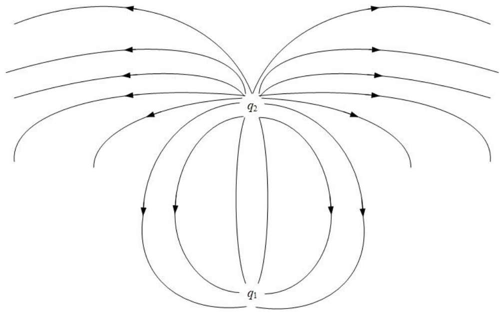
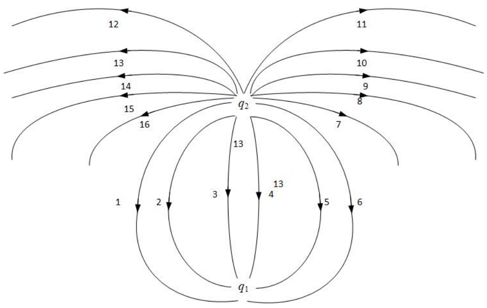
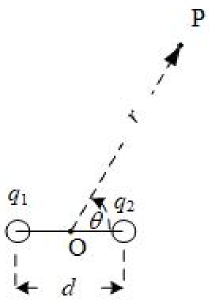
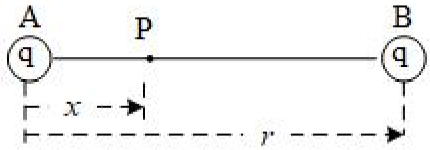
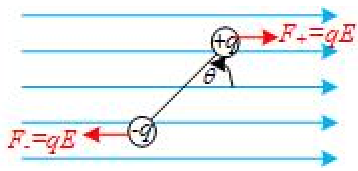
Electromagnetism: Electrostatics-Electric Fiend and Potential

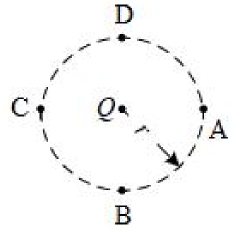
(Selected Questions)

Abbreviations: Q- Question, A- Answer, I – Illustration to the solution

Q-1	Consider the situation shown in the figure. What are signs of q_1 and q_2? If the lines are drawn in proportion to the charge, what is the ratio $\frac{q_1}{q_2}$? 
A-1	q_2 is (+)ve and q_1 is (-)ve.; $\frac{q_1}{q_2} = \frac{1}{3}$
I-1	Since electric lines of force are emanating for $\boldsymbol{q_2}$ hence it **is (+)ve** and entering into $\boldsymbol{q_1}$ hence it is **(-)ve** Electric lines of force emanating or entering a charge are 4π times the charge $\phi = 4\pi q \Rightarrow \frac{\phi_1}{\phi_2} = \frac{4\pi q_1}{4\pi q_2} = \frac{q_1}{q_2}$…(1) It is seen from the figure that the lines of force numbered 1 to 6 only emanating from charge q_2 are entering into charge q_1 i.e. $\phi_1 = 6$. While all the lines of force numbered 1 to 18 are emanating from charge q_2 and $\phi_2 = 18$. Lines of force numbered 7 to 18 are terminating on unknown charges. Therefore, using the available data in (1) $\frac{q_1}{q_2} = \frac{6}{18} \Rightarrow \boldsymbol{\frac{q_1}{q_2} = \frac{1}{3}}$ **is the answer.** 
Q-2	It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what?
A-2	Dipole moment of an electric dipole Distance of a point under consideration from mid- point of the two charges constituting dipole.
I-2	Electric field at any point is $\vec{p} = q\vec{d}$ …(1), here d is the separation between two charges constituting dipole. Influence of a dipole at any point P as shown in the figure is $V = \frac{1}{4\pi\varepsilon_0} \times \frac{p\cos\theta}{r^2}$…(2). In (2) r is the distance of the point under consideration from midpoint of charge O. The mathematical expression (2) is valid for $\boldsymbol{d \ll r}$**, is the answer.** 
Q-3	Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving it from A to B. The potential (a) continuously increase (b) continuously decrease (c) increase and then decrease (d) decrease and then increase
A-3	(d)

