LET US DO SOME PROBLEMS-XXXIII

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The National Defence Academy (NDA) examination is conducted by UPSC for admission to the Army, Navy and Air Force wings of the NDA. This exam is a gateway for candidates looking forward to join Defence Forces including Army, Navy and Air Force. This is a national level examination conducted twice a year: NDA I and NDA II to help candidates make a career in Defence services.

National Defence Academy & Naval Academy (NDA & NA) Examination is a two-step process where candidates first need to take the examination and then qualify the personality test or interview conducted by the Service Selection Board (SSB).

The UPSC is solely responsible for issuing guidelines for selection and the final conduct of the Entrance Examination.

The more can be had from the website: https://www.careerindia.com/upsc/nda-na-i-and-ii-exam-e24.html

Some questions from the latest examination have been selected for the readers with answers to understand the standard of the questions. The detailed solutions are not given here. If any reader wants to have the solution or solutions of the problems, he or she can request the Coordinator's desk for that.

QUESTIONS

Q1. The smallest positive integer *n* for which $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$ where $n = \sqrt{-1}$, is

(a) 2 (b) 4
(c) 6 (d) 8

Ans. (a)

Q2. The value of *x*, satisfying the equation $log_{cos} \sin x = 1$, where $0 < x < \frac{\pi}{2}$ is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Ans. (c)

Q3. If Δ is the determinant $\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$ then what is the value of the following determinant? ($p \neq 0$ *or* 1, $q \neq 0$ *or* 1)

(a)pΔ (b)qΔ
(c)(p+q) Δ (d)pqΔ

Ans. (d)

Q4. If $a+b+c=4$ and $ab+ bc+ca=0$, then what is the value of the following determinant?

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a)32 (b)-64
(c)-128 (d)64

Ans. (b)

Q5. The number of integer values of *k*, for which the equation $2sinx=2k+1$ has a solution, is (a)zero (b)one
(c)two (d)four

Ans. (c)

Q6. If the roots of the quadratic equation $x^2+2x+k=0$ are real, then

(a)k<0 (b)k≤0
(c)k<1 (d)k≤1
Ans. (d)

Q7. If *n*=100!,then what is the value of the following?

$$\frac{1}{log_2 n}+\frac{1}{log_3 n}+\frac{1}{log_4 n}+\cdots+\frac{1}{log_{100} n}$$

(a)0 (b)1
(c)2 (d)3
Ans. (b)

Q8. Let $A=\begin{vmatrix} p & q \\ r & s \end{vmatrix}$ where *p,q,r* and *s* are any four different prime numbers less than 20. What is the maximum value of the determinant? (a)215
(b)311
(c)317 (d)323
Ans. (c)

Q9. What is *cot2xcot4x - cot4xcot6x - cot6xcot2x* equal to?
(a)-1 (b)0
(c)1 (d)2
Ans. (c)

Q10. What is the value of the following?
$tan31^0 tan33^0 tan35^0 \ldots\ldots tan57^0 tan59^0$
(a)-1 (b)0
(c)1 (d)2
Ans. (c)

Q11. If $f(x)=$

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

then what is *f(-1)+f(0)+f(1)* equal to?
(a)0 (b)1
(c)100 (d)-100
Ans. (a)

Q12. The equation $sin^{-1}x-cos^{-1}x=\frac{\pi}{6}$ has
(a)no solution
(b)unique solution
(c)two solutions
(d)infinite number of solutions
Ans. (b)

Q13. What is $\frac{1+tan^2\theta}{1+cot^2\theta}-\left(\frac{1-tan\theta}{1-cot\theta}\right)^2$ equal to?
(a)0 (b)1
(c)2tanθ (d)2cotθ
Ans. (a)

Q14. What is the interior angle of a regular octagon of side length 2 cm?
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$
(c) $\frac{3\pi}{5}$ (d) $\frac{3\pi}{8}$
Ans. (b)

Q15. Consider the following statements:
1.the null set is a subset of every set.
2.every set is a subset of itself.
3.If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?
(a)1 and 2 only (b)2 and 3 only
(c)1 and 3 only (d)1,2 and 3
Ans. (d)

Q16. Consider the following statements in respect of an arbitrary complex number *Z*:
1.The difference of *Z* and its conjugate is an imaginary number.
2.The sum of *Z* and its conjugate is a real number.

