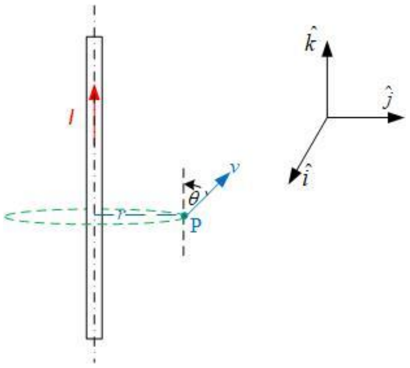
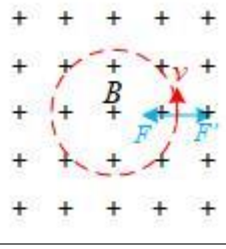
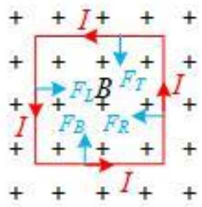
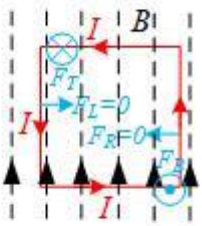
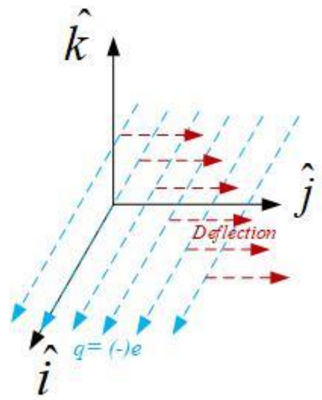
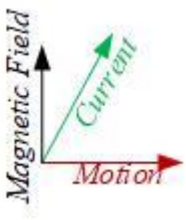
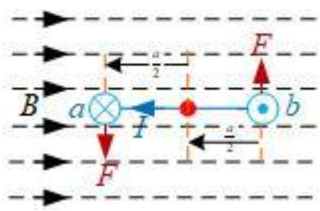
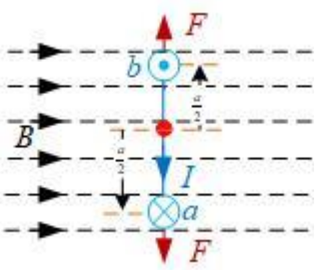
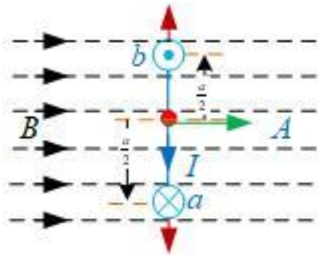
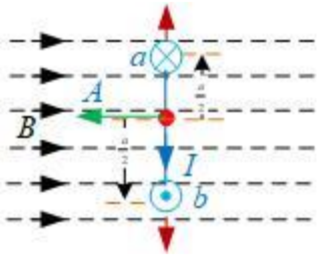
Electromagnetism: Magnetic Effect of Electric Current (Set-3)

Illustrations Only

I-01	Given system is shown in figure. Conductor carrying current I is oriented along $\hat{k}$. A particle P carrying charge q, at distance r from the conductor is moving with a velocity $\vec{v}$ at an angle θ w.r.t. to conductor, as shown in the figure. Magnetic field, as per Biot-Savart's Law, at the location of charge. is $\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{\imath})\ldots(1)$.  Therefore, magnetic force experienced by the charge, as per Lorentz's Force Law would be $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB\sin\alpha\ldots(2)$. Here, is angle between magnetic field and velocity of the particle. It can be visualized from the figure that $\alpha = 90^0 \Rightarrow \sin 90^0 = 1\ldots(3)$. Combining (1), (2) and (3), force $F = qv \times \frac{\mu_0 I}{2\pi r}\ldots(4)$. None of the parameters in (4) are zero and hence the charged particle would experience a force. The problem further states that the particle is seen from a frame of reference which is moving with velocity v in the same direction as that of conductor i.e. velocity of the observer is also $\vec{v}_0 = \vec{v}$. It implies that relative velocity of the observer w.r.t. particle is $v_r = \vec{v}_0 - \vec{v} \Rightarrow v_r = \vec{v} - \vec{v} = 0$. Now the analysis of the two parts is as under – **Part (a):** Force experienced by the particle in (2) depends upon its velocity at the point w.r,t. magnetic field at the point. It has nothing to do with velocity of the observer. Since both velocity and magnetic fields are non-zero and hence force would exist. Hence, **answer to part(a) is NO.** **Part (b):** Magnetic field at any point around a current carrying conductor is regulated by (1). It has nothing to do with velocity of either of particle or the observer. Moreover, none of the parameters in (1) are zero and magnetic field would exist. Hence, **answer to part(a) is Yes.** Thus, a**nswers are (a) No (b) Yes.**
I-02	As per Lorentz's Force Law force experienced by a charged particle is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB\sin\alpha\,\hat{n}\ldots(1)$. In this variables that affects the force are – **(a)** q is the charge of the particle, which is non-zero as per statement of the problem, **(b)** v is the relative velocity of particle w.r.t. magnetic field in which it is placed, **(c)** B is intensity of magnetic field in which it is placed, this is non-zero by the statement of the problem, and **(d)** α is the angle of velocity vector$\vec{B}$ w.r.t to magnetic vector$\vec{v}$. The direction of force is $\hat{n}$ which perpendicular to vectors $\vec{v}$ and $\vec{B}$. Thus for acceleration of the particle, as per Newton's Second Law of Motion $\vec{a} = \frac{\vec{F}}{m}$force must be zero. This is possible only if – **(i)** charge is in state of rest $\vec{v} = 0$, **(ii)** $\sin\alpha = 0 \Rightarrow \alpha \in \{0, \pi\}$ i.e. if $\vec{v} \neq 0$ the velocity of the particle is either in direction of magnetic field or opposite to it, or **(iii)** both $\vec{v} = 0 \cap \sin\alpha = 0$. Hence **answer to first part is Yes**. If conditions of acceleration (i) or (ii) discussed are **False,** then particle can be accelerated. If particle is accelerated then direction of force is along $\hat{n}$ and hence it will act like a centripetal force. Thereby, particle will perform uniform circular motion of radius $F = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{F}$, without change of velocity, as shown in the figure. Hence, **answer to the second part is No**. 

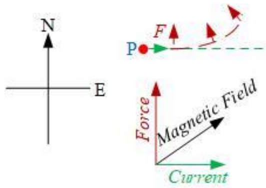
	Thus **answers are Yes, No.**
I-03	Current loop is a closed path in which some current $\underline{I}$ is flowing. The loop when placed in magnetic field it would experience force as per Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\alpha\, \hat{n}$…(1). In this variables that affects the force are – **(a)** I is the current in the loop which is non-zero as per statement of the problem, **(b)** l is the length of the loop; it will have to considered in segments since loop is closed, **(c)** B is intensity of magnetic field in which it is placed, this is non-zero by the statement of the problem, and **(d)** α is the angle of magnetic field vector $\vec{B}$ w.r.t to length vector $\vec{l}$. The direction of force is $\hat{n}$ which perpendicular to vectors $\vec{l}$ and $\vec{B}$. For force experienced by loop to be zero, it possible only if – **(i)** current in the loop is $I \neq 0$ of rest $\vec{v} = 0$, **(ii)** magnetic field $B \neq 0$ (iii) length of any segment $l \neq 0$, **(iv)** coil is so placed that for every elemental length $\sin\alpha = 0 \Rightarrow \alpha \in \{0, \pi\}$ for the segment; this is not possible even if loop is ***(a)*** either placed such that its area vector is perpendicular to magnetic field or ***(b)*** along the magnetic field, and (v) net force on the loop is zero.  For simplicity a square loop in two extreme positions as brought out at (iv) is being discussed.  In case of orientation at (a), shown in figure, each of the side is experiencing equal force, yet forces in left side F_L and right side F_R of the loop are opposite. Likewise forces on top side F_T and bottom side F_B are opposite. Thus net force on the loop is zero. Now analyzing forces on sides in orientation (b), forces experienced by top side F_T and bottom side F_B are equal and opposite. Thus, these two sides contribute to each side are equal. However, forces on left side F_L and right side F_R, both are equal to zero. Same applies to loop placed in any orientation. Thus force experienced by a current carrying loop placed ina magnetic field is Zero. Hence, **answer is Yes.**
I-04	Free electrons in a conductor, vis-a-vs normal conducting wire perform thermal motion, defined as Brownian Motion, such that net number of electrons passing through across-section is zero. Thus current through a cross-section wire is $I = 0$…(1) Yet, the only charge carrier capable of causing current are electrons. Therefore, as per Lorentz's Force law each electron having charge $q$in motion moving with a velocity $\vec{v}$ would experience a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ …(2), when placed in magnetic field $\vec{B}$. Hence, **answer to the first part is Yes.** But, as regards force experienced by the conducting wire, extension of the Lorentz's Force Law would be applicable. Accordingly, $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B}$…(3). Combining (1) and (3), the force experience by the conducting wire would be zero. Thus, **answer to the second part is No.** Thus **answers are (a) Yes and (b) No.**
I-05	In a cubical region of side a, largest circle that can be drawn is of radius $\frac{a}{2}$. A particle carrying charge q moving with a velocity $\vec{v}$ in magnetic field $\vec{B}$ experiences a force $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\alpha\, \hat{n}$ …(1). Here, and α is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$. The direction of force is $\hat{n}$ which perpendicular to vectors $\vec{v}$ and $\vec{B}$. This force acts as centripetal force, a necessary condition for circular motion of radius r. Combining such that $F = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{qv \sin\alpha}$…(2). In case $r \leq \frac{a}{2}$ the charged particle will continue describe circular motion in the cubical region. Thus, **answer is Yes, if radius of circle is less than or equal to half of the side of the cube.**

I-06	Given that electron beam having charge of each electron $q = (-)e$ …(1), is projected along X-axis with some velocity $\vec{v} = v\hat{\imath}$…(2). Deflection of the beam is given to be along Y axis i.e. in direction $\hat{\jmath}$. Such a deflection is possible only when the electron beam experiences a force $\vec{F} = F\hat{\jmath}$ i.e. along $\hat{\jmath}$.  This system can be analyzed using Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B}$…(3). Combining (1) and (2) in (3) we have $F\hat{\jmath} = (-e)(v\hat{\imath}) \times \vec{B}$…(4). We know that vector algebra does not permit division. However, using  Flemings's Left Hand Rule (FLHR) as shown in the figure, a simple way of analysis derived from Lorentz's Force Law, (4) has to be satisfied. Accordingly, figure of FLHR is oriented to align with the given directions of current (it is opposite to direction of electron beam since electrons are (-)ve charge carriers) and deflections. It leads to the conclusion that magnetic field is along Z-axis i.e. in direction $\hat{k}$. Thus, **magnetic field is along $\hat{k}$ answer to part (a)**. This implies that **magnetic field is parallel to Z-axis, is answer to part (b)**. Thus answers are **(a) Magnetic field along $\hat{k}$, (b) Magnetic field is parallel to Z-axis**
I-07	Rotation of a system is effect of a torque $\vec{\Gamma} = \vec{r} \times \vec{F} \Rightarrow \vec{\Gamma} = rF \sin\alpha\ \hat{n}$…(1). Here, r is the distance of point of application of force F from the point of rotation, α angle of vector $\vec{F}$ w.r.t. to vector $\vec{r}$. Therefore, it is possible for a current loop to stay without rotating only if the loop is just placed without a point of rotation fixed. But, in electrical devices current loops are mounted on a shaft which serves as point of rotation of loop. And sides of current loop, which may experience force, would experience force when placed in magnetic field. Thus, it tends to satisfy requirements of (1). Hence, **answer for first part is Yes.** In respect of orientation of loop in magnetic field coil, for convenience, is considered to be rectangular. Two extreme orientations are shown in figure.  In first orientation side ab of the rectangular loop is along the magnetic field and  it is carrying current I form b to a. Thus, as per Flemings Left Hand Rule (FLHR) the side of loop in which current is entering at end 'a' would experience a downward force F as per Lorentz's Force Law (LFL). Likewise, side of the loop in which current is coming out at end 'b' would experience an upward force F as per LFL. This in this orientation when area vector of the loop is perpendicular to the magnetic fields. The loop would experience a torque. But, in orientation when area vector is parallel to the magnetic field though sides of coil at 'a' and 'b' would experience force as per LFL, but force distance vector being collinear $\alpha = 0 \Rightarrow \sin\alpha = 0$, torque as per (1) would be zero. Thus, **answer for second part is when area vector of loop is parallel or anti-parallel to magnetic field.**
I-08	Despite net charge on a current carrying wire being zero, cloud of free electron in wire experience a unidirectional drift. This drift is responsible for current in the wire. When current carrying wire is placed in magnetic field, as per Bio-Savart's Law it produces a magnetic field around it. Interaction of these two magnetic field produces reorientation of magnetic field causing a force F on conductor as per Lorentz's Force Law expressed as $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\alpha\,\hat{n}$. Here, I is the current through wire, l is length of wire, B is magnetic field in which wire is placed, and α is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$. The three parameters I, l, B are non- zero as per statement of problem. Therefore, if $\alpha \neq 0$, i.e. wire is not parallel to magnetic field then it will experience force.

I-09	Despite net charge on a current carrying wire being zero, cloud of free electron in wire experience a unidirectional drift. This drift is responsible for current in the wire. When current carrying wire is placed in magnetic field, as per Bio-Savart's Law it produces a magnetic field around it. Interaction of these two magnetic field produces reorientation of magnetic field causing a force F on conductor as per Lorentz's Force Law expressed as $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \ \vec{F} = IlB \sin\alpha\, \hat{n}$. Here, I is the current through wire, l is length of wire, B is magnetic field in which wire is placed, and α is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$. The three parameters I, l, B are non- zero as per statement of problem. Therefore, if $\alpha \neq 0$, i.e. wire is not parallel to magnetic field then it will experience force. Thus, in the given system for force to be zero there are two possibilities –  **(i)** Angle $\alpha = 0 \Rightarrow \sin\alpha = 0$ between area vector $\vec{A}$ and magnetic field vector $\vec{B}$. This case is shown in figure where 'a' of the loop through which current is entering is below and 'b' of loop through which current is leaving is above. This is the natural position of least potential energy $U_i = 0$ of the current carrying loop**, a state of stable equilibrium.**  **(ii)**The angle $\alpha = \pi \Rightarrow \sin\alpha = 0$. his case is shown in figure where 'a' of the loop through which current is entering is above and 'b' of loop through which current is leaving is below. Position of current carrying loop in this case requires work to be done by external force to rotate coil for position in case (i). Thus potential energy of the current carrying loop in this case $U_i > 0$. Hence, current carrying loop in this case despite forces on the loop being in equilibrium, **the loop is in instable equilibrium.** Thus, **answer is $\boldsymbol{\alpha = 0}$** **N.B.:** Area vector $\vec{A}$ is of magnitude equal to the area A under consideration and in a direction such that direction of perimeter is anti-clockwise direction. This is consistent with angles measured (+)ve in anticlockwise direction
I-10	Comparison of two units of different physical quantities require to convert them into to their units. This exercise is similar to that determining dimensions of any physical quantities into fundamental dimensions. Same process will be followed for units weber and volt-second. As per Lorentz Force Law force F experienced by a charge q moving with a velocity v in magnetic field B is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow B = \frac{F}{qv}$…(1). Unit of force is $N = kg.m.s^{-2}$…(2), unit of charge is $q = C$…(3), unit of velocity is $v = m.s^{-1}$…(4). Thus, combining (2), (3) and (4) in (1) we have unit of flux density is $B = \frac{kg.m.s^{-2}}{(C)(m.s^{-1})} \Rightarrow B = kg.C^{-1}.s^{-1}$…(5) Further, flux density $B = \frac{Wb}{m^2} \Rightarrow Wb = B.m^2$….(6). Combining (5) and (6) $Wb = kg.m^2.s^{-1}.C^{-1}$…(7). Now, let us derive unit Volt-s in basic units. Volt is uit of electric potential at a point is equal to amount of work done in moving a unit charge from infinity to that point. Thus, $V = \frac{Joule}{Coulomb} \Rightarrow V = \frac{N.m}{C}$ …(8). Combining (2) and (8), we have $V = \frac{kg.m.s^{-2}.m}{C} \Rightarrow V = kg.m^2.s^{-2}.C^{-1}$…(9). Therefore, using (9) unit of $Volt.Sec = (kg.m^2.s^{-2}.C^{-1})s \Rightarrow Volt.Sec = kg.m^2.s^{-1}.C^{-1}$…(10). Thus, **from (7) and (10) it is verified that unit of Weber and Volt-Sec are same.**
I-11	Given system with directional orientations as stipulated in the problem is shown in the figure. The positively charged particle is projected towards east and hence direction of current would also be east. Further, the particle is stated to be deflected towards north. Deflection is in direction of force and hence force F acting on the particle is along north direction.

Force experienced by a chargedparticle in motion in magnetic field is defined by Lorentz's Force Law expresed as $\vec{F} = q\vec{v} \times \vec{B}$…(1). These vectors are simplied for use in Flamming's Left Hand Rule (FLHR). Accordingly, figure shows FLHR aligned to directions of current and deflections.

Thus as per figure directin of magnetic field is away from the observer, or in parlance of the given directions downward. Hence, **answer is option (d).**

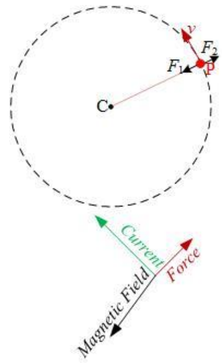


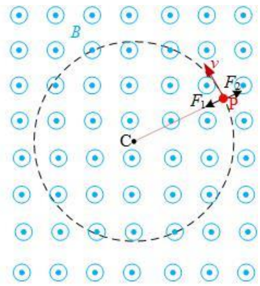
Flamming's Left Hand Rule

I-12

Given system of a charged particle in horizontal circular motion on a frictionless table is shown in the figure. The direction of rotation is initially taken to be anticlockwise. The string attached to the center C of the circle and particle P will exert a centripetal force F_1 on the particle. As a result of it, the particle will experience an equal opposite centrifugal force F_2. Thus, the particle, in a state of equilibrium would continue to perform uniform circular motion.

Next, magnetic field B in vertical direction is switched ON as shown in adjoining figure. The moving charged particle in accordance with Lorentz's Force Law (LFL), expressed as $\vec{F} = q\vec{v} \times \vec{B}$, would experience a force.





Simplistic representation of directions of vectors is given by Flamming's Left Hand Rule (FLHR). Accordingly, figure shows FLHR aligned to directions of current and magnetic field, as given.

Accordingly, additional force F experienced by the charged particle will be in the direction of F_2. As a result net centripetal force would be $F_2{}' = F_2 + F$. As a result it will tend to increase radius of the circle. String, though not specified is considered non-strechable, would experience tension $F_1' = F_2' \Rightarrow F_1' > F_1$.

Thus, *tension in the string would increase in this case of uniform circular motion where direction of rotation is anticlockwise.* However, if particle is performing clockwise motion, the direction of force would reverse. Therefore, centripetal force would become$F_2' = F_2 - F$. In state of equilibrium for uniform circular motion $F_1' = F_2' \Rightarrow F_1' < F_1$; *tension in the string would decrease in this case of uniform circular motion where direction of rotation is clockwise.*

Thus, **answer is tension in string may increase or decrease as provided in option (d)**

I-13

Force F experienced by charged particle with a velocity v, perpendicular to magnetic field B, as per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB \sin\theta$…(1), here θ is the angle of velocity vector $\vec{v}$ relative to magnetic field vector $\vec{B}$. Further, and θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$. it is given that the vectors $\vec{v}$ and $\vec{B}$ are perpendicular to each other hence $\theta = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1$…(2). Thus, combing (1) and (2), and that velocity v and magnetic field B are same for all the particles, we have $F \propto q$.

Taking absolute magnitude of charge of electron to be e for the given particles –

(a) Electron has charge $q_E = -e \Rightarrow F_E \propto -e$.
(b) Proton has charge $q_P = e \Rightarrow F_E \propto e$
(c) He^+ ion has charge $q_H = e \Rightarrow F_E \propto e$
(d) Li^{++} ion has charge $q_L = 2e \Rightarrow F_E \propto 2e$

It is seen that –

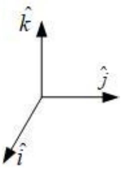
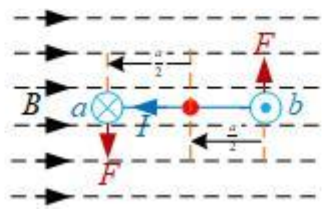
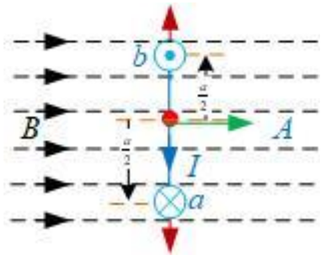
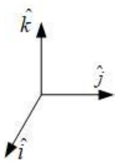
(i) Direction of experienced by electron is opposite to that other three given particles,
(ii) Magnitude of forces experienced by electron, proton and Helium ions are equal. But force experienced by Li^{++} ion is twice of the magnitude of other three particles.
(iii) Hence, *maximum magnitude of magnetic force is experienced by Li^{++} ion, matches with the option (d).*

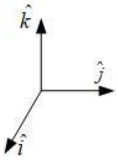
Thus, **answer is option (d).**

I-14

Force F experienced by charged particle with a velocity v, perpendicular to magnetic field B, as per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB \sin\theta$…(1), here θ is the angle of velocity vector $\vec{v}$ relative to magnetic

	field vector $\vec{B}$. Further it is given that the vectors $\vec{v}$ and $\vec{B}$ are perpendicular to each other hence $\theta = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1$…(2). Thus, combing (1) and (2), and that velocity v and magnetic field B are same for all the particles, magnetic force experienced by the charged particle is $F_m = F = qvB$…(3) Taking absolute magnitude of charge of electron to be e for the given particles – (a) Electron has charge $q_E = -e \Rightarrow F_E \propto -e$. (b) Proton has charge $q_P = e \Rightarrow F_E \propto e$ (c) He^+ ion has charge $q_H = e \Rightarrow F_E \propto e$ (d) Li^+ ion has charge $q_L = e \Rightarrow F_E \propto e$ In the given situation charged particle will describe circular motion or radius r that centripetal force $F_C = \frac{mv^2}{r}$ …(4) is experienced by each particle is equal to magnetic force i.e. $F_C = F_m$…(5). Combining (3), (4) and (5) we have $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$. Given that velocity v and magnetic field B are same for all the four particles and charge of each particle as discussed above is also same, we have $r \propto m$…(6). We know that mass of electron is minimum among the given four particles. Hence, *radius of circle described electron is minimum, as given option (a).* Thus, **answer is option (a).**
I-15	Force F experienced by charged particle with a velocity v, perpendicular to magnetic field B, as per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB\sin\theta$…(1), here θ is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{v}$. Further, it is given that the vectors $\vec{v}$ and $\vec{B}$ are perpendicular to each other hence $\theta = \frac{\pi}{2} \Rightarrow \sin\frac{\pi}{2} = 1$…(2). Thus, combing (1) and (2), and that velocity v and magnetic field B are same for all the particles, magnetic force experienced by the charged particle is $F_m = F = qvB$…(3) Taking absolute magnitude of charge of electron to be e for the given particles – (a) Electron has charge $q_E = -e \Rightarrow F_E \propto -e$. (b) Proton has charge $q_P = e \Rightarrow F_E \propto e$ (c) He^+ ion has charge $q_H = e \Rightarrow F_E \propto e$ (d) Li^+ ion has charge $q_L = e \Rightarrow F_E \propto e$ In the given situation charged particle will describe circular motion or radius r that centripetal force $F_C = \frac{mv^2}{r}$ …(4) is experienced by each particle is equal to magnetic force i.e. $F_C = F_m$…(5). Combining (3), (4) and (5) we have $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$. Given that velocity v and magnetic field B are same for all the four particles and charge of each particle as discussed above is also same, we have $r \propto m$…(6). It is given that velocity v of projection of each of the charged particle is same and hence frequency of revolutions is nothing but number of revolutions-per-second $f = n = \frac{v}{2\pi r} \Rightarrow f \propto \frac{1}{r}$…(7). We know that mass of electron is minimum among the given four particles. Whereas mass of Li^+ ion is maximum. Hence, radius of circle described electron R_e is minimum R_{min}, while radius of circle of Li+ ion R_L is maximum R_{max}. Applying these discussions in (7) minimum frequency shall be of the particle describing circle of longest radius i.e. R_{max}. This conclusion conforms with the option (d). Hence, **answer is option (d).**
I-16	To understand torque experienced by a loop carrying current $I = 10$A placed in magnetic field $B = 0.1$ T, a simple case of a rectangular loop abcd is taken as shown in the figure. In this case the side ab of the loop is facing observer and a current is entering at 'a' and after passing through the coil it is returning from 'b'.

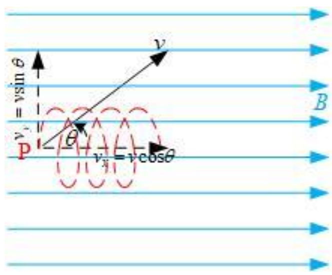
	 As per Lorentz's Force Law, force F will be experienced by the sides ad and bc of length l such that $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\alpha\, \hat{n}$…(1). Here, α is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$. In this case $\alpha = \frac{\pi}{2} \Rightarrow \sin\alpha = \sin\frac{\pi}{2} = 1$. Therefore, $\vec{F} = IlB\hat{n}$. Torque experienced by the loop at its axis would be sum of torques $\vec{\Gamma}$ on both the sides ad and bc. Thus, $\vec{\Gamma} = \vec{\Gamma}_{ad} + \vec{\Gamma}_{bc} = \left(IlB(-\hat{k})\right) \times \left(\frac{a}{2}(-\hat{j})\right) + \left(IlB(\hat{k})\right) \times \left(\frac{a}{2}(\hat{j})\right) = \frac{IlBa}{2}(\hat{\imath} + \hat{\imath})$. It leads to $\Gamma = I(la)B \Rightarrow \Gamma = IAB$. ..(2). Here, $A = ab$ is the area of the loop and the 3-D vectors used as reference are shown in the figure.  In the problem it is stated that magnetic field is perpendicular to the plane of the loop and this situation is shown in the figure. In this case total torque on the loop is $\vec{\Gamma} = \vec{\Gamma}_{ad} + \vec{\Gamma}_{bc} = \left(IlB(-\hat{k})\right) \times \left(\frac{a}{2}(-\hat{k})\right) + \left(IlB(\hat{k})\right) \times \left(\frac{a}{2}(\hat{k})\right)$. It leads to  $\vec{\Gamma} = \frac{IAB}{2}(\hat{k} \times \hat{k})$….(3). Thus, for two unidirectional unit vectors is $\hat{k} \times \hat{k} = 0$ …(4). Combining (3) and (4) $\Gamma = 0$. *This conclusion matches with option (a).* Hence. **answer is option (a).**
I-17	Given are beams of electron and protons moving with As per Lorentz's Force Law, force F will be experienced by the sides ad and bc of length l such that $\vec{F} = q\vec{v} \times \vec{B} = qvB \sin\alpha\, \hat{n}$…(1). Here, α is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$, which is stated to be perpendicular i.e. $\alpha = \frac{\pi}{2} \Rightarrow \sin\alpha = 1$ …(2). Thus, combining (1) and (2) we have $\vec{F} = qvB\hat{n}$…(3). Charge and mass of electrons are $q_e = -e$ and $m_e = m$ while that of proton are $q_e = e$ and $m_e = 1836m$. Thus forces experienced by electrons and protons are, $\vec{F_e} = -evB\hat{n}$ and $\vec{F_p} = evB\hat{n}$ and accelerations as per Newton's Second Law of Motion would be $\vec{a}_e = \frac{\vec{F_e}}{m_e} \Rightarrow \vec{a}_e = -\frac{evB}{m}\hat{n}$ and $\vec{a}_p = \frac{\vec{F_p}}{m_p} \Rightarrow \vec{a}_p = \frac{evB}{1876m}\hat{n}$…(4) It is seen from (4) that – *(i)* Accelerations of electrons and protons are in opposite direction, would *cause separation of the two beams* (ii) Magnitude of acceleration of electron $a_e = 1876a_p$, and hence angle of deviation, *(iii)* Combined effect of (i) and (ii)) in First Equation of Motion $\vec{v} = \vec{u} + \vec{a}t$ at any instant t *deviations of the two beams would also be different.* These conclusions match with option (c). Hence, **answer is option (c).**
I-18	 Given that a charged particle is projected with a velocity v making an acute angle θ with the uniform magnetic field, as shown in the figure. Let us consider the particle to be positively e positively charged. A charged particle in motion in magnetic field, as per Lorentz's Force Law experiences a force $\vec{F} = q\vec{v} \times \vec{B}$…(1). The 3D unit vectors are shown separately. Accordingly, for simplicity, the velocity vector $\vec{v} = v\cos\theta\,\hat{j} + v\sin\theta\,\hat{k}$ …(2) while magnetic field is $\vec{B} = B\hat{j}$…(3). Here, θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$ Combing (1), (2) and (3) $\vec{F} = q\left(v\cos\theta\,\hat{j} + v\sin\theta\,\hat{k}\right) \times B\hat{j} \Rightarrow \vec{F} = qvB\left[\cos\theta\,(\hat{j} \times \hat{j}) + \sin\theta\,(\hat{k} \times \hat{j})\right]$…(4) Since, $\hat{j} \times \hat{j} = 0$ and $\hat{k} \times \hat{j} = -\hat{\imath}$ the (4) leads to $\vec{F} = qvB(-\hat{\imath})$. Thus the particle P would perform a circular motion in $\hat{\imath} - \hat{j}$ plane. Yet, the velocity component $v\cos\theta\,\hat{j}$ would cause a constant drift of the circular motion along $\hat{j}$. Hence, *resultant motion of the particle would be a uniformly placed helix or a helix of uniform pitch, as provided in option (c).* **Thus, answer is option (c).**

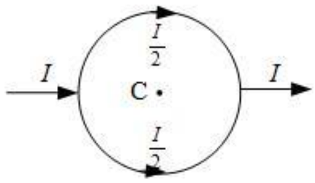
<table>
<tr><td>I-19</td><td>Given that a charged particle moves in a uniform magnetic field and a parallel uniform electric field. For convenience unit direction vectors are indicated separately.


