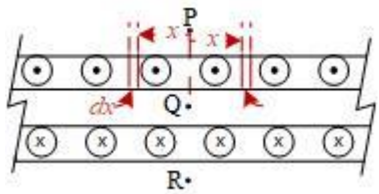
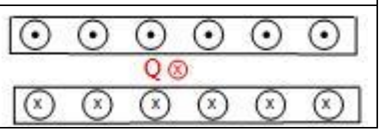
| | **Magnetic Field at C:** Taking forward analysis for point A above, here $b = 0.20$ m and $b > r$ i.e. outside the wire carrying current $I$. As per Ampere's Law we have $\mu_0 I = \oint B_C\, dl \Rightarrow \mu_0 i_p = B_C \oint dl$….(2). The value of $\oint dl = 2\pi b$…(3) Combining (1), (2) and (3), $\mu_0 I = B_C \times (2\pi b) \Rightarrow B_C = \frac{\mu_0 I}{2\pi b}$ …(4).Using the available data, $B_C = \frac{(4\pi\times10^{-7})\times5.0}{2\pi\times0.20} \Rightarrow B_C = 5\times10^{-6}$ or **5.0 μT is the answer of part (c).**<br><br>(Graph Not to the scale)<br>Using the above illustration graph of magnetic field $B - x$ is as shown here.<br>Thus, **answers are (a) 2.0 μT (b) 10 μT (c) 5.0 μT.** |
| I-27 | An idealized magnetic field $\vec{B} = B\hat{\jmath}$ is shown in the figure which is unform between lines if force KL and MN, but on the left of KL it is zero and so also on the right of MN. It is required to prove using Ampere's Circuital Law that $\oint \vec{B}.d\vec{l} = \mu_0 i$…(1).<br><br>In the closed path PQRS of the given system shown in figure, length vectors are PQ $= a\hat{\imath}$, QR $= a(-\hat{\jmath})$, RS $= b(-\hat{\imath})$ and SP $= a\hat{\jmath}$. Here, for simplicity QR is taken at the brink of idealized magnetic field, where $B = 0$, and sides PQ and RS are in region of magnetic field $B$. Further, there is no current passing through PQRS and hence $i = 0$…(2)<br>Let current in the hypothetical loop PQRS is $i$. Conjuring LHS of (1) as required in the problem $\oint \vec{B}.d\vec{l}_{\text{PQRS}} = \int_0^a \vec{B}.d\vec{l}_{\text{PQ}} + \int_b^0 \vec{B}.d\vec{l}_{\text{QR}} + \int_a^0 \vec{B}.d\vec{l}_{\text{RS}} + \int_0^b \vec{B}.d\vec{l}_{\text{PQ}}$.<br>This leads to $\oint \vec{B}.d\vec{l}_{\text{PQRS}} = \int_0^a B\cos\frac{\pi}{2} dl + \int_b^0 B\cos 0\, dl + \int_a^0 B\cos\frac{\pi}{2} dl + \int_0^b B\cos 0 dl$.<br>It further, solves to$\oint \vec{B}.d\vec{l}_{\text{PQRS}} = B\int_0^b dl \Rightarrow \oint \vec{B}.d\vec{l}_{\text{PQRS}} = Bb$…(3)<br>Combining (2) and (3) in (1), we should have $Bb = \mu_0 \times 0 \Rightarrow Bb = 0$ …(4).<br>In (4) both the multiplicands are distinct and non-zero and hence the equation is not valid, t**he proposition that the idealized magnetic field is not possible is proved** |
| I-28 | Magnetic field at a point near a large metal sheet carrying uniform surface current has been analyzed in **Appendix-IV**. It is seen that magnetic field is **(a)** proportional to surface current density, **(b)** *independent to the distance from the large sheet and* ***(c)*** *direction of magnetic field can be determined applying Ampere's Right Hand Thumb Rule. It is summarized as* $B = \frac{\mu_0 K}{2}$…(1).<br><br>Above inferences are applied to the given problem where at each of the points P, Q and R, magnetic field is resultant of the magnetic fields, *due to two sheets carrying currents in opposite direction*, at point P is $B_{\text{P}} = B_{\text{R}} = \frac{\mu_0 K}{2} + (-)\frac{\mu_0 K}{2}$…(2). While at point Q, in the space between the two sheets carrying currents in opposite directions is $B_{\text{Q}} = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} \Rightarrow B_{\text{Q}} = \mu_0 K$…(3), in a direction towards the Right Hand side.<br>Thus, **answer at point P is Zero, at Q is $\mu_0 K$towards right in the figure, and at R is Zero**<br>***N.B.:*** *Appendix IV elaborates analysis of magnetic field by a metal sheet carrying uniform current. This can be also analyzed using Ampere's Circuital Law $\oint B.dl = \mu_0 I$ to determine magnetic field at any point near a large sheet of width $x$, current is $I = kx$ and length of the path surrounding a large sheet is $l = 2x$. Taking $\vec{B}$ along $d\vec{l}$. As per the law it leads to $B \times 2x = \mu_0 \times KX \Rightarrow B = \frac{\mu_0 K}{2}$. In the problem sheet is taken to be large and hence end effect on path length can be ignored.* |
| I-29 | Extension of Appendix-IV to the given problem provides magnetic field at point Q in space between two large metal sheets carrying uniformly<br> |