I-3	Potential at a point, at a distance R from a charge Q is $V=\frac{Q}{4\pi\varepsilon_0 R}$...(1). Let displacement between two charges be r and point between two equal (+q) charges placed at A and B. A point P between A and B under consideration, as shown in the figure, is at a distance x such that $0<x<r$ from the charge A, then the distance of the point from B would be equal to $(r-x)$.  It is required to determine variation in potential for $0<x<r$ Accordingly, as per (1) net potential at P due to charges at A and B is $V=V_A+V_B \Rightarrow V=\frac{q}{4\pi\varepsilon_0 x}+\frac{q}{4\pi\varepsilon_0 (r-x)}$. Thus, $V=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x}+\frac{1}{r-x}\right) \Rightarrow V=\frac{q}{4\pi\varepsilon_0}\times\left\{\frac{1}{x}+\frac{1}{(r-x)}\right\}$. Variation of potential V w.r.t. x is $\frac{dV}{dx}=\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{x^2}-\frac{1}{(r-x)^2}(-1)\right] \Rightarrow \frac{dV}{dx}=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-x)^2}-\frac{1}{x^2}\right] \Rightarrow \frac{dV}{dx}=\frac{q}{4\pi\varepsilon_0}\left[\frac{x^2-(r-x)^2}{\left(x(r-x)\right)^2}\right] \Rightarrow \frac{dV}{dx}=\frac{q}{4\pi\varepsilon_0}\times\frac{r(2x-r)}{\left(x(r-x)\right)^2}$...(2). In this expression $\frac{dV}{dx}$ all terms except $(r-2x)$ are (+)ve, and this terms changes as analyzed below – **(a)** $\boldsymbol{0<x<\frac{r}{2}}$: The factor $(2x-r)<0$. Hence the slope is (-)ve. It implies that potential decreases with the increase of x. **(b)** $\boldsymbol{x=\frac{r}{2}}$**:** The factor $(2x-r)=0$. Hence, slope becomes zero. It implies that potential is at its minimum, using concept of maxima-minima **(c)** $\boldsymbol{\frac{r}{2}<x<r}$**:** The factor $(2x-r)>0$. Hence the slope is (+)ve. It implies that potential increases with the increase of x. Thus analysis reveals that potential initially decreases and then increases as provided in **option (d), is the answer.**
Q-4	An electric dipole is placed in a uniform electric field. Net electric force on dipole (a) always zero (b) depends on the orientation of the dipole (c) can never be zero (d) depends on the strength of the dipole
A-4	(a)
I-4	Magnitude of force of $+q$ and $-q$ charges of the dipole due to uniform electric field is $F=qE$, for any angular position (θ) of the dipole w.r.t. electric field, but direction of the two forces as shown in the figure are opposite to each other. In an electric dipole the two charges coexist and hence net force on the dipole due to uniform electric field is $F=F_+ + F_- = qE-qE=0$.. This analysis is supported by the **option (a), is the answer.** 
Q-5	A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is - (a) Zero (b) Positive (c) Negative (d) Zero if charge Q is at the center and nonzero otherwise
A-5	(a)
I-5	Electric potential is a scalar quantity and it is quantified for a charge Q at any point at a distance r from it as $V=\frac{Q}{4\pi\varepsilon_0 r}$. And work done moving a charge q from point A to B is $W=q(V_B-V_A)$.

<table>
<tr><td></td><td>In rotating a charge around a circle of radius, to complete one evolution is $W = q(V_A - V_A) \Rightarrow W = 0$. This conclusion matches with option (a) is the answer.

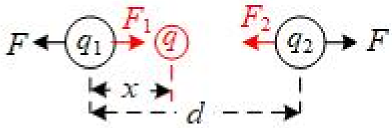
N.B.: An isolated charge creates circular equipotential surfaces centered at the charge. Hence, work done in moving a point charge from one point on an equipotential surface to any other point it is zero. It is not necessary to rotate the charge in one complete revolution.
</td></tr>
<tr><td>Q-6</td><td>The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking electric potential at the origin to be zero
(a) It is uniform region
(b) It is proportional to r
(c) It is proportional to r^2
(d) It increases as one goes away from the origin</td></tr>
<tr><td>A-6</td><td>(c)</td></tr>
<tr><td>I-6</td><td>Electric field at a point due to a system charges is $\vec{E} = \sum \vec{E}_i$ where due to a a particular charge q_i electric field at a point is $\vec{E} = \frac{q_i}{4\pi\varepsilon_0 r_i^2}\hat{r}_i \Rightarrow E = \frac{q_i}{4\pi\varepsilon_0 r_i^2} \Rightarrow E \propto \frac{1}{r^2}$. But, it is stated in the problem that $E \propto r$. It is not a hypothetical case. Further, potential at a point is $V = -\int E dr \Rightarrow V \propto \int r dr \Rightarrow V \propto r^2$. This conclusion, matches with only option (c), is the answer.

N.B.: In this question analysis leads to discrete answer matching with one of the options given. Hence, it is not necessary to analyze each of the option.</td></tr>
<tr><td>Q-7</td><td>At what separation should two equal charges 1.0 C each be placed so that force between them must be equal to the weight of a 50 kg person?</td></tr>
<tr><td>A-7</td><td>424 m</td></tr>
<tr><td>I-7</td><td>As per Coulomb's Law force between two charges is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^2C^{-2}, and given that $q_1 = q_2 = 1.0$ C. Separation between the charges r is to be determined for a force equal to weight of a 50 kg person. It is to be noted that weight has a unit N, while given data is in kg, a unit of mass. Applying Newton's Second Law of motion $F = ma$ here $a = g = 10$ m/s^2 is used to convert given data in force.