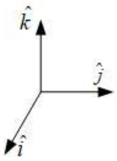
As per mechanics, motion of a particle is influenced by forces acting on it. And forces on a charged particle in this case are –
(a) Force due to magnetic field is governed by Lorentz's Force Law, thus $\vec{F}_M = q\vec{v} \times \vec{B}$…(1).
(b) Force due to electric field is governed by Coulomb's Force Law. Thus, $\vec{F}_E = q\vec{E}$…(2).
This problem is solved by first considering motion of the charged particle in uniform magnetic field. On this motion effect of uniform electric field is superimposed.

Let, initial velocity of the particle at an instant is v making an acute angle θ with the magnetic field, as shown in the figure. Let us consider the particle to be positively charged.

The 3D unit vectors are shown separately. Accordingly, for simplicity, the velocity vector $\vec{v} = v\cos\theta\,\hat{j} + v\sin\theta\,\hat{k}$ …(3) while magnetic field is $\vec{B} = B\hat{j}$…(4)

Combing (1), (3) and (4) $\vec{F} = q\left(v\cos\theta\,\hat{j} + v\sin\theta\,\hat{k}\right) \times B\hat{j}$. Solving this we have $\vec{F} = qvB\left[\cos\theta\,(\hat{j}\times\hat{j}) + \sin\theta\,(\hat{k}\times\hat{j})\right] \Rightarrow \vec{F} = qvB(-\hat{\imath})$…(5). It is uses vector products $\hat{j}\times\hat{j} = 0$ and $\hat{k}\times\hat{j} = -\hat{\imath}$. Thus the particle P would perform a circular motion in $\hat{\imath} - \hat{j}$ plane. Yet, the velocity component $v\cos\theta\,\hat{j}$ would cause a constant drift of the circular motion along $\hat{j}$. Hence, resultant motion of the particle would be a uniformly placed helix or a helix of uniform pitch without change in speed of the particle.


Effect of electric field is change in acceleration $\vec{a} = \frac{\vec{F}_E}{m}$. Thus, as per mechanics $v^2 = u^2 + 2as$. Thus, as particle traverses its velocity would change. As a result, the trajectory of the charged particle would be a helix whose (i) radius is increasing, and (ii) pitch is increasing. The option (d) is closest to the conclusions at (i) and (ii). Hence, answer is option (d).</td></tr>
<tr><td>I-20</td><td>Magnetic field B due to current I in a circular wire as per Biot-Savart's Law is $B = \frac{\mu_0 I}{2a}$…(1). For convenience 3D vectors are shown in a separate figure. In this system, as shown in the figure, current I across the circular wire in $(\hat{\imath} - \hat{j})$ plane


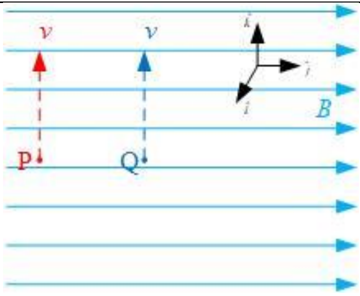


takes route through two semicircular arcs each carrying current $\frac{I}{2}$. Thus, using (1) magnitude of the magnetic field at C due to either of the semicircular arcs would be $B' = \frac{1}{2}\frac{\mu_0\frac{I}{2}}{2a} \Rightarrow \frac{I}{2} = \frac{\mu_0 I}{8a}$…(2). Yet as per Ampere's Right Hand Thumb rule magnetic field at C due to upper semicircular arc would be in direction $(-\hat{\imath})$ while due to lower semicircular arc would be in direction $(\hat{\imath})$.
Thus, net magnetic field at the center C would be $\vec{B}_C = \vec{B}_U + \vec{B}_L \Rightarrow \frac{\mu_0 I}{8a}(-\hat{\imath}) + \frac{\mu_0 I}{8a}(\hat{\imath}) = 0$…(1)
Further, it is given that a charge q is passing through the center with a speed v. It can be any angle say θ with direction $\hat{\imath}$.
Therefore, force experienced by the charge as per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B}$. In the instant case, using the available data it would be $\vec{F} = q(v\hat{\imath}) \times 0 = 0$. Thus, the moving charge would not experience any force as provided in option (d). Hence, answer is option (d).</td></tr>
<tr><td>I-21</td><td>Complete statement of Lorentz's Force Law for electro-magnetic force is $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$…(1). It is required to find conditions for the charged particle to be at rest i.e. $v = 0$.

Equation (1) can be split into$\vec{F} = \vec{F}_e + \vec{F}_m$ …(2) Here, electric force is $\vec{F}_e = q\vec{E}$…(3) and $\vec{F}_m = q\left(\vec{v} \times \vec{B}\right) \Rightarrow \vec{F}_m = qvB\sin\theta\,\hat{n}$…(4). Here, $\hat{n}$ unit vector perpendicular to the plane of vectors $\vec{v}$ and $\vec{B}$ and θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$. Thus, charged particle to remain at rest necessary condition as per mechanics is that the two forces must be in equilibrium. Thus combining (2), (3) and (4) we have $q\vec{E} + qvB\sin\theta\,\hat{n} = 0 \Rightarrow q\vec{E} = qvB\sin\theta\,(-\hat{n})$…(5).</td></tr>
</table>

	For particle to be at rest, (5) necessarily leads to $v=0 \Rightarrow F_m=0$…(6). Accordingly, possibilities that emerge are – (i) $E=0$ makes both the addends to be zero as *provided in option (a) is correct.* (ii) Discussions at (i) invalidates possibility of $E\neq 0$. Hence, *option (c) is incorrect.* (iii) In light of given (6), value of B, zero or non-zero does not matter. Hence it is not necessary that $B=0$. Hence, *option (c) is incorrect.* (iv) In view of discussions at (iii) above, the *option (d) is correct.* Thus, **answer is options (a) and (d)**
I-22	Given that a charged particle at rest $v=0$ experiences experiences a an electromagnetic force. As per Lorentz's Force Law for electro-magnetic force is $\vec{F}_{em}=q(\vec{E}+\vec{v}\times\vec{B}) \Rightarrow \vec{F}=q\vec{E}+q(\vec{v}\times\vec{B})$. It leads to $\vec{F}=\vec{F}_e+\vec{F}_m$…(1). It leads to $\vec{F}_e=q\vec{E}$…(2) and $\vec{F}_m=q(\vec{v}\times\vec{B}) \Rightarrow \vec{F}_m=qvB\sin\theta\,\hat{n}$…(3), is required to find conditions for the charged particle to be at rest i.e. $v=0 \Rightarrow a=0 \Rightarrow F=0 \Rightarrow \vec{F}_e+\vec{F}_m=0$, as per mechanics. This is possible whent either both $F_e=0$ and $F_m=0$ or $\vec{F}_e=-\vec{F}_m$ …(4), i.e. both are equal and opposite. These discussions lead to following possibilities- (i) If $E=0$ then it is must that $qvB\sin\theta=0$. In the given system, it is not necessary for either $B=0$. Thus, *option (a) is correct.* (ii) If $E\neq 0$ then for (4) to satisfy $qe=qvB\sin\theta$ either $B=0$ or $\sin\theta=0$ if $B\neq 0$. Thus, *option (d) is correct*. (iii) If $E\neq 0$ then for (4) to satisfy $qe=qvB\sin\theta$ condition provided in option (b), $B\neq 0$ alone would not suffice. Hence, *option (b) is ncorrect*. (iv) Option (c) includes possiblities at (i) and (ii) above which are contracdictory. Hence, *option (c) is incrrect.* Thus, **answer is option (a) and (d)**
I-23	Given that a charged particle deflects in a gravity free room. It implies that there is no gravitational force and only electromagnetic forces $\vec{F}_{em}\neq 0$. As per Lorentz's Fore Law $\vec{F}_{em}=q(\vec{E}+\vec{v}\times\vec{B})$. This can be decomposed as $\vec{F}_{em}=\vec{F}_e+\vec{F}_m$…(1). Here, $\vec{F}_e=q\vec{E}$…(2) and $\vec{F}_m=q(\vec{v}\times\vec{B}) \Rightarrow \vec{F}_m=qvB\sin\theta\,\hat{n}$…(3). Here, θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$ Given that the moving charged particle moving is deflected i.e. $v\neq 0$. It has following - (i) Electric field is either zero or along the direction of velocity of the charged particle. This will not change speed of particle but would deflect it due to $\vec{F}_m$. (ii) Electric field is non-zero, and not in the direction of motion of the charged particle, it deflect motion of the charged particle as well as speed. (iii) If there is magnetic field, as per (3) it will change direction of the motion without change in speed. Thus there will be deflection. In light of the above possibilities each of the given option is being analyzed – **Option (a):** This does not guarantee that $B=0$, henceas per possibility (ii)there would be deflection. Hence, *this option is incorrect.* **Option (b):** While $B\neq 0$, it does not guarantee that $E=0$. Therefore, possibility cancellation of deflection by $\vec{F}_m$ as per (3) due to deflection cause by $\vec{F}_e$ as per (2) is not assured. Thus if would be there due to $\vec{F}_{em}=\vec{F}_e+\vec{F}_m\neq 0$ and direction of $\vec{F}_{em}$ is not same as direction of motion of the charged particle deflection would be there as per (ii). Hence, alone $B\neq 0$ is not sufficient. Hence, *this option is incorrect.* **Option (c):** The three possibilities (i), (ii) and (iii) together would satisfy conditions of deflection. Hence, *this option is correct.*

	Option (d): In this case also the three possibilities (i), (ii) and (iii) together would satisfy conditions of deflection. Hence, *this option is correct.* Hence, **answer is option (c) and (d)**
I-24	Net force $\vec{F}$ experienced by particle of mass m, carrying charge $q \neq 0$ moving with a velocity $\vec{v}$ in magnetic field $\vec{B}$, electric field $\vec{E}$ and gravitational acceleration $\vec{g} = 0$ can be expressed by combining Newton's Laws of Gravitation $\vec{F_g} = m\vec{g}$ …(1) and Lorentz's Fore Law $\vec{F_{em}} = q(\vec{E} + \vec{v} \times \vec{B})$…(2). Thus, $\vec{F} = \vec{F_g} + \vec{F_{em}}$…(3). It is given that charged particle of mass $m \neq 0$ moves in gravity free space. It means $\vec{g} = 0 \Rightarrow \vec{F_g} = 0$…(4). Combining (2), (3) and (4), net force would be $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \Rightarrow \vec{F} = q\vec{E} + qvB \sin\theta\, \hat{n}$…(5). Here, $\hat{n}$ unit vector perpendicular to the plane of vectors $\vec{v}$ and $\vec{B}$ and θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$. Further, it is given that the particle is moving with uniform velocity, therefore, as per Newton's Laws of Motion $\vec{F} = 0$. This possible only when $q\vec{E} = -qvB \sin\theta\, \hat{n}$…(6)This leads to following possibilities- (i) If $E = 0$ and $B = 0$, or (ii) If $E = 0$ and $\theta = 0$; in this case condition $B \neq 0$ is valid, or (iii) If $E = 0$ and $\theta = \pi$; in this case condition $B \neq 0$ is valid, or (iv) If $E \neq 0$ and $B \neq 0$ but the electric field is in direction of $\hat{n}$ i.e. both electrical force and magnetic force are equal and opposite. These discussions are compared with the options given **Option (a):** Satisfies possibility (i), hence *this option is correct*. **Option (b):** Satisfies possibility (ii), hence *this option is correct*. **Option (c):** In this case there will acceleration due to electrostatic force, but magnetic force would be zero. Thus, equation (6) would not be satisfied. Hence, this option is incorrect. **Option (d):** Satisfies possibility (iv), hence *this option is correct*. Thus, **answers is option (a), (b) and (d).**
I-25	Given that a particle carrying charge q moves along a circle in presence of constant electric and magnetic field. It implies that the particle is experiencing a centripetal force $\vec{F_c}$. Charged particle in presence of given fields as per Lorentz's Fore Law would experience $\vec{F_{em}} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \vec{F_{em}} = \vec{F_e} + \vec{F_m}$…(1). Here, $\vec{F_e} = q\vec{E}$…(2) and $\vec{F_m} = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F_m} = qvB \sin\theta\, \hat{n}$…(3). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. velocity vector $\vec{v}$ Effect of $\vec{F_e}$ is an acceleration $\vec{a}_e = \frac{\vec{F_e}}{m} \Rightarrow \vec{a}_e = \frac{q\vec{E}}{m}$ …(4) which in accordance with the equations of motion $\vec{v} = \vec{u} + \vec{a}t$ would change velocity of particle. Thus, for $v = \text{Const}$ electric field $E = 0$. This contradicts conditions, necessary for a circular motion. Thus, *options (c) and (d) where given that $E \neq 0$ turn out to be incorrect.* Taking discussion while invalidating options (c) and (d) due to $E \neq 0$, options (a) and (b) are open for consideration where) $E = 0$. For circular motion, centripetal force is created by magnetic field ad hence $\vec{F_m} \neq 0$. It, using (3), leads to $qvB \sin\theta \neq 0$. From the statement of the problem cahrged particle performing circular motion $q \neq 0$ && $v \neq 0$. Then the only requirements for circular motion are $B \neq 0$ && $\theta \neq 0$. Thus, *option (a) turns out to be incorrect.* As regards option (b) where given that $E = 0$, yet it is silent on angle between vectors $\vec{v}$ and $\vec{B}$. But, it certainly stipulates $B \neq 0$ a necessary condition for circular motion. Hence, *option (b) is correct.* Hece, **answer is option (b).**

I-26	Force experienced by a charged particle in electric and magnetic field, as per Lorentz's Force Law is $\vec{F}_{em} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \vec{F}_{em} = \vec{F}_e + \vec{F}_m$…(1). Here, $\vec{F}_e = q\vec{E}$…(2) and $\vec{F}_m = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F}_m = qvB \sin\theta\, \hat{n}$…(3). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{v}$ For the particle to go undeflected in the system ; it implies that $q \neq 0$, $v \neq 0$, $E \neq 0$ and $B \neq 0$.Thus it has to satisfy two conditions $\vec{F}_m \neq 0$ && $\vec{F}_e$. Requirements for compliance of each option is being discussed- **Option (a):** For $\vec{F}_e \neq 0$: it must have $\boldsymbol{\vec{E}}\|\boldsymbol{\vec{v}}$. And for $\vec{F}_m = 0$ it must have $\sin\theta = 0 \Rightarrow \theta = 0$ i.e. $\vec{v} \times \vec{B}$. Using Euclid's postulates that since $\vec{E}\|\vec{v}$ && $\vec{v}\|\vec{B}$ it leads to $\boldsymbol{\vec{E}}\|\boldsymbol{\vec{B}}$. *This satisfies conditions in option (a), hence it is correct.* **Option (b):** For $\vec{F}_{em} \neq 0$ net force $\vec{F}_e + \vec{F}_m$ is along $\vec{v}$. This necessitates that $\vec{E}\|\vec{v}$ and for $\vec{F}_m = qvB \sin\theta\, \hat{n}$ $\theta \neq 0$, Thus, $\vec{E}$ is not parallel to $\vec{B}$ which satisfies conditions in option (b). Hence, option (b) is correct. **Option (c):** In one part of the option $\vec{v}\|\vec{B} \Rightarrow \theta = 0 \Rightarrow \sin\theta = 0$ then as per (3) $\vec{F}_m = 0$ satisfies condition of undeflected motion of the particle. But in secord part of the option states that $\vec{E}$ *is not parallel to* $\vec{B}$. *Hence, the particle will experience an acceleration in direction other that of* $\vec{v}$. *Thus particle will experience deflection. Hence, option (c) is incorrect.* **Option (d): From the above discussions for particle to move undeflected** $\vec{E}\|\vec{B}$ not enough. *It must also have* $\vec{v}\|\vec{E}$ *which is denied in secod part of the option. Hence, option (d) is incorrect.* Thus. **answer is option (a) and (b)**.
I-27	Given that a particle carrying charge q is un-accelerated i.e. $\vec{a} = 0$ and moves a in region which contains electric field $\vec{E} \neq 0$ and magnetic field $\vec{B} \neq 0$. Thus, as per Lorentz's Force Law is $\vec{F}_{em} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \vec{F}_{em} = \vec{F}_e + \vec{F}_m = 0$…(1). Here, $\vec{F}_e = q\vec{E}$…(2) and $\vec{F}_m = q(\vec{v} \times \vec{B}) \Rightarrow \vec{F}_m = qvB \sin\theta\, \hat{n}$…(3). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. velocity vector $\vec{l}$, while unit vector $\hat{n}$ is perpendicular to plane containing vectors $\vec{v}$ & $\vec{B}$. It implies that $\hat{n} \perp \vec{v}$ && $\hat{n} \perp \vec{B}$…(4). In this context each of the option is being analyzed. **Option (a):** Equation (1) leads $\vec{F}_e = -\vec{F}_m \Rightarrow q\vec{E} = -qvB \sin\theta\, \hat{n}$…(5). Further, magnetic field causes acceleration along $\hat{n}$ then as per (5) we have $\vec{E} \to \hat{n}$ (6). Combining (1) and (6), *we have* $\vec{E} \perp \vec{B}$...(7). *Thus, option (a) is correct.* **Option (b):** Using the discussions at option (a) together with $\hat{n} \perp \vec{v}$ in (4), leads to $\vec{E} \perp \vec{v}$. *Thus, option (b) is correct.* **Option (c):** As provided $\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1 \Rightarrow \vec{F}_m = qvB\hat{n}$ as per (3). But, *this alone is not sufficient unless (5) is satisfied. Stipulation in this option is silent in respect of* $\vec{E}$. *Hence, this option (c) is incorrect.* **Option (d):** Since *all the three quantities are* $\vec{E}, \vec{v}$ & $\vec{B}$ *mere stating* $E = vB$, *as stated in the option is insufficient to satisfy (5). Hence, option (d) is incorrect.* Thus. **answer is option (a) and (b)**.
I-28	As stated in problem charge on ion P is $q_P = e$ and ion Q is $q_Q = 2e$ and projected in uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$ such that $\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2}$…(1). Though both the particles of masses m are projected from the same place, however, for better understanding of underlying concepts the particles are shown separately in the figure.  As per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB \sin\theta\, \hat{n}$…(2). Combining (1) and (2) we get $\vec{F} = qvB\hat{n}$…(3). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. velocity vector $\vec{v}$

This force $\vec{F}$, as per mechanics acts as centripetal force and would cause a circular motion of radius r such that $\vec{F} = m\vec{a}$…(4). Combining (3) and (4), $\vec{a} = \frac{qvB}{m}\hat{n} \Rightarrow a = \frac{qvB}{m}$…(5).

Again, as per mechanics, centripetal acceleration $a = \frac{v^2}{r}$…(6). Combining (5) and (6), $\frac{v^2}{r} = \frac{qvB}{m} \Rightarrow q = \left(\frac{mv}{B}\right)\frac{1}{r}$ …(7)

Applying the given data to the charged particle P and Q we have $e = \left(\frac{mv}{B}\right)\frac{1}{r_p}$…(8) and $2e = \left(\frac{mv}{B}\right)\frac{1}{r_q}$…(9).

Taking ratio of (8) and (9) we have $\frac{e}{2e} = \frac{\left(\frac{mv}{B}\right)\frac{1}{r_p}}{\left(\frac{mv}{B}\right)\frac{1}{r_q}} \Rightarrow \frac{r_q}{r_p} = \frac{1}{2}$…(10). This final form is being used to analyze the given options-

(a) Both ions will go along circle of equal radii
(b) The circle described by the single-ionized charge will have a radius double that of the other circle
(c) The two circles do not touch each other
(d) The two circles touch each other

Option (a): The ration of the two radii $\frac{r_q}{r_p} \neq 1$, hence *option (a) is incorrect.*

Option (b): Equation (10) leads to $r_p = 2r_q$, it satisfies statement in option (b). Hence, *option (b) is correct.*

Option (c): Given that the two particles are projected in the given way from same place and hence trajectories, of the two particles would touch each other. This contradicts statement in this option. Hence, *option (c) is incorrect.*

Option (d): The analysis at option (c) asserts statement in this option. Hence, *option (d) correct.*

Thus, **answer is option (b) and (d).**

I-29

It is required to determine direction of magnetic field. Given that charge of an electron is $q = -e$ moving with a velocity along X-axis with a velocity $\vec{u} = u\hat{\imath}$. It is required that velocity of the particle in a short time t reverses and particle start moving along $(-\hat{\imath})$; such that $\vec{v} = v(-\hat{\imath})$. Thus, as per equation of motion $\vec{v} = \vec{u} + \vec{a}t \Rightarrow \vec{a} = \frac{\vec{v}-\vec{u}}{t} \Rightarrow \vec{a} = \frac{\vec{v}-\vec{u}}{t} \Rightarrow \vec{a} = \frac{v(-\hat{\imath})-u\hat{\imath}}{t} \Rightarrow \vec{a} = \frac{v+u}{t}(-\hat{\imath})$…(1)

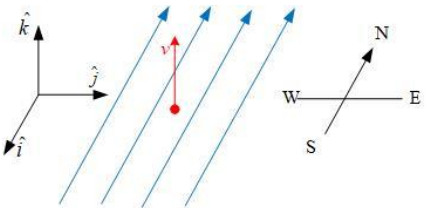
As per Lorentz's Force Law is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = (-e)vB\sin\theta\,\hat{n}$…(2). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. velocity vector $\vec{v}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$. Combining (1) and (2) we get $\vec{F} = qvB\hat{n}$…(3).

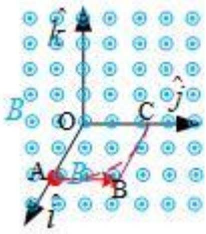
As per mechanics $\vec{F} = m\vec{a}$..(4). Comparing (3) and (4), $m\vec{a} = qvB\hat{n} \Rightarrow \vec{a} = \frac{(-e)vB}{m}\hat{n}$…(5).

Now comparing (1) and (5), $\frac{(-e)vB}{m}\hat{n} = \frac{v+u}{t}(-\hat{\imath}) \Rightarrow \hat{n} \to \hat{\imath}$…(6). It implies that magnetic force is along X-axis. Going back to (2) which stipulates that unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$. In the instant case this is $\mathrm{X}-\mathrm{Y}$ plane. Thus, *it is not necessry for magnetic field to be along Y-axis only or Z-axis only. Hence, option (c) and (d)are incorrect.*

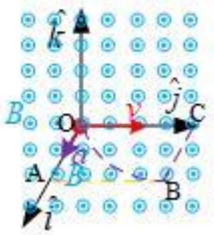
Above discussions lead to magnetic flux $\vec{B}$ can be anywhere on the X-Y plane that make Option (a) and (b) correct.

Hence, **answeris option (A) and (b).**

I-30	As per Electromagnetic Field Theory, magnetic field $\vec{B} = B\hat{b}$ and Electric field $\vec{E} = E\vec{e}$ produces electromagnetic wave $c\hat{v}$ which propagates with velocity c in direction $\hat{v} \perp \vec{e}$ && $\hat{v} \perp \hat{b}$, while $\vec{e} \perp \hat{b}$. Dimensionally, $[E] = \mathrm{MLT^{-3}I^{-1}}$ …(1), $[B] = \mathrm{MI^{-1}T^{-2}}$…(2) and $[v] = \mathrm{LT^{-1}}$…(3). Each of the option is being analyzed dimensionally – **Option (a):** $B'_y = B_y + \frac{vE_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \mathrm{MI^{-1}T^{-2}}$. Since both the addends on the RHS have same dimensions hence $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}}$. *Dimensionally, $[LHS] = [RHS]$, these are not wrong. Hence, as desired option (a) is incorrect.* **Option (b):** $B'_y = E_y - \frac{vB_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{MI^{-1}T^{-2}}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.* **Option (c):** $B'_y = B_y + vE_z \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + (\mathrm{LT^{-1}}) \times (\mathrm{MLT^{-3}I^{-1}})$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{ML^2I^{-1}T^{-4}}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.* **Option (d):** $B'_y = E_y + vB_z \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + (\mathrm{LT^{-1}}) \times (\mathrm{MI^{-1}T^{-2}})$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{MLI^{-1}T^{-3}}$. Both the addends on the RHS have same dimensions hence $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}}$. *Dimensionally, $[LHS] = [RHS]$, these are not wrong. Hence, as desired option (a) is incorrect.* Hence, **answer is option (b) and (c)** **N.B.:** This problem requires understanding of electromagnetic waves. Yet, despite E, B and v being discretely different physical quantities, correctness of relations between them has been solved dimensionally.
I-31	Given that α-particle having charge $q = 2e \Rightarrow q = 2 \times (1.6 \times 10^{-19})$ C is moving, in a region having magnetic field $B = 1.0$ T in a direction south to north i.e. along $(-\hat{\imath})$, with a velocity $v = 3 \times 10^7$m/s in a direction upward i.e. along $\hat{k}$. With the given data angle θ between vectors $\vec{v}$ and $\vec{B}$ is $\theta = \frac{\pi}{2}$.  In this case magnetic force experience by the α-particle can be determined using Therefore, magnetic force using Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB\sin\theta\,\hat{n}$…(1). Using the available data in (1), $\vec{F} = \left(2 \times (1.6 \times 10^{-19})\right) \times (3 \times 10^7)\hat{k}.\times 1.0(-\hat{\imath}) \times \sin\frac{\pi}{2}$. This resolves into $\vec{F} = 9.6 \times 10^{-12}(-\hat{\jmath})$ N. It implies that a force $\mathbf{9.6 \times 10^{-12}}$ **N will act towards the west is the answer.**
I-32	Given system is shown in figure where an electron P having charge $q = (-)e = (-)1.6 \times 10^{-19}$C…(1). It is stated to be moving horizontally i.e. in $(\hat{\imath} - \hat{\jmath})$ plane parallel to $\hat{\jmath}$ with a kinetic energy $KE = 10$ keV…(2) As per mechanics $KE = \frac{1}{2}mv^2 \Rightarrow \vec{v} = \left(\sqrt{\frac{2\times KE}{m}}\right)\hat{\jmath}$…(3) And as per electrostatics 1J = 1CV…(4) Thus, combining (1), (2) and (4) we have $KE = 10 \times 10^3 \times 1.6 \times 10^{-19}\mathrm{CV} \Rightarrow KE = 1.6 \times 10^{-15}$J…(5). It is also given that magnetic field $\vec{B} = 1.0 \times 10^{-7}\hat{k}$ is in vertically upward direction. This makes it possible to analyze motion of electron using Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB\sin\theta\ \hat{n}$…(6). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. velocity vector $\vec{v}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$.