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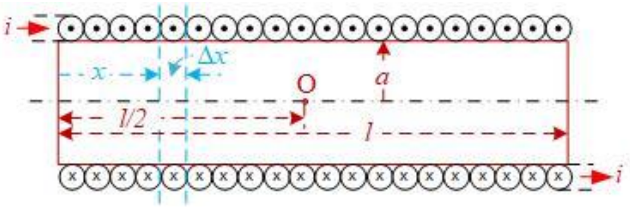
| | distributed current density $K$, in opposite directions, $B_{\mathrm{Q}} = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} \Rightarrow B_{\mathrm{Q}} = \mu_0 K \ldots(1)$<br><br>Further problem states that a particle carrying a charge $q$ and mass $m$ is projected into the plane of the diagram as shown in the figure. Therefore, as per Lorentz's Force Law, $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB_{\mathrm{Q}} \sin\frac{\pi}{2}\hat{n} \ldots(2)$. Combining (1) and (2), magnitude of force is $F = (qv) \times (\mu_0 K) \ldots(3)$.<br><br>Further, it is stated that particle is describes a circle of radius $r$. This is possible when, as per principle of circular motion $F = \frac{mv^2}{r} \ldots(4)$. Equating (3) and (4), $\frac{mv^2}{r} = qv\mu_0 K \Rightarrow \boldsymbol{v} = \frac{\boldsymbol{q\mu_0 Kr}}{\boldsymbol{m}}$ **is the answer.**<br><br>**N.B.: To correlate (1) with basic concepts refer to Appendix-IV.** |
| I-30 | Magnetic field inside a compact coil is carrying current $i$ is $B = \mu_0 Ni$, in case of a solenoid $B = \mu_0 nli \ldots(1)$. Here $n$ is number of turns per unit length, $l$ is the length of the solenoid. Using the given data in (1) we have, $3.14 \times 10^{-2} = (4\pi \times 10^{-7}) \times n \times 5.00 \Rightarrow n = \frac{3.14\times10^{-2}}{20.0\times\pi\times10^{-7}} \Rightarrow n = \frac{10^5}{20.0} \Rightarrow n = \mathbf{5000}$ **turns/m is the answer.** |
| I-31 | Magnetic field inside a compact coil is carrying current $i$ is $B = \mu_0 Ni$, in case of a solenoid $B = \mu_0 nli \ldots(1)$. Here $n$ is number of turns per unit length, $l$ is the length of the solenoid. Given that solenoid is closely wound using a wire of radius $r = 0.5 \times 10^{-3}$ m. Therefore number of turns per meter length would be $n = \frac{1}{2r}$, here $2r$ is the center-to-center spacing between two turns of the solenoid. Thus using the given data in (1) we have, $B = (4\pi \times 10^{-7}) \times \frac{1}{2\times(0.5\times10^{-3})} \times 5 \Rightarrow n = 20\pi \times 10^{-4} \Rightarrow n = \mathbf{2\pi \times 10^{-3}}$**T is the answer.**<br><br>**N.B.:** Here answer is reported on the principle of significant digits. |
| I-32 | Magnetic field $B$ inside a solneoid carrying current $I$ is $B = \mu_0 nI \ldots(1)$. Here, $n$ is number of turns per–meter length and $I$ is current through the solnoid. In problem no mention has been made of radius of wire and there fore it can be safely assumed the solenoid is having a single layer of $N = 400$ turns over a length $l = 0.20$ m. Thus, $n = \frac{N}{l} \Rightarrow n = \frac{400}{0.20} = 2.0 \times 10^3$<br><br>Further, it is given that radius of the solenoid radius $r = 1.0 \times 10^{-3}$. Therefore, length of wire having $N$ turns is $L = 2\pi rn \ldots(2)$. Accordingly, with the given resistance of wire per unit length $R' = 0.01$ Ω/m net resistance of the wire is $R = LR' \Rightarrow R = 2\pi rnR' \ldots(3)$. Thus, using the available data $R = 2\pi \times (1.0 \times 10^{-2}) \times 400 \times 0.01 \Rightarrow R = 8.0\pi \times 10^{-2}\Omega$...(2)<br><br>Magnetic field $B$ inside a solneoid carrying current $I$ is $B = \mu_0 nI$. No mention has been made of radius of wire and there fore it can be safely assumed the solenoid is having a single layer of $N = 400$ turns over a length $l = 0.20$ m.<br><br>$\Rightarrow I = \frac{B}{\mu_0 n} \Rightarrow I = \frac{1.0\times10^{-2}}{(4\pi\times10^{-7})\times(2.0\times10^3)} \Rightarrow I = \frac{10^2}{8\pi} \ldots(3)$.<br><br>As per Ohm's Law $E = IR \ldots(4)$. Using data in (2) and (3) in (4), we have $E = \left(\frac{10^2}{8\pi}\right) \times (8.0\pi \times 10^{-2})$. It solves into $E = \mathbf{1.0}$ **V is the answer.**<br><br>**N.B.:** Answer is reported using principle of SDs. |
| I-33 | Given system is shown in the figure where number of turns per unit length of a tightly wound solenoid is $n$/m. This is implicit in statement that number of turns in small length of coil $dx$ and is approximated to current $I = nidx \ldots(1)$ in a loop.<br><br><br><br><br>As per Biot=Savart's Law magnetic field at a point displaced by $x$ from center of the loop the axis of loop of radius $r$ , as per Appendix-II, is $B = \frac{\mu_0 I}{2} \times \frac{r^2}{(x^2+r^2)^{\frac{3}{2}}}$ $\ldots(2)$. |

In the instant case the point at which magnetic field is to be determined is O and $r \to a$, while distance of the loop from O is $x \to \frac{l}{2} - x$ and, therefore, transformation of variables together with (1) leads to-

$$dB_0 = \frac{\mu_0 ni dx}{2} \times \frac{a^2}{\left(\left(\frac{l}{2}-x\right)^2+a^2\right)^{\frac{3}{2}}} \Rightarrow dB_0 = \frac{\mu_0 nia^2}{2} \times \frac{1}{\left(\left(\frac{l}{2}-x\right)^2+a^2\right)^{\frac{3}{2}}} dx \ldots(3).$$

Here, $x$ varies from $x = 0$ to $x = l$, and net magnetic field at O is $B_0 = \frac{\mu_0 nia^2}{2} \int_0^l \frac{1}{\left(\left(\frac{l}{2}-x\right)^2+a^2\right)^{\frac{3}{2}}} dx \ldots(4)$, as desired in the problem.

Integrating (4), $B_0 = \frac{\mu_0 nia^2}{2} \left[\int \frac{1}{((a\tan\theta)^2+a^2)^{\frac{3}{2}}} dx\right]_0^l$, here $a\tan\theta = \frac{l}{2} - x \Rightarrow a\sec^2\theta\, d\theta = -dx$. Also, $\tan\theta = \frac{l-2x}{2a}$. Thus, we have, $B_0 = \frac{\mu_0 nia^2}{2} \left[\int \frac{(-)a\sec^2\theta}{((a\tan\theta)^2+a^2)^{\frac{3}{2}}} d\theta\right]_0^l \Rightarrow B_0 = (-)\frac{\mu_0 nia^2}{2}\left[\int \frac{a\cos\theta}{(a\sec\theta)^3} d\theta\right]_0^l \Rightarrow B_0 = (-)\frac{\mu_0 ni}{2}\left[\int \cos\theta\, d\theta\right]_0^l$. It solves into $B_0 = \frac{\mu_0 ni}{2}[\sin\theta]_l^0 \ldots(5)$.

Transforming $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \Rightarrow \sin\theta = \frac{\frac{l-2x}{2a}}{\sqrt{1+\left(\frac{l-2x}{2a}\right)^2}} \Rightarrow \sin\theta = \frac{l-2x}{\sqrt{4a^2+(l-2x)^2}} \ldots(6)$.

Combining (5) and (6), $B_0 = \frac{\mu_0 ni}{2}\left[\frac{l-2x}{\sqrt{4a^2+(l-2x)^2}}\right]_l^0 \Rightarrow B_0 = \frac{\mu_0 ni}{2}\left[\frac{l}{\sqrt{4a^2+l^2}} - \frac{l-2l}{\sqrt{4a^2+(l-2l)^2}}\right]$.

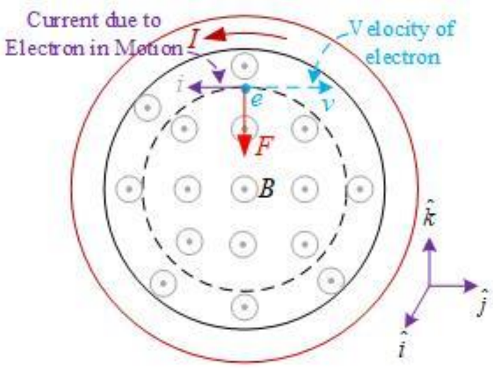
It solves into $B_0 = \frac{\mu_0 ni}{2}\left[\frac{l}{\sqrt{4a^2+l^2}} + \frac{l}{\sqrt{4a^2+l^2}}\right] \Rightarrow B_0 = \frac{\mu_0 nil}{\sqrt{4a^2+l^2}} \Rightarrow B_0 = \frac{\boldsymbol{\mu_0 ni}}{\sqrt{\mathbf{1}+\left(\frac{2a}{l}\right)^2}}$ **is the answer of part (a).**

Taking $l \gg a$ then $\frac{2a}{l} \to 0$, it leads to $B_0 = \frac{\boldsymbol{\mu_0 ni}}{\sqrt{\mathbf{1}+(\mathbf{0})^2}} \Rightarrow B_0 = \boldsymbol{\mu_0 ni}$**,** and if $a \gg l$ then, $1 + \left(\frac{2a}{l}\right)^2 \to \left(\frac{2a}{l}\right)^2$. It leads to $B_0 = \frac{\boldsymbol{\mu_0 ni}}{\sqrt{\left(\frac{2a}{l}\right)^2}} \Rightarrow B_0 = \frac{\boldsymbol{\mu_0 ni}}{\frac{2a}{l}} \Rightarrow B_0 = \frac{\boldsymbol{\mu_0 nil}}{\boldsymbol{2a}}$**, both the cases are proved.**

**I-34**

Electric field inside a long, tightly-wound solenoid carrying current $I = 2.00$ A is $B = \mu_0 nI \ldots(1)$, here $n$ is number of turns per unit length. If an electron is found to perform uniform circular motion of frequency $f = 1.00 \times 10^8$rev/s then it implies that –

(a) Uniform circular motion of electron constitutes a current $i = ev \Rightarrow i = e \times 2\pi rf \ldots(2)$
(b) Uniform circular motion would cause a centripetal force $F = mr\omega^2 \Rightarrow F = mr(2\pi f)^2 \ldots(3)$
(c) As per Lorentz's Force Law force on electron $\vec{F} = e\vec{v} \times \vec{B} \Rightarrow F = qvB$. Combining this with (2) we have $F = 2\pi qrfB \ldots(4)$




Lorentz's force at (4) constitutes centripetal force at (3) as shown in the figure. Accordingly, $mr(2\pi f)^2 = 2\pi erfB \Rightarrow eB = 2m\pi f \ldots(5)$.