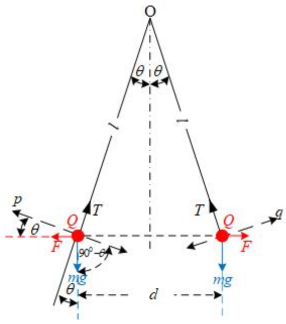
Accordingly, using the available data force between the two charges is $F = W \Rightarrow 50 \times 10 = 8.99 \times 10^9 \times \frac{1.0 \times 1.0}{r^2} \Rightarrow r^2 = 1.796 \times 10^3 \Rightarrow r^2 = 17.96 \times 10^2 \Rightarrow r = 4.238 \times 10^2$ m, say $r =$ 424 m is the answer of first part.

N.B.:(a) In this case precise answer would depend upon the value of g (not specified in the question) considered in the solution. Moreover, this value of g is in agreement with the SDs of the data given in the problem

(b) Since data is given in conventional notation and hence, for convenience, the same is used in answer.</td></tr>
</table>

Q-8	Two charges 2.0×10^{-6} C and 1.0×10^{-6} C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges?
A-8	5.9 cm from the larger charge in between the two charges.
I-8	The system given in the problem is shown in the figure. As per Coulomb' law of force between two electrical between charges $q_1 = 2.0 \times 10^{-6}$ and $q_2 = 1.0 \times 10^{-6}$is $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ Nm^{2}C^{-2}, and separation between them $d = 10^{-1}$m..  Both the charges q_1 and q_2 being positive would experience a single force of repulsion $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{d^2}$…(1) Making net force on these A third charge q is introduced in the system of the two charges, such that net force on the two charges is zero. Thus, eventually both the charges would experience two forces of which second force on q_1 is $F_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q}{x^2}$ …(2), and the second force on q_2 is $F_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q}{(d-x)^2}$…(3) Thus, require of zero net force on both the charges q_1 is $F + F_1 = 0 \Rightarrow F = -F_1$. In this combining (1) and (2) we have $\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{d^2} = -\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q}{x^2} \Rightarrow \frac{q_2}{d^2} = -\frac{q}{x^2} \Rightarrow q = q_2 \left(\frac{x}{d}\right)^2$…(4). Likewise, net force q_2 is $F + F_2 = 0 \Rightarrow F = -F_2$. In this combing combining (1) and (2) we have $\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{d^2} = -\frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q}{(d-x)^2} \Rightarrow \frac{q_1}{d^2} = -\frac{q}{x^2} \Rightarrow q = q_1 \left(\frac{d-x}{d}\right)^2$…(5). Combining, (4) and (5), $q_2 \left(\frac{x}{d}\right)^2 = q_1 \left(\frac{d-x}{d}\right)^2 \Rightarrow \left(\frac{d-x}{x}\right)^2 = \frac{q_2}{q_1} \Rightarrow \frac{d-x}{x} = \sqrt{\frac{q_2}{q_1}}$. Using the available data $\frac{0.1-x}{x} = \sqrt{\frac{1.0\times10^{-6}}{2.0\times10^{-6}}} \Rightarrow \frac{0.1-x}{x} = \frac{1}{\sqrt{2}}$. Applying componendo we have $\frac{0.1}{x} = \frac{1+\sqrt{2}}{\sqrt{2}} \Rightarrow x = \frac{0.1\times\sqrt{2}}{1+\sqrt{2}} \Rightarrow x = 0.063$ m, or from q_1 **6.3 cm, the large charge is the answer**.. **N.B.:** Unless must, arithmetic calculations should be kept until the last; it leads to highly simplified calculations. This problem is a good example of it.
Q-9	Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons?
A-9	3.35×10^{25}, 5.35×10^6 C
I-9	One mol of an atom is equal to mass of substance in grams equal to atomic number $A = N + Z$, where N is number of electrons and Z is number of protons. Molecular mass of water (H_2O) is $A_W = 2 \times A_H + A_O$and it is $A_W = 2 \times 1 + 16 = 18$ Further, number of molecules in A gram water is Avogadros' number (6.023×10^{23}). Therefore, number of atoms in 100 g water $N = \frac{100}{18} \times 6.023 \times 10^{23}$. Further, number of electrons in a water molecule $n_W = 2 \times n_H + n_O = 2 \times 1 + 8 = 10$. Therefore, total number of electrons in given quantity of water is $N_e = N \times n_W = \left(\frac{100}{18} \times 6.023 \times 10^{23}\right) \times 10 = 3.346 \times 10^{25}$ say $\mathbf{3.35 \times 10^{25}}$.