	 With the given geometry of the system $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$. This together with the available data (6) leads to $\vec{F} = (-e)(v\hat{\jmath}) \times (B\hat{k}) \Rightarrow \vec{F} = evB(-\hat{\imath}) \Rightarrow \vec{a} = \frac{evB}{m}(-\hat{\imath})$…(7). This leads to conclusion that *the electron would **deflect towards left** of the direction of its motion as shown in the figure* and is ***answer of the part (a)*** Further, in part (b) it is required to determine sideways deflection of the electron when it travels through a distance, shown in figure as AB, $y = 1m$…(8) This can be analyzed on the lines of projectile motion where motion along $(\hat{\jmath})$ is accelerated, i.e. with uniform velocity, and to travel a distance y it would take time $t = \frac{y}{v}$…(9). During this period magnitude of deflection of electron along $(-\hat{\imath})$, shown in figure as BC as per second equation of motion, would be equal $x = 0 \times t + \frac{a}{2}t^2 \Rightarrow x = \frac{a}{2}\left(\frac{y}{v}\right)^2$…(10). Combining, (3),(5), (7), (8) and (10) we have $x = \frac{1}{2}\left(\frac{evB}{m}\right)\left(\frac{y}{v}\right)^2 \Rightarrow x = \frac{1}{2}\left(\frac{eBy^2}{mv}\right) \Rightarrow x = \frac{1}{2}\left(\frac{eBy^2}{m\left(\sqrt{\frac{2\times KE}{m}}\right)}\right)$. It simplifies into $x = \frac{1}{2}\left(\frac{eBy^2}{\sqrt{2\times m\times KE}}\right)$, mass of electron $m = 9.1 \times 10^{-31}$kg. Using the available data $x = \frac{1}{2}\left(\frac{(1.6\times10^{-19})\times(1.0\times10^{-7})\times1^2}{\sqrt{2\times(9.1\times10^{-31})\times(1.6\times10^{-15})}}\right) \Rightarrow x = \frac{0.80\times10^{-26}}{\sqrt{2.9\times10^{-46}}} \Rightarrow x = 0.148 \times 10^{-3}$m or *0.015cm is the answer of part (b)* Thus, **answer is (a) Left, (b) 0.015 cm** **N.B.:** Though the problem is simple it involves integration multiple concepts and conversion of units
I-33	Given is that particle having charge $q = 1.0 \times 10^{-9}$C in a magnetic field $\vec{B} = (4.0 \times 10^{-3})\hat{k}$ experiences a force $\vec{F} = (4.0\hat{\imath} + 3.0\hat{\jmath}) \times 10^{-10}$N. It is required to find velocity v of the charged particle. As per Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B}$, here velocity is a vector say $\vec{v} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$. Using the available data, $(4.0\hat{\imath} + 3.0\hat{\jmath}) \times 10^{-10} = (1.0 \times 10^{-9})(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) \times (4.0 \times 10^{-3})\hat{k}$. It leads to $(4.0\hat{\imath} + 3.0\hat{\jmath}) \times 10^{-10} = (4.0 \times 10^{-12})\left(a(\hat{\imath} \times \hat{k}) + b(\hat{\jmath} \times \hat{k}) + c(\hat{k} \times \hat{k})\right)$. Using principle of cross-product of vectors $(4.0\hat{\imath} + 3.0\hat{\jmath}) = 0.04(a(-\hat{\jmath})) + b\hat{\imath} \Rightarrow 4.0\hat{\imath} + 3.0\hat{\jmath} = 0.04b\hat{\imath} - 0.04a\hat{\jmath}$. Equating each component of vectors we have $0.04b = 4.0 \Rightarrow b = 100$ and $-0.04a = 3.0 \Rightarrow a = -75$. Thus, velocity vector is $\vec{v} = (\mathbf{-75\hat{\imath} + 100\hat{\jmath}})$**m/s. is the answer.**
I-34	Given that $\vec{B} = (7.0\hat{\imath} - 3.0\hat{\jmath}) \times 10^{-3}$T and acceleration of a charged particle is $\vec{a} = (x\hat{\imath} + 7.0\hat{\jmath}) \times 10^{-6}$ m/s². Acceleration of a particle, as per mechanics is $\vec{F} = m\vec{a}$….(1). Here, mass of the charged particle and $\vec{F}$ force on the particle, in this case, as per Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB\sin\theta\ \hat{n}$…(2). Here, θ and α is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$. Form these discussions t is clear that for force to exist the angle is in the range $0 < \theta < \pi$.As long as angle is in this range $\vec{a} \perp \vec{v}$ && $\vec{a} \perp \vec{B}$. In this case vector-dot products $\vec{F} \cdot \vec{v} = 0$…(3)and $\vec{F} \cdot \vec{B} = 0$…(4) Since, only $\vec{F}$ and $\vec{B}$ are specified and hence taking (1) and (4) with the given data it is necessary that we have $(7.0\hat{\imath} - 3.0\hat{\jmath}) \cdot (x\hat{\imath} + 7.0\hat{\jmath}) = 0 \Rightarrow 7.0x(\hat{\imath} \cdot \hat{\imath}) - 3.0x(\hat{\jmath} \cdot \hat{\imath}) + 49(\hat{\imath} \cdot \hat{\imath}) - 21(\hat{\jmath}.\hat{\jmath}) = 0$. Applying principle of dot-product of vectors we have $7.0x - 21 = 0 \Rightarrow x = \frac{21}{7.0} \Rightarrow \boldsymbol{x = 3.0}$ **is the answer**.

I-35 Given that a bullet of mass $m = 0.01$ kg carries a charge $q = 4.00 \times 10^{-6}$ C is fired along horizontal direction with a speed $v = 270$ m/s. There is a vertical magnetic field $B = 500 \times 10^{-6}$ T. For convenience initial position of the particle is taken at origin O and its motion along $(\hat{\imath} - \hat{\jmath})$ velocity vector as $\vec{v} = v\hat{\jmath}$…(1) as shown in the figure. Accordingly, $\vec{B} = B\hat{k}$…(2).



As per Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B}$…(3). Combining (1), (2) and (3), $\vec{F} = q\left(v\hat{\jmath} \times B\hat{k}\right) \Rightarrow \vec{F} = qvB\left(\hat{\jmath} \times \hat{k}\right) \Rightarrow \vec{F} = qvB\hat{\imath}$…(4), is arrived at using principle of cross-product of vectors. Therefore, acceleration of the particle taking (4), as per mechanics, is $\vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{a} = \frac{qvB}{m}\hat{\imath}$…(5).

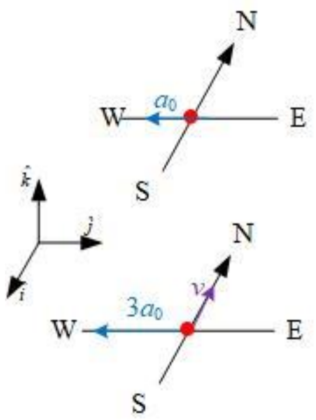
Using available data, $\vec{a} = \frac{(4.00\times10^{-6})(270)(500\times10^{-6})}{0.01}\hat{\imath} \Rightarrow \vec{a} = 5.4 \times 10^{-5}\hat{\imath}$ m/s²…(6). Rest of the problem is simple application of concepts of projectile motion in this case.

While particle is accelerated with magnetic force along $\hat{\imath}$, its travel of $y = 100$ m along $\hat{\jmath}$ is un-accelerated. Hence, hence time t taken in this travel, using available data, is $t = \frac{y}{v} \Rightarrow t = \frac{100}{270} \Rightarrow t = \frac{10}{27}$ s…(7).

Motion of the particle with acceleration $\vec{a}$ with initial velocity $u = 0$ in time t is deflection $x = BC = OA$ of the particle along $\hat{\imath}$. This can be determined with equation of motion, $x = 0 \times t + \frac{at^2}{2}$…(8). Using available data in (8) we have $x = \frac{(5.4\times10^{-5})\left(\frac{10}{27}\right)^2}{2} \Rightarrow$ $\mathbf{x = 3.70 \times 10^{-6}}$**m is the answer.**

N.B.: This problem involves integration of concepts of mechanics with Lorentz' Force Law.

I-36 Given that in a room there are electric and magnetic fields. Acceleration of proton in two different cases proton is given as shown in the figure. Reference unit direction vectors are also indicated in the figure to facilitate analysis.



Motion of a proton of mass m and charge e can be analyzed in accordance with Lorentz's Force Law $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \Rightarrow \vec{a} = \frac{e}{m}\vec{E} + \frac{evB}{m}\sin\theta\ \hat{n}$…(1). Here, θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$.

Given are two cases as under -

Case 1: When proton is released from the state of rest $v = 0$, acceleration of the particle as per (1) would be $\vec{a}_1 = \frac{e}{m}\vec{E}$…(2). It is given that acceleration is towards west, $\vec{a}_1 = a_0(-\hat{\jmath})$…(3). Combining (2) and (3) we get $\frac{e}{m}\vec{E} = a_0(-\hat{\jmath}) \Rightarrow \vec{E} = \frac{a_0 m}{e}(-\hat{\jmath})$…(4). Using reference vectors **electric field is of magnitude** $\boldsymbol{\frac{a_0 m}{e}}$ **towards west.**

Case 2: When proton projected towards north with a velocity $\vec{v} = v_0(-\hat{\imath})$, it experiences an acceleration $\vec{a}_2 = 3a_0(-\hat{\jmath})$…(5). As per (1) together with (4) is $\vec{a}_2 = \frac{e}{m}\left(\frac{a_0 m}{e}\right)(-\hat{\jmath}) + \frac{evB}{m}\sin\theta\ \hat{n}$…(6). Combining (5) and (6), $(-)3a_0\hat{\jmath} = (-)a_0\hat{\jmath} + \frac{qeB}{m}\sin\theta\ \hat{n} \Rightarrow \frac{ev_0B}{m}\sin\theta\ \hat{n} = (-)2a_0\hat{\jmath}$. It leads to $B\ \hat{n} = \frac{2ma_0}{ev\sin\theta}(-\hat{\jmath})$…(7).

Going back to discussions following (1) and that velocity v is along $(-\hat{\imath})$the magnetic field $\vec{B}$ would be on $\left(\hat{\imath} - \hat{k}\right)$ plane. Accordingly, angle θ is of magnetic field with velocity in $\left(\hat{\imath} - \hat{k}\right)$. The equation (8), where parameters m, e, a_0 and , v_0 are constant, be written as $B = K\frac{1}{\sin\theta}$. Therefore, for maximum B, let us apply concept of maxima-minima $\frac{dB}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right) = 0$…(8) Now substitute $u = \sin\theta \Rightarrow \frac{du}{d\theta} = \frac{d}{d\theta}\sin\theta \Rightarrow \frac{du}{d\theta} = \cos\theta$. Manipulating (8) $\frac{d}{d\theta}\left(\frac{1}{u}\right) = \frac{d}{du}\left(\frac{1}{u}\right) \times \frac{du}{d\theta}$. It further leads to $\frac{d}{d\theta}\left(\frac{1}{u}\right) = (-)\frac{1}{u^2} \times \cos\theta \Rightarrow \frac{dB}{d\theta} = (-)\frac{\cos\theta}{\sin^2\theta}$…(9). Further, $\frac{\cot\theta}{\sin\theta} = 0 \Rightarrow \cot\theta = 0$. It is a trigonometric equation and its principal solution for either maxima or minima is $\theta \pm \frac{\pi}{2}$. It requires to choose among the two possible values of θ for maximum value of B. This is ascertained by taking second derivative of (8). If $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) > 0$ the its solution among the two values will give minimum B, else if $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) < 0$ then minima.

Accordingly, $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{d}{d\theta}\left(-\frac{\cot\theta}{\sin\theta}\right) \Rightarrow \frac{d}{d\theta}\left(-\frac{\cot\theta}{\sin\theta}\right) = (-)\frac{\sin\theta\left(\frac{d}{d\theta}\cot\theta\right) - \cot\theta\left(\frac{d}{d\theta}\sin\theta\right)}{\sin^2\theta}$. It, further, solves into $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = (-)\frac{\sin\theta\times(-\operatorname{cosec}^2\theta) - \cot\theta\times\sin\theta}{\sin^2\theta} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{\operatorname{cosec}\theta + \cot\theta\times\sin\theta}{\sin^2\theta}$…(10).

This is where value of $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right)$ in (10) needs to be examined for solution $\theta \pm \frac{\pi}{2}$ of obtained from (9), Taking each of the values-

(i) $\theta = (+)\frac{\pi}{2}$: Then, $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{1-0\times1}{1} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = 1 \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) > 0$ is condition of minima.

(ii) $\boldsymbol{\theta = (-)\frac{\pi}{2}}$: Then, $\boldsymbol{\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{-1-0\times(-1)}{(-1)^2} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = -1 \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) < 0}$ is condition of maxima, as desired

As discussed following (1) above, θ is angular displacement of vector $\vec{B}$ w.r.t. $\vec{v}$ and that the angle is (+)ve in anticlockwise direction, while it is (-)ve in clockwise direction. Thus, going back to the figure **magnetic field $\vec{B}$** will be along $(\hat{k})$ i.e. **downward** .

Thus, **answers are $\boldsymbol{E = \frac{ma_0}{e}}$ toward west and $\boldsymbol{B = \frac{2ma_0}{ev_0}}$ downward.**

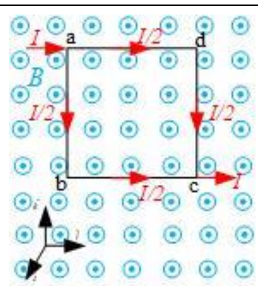
N.B.: This problem integrates concepts of electromagnetic force along with mathematics of vector algebra, trigonometric equations, maxima-minima, direction of angular displacement, It is a good example to appreciate beauty of mathematics in analysis of physical situations correctly, and in an unambiguous manner. Proficiency and confidence at it is acquired through understanding of concepts and its practice in problem solving.

I-37

It is required to find magnetic force, which as per Lorentz's Force Law is $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (I\vec{l}) \times \vec{B} \Rightarrow \vec{F} = IlB\sin\theta\ \hat{n}$…(1). Here, $I = 10$A, $l = 0.10$m, $B = 0.1$T and $\theta = 53^0$ is the anglual deviation of vector of $\vec{v}$ w.r.t. to $\vec{B}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$.

Using avaliable data in (1) we have $\vec{F} = (IlB\sin 53^0)\,\hat{n} \Rightarrow \vec{F} = (10 \times 0.10 \times 0.1 \times 0.798)\,\hat{n} \Rightarrow \vec{F} = 0.08\hat{n}$. Thus, **answer is 0.08 Nperpendicular plane of containing vectors $\boldsymbol{\vec{l} - \vec{B}}$** .

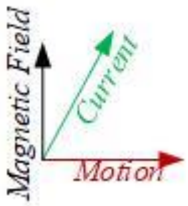
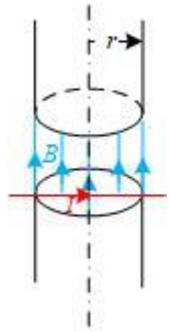
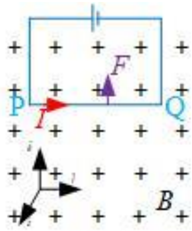
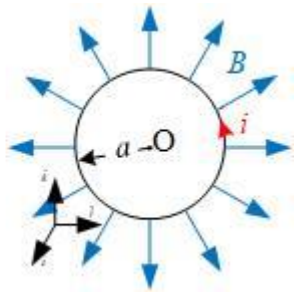
I-38



Given system is shown in the figure where square wire frame abcd on $(\hat{\imath} - \hat{\jmath})$ plane has each side of length $l = 0.30$ m. A current $I = 2$A enters the frame at vertex 'a' and leaves at vertex 'c'. The frame forms a parallel combination of equal resistances such that each side carries a current $i = \frac{I}{2}$. Further, given that magnetic field is perpendicular to the plane of the frame $\vec{B} = B\hat{k}$ as shown in the figure.

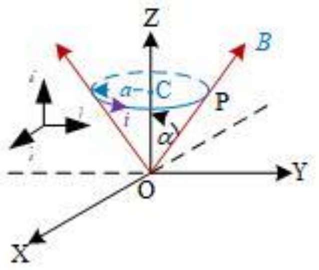
Magnetic force experienced by a wire as per Lorentz's Force Law is mathematically $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\vec{l}) \times \vec{B} \Rightarrow \vec{F} = \text{i}lB\sin\theta\ \hat{n}$…(1). Here, θ is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$

In the instant case each side of the wire is perpendicular to the magnetic field and hence $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$. Thus, rewriting (1) we have $\vec{F} = ilB\,\hat{n}$ …(2).

	 Using the available data and the symmetry in geometry each side of the frame would experience a force $F = IlB$. Using the available data $F = \frac{2}{2} \times 0.20 \times 0.1 \Rightarrow \boldsymbol{F = 0.02}$ N. But, direction of force on each side can be determined using Flamming's Left Hand Rule by orienting the figure appropriately. Accordingly, forces experienced by sides ab and dc would be along $(-\hat{\jmath})$ i.e. towards left., while force experienced by sides ad and bc would along $(-\hat{\imath})$ i.e. downward. Hence, **answer is 0.02 N on each wire, on ab and dc towards left and on dc and ab downward.**
I-39	Given system is shown in figure where two strong cylindrical magnets produce a magnetic field $B = 1.0$ T. A wire carrying current $I = 2.0$ A is placed perpendicular to the magnetic field intersecting axis of the region. It implies that it passes through its center. This lead to length of wire intercepting magnetic field is $l = 2r = 2 \times 0.04$m and it is only responsible for the force experienced by the wire.  Magnetic force experienced by a wire as per Lorentz's Force Law is mathematically $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\vec{l}) \times \vec{B} \Rightarrow \vec{F} = ilB \sin\theta\ \hat{n}$…(1). In the instant case each side of the wire is perpendicular to the magnetic field and hence $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$. Thus, rewriting (1) we have $\vec{F} = i(2r)B\,\hat{n}$ …(2). Using the available data and the symmetry in geometry each side of the frame would experience a force $F = IlB$. Using the available data $F = 2.0 \times (2 \times 0.04) \times 1.0 \Rightarrow \boldsymbol{F = 0.16}$ **N is the answer.**
I-40	As per Lorentz's Force Law is mathematically $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (I\vec{l}) \times \vec{B}$…(1). As given $\vec{l} = l\hat{\imath}$ and $\vec{B} = B_0(\hat{\imath} + \hat{\jmath} + \hat{k})$. Accordingly, (1) leads to $\vec{F} = I(l\hat{\imath}) \times B_0(\hat{\imath} + \hat{\jmath} + \hat{k}) \Rightarrow \vec{F} = B_0 Il(\hat{\imath} \times \hat{\imath} + \hat{\imath} \times \hat{\jmath} + \hat{\imath} \times \hat{k})$. Applying concepts of vector products $\vec{F} = B_0 Il(\hat{k} - \hat{\jmath}) \Rightarrow \boldsymbol{F = \sqrt{2}B_0 Il}$ **is the answer**.
I-41	As per Lorentz's Force Law is mathematically $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (I\vec{l}) \times \vec{B}$…(1). Given system with 3D direction vectors are shown in the figure where length $l = 0.50$m of wire PQ on which magnetic force is to be determined $\vec{l} = l\hat{\jmath}$ is placed in magnetic field $B = 0.20$ T is $\vec{B} = B(-\hat{k})$. The wire is carrying current $I = 5.0$ A. Using the available data $\vec{F} = 5.0 \times (0.50\hat{\jmath}) \times \left(0.20(-\hat{k})\right) \Rightarrow \vec{F} = -0.50(\hat{\jmath} \times \hat{k}) \Rightarrow \vec{F} = 0.50(-\hat{\imath})$ N, i.e. **0.50 N inward the circuit as shown in the figure is the answer.** 
I-42	Given system is shown in the figure alongwith unit dimension vectors. Force experience by a current carrying wirer as per Lorentz's Force Law is $\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\vec{l}) \times \vec{B} \Rightarrow \vec{F} = i\vec{l} \times \vec{B}$…(1).  This system has cylindrical geometry and hence magnetic field, at point of consideration, $\vec{B} = B\hat{r}$…(2). Here, $\hat{r}$ is unit radial vector. Taking an element of length Δl vectorially $\vec{l} = \Delta l\hat{t}$…(3). Here $\hat{t}$ is tangential at the point of consideration. Thus, $\hat{r} \perp \hat{t}$…(4). Accordingly, (1) can be rewritten $\overline{\Delta F} = i(\Delta l\hat{t}) \times (B\hat{r}) \Rightarrow \Delta\vec{F} = \Delta liB(\hat{t} \times \hat{r})$…(5). Thus, as per (4), equation (5) can be rewritten as $\Delta\vec{F} = \Delta liB\hat{n}$…(6). In (6), as per geometry of the system considered with unit direction vectors $\hat{n} \to (-\hat{\imath})$ i.e. uni-directionally ***perpendicularly into the plan of the figure***. Therefore, magnitude of the net force on the circular wire would lead to $F = iB\oint dl \Rightarrow F = iB(2\pi a) \Rightarrow \boldsymbol{F = 2\pi aiB}$. Thus, **answer is $\boldsymbol{2\pi aiB}$ perpendicularly into the plan of the figure.**

I-43

Given is a circular loop placed parallel to X-Y plane with it center $C(0,0.d)$, it implies $OC = d\ldots(1)$ and a hypothetical magnetic field at every point on the perimeter of the loop $\vec{B} = B_0\vec{e}_r\ldots(2)$, here unit vector is along the line joining origin O and point of consideration on the perimeter of the loop; in case of point P it is $\vec{e}_r||OP$. Correspondence of X,Y,Z axes with unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ is shown in the figure.



It is seen that angle α is between vectors $\hat{k}$ & $\hat{e}_r$ and is uniform at every point on the perimeter of the circular loop.

Since loop is not a straight wire hence force on the loop can be determined by initially taking force experienced by an element length $\Delta\vec{l} = \Delta\vec{\theta} \times \vec{r} \Rightarrow \Delta\vec{l} = \Delta\vec{\theta} \times r\hat{r} \Rightarrow \Delta\vec{l} = r(\Delta\vec{\theta} \times \hat{r}) \Rightarrow \Delta\vec{l} = r(\Delta\theta\hat{k} \times \hat{r})\ldots(3)$. here $r = OP$ and $\vec{\theta}$ is along $\hat{k}$.

Accordingly, $\Delta\vec{l} = r\Delta\theta(\hat{k} \times \hat{r}) \Rightarrow \Delta\vec{l} = \Delta l(\hat{k} \times \hat{r})\ldots(4)$.

In the given geometry, $\vec{B} = B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k}\ldots(5)$.

Magnetic force as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \Delta\vec{F} = (i\Delta\vec{l}) \times \vec{B} \Rightarrow \Delta\vec{F} = i(\Delta\vec{l} \times \vec{B})\ldots(6)$. Here, geometrically $\vec{B} = B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k}\ldots(7)$

Combining (4), (5) and (6) we have $\Delta\vec{F} = i\left(\Delta l(\hat{k} \times \hat{r}) \times (B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k})\right)\ldots(8)$

Eqn. (8) leads to $\Delta\vec{F} = \Delta liB\left(\sin\alpha\left(\hat{k} \times \hat{r} \times \hat{r}\right) + \cos\alpha\left(\hat{k} \times \hat{r} \times \hat{k}\right)\right)$,,,(9). This turns out to be problem of triple cross product of vectors. Instead considering symmetry of the geometry, as shown in figure, let us simplify using (3) at P $\Delta\vec{l} = a\Delta\theta(-\hat{\imath})$ and $\hat{r} = \hat{\jmath}$. With this, combining (6) and (7), we have –

$$\Delta\vec{F} = i(a\Delta\theta(-\hat{\imath})) \times (B\sin\alpha\,\hat{\jmath} + B\cos\alpha\,\hat{k}) \Rightarrow \Delta\vec{F} = (-)iaB\Delta\theta\left(\sin\alpha\,(\hat{\imath} \times \hat{\jmath}) + \cos\alpha\,(\hat{\imath} \times \hat{k})\right)$$

$$\Delta\vec{F} = (-)iaB\Delta\theta(\sin\alpha\,\hat{k} + \cos\alpha\,\hat{\jmath}) \Rightarrow \Delta\vec{F} = \left(iaB\sin\alpha\,(-\hat{k}) + iaB\cos\alpha\,\hat{\jmath}\right)\Delta\theta$$

$$\Delta\vec{F} = \Delta F_a(-\hat{k}) + \Delta F_r(-\hat{r}),\text{ here } \Delta F_a = iaB\sin\alpha\,\Delta\theta \text{ and } \Delta F_r = iaB\cos\alpha\,\Delta\theta\ldots(9)$$

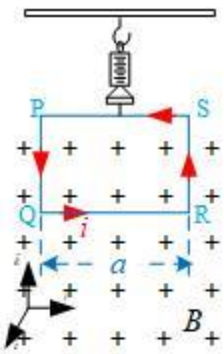
An observation of the symmetrical geometry t is to be noted ΔF_r component acting along $\hat{\jmath} \rightarrow \hat{r}$ would mutually cancel with geometrically opposite points leading to $F_r = \oint \Delta F_r = 0\ldots(10)$.

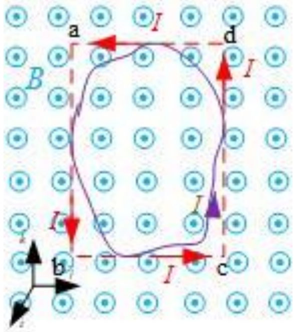
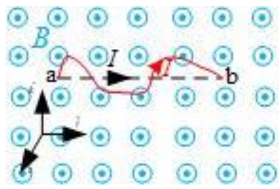
In respect of axial force ΔF_a is along $(-\hat{k})$ i.e. downward and net force over the circular loop would be $F_a = \oint \Delta F_a \Rightarrow F_a = iaB\sin\alpha \oint \Delta\theta \Rightarrow F_a = 2\pi iaB\sin\alpha\ldots(11)$. Going back to the geometry $\sin\alpha = \frac{a}{\sqrt{a^2+d^2}}\ldots(12)$. Combining (11) and (12), together with the direction discussed above, net force on the circular loop is $F = F_a = 2\pi iaB \times \frac{a}{\sqrt{a^2+d^2}} \Rightarrow \boldsymbol{F} = \frac{\boldsymbol{2\pi ia^2B}}{\sqrt{\boldsymbol{a^2+d^2}}}$ **downward is the answer.**

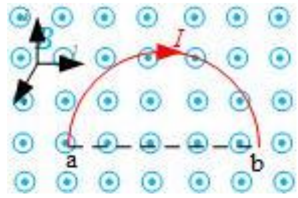
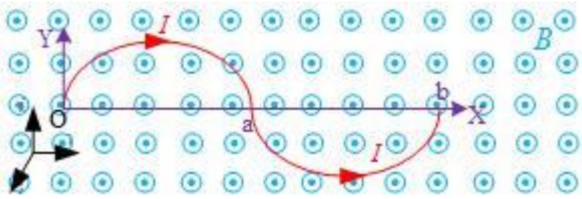
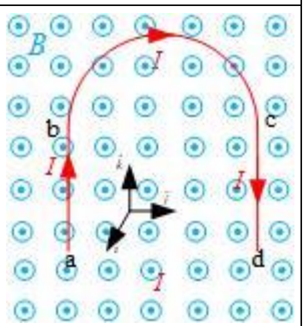
N.B.: Though this problem is for a hypothetical magnetic field, yet it is a gives good practice to gain proficiency in handling three dimensional vectors. Further, mathematics is an effective analytical tool problem can and should be simplified using symmetries wherever possible.