Combine (1) and (5), $e(\mu_0 nI) = 2m\pi f \Rightarrow n = \left(\frac{m}{e}\right) \times \frac{2\pi f}{(4\pi\times10^{-7})\times I} \ldots(6)$.

Using the available data in (6) together with mass of electron $m = 9.1 \times 10^{-31}$kg and magnitude of charge of an electron $q = 1.6 \times 10^{-19}$ C we have $n = \frac{9.1\times10^{-31}}{1.6\times10^{-19}} \times \frac{1.00\times10^8}{2\times2.0\times10^{-7}} \Rightarrow 1.42 \times 10^3$turns or **1420 Turns is the answer.**

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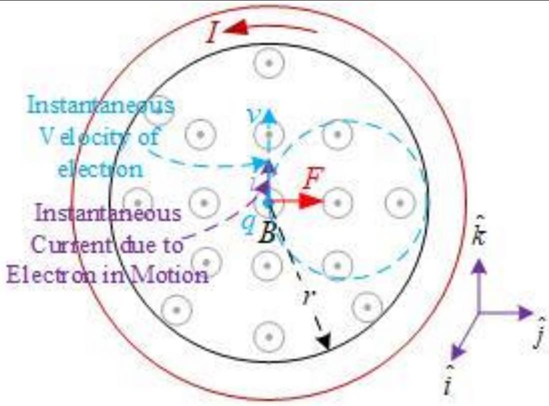
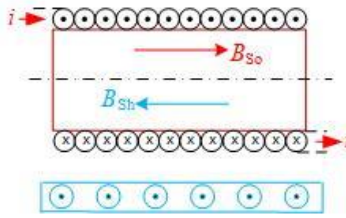
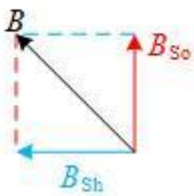
| | **N.B.:** The problem apparently looks quite complex. Yet a systematic resolutions of problems in a step-by-step leads to cancellation of many parameters, and a simple solution. |
| I-35 | Given system is shown in the figure. A tightly-wound solenoid of radius $r$ having $n$ turns per meter length is carrying current $I$. Therefore, magnetic field throughout the cross-section of the coil is $B = \mu_0 nI$…(1).<br><br><br>A particle carrying charge $q$ projected from a point on the axis of the solenoid with some velocity $v$., perpendicular to the axis.<br>The constraint of velocity is that it is maximum velocity yet the particle does not strike the solenoid.<br>It is a fit case for application of Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B}$. This force is perpendicular to the velocity of the charged particle such that $F = qvB$…(2). The latter makes it a fit case of circular motion having radius of the path $r$ of the charged particle, such that centripetal force e $F = \frac{mv^2}{r}$…(3).<br>Combining (1), (2) and (3) $\frac{mv^2}{r} = qv(\mu_0 nI) \Rightarrow v = \frac{\mu_0 qnIr}{m}$…(4). It is seen that $v \propto r$. It is required to find maximum velocity such that the particle does no touch the solenoid or $2r_{max} = R \Rightarrow r_{max} = \frac{R}{2}$. Therefore, for $v \to v_{max} \Rightarrow r \to r_{max}$. Accordingly for the limiting value $v_{max} = \frac{\mu_0 qnIr_{max}}{m} \Rightarrow v_{max} = \boldsymbol{\frac{\mu_0 qnIR}{2m}}$ **is the answer.**<br>**N.B.:** Creating a figure illustrating the system makes it easy to visualize the physics that goes into the problem and thus evolve solution to a problem apparently complicated. |
| I-36 | Given system is shown in the figure along with the direction of magnetic fields $B_{So}$ produced by solenoid, having tightly-wound $n$ turns per meter. carrying current $i$ and $B_{Sh}$ produced by sheet carrying surface current $I = kdl$ through width $dl$. It is proved that $B_{So} = \mu_0 ni$…(1) and in Appendix IV we see that $B_{Sh} = \frac{\mu_0 K}{2}$…(2).<br><br><br>It is seen in the figure that, for the direction of currents as considered, $B_{Sh}$ and $B_{So}$ are in opposite directions. Therefore magnetic field near center of the solenoid to be zero, we have from (1) and (2), $\mu_0 ni = \frac{\mu_0 K}{2} \Rightarrow i = \boldsymbol{\frac{K}{2n}}$ **is the answer of part (a).**<br>Part (b) of the problem requires to determine magnetic field when solenoid carrying current $i = \frac{K}{2n}$…(3), in earlier case is turned through perpendicular to the axis of the sheet. It implies that axis of the solenoid is parallel to the surface current in the sheet. It leads to a situation two magnetic fields $B_{Sh}$ and $B_{So}$ of equal magnitude are perpendicular to each other, whichever way, as shown here. Therefore, resultant magnetic field would be $B = \sqrt{{B_{So}}^2 + {B_{Sh}}^2}$…(4).<br><br><br>Combining (1)..(3) in (4), we have $B = \sqrt{\left(\mu_0 n \times \frac{K}{2n}\right)^2 + \left(\frac{\mu_0 K}{2}\right)^2} \Rightarrow B = \sqrt{\left(\frac{\mu_0 K}{2}\right)^2 + \left(\frac{\mu_0 K}{2}\right)^2} \Rightarrow B = \boldsymbol{\frac{\mu_0 K}{\sqrt{2}}}$ **is the answer of part (b)**<br>Thus, **answers are (a)** $\boldsymbol{\frac{K}{2n}}$ **(b)** $\boldsymbol{\frac{\mu_0 K}{\sqrt{2}}}$ |
| I-37 | In absence of any other data it is safe to assume that the capacitor capacitance $C = 100 \times 10^{-6}$ F and the long solenoid form an RC circuit. The capacitor is kept fully charged to a potential difference $V_0 = V = 20$ Volts. |

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| | Thus drop of voltage across the capacitor to $V_t = 0.90V_0$, i.e. 90% of maximum value $V_0 = V$ value, during $t = 2.0$ s is considered to be linear.<br><br>Charge on a capacitor is $Q = CV$…(1). Accordingly, initial charge on the capacitor $Q_0 = CV_0$...(2) After time $t$ charge on the capacitor is $Q_t = CV_t = C(0.90V_0)$…(3). Therefore, average current through the solenoid having $n = 4000$ turns/mteterduring the period $0 < t < 2.0$ is $i = \frac{Q_t - Q_0}{t}$…(4).<br><br>Magnetic field at the center of a solenoid carrying current $i$ is $B = \mu_0 ni$…(5). Combining (1)..(4) in (5) we have $B = (4\pi \times 10^{-7}) \times 4000 \times \left(\frac{Q_t - Q_0}{t}\right) \Rightarrow B = (4\pi \times 10^{-7}) \times 4000 \times \frac{CV_0 - 0.90CV_0}{t}$. Using the available data $B = (16\pi \times 10^{-4}) \times \frac{0.10 \times (100 \times 10^{-6}) \times 20}{2.0} \Rightarrow B =$ $\mathbf{16\pi \times 10^{-8}}$**T is the answer.** |

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# Appendix-I

# Torque on Small Current Carrying Loop Placed Inside Another Current Carrying Loop

**Part A - Magnetic Field at the Center of a Current Carrying Loop:** Let us consider a loop of radius $R$ carrying current $I$ in anti-clockwise direction. The loop is taken to be in $\hat{\imath} - \hat{\jmath}$ plane and in accordance with the three unit directions vectors are also shown for reference in Fig. 1.

