LET US DO SOME PROBLEMS IN MATHEMATICS- XXIX

The following problems are from the Entrance Examination Question Paper of Indian Statistical Institute for undergraduate courses. The detailed solutions are not given here except the answer. If some reader needs the detailed solution of any question or questions, he or she may request the HELP Desk of the Coordinator.

Q1. The number of subsets of 1, 2, 3, …., 10 having an odd number of elements is
(a)1024 (b)512 (c)256 (d)50
Ans.(b)

Q2. For the function on the real line *R* given by $(x) = |x| + |x+1| + e^x$, which of the following is true?
(a) It is differentiable everywhere
(b) It is differentiable everywhere except at $x = 0$ and $x = -1$
(c) It is differentiable everywhere except at $x = \frac{1}{2}$
(d) It is differentiable everywhere except at $x = -\frac{1}{2}$
Ans.(b)

Q3. If f, g are real-valued differentiable functions on the real line *R* such that $f(g(x))=x$ and $f'(x)=1+(f(x))^2$, then $g'(x)$ equals

(a)$\frac{1}{1-x^2}$ (b)$1+x^2$ (c) $\frac{1}{1+x^4}$ (d)$1+x^4$
Ans.(a)

Q4. The number of real solutions of $e^x=sin(x)$ is
(a)0 (b)1 (c)2 (d) Infinite
Ans.(d)

Q5. What is limit of $\sum_{k=1}^{n} \frac{e^{-k/n}}{n}$ as *n* tends to infinity?
(a) The limit does not exist (b)Infinity (c)$1-e^{-1}$ (d)$e^{-0.5}$
Ans.(c)

Q6. A group of 64 players in a chess tournament needs to be divided into 32 groups of 2 players each. In how many ways can this be done?
(a)$\frac{64!}{32!\,2^{32}}$ (b)$\binom{64}{2}\binom{62}{2}\cdots\binom{4}{2}\binom{2}{2}$
(c) $\frac{64!}{32!\,32!}$ (d) $\frac{64!}{2^{64}}$
Ans.(a)

Q7. The integral part of $\sum_{n=2}^{9999} \frac{1}{\sqrt{n}}$ equals
(a)196 (b)197 (c)198 (d)199
Ans.(b)

Q8. There are 128 numbers 1, 2, 3,…., 128 which are arranged in a circular pattern in clockwise order. We start deleting numbers from this set in a clockwise fashion as follows. First delete the

number 2, then skip the next available number (which is 3) and delete 4. Continue in this manner, that is, after deleting a number, skip the next available number clockwise and delete the number available after that, till only one number. What is the last number left?
(a)1 (b)63 (c)127 (d)None of the above
Ans.(a)

Q9. Let z and w be complex numbers lying on the circles of radii 2 and 3 respectively, with centre (0,0). If the angle between the corresponding vectors is 60 degrees, then the value of $\frac{|z+\omega|}{|z-\omega|}$ is
(a) $\frac{\sqrt{19}}{\sqrt{7}}$ (b) $\frac{\sqrt{7}}{\sqrt{19}}$ (c) $\frac{\sqrt{12}}{\sqrt{7}}$ (d) $\frac{\sqrt{7}}{\sqrt{12}}$
Ans.(a)

Q10. Two vertices of a square lie on a circle of radius r and the other two vertices lie on a tangent to this circle. Then the length of the side of the square is
(a) $\frac{3r}{2}$ (b) $\frac{4r}{3}$ (c) $\frac{6r}{5}$ (d) $\frac{8r}{5}$
Ans.(d)

Q11. For a real number x, let [x] denote the greatest integer less than or equal to x. then the number of real solutions of $|2x-[x]|=4$ is
(a)4 (b)3 (c)2 (d)1
Ans.(a)