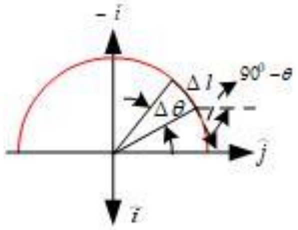
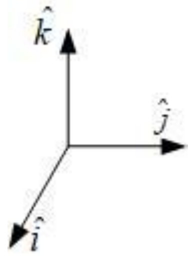
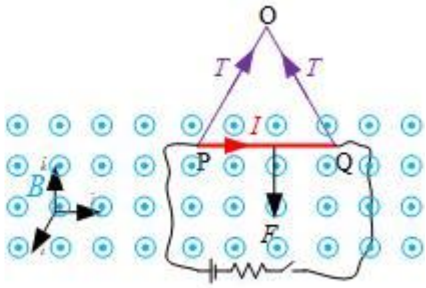
I-44

Given system is shown with current i in anticlockwise direction suspended in a region having uniform magnetic field $\vec{B} = B(-\hat{\imath})$. For convenience of analysis unit-vectors in 3D are also shown in the figure. Accordingly, width of rectangular loop QR inside magnetic field is $\vec{l} = a\hat{\jmath}$, while heights of the loop along PQ and RS inside magnetic field is $\vec{l}_1 = b(-\hat{k})$ and $\vec{l}_2 = b\hat{k}$ respectively. Let $\vec{F}_g = F_g\hat{k}\ldots(1)$ is the tension experienced by the spring when the rectangular loop is suspended in magnetic field, without any current flowing through the loop.



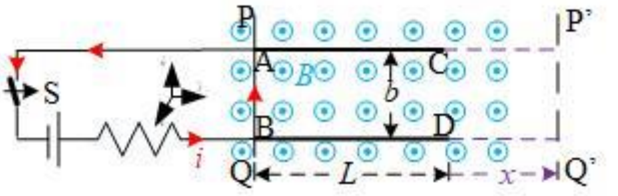
	Magnetic force as per Lorentz's Force Law is $\vec{F}=(q\vec{v})\times\vec{B} \Rightarrow \vec{F}=\left(i\vec{l}\right)\times\vec{B} \Rightarrow \vec{F}=i\left(\vec{l}\times\vec{B}\right)$. ...(2). Analyzing forces on sides of the loop in the given magnetic field we have- $\vec{F}_{\mathrm{PQ}}=ib\left(-\hat{k}\right)\times B(-\hat{\imath})$, $\vec{F}_{\mathrm{QR}}=ia(\hat{\jmath})\times B(-\hat{\imath})$ and $\vec{F}_{\mathrm{RS}}=ia\left(\hat{k}\right)\times B(-\hat{\imath})$, while $\vec{F}_{\mathrm{SP}}=0$ since this side is outside magnetic field. Thus net force on the spring due to anticlockwise flow current is $\vec{F}_1=F_g\hat{k}+\vec{F}_{\mathrm{PQ}}+\vec{F}_{\mathrm{QR}}+\vec{F}_{\mathrm{RS}}$. Combining above set of equation we have$\vec{F}_1=F_g\hat{k}+ibB\left(\hat{k}\times\hat{\imath}\right)+iaB\left(\hat{\jmath}\times(-\hat{\imath})\right)+ibB\left(-\hat{k}\times\hat{\imath}\right) \Rightarrow \vec{F}_1=\left(F_g+iaB\right)\hat{k}$ …(3). From the above analysis, when current in the loop is reversed change force only in side would affect the tension and as such $\vec{F}_2=F_g\hat{k}+iaB\left((-\hat{\jmath})\times(-\hat{\imath})\right) \Rightarrow \vec{F}_2=\left(F_g-iaB\right)\hat{k}$…(4) Thus change in magnitude of the tension of the spring would $\Delta F=\left\|\vec{F}_2-\vec{F}_1\right\| \Rightarrow \Delta F=\left\|\left(F_g-iaB\right)-\left(F_g-iaB\right)\right\|$. It leads to $\boldsymbol{\Delta F=2iaB}$ **is the answer.**
I-45	 An arbitrary loop carrying current I has been shown placed perpendicular to the magnetic field $\vec{B}=B\hat{k}$ ..(1). Magnetic force experience by a conductor as per Lorentz's Force Law is $\vec{F}=(q\vec{v})\times\vec{B} \Rightarrow \vec{F}=\left(i\vec{l}\right)\times\vec{B} \Rightarrow \Delta\vec{F}=i\left(\Delta\vec{l}\times\vec{B}\right)$…(2). A circumscribing rectangular loop with the same current is also shown in the figure. In (2) an element of the arbitrary loop can be resolved as $\Delta\vec{l}=\Delta x\hat{\imath}+\Delta y\hat{\jmath}$…(3). Combining (2) and (3) we have $\Delta\vec{F}=i\left((\Delta x\hat{\imath}+\Delta y\hat{\jmath})\times B\hat{k}\right)$…(4). This can be simplified into - $\Delta\vec{F}=iB\left(\Delta x\left(\hat{\imath}\times\hat{k}\right)+\Delta y\left(\hat{\jmath}\times\hat{k}\right)\right) \Rightarrow \Delta\vec{F}=iB\left(\Delta x(-\hat{\jmath})+\Delta y(\hat{\imath})\right)$…(5) Thus, magnetic force experienced by the loop is $\oint df=iB\left((\oint dx)(-\hat{\jmath})+(\oint dy)(\imath)\right)$…(6). Line integral in a closed loop $\oint dx=0$ && $\oint dy=0$…(7). Combining (5), (6) and (7) $\Delta F=0$, hence proved.
I-46	 A wire in some arbitrary shape connects two points a and b. For convenience unit direction vectors are shown in the figure. Further, for simplicity the wire is considered in $(\hat{\imath}-\hat{\jmath})$ plane and line joining ends a and b of length $\vec{\lambda}=\lambda\hat{\jmath}$…(1). Wire is considered to be carrying current i is taken to be perpendicular to the magnetic field $\vec{B}=B\hat{k}$ ..(2), as shown in the figure. Magnetic force experience by a conductor as per Lorentz's Force Law is $\Delta\vec{F}=(q\vec{v})\times\vec{B} \Rightarrow \vec{F}=\left(i\overrightarrow{\Delta l}\right)\times\vec{B} \Rightarrow \Delta\vec{F}=i\left(\Delta\vec{l}\times\vec{B}\right)$…(2). An element of the arbitrary loop can be resolved as $\Delta\vec{l}=\Delta x\hat{\imath}+\Delta y\hat{\jmath}$…(3). Combining (2) and (3) we have $\Delta\vec{F}=i\left((\Delta x\hat{\imath}+\Delta y\hat{\jmath})\times B\hat{k}\right)$…(4). This can be simplified into - $\Delta\vec{F}=iB\left(\Delta x\left(\hat{\imath}\times\hat{k}\right)+\Delta y\left(\hat{\jmath}\times\hat{k}\right)\right) \Rightarrow \Delta\vec{F}=iB\left(\Delta x(-\hat{\jmath})+\Delta y(\hat{\imath})\right)$…(5). Hence, total magnetic force on the wire is $\vec{F}=iB\left(\left(\int_0^0 dx\right)(-\hat{\jmath})+\left(\int_a^b dy\right)\hat{\imath}\right) \Rightarrow F=i\lambda B$. While, actual length of wire is $l=\int_0^l dl$. As per Euclid's postulates, actual length of line $l\geq\lambda$ shortest distance λ joining the two points. Thus, **magnetic force on a current carrying wire between two points, placed in magnetic field, is independent of the length of the wire. Hence proved.**

I-47	A semicircular wire has two ends a and b. with radius of semicircle $R = 0.05$m. For convenience unit direction vectors are shown in the figure. Further, for simplicity the wire is considered in $(\hat{\imath} - \hat{\jmath})$ plane and line joining ends a and b of length $\vec{\lambda} = 2R\hat{\jmath}\ldots(1)$. Wire is considered to be carrying current $I = 5.0$A is taken to be perpendicular to the magnetic field $\vec{B} = B\hat{k}$ ..(2), as shown in the figure.  Magnetic force experience by a conductor as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\overrightarrow{\Delta l}) \times \vec{B} \Rightarrow \Delta\vec{F} = i(\Delta\vec{l} \times \vec{B})\ldots(2)$. An element of the arbitrary loop can be resolved as $\Delta\vec{l} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}\ldots(3)$. Combining (2) and (3) we have $\Delta\vec{F} = i\left((\Delta x\hat{\imath} + \Delta y\hat{\jmath}) \times B\hat{k}\right)\ldots(4)$. This can be simplified into - $\Delta\vec{F} = iB\left(\Delta x(\hat{\imath} \times \hat{k}) + \Delta y(\hat{\jmath} \times \hat{k})\right) \Rightarrow \Delta\vec{F} = iB(\Delta x(-\hat{\jmath}) + \Delta y(\hat{\imath}))\ldots(5)$. Hence, total magnetic force on the wire is $\vec{F} = iB\left(\left(\int_0^0 dx\right)(-\hat{\jmath}) + \left(\int_0^{2R} dy\right)\hat{\imath}\right) \Rightarrow \vec{F} = 2IRB\hat{\imath}.\ \ldots(6)$. While, actual length of wire is $l = \int_0^l dl$. As per Euclid's postulates, actual length of semicircular arc $l \geq 2R$, the latter is the shortest distance joining the two points. Using the available data $\vec{F} = 2 \times 5.0 \times 0.05 \times 0.50\hat{\imath} \Rightarrow$ $\boldsymbol{F = 0.25}$ **is the answer.**
I-48	A wire in shape of a curve defined by $y = \sin\left(\frac{2\pi}{\lambda}x\right)\ldots(1)$ is placed in X-Y plane and magnetic field $\vec{B} = B\hat{k}\ldots(2)$ is Z-direction. For convenience direction vectors $\hat{\imath} \rightarrow \mathrm{X-axis}, \hat{\jmath} \rightarrow \mathrm{Y-axis}, \hat{k} \rightarrow \mathrm{Z-axis}$ and distance Ob along X-axis corresponds to $\lambda\hat{\imath}$. Wire is carrying current I.  Magnetic force experience by a conductor as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\overrightarrow{\Delta l}) \times \vec{B} \Rightarrow \Delta\vec{F} = i(\Delta\vec{l} \times \vec{B})\ldots(2)$. An element of the arbitrary loop can be resolved as $\Delta\vec{l} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}\ldots(3)$. Combining (2) and (3) we have $\Delta\vec{F} = i\left((\Delta x\hat{\imath} + \Delta y\hat{\jmath}) \times B\hat{k}\right)\ldots(4)$. This can be simplified into - $\Delta\vec{F} = iB\left(\Delta x(\hat{\imath} \times \hat{k}) + \Delta y(\hat{\jmath} \times \hat{k})\right) \Rightarrow \Delta\vec{F} = iB(\Delta x(-\hat{\jmath}) + \Delta y(\hat{\imath}))\ldots(5)$. Hence, total magnetic force on the wire is $\vec{F} = iB\left(\left(\int_0^{\lambda} dx\right)(-\hat{\jmath}) + \left(\int_0^0 dy\right)\hat{\imath}\right) \Rightarrow \vec{F} = I\lambda B(-\hat{\jmath}).\ \ldots(6)$. Thus, magnitude of the magnetic force experienced by the wire is $\boldsymbol{I\lambda B}$ **is the answer.**
I-49	Given system is shown in the figure. A rigid wires is shaped such that it has two straight and parallel portions of equal lengths cb and cd $\vec{l}_{ab} = l(-\hat{\imath})\ldots(1)$, and $\vec{l}_{cd} = l\hat{\imath}\ldots(2)$ respectively. The length vectors, though parallel are taken in direction of currents and unit-direction vectors, as shown in the figure. These two portions are connected through a portion bc in semicircular shape of radius R. Vectorially, a small length of the arc $\Delta\vec{l}_{bc} = R\hat{r} \times \Delta\theta(-\hat{k})\ldots(3)$; here $\hat{r}$ unit-direction vector of the element $\Delta\vec{l}_{bc}$and unit-direction vector of $\Delta\theta$ is taken along $(-\hat{k})$ since current in the semicircular portion is in clockwise direction.  The wire is taken to be on $(\hat{\imath} - \hat{\jmath})$ plane while magnetic field, as shown in the figure, is $\vec{B} = B\hat{k}\ldots(4)$. Magnetic force experience by a conductor as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\overrightarrow{\Delta l}) \times \vec{B} \Rightarrow \Delta\vec{F} = i(\Delta\vec{l} \times B\hat{k}) \Rightarrow \Delta\vec{F} = iB(\vec{l} \times \hat{k})\ldots(5)$. Combining above equations it leads to –

	 $\vec F = \vec F_{ab} + \vec F_{bc} + \vec F_{ca}$ $\vec F = IB\left(l(-\hat\imath)\times B\hat k + \int_\pi^0 \left(R\hat r \times d\theta(-\hat k)\right)\times \hat k + l(\hat\imath)\times\hat k\right)$ $\vec F = (-)IRB\left(\int_\pi^0(\hat l d\theta)\times\hat k\right) \Rightarrow \vec F = (-)IRB\left(\int_\pi^0(\hat l d\theta)\times \hat k\right)$; here unit vector $\vec l = \cos(90^0-\theta)\,\hat\jmath - \sin(90^0-\theta)\,\hat\imath$. Thus, $\vec F = (-)IRB\left(\int_\pi^0((\sin\theta\,\hat\jmath - \cos\theta\,\hat\imath)d\theta)\times\hat k\right)$. It leads to - $\vec F = (-)IRB\left(\int_\pi^0\left(\left((\hat\jmath\times\hat k)\sin\theta - (\hat\imath\times\hat k)\cos\theta\right)d\theta\right)\right) \Rightarrow \vec F = (-)IRB\left(\int_\pi^0((\hat\imath\sin\theta + \hat\jmath\cos\theta\,\hat\imath)d\theta)\right)$ $\vec F = (-)IRB([\hat\imath\sin\theta - \hat\jmath\cos\theta]_\pi^0)\hat\imath \Rightarrow \vec F = IRR[\cos 0 - \cos\pi]\hat\imath \Rightarrow \vec F = 2IRB\hat\imath$, Thus, **force is $2IRB$ downward is the answer.** **N.B.:** It is an example of proficiency in analysis using mathematics as an unambiguous tool of clarity.
I-50	 Given is a straight wire, placed horizontally, has mass $m = 10\text{mg} \Rightarrow m = 10^{-5}\text{kg}$ and length $l = 1.0$ m, carries a current $I = 2.0$ A. It is required to find magnetic field required in the region such that magnetic force on the wire balances its weight. For convenience of analysis 3D unit vectors are shown in the figure, and wire is so placed on a parallel to $(\hat\imath - \hat\jmath)$ plane and oriented along $\hat\jmath$ such that length vector is $\vec l = l\hat\jmath$…(1).\ As per Lorentz's Force Law magnetic force experience by a conductor is $\Delta\vec F = (q\vec v)\times\vec B \Rightarrow \vec F = (i\vec l)\times\vec B \Rightarrow \vec F = i(\vec l\times B\hat k) \Rightarrow \vec F = iB(l\hat\jmath\times\hat B) \Rightarrow \vec F = iBl(\hat\jmath\times\hat B)$…(2). Gravitational force is vertically downward, therefore gravitational force $\vec F_g = mg(-\hat k)$…(3). Taking $g = 10$ m.s[2]. Thus, using available data and equation of equilibrium would be $\vec F + \vec F_g = 0 \Rightarrow ilB(\hat\jmath\times\hat B) = -mg(-\hat k)$. It leads to $2.0\times 1.0\times B(\hat\jmath\times\hat B) = 10^{-5}\times 10\hat k \Rightarrow B(\hat\jmath\times\hat B) = \frac{0.10}{2}\hat k \Rightarrow B\sin\theta\,\hat n = 5\times 10^{-5}\hat k$. Here, $\hat n$ is the unit vector perpendicular to the plane containing vector $(\hat\jmath - \hat B)$ and in this case θ is the angle of unit vector $\hat B$ and unit vector $\hat\jmath$. Since, vectorially $\hat n \to \hat k \Rightarrow B = \frac{5\times10^{-5}}{\sin\theta}$. Hence, minimum value of B depends upon maximum value on $\sin\theta = 1 \Rightarrow \boldsymbol{B_{min} = 5\times 10^{-5}}$**T is the answer.**
I-51	 Given system is shown in the figure and with given data ΔOPQ is equilateral of side $l = 0.20$ m. For convenience 3D unit vectors are shown in the figure. Hence,$2T\sin\frac{\pi}{3} = F \Rightarrow 2T\left(\frac{\sqrt3}{2}\right) = F \Rightarrow T = \frac{F}{\sqrt3}$. Given that mass of wire $m = 0.200$ kg and $\vec B = 0.500\hat\imath$ T, and acceleration due to gravity is not specified it's magnitude is taken as $g = 10$ m/s². It leads to $\vec g = 10(-\hat k)$ ms². Both the parts are solved below – **Part (a):** When switch is open $\vec F = \vec F_g = m\vec g$; using given data $\vec F = 0.200\times 10(\hat\imath) \Rightarrow F = 2.00\text{ N} \Rightarrow T = \frac{2.00}{\sqrt3} \Rightarrow T = 1.15$ N or $T = 1.2$ N **Part (b):** When switch is closed net force on wire would be $\vec F = \vec F_g + \vec F_m$. Here, $\vec F_m = (i\vec l)\times\vec B \Rightarrow \vec F_m = i(l\hat\jmath\times B\hat k) \Rightarrow \vec F_m = ilB\hat\imath$. Thus, using available data $\vec F = 2.00\hat\imath + 2.0\times 0.20\times 0.500\hat\imath \Rightarrow \vec F = 2.2\hat\imath$ N. Hence, tension in the strings would be $T = \frac{2.2}{\sqrt3} \Rightarrow T = 1.3$ N. Hence, **answers are (a) $\mathbf{1.2}$ N and (b) $\mathbf{1.3}$ N.**

N.B.: Reporting of answers is using principle of significant digits.

I-52



For convenience of analysis unit vectors in 3D asre shown in the figure. Given system is placed in $(\hat{\imath}-\hat{\jmath})$ plane and magnetic field is $\vec{B}=B\hat{k}$…(1). At $t=0$, switch S is closed wire PQ is at ends AB of the two strips AC||BD clamped at a searation $\vec{l}=b(-\hat{\imath})$. It will establish a current i in the circuit as shown in the figure. This current flows in portion BA of the wires from B to A whose length is $l=b$…(2), i.e. separation between the two strips ACand BD, each of length L that are clamped. The strips AC and BD are firction less, but while after traversing the length

As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F}=i\vec{l}\times\vec{B}$. Using the avaliable data $\vec{F}=i\left(b(-\hat{\imath})\times\left(B\hat{k}\right)\right)\Rightarrow\vec{F}=ibB\left((-)\hat{\imath}\times\hat{k}\right)\Rightarrow\vec{F}=ibB\hat{\jmath}$…(3). Thus wire of mass m would experience a force and acceleration $F=ibB\Rightarrow a=\frac{F}{m}\Rightarrow a=\frac{ibB}{m}$…(4) along right side i,e, toward ends C-D.

Strips are of hiher crossection and hence considered to be of negligible resistance. Therefore, while wire under amgnetic force would slip along the length L of the strip there would be no change of current and consequently force F and acceleration a would remain constant.

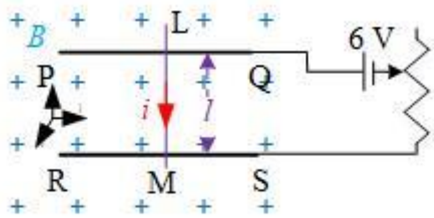
Thus velocity attained by the wire, starting from state of rest frpm position PQ with $u=0$, as it reaches position P'Q' and touches the floor with a velocity v, as per 3rd equation of motion, $v^2=u^2+2as$…(5). It, wiith available data, leads to $v^2=0+2\left(\frac{ibB}{m}\right)L\Rightarrow v^2=\frac{2ibBL}{m}$…(6).

As soon as wire touches ground having coefficient of friction μ, it experiences a frictional force $f=-\mu mg$ …(7), here is acceleration due to gravity. Thus wire would experience a frictional$a_f=\frac{f}{m}\Rightarrow a_f=(-)\frac{\mu mg}{m}$. It leads to $a_f=(-)\mu g$…(8).

Again applying (5), in this case with $u^2=v^2=\frac{2ibBL}{m}$, $v^2=0$ and $a_f=(-)\mu g$, distance $s=x$ travelled by the wire, as shown in the figure, until it stops is $0=\frac{2ibBL}{m}+2\left((-)\mu g\right)x\Rightarrow x=\frac{\frac{2ibBL}{m}}{2\mu g}\Rightarrow \boldsymbol{x=\frac{ibBL}{\mu mg}}$ **is the answer.**

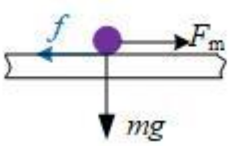
N.B.: This problems integrates electromagnetism with mechanics.

I-53

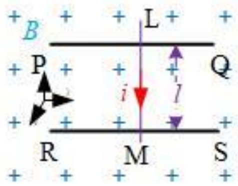
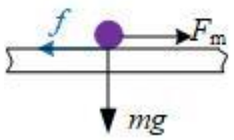
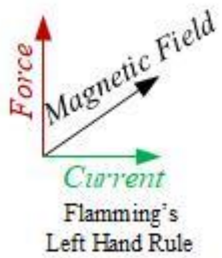
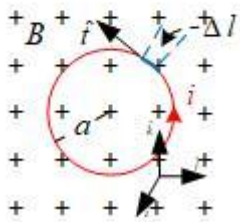
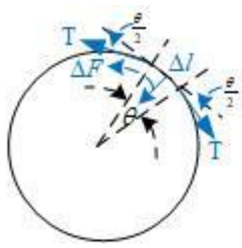


For convenience of analysis unit vectors in 3D asre shown in the figure. Given system is placed in $(\hat{\imath}-\hat{\jmath})$ plane and magnetic field is $\vec{B}=8.00\times10^{-1}(-\hat{k})$T…(1). Two rails PQ||RS having a separation $\vec{l}=4.9\times10^{-2}(-\hat{\imath})$ complete the circuit through a metal wire LM of mass $m=1.0\times10^{-2}\ kg$. Resistance of the circuit slowly decreased and when it is $r=20.0\Omega$ the wire starts sliding on the rail.

As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F}=i\vec{l}\times\vec{B}$. Using the avaliable data $\vec{F}=i\left(l(\hat{\imath})\times B\left(-\hat{k}\right)\right)\Rightarrow\vec{F}=ilB\left(\hat{\imath}\times(-)\hat{k}\right)\Rightarrow\vec{F}=ilB\hat{\jmath}$…(3). Thus, wire would experience a force and acceleration $F=ilB$…(4) along right side i.e. toward ends Q-S.



As per Ohm's law slipping of wire does not take place at $i<\frac{v}{r}$ here voltage of source is $v=6$V. It implies that there is friction between rails and wire and at limiting condition $i=\frac{v}{r}$ …(5), the frictional force is in direction opposite to the magnetic force in (4) is $f=\mu mg$…(6), as shown in the figure. Here, acceleration due to gravity is taken as $g=10$ m/s^2.

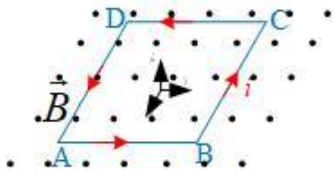
	Until limiting condition is reaheed there is equilibrium of forces such that $F = f$. In this equations (4) and (5) are combined, and with the avalable data $ilB = \mu mg \Rightarrow \mu = \frac{\left(\frac{v}{r}\right)lB}{mg} \Rightarrow \mu = \frac{6\times(4.9\times10^{-2})\times(8.00\times10^{-1})}{(1.0\times10^{-2})\times10\times20.0}$. It solves to $\mu = 0.117$ A. Using principle of significant digits $\boldsymbol{\mu = 0.1}$ **is the answer.**
I-54	For convenience of analysis unit vectors in 3D asre shown in the figure. Given system is placed in $(\hat{\imath} - \hat{\jmath})$ plane and space has a magnetic field is $\vec{B}$. Its direction is not defined. A straight wire LM of length $\vec{l} = l\hat{\imath}$ and mass m is placed on two plastic rails, shown in the figure as PQ||RS. The wire carries a current i from L to M.  As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F} = i\vec{l} \times \vec{B}$. Using the avaliable data $\vec{F} = i\left(l(\hat{\imath}) \times B(\hat{b})\right) \Rightarrow \vec{F} = ilB(\hat{\imath} \times \hat{b}) \Rightarrow \vec{F} = ilB\sin\theta\,\hat{n}$…(1). Since, wire is to slide on rails along $\hat{\jmath}$ direction, minimum force must be along such that $\hat{n} \rightarrow \hat{\jmath}$ sothat angle between these two unit vectors $\alpha = 0 \Rightarrow \cos\alpha = 1$.  Frictional force experienced by the wire, in direction opposite to the magnetic force tending to cause slipping is $f = \mu mg$…(2), as shown in the figure. Here, g is acceleration due to gravity. At limiting condition is reaheed there is equilibrium of forces such that $F = f$. Accordingly, combining (1) and (2), we have $ilB\sin\theta = \mu mg \Rightarrow B = \left(\frac{\mu mg}{il}\right)\frac{1}{\sin\theta}$….(3). It implies that $B \propto \frac{1}{\sin\theta}$. Thus, for minimum vale of B, as dsired, it would occur when $\lvert\sin\theta\rvert_{max} = 1$…(4). Combining (3) and (4), $\boldsymbol{B_{min} = \frac{\mu mg}{il}}$ **is the answer.**
I-55	As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F} = i\vec{l} \times \vec{B}$. In this case a small length of the circular loop $\Delta\vec{l} = \Delta l\hat{t}$ and magnetic field $\vec{B} = B(-\hat{k})$. Accordingly, $\Delta\vec{F} = i\left((\Delta l\hat{t}) \times B(-\hat{k})\right)$…(1). Here, unit tangent vector for Δl is $\hat{t}$ and is $\perp$ to $\vec{B}$. Hence, asper Flemmings Left Hand Rule, as shown in the figure, is $\Delta\vec{F} = i\Delta lB(-\hat{r})$…(2). Since, $\hat{r}$ is unit vector along radius i.e. ouward and hence magnetic force along $(-\hat{r})$ toward the center of the cirle. Thus, **answer of part (a) wire is** $\boldsymbol{i\Delta lB}$ **towards the center.**   **Part (b)** requires to determine fore of compression on wire and is being analyzed with priniples of statics of forces. As determined in part (a) force on small part of the circualr wire which subtends an angle θ at its center O is $\Delta F = i\Delta lB$ towards the center as per (2). This force is result of tensile tension T experienced by the small part of the wire. Geometrically tension T is at an angle $\left(90^0 - \frac{\theta}{2}\right)$ with the force ΔF. Therefore, vectorially $\Delta F = 2T\cos\left(90^0 - \frac{\theta}{2}\right) \Rightarrow \Delta F = 2T\sin\frac{\theta}{2}$….(3)  Since, $\Delta l \ll\Rightarrow \theta \rightarrow 0 \Rightarrow \sin\theta \rightarrow \theta$. Therefore, $\sin\frac{\theta}{2} \rightarrow \frac{\theta}{2}$…(4). It leads to $\Delta F = 2T\frac{\theta}{2} \Rightarrow \Delta F = T\theta$…(5). Combining (2) and (5), $i\Delta lB = T\theta$. Since length of the small part is $\Delta l = a\theta$. It leads to $T\theta = i(a\theta)B$. It leads to $\boldsymbol{T = iaB}$ **is answer of the part (b).** **N.B.:** It is the application of principle of Hoop Stress, in mechanics, into electromagnetiŧsm.
I-56	Tension in the wire for the system is determined as $T = iaB$ **…(1),** in illustration I-55.