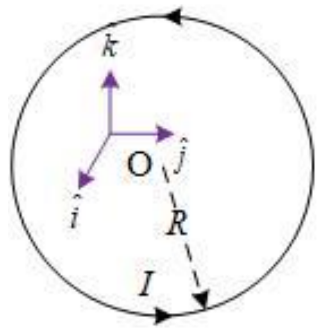

Fig. 1

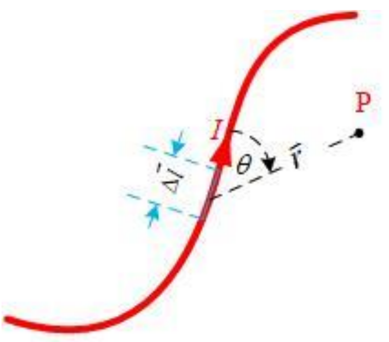

Fig. 2

Biot-Savart's Law stipulated direction and magnitude of magnetic field at a point situated at a distance $r$ from a wire of length $\Delta\vec{l} = \Delta l\hat{l}$ carrying current $I$ along $\hat{l}$ . According to the law, as shown in Fig. 2.-

$$\Delta\vec{B} = \frac{\mu_0 I}{4\pi R^2}\Delta\vec{l} \times \hat{r} \Rightarrow \Delta\vec{B} = \frac{\mu_0 I}{4\pi R^2}\Delta l\hat{l} \times \hat{r}$$

$$\Delta\vec{B} = \frac{\mu_0 I\Delta l}{4\pi R^2}\hat{k}\ldots(1).$$

Taking the elemental length $\Delta l$ is part of a circle of radius $R$ which subtends an angle $\Delta\theta$ at the circle of the circle such that $\Delta l = R \times \Delta\theta\ldots(2)$.

Combining (1) and (2), $\Delta\vec{B} = \frac{\mu_0 IR\Delta\theta}{4\pi R^2}\hat{k}$. Thus, in the instant case, having ascertained direction of the magnetic field at O as $\hat{k}$.its magnitude is $\Delta B = \frac{\mu_0 I\Delta\theta}{4\pi R}\ldots(3)$.

Accordingly, magnitude of the net magnetic field at the center of the circular loop is $B = \int_0^{2\pi}\frac{\mu_0 I}{4\pi R}d\theta \Rightarrow B = \frac{\mu_0 I}{4\pi R}[\theta]_0^{2\pi} \Rightarrow B = \frac{\mu_0 I}{4\pi R} \times 2\pi$. It leads to $\boldsymbol{B} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2R}}\ldots(4)$. This magnetic field at O is in along $\hat{\boldsymbol{k}}$ .

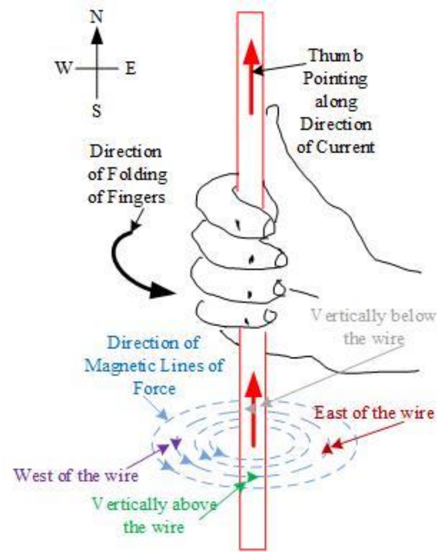

Fig. 3

The direction of magnetic field in this can also ascertained using **Right-Hand-Thumb-Rule** applied to circular loop for convenience as shown in Fig. 3, which when applied to loop or coil is as per Fig.4.

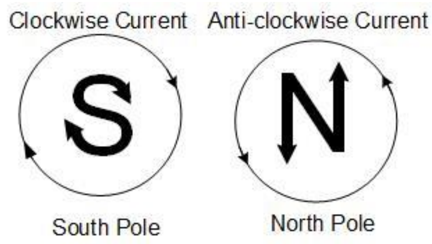

Fig. 4

Area of a loop of radius $R$ is $A = \pi R^2$. This loop if carries ***current i in anticlockwise direction*** then in vector form $\vec{A} = \pi R^2\hat{k}$. Since, current in isolation is treated as a scalar and hence for a current carrying loop a new term is **magnetic dipole moment of a current carrying loop** which is defined as $\vec{\boldsymbol{\mu}} = \boldsymbol{i}\vec{\boldsymbol{A}}$ and in instant case $\vec{\mu} = i\pi R^2\hat{k}\ldots(5)$.

**Part B – Interaction of Magnetic Fields of Two Current Carrying Concentric Loops:** Two current carrying concentric loops A and B having magnetic dipole moments $\vec{\mu}_A$ and $\vec{\mu}_B$,, respectively will exhibit resultant magnetic dipole moment $\vec{\mu} = \vec{\mu}_A + \vec{\mu}_B\ldots(6)$

**Part C – Torque on Current Carrying Loops:** Taking two loops in a state of rest, they would experience equal and opposite torques as per Newton's Third Law of Motion. Generally concern is about a small loop of radius $r$ carrying current at the center of an outer loop having much larger radius $R \gg r$. In case of coplanar loop discussions at part B above. In case of coplanar loop resultant magnetic dipole moments are either arithmetic sum or difference of the two dipole moments depending upon direction of their currents as discussed in part A.

But, it becomes an important pair of loops only when they are inclined with respect to each other a torque appears and there. The loops are taken to be concentric and they have radial symmetry about the common axis. There are two possibilities, each of them are analyzed below –

**Case 1- Planes of loops are perpendicular to each other:**

A circular loop of radius $R$ carrying a current $I$ is placed in $\hat{\imath}-\hat{\jmath}$ plane as shown in the figure. Axis of the loop is along $\hat{k}$. As per (4), in accordance with Biot-Savart's law it will produce magnetic field $\vec{B}=\frac{\mu_0}{2R}\hat{k}$, at the center of the loop. Further, it is given that another circular loop of radius $r$ carrying current $i$ in anti-clockwise direction as seen against $\hat{\imath}$. The small loop, as shown in the Fig. 5, is in $\hat{\jmath}-\hat{k}$ plane.

From the statement of the system shown in the figure on the left, it is observed that –

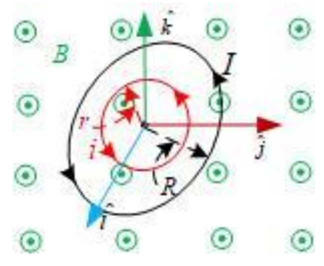

Fig. 5

a) magnetic field at the center of the outer loop of radius $R$ is $\vec{B}$.
b) plane of the smaller loop of radius $r$ is along the magnetic field.
c) given that $r \ll R$, and geometrical symmetry of the loop, force experienced by inner coil as per will produce a torque about diameter of smaller coil. We take for convenience diameter of loop along Y-Y' i.e. $\hat{\jmath}$.