	It is known that charge of an electron is $e = (1.6 \times 10^{-19})$ C and hence total negative charge of electrons in the given quantity of water is $Q_e = N_e \times e = (3.346 \times 10^{25}) \times (1.6 \times 10^{-19}) \Rightarrow Q_e = 5.354 \times 10^6$ say $\mathbf{5.35 \times 10^6}$ **C.** **N.B.: (a)** Principle of SDs should be applied at last stage of reporting the answer, thus N_e is reported using SDs. **(b)** In calculating Q_e calculated value of $N_e = 3.346 \times 10^{25}$ is used but final value of Q_e is reported applying principle of SDs at the last stage. **(c)** This problem is a good example of application of principle of SDs.
Q-10	Suppose all electrons of 100 g of water are lumped together to form a negatively charged particle and all nuclei are lumped to form a positively charged particle. If these two charged particles are placed 10.0 cm away from each other, find the force of attraction between them. Compare it to your weight.
A-10	2.56×10^{25}N
I-10	Any matter in its natural state is electrically neutral, it implies that number of oppositely charged atomic particles is electrons and protons, electrically complementary to each other, are equal in number and quantity of charge. One mol of an atom is equal to mass of substance in grams equal to atomic number $A = N + Z$, where N is number of electrons and Z is number of protons. Molecular mass of water (H_2O) is $A_W = 2 \times A_H + A_O$and it is $A_W = 2 \times 1 + 16 = 18$ Further, number of molecules in A gram water is Avogadros' number (6.023×10^{23}). Therefore, number of atoms in 100 g water $N = \frac{100}{18} \times 6.023 \times 10^{23}$. Further, number of electrons in a water molecule $n_W = 2 \times n_H + n_O = 2 \times 1 + 8 = 10$. Therefore, total number of electrons in given quantity of water is $N_e = N \times n_W = \left(\frac{100}{18} \times 6.023 \times 10^{23}\right) \times 10 = 3.346 \times 10^{25}$ say $\mathbf{3.35 \times 10^{25}}$. It is known that charge of an electron is $e = (1.6 \times 10^{-19})$ C and hence total negative charge of electrons in the given quantity of water is $Q_e = N_e \times e = (3.346 \times 10^{25}) \times (1.6 \times 10^{-19}) \Rightarrow Q_e = 5.354 \times 10^6$ say $\mathbf{5.35 \times 10^6}$ **C.** Separation between the concentrated (+) ve and (-)ve charges is given to be 0.10 m. As per Coulomb's Law of Force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$. In the expression value of constant $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2C^{-2}, and with the given data $F = (9 \times 10^9) \times \frac{(5.35\times10^6)(-5.35\times10^6)}{(0.1)^2} \Rightarrow F = \mathbf{-2.56 \times 10^{16}}$ N attractive force is the answer. My weight is 50 kg, the unit used is of mass and hence quantitatively weight is $W = mg = 50 \times 10 = 500$ N. Hence, the required ratio is $\frac{F}{W} = \frac{2.56\times10^{16}}{5\times10^2} = \mathbf{5.12 \times 10^{13}}$**times my weight.** **N.B: (a)** Answer of second part is subjective and depends upon weight of person answering the question. In such questions it is advisable to use a value for it convenient for calculations. **(b)** It is also to be noted that weight of persons is scaled in mass and accordingly unit used is kg. The reason being mass remain unchanged even in change of place. Therefore, it needs to be appropriately converted in Newton.
Q-11	Two identical balls, each having a charge 2.00×10^{-7} C and a mass 100 g are suspended from a common point by two insulating strings each 50 cm long. The balls are held at separation 5.0 cm apart and then released. At the instant just after release of the balls find – (a) The electric force on one of the charged balls

	(b) The component of resultant force on it along and perpendicular to the string (c) The tension in the string (d) The acceleration of one of the balls.
A-11	(a) 0.144 N, (b) zero., 0.094 N away from other charge (c) 1.01 N, (d) 0.94 m.s^{-2} perpendicular to the string and going away from the other charge
I-11	Given system is shown in the figure where charge on the two identical balls is $Q = 2.0 \times 10^{-7}$ C having mass $m = 0.1$ kg and length of strings $l = 0.5$ m. They are held in suspended state at a distance $d = 5.0 \times 10^{-2}$ m.  The system being symmetrical about the vertical axis analysis of one ball would apply on the other. In state of free motion, ball is subjected to three forces – (a) gravitational force $F_g = mg$ N, here acceleration due to gravity is taken to be $g = 10$ m/s^2, (b) electrostatic force $F = \frac{1}{4\pi\varepsilon_0} \times \frac{QQ}{d^2}$N, here $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}, and (c) Resultant of gravitational and electric forces $\vec{F_R} = \sqrt{F_g{}^2 + F^2}\hat{\imath}$ …(1) . Since, string, after the balls are released, remain taught therefore there is an equilibrium of forces. $\vec{T} + \vec{F_R} = 0$…(2). Geometrically, $\sin\theta = \frac{d/2}{l} = \frac{d}{2l} \Rightarrow \sin\theta = \frac{5.0\times10^{-2}}{2\times0.5} \Rightarrow \sin\theta = 5.0 \times 10^{-2}$…(3) and $\cos\theta = \sqrt{1-\sin^2\theta} \Rightarrow \cos\theta = \sqrt{1-(5.0\times10^{-2})^2} \Rightarrow \cos\theta = 0.999$ …(4). With this analysis, each of the part of the problem is being solved – **Part (a):** Electrostatic force on charge is $F = (9\times10^9) \times \frac{(2.0\times10^{-7})^2}{(5.0\times10^{-2})^2} = (9\times10^9)\times(4.0\times10^{-6})^2 = 1.44\times10^{-1}$N say **0.144 N, is the answer.** **Part (b):** Net force along the string as per (2) above as per (2) above is zero. But, net force perpendicular to the string is $F_p = mg\sin\theta - F\cos\theta$…(5). Combining (5) with (3) and (4) we have $F_p = (0.1\times10)\times(5.0\times10^{-2}) - (0.144)\times(0.999) = \mathbf{-0.094}$ **N, is the force perpendicular to the string, separating from the other charge.** **Part (c):** Tension in the string from (1), (2) and result in part (a) above, $\lvert\vec{T}\rvert = \lvert\vec{F}\rvert = \sqrt{F_g{}^2 + F^2} = \sqrt{(0.144)^2 + (0.1\times10)^2} = \mathbf{1.01}$ **N is the answer.** **Part (d):** Acceleration of the ball is determined from net of force F_p on the ball perpendicular to the string obtained in part (b) using Newton's Second Law of motion and $a_p = \frac{F_p}{m} \Rightarrow a_p = \frac{0.094}{0.1} = \mathbf{0.94}$ **m/s^2.** **Answers** are, (a) 0.144 N, (b) zero., 0.094 N away from other charge, (c) 1.01 N, (e) 0.94 m.s^{-2} perpendicular to the string and going away from the other charge **N.B.:** Value of g is not provided and hence value taken in solution would marginally influence the answer.
Q-12	Two particles A and B having charge Q and $2Q$ respectively are placed on a smooth table with a separation d. A third particle C is clamped on the table in such a way that the particles A and B remain at rest on the table under the electrical forces. What should be the charge on C and where should it be clamped?
A-12	$-(6-4\sqrt{2})Q$ between Q and 2Q at a distance $(\sqrt{2}-1)d$ from Q