As per mechanics, Young Modulus $Y=\frac{stress}{strain}\Rightarrow Y=\frac{\frac{T}{A}}{\frac{\Delta p}{p}}\Rightarrow Y=\frac{Tp}{A\Delta p}\Rightarrow \Delta p=\frac{Tp}{AY}$…(2) . Here, area of cross-section of the wire forming the circular loop $A=\pi r^2$…(3), and perimeter of the circular loop is $p=2\pi a$…(4)
Accordingly, $\Delta p=\frac{(iaB)(2\pi a)}{(\pi r^2)Y}\Rightarrow \Delta p=\frac{2ia^2B}{r^2Y}$…(5)

Taking derivative of (4), $\Delta p=2\pi\Delta a$…(6). Combining (5) and (6) we have $2\pi\Delta a=\frac{2ia^2B}{r^2Y}\Rightarrow \boldsymbol{\Delta a}=\boldsymbol{\frac{ia^2B}{\pi r^2Y}}$ **is increase in radius, is the answer.**

I-57

Given that a square loop ABCD with side length l is placed in X-Y plane corresponding to plane $\hat{\imath}-\hat{\jmath}$ in vector space such vettex A is at origin, and sides AB is along Y-axis and AD alons –Y axishaving magnetic field expressed as $\vec{B}=B_0\left(1+\frac{x}{l}\right)\hat{k}$…(1). Vectorially, magnetic field unidirectional along $\hat{k}$ but it is non-uniform and its depnedent upon value of x at each point on the plane of the loop.



Length vector of each side $\vec{l}_{\mathrm{AB}}=l\hat{\jmath}$, $\vec{l}_{\mathrm{BC}}=l(-\hat{\imath})$, $\vec{l}_{\mathrm{CD}}=l(-\hat{\jmath})$ and $\vec{l}_{AB}=\vec{l}_{\mathrm{DA}}=l\hat{\imath}$…(2).

As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F}=i\vec{l}\times\vec{B}$…(3)

Therefore, net force experienced by a small length Δl of each side of the loopthe loop carrying current i, as per (1), (2) and (3) would be, $\vec{F}=\vec{F}_{\mathrm{AB}}+\vec{F}_{\mathrm{BC}}+\vec{F}_{\mathrm{CD}}+\vec{F}_{\mathrm{DA}}$…(4)

Taking each component in (4) separately –

$\boldsymbol{\vec{F}_{AB}}\big|_{\boldsymbol{x=0}}=i\left(\vec{l}_{\mathrm{AB}}\times\vec{B}\right)\Rightarrow \vec{F}_{\mathrm{AB}}=i\left(l\hat{\jmath}\times B_0\left(1+\frac{x}{l}\right)\hat{k}\right)\Big|_{x=0}\Rightarrow \vec{F}_{\mathrm{AB}}=ilB_0\left(\hat{\jmath}\times\hat{k}\right)\Rightarrow \vec{F}_{\mathrm{AB}}=\boldsymbol{ilB_0\hat{\imath}}$…(5)

$$\boldsymbol{\vec{F}_{BC}}=i\left(\vec{l}_{\mathrm{BC}}\times\vec{B}\right)\Rightarrow \vec{F}_{\mathrm{BC}}=iB_0\left(\int_0^i\left(1+\frac{x}{l}\right)dx\right)(-\hat{\imath})\times\hat{k}\Rightarrow \vec{F}_{\mathrm{BC}}=iB_0\left(\int_0^i\left(1+\frac{x}{l}\right)dx\right)\hat{\jmath}$$

$\Rightarrow \vec{F}_{\mathrm{BC}}=iB_0\left[x+\frac{x^2}{2l}\right]_0^l\hat{\jmath}\Rightarrow \vec{F}_{\mathrm{BC}}=iB_0\left[l+\frac{l^2}{2l}\right]\hat{\jmath}\Rightarrow \vec{F}_{\mathrm{BC}}=iB_0\left[l+\frac{l}{2}\right]\hat{\jmath}\Rightarrow \vec{F}_{\mathrm{BC}}=\boldsymbol{\frac{3}{2}ilB_0}\hat{\jmath}$…(6)

$\boldsymbol{\vec{F}_{CD}}\big|_{\boldsymbol{x=-l}}=i\left(\vec{l}_{\mathrm{CD}}\times\vec{B}\right)\Rightarrow \vec{F}_{\mathrm{CD}}=i\left(l(-\hat{\jmath})\times B_0\left(1+\frac{-l}{l}\right)\hat{k}\right)\Big|_{x=0}\Rightarrow \vec{F}_{\mathrm{CD}}=(-)ilB_0(-)\hat{\imath}\Rightarrow \vec{F}_{\mathrm{CD}}=\boldsymbol{0}$ …(7)

$$\vec{F}_{\mathrm{DA}}=i\left(\vec{l}_{\mathrm{DA}}\times\vec{B}\right)\Rightarrow \vec{F}_{\mathrm{DA}}=iB_0\left(\int_0^i\left(1+\frac{x}{l}\right)dx\right)(\hat{\imath})\times\hat{k}\Rightarrow \vec{F}_{\mathrm{DA}}=iB_0\left(\int_0^i\left(1+\frac{x}{l}\right)dx\right)(-\hat{\jmath})$$

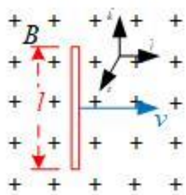
$$\Rightarrow \vec{F}_{\mathrm{DA}}=iB_0\left[x+\frac{x^2}{2l}\right]_0^l(-\hat{\jmath})\Rightarrow \vec{F}_{\mathrm{DA}}=iB_0\left[l+\frac{l^2}{2l}\right](-\hat{\jmath})\Rightarrow \vec{F}_{\mathrm{DA}}=iB_0\left[l+\frac{l}{2}\right](-\hat{\jmath})$$

$$\Rightarrow \vec{F}_{\mathrm{DA}}=(-)\boldsymbol{\frac{3}{2}ilB_0\hat{\jmath}}\text{…(8)}$$

Combining (4)…(8), $\vec{F}=ilB_0\hat{\imath}+\frac{3}{2}ilB_0\hat{\jmath}+0(-)\frac{3}{2}ilB_0\hat{\jmath}\Rightarrow \vec{F}=ilB_0\hat{\imath}$. Thus, magnitude of the force is $\boldsymbol{ilB_0}$ **is the answer.**

I-58

For convenience 3D vectors are shown in the figure. Accordingly, orientations are such that conductor of length $\vec{l}=l\hat{\imath}$, is placed in magnetic field $\vec{B}=B\left(-\hat{k}\right)$ is moving with a velocity $\vec{v}=v\hat{\jmath}$. Charge of an electron is $q=-e$.



Magnetic force on a charge, as per Lorentz's Force Law, is $\vec{F}=q\vec{v}\times\vec{B}$…(1).

Each part of the problem is being solved separately –

Part (a): Average magnetic force on each electron as per (1) with the available data is $\vec{F}_m=(-e)v\hat{\jmath}\times B\left(-\hat{k}\right)\Rightarrow \vec{F}_m=evB\left(\hat{\jmath}\times\hat{k}\right)\Rightarrow \vec{F}_m=evB\hat{\imath}$…(2). Thus force is $\boldsymbol{evB}$ **along X-axis is answer of the part (a)**.

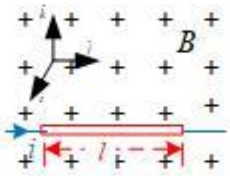
Part (b): To stop the flow electric field E that would develop inside the conductor that would stop redistribution of electrons should develop a force $\vec{F_e} = q\vec{E} \Rightarrow \vec{F_e} = (-e)\vec{E}$…(3), such electrns are in equilibrium. Accordingly, $\vec{F_m} + \vec{F_e} = 0$…(4). Combining (2)…(4) we have $evB\hat{\imath} + (-e)\vec{E} = 0$. It leads to $e\vec{E} = evB\hat{\imath} \Rightarrow E = vB$…(5) along $\hat{\imath}$ is the answer of part (b).

Part (c): Potential difference V along a wire is $V = lE \Rightarrow V = \boldsymbol{lvB}$ **is the answer of part (c).**

Thus, **answers are (a) $\boldsymbol{evB}$, (b) $\boldsymbol{vB}$, (c) $\boldsymbol{lvB}$**

I-59

For convenience 3D vectors are shown in the figure. Accordingly, orientations are such that silver strip of length say $\vec{l} = l\hat{\jmath}$ and area of cross-section A is placed in magnetic field $\vec{B} = B(-\hat{k})$. The strip having number of electrons per unit volume n is carrying a current i along its length i.e. $\hat{\jmath}$. Charge of an electron is $q = -e$.



Solving each part –

Part (a): Current in a conductor is $i = \frac{dQ}{dt} \Rightarrow i = \frac{d(nVq)}{dt} \Rightarrow i = \frac{d(nAl(-e))}{dt} \Rightarrow |i| = nAe\frac{d}{dt}l \Rightarrow |i| = nAev$. Here, $v = \frac{d}{dt}d$ is the drift velocity of free-electrons. Hence, $\boldsymbol{v = \frac{i}{nAe}}$ **is the answer of part (a).**

Part (b): Since charge carriers are electrons having (-) ve charge, hence drift velocity of electrons is opposite to the direction of current. Accordingly, $\vec{v} = (-)\frac{i}{nAe}\hat{\jmath}$….(1)

Magnetic force on a charge, as per Lorentz's Force Law, is $\vec{F} = q\vec{v} \times \vec{B}$…(2). Combining (1) and (2) magnetic force of free electrons is $\vec{F}_{me} = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_{me} = (-e)(-)\frac{i}{nAe}\hat{\jmath} \times B(-\hat{k})$. It leads to $\vec{F}_{me} = (-)\frac{ieB}{nAe}\hat{\jmath} \times \hat{k} \Rightarrow \vec{F}_{me} = \frac{iB}{nA}(-\hat{\imath})$…(3) Thus, magnitude of magnetic force experienced by free electron is $\boldsymbol{\frac{iB}{nA}}$ **upward is the answer of part (b)**.

Part (c): Force on a charge in an electric field $\vec{E}$ is $\vec{F_e} = q\vec{E} \Rightarrow \vec{E} = \frac{\vec{F_e}}{q}$…(4). In the instant case charges are electrons and and force is $\vec{F_e} = \vec{F}_{me}$. Hence combining (3) and (4) we have $\vec{E} = \frac{\frac{iB}{nA}(-\hat{\imath})}{(-e)} \Rightarrow \vec{E} = \frac{iB}{nAe}\hat{\imath}$ …(5). Thus, **answer of part (c) is $\boldsymbol{\frac{iB}{nAe}}$**

Part (d): It is required to determine transerse emf V_{w}, as per Hall Effect, produced along the width of the conductor, when a current-carrying wire is placed in a magnetic field. Let width of the silver strip along $\hat{\imath}$ i.e. direction of is d, $V_{\mathrm{w}} = Ed$, here magnitude of $\vec{E}$ is obtained at (5). Accordingly, we have $V_{\mathrm{w}} = \left(\frac{iB}{nAe}\right)d \Rightarrow V_{\mathrm{w}} = \boldsymbol{\frac{iBd}{nAe}}$ **is the answer of part (d).**

Thus, **answers are (a) $\boldsymbol{\frac{i}{Ane}}$ (b) $\boldsymbol{\frac{iB}{An}}$ upward (c) $\boldsymbol{\frac{iB}{nAe}}$ (d) $\boldsymbol{\frac{iBd}{Ane}}$**.

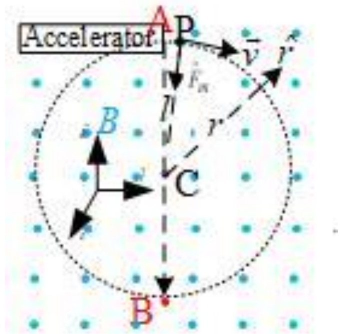
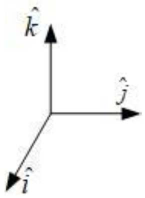
I-60

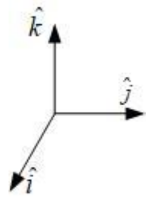
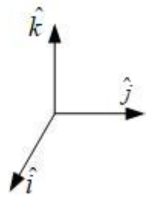
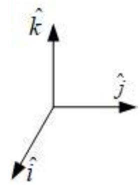
Given that a particle carrying charge $q = 2.0 \times 10^{-8}$ C and mass $m = 2.0 \times 10^{-13}$kg is projected with a velocity with a specified magnitude taken vectorially $\vec{v} = 2.0 \times 10^{3}\hat{\jmath}$ m/s is perpenducularly projected in a uniform magnetic field of specified magnitude is taken vectorially $\vec{B} = 0.10\hat{k}$ T.

In the given syetem the charged particle will experience a magnetic force as per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B}$…(1). Using the available data, $\vec{F} = (2.0 \times 10^{-8})(2.0 \times 10^{3}\hat{\jmath}) \times (0.10\hat{k})$. This leads to $\vec{F} = 4.0 \times 10^{-6}(\hat{\jmath} \times \hat{k}) \Rightarrow \vec{F} = 4.0 \times 10^{-6}\hat{\imath}$ N…(2)

It is seen that this $\vec{F} \perp \vec{v}$ and the charged partcile would continue to move constant speed $v = 2.0 \times 10^{3}$ m/s. It implies that $\vec{F}$ acts as centripetal force and particle would describe a circular motion of radius r such that $F = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{F}$…(3).

	Using the avaialble data **radius of the circle would be** $\boldsymbol{r} = \frac{(2.0\times10^{-13})(2.0\times10^{3})^2}{4.0\times10^{-6}} \Rightarrow r = 2.0\times10^{-1}$m…(4), or **20 cm.** Therfore, time period would be $T = \frac{perimeter\ of\ the\ circle}{velocity\ of\ the\ particle} \Rightarrow T = \frac{2\pi r}{v} \Rightarrow T = \frac{2\pi(2.0\times10^{-1})}{2.0\times10^{3}}$. It leds to a time period $\boldsymbol{T} = 2\pi\times10^{-4}$ **s.** or 6.3×10^{-4}s Thus, **answers are 20 cm,** $\mathbf{6.3\times10^{-4}}$**s.**
I-61	Given that radius of circle described by a proton in magnetic field $B = 0.10$ T is $r_p = 1$ cm, It is required to find radius r_α of an α-particle moving with the same speed in the same magnetic field. This problem involves concept of magnetic force as per Lorentz's Force Law, which is $\vec{F} = q\vec{v}\times\vec{B} \Rightarrow F = qvB\sin\theta\,\hat{n}$…(1) and mechanics of uniform cicular motion $F = \frac{mv^2}{r}$…(2). For uniform cicular motion combining (1) and (2) $qvB\sin\theta = \frac{mv^2}{r} \Rightarrow \frac{v}{B\sin\theta} = \frac{qr}{m}$…(3). With identical $\vec{v}$ and $\vec{B}$ for proton and α-particle L.H.S is same for both the paticles accordingly, $\frac{q_p r_p}{m_p} = \frac{q_\alpha r_\alpha}{m_\alpha} \Rightarrow r_\alpha = \left(\frac{q_p m_\alpha}{q_\alpha m_p}\right) r_p$…(4) Given that $r_p = 1$ cm , and we know that charge of proton $q_p = e$ and $q_\alpha = 2e$, while taking mass of proton $m_p = m$, mass of alpha particle is $m_\alpha = 4m$. Accordingly, $r_\alpha = 1\times\frac{e\times4m}{2e\times m} \Rightarrow r_\alpha =$ **2cm is the answer**. **N.B.:** In the problem value of B is notional and is not rrequired in arriving at results. Secondly, though radius of proton is given in CGS unit, deliberately it has not been converted in SI, because what is required to be detrmined is another radius. Thirdly, all quantities of coefficient in (4) are ratios of identical quantities of the two particles. Thus, this problem gets automatically simplified, without involving apparent calculations.
I-62	Given that an electron having charge $q = -1.6\times10^{-19}$J circulates in a path of radius $r = 0.10$ m possesses kinetic energy $K = \frac{1}{2}mv^2 = 100eV$….(1). One $1eV = qV\|_{V=1}$ is the energy in Joules gained by an electron is moved against electric field created by a potential difference $\Delta V = 1$V.Therefore, $1eV = (1.6\times10^{-19})\times 1 \Rightarrow 1eV = 1.6\times10^{-19}$J…(2). Combining (1) and (2), Therefore, $\frac{1}{2}mv^2 = 100\times(1.6\times10^{-19}) \Rightarrow mv^2 = 3.2\times10^{-17}$…(3). And, speed of electron is $v = \sqrt{\frac{3.2\times10^{-17}}{m}} \Rightarrow v = \sqrt{\frac{3.2\times10^{-17}}{9.1\times10^{-31}}} \Rightarrow v = \sqrt{35.16}\times10^{6}$…(4). Here, mass of electron $m = 9.1\times10^{-31}$. A charged particle in motion with a velocity $\vec{v}$ inside a magnetic field $\vec{B}$ experiences a magnetic force, as per Lorentz's Force Law, which is $\vec{F} = q\vec{v}\times\vec{B} \Rightarrow \vec{F}_m = qvB\sin\theta\,\hat{n}$…(5) Here, θ is angle of $\vec{B}$ w.r.t. $\vec{v}$ and unit vector $\hat{n}$ is perpendicular to the plane contaning vectors $\vec{v}$ and $\vec{B}$. Since the electron is describing circular motion of radius r, hence centifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(6) is experienced by it . In state of uniform motion, i.e. equilibrium, $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB\sin\theta = \frac{mv^2}{r}$. It leas to $B\sin\theta = \frac{mv^2}{qvr} \Rightarrow B\sin\theta = \frac{mv}{qr}$…(7). It is seen that magnitude of magnetic field is dependent upon $\sin\theta$ and maximum value of $\sin\theta_{max} = 1 \Rightarrow B_{min} = \frac{mv}{qr}$…(8). Using the available data in (8), $B_{min} = \frac{(9.1\times10^{-31})(\sqrt{35.16}\times10^{6})}{(1.6\times10^{-19})\times0.10} \Rightarrow \boldsymbol{B_{min}} = 33.7\times10^{-5}$T or $\mathbf{3.4\times10^{-4}}$**T is answer of the first part.** Number of revolutions per second made by electron is $n = \frac{speed\ of\ electron}{Permeter\ of\ the\ circular\ orbit} \Rightarrow n = \frac{v}{2\pi r}$…(9). Using the available data $n = \frac{\sqrt{35.16}\times10^{6}}{2\pi\times0.10} \Rightarrow n =$ $\mathbf{9.4\times10^{6}}$ **is answer of the second part,** Thus, **answers are** $\mathbf{3.4\times10^{-4}}$**T,** $\mathbf{9.4\times10^{6}}$.

I-63	For convenience of analysis, unit vectors in 3D are shown in the figure. Protons having charge $q = e$ come out of an accelerator rom A with kinetc energy $K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$…(1). Here, m is mass of proton and $\vec{v} =$ is the velocity. The proton beam bends while passing through magnetic field $\vec{B} \perp \vec{v}$.  As per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = q\vec{v} \times B\hat{k} \Rightarrow \vec{F}_m = qvB\sin\theta\,(-\hat{r})$ .Here, angle of $\vec{B}$ w.r.t. $\vec{v}$ is given to be $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$ and unit radial vector $\hat{r}$ as shown in the figure. Accordingly, $\vec{F}_m = qvB(-\hat{r})$…(2), centripetal force responsible for circular motion of the proton. Magnetic force at A is in a direction perpendicular to the velocity of ejection by accelerator. It will not will change the speed and circular trajectory of protons is shown in the figure. The proton while describing circular it will experience it will experience centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(3). . In state of uniform motion, i.e. equilibrium, $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r}$. It leas to $B = \frac{mv^2}{qvr} \Rightarrow B = \frac{mv}{qr}$…(4). Using the available data, $B = \frac{m\left(\sqrt{\frac{2K}{m}}\right)}{er} \Rightarrow B = \frac{\sqrt{2mK}}{er}$…(5). It is given that magnitude of the magnetic field is such that it just misses a target placed at distance l from the accelerator. From the the geometry of of the circular path of proton in magnetic field it isseen that $r = \frac{l}{2}$, accordingly (5) gets transformed to $B = \frac{\sqrt{2mK}}{e\frac{l}{2}} \Rightarrow B = \frac{2\sqrt{2mK}}{el} \Rightarrow \boldsymbol{B = \frac{\sqrt{8mK}}{el}}$ **is the answer.** **N.B.:** This problem needs careful analysis of motion of the charged particle. Accordingly, the target along the accelerator will always be missed. It must be along a line perpendicular to the initial velocity at A, the instant of ejection from the accelerator. Rest of the problem is application of electromagnetic forces and mechanics of circular motion.
I-64	Let a particle having mass m and carrying a charge q is accelerated through a potential difference $V = 12 \times 10^3$ V. At the instant of injection into perpendicular magnetic field $\vec{B} = 0.2\hat{k}$ velocity attained by the particle, taken to be along Y-axis, is $\vec{v} = v\hat{j} = (1.0 \times 10^6)\hat{v}$ m/s. For convenience 3D unit vectors are shown in the figure.  As per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB(\hat{v} \times \hat{k}) \Rightarrow \vec{F}_m = qvB(-\hat{r})$ …(1). Since $\vec{F}_m \perp \vec{v}$ it a condition of uniform circular motion. It is required to find radius of circle described by the particle. During circular motion particle would experience a centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$…(2). During circular motion of the particle forces are in equilibrium and hence $\vec{F}_m + \vec{F}_c = 0$ …(3). Combining (1)..(3), $qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \left(\frac{m}{q}\right)\left(\frac{v}{B}\right)$…(4). In (4) $\frac{m}{q}$ is unknown and for this acceleration of particle in electric field is being analyzed. Energy balance of the particle after its acceleration in electric field. Accordingly, $qV = \frac{1}{2}mv^2 \Rightarrow \frac{m}{q} = \frac{2V}{v^2}$…(5). Combining (4) and (5), $r = \left(\frac{2V}{v^2}\right)\left(\frac{v}{B}\right) \Rightarrow r = \frac{2VB}{vB}$ Using the available data, $r = \frac{2(12\times10^3)}{(1.0\times10^6)(0.2)} \Rightarrow \boldsymbol{r} = 12 \times 10^{-2}$m or **12 cm is the answer.**

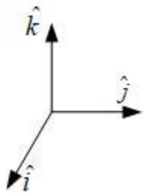
I-65	 Mass of helium ion is four times the mass of a proton accordingly $m = 4(1.6 \times 10^{-27})$kg and charge on it is $q = 2e = 2 \times (1.6 \times 10^{-19})$ C. For convenience of analysis 3D unit vectors are shown in the figure. Given is speed of projection of the ion, taken along $\hat{j}$, is $\vec{v} = v\,\hat{j} = 10 \times 10^3\,\hat{j}$ m/s. The ion is projectedinto magnetic field perpendicularly. Accordingly, taking $\vec{B} = B\hat{k} = 1.0\hat{k}$T. This system would cause a magnetic force on the ion as per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB(\hat{v} \times \hat{k}) \Rightarrow \vec{F}_m = qvB\hat{n}$. Since $\vec{F}_m \perp \vec{v}$ it a condition of uniform circular motion, therefore, $\hat{n} \rightarrow (-\hat{r})$ hence, $\vec{F}_m = qvB(-\hat{r})$…(1). Using the available data $F_m = 2 \times (1.6 \times 10^{-19})(10 \times 10^3)(1.0) \Rightarrow F_m = \mathbf{3.2 \times 10^{-15}}$ **N is answer of part (a).** During circular motion the ion would experience a centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$…(2). During circular motion of the particle forces are in equilibrium and hence $\vec{F}_m + \vec{F}_c = 0$ …(3). Combining (1)..(3), $qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \left(\frac{m}{q}\right)\left(\frac{v}{B}\right)$…(4). Using available data $r = \left(\frac{4\times1.6\times10^{-27}}{2\times(1.6\times10^{-19})}\right)\left(\frac{10\times10^3}{1.0}\right)$. It leads to $\boldsymbol{r = 2.0 \times 10^{-4}}$**m is answer of the part (b)** Thus, time taken by the the ion complete one revolution $T = \frac{Perimeter\ of\ the\ circle}{Speed\ of\ the\ ion} \Rightarrow T = \frac{2\pi r}{v}$. Using the available data $T = \frac{2\pi(\mathbf{2.0\times10^{-4}})}{10\times10^3} \Rightarrow T = 1.3 \times \mathbf{10^{-7}}$**s.** **Thus, answers are (a) $\mathbf{3.2 \times 10^{-15}}$ N (b) $\mathbf{2.1 \times 10^{-4}}$m (c) $\mathbf{1.3 \times 10^{-7}}$ s**
I-66	 Mass of a proton is $m = (1.6 \times 10^{-27})$kg and charge on it is $q = e = (1.6 \times 10^{-19})$ C. For convenience of analysis 3D unit vectors are shown in the figure. Given is speed of projection of the ion, taken along $\hat{j}$, is $\vec{v} = v\,\hat{j} = 3 \times 10^6\,\hat{j}$ m/s. The ion is projectedinto magnetic field perpendicularly. Accordingly, taking $\vec{B} = B\hat{k} = 0.6\hat{k}$T. This system would cause a magnetic force on the ion as per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB(\hat{v} \times \hat{k}) \Rightarrow \vec{F} = qvB\hat{n}$. Since $\vec{F}_m \perp \vec{v}$ it a condition of uniform circular motion, therefore, $\hat{n} \rightarrow (-\hat{r})$ hence, $\vec{F} = F(-\hat{r}) = qvB(-\hat{r})$…(1). Therefore, centripetal acceleration experienced by the proton is $a = \frac{F_m}{m} \Rightarrow a = \frac{qvB}{m}$…(2). Using the available data $a = \frac{(1.6\times10^{-19})(3\times10^6)(0.6)}{1.6\times10^{-27}} \Rightarrow a = \mathbf{1.8 \times 10^{14}}$ **m/s² is the answer.**
I-67	 Given that an electron carrying charge $q = (1.6 \times 10^{-19})$ C mives in a circular path of radius $\hat{r} = r\hat{r} = 1\hat{r}$ m moves with a velocity $\vec{v} = v\hat{v}$ is perpendicular to the magnetic field $\vec{B} = B\hat{k} = 0.50\hat{k}$T. For convenience of analysis 3D unit vectors as shown in the figure have been used.. Electron in this system would cause a magnetic force as per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = qvB(\hat{v} \times \hat{k}) \Rightarrow \vec{F} = qvB\hat{n}$. Since $\vec{F}_m \perp \vec{v}$ it a condition of uniform circular motion, therefore, $\hat{n} \rightarrow (-\hat{r})$ hence, $\vec{F} = F(-\hat{r}) = qvB(-\hat{r})$…(1). Therefore, centripetal acceleration experienced by the proton is $a = \frac{F_m}{m} \Rightarrow a = \frac{qvB}{m}$…(2). Mass of electron $m = 9.1 \times 10^{-31}$kg. In circular motion eectron of mass m would experience a centrfugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(3). During uniform circular motion $\vec{F} + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m}$…(4). Using the available data we have $v = \frac{(1.6\times10^{-19})\times0.5\times1}{9.1\times10^{-31}} \Rightarrow \boldsymbol{v = 8.8 \times 10^{10}}$ **m/s. is the answer of part (a), but it is unreasonable** since it is greater than velocity of light $c = 3 \times 10^8$m/s.