Q12. Let f, g be differentiable functions on the real line R with $f(0)>g(0)$. Assume that the set $M= t\in R$ such that $f(t)= g(t)$ is non-empty and that $f'(t) \geq g'(t)$ for all $t\in M$. then which of the following is necessarily true?
(a) If $t \in M$, then $t<0$ (b)For any $t\in M$, $f'(t)>g'(t)$
(c) For any $t \notin M$, $f(t) > g(t)$ (d)None of these
Ans.(c)

Q13. Let $A = \{x_1, x_2, \ldots, x_{50}\}$ and $B = \{y_1, y_2, \ldots, y_{20}\}$ be two sets of real numbers. What is the total number of functions $f:A\rightarrow B$ such that f is onto and $f(x_1) \leq f(x_2) \leq \ldots \leq f(x_{50})$?
(a) $\binom{49}{19}$ (b) $\binom{49}{20}$ (c) $\binom{50}{19}$ (d) $\binom{50}{20}$
Ans.(a)

Q14. The number of complex roots of the polynomial z^5-z^4-1 which have moulus 1 is
(a)0 (b)1 (c)2 (d)More than2
Ans.(c)

Q15. The number of real roots of the polynomial $P(x)=(x^{2020}+2020x^2+2020)(x^3-2020)(x^2-2020)$ is
(a)2 (b)3 (c)2023 (d)2025
Ans.(b)

Q16. Which of the following is the sum of an infinite geometric sequence whose terms come from the set $\left\{1,\frac{1}{2},\frac{1}{4},\ldots,\frac{1}{2^n},\ldots\right\}$?
(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{1}{9}$ (d) $\frac{1}{11}$
Ans.(b)

Q17. if a, b, c are distinct odd natural numbers, then the number of rational roots of the polynomial ax^2+bx+c

(a)must be 0 (b) must be 1

(c)must be 2 (d) cannot be determined from the given data

Ans.(a)

Q18. Shubhangi thinks she may be allergic to Bengal gram and takes a test that is known to give the following results:

(i) For people who really do have the allergy, the test says "Yes" 90% of the time

(ii) For people who do not have the allergy, the test says "Yes" 15% of the time

If 2% of the population has the allergy and Shubhangi's test says "Yes" then the chances that Shubhangi does really have the allergy are

(a) 1/9 (b)6/55

(c)1/11 (d) Cannot be determined from the given data

Ans.(b)

Q19. If $\sin(tan^{-1}(x)) = \cot\left(sin^{-1}\sqrt{\frac{13}{17}}\right)$ then x is

(a) $\frac{4}{17}$ (b) $\frac{2}{3}$

(c) $\sqrt{\frac{17^2-13^2}{17^2+13^2}}$ (d) $\sqrt{\frac{17^2-13^2}{17\ x\ 13}}$

Ans.(b)

Q20. The word **PERMUTE** is permuted in all possible ways and the different resulting words are written down in alphabetical order (also known as dictionary form), irrespective of whether the word has meaning or not, then the 720th word would be:

(a)EEMPRTU (b)EUTRPME

(c)UTRPMEE (d)MEETPUR

Ans.(b)

Q21. Let ***f(x), g(x)*** be functions on the real line R such that both $f(x)+g(x)$ and $f(x)g(x)$ are differentiable. Which of the following is FALSE?

(a) $f(x)^2+g(x)^2$ is necessarily differentiable

(b) $f(x)$ is differentiable if and only if g(x) is differentiable

(c) $f(x)$ and $g(x)$ are necessarily continuous

(d) If $f(x)>g(x)$ for all $x \in R$ then $f(x)$ is differentiable

Ans.(d)

Q22. Let S be the set consisting of all those real numbers that can be written as $p-2a$ here p and a are the perimeter and area of a right angled triangle having base length 1, then S is

(a) $(2, \infty)$ (b) $(1,\infty)$

(c) $(0,\infty)$ (d) The real line R

Ans.(a)

Q23. For any real number, let [x] be the greatest integer m such that $m \le x$, then the number of points of discontinuity of the function $g(x)=[x^2-2]$ on the interval (-3,3) is

(a)5 (b)9

(c)13 (d)16

Ans.(d)