I-12

In the instant problem, as shown in the figure, all charges are placed on a smooth table they would experience electric force as per Coulomb's Law $F = \frac{q_1 \times q_2}{4\pi\varepsilon_0 d^2}$ …(1), where d is separation between the two charges under consideration.

It is given that a particle C which carries some charge say q is clamped on the table such that the particles A and B having charge Q and $2Q$ respectively remain at rest. It is seen that each of the three charge experiences Two forces due to other two charges. Thus for particle A and B to remain at rest the two forces on each charge must be in equilibrium. Taking forces on each particle separately -

Analysis of equilibrium of particle A having Charge $q_1 = Q$:

It experiences two forces –

(a) mutual repulsive force between particles A due to particle B having charge $q_2 = 2Q$ such that $F = \frac{Q \times 2Q}{4\pi\varepsilon_0 d^2}$ …(1)

(b) force due to charge q on particle C at a distance x from A, it has to be opposite to the F such that $F_1 = \frac{Q \times q}{4\pi\varepsilon_0 x^2}$…(2)

Therefore, under equilibrium from (1 & 2), $F + F_1 = 0 \Rightarrow \frac{Q \times 2Q}{4\pi\varepsilon_0 d^2} = -\frac{Q \times q}{4\pi\varepsilon_0 x^2} \Rightarrow q = -2Q\left(\frac{x}{d}\right)^2$ …(3)

Analysis of equilibrium of particle B having Charge $q_2 = 2Q$:

It experiences two forces –

(a) mutual repulsive force F between particles B due A is same as at (1).

(b) force due to charge q on particle C at a distance $(d - x)$ from B is $F_2 = \frac{2Q \times q}{4\pi\varepsilon_0 (d-x)^2}$….(4)

Therefore, under equilibrium from (1 & 4), $F + F_2 = 0 \Rightarrow \frac{Q \times 2Q}{4\pi\varepsilon_0 d^2} = -\frac{2Q \times q}{4\pi\varepsilon_0 (d-x)^2} \Rightarrow q = -Q\left(\frac{d-x}{d}\right)^2$ …(5).

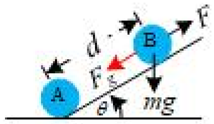
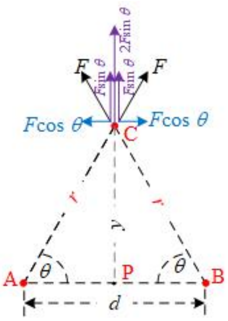
Combining (3 & 5) we have, $-2Q\left(\frac{x}{d}\right)^2 = -Q\left(\frac{d-x}{d}\right)^2 \Rightarrow \left(\frac{d-x}{x}\right)^2 = 2 \Rightarrow d - x = \sqrt{2}\,x$. It leads to $x = \frac{d}{\sqrt{2}+1}$. This is further resolved $x = \frac{d}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} \Rightarrow x = (\sqrt{2} - 1)d$ is the answer.

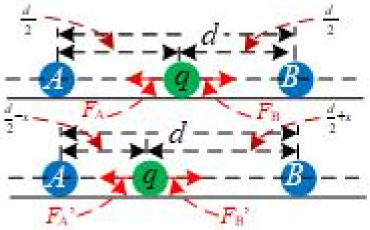
Using value of x in (3) $q = -2Q\left(\left(\sqrt{2} - 1\right)d\right)^2 \Rightarrow q = -2Q\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 \Rightarrow q = -2\left(3 - 2\sqrt{2}\right)d^2 Q$. It leads to $q = -\left(6 - 4\sqrt{2}\right)d^2 Q$ **is the answer**.