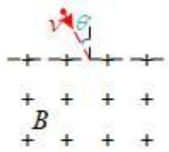
In case the particle is proton velocity would be determined using (4) where $m = m_p = 1.6 \times 10^{-27}$. Accordingly, velocity of proton $v_p = \frac{qBr}{m_p} \Rightarrow v_p = \frac{(1.6\times10^{-19})\times0.5\times1}{1.6\times10^{-27}} \Rightarrow \boldsymbol{v_p = 5.0 \times 10^7}$**m/s is answer of part (b).**

Thus, **answers are (a)** $\boldsymbol{8.8 \times 10^{10}}$**m/s (b)** $\boldsymbol{5.0 \times 10^7}$**m/s** .

I-68

This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass mass m and charge q moving with a velocity $\vec{v} = v\hat{v}$ enters a magnetic field $\vec{B} = B(-\hat{k})$ as shown in the figure. It is seen that velocity vector $\vec{v} \perp \vec{B}$. Therefore, magnetic force experienced by the particle $\vec{F}_m = q\vec{v} \times \vec{B}$. It leads to $\vec{F}_m = qvB(\hat{n})$….(1). Here, $\hat{n}$ is perpendicular to both the vectors $\vec{v}$ and $\vec{B}$. This is a condition of circular motion where $\vec{F}_m$ acts as centripetal force such that $\hat{n} \rightarrow (-\hat{r})$ …(2), and $\hat{r}$ is the radius vector of the circular path.

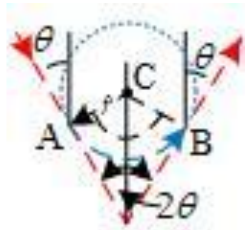




With this pre-analysis of the system, each part is being solved separately –

Part (a): The particle during circular motion would experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(3). During the uniform circular motion forces are in equilibrium. Thus, combining (1), (2) and (3) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow \boldsymbol{r = \frac{mv}{qB}}$**…(4), is answer of part (a)**

Part (b): Let A is the point at which charged particle is entering the magnetic field at an angle θ and after taking a circular path of radius r, having center at C, it exits the magnetic field at B, as shown in the figure $\angle ACB = \alpha = (\pi - 2\theta)$…(5). Thus, geometrically the arc AB subtends an angle $(\boldsymbol{\pi - 2\theta})$**) at its center C is the answer of part (b).**



Part (c): Time spend by the particle in the magnetic field which is performing uniform circular motion with speed v is $t = \frac{rength\ of\ the\ arc}{speed\ of\ the\ particle}$…(6). Length of the arc AB $l_{AB} = r\alpha$…(7). Thus combining (4)…(7) we have $t = \frac{r(\pi-2\theta)}{v} \Rightarrow t = \frac{\frac{mv}{qB}(\pi-2\theta)}{v} \Rightarrow t = \frac{m(\pi-2\theta)}{qB}$…(8).Thus, **answer of part (c) is** $\boldsymbol{\frac{m(\pi-2\theta)}{qB}}$

Part (d): In this case charge of the particle is $(-q)$. Therefore, analysis would be on the lines in part (a)..(c) except in all the equations. Therefore direction of magnetic force would reverse leading to the trajectory of the path of the particle as shown in the figure.

Since, magnitude of the magnetic force and counterbalancing centrifugal force remain unchanged. Hence, radius of the path of uniform circular motion would remain same as $\boldsymbol{r = \frac{mv}{qB}}$. Further, parajectory of the particle is major arc of the circle and hence geometrically angle formed by the major arc at the centre C of the trajectory is $\boldsymbol{\pi + 2\theta}$. As regards speed of the particle as weel as radius of the circular path remain unchanged. Hence, **time taken by the particle to come out of the magnetic field is** $\boldsymbol{\frac{m(\pi+2\theta)}{qB}}$**.**

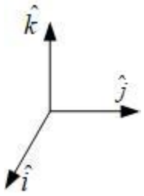
Thus, **answer of the part (d) is** $\boldsymbol{\frac{mv}{qB}, \pi + 2\theta, \frac{m(\pi+2\theta)}{qB}}$

Thus, **answers are (a)** $\boldsymbol{\frac{mv}{qB}}$ **(b)** $\boldsymbol{\pi - 2\theta}$ **(c)** $\boldsymbol{\frac{m}{qB}(\pi - 2\theta)}$ **(d)** $\boldsymbol{\frac{mv}{qB}, \pi + 2\theta, \frac{m}{qB}(\pi + 2\theta)}$**.**

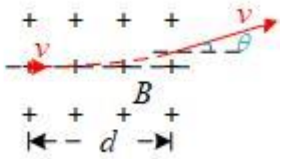
N.B.: (1) In such in part (d) analytical equations remain same as in part (a)…(c), except for the change in charge from $q \rightarrow (-q)$. Accordingly there is change in trajectory of the charged particle. Thus affecting change in geometry, wherever necessary, symmetry of equations can be utilized to abridge the answer, unless Part (d) is an independent question.

(2) This problem involves uniform speed of particle along a circular trajectory. Hence, $t = \frac{rength\ of\ the\ arc}{speed\ of\ the\ particle}$ is correct. However, solving it as, on the lines of projectile motion $t = \frac{Displacement\ AB}{Component\ of\ velocity\ vector\ along\ AB}$ would be incorrect.

I-69



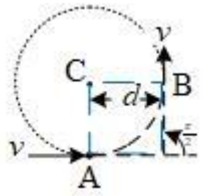
This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass mass m and charge q moving with a velocity $\vec{v} = v\hat{v}$ enters a magnetic field $\vec{B} = B(-\hat{k})$ as shown in the figure. It is seen that velocity vector $\vec{v} \perp \vec{B}$. Therefore, magnetic force experienced by the particle $\vec{F}_m = q\vec{v} \times \vec{B}$. It leads to $\vec{F}_m = qvB(\hat{n})$….(1). Here, $\hat{n}$ is perpendicular to both the vectors $\vec{v}$ and $\vec{B}$. This is a condition of circular motion where $\vec{F}_m$ acts as centripetal force such that $\hat{n} \rightarrow (-\hat{r})$ …(2), and $\hat{r}$ is the radius vector of the circular path.



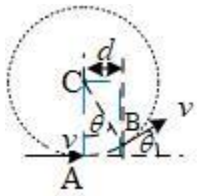
The particle during circular motion would experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(3). During the uniform circular motion forces are in equilibrium. Thus, combining (1), (2) and (3) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$…(4).

With this pre-analysis of the system, each part is being solved separately –

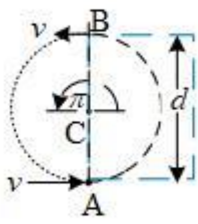
(a) $\frac{mv}{qB}$ (b) $\frac{mv}{2qB}$ (c) $\frac{2mv}{qB}$



Part (a) - $\boldsymbol{d}$ is slightly smaller than $\boldsymbol{\frac{mv}{qB}}$: Here, d is width of the magnetic field and radius of the circular trajectory of the particle insidethe magnetic field is $r = \frac{mv}{qB}$. It implies that $d \rightarrow r$. Thus, geometry approach to as shown in the figure. Thus, angle of deviation of the particle is $\boldsymbol{\theta = \frac{\pi}{2}}$ **is answer of the part (a).**



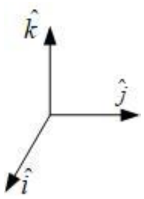
Part (b) - $\boldsymbol{d}$ is slightly smaller than $\boldsymbol{\frac{mv}{2qB}}$: Here, $d \rightarrow \frac{mv}{2qB} \Rightarrow d \rightarrow \frac{r}{2}$. In this case angle of deviation is $\theta = \angle ACB$ and $\sin\theta = \frac{d}{r} \Rightarrow \sin\theta \rightarrow \frac{\frac{r}{2}}{r} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \boldsymbol{\frac{\pi}{6}}$ **is answer of the part (a).**



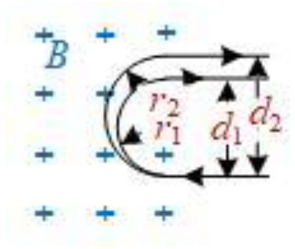
Part (c) - $\boldsymbol{d}$ is slightly smaller than $\boldsymbol{\frac{2mv}{qB}}$: Here, $d \rightarrow \frac{2mv}{qB} \Rightarrow d \rightarrow 2r$. In this case magnetic field is spread over a length which allows projectile to just as shown in the figure. Hence, angle of deviation of the particle is $\boldsymbol{\theta = \pi}$ **is answer of the part (c).**

Thus, **answers are (a)** $\boldsymbol{\frac{\pi}{2}}$ **(b)** $\boldsymbol{\frac{\pi}{6}}$ **(c)** $\boldsymbol{\pi}$

I-70



This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass mass m and charge q moving with a velocity $\vec{v} = v(-\hat{j}) = 6.0 \times 10^4(-\hat{j})$ enters a magnetic field $\vec{B} = B(-\hat{k}) = 0.5(-\hat{k})$T as shown in the figure. It is seen that velocity vector $\vec{v} \perp \vec{B}$. Therefore, magnetic force experienced by the particle, as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = q\left(6.0 \times 10^4(-\hat{j})\right) \times \left(0.5(-\hat{k})\right) \Rightarrow \vec{F}_m = q(3.0 \times 10^4)\hat{n}$…(1).



It is case of circular motion where $\hat{n}$ is perpendicular to both vectors $\vec{v}$ and $\vec{B}$ and $\hat{n} = (-\hat{r})$…(2) Moreover, ions are singly charged, yet initial direction of deflection as given is along $(-\hat{\imath})$ and hence charge on ions must be negative. It leads to $\vec{F}_m = qvB(-\hat{r})$….(3).

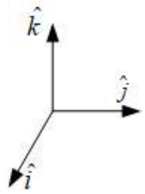
The particle during circular motion would experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(4). During the uniform circular motion forces are in equilibrium. Thus, combining (1), (2) and (3) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$…(5), is radius of the circular trajectory of the ions.

We are given two isotopes whose masses are m_1 and m_2 and, therefore, ratio of their radii is $\frac{r_1}{r_2} = \frac{\frac{m_1 v}{qB}}{\frac{m_2 v}{qB}}$. It leads to $\frac{r_1}{r_2} = \frac{m_1}{m_2}$…(6). It is given that ions emerge out of the magnetic field in backward direction and hence seperation of incident and emergent ions is diameter of the circular trajectory of the ions where $d = 2r$ and likewise atomic mass is $m = A(1.6 \times 10^{-27})$, here A is atomic number of the atom. Accordingly, (6) is transformed into $\frac{d_1}{d_2} = \frac{m_1}{m_2} \Rightarrow \frac{2\times r_1}{2\times r_2} = \frac{A_1\times(1.6\times10^{-27})}{A_2\times(1.6\times10^{-27})} \Rightarrow \frac{d_1}{d_2} = \frac{Z_1}{Z_2}$…(7).

Usimg the given data in (7), $\frac{3}{3.5} = \frac{m_1}{m_2} \Rightarrow \frac{6}{7} = \frac{m_1}{m_2}$…(8). Considering the atomic structure ions having mass $m_1$is ^{12}C and the other ion is $m_1$is ^{14}C whose atomic numbers are 12 and 14 respectively. Thus, answer is ^{12}C and ^{14}C.

N.B.: It is seen from the illustration that all the given data is notional and is not required when solving the problem algebriacally. It, however, requires understanding of atomic numbers for iosotopes. is seen from the illustration that all the given data is notional and is nome of it is required when solving the problem algebriacally. It is, therefore, advised that numerical solution should not be attempted unless it is essential. It saves time and brings in accuracy of results. It, however, requires understanding of atomic numbers for iosotopes.

I-71 This problem involves 3D vectors and hence unit vectors are shown in the figure. An Fe^{+} ion having charge $q = e$, having atomic number A, is accelerated, say along $\hat{\jmath}$, through potential difference $PD = 500$. Therefore, kinetic energy of the ion being injected in the magnetic field $\vec{B} = B\hat{k} = (20 \times 10^{-3})\hat{k}$, taken to be along $\hat{k}$, $K = PD \times e$…(1). Let the ion acquires a velocity $\vec{v} = v\hat{\jmath}$ then $K = \frac{1}{2}mv^2$…(2). Here m is the mass of the ion.



Combining (1) and (2), $\frac{1}{2}mv^2 = PD \times e \Rightarrow v = \sqrt{\frac{2PD\times e}{m}}$…(3), is velocity of the ion.

It is seen that velocity vector $\vec{v} \perp \vec{B}$. This ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = e\left(\sqrt{\frac{2PD\times e}{m}}\hat{\jmath}\right) \times B\hat{k} \Rightarrow \vec{F}_m = \left(\sqrt{\frac{2PD\times e^3 B^2}{m}}\right)\hat{\imath}$. This is a case of circular motion of ion where $\vec{v} \perp \vec{F}_m$, accordingly ceptripetal acceleration would acts along $(-\hat{r}) = \hat{\imath}$. This concludes to $\vec{F}_m = \left(\sqrt{\frac{2PD\times e^3 B^2}{m}}\right)(-\hat{r})$…(4).

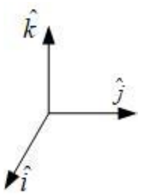
While the ion describes circular motion wouldit experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(5). During the uniform circular motion the two forces $\vec{F}_m$ and $\vec{F}_C$ are in equilibrium. Thus, combining(3), (4) and (5) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow \left(\sqrt{\frac{2PD\times e^3 B^2}{m}}\right)(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow \left(\sqrt{\frac{2PD\times e^3 B^2}{m}}\right) = \frac{m\left(\frac{2PD\times e}{m}\right)}{r} \Rightarrow r = \sqrt{\frac{2PD\times m}{e\times B}}$…(6).

We are given two isotopes whose masses are $m_1 = A_1 \times (1.6 \times 10^{-27})$ and $m_2 = A_2 \times (1.6 \times 10^{-27})$ and, give that $A_1 = 57$ and $A_2 = 58$. Accordingly, using the available data, $\boldsymbol{r_1} = \sqrt{\frac{2\times500\times(57\times(1.6\times10^{-27}))}{(1.6\times10^{-19})\times(20\times10^{-3})}} = \mathbf{119}$ **cm**. Likewise, $\boldsymbol{r_2} = \sqrt{\frac{2\times500\times(58\times(1.6\times10^{-27}))}{(1.6\times10^{-19})\times(20\times10^{-3})}} = \mathbf{120}$ **cm**.

Thus, **answers are 119 cm and 120 cm.**

I-72

This problem involves 3D vectors and hence unit vectors are shown in the figure.Given are single charged potessioum ions having kinetic energy $K = 32 \times 10^3$eV. For reference energy $1eV = 1.6 \times 10^{-19}$J, thus $K = (32 \times 10^3)(1.6 \times 10^{-19})$ J. Such ions are injected along $(\hat{j})$ into a magnetic field $B = 0.500(-\hat{k})$ T …(1) of width $d = 1.00 \times 10^{-2}$m.

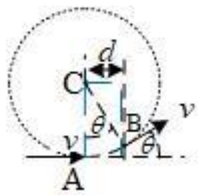


Let the ion acquires a velocity $\vec{v} = v\hat{j}$ then $K = \frac{1}{2}mv^2$…(2). Here m is the mass of the ion. Accordingly, $\frac{1}{2}mv^2 = K \Rightarrow v = \sqrt{\frac{2K}{m}}$…(2),. Hence, velocity vector of the ion. Is $\vec{v} = v\hat{j}$…(3).

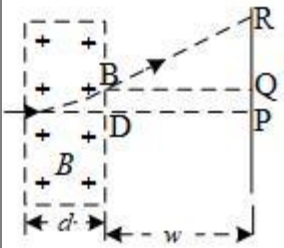
It is seen that velocity vector $\vec{v} \perp \vec{B}$. This ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = e\left(\sqrt{\frac{2K}{m}}\hat{j}\right) \times B\hat{k} \Rightarrow \vec{F}_m = \left(eB\sqrt{\frac{2K}{m}}\right)\hat{\imath}$. This is a case of circular motion of ion where $\vec{v} \perp \vec{F}_m$, accordingly ceptripetal acceleration would acts along $(-\hat{r}) = \hat{\imath}$. This concludes to $\vec{F}_m = \left(eB\sqrt{\frac{2K}{m}}\right)(-\hat{r})$…(4).

While the ions describe circular motion wouldit experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(5). We know that mass number of potassium is A, therefore, $m = A \times (1.6 \times 10^{-27})$kg.

During the uniform circular motion the two forces $\vec{F}_m$ and $\vec{F}_C$ are in equilibrium. Thus, combining (4) and (5) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right)(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right) = \frac{m\left(\sqrt{\frac{2K}{m}}\right)^2}{r}$. It leads to $\Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right) = \frac{2K}{r} \Rightarrow r = \frac{\sqrt{2K\times m}}{eB}$…(6).



Circular trajectory of the ion through narrow magnetic field is shown in the figure. Ion during motion inside magnetic field is deflected through an angle θ, which trigonometrically is $\sin\theta = \frac{d}{r}$…(7), as shown in the figure. Taking $\theta \ll \Rightarrow \sin\theta \to \theta \Rightarrow \theta = \frac{d}{r}$…(8)



Outside the magnetic field it reaches the screen along BR as shown in the figure. Therefore, for ions of kotassium with $A_1 = 39$ and $A_2 = 41$, their striking points R on the screen at a distance wf from the magnetic field would be, above point P, at height $h = \mathrm{RP} - \mathrm{QP} \Rightarrow \frac{h}{w} = \tan\theta$…(9). With the approximation at (8), we have $\tan\theta \to \theta = \frac{d}{r}$. Accordingly, $\frac{h}{w} = \frac{d}{r} \Rightarrow h = \frac{d\times w}{r}$ …(11). This for the two ions is h_1 and h_2.

Accordingly, seperation between the two ions striking on the screen is $\Delta h = \lvert h_1 - h_2\rvert$. It further solves into $\Delta h = \left\lvert\frac{d\times w}{r_1} - \frac{d\times w}{r_2}\right\rvert \Rightarrow \Delta h = (d \times w)\left\lvert\frac{1}{r_1} - \frac{1}{r_2}\right\rvert$ ...(12).

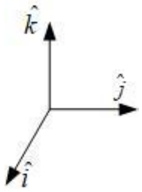
Combining (6) and (12), $\Delta h = (d \times w)\left|\frac{1}{\frac{\sqrt{2K\times(A_1\times(1.6\times10^{-27}))}}{e\times B}} - \frac{1}{\frac{\sqrt{2K\times(A_2\times(1.6\times10^{-27}))}}{e\times B}}\right|$. It further solves into $\Delta h = \frac{(d\times w)\times(e\times B)}{\sqrt{2K\times(1.6\times10^{-27})}}\left|\frac{1}{\sqrt{A_1}} - \frac{1}{\sqrt{A_2}}\right|$.

Using the available data, $\Delta h = \frac{((1.00\times10^{-2})\times(95.5\times10^{-2}))\times((1.6\times10^{-19})\times(0.500))}{\sqrt{2\times((32\times10^{3})\times(1.6\times10^{-19}))\times(1.6\times10^{-27})}}\left|\frac{1}{\sqrt{39}} - \frac{1}{\sqrt{41}}\right|$

$$\Rightarrow \Delta h = (1.89\times10^{-1})\times(3.954\times10^{-3}) \Rightarrow \Delta \boldsymbol{h} = \mathbf{0.75\ mm\ is\ the\ answer.}$$

N.B. This illustration uses approximation in arriving at values of θ and h, and has been accordingly brought out in it.

I-73

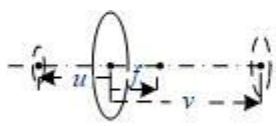


This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass $m = 2.0\times10^{-5}$kg, carries a charge $q = 2.0\times10^{-3}$ C moves with a velocity $\vec{v} = v\hat{v} \Rightarrow \vec{v} = 4.8\hat{k}$ m/s…(1), inside magnetic field is $\vec{B} = B\hat{\jmath} \Rightarrow \vec{B} = 1.2\hat{\jmath}$ T…(2).

It is seen that velocity vector $\vec{v} \perp \vec{B}$. This ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F_m} = q\vec{v}\times\vec{B} \Rightarrow \vec{F_m} = e(v\hat{k})\times B\hat{\jmath} \Rightarrow \vec{F_m} = (evB)(-\hat{\imath})$. This is a case of circular motion of ion where $\vec{v} \perp \vec{F_m}$, accordingly ceptripetal acceleration would acts along $(-\hat{r}) = (-\hat{\imath})$. This concludes to $\vec{F_m} = (qvB)\hat{r}$…(3). Here, $\hat{r}$ is the radius vector of the circular trajectory of the charged particle.

In state of circular motion the particle would experience centripetal force $F_c = \frac{mv^2}{r}$...(4). In case of uniform circular motion there is equilibrium of forces acting on the particle. Accordingly, $F_m = F_c \Rightarrow qvB = \frac{mv^2}{r}$. It leads to $r = \frac{mv}{qB}$…(5).

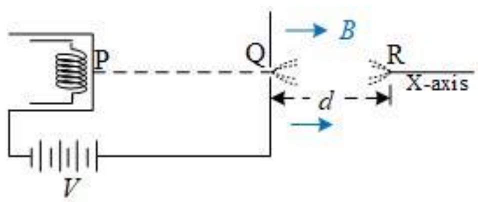
Using the available data radius of the circular trajectory of the particle is $r = \frac{(2.0\times10^{-5})\times4.8}{(2.0\times10^{-3})\times1.2} \Rightarrow r = 4$ cm…(6)



Further, given is a lens of focal length $f = 0.12$m and circular trajectory of the particle at a distance $u = -0.18$ m. Therefore, image would also be circular and its location v from the lens is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$. It leads to $\frac{1}{v} = \frac{1}{0.12} - \frac{1}{0.18} \Rightarrow v = \frac{0.18\times0.12}{0.18-0.12} \Rightarrow v = 0.36$m. Therefore, amplification factor $m = -\frac{v}{u}$. It leads to $m = -\frac{0.36}{(-0.18)} \Rightarrow m = 2$. Accordingl, radius of the circular image would be $r' = r\times m \Rightarrow r' = 4\times2 =$ **8 cm is the answer.**

I-74

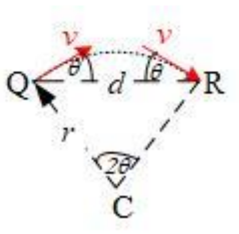
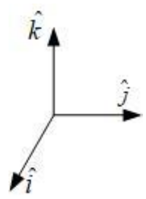
Let, mass of electron is m, carries a charge $q = -e$ moves is emitted with a velocity $u \ll$ and is accelerated by a electric potential difference V along a seperation $\vec{d} = d\hat{\imath}$ as shown in the figure.



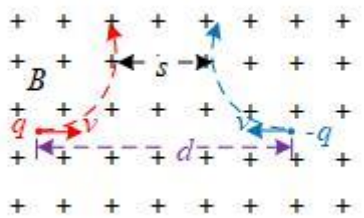
Kinetic energy gained by the electron during travel from P to Q would be $K = eV = \frac{1}{2}mv^2$. Hence, $v = \sqrt{\frac{2eV}{m}}$…(1).

Further, there is magnetic field alongmagnetic field is $\vec{B} = B\hat{\imath}$ …(2).

These aligned electrons ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F_m} = q\vec{v}\times\vec{B} \Rightarrow \vec{F_m} = cvB\sin\theta\,\hat{n} \Rightarrow \vec{F_m} = e(v\hat{v})\times B\hat{\imath} \Rightarrow \vec{F_m} = F_m\hat{n} \Rightarrow F_m = evB$…(3).

	Non-divergent electrons would not experience magnetic force since both $\hat{v}=\hat{\imath}$ and hence from (3) we have $\vec{F}_m=e(v\hat{\imath})\times B\hat{\imath}\Rightarrow \vec{F}_m=evB(\hat{\imath}\times\hat{\imath})=0$, since $\hat{\imath}\times\hat{\imath}=0$. Accordingly, time taken by the electron along QR would be $t=\frac{d}{v}\Rightarrow t=\sqrt{\frac{md^2}{2eV}}$…(4).  But, electrons diverted by any angle say θ would experience, as per (3), magnetic force $\vec{F}_m=evB\sin\theta\,\hat{n}$ ... (4). along $(\hat{n})$ is perpendicular to both the vectors $\hat{v}$ and $\hat{\imath}$ and hence eventually $\hat{v}\perp\hat{n}$ is a valid case of circular motion. Hence, divergent electron would experience a uniform circular motion of radius r such that $F_c=\frac{mv^2}{r}$…(5). In case of uniform circular motion of radius r, there would be equilibrium of forces. Hence, $F_m=F_c\Rightarrow evB\sin\theta=\frac{mv^2}{r}\Rightarrow r=\frac{mv}{eB\sin\theta}$…(6). Electrons emerging at Q have same kinetic energy K and hence same speed v. Yet, possibility of slight diversion by an angle θ in the merging electrons is not ruled out. Electro-mechanics of such electrons reaching R reveal a geometrical symmetry as shown in the figure. Yet, dependence of $F_m\propto\sin\theta$ in (3) and radius of curvature of the arc described by the electrons $r\propto\frac{1}{\sin\theta}$ in (6) makes $d=QR$ independent of θ. It is discussed in footnote to the illustration. Accordingly length of the chord would be $QR=2r\sin\theta\Rightarrow QR=2\left(\frac{m\left(\sqrt{\frac{2eV}{m}}\right)}{eB\sin\theta}\right)\sin\theta$. It leads to $QR=\sqrt{\frac{8mV}{eB^2}}$**, is the answer.** **N.B.:** Electro-mechanical analysis involved in the problem is of intresting relevance, and is being discussed. **C**hange in momentum during time Δt taken by deviated electron to describeng motion along the arc QR is $\Delta p=mv\left((\sin\theta\ \hat{\jmath}+\cos\theta\ \hat{\imath})-(-\sin\theta\ \hat{\jmath}+\cos\theta\ \hat{\imath})\right)\Rightarrow\Delta p=2mv\sin\theta\ \hat{\jmath}$. Let then as per mechanics $\frac{\Delta p}{\Delta t}=F_m\Rightarrow\frac{2mv}{\Delta t}\sin\theta\ =evB\sin\theta\ \Rightarrow\Delta t=\frac{2m}{eB}$ …(7). It is seen that time taken to reach R by all electrons is independent of angle of diversion Electron, is describing circular motion with speed v with a radius r and time period of the circular motion is $T=\frac{Perimeter\ of\ the\ circular\ trajectory}{Speed\ of\ the\ electron}\Rightarrow T=\frac{2\pi r}{v}$. Using (6) it leads to $T=\frac{2\pi\left(\frac{mv}{eB\sin\theta}\right)}{v}\Rightarrow T=\frac{2\pi m}{eB\sin\theta}$. It leads to length of the arc QR is $\frac{\Delta t}{T}=\frac{PQ_{arc}}{2\pi r}\Rightarrow QR_{arc}=\left(\frac{\Delta t}{T}\right)2\pi r\Rightarrow QR_{arc}=2\pi\left(\frac{\frac{2m}{eB}}{\frac{2\pi m}{eB\sin\theta}}\right)\left(\frac{mv}{eB\sin\theta}\right)$. It solves into $QR_{arc}=\left(\frac{2mv}{eB}\right)$. Accordingly length of the chord would be $QR=2r\sin\theta\Rightarrow QR=2\left(\frac{m\left(\sqrt{\frac{2eV}{m}}\right)}{eB\sin\theta}\right)\sin\theta$. It leads to $QR=\sqrt{\frac{8mV}{eB^2}}$.
I-75	This problem involves 3D vectors and hence unit vectors are shown in the figure. Given are twoa particles of mass m, carries a charges $q_1=q$ and $q_2=(-q)$ moves with initial velocities $\vec{v}_1=v\hat{\jmath}$ and $\vec{v}_2=v(-\hat{\jmath})$ …(1), inside magnetic field is $\vec{B}=B(-\hat{k})$ T…(2).  It is seen that velocity vector $\vec{v}\perp\vec{B}$. This q_1 in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F}_{m1}=q\vec{v}\times\vec{B}\Rightarrow\vec{F}_{m1}=qvB(\hat{\jmath})\times(-\hat{k})\Rightarrow\vec{F}_m=qvB(-\hat{\imath})$ …(3). This is a case of circular motion of ion where $\vec{v}\perp\vec{F}_m$, accordingly ceptripetal acceleration would acts along $(-\hat{r})=(-\hat{\imath})$. This concludes to $\vec{F}_m=(qvB)\hat{r}$…(3). Here, $\hat{r}$ is the radius vector of the circular trajectory of the charged particle. Likewise, q_1 in the magnetic field would experience a force $\vec{F}_{m2}=q_2\vec{v}_2\times\vec{B}\Rightarrow\vec{F}_{m2}=(-q)vB(-\hat{\jmath})\times(-\hat{k})\Rightarrow\vec{F}_{m_2}=qvB(-\hat{\imath})$ …(4).