Taking forward analysis force on a small element of inner loop of length $\Delta\vec{l}=r\Delta\theta\hat{l}$, as per (2), carrying current $i$, as per Fig. 6, is as per limited version of Lorentz's Force Law $\Delta\vec{F}=i\vec{l}\times\vec{B}\Rightarrow\Delta\vec{F}=i\left(r\Delta\theta\hat{l}\right)\times\vec{B}$…(7). Here, the other part of the law which defines force on free charges, as none is there, is ignored. The force as a result of cross-product is $\Delta\vec{F}=irB\sin\theta\,\Delta\theta(-\hat{\imath})$….(7).

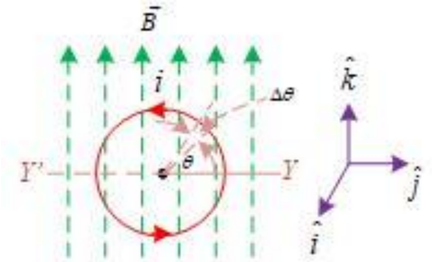

Fig. 6

Therefore, torque experienced by the element of loop about diameter Y-Y' would be $\Delta\vec{\Gamma}=\overrightarrow{QP}\times\Delta\vec{F}$ as per Fig. 7.

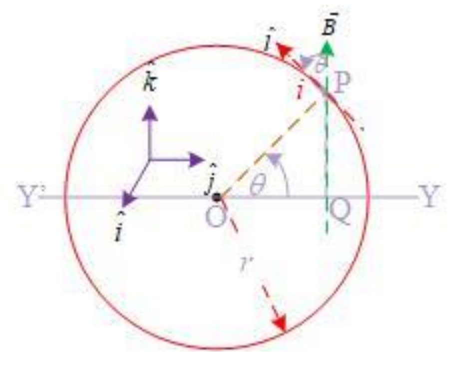

Fig. 7

Combining (7) with the geometry, it leads to $\Delta\vec{\Gamma}=\left(r\sin\theta\,\hat{k}\right)\times\left(irB\sin\theta\,\Delta\theta(-\hat{\imath})\right)$….(8). This expression simplifies into $\Delta\vec{\Gamma}=iBr^2\sin^2\theta\,\Delta\theta(-\hat{\jmath})\Rightarrow\Delta\vec{\Gamma}=\frac{iBr^2}{2}(1-\cos 2\theta)\Delta\theta(-\hat{\jmath})$ ...(9). Thus, net torque would be $\vec{\Gamma}=\left[\int_0^{2\pi}\frac{iBr^2}{2}(1-\cos 2\theta)\Delta\theta\right](-\hat{\jmath})$. The integral simplifies to $\vec{\Gamma}=\frac{iBr^2}{2}\left[\int_0^{2\pi}d\theta-\int_0^{2\pi}\cos 2\theta\;d\theta\right](-\hat{\jmath})\Rightarrow\vec{\Gamma}=\frac{iBr^2}{2}\times 2\pi(-\hat{\jmath})$. Thus, net torque on the inner loop is $\vec{\Gamma}=iB\pi r^2(-\hat{\jmath})$…(10).

Combining (4) and (10), magnitude of the torque is $\Gamma=i\left(\frac{\mu_0 I}{2R}\right)\pi r^2\Rightarrow\boldsymbol{\Gamma}=\boldsymbol{\frac{\mu_0\pi i I r^2}{2R}}$**….(11)**

Using (5), expression in (11) can be expressed as $\vec{\Gamma}=i\vec{A}\times\vec{B}\Rightarrow\vec{\Gamma}=\vec{\mu}\times\vec{B}$…(12). Here, magnetic dipole moment of a loop is $\vec{\mu}=i\vec{A}$ and in case of coil $\vec{\mu}=ni\vec{A}$…(13).

**Case 1- Planes of loops are inclined at an angle $\boldsymbol{\alpha}$:** Plane of the larger loop creating magnetic field $\vec{B}=B\hat{k}$ is along plane $\hat{\imath}-\hat{\jmath}$. is shown in the Fig 8 (View from $\hat{k}$). Let plane of the smaller loop of radius $r$ is inclined to the plane of larger loop at an angle $\alpha$, as shown in the Fig. 8 (View from $\hat{\imath}$). Considering radial symmetry the inclination is taken w.r.t, Y-Y' as shown in the figure i.e. $\hat{\jmath}$. In this diameter along X-X' remains aligned to $\hat{\imath}$. Thus, area vector of the loop, with respect to direction current $i$ in it as discussed in part A, is $\vec{A}$ and inclined at an angle $\alpha$ w.r.t.$\vec{B}$.

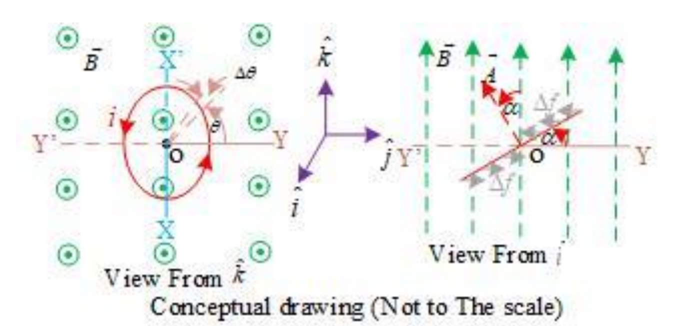

Fig. 8

Each small length of loop $\Delta\vec{l} = r\Delta\theta\hat{l}$ , due to current $i$, would experience force $\Delta\vec{f} = i(r\Delta\theta\hat{l}\,) \times \vec{B}$ as per (7). In this orientation $\Delta\vec{f} = \Delta f(-\hat{j})$ on the semicircular arc on the right of X-X'. Likewise, $\Delta\vec{f} = \Delta f(\hat{j})$ on the semicircular arc on the left of X-X' to current $i$ in the loop.

These distributed forces along the two semicircular arc would form a couple about Y-Y' causing rotation of loop about O along $\hat{\imath}$. In an effort to quantify torque on the inclined loop elemental force $\Delta\vec{f}$ is resolution of QP perpendicular to Y-Y' and it is PR = QP $\sin\alpha$…(14), as shown in the Fig. 9. Accordingly, torque equation in (9) is moderated into-

$$\Delta\vec{\Gamma} = \overrightarrow{PR} \times \left(irB\sin\theta\,\Delta\theta(\hat{l})\right) \Rightarrow \Delta\vec{\Gamma} = \left(QP\sin\alpha\,\hat{k}\right) \times \left(irB\sin\theta\,\Delta\theta(\hat{l})\right)$$

$\Delta\vec{\Gamma} = \left((r\sin\theta)\sin\alpha\,\hat{k}\right) \times \left(irB\sin\theta\,\Delta\theta(\hat{l})\right)$. On integration it solves into

$$\vec{\Gamma} = iB\pi r^2 \sin\alpha\,(\hat{\imath}) \Rightarrow \vec{\Gamma} = i\vec{A} \times \vec{B} \Rightarrow \vec{\Gamma} = \vec{\mu} \times \vec{B} \ldots(15)$$

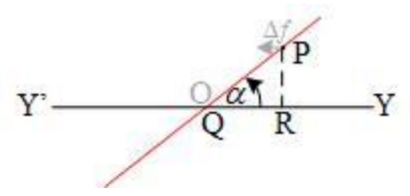

Fig. 9

It is seen that torque on a loop carrying current loop/coil produced (by an external magnetic field derived in (15) and is identical to that derived in (12). In the latter case $\alpha = \frac{\pi}{2}$ when both coils are perpendicular.