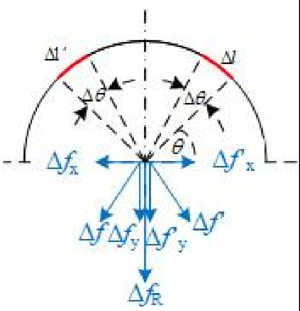
N.B.: The forces $F_1 = F_2 = F$ on particle A and B respectively, would produce equal and opposite forces on particle C having charge q. *Hence, it is not necessary to clamp charged particle C and in the final solution all the three charged particles charges would stay at rest, in equilibrium of forces.*

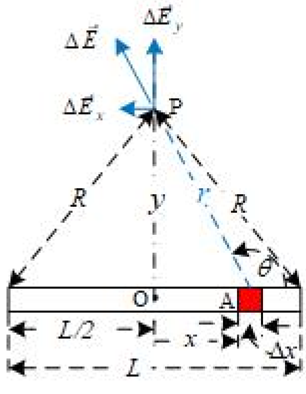
Q-13

A particle A having a charge 2.0×10^{-6} C and a mass 100 g is placed at the bottom of a smooth inclined plane of inclination 30^0. Where should another particle B, having same charge and mass be placed on the incline so that it may remain in equilibrium?

A-13	27 cm from bottom
I-13	Given that two particles A and B have same mass $m = 0.1\text{kg}$ and carrying same charge $q = 2.0 \times 10^{-6}$ C are placed on incline at an angle $\theta = 30^0$. Particle A is on the bottom of the inline and particle B is up on the incline, as shown in the figure. It is required to find distance d when mass B stays in equilibrium.  Physics involved in the problem is as under – (a) Mass A is on the ground and offer equal and opposite reaction to forces acting on it. Thus it stays at rest (b) Electrostatic repulsive-force $F = \frac{q\times q}{4\pi\varepsilon_0 d^2}$ …(1), on B as per Coulomb's Law tends to move it up is the electrostatic force as per Coulomb's law. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times 10^9)$ Nm^{2}C^{-2}, and separation with charged particle $d = 0.1$ m. (c) The inclined plane being smooth cannot cause restraining force on B to maintain equilibrium. (d) Weight of the ball B is $W = mg$ has its component $F_g = W\cos(90-\theta) \Rightarrow F_g = mg\sin\theta$ …(2). It is equal acts in direction opposite to F to stay it in equilibrium. Thus combining (1 & 2) $\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{d^2} = mg\sin\theta$. Using the available data $(9\times10^9)\times\frac{(2.0\times10^{-6})^2}{d^2} = 0.1\times10\times\sin 30^0$. It simplifies to $d^2 = 7.2\times10^{-2} \Rightarrow d = 2.68\times10^{-1}$m say **27 cm is the answer**.
Q-14	Two particles A and B, each having a charge Q are placed at a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of the maximum force?
A-14	$\frac{d}{2\sqrt{2}}$; $\boldsymbol{\frac{4qQ}{3\sqrt{3}\pi\varepsilon_0 d^2}}$ or $3.08\times\frac{Qq}{4\pi\varepsilon_0 d^2}$
I-14	Given system is shown in figure and it is symmetrical about line PC with two charges Q at A and B separated at a distance d and another charge q at a distance on perpendicular bisector of line AB at a distance y from P. Geometrically C is equidistant from A and B. It is required to find y such that force acting on charge q is maximum.  Distance of charge q at point C from charges Q at points A and B is $r = \sqrt{y^2+\left(\frac{d}{2}\right)^2} \Rightarrow r = \frac{\sqrt{d^2+4y^2}}{2}$ and $\alpha = 90^0-\theta$ while $\sin\theta = \frac{y}{r} = \frac{d}{2r}$ and $\cos\theta = \frac{d}{2r}$. As per Coulomb's Law magnitude of forces on charge q at C due to charges Q at A and B are of magnitude $F = \frac{Q\times q}{4\pi\varepsilon_0 r^2}$. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times10^9)$ Nm^{2}C^{-2}. Resolution of the two force vectors parallel to AB are of magnitude $F\cos\theta$ but in opposite directions with zero resultant. But, the resolution of vectors along PC is equal and additive such that $F_R = 2F\sin\theta = 2\times\frac{Q\times q}{4\pi\varepsilon_0 r^2}\times\frac{y}{r}$. It simplifies to $F_R = 2F\sin\theta = \frac{qQy}{2\pi\varepsilon_0 r^3} \Rightarrow$ $F_R = \frac{qQy}{2\pi\varepsilon_0\left(\frac{\sqrt{d^2+4y^2}}{2}\right)^3} = \frac{4qQ}{\pi\varepsilon_0}\times\frac{y}{(d^2+4y^2)^{\frac{3}{2}}}$…(1) It is seen that in F_R the only variable is y therefore for