The problms states that Coulomb's force F_e between the two charges is switched off. It implies that we have to consider $F_e = 0$, between the two opposite charges.



It is seen that magnitude of both the centripetal forces $F_{m1} = F_{m1} = qvB$…(5). In state of circular motion the particle would experience centripetal force $F_c = \frac{mv^2}{r}$...(6). In case of uniform circular motion there is equilibrium of forces acting on the particle. Accordingly, $F_m = F_c \Rightarrow qvB = \frac{mv^2}{r}$. It leads to $r = \frac{mv}{qB}$…(7).

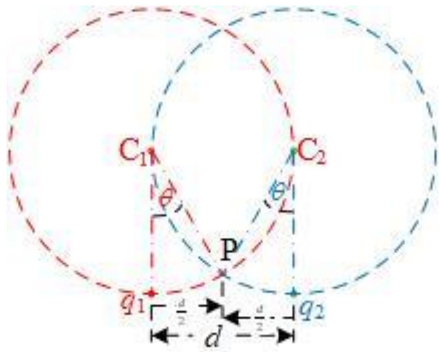
With this pre-analysis each part of the problem is being solved separately-

Part (a): It is seen from (7), $v = \frac{rqB}{m} \Rightarrow v \propto r$….(8) Therefore, for maximum velocity of the charges such that they do not collide, $r_{max} = \frac{d}{2}$…(9). Combining (8) and (9), $v_{max} = \frac{r_{max}qB}{m} \Rightarrow v_{max} = \frac{\left(\frac{d}{2}\right)qB}{m} \Rightarrow v_{max} = \boldsymbol{\frac{qBd}{2m}}$…(10)**, is the answer of part (a).**

Part (b): It is seen that initially seperation s between the charges decrease as long as deviation of the charges from intital velocity is $0 < \theta < \frac{\pi}{2}$ as shown in the figure. Further, $s_{min} = d - 2r$…(11) at $\theta < \frac{\pi}{2}$. Thereafter, for $\frac{\pi}{2} < \theta < 3\frac{\pi}{2}$ separation stars increasing. At given velocity $v = \frac{v_{max}}{2}$, combining (7) and (10), $r = \frac{m\left(\frac{v_{max}}{2}\right)}{qB} \Rightarrow r = \frac{m\left(\frac{qBd}{2m}\right)}{2qB} \Rightarrow r = \frac{d}{4}$. Hence, using (11), minimum seperation between charges would be $s_{min} = d - 2\left(\frac{d}{4}\right) \Rightarrow \boldsymbol{s_{min}} = \boldsymbol{\frac{d}{2}}$…(12).

Likewise, maximum seperation would be $s_{max} = d + 2r \Rightarrow s_{max} = d + 2\left(\frac{d}{4}\right) \Rightarrow s_{max} = d + \frac{d}{2}$. It leads to $\boldsymbol{s_{max}} = \boldsymbol{\frac{3d}{2}}$…(13).

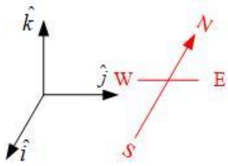
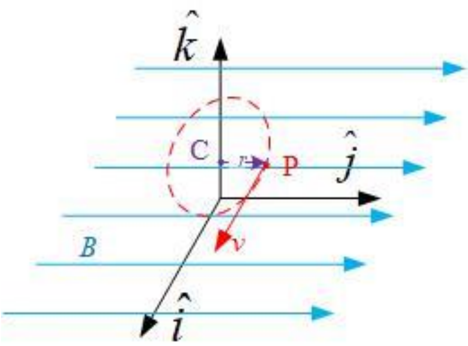
Thus, **answer to part (b) is $\boldsymbol{\frac{d}{2}}$ and $\boldsymbol{\frac{3d}{2}}$**.



Part (c): When $v = 2v_m$, as per (7) and (10), radius of the trajectory of the charged particles would be $r = \frac{m(2v_m)}{qB} \Rightarrow r = \frac{m\left(2\left(\frac{qBd}{2m}\right)\right)}{qB} \Rightarrow r = d$…(14). Geometrically, this situation can be expressed as shown in the figure and the two charges collide after describing an angular displacement θ. Here, $\sin\theta = \frac{\frac{d}{2}}{r} \Rightarrow \sin\theta = \frac{d}{2r} \Rightarrow \sin\theta = \frac{d}{2d} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$…(15).

For uniform circular motion time to complete angular displacement 2π is $T = \frac{2\pi r}{v}$. Combining (8) and (14) we have $T = \frac{2\pi d}{\frac{dqB}{m}} \Rightarrow T = \frac{2\pi m}{qB}$. Therefore, time t taken for angular displacement $\frac{\pi}{6}$ would be $\frac{t}{T} = \frac{\frac{\pi}{6}}{2\pi} \Rightarrow t = \frac{T}{12} \Rightarrow t = \frac{\frac{2\pi m}{qB}}{12} \Rightarrow t = \boldsymbol{\frac{\pi m}{6qB}}$ **is answer of part (c).**

Part (d): In case (c), collision between the two oppoite charges is considered to be inellastic. In such a situationcharges on collision would become single mass, unlike the case of elastic collision. Therefore, result of collision is a single mass $m' = 2m$ and charge $q' = q + (-q) \Rightarrow q' = 0$. Therefore, as per (3)$\overrightarrow{F'_m} = (q'vB)\hat{r} \Rightarrow \overrightarrow{F'_m} = (0vB)\hat{r} \Rightarrow \overrightarrow{F'_m} = 0$. Hence, **after collision onward motion is of a neutral particle of mass $\boldsymbol{2m}$ would move upward perpendicular to $\boldsymbol{d}$.**

	Thus, **answer are (a) $\frac{qBd}{2m}$ (b) $\frac{d}{2}$, $\frac{3d}{2}$ (c) $\frac{\pi m}{6qB}$ (d) the particles stick together and the combined mass moves with constant speed v_m along the straight line drawn upward in the plane of figure through the point of collision**
I-76	For convenience 3D unit vectors alongwith geographical direction in $\hat{\imath}-\hat{\jmath}$ plane are shown in figure. Accordingly, given data vectorially is $\vec{B}=B(-\hat{\jmath})=0.20(-\hat{\jmath})$ T is some region in space in which a particle of mass $m=0.020\times10^{-3}$kg carrying a charge $q=1.0\times10^{-5}$ C is projected with a velocity $\vec{v}=v(-\hat{\imath})$.  A charged particle in magnetic field is the case of magnetic force, which as per Lorentz's Force Law, $\vec{F}_{m1}=q\vec{v}\times\vec{B}\Rightarrow\vec{F}_{m1}=qv(-\hat{\imath})\times B(-\hat{\jmath})\Rightarrow\vec{F}_m=qvB(\hat{k})$ …(3). It is seen that this magnetic force is a direction verticallu upwards. The particle would continue to perform uniform circular motion, it must be counter balanced by some force $\vec{F}$ such that $\vec{F}_m+\vec{F}=0$. It leads to a condition where $\vec{F}=qvB(-\hat{k})$…(4) Since particle has a mass a nd hence it would experience a gravitational force $\vec{F}=mg(-\hat{k})$…(5). Here, $g=10$ m/s^2 is acceleration due to gravity which is down ward. Combining (4) and (5), and using the available data $qvB=mg\Rightarrow v=\frac{mg}{qB}\Rightarrow v=\frac{(0.020\times10^{-3})\times10}{(1.0\times10^{-5})\times(0.20)}$. It solves to $\boldsymbol{v}=$ **100m/s is the answer.**
I-77	For convenience 3D unit vectors are shown in figure. Given is a particle describing circular motion of radius $r=1.0\times10^{-2}$m. Axis of the circular motion, since not specified, is arbitratily taken to be along $\hat{k}$, perpendicular to the magnetic field for simplicification, in the region where exists a magnetic field B, which is considered such that $\vec{B}=B\hat{\jmath}=0.40\hat{\jmath}$ T…(1).  If particle, in motion in clockwise direction with a velocity $\vec{v}$, in uniform magnetic field $\vec{B}$ then it will experience magnetic force as per Lorentz's Force Law, $\vec{F}_m=q\vec{v}\times\vec{B}\Rightarrow\vec{F}_m=qvB\hat{n}$…(2). Here, $\hat{n}$ is vector perpendicular to plane of vectors $\vec{v}$ and $\vec{B}$. In the intant case a particle is taken, for convenience, to be at P where radial position vector $\vec{r}$ is along $\hat{\jmath}$. Accordingly, velocity of the particle is $\vec{v}=v\hat{\imath}$ as shown in the figure. Therefore, magnetic force as per (2) would be $\vec{F}_m=q(v\hat{\imath})\times(B\hat{\jmath})\Rightarrow\vec{F}_m=qvB(\hat{\imath}\times\hat{\jmath})\Rightarrow\vec{F}_m=qvB\hat{k}$…(3). The particle during circular motion, as stipulated, would experience a centrifugal force $\vec{F}_c=\frac{mv^2}{r}\hat{r}$…(4). Uniform cicular motion prior to application of electric would have equilibrium of forces on the particle., as per Newton's First Law of Motion. And as per Newton's Third Law of Motion while $\vec{F}_m$ is the cause, reation is $\vec{F}_c$. Thus, $\vec{F}_m+\vec{F}_c=0\Rightarrow qvB\hat{k}+\frac{mv^2}{r}\hat{r}\Rightarrow qvB\hat{k}=-\frac{mv^2}{r}\hat{r}\Rightarrow\frac{mv^2}{r}=qvB\Rightarrow v=\left(\frac{q}{m}\right)rB$…(5). It is desired that on the charged particle an electric field is superimpsed $\vec{E}=E\hat{e}=200\hat{e}$V/m…(6), so that it moves in a straight path. This Velocity of particle at P is $\vec{v}=v\hat{\imath}$.,,(7). It is possible only if $\vec{F}_e=q\vec{E}\Rightarrow\vec{F}_e=qE\hat{e}$…(8), counter balances the cause. Accordingly, $\vec{F}_m+\vec{F}_e=0\Rightarrow qvB\hat{k}+qE\hat{e}=0\Rightarrow qvB\hat{k}=-qE\hat{e}\Rightarrow qvB=qE$ It leads to $v=\frac{E}{B}$…(9). Combining (5) and (9), $\left(\frac{q}{m}\right)rB=\frac{E}{B}\Rightarrow\frac{q}{m}=\frac{E}{rB^2}$…(10). Using the available data, $\frac{q}{m}=\frac{E}{rB^2}$. Using the available data $\boldsymbol{\frac{q}{m}}=\frac{200}{(1.0\times10^{-2})(0.4)^2}=\boldsymbol{1.25\times10^5}$ **C/kg is the answer.**

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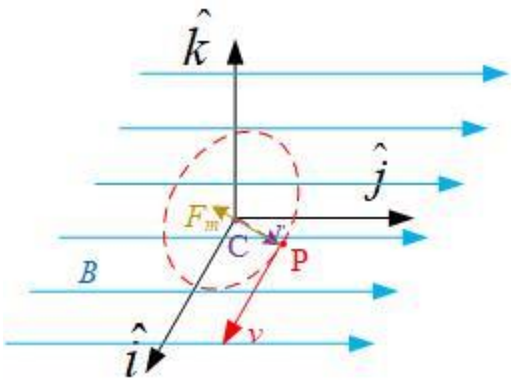

For convenience 3D unit vectors are shown in figure. It is required to analyze undeflected motion $\vec{v} = v\hat{v} \Rightarrow \vec{v} = (2.0 \times 10^5)\hat{v}$ m/s of protron having charge $q = e = 1.6 \times 10^{-19}$ C and mass $m = 1.6 \times 10^{-27}$kg in fields say $\vec{E} = E\hat{e}$ and $\vec{B} = B\hat{\jmath}$ which are mutually perpendicular i.e. $\hat{e} \perp \hat{\jmath}$.

However, problem states that when electric field $\vec{E} = E\hat{e}$ is switched off, only magneic field $\vec{B}$ remains in place. And the charge has a circular trajectory of radius $\vec{r} = r\hat{r} \Rightarrow \vec{r} = 4.0 \times 10^{-2}\hat{r}$. Accordingly, problems is being first solved for motion of proton under magnetic field. This motion is considered in $(\hat{\imath} - \hat{k})$ plane. After this effect of electric field is considered to satisfy required condition of undeflected motion of the charge.

For convenience of analysis point P on the circular trajectory, as shown in the figure, is considered such that $\hat{v} = \hat{\imath}$ …(1).

As per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = qvB\hat{n}$…(2). Here, $\hat{n} \to \hat{k}$ is vector perpendicular to plane of vectors $\vec{v}$ and $\vec{B}$. Moreover, (2) essentially requires charge to be in motion and therefore $v \neq 0$.

For uniform circular motion of the proton while $\vec{F}_m$ is the cause, centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$…(3) is reaction which creates an equilibrium of forces such that $\vec{F}_m + \vec{F}_c = 0$…(4), as per Newton's Third Law of Motion. Thus, in accordance with Newton's First Law of Motion its speed of rotation v and radius r of the circular trajectory remains constant.

Accordingly, combining (2), (3) and (4), we have $qvB\hat{k} + \frac{mv^2}{r}\hat{r} \Rightarrow qvB\hat{k} = -\frac{mv^2}{r}\hat{r} \Rightarrow \frac{mv^2}{r} = qvB$. It leads to $B = \frac{vm}{qr}$…(5). Using the available data, $B = \frac{(2.0\times10^5)(1.6\times10^{-27})}{(1.6\times10^{-19})(4.0\times10^{-2})} \Rightarrow \boldsymbol{B = 0.05}$ **T, is one part of the answer**.

For proton to describe undeflected motion without $\vec{F}_c$, requires an external force that creates an equilibrium with with $\vec{F}_m$. In this case it is electrostatic force which is $\vec{F}_e = q\vec{E} \Rightarrow \vec{F}_e = qE\hat{e}$…(6). Accordingly combining (2) and (6), $qvB\hat{n} + qE\hat{e} = 0 \Rightarrow qvB\hat{n} = -qE\hat{e} \Rightarrow v = \frac{E}{B}$…(7).

Combining (5) and (7), we have $\left(\frac{q}{m}\right)rB = \frac{E}{B} \Rightarrow \left(\frac{q}{m}\right) = \frac{E}{rB^2}$…(8).

Using the available data in (6), $\left(\frac{1.6\times10^{-19}}{1.6\times10^{-27}}\right) = \frac{E}{(4.0\times10^{-2})B^2} \Rightarrow E = (4.0 \times 10^6)(0.05)^2 \Rightarrow \boldsymbol{E = 1.0 \times 10^4}$ **N/C is second part of the answer.**

Thus, **answers are $\boldsymbol{1.0 \times 10^4}$ N/C and $\boldsymbol{0.05}$ T.**

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Given a particle having a charge $q = 5.0 \times 10^{-6}$ C and mass $m = 5.0 \times 10^{-12}$ kg is projected with a velocity $\vec{v} = v\hat{v} \Rightarrow \vec{v} = 1.0 \times 10^3\hat{v}$ m/s in a magneic field $\vec{B} = B\hat{\jmath} \Rightarrow \vec{B} = 5.0 \times 10^{-3}$T such that magnetic field subtends with velocity vector an angle $\theta = \sin^{-1}(0.90) \Rightarrow \sin\theta = 0.90$.

This situation as shown in the figure can be analyzed by resolving velocity in two perpndicular components $\vec{v} = v_j\hat{\jmath} + v_k\hat{k} \Rightarrow \vec{v} = v\cos\theta\,\hat{\jmath} + v\sin\theta\,\hat{k}$…(1) is shown in the figure along with 3D unit direction vectors.

As per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B}$…(2). Here, $\hat{n} \to \hat{k}$ is vector perpendicular to plane of vectors $\vec{v}$ and $\vec{B}$. The equation (2) essentially requires charge to be in motion and therefore $v \neq 0$. Therefore, combining (1) and (2) we have $\vec{F}_m = qvB(\cos\theta\,\hat{\jmath} + \sin\theta\,\hat{k}) \times \hat{\jmath} \Rightarrow \vec{F}_m = qvB(\cos\theta\,\hat{\jmath} \times \hat{\jmath} + \sin\theta\,\hat{k} \times \hat{\jmath})$. It leads to $\vec{F}_m = qvB(0 + \sin\theta\,(-\hat{\imath}))$…(3).

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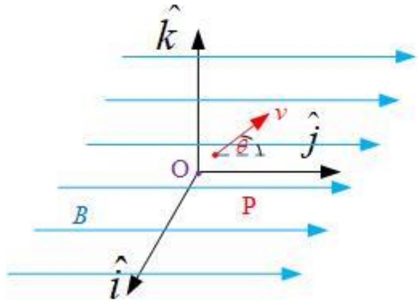
Analysis of (3) reveals that it is only velocity component $v_k\hat{k} = v\sin\theta\,\hat{k}$ which affects creates a $\vec{F}_m \perp v_k\hat{k}$, force motion of the charge and thus qualifies for a uniform circular motion. While, $v_j\hat{j}$ being along $\vec{B}$ does not affect motion of the charge. Thus, it leads to resultant motion of charge which is superimposition of translation motion on circular motion, eventually it is of helix form which a radius and a pitch.



Therefore, both circular motion is being analyzed to determine radius of the helx. This radius will be used to determine time period of circular motion, which in turn will help to determine pitch of helix using translational motion of the particle.

Circular Motion: Magnetic force $\vec{F}_m$is the cause of uniform circular motion, while centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}\ldots(4)$ is the reaction which creates an equilibrium of forces such that $\vec{F}_m + \vec{F}_c = 0\ldots(5)$, as per Newton's Third Law of Motion. Thus, in accordance with Newton's First Law of Motion its speed of rotation v and radius r of the circular trajectory remains constant.

Comining (3), (4) and (5), $qvB\sin\theta\,(-\hat{\imath}) + \frac{m(v\sin\theta)^2}{r}\hat{r} = 0 \Rightarrow r = \frac{mv\sin\theta}{qB}\ldots(6)$. Using the available data it leads to $r = \frac{(5.0\times10^{-12})(1.0\times10^{3})(0.90)}{(5.0\times10^{-6})(5.0\times10^{-3})} \Rightarrow r = 0.18$ m hence **diamteter is 36 cm is the answer of the first part.**

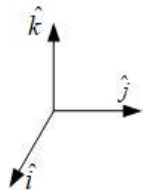
The charge is since describing circular motion of radius r with a uniform speed v, its time period is $T = \frac{2\pi r}{v\sin\theta}\ldots(7)$. Hence, using available data $T = \frac{2\pi\times0.18}{(1.0\times10^{3})(0.9)} \Rightarrow T = 1.2\times10^{-3}\text{s}\ldots(8)$.

Translational Motion: While describing circular motion the charged particle continues to traverse with translational velocity $v_j\hat{j} = v\cos\theta\,\hat{j}$ and traverses along $\hat{j}$ a distance $\lambda = v_j \times T \Rightarrow \lambda = v\cos\theta\,T$. Using the available data $\lambda = (1.0\times10^{3})\left(\sqrt{1-\sin^2\theta}\right)(1.2\times10^{-3}) \Rightarrow \lambda = 1.4\times\left(\sqrt{1-(0.9)^2}\right) \Rightarrow$ $\boldsymbol{\lambda = 0.61}$ **m is the pitch of the helix is answer of the second part.**

Thus, **answer is 36 cm and 53 cm.**

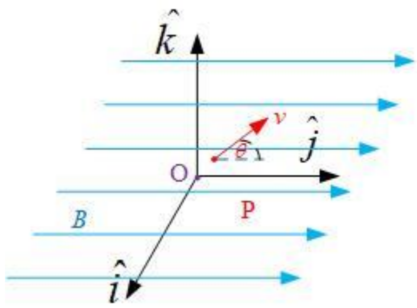
I-80

For convenience of analysis 3D unit vectorss are shown in the figure. Given is a proton having mass $m = 1.6\times10^{-27}$kg and charge $q = 1.6\times10^{-19}$ C. It is projected in magnetic field $\vec{B} = B\hat{b} \Rightarrow \vec{B} = 0.020\hat{j}$. Trajectory of the proton is helical with radius $r = 5.0\times10^{-2}$m and pitch $\lambda = 2.0\times10^{-1}$m.



Let, the proton is projcted with a velocity v making an angle θ with the magnetic field such that $\vec{v} = v_j\hat{j} + v_k\hat{k} \Rightarrow \vec{v} = v\cos\theta\,\hat{j} + v\sin\theta\,\hat{k}\ldots(1)$.

A moving charge would experience magnetic force as per Lorentz's Force Law, $\vec{F}_m = q\vec{v}\times\vec{B}\ldots(2)$. Here, $\hat{n} \to \hat{k}$ is vector perpendicular to plane of vectors $\vec{v}$ and $\vec{B}$. The equation (2) essentially requires charge to be in motion and therefore $v \neq 0$. Therefore, combining (1) and (2) we have $\vec{F}_m = qvB\left(\cos\theta\,\hat{j} + \sin\theta\,\hat{k}\right)\times\hat{j} \Rightarrow \vec{F}_m = qvB\left(\cos\theta\,\hat{j}\times\hat{j} + \sin\theta\,\hat{k}\times\hat{j}\right)$. It leads to $\vec{F}_m = qvB\left(0 + \sin\theta\,(-\hat{\imath})\right)\ldots(3)$.



Analysis of (3) reveals that it is only velocity component $v_k\hat{k} = v\sin\theta\,\hat{k}$ which affects creates a $\vec{F}_m \perp v_k\hat{k}$, force motion of the charge and thus qualifies for a uniform circular motion. While, $v_j\hat{j}$ being along $\vec{B}$ does not affect motion of the charge. Thus, it leads to resultant motion of charge which is superimposition of translation motion on circular motion, eventually it is of helix form which a radius and a pitch.

Therefore, both circular motion is being analyzed to determine radius of the helx. This radius will be used to determine time period of circular motion, which in turn will help to determine pitch of helix using translational motion of the particle.

Circular Motion: Magnetic force $\vec{F}_m$is the cause of uniform circular motion, while centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$…(4) is the reaction which creates an equilibrium of forces such that $\vec{F}_m + \vec{F}_c = 0$…(5), as per Newton's Third Law of Motion. Thus, in accordance with Newton's First Law of Motion its speed of rotation v and radius r of the circular trajectory remains constant.

Comining (3), (4) and (5), $qvB\sin\theta\,(-\hat{\imath}) + \frac{m(v\sin\theta)^2}{r}\hat{r} = 0 \Rightarrow v\sin\theta = \frac{(5.0\times10^{-2})(1.6\times10^{-19})(0.020)}{1.6\times10^{-27}}$ …(6). It solves into $v\sin\theta = \mathbf{1.0\times10^3}$**m/s is the velocity o the proton perpendicular the the magntic field, is one part of the solution.**

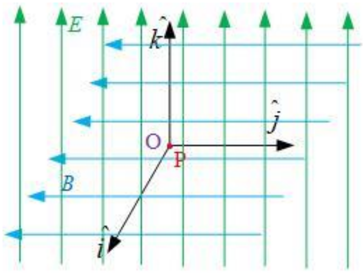
The charge is since describing circular motion of radius r with a uniform speed v, its time period is $T = \frac{2\pi r}{v\sin\theta}$ …(7). Hence, using available data $T = \frac{2\pi\times(5.0\times10^{-2})}{(1.0\times10^3)} \Rightarrow T = 3.14\times10^{-4}$s…(8).

Translational Motion: While describing circular motion the charged particle continues to traverse with translational velocity $v_j\hat{\jmath} = v\cos\theta\,\hat{\jmath}$ and traverses along $\hat{\jmath}$ a distance $\lambda = v_j \times T \Rightarrow \lambda = v\cos\theta\,T$. Therefore, velocity of charge along the magnetic field is $v\cos\theta = \frac{\lambda}{T}$. Using the available data we have $v\cos\theta = \frac{2.0\times10^{-1}}{3.14\times10^{-4}} \Rightarrow v\cos\theta = 6.4\times10^2$m/s.

Thus, **answer is $\mathbf{1.0\times10^3}$ m/s and $\mathbf{6.4\times10^2}$m/s.**

I-81

A particle of mass m and charge q is released from origin, it implies that initial velocity is zero. Accordingly, $\vec{u} = u\hat{u} = 0$, in a region having electric and magnetic fields $\vec{B} = (-)B_0\hat{\jmath}$ and $\vec{E} = E_0\hat{k}$. The system with 3D-unit vectors is shown in the figure.

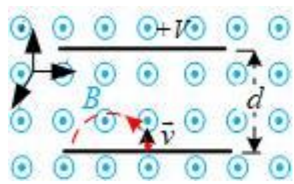


Forces on a charged particle would influence its motion. Thus, as per Lorentz's Force Law $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \Rightarrow \vec{F} = q(E_0\hat{k} + u\hat{u}\times(-)B_0\hat{\jmath})$. It leads to $\vec{F} = q(E_0\hat{k} - (0\times B_0)\hat{u}\times\hat{\jmath}) \Rightarrow \vec{F} = (qE_0)\hat{k} \Rightarrow \vec{a} = \left(\frac{qE_0}{m}\right)\hat{k}$….(1).

It is seen from (1) acceleration of the particle is along $\hat{k}$ i.e. z-coordinate and, therefore, velocity of the particle as a function of z-coordinate, as per kinematics, is $v^2 = u^2 + 2az$…(2). Combining (1) and (2), with the available data, $v^2 = 0 + 2\left(\frac{qE_0}{m}\right)z \Rightarrow \boldsymbol{v} = \sqrt{\frac{\mathbf{2qE_0z}}{\boldsymbol{m}}}$ **is the answer**.

I-82

Given system is shown in the figure with 3D-unit vectors for reference.In the region there is magmetic field $\vec{B} = B_0\hat{\imath}$ and electric field $\vec{E} = E_0(-\hat{k})$ where $E_0 = \frac{V}{d}$…(1). An electron having charge $q = -e$ is emitted with a negligible velocity $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$, here u_x, u_y *and* u_z are negilgible.



In the system the electron would experience force as per Lorentz's Force Law combines electrostatic and magnetic forces on a charge $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \Rightarrow \vec{F} = (-e)(E_0(-\hat{k}) + vB_0\,\hat{n})$. It solves into $\vec{F} = \vec{F}_E + \vec{F}_M = (eE_0)\hat{k} - (evB_0)\hat{n}$ …(2). Here, $\hat{n}$ direction of magnetic force $\vec{F}_M$ is perpendicular to vector $\vec{B}$ and velocity vector $\vec{v}$. In the system since $\vec{v}\perp\vec{B}$, the factor $\sin\theta = 1$, where θ is the angle of $\vec{B}$ w.r.t. $\vec{v}$. It forms a case of circular motion as shown in the figure.

Ananlysis, of (2) reveals that instantaneous velocity of the electron depends upon $\vec{F}_E$ and consequent acceleration $\vec{a}_E = \frac{\vec{F}_E}{m_e} \Rightarrow \vec{a}_E = \left(\frac{eE_0}{m_e}\right)\hat{k}$…(3). Accordingly, as per kinematics velocity of electron, as it moves along $\hat{k}$ towards plate at $+V$ in Z-dircetion, is $v^2 = {u_z}^2 + 2a_Ez \Rightarrow v^2 = 0 + 2a_Ez \Rightarrow v = \sqrt{2\left(\frac{eE_0}{m_e}\right)z}$…(4).