**Conclusion:** In general form torque experienced by a current carrying loop placed in a uniform electric field produced by a large loop carrying current $I$, as per (12) and (16), would be $\vec{\Gamma} = \vec{\mu} \times \vec{B} = \mu B\sin\alpha\,\hat{\Gamma} \Rightarrow \vec{\Gamma} = \boldsymbol{\frac{\mu_0 \pi i I r^2}{2R}\sin\alpha}\,\hat{\Gamma}$…(16). Here, $\mu = i(\pi r^2)$ and $\mu\sin\alpha = i(\pi r^2\sin\alpha)$. Thus, in (16) $\pi r^2\sin\alpha = A\sin\alpha$ is the area of the current carrying turn, resolved perpendicular to the magnetic field perpendicular that it intercepts. Accordingly, $\boldsymbol{\vec{\Gamma} = \vec{\mu} \times \vec{B}}$ is the general expression of torque experienced by a current carrying urn of any shape when placed in a uniform magnetic field and simplifies analysis in complex situations

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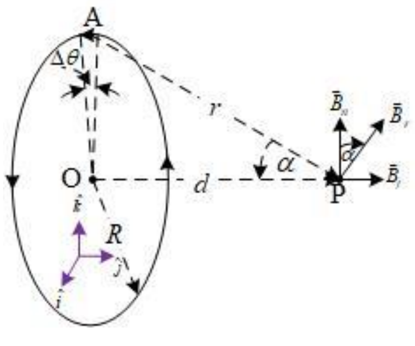
## Appendix-II

## Magnetic Field at a Point on the Axis a Coil

Given is a circular coil containing $n = 200$ turns of radius $R = 0.10$ m in $\hat{\imath} - \hat{k}$ plane carrying current $I = 2.0$ A. As per Biot-Savart's Law, magnetic field at point P on the axis of the loop at a distance $d$ from the center of the loop O; a distance $r$ from a small element of wire of length $\Delta\vec{l} = r\Delta\theta\hat{l}$ will be $\Delta\vec{B}_P = \frac{\mu_o nIR\Delta\theta}{4\pi r^2}\widehat{B_r}$, here $\Delta B_P = \frac{\mu_o nIR\Delta\theta}{4\pi r^2}$…(1)This magnetic field has two components $\Delta\vec{B}_P = \Delta\vec{B}_j + \Delta\vec{B}_N \Rightarrow \Delta\vec{B}_P = \Delta B_P\cos\left(\frac{\pi}{2} - \alpha\right)\hat{\jmath} + \Delta B_P\sin\left(\frac{\pi}{2} - \alpha\right)\hat{N}$. It simplifies into $\vec{B}_P = \Delta B_P\sin\alpha\,\hat{\jmath} + \Delta B_P\cos\alpha\,\hat{N}$…(2), as shown in the figure. With the symmetry of the loop about its axis O, the component of magnetic field along the loop $\hat{N}$ will cancel out leaving the component along $\hat{\jmath}$ to be only effective.

Thus, combining (1) and (2), net magnetic field at point P, due to the loop is $B_P = \int_0^{2\pi}\frac{\mu_o nIR\Delta\theta}{4\pi r^2}\sin\alpha \Rightarrow B_P = \frac{\mu_o nIR\sin\alpha}{4\pi r^2}\int_0^{2\pi}d\theta \Rightarrow B_P = \frac{\mu_o nIR\sin\alpha}{4\pi r^2} \times 2\pi \Rightarrow B_P = \frac{\mu_o nIR\sin\alpha}{2r^2}$…(3). It is seen from the figure that $\sin\alpha = \frac{R}{r}$, therefore, $B_P = \frac{\mu_o nIR^2}{2r^3}$. Further, $r = \sqrt{d^2 + R^2}$ therefore, $B_P = \frac{\mu_o nIR^2}{2(d^2+R^2)^{\frac{3}{2}}}$ …(4).




At the center of the coil P → 0, $r \to R$ and $\alpha \to \frac{\pi}{2}$ and, therefore, using (3) $B_O = \frac{\mu_o nIR\sin\frac{\pi}{2}}{2R^2}$. It solves into $B_O = \frac{\mu_o nI}{2R}$…(5).

In case of a circular loop $n = 1$ accordingly equations (4) and (5) become $B_P = \frac{\mu_o I R^2}{2(d^2+R^2)^{\frac{3}{2}}}$ ...(6) and $B_O = \frac{\mu_o I}{2R}$...(7), respectively.

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**Appendix-III**

## Magnetic Field at Any Point Inside a Circular Loop Carrying Current

### *Synopsis*

*This paper is an outcome of discussions with students of class 9th to 12th on electromagnetism during which it was observed that all texts and references cover derivation of magnetic field at the center of a current carrying circular loop and at any point on axis of the loop, perpendicular to the plane of the loop. An obvious question cropped up 'what could be magnetic field at any point inside the loop lying in its plane?' Interactive Online Mentoring Sessions (IOMS), flagship of Gyan Vigyan Sarita, which focuses on grooming competence to compete among unprivileged children with a sense of Personal Social Responsibility (PSR) in a non-organizational, non-remunerative, non-commercial and non-political manner. As a mentor of the initiative where students are prompted to come out of rote-learning and explore mathematics and science with an out-of-box perspective in their day-today experiences, the obvious question could not be averted. Accordingly, an illustration of the solution to the question has been evolved within the scope of understanding of target students.*

**Problem Formulation:** Consider a point P inside a circular loop of radius $R$ in $\hat{j} - \hat{k}$ plane carrying a current $I$. The point P is at a distance $a$ from the center of the loop. It is required to determine magnetic field $B$ at P, as shown in the figure.

As per Biot-Savart's Law magnetic field at a point P at a distance $\vec{r} = r\hat{r}$ from a small length of loop $\Delta\vec{l}$ carrying current $I$ is $\Delta\vec{B} = \left(\frac{\mu_0 I}{4\pi}\right)\frac{\Delta\vec{l}\times\hat{r}}{r^2}$....(1). In the system small length of wire is RS such that $\Delta\vec{l} = (R\Delta\theta)\hat{l}$ and point is P at which magnetic flux density is to be determined is displaced by $\vec{r}$. As $\Delta\theta \to 0$ the element $\Delta\vec{l}$ becomes tangential to radial OA and hence (1) can be written as $\Delta\vec{B} = \left(\left(\frac{\mu_0 IR}{4\pi}\right)\frac{\sin\left(\frac{\pi}{2}+\alpha\right)}{r^2}\Delta\theta\right)\hat{\imath} \Rightarrow \Delta\vec{B} = \left(\left(\frac{\mu_0 IR}{4\pi}\right)\frac{\cos\alpha}{r^2}\Delta\theta\right)\hat{\imath}$...(2).

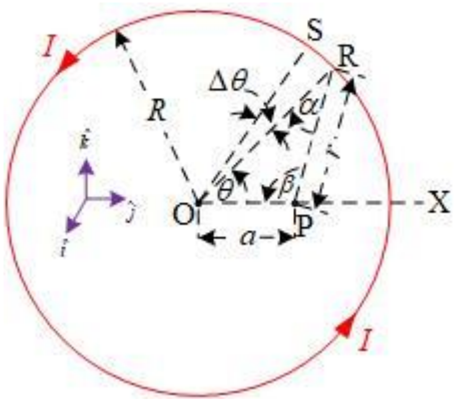



Fig. 1

**Problem Resolution:** It is seen that (2) has three variables such that $B = f(r, \alpha, \theta)$ and $B$ at P can be obtained by integrating (2) w.r.t. $\theta$ in the interval $[0,2\pi]$ to arrive at net magnetic field at the point due to the loop. Therefore, in function only of $\theta$, by eliminating $r$ and $\alpha$, with the related parameters $R$ and $a$ which are geometrical constants.