	maximum value of F_R . As per differential calculus if $\frac{dF_R}{dy}=0$ then F_R may be maximum or minimum, taking value of y so arrived at and using it in $\frac{d^2F_R}{dy^2}=\frac{d}{dy}\left(\frac{dF_R}{dy}\right)$ if it is (-)ve then F_R is maximum. Observation figure and (1) reveals that minimum value of $F_R=0$. And, when charge q is either at P, $y=0\Rightarrow F_R=0$ or $y\to\infty\Rightarrow(d^2+4y^2)^{\frac{3}{2}}\gg y\Rightarrow F_R=0$. Therefore, taking $\frac{dF_R}{dy}=0$, and if either $y\neq 0$ or $y\neq\infty$, then the value of y arrived at would correspond to maximum value of F_R. With little of differential calculus performing differentiation in two stages $\frac{d}{dy}\frac{y}{(d^2+4y^2)^{\frac{3}{2}}}=0$, it leads to Substituting $u=d^2+4y^2\Rightarrow\frac{du}{dy}=8y$. Accordingly, $\left(\frac{d}{du}u^{-\frac{3}{2}}\times\frac{du}{dy}\right)y+u^{-\frac{3}{2}}\times 1=0$. It further simplifies to $\left(-\frac{3}{2}y\times u^{-\frac{5}{2}}\times 8y\right)+u^{-\frac{3}{2}}=0\Rightarrow 1-\frac{12y^2}{d^2+4y^2}=0\Rightarrow y^2=\left(\frac{d}{8}\right)^2\Rightarrow y=\pm\frac{d}{2\sqrt{2}}$...(2) Combining (1 & 2), $F_R=\frac{4qQ}{\pi\varepsilon_0}\times\frac{y}{(d^2+4y^2)^{\frac{3}{2}}}=\frac{4qQ\times\frac{d}{2\sqrt{2}}}{\pi\varepsilon_0\left(d^2+4\left(\frac{d}{2\sqrt{2}}\right)^2\right)^{\frac{3}{2}}}=\frac{\sqrt{2}qQ}{\pi\varepsilon_0 d^2}\times\frac{1}{\left(1+\frac{1}{2}\right)^{\frac{3}{2}}}\Rightarrow F_R=\boldsymbol{\frac{4qQ}{3\sqrt{3}\pi\varepsilon_0 d^2}}$...(3). In electrostatics $\left(\frac{1}{4\pi\varepsilon_0}\right)$ occurs as a generic coefficient hence (3) is resolved as It solves into $F_R=\left(\frac{4\times 4}{3\sqrt{3}}\right)\frac{qQ}{4\pi\varepsilon_0 d^2}=\boldsymbol{3.08\times\frac{qQ}{4\pi\varepsilon_0 d^2}}$ **is more appropriate answer.** **N.B.:** (a) Presenting answer in more general form $3.08\times\frac{qQ}{4\pi\varepsilon_0 d^2}$ is more appropriate than as derived $\frac{4qQ}{3\sqrt{3}\pi\varepsilon_0 d^2}$. (b) Units are not used in the answer being in algebraic form.
Q-15	Two particles A and B each having a charge Q are held fixed with a separation d between them. A particle C having a mass m and a charge q is kept at the middle point of the line joining A,B . (a) If it is displaced through a distance x, along the line AB, what would be the electric force experienced by it. (b) Assuming $x\ll d$ show that this force is proportional to x. (c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement as at (b)? Find the time period of the oscillations under conditions stipulated at (c).
A-15	(a) $\frac{qQd}{2\pi\varepsilon_0}\times\frac{x}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}$ **(b) Proved** **(c)** Time period $=\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$
I-15	Given system is shown in figure and it is symmetrical about line PC with two charges Q at A and B separated at a distance d and another particle of mass m and charge q is placed at C, a midpoint of A and B. Particle C is a displaced by x from the midpoint, thus its distance from particle A becomes $\left(\frac{d}{2}-x\right)$ and from particle B is $\left(\frac{d}{2}-x\right)$. The problem is analysis of forces and motion of the particle C when it is displaced. 