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As regards radius of circular motion due to $\vec{F}_M$ the condition of equlibrium is $\vec{F}_M + \vec{F}_C = 0 \Rightarrow \vec{F}_M = -\vec{F}_C$, where $\vec{F}_C = \frac{m_e v^2}{r}$ is the centripetal force. Accordingly, $evB_0 = \frac{m_e v^2}{r} \Rightarrow r = \frac{m_e v}{eB_0}$...(5). Combining (4) and (5), $r = \left(\frac{m_e}{eB_0}\right)\sqrt{2\left(\frac{eE_0}{m_e}\right)z} \Rightarrow r = \sqrt{2\left(\frac{m_e E_0}{eB_0^2}\right)z}$...(6). Combining (1) and (6) $r = \sqrt{2\left(\frac{m_e \frac{V}{d}}{eB_0^2}\right)z}$. Here, maximimum value of $z \to d$ and, therefore, $r = \sqrt{2\left(\frac{m_e \frac{V}{d}}{eB_0^2}\right)d} \Rightarrow r = \sqrt{2\left(\frac{m_e V}{eB_0^2}\right)}$...(7).

Thusm essential condition for the electron fails to reach uper plate is $d > r \Rightarrow \boldsymbol{d} > \sqrt{\boldsymbol{2\left(\frac{m_e V}{eB_0^2}\right)}}$, **proved.**

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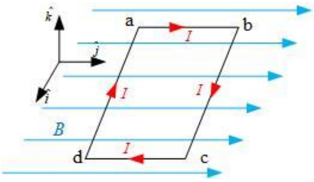
Given system is shown in the figure with 3D-unit vectors The coil has 100 torns of in rectangular shape having length $l = 5 \times 10^{-2}$m and width $w = 4 \times 10^{-2}$. The coil carries a current $I = 2$ A. It is required to finf magnetic field B whereby the the coil exeriences a net torque $\Gamma_{net} = 0.2$ Nm.



A current carrying conductor placed in magnetic field, as [er Lorentz's Force Law experiences magnetic force $\vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\theta\, \hat{n}$...(1). Here, θ is angle of $\vec{B}$ w.r.t. $\vec{l}$ and $\hat{n}$ is unit direxction vector perpendicular to the plane containing vectors $\vec{l}$ and $\vec{B}$. It is to be noted that length vector is taken along the direction of the current in it. Accordingly for two opposite sides, $\vec{l}_{ab} = l_{ab}\hat{\jmath}$ while $\vec{l}_{cd} = l_{cd}(-\hat{\jmath})$. Same principle is used for other two sides of the rectangular coil.

Given is since a coil of turns $n = 100$ in shape abcd and hence to take n times the forces on each side of rectangle abcd, using (1). Accordingly forces –

Side ab: $\vec{F}_{ab} = I(l_{ab}\hat{\jmath}) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{ab} = Il_{ab}B(\hat{\jmath} \times \hat{\jmath}) = 0$, since $\hat{\jmath} \times \hat{\jmath} = 0$.

Side bc: $\vec{F}_{bc} = I(l_{bc}\hat{\imath}) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{k})$..

Side cd: $\vec{F}_{cd} = I\left(l_{cd}(-\hat{\jmath})\right) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{cd} = -Il_{cd}B(\hat{\jmath} \times \hat{\jmath}) = 0$, since $\hat{\jmath} \times \hat{\jmath} = 0$.

Side da: $\vec{F}_{da} = I\left(l_{da}(-\hat{\imath})\right) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{da} = -Il_{da}B(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{F}_{da} = Il_{bc}B(-\hat{k})$.

It is to be noted that while $\vec{F}_{ab}$ and $\vec{F}_{cd}$ the other two forces $\vec{F}_{bc}$ and $\vec{F}_{da}$ are-

(a) equal in magnitude $F_{bc} = F_{da} = Il_{bc}B$,
(b) opposite directions
(c) seperated by a distance equal to width of the coil w.
(d) The above three are a valid case of a torque on the turn of the coil.

Accordingly, torque on the couple $\vec{\Gamma} = \vec{w} \times \vec{F} \Rightarrow \vec{\Gamma} = \vec{l}_{ab} \times \vec{F}_{bc} \Rightarrow \vec{\Gamma} = (l_{ab}\hat{\jmath}) \times \left(Il_{bc}B\hat{k}\right)$. It further solves to $\vec{\Gamma} = Il_{ab}l_{bc}B\left(\hat{\jmath} \times \hat{k}\right) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$...(2). Here, $A = Il_{ab}l_{bc}$...(3), is the area of turns of the coil. Given that the coil has n turns of same area and hence net torque would be $\vec{\Gamma}_{net} = n\vec{\Gamma} \Rightarrow \vec{\Gamma}_{net} = nIAB\hat{\imath} \Rightarrow B = \frac{\Gamma_{net}}{nIA}$...(4). Using the available data, $B = \frac{0.2}{100 \times 2 \times \left((5 \times 10^{-2})(4 \times 10^{-2})\right)} \Rightarrow \boldsymbol{B = 0.5}$**T is the answer.**

N.B.: Equation (2) can also be written as $\vec{\Gamma} = IAB\hat{\imath} = I\vec{A} \times \vec{B}$ ...(5). In this case $\vec{A} = A\hat{a}$ where unit vector $\hat{a}$ is along perpendicular to the area A such that for observer if current in the loop is anti-clockwise then $\hat{a}$ is (+)ve i.e. towards the observer. Whereas, if current in the loop is clockwise then $\hat{a}$ is (-)ve i.e. away from the observer.

In the instant case, as shown in the figure, when we observe the coil from the top current is clockwise and hence $\hat{a} = \left(-\hat{k}\right)$. Accordingly, $\vec{\Gamma} = IA\left(-\hat{k}\right) \times (B\hat{\jmath}) \Rightarrow \vec{\Gamma} = IAB\left(-\hat{k}\right) \times (\hat{\jmath}) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$...(6). It is seen that conclusion at (6) conforms to (2), used to solvethe problem.

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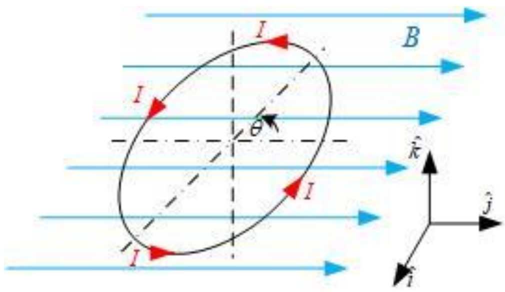
Thus, ***equation (6) alongwith direction vector of area can be use as takeaway for handling problems involving torque experienced by a current carrying coil placed in uniform magnetic field.***

I-84

Given system is shown in the figure alongwith 3D-Unit vectors. It comprises of a circular coil of radius $r = 2.0 \times 10^{-2}$m having turns $n = 50$, carrying a current $I = 5.0$ A placed in magnetic field $\vec{B} = 0.20\hat{\jmath}$.



The area of the coil is $\vec{A} = (\pi r^2)\hat{a}$ ...(1), where $\hat{a}$ is the unit direction vector of the area. Accordingly, torque experienced by the coil is $\vec{\Gamma} = (nI)\vec{A} \times \vec{B} \Rightarrow \vec{\Gamma} = (nIAB)\hat{a} \times \hat{\jmath}$...(2).

Taking position of the coil such that it forms an angle θ with the magnetic field, angle of $\vec{B}$ relative to the area vector $\hat{a}$ will be $\alpha = \frac{\pi}{2} + \theta$...(3). Accordingly, equation (2) transforms into $\vec{\Gamma} = (nIAB)\sin\alpha\,(\hat{\imath})$...(4).

In (4) the only variable is $\alpha = \frac{\pi}{2} + \theta$ and maximum torque would occur for $\sin\alpha = 1 \Rightarrow \alpha = \frac{\pi}{2} \Rightarrow \theta = 0$...(5). It implies that the plane of the coil is parallel to the magnetic field.

Nm.

Accordingly, maximum torque is $\Gamma_{max} = nIAB$. Using the available data, $\Gamma_{max} = 50 \times 5.0 \times \pi \times (2.0 \times 10^{-2})^2 \times 0.20 = 6.24 \times 10^{-2}$ N-m say $\mathbf{6.2 \times 10^{-2}}$**N-m is the answer of part (a).**

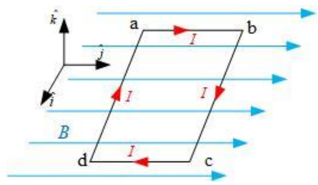
Position of the coil experiencing torque half of the maximum $\Gamma = (nIAB)\sin\alpha \Rightarrow \Gamma = \Gamma_{max}\sin\alpha \Rightarrow \Gamma = \frac{\Gamma_{max}}{2} \Rightarrow \sin\alpha = \frac{1}{2} \Rightarrow \frac{\pi}{2} + \theta = \frac{\pi}{6} \Rightarrow \theta = \left|\frac{\pi}{2} - \frac{\pi}{6}\right| \Rightarrow \theta = \frac{\pi}{3}$ or $\mathbf{60^0}$ **is answer of the part (b).**

Thus, **answers are (a)** $\mathbf{6.2 \times 10^{-2}}$**N-m, (b)** $\mathbf{60^0}$**.**

N.B.: This problem has been deliberately solved using derivation From First-principle in illustrations. It is believed that it will help students to resort to phased practice of solving problems, and develop required proficiency, accuracy and speed to achieve their vision.

I-85

Given system is shown in the figure with 3D-unit vectors. The rectangular loop has length $l = 2.0 \times 10^{-1}$m and width $w = 1.0 \times 10^{-1}$. The coil carries a current $I = 5.0$ A. It is required to finf magnetic field $\vec{B} = 0.20\hat{\jmath}$ T.



A current carrying conductor placed in magnetic field, as [er Lorentz's Force Law experiences magnetic force $\vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB\sin\theta\,\hat{n}$...(1). Here, θ is angle of $\vec{B}$ w.r.t. $\vec{l}$ and $\hat{n}$ is unit direxction vector perpendicular to the plane containing vectors $\vec{l}$ and $\vec{B}$. It is to be noted that length vector is taken along the direction of the current in it. Accordingly for two opposite sides, $\vec{l}_{ab} = l_{ab}\hat{\jmath}$ while $\vec{l}_{cd} = l_{cd}(-\hat{\jmath})$. Same principle is used for other two sides of the rectangular coil.

Thus, forces on each side of rectangle abcd, using (1) are –

Side ab: $\vec{F}_{ab} = I(l_{ab}\hat{\jmath}) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{ab} = Il_{ab}B(\hat{\jmath} \times \hat{\jmath}) = 0$, since $\hat{\jmath} \times \hat{\jmath} = 0$.

Side bc: $\vec{F}_{bc} = I(l_{bc}\hat{\imath}) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{k})$..

Side cd: $\vec{F}_{cd} = I\big(l_{cd}(-\hat{\jmath})\big) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{cd} = -Il_{cd}B(\hat{\jmath} \times \hat{\jmath}) = 0$, since $\hat{\jmath} \times \hat{\jmath} = 0$.

Side da: $\vec{F}_{da} = I\big(l_{da}(-\hat{\imath})\big) \times (B\hat{\jmath}) \Rightarrow \vec{F}_{da} = -Il_{da}B(\hat{\imath} \times \hat{\jmath}) \Rightarrow \vec{F}_{da} = Il_{bc}B(-\hat{k})$.

Therefore, net force on the current carrying loop placed in magnetic field is $\vec{F} = \vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{cd} + \vec{F}_{da}$. It leads to $\vec{F} = 0 + Il_{bc}B(\hat{k}) + 0 + Il_{bc}B(-\hat{k}) \Rightarrow \vec{F} = \mathbf{0}$ **is answer of the part (a).**

Accordingly, torque on the couple $\vec{\Gamma} = \vec{w} \times \vec{F} \Rightarrow \vec{\Gamma} = \vec{l}_{ab} \times \vec{F}_{bc} \Rightarrow \vec{\Gamma} = (l_{ab}\hat{\jmath}) \times (Il_{bc}B\hat{k})$. It further solves to $\vec{\Gamma} = Il_{ab}l_{bc}B(\hat{\jmath} \times \hat{k}) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$...(2). Here, $A = Il_{ab}l_{bc}$...(3), is the area of turns of the coil. Given that the coil has n turns of same area and hence net torque would be $\vec{\Gamma}_{net} = n\vec{\Gamma} \Rightarrow \vec{\Gamma}_{net} = nIAB\hat{\imath} \Rightarrow B = \frac{\Gamma_{net}}{nIA}$…(4). Using the available data, $B = \frac{0.2}{100\times2\times((5\times10^{-2})(4\times10^{-2}))} \Rightarrow$ $\boldsymbol{B = 0.5}$**T is the answer of part (a)**

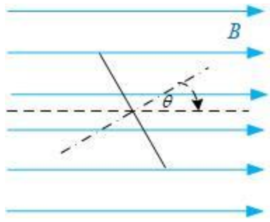
As regards the torque acting on the loop, extrapolation of Lorentz's Force law leads to $\vec{\Gamma} = IAB\hat{\imath} = I\vec{A} \times \vec{B}$ …(5). In this case $\vec{A} = A\hat{a}$ where unit vector $\hat{a}$ is along perpendicular to the area A such that for observer if current in the loop is anti-clockwise then $\hat{a}$ is (+)ve i.e. towards the observer. Whereas, if current in the loop is clockwise then $\hat{a}$ is (-)ve i.e. away from the observer.

In the instant case, as shown in the figure, when we observe the coil from the top current is clockwise and hence $\hat{a} = (-\hat{k})$. Accordingly, $\vec{\Gamma} = IA(-\hat{k}) \times (B\hat{\jmath}) \Rightarrow \vec{\Gamma} = IAB(-\hat{k}) \times (\hat{\jmath}) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$…(6). Using the available data $\Gamma = IAB \Rightarrow \Gamma = 5.0 \times ((2.0 \times 10^{-1}) \times (1.0 \times 10^{-1})) \times 0.20 \Rightarrow \Gamma = \boldsymbol{0.02}$ **Nm on the shorter side is the answer of part (a)**

Thus , **answers are (a) Zero (b) 0.02 Nm parallel to the shorter side**

I-86

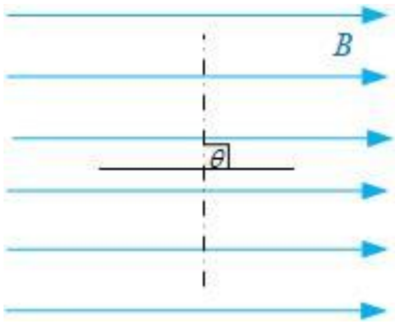
A coil having turns $n = 500$ and carrying current $I = 1.0$ A placed in a magnetic field $B = 0.40$ T and axis of the coil makes an angle $\theta = 30^0$, as shown in the figure. Torque experienced by coil, using Lorentz's Force Law, is $\vec{\Gamma} = nI\vec{A} \times \vec{B} \Rightarrow \Gamma = nIAB\sin\theta$…(1). Here, $\vec{A}$ is the area vector whose magnitude is $A = \pi r^2$, given that $r = 2.0 \times 10^{-2}$m.



Using available data in (1) we have $\Gamma = 500 \times 1.0 \times (\pi \times (2.0 \times 10^{-2})^2) \times 0.40 \times \sin 30^0 \Rightarrow \Gamma = \boldsymbol{0.13}$ **Nm is the answer.**

I-87

Given is a circular loop made of wire of length L. Hence, radius of the loop is such that $L = 2\pi r \Rightarrow r = \frac{L}{2\pi}$. Hence, area of the coil is $A = \pi r^2 \Rightarrow A = \pi\left(\frac{L}{2\pi}\right)^2 \Rightarrow A = \frac{L^2}{4\pi}$…(1) Coil is placed in magnetic field B such that plane of the coil is parallel to magnetic field. It leads to angle between area vector, which is ⊥to the plane of the coil leading to $\theta = \frac{\pi}{2}$…(2).



Torque experienced by a loop, using Lorentz's Force Law, is $\vec{\Gamma} = i\vec{A} \times \vec{B} \Rightarrow \Gamma = iAB\sin\theta$…(3). Applying (3) to both the cases.

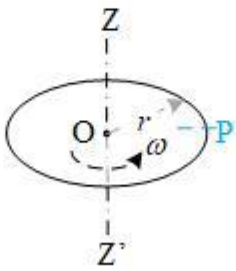
Circular Loop: Taking area as in (1), using (3), torque is $\Gamma = i\left(\frac{L^2}{4\pi}\right)B\sin\frac{\pi}{2} \Rightarrow \Gamma = \boldsymbol{\frac{iL^2B}{4\pi}}$ **is answer of the part (a).**

Square Loop: Area of the square loop made out of the same wire would be $A = \frac{L}{4} \times \frac{L}{4} \Rightarrow A = \frac{L^2}{16}$. Accordingly, torque is $\Gamma = i\left(\frac{L^2}{16}\right)B\sin\frac{\pi}{2} \Rightarrow \Gamma = \boldsymbol{\frac{iL^2B}{16}}$ **is answer of the part (b).**

Thus, **answers are (a)** $\boldsymbol{\frac{iL^2B}{4\pi}}$, **(b)** $\boldsymbol{\frac{iL^2B}{16}}$

I-89

Given is a non-conducting ring of radius r and mass m carries a uniformly distributed charge q. The ring is rotated with an angular velocity ω about its axis Z-Z' as shown in the figure.



Since current $i = \frac{\Delta Q}{\Delta t}$, Since, angular speed of the ring is $\omega = 2\pi N \Rightarrow N = \frac{\omega}{2\pi}$, here N is number revolutions per second. Therefore, time of one revolution $T = \frac{1}{N} \Rightarrow T = \frac{2\pi}{\omega}$.

In time $\Delta t = T$ the perimeter of the rings passes through a point and hence charge on the ring passes through the point P under consideration is $\Delta Q = q$. Accordingly, equivalent current in the ring is $i = \frac{q}{\frac{2\pi}{\omega}} \Rightarrow i = \boldsymbol{\frac{q\omega}{2\pi}}$ **is the answer of part (a).**

Magnetic moment of a coil is $\vec{\mu} = i\vec{A} \Rightarrow \vec{\mu} = iA\hat{a} \Rightarrow \mu = iA \Rightarrow \mu = \left(\frac{q\omega}{2\pi}\right)(\pi r^2) \Rightarrow \mu = \boldsymbol{\frac{q\omega r^2}{2}}$ **is answer of the part (b).**

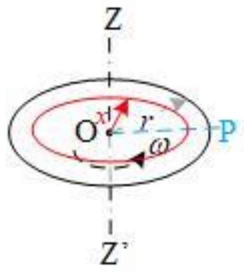
Angular momentum of a ring is $L = I\omega$, here moment of inertia of a ring is $I = mr^2$. Therefore, $L = mr^2\omega \Rightarrow \omega r^2 = \frac{L}{m}$. Combining this in value of μ we have $\mu = \left(\frac{q}{2}\right)\left(\frac{L}{m}\right) \Rightarrow \boldsymbol{\mu = \frac{q}{2m}L}$**, hence proved.**

Thus, **answers are (a)** $\boldsymbol{\frac{q\omega}{2\pi}}$ **(b)** $\boldsymbol{\frac{q\omega r^2}{2}}$ **(c) Proved.**

N.B.: Non-conducting material has a property that at normal conditions charges remain at place and do not flow. Therefore, current is produced by charges distributed on non-conducting geometry only when the geometry changes its position. In this case geometry of non-conducting is a ring and displacement of charges is created by angular motion of the ring about its axis. This principle can be applied to any geometry of non-conducting material.

I-90

Given is a non-conducting disc of radius r and mass m carries a uniformly distributed charge q. The ring is rotated with an angular velocity ω about its axis Z-Z' as shown in the figure.



Since current $i = \frac{\Delta Q}{\Delta t}$, Since, angular speed of the disc is $\omega = 2\pi N \Rightarrow N = \frac{\omega}{2\pi}$, here N is number revolutions per second. Therefore, time of one revolution of the disc is $T = \frac{1}{N} \Rightarrow T = \frac{2\pi}{\omega}$.

In time $\Delta t = T$ the complete disc passes through a radial OP and hence charge on the ring passes through the OP under consideration is $\Delta Q = q$. Accordingly, equivalent current in the ring is $i = \frac{q}{\frac{2\pi}{\omega}} \Rightarrow i = \frac{q\omega}{2\pi}$…(1).

We know that magnetic moment of a coil is $\vec{\mu} = i\vec{A}$…(2). Therefore, current due to distributed charge needs to be analyzed by decomposing the disc into elemental rings of infinitesimal thickness and then taking cumulative effect of all such rings.

Accordingly let us take an elemental ring of radius $0 < x < r$ of radial thickness $\Delta x \to 0$. Therefore, charge on the ring $\Delta q = 2\pi x\Delta x\left(\frac{q}{\pi r^2}\right) \Rightarrow \Delta q = \frac{2q}{r^2}x\Delta x$….(3). Accordingly, using (1), current established due to angular motion of disc vis-à-vis ring is $\Delta i = \frac{\left(\frac{2q}{r^2}x\Delta x\right)\omega}{2\pi} \Rightarrow \Delta i = \left(\frac{2q\omega}{2\pi r^2}\right)x\Delta x$…(4).

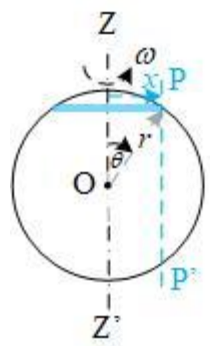
Combining (2) and (4), magnetic moment of the ring is $\Delta\mu = \Delta i(\pi x^2) \Rightarrow \Delta\mu = \left(\left(\frac{2q\omega}{2\pi r^2}\right)x\Delta x\right)(\pi x^2)$. Here, πx^2 is the area enclosed by the ring. Accordingly, $\Delta\mu = \left(\frac{q\omega}{r^2}\right)x^3\Delta x$…(5). Therefore, magnetic effect of the disc is $\mu = \int_0^r \left(\frac{q\omega}{r^2}\right)x^3 dx \Rightarrow \mu = \left(\frac{q\omega}{r^2}\right)\int_0^r x^3 dx \Rightarrow \mu = \left(\frac{q\omega}{r^2}\right)\left[\frac{x^4}{4}\right]_0^r \Rightarrow \mu = \left(\frac{q\omega}{4r^2}\right)r^4 \Rightarrow \mu = \frac{q\omega r^2}{4}$…(6).

Angular momentum of a ring is $L = I\omega$, here moment of inertia of a ring is $I = \frac{mr^2}{2}$. Therefore, $L = \frac{mr^2\omega}{2} \Rightarrow \omega r^2 = \frac{2L}{m}$…(7). Combining (6) and (7) we have $\mu = \left(\frac{q}{4}\right)\left(\frac{2L}{m}\right) \Rightarrow \boldsymbol{\mu = \frac{q}{2m}L}$**, hence proved.**

N.B.: Non-conducting material has a property that at normal conditions charges remain at place and do not flow. Therefore, current is produced by charges distributed on non-conducting geometry only when the geometry changes its position. In this case geometry of non-conducting is a disc and displacement of charges is created by angular motion of the disc about its axis. This principle can be applied to any geometry of non-conducting material.

I-91

Given is a non-conducting solid sphere of radius r and mass m carries a uniformly distributed charge q. The ring is rotated with an angular velocity ω about its axis Z-Z' as shown in the figure. Thus charge density in the sphere is $\rho = \frac{q}{\frac{4}{3}\pi r^3} \Rightarrow \rho = \frac{3q}{4\pi r^3}$…(1).



Since current $i = \frac{\Delta Q}{\Delta t}$…(2). Since, angular speed of the disc is $\omega = 2\pi N \Rightarrow N = \frac{\omega}{2\pi}$, here N is number revolutions per second. Therefore, time of one revolution of the disc is $T = \frac{1}{N} \Rightarrow T = \frac{2\pi}{\omega}$…(3).

In time $\Delta t = T$ the complete sphere passes through a line PP' parallel to ZZ', axis of rotation and hence charge on the ring passes through the OP under consideration is $\Delta Q = q$. Accordingly, equivalent current in the ring is $i = \frac{q}{\frac{2\pi}{\omega}} \Rightarrow i = \frac{q\omega}{2\pi}$…(4).

We know that magnetic moment of a coil is $\vec{\mu} = i\vec{A}$…(5). Therefore, current due to distributed charge needs to be analyzed by decomposing the disc into elemental cylinders of radius $0 < x < r$ of radial thickness $\Delta x \to 0$. Here, $x = r\sin\theta \Rightarrow \Delta x = r\cos\theta\,\Delta\theta$…(6).

Accordingly let us take an elemental cylinder of radius $0 < x < r$ of radial thickness $\Delta x \to 0$ and height $h = 2r\cos\theta$. Therefore, charge on the ring $\Delta q = (2\pi x\Delta x \times h)\rho \Rightarrow \Delta q = (2\pi x\Delta x \times 2r\cos\theta)\left(\frac{q}{\frac{4}{3}\pi r^3}\right)$. It leads to $\Delta q = \frac{3q}{r^2}\cos\theta\,(r\sin\theta)(r\cos\theta\,\Delta\theta) \Rightarrow \Delta q = 3q\cos^2\theta\sin\theta\,\Delta\theta$…(7). Let, $\cos\theta = u \Rightarrow -\sin\theta\,\Delta\theta = \Delta u$ …(8). Combining (7) and (8), we get $\Delta q = -3qu^2\Delta u$…(9).

Combining (2), (3) and (9), current in the cylinder is $\Delta i = \frac{\Delta q}{T} \Rightarrow \Delta i = \frac{-3qu^2\Delta u}{\frac{2\pi}{\omega}} \Rightarrow \Delta i = -\frac{3q\omega}{2\pi}u^2\Delta u$…(10).

Therefore, as per (5) magnet moment of the elemental cylinder is $\Delta\mu = \Delta iA \Rightarrow \Delta\mu = \left(-\frac{3q\omega}{2\pi}u^2\Delta u\right)(\pi x^2)$ …(11).

Combining (6) in (11), $\Delta\mu = \left(-\frac{3q\omega}{2}u^2\Delta u\right)(r^2\sin^2\theta) \Rightarrow \Delta\mu = \left(-\frac{3q\omega r^2}{2}u^2\Delta u\right)(1-u^2)$. It further solves into $\Delta\mu = \frac{3q\omega r^2}{2}(u^4 - u^2\Delta u)\Delta u$…(12). Integrating (12), $\mu = \frac{3q\omega r^2}{2}\left(\frac{u^5}{5} - \frac{u^3}{3}\right)$…(13). Reverting back to the variable $u \to \cos\theta$ and limits of $\theta = 0$ to $\theta = \frac{\pi}{2}$ we get $\mu = -\frac{3q\omega r^2}{2}\left[\frac{\cos^5\theta}{5} - \frac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$. It, further, reduces to $\mu = \frac{3q\omega r^2}{2}\left(-\left(\frac{1}{5} - \frac{1}{3}\right)\right) \Rightarrow \mu = \frac{3q\omega r^2}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{3q\omega r^2}{2}\left(\frac{2}{15}\right) \Rightarrow \mu = \frac{q\omega r^2}{5}$…(14)

Angular momentum of a ring is $L = I\omega$, here moment of inertia of a sphere about its axis is $I = \frac{2mr^2}{5}$. Therefore, $L = \frac{2mr^2\omega}{5} \Rightarrow \omega r^2 = \frac{5L}{2m}$…(15). Combining (14) and (15) we have $\mu = \left(\frac{q}{5}\right)\left(\frac{5L}{2m}\right) \Rightarrow \boldsymbol{\mu = \frac{q}{2m}L}$, **hence proved.**

N.B.: Non-conducting material has a property that at normal conditions charges remain at place and do not flow. Therefore, current is produced by charges distributed on non-conducting geometry only when the geometry changes its position. In this case geometry of non-conducting is a disc and displacement of charges is created by angular motion of the disc about its axis. This principle can be applied to any geometry of non-conducting material.

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