Using properties of triangle in ΔORP, $\frac{OP}{\sin\alpha} = \frac{RP}{\sin\theta} = \frac{OR}{\sin\beta} \Rightarrow \frac{a}{\sin\alpha} = \frac{r}{\sin\theta} = \frac{R}{\sin(\pi-(\alpha+\theta))} \Rightarrow \frac{a}{\sin\alpha} = \frac{r}{\sin\theta} = \frac{R}{\sin(\alpha+\theta)}$.

Accordingly, $r = a\frac{\sin\theta}{\sin\alpha}$ ...(3) and $\sin\alpha = \frac{a}{R}\sin(\alpha+\theta)$....(4).

It, further, solves into

$$\sin\alpha = \frac{a}{R}(\sin\alpha\cos\theta + \cos\alpha\sin\theta) \Rightarrow \left(1 - \frac{a}{R}\cos\theta\right)\sin\alpha = \frac{a}{R}\sin\theta\cos\alpha.$$

Introducing a normalization parameter $t = \frac{a}{R}$ which defines relative position of point P in the plane of loop w.r.t. its center O we have -

$$\Rightarrow (1 - t\cos\theta)\sin\alpha = t\sin\theta\,.\cos\alpha \Rightarrow (1 - t\cos\theta)\sin\alpha = t\sin\theta\sqrt{1-\sin^2\alpha}$$

$$(1 + t^2\cos^2\theta - 2t\cos\theta)\sin^2\alpha = t^2\sin^2\theta\,(1-\sin^2\alpha)$$

$$\Rightarrow (1 + t^2(\cos^2\theta + \sin^2\theta) - 2t\cos\theta)\sin^2\alpha = t^2\sin^2\theta$$

$$\Rightarrow \sin^2\alpha = \frac{t^2\sin^2\theta}{(1 + t^2 - 2t\cos\theta)}$$

$$\Rightarrow \sin\alpha = \frac{t\sin\theta}{\sqrt{(1+t^2-2t\cos\theta)}} \ldots(5)$$

$$\Rightarrow \cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{t^2\sin^2\theta}{(1+t^2-2t\cos\theta)} = \frac{(1+t^2-2t\cos\theta) - t^2\sin^2\theta}{(1+t^2-2t\cos\theta)}$$

$$\Rightarrow \cos^2\alpha = \frac{(1+t^2(1-\sin^2\theta) - 2t\cos\theta)}{(1+t^2-2t\cos\theta)} = \frac{(1+t^2\cos^2\theta - 2t\cos\theta)}{(1+t^2-2t\cos\theta)} = \frac{(1-t\cos\theta)^2}{(1+t^2-2t\cos\theta)}$$

$$\Rightarrow \cos\alpha = \frac{1-t\cos\theta}{\sqrt{(1+t^2-2t\cos\theta)}} \ldots(6)$$

Combining (3) $\left[r = a\frac{\sin\theta}{\sin\alpha}\right]$ and (5) $\left[\sin\alpha = \frac{t\sin\theta}{\sqrt{(1+t^2-2t\cos\theta)}}\right]$, we have –

$$r = a\frac{\sin\theta}{\frac{t\sin\theta}{\sqrt{(1+t^2-2t\cos\theta)}}} \Rightarrow r = R\sqrt{(1+t^2-2t\cos\theta)} \Rightarrow r^2 = R^2(1+t^2-2t\cos\theta)\ldots(7)$$

Combining (2),(6) and (7) we get –

$$\Delta B_t = \left(\left(\frac{\mu_0 IR}{4\pi}\right)\frac{\cos\alpha}{r^2}\Delta\theta\right) = \left(\frac{\mu_0 IR}{4\pi}\right)\frac{\frac{1-t\cos\theta}{\sqrt{(1+t^2-2t\cos\theta)}}}{R^2(1+t^2-2t\cos\theta)}\Delta\theta \Rightarrow \Delta B_t = \left(\frac{\mu_0 I}{4\pi R}\right)\frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}}\Delta\theta\ldots(8)$$

Therefore, net magnetic field at P is –

$$B_t = \int_0^{2\pi}\left(\frac{\mu_0 I}{4\pi R}\right)\frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}}\Delta\theta \Rightarrow B_t = \left(\frac{\mu_0 I}{4\pi R}\right)\int_0^{2\pi}\frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}}d\theta.$$

Taking the limits outside the integration, for convenience of substitution we get –

$$B_t = \left(\frac{\mu_0 I}{4\pi R}\right)\left[\int\frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}}d\theta\right]_0^{2\pi} \ldots(9)$$

The integration in (9), $F(t) = \left[\int\frac{(1-t\cos\theta)}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}}d\theta\right]_0^{2\pi}$ …(10), at the center of the loop O where $a = 0 \Rightarrow t = \frac{a}{R} = 0$ it reduces to the expression in (10) reduces to $F(0) = [\int d\theta]_0^{2\pi} \Rightarrow F(0) = 2\pi$ …(11). Thus, magnetic field at O, combining (9) and (11) is $B_O = \left(\frac{\mu_0 I}{4\pi R}\right)2\pi \Rightarrow B_O = \frac{\mu_0 I}{2R}$…(12), is in conformity with the known value of $B$ at the center of a current carrying loop.

Likewise, magnetic field at $a = R^- \Rightarrow t = \frac{R-h}{R}\Big|_{h\to 0} = 1 - \frac{h}{R}\Big|_{h\to 0}$, we have –

$$F(1) = \left[\int \frac{(1-\cos\theta)}{(1+1-2\cos\theta)^{\frac{3}{2}}} d\theta\right]_0^{2\pi} = \frac{1}{2\sqrt{2}}\left[\int \frac{1}{\sqrt{1-\cos\theta}} d\theta\right]_0^{2\pi} = \frac{1}{2\sqrt{2}}\left[\int \frac{1}{\sqrt{2\sin^2\frac{\theta}{2}}} d\theta\right]_0^{2\pi} \Rightarrow F(1) = \frac{1}{4}\left[\int \frac{1}{\sin\frac{\theta}{2}} d\theta\right]_0^{2\pi}$$

Taking $\frac{\theta}{2} = u \Rightarrow d\theta = 2du$ it leads to-

$$F(1) = \frac{1}{4}[\int \operatorname{cosec} u\,(2du)]_0^{2\pi} = \frac{1}{2}[\int \operatorname{cosec} u\, du]_0^{2\pi} = (-)\frac{1}{2}[\operatorname{cosec} u \cot u]_0^{2\pi} \ldots(13)$$

Making reverse substitution (13) we have -

$$F(1) = (-)\frac{1}{2}\left[\operatorname{cosec}\frac{\theta}{2}\cot\frac{\theta}{2}\right]_0^{2\pi} = \frac{1}{2}[\operatorname{cosec} 0 \cot 0 - \operatorname{cosec}\pi \cot\pi] = \frac{1}{2}[(\infty)\times(\infty) - (\infty)\times(\infty)] \ldots(14)$$

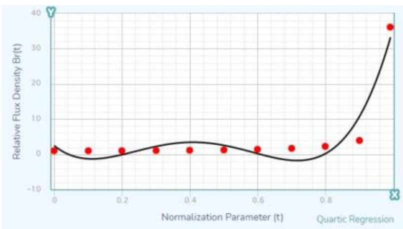
Thus, from (14), $F(1)$ is indeterminate.