Which of the above statements is/are correct?
(a)1 only (b)2 only
(c)Both 1 and 2 (d)Neither 1 nor 2
Ans. (c)

Q17. What is *C(n,1)+C(n,2)+…….+C(n,n)* equal to?
(a)$2+2^2+2^3+\ldots+2^n$
(b)$1+2+2^2+2^3+\ldots+2^n$
(c) $1+2+2^2+2^3+\ldots+2^{n-1}$

(d) $2+2^2+2^3+\ldots+2^{n-1}$

Ans. (c)

Q18. If the first term of an *AP* is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms?

(a)-500 (b)-250
(c)500 (d)250

Ans. (b)

Q19. If the lines $y+px=1$ and $y-gx=2$ are perpendicular, then which one of the following is correct?

(a)pq+1=0
(b)p+q+1=0 (c)pq-1=0 (d)p-q+1=0

Ans. (c)

Q20. If the image of the point (-4, 2) by a line mirror is (4, -2), then what is the equation of the line mirror? (a)y=x
(b)y=2x (c)4y=x (d) y=4x

Ans. (b)

Q21. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola?

(a)$\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{\sqrt{11}}{2}$ (d)$\frac{\sqrt{13}}{2}$

Ans. (d)

Q22. A coin is tossed twice. If *E* and *F* denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

Ans. (c)

Q23. If *A* and *B* are two events such that P(A)= $\frac{3}{4}$ and P(B)= $\frac{5}{8}$, then consider the following statement:

1.the minimum value of P(A∪B) is $\frac{3}{4}$

2.the maximum value of P(A∩B) is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a)1 only (b)2 only
(c)both 1 and 2 (d)neither 1 nor 2

Ans. (c)

Q24. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, *x* is 15, then what is the value of x?

(a)10 (b)12
(c)13 (d)15

Ans. (d)

Q25. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is

(a)810 (b)900
(c)30000 (d)81000

Ans. (c)

Q26. What is the sum of the deviations of the variate values 73, 85, 92, 105, 120 from their mean?

(a)-2 (b)-1
(c)0 (d)5

Ans. (c)

Q27. What is $\int e^{(2\ln x+\ln x^2)}dx$ equal to?

(a)$\frac{x^4}{4}+C$ (b) $\frac{x^3}{3}+C$
(c) $\frac{2x^5}{5}+C$ (d) $\frac{x^5}{5}+C$

Ans. (d)

Q28. The general solution of a differential equation is $y^2+2cy-cx+c^2=0$, where *C* is an arbitrary constant, then what is the order of the differential equation?

(a)1 (b)2
(c)3 (d)4

Ans. (a)

Q29. If $x = e^t \cos t$ and $y = e^t \sin t$, then what is $\frac{dx}{dy}$at t=0 equal to?

(a)0 (b)1

(c)2e (d)-1

Ans. (b)

Q30. What is the derivative of *sin(ln x)+cos(ln x)* with respect to *x* at *x=e*?

(a)$\frac{cos1-sin1}{e}$

(b) $\frac{sin1-cos1}{e}$

(c) $\frac{cos1+sin1}{e}$ (d) 0

Ans. (a)

Q31. If $f(x) = e^{|x|}$, then which one of the following is correct?

(a)f'(0)=1 (b)f'(0)=-1

(c)f'(0)=0 (d)f'(0) does not exist

Ans. (d)

Q32. A 24 cm long wire is bent to form a triangle with one of the angles as 60degrees. What is the altitude of the triangle having the greatest possible area?

(a)$4\sqrt{3}$ cm (b)$2\sqrt{3}$ cm

(c)6cm (d)3cm

Ans. (a)

Q33. If $\lim_{x\to a} \frac{a^x - x^a}{x^a - a^a} = -1$ then what is the value of *a*?

(a)-1 (b)0

(c)1 (d)2

Ans. (c)

Q34. What is the projection of the line segment joining A(1,7,-5) and B(-3,4,-2) on y-axis?

(a)5 (b)4

(c)3 (d)2

Ans. (c)

Q35. If *l,m,n* are the direction cosines of the line *x=1=2(y+3)=1-z*, then what is $l^4+m^4+n^4$ equal to?

(a)1 (b) $\frac{11}{27}$

(c) $\frac{13}{27}$ (d)4

Ans. (b)