Therefore, instead of calculating of determining pattern of $F(t)$, relative flux density $B_a = \frac{B_t}{B_0}$ is, combining (9) and (12) is –

$$B_{r_t} = \frac{\left(\frac{\mu_0 I}{4\pi R}\right)\left[\int \frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}} d\theta\right]_0^{2\pi}}{\frac{\mu_0 I}{2R}} = \frac{1}{2\pi}\left[\int \frac{1-t\cos\theta}{(1+t^2-2t\cos\theta)^{\frac{3}{2}}} d\theta\right]_0^{2\pi} \ldots(15).$$

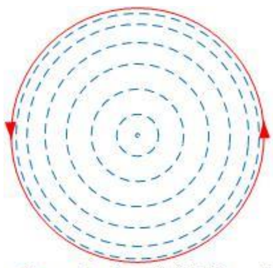
Combining (10) and (15), it leads to $B_{r_t} = \frac{1}{2\pi} \times F(t) \ldots(16)$

The integration $F(t)$ in (10), a part of (16), is not solvable by normal methods and pattern of flux density distribution has been determined using Trapezoidal Rule numerical method, using MS-Excel, in an interval $t = [0, 0.99)$. Results are plotted in Fig. 2 using MyCurveFit, Online Curve Fitting software (https://mycurvefit.com/). As $a \to R \Rightarrow t \to 1$ the integration $F(1)$ tends to be indeterminate and hence not plotted. Thus distribution of magnetic field over the cross-section of the loop , which is denser near the perimeter of the loop and rarer at the center, in the form of circular contours of uniform magnetic fields, is shown in Fig. 3.




Variation of Flux Density with Normalization Parameter (t)

Fig. 2



Conceptual graph Of Magnetic Contours in A Loop Carrying Current (Not to the scale)

Fig. 3

**Data Calculated Numerically:** Data used in Fig. 2, is as under -

| $t=\frac{a}{R}$ | $Br_t$ | $t=\frac{a}{R}$ | $Br_t$ | $t=\frac{a}{R}$ | $Br_t$ |
|---|---|---|---|---|---|
| 0 | 1 | 0.4 | 1.141324 | 0.8 | 2.257082 |
| 0.1 | 1.006735 | 0.5 | 1.245621 | 0.9 | 3.925924 |
| 0.2 | 1.031171 | 0.6 | 1.410594 | 0.99 | 36.10549 |
| 0.3 | 1.073742 | 0.7 | 1.692237 | 1 | Indeterminate |

**Conclusion:** Non-uniform magnetic field in the cross-section of the loop will impact philosophy of design of transformer core which is currently using uniform magnetic material in transformer core. Thus, this paper opens an opportunity to review overall design of transformer specially those used in instrumentation and control where errors due to core losses and magnetizing current are significant at macro level. At micro level it calls for review of magnetic forces that would influence configuration of orbital motion of electrons in atoms. Thus, review of overall spectrum of physics.

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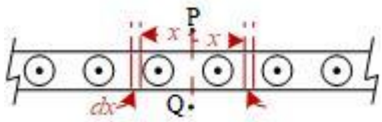
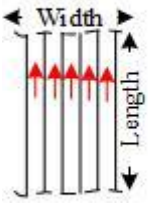
## Appendix-IV

# Magnetic Field at Any Point Near a Large Metal Sheet Having Uniform Electric Current




A large metal carry surface current such that it can be approximated thin strips $dx$ of width as given in the figure carrying current $di = Kdx$…(1),as considered in ths formulation. Effect of magnetic of the metal sheet at any point of the points P, Q and R can be determined by considering lay of the wires along long straight wire spread across the width of the sheet, as shown in the figure.




Using (1) net current in the strip of width $2x$ is as under –

$$i = \int di = \int_0^{2x} Kdx \Rightarrow i = 2Kx\ldots(2)$$


Since sheet is large, therefore, for determination of magnetic field at any point we can safely model two elemental wires of the sheet symmetrically placed on both sides of the point. Accordingly, two hypothetical points A and B are taken for determination of magnetic field by elemental wires C and D carrying current, in a direction coming out of the surface of the figure. Applying Biot-Savart's Law for magnetic at a point due to a long straight wire $B = \frac{\mu_0 I}{2\pi r}$…(3).

Accordingly, at point A magnetic field due to current in C is $dB_{CA} = \frac{\mu_0}{2\pi\sqrt{x^2+a^2}}di$, along perpendicular to CA and likewise due to current in D is $dB_{DA} = \frac{\mu_0}{2\pi\sqrt{x^2+a^2}}di$. along perpendicular to DA.

Thus,$dB_{\mathrm{A}} = dB_{CA}\cos\left(\frac{\pi}{2}-\alpha\right) + dB_{CA}\cos\left(\frac{\pi}{2}-\alpha\right)$

$\Rightarrow dB_{CA}\sin\alpha + \mathrm{d}B_{CA}\sin\alpha$.

Using geometry shown I the figure and (1), $dB_{\mathrm{A}} = 2\times\frac{\mu_0}{2\pi\sqrt{x^2+a^2}}\times\frac{a}{\sqrt{x^2+a^2}}di \Rightarrow dB_{\mathrm{A}} = \frac{\mu_0 a}{\pi(x^2+a^2)}Kdx\ldots(4)$

Applying results in (4), of this model, to the given problem, we have $dB_{\mathrm{A}} = \frac{\mu_0 Ka}{\pi(x^2+a^2)}dx\ldots(5)$. Therefore, $B_{\mathrm{A}} = \int_o^x \frac{\mu_0 Ka}{\pi(x^2+a^2)}dx \Rightarrow B_{\mathrm{A}} = \frac{\mu_0 Ka}{\pi}\int_o^x \frac{dx}{x^2+a^2}$. Geometrically $\tan\alpha = \frac{a}{x} \Rightarrow x = a\cot\alpha \Rightarrow dx = -a\,\mathrm{cosec}^2\,\alpha\, d\alpha$. It is seen in the figure that for a large sheet as $x\to 0 \Rightarrow \alpha \to \frac{\pi}{2}$, and as $x\to\infty \Rightarrow \alpha\to 0$. Accordingly, using the limits, $B_{\mathrm{A}} = \frac{\mu_0 Ka}{\pi}\left[\int \frac{1}{a^2\mathrm{cosec}^2\,\alpha} a\,\mathrm{cosec}^2\,\alpha\, d\alpha\right]_{\frac{\pi}{2}}^{0} \Rightarrow B_{\mathrm{A}} = \frac{\mu_0 K}{\pi}[\int d\alpha]_{\frac{\pi}{2}}^{0} \Rightarrow B_A = \frac{\mu_0 K}{\pi}[\alpha]_{\frac{\pi}{2}}^{o}$. This reduces to $B_{\mathrm{A}} = \frac{\mu_0 K}{\pi}\left[0-\frac{\pi}{2}\right]$. It leads to $B_{\mathrm{A}} =$

$-\frac{\mu_0 K}{2}$...(6). Likewise, magnetic field at point B would be $B_B = \frac{\mu_0 K}{2}$...(7), but it is in a direction opposite to $B_A$ and can be verified with Ampere's Right Hand Thumb Rule.

***Important observation:*** *Results at (6) to (9) lead to a conclusion that magnitude of magnetic field around a large sheet carrying current is proportional to current density and is independent of distance from the sheet.* Directions of the magnetic fields can be verified with Ampere's Right Hand Thumb Rule.

*This analysis can be simplified using Ampere's Circuital Law $\oint B.dl = \mu_0 I$ to determine magnetic field at any point near a large sheet of width $x$, current is $I = kx$ and length of the path surrounding a large sheet is $l = 2x$. Taking $\vec{B}$ along $d\vec{l}$. As per the law it leads to $B \times 2x = \mu_0 \times KX \Rightarrow B = \frac{\mu_0 K}{2}$. In the problem sheet is taken to be large and hence end effect on path length can be ignored.*

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**Important Note:** You may encounter need of clarification on contents and analysis or an inadvertent typographical error. We would gratefully welcome your prompt feedback on mail ID: subhashjoshi2107@gmail.com. If not inconvenient, please identify yourself to help us reciprocate you suitably.

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