	As per Coulomb's Law magnitude of forces on charge q at C due to charges Q at A and B are of magnitude $F=\frac{Q\times q}{4\pi\varepsilon_0 r^2}$ …(1) Here, $\frac{1}{4\pi\varepsilon_0}=(9\times 10^9)$ Nm^{2}C^{-2} . All charges expressed algebraically are taken to be (+)ve and hence forces exerted on particles C by charges on particles A and B are repulsive, but in opposite directions. Therefore, only forces due to charged particles A and B are considered in the analysis of each part as under - **Party (a):** When particle is displaced by a distance x towards from the midpoint, net force is $F_x=F_A'-F_B'$. Accordingly, as per (1) $F_x=\frac{Q\times q}{4\pi\varepsilon_0\left(\frac{d}{2}-x\right)^2}-\frac{Q\times q}{4\pi\varepsilon_0\left(\frac{d}{2}+x\right)^2}=\frac{qQ}{4\pi\varepsilon_0}\times\frac{2dx}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}\Rightarrow F_x=\frac{qQd}{2\pi\varepsilon_0}\times\frac{x}{\left(\left(\frac{d}{2}\right)^2-x^2\right)^2}$ **is the answer of part (a).** **Part (b):** When $x\ll d\Rightarrow x\ll\frac{d}{2}\Rightarrow x^2\ll\left(\frac{d}{2}\right)^2\Rightarrow\left(\left(\frac{d}{2}\right)^2-x^2\right)\to\frac{d^4}{16}$. Applying the limiting value in expression of F_R arrived at part (a) we have$F_x=\frac{qQd}{2\pi\varepsilon_0}\times\frac{x}{\frac{d^4}{16}}\Rightarrow F_x=\left(\frac{8qQ}{\pi\varepsilon_0 d^3}\right)x$. In this expression except x all other values are constant. Therefore, $F_R\propto x$ **is proved.** **Part (c):** When particle C is moved towards particle A net force on particle C is towards particle B, i.e. it in direction opposite to the displacement. Thus this together with derivation in part (b) satisfies necessary conditions for Simple Harmonic Motion. Acceleration of a particle performing SHM at any displacement y is $F_x=m\omega^2 x$,. Combining results in Part(a) and (b), we have $\omega^2=\frac{8qQ}{\pi\varepsilon_0 d^3 m}$, here $\omega=2\pi f=\frac{2\pi}{T}$. It leads to $\left(\frac{2\pi}{T}\right)^2=\frac{8qQ}{\pi\varepsilon_0 d^3 m}\Rightarrow T=\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$ **is the answer.** **N.B.:** In part (d) answer is not written as $\left(\frac{2\pi}{T}\right)^2=\frac{8qQ}{\pi\varepsilon_0 d^3 m}\Rightarrow T=\left(\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}\right)^{\frac{1}{2}}=\pm\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$. Since, time period is always positive as much as surd $\sqrt{\frac{\pi^3\varepsilon_0 d^3 m}{2qQ}}$ is. In either case absolute value remain the same, but meaning of sign makes a difference.
Q-16	A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the center of curvature of the semicircle.
A-16	$\frac{Q}{2\pi L^2}$
I-16	Given is a semi-circular ring made of a rod of length L is carrying a uniformly distributed positive charge Q. It is required to find electric field at the center of curvature O of the semicircle.  As per Coulomb's Law force between two charges Q and q is $\vec{F}=\frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$. And electric field due to charge is force caused by it on unit positive charge. Accordingly, $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$…(1) The system, as given, is shown in the figure with detailing for the purpose of analysis. Let a small elemental arc of length $\Delta l=R\Delta\theta$ is considered in first quadrant, here radius of curvature of the semicircle is $R=\frac{L}{\pi}$. Charge on the element would be $\Delta Q=\frac{Q}{\pi R}\times R\Delta\theta=\frac{Q}{\pi}\Delta\theta$. Thus field $\Delta\vec{E}=\Delta\vec{E}_x+\Delta\vec{E}_y$…(2).

	Component of force $\vec{E}_x$ is along axis $-\hat{\imath}$ while the component $\vec{E}_y$ is perpendicular to it. Likewise, another complementary element $\Delta l'$, in second quadrant, causes force on the particle $\Delta\vec{E'} = \Delta\vec{E'}_x + \Delta\vec{E'}_y$…(3). Components along X-axis $\Delta\vec{E}_x + \Delta\,\vec{E'}_x = 0$ …(4) being equal and opposite cancel out each other. While components along Y-axis are equal and unidirectional $\Delta\vec{E}_y = \Delta\vec{E'}_y$…(5). Thus on combining (2,4, 5 & 6), $\Delta\vec{E}_R = 2\Delta\vec{E}_y = 2\Delta E\cos\theta\,(-\hat{\jmath})$…(6). Here, using (1) $\Delta\vec{E} = \frac{\Delta Q}{4\pi\varepsilon_0 R^2}(-\hat{\jmath}) = \frac{\left(\frac{Q}{\pi}\Delta\theta\right)}{4\pi\varepsilon_0 R^2}(-\hat{\jmath}) = \frac{Q}{4\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath})$. Accordingly, using these values in (6) $\Delta f_R = 2\times\frac{Q\cos\theta}{4\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath}) = \frac{Q\cos\theta}{2\pi^2\varepsilon_0 R^2}\Delta\theta(-\hat{\jmath})$ Therefore, net electric field $\vec{E}_R = \int d\vec{f}_R = (-\hat{\jmath})\int_0^{\frac{\pi}{2}}\frac{Q\cos\theta}{2\pi^2\varepsilon_0 R^2}d\theta \Rightarrow \vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta$. It is to be noted that since each element Δl is being considered with complementary element Δl ‘ and therefore limit of integration is taken from 0 to $\frac{\pi}{2}$. It simplifies to using $\vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}[\sin\theta]_0^{\frac{\pi}{2}} = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}[1-0] = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0 R^2}$. Substituting the value of $R = \frac{L}{\pi}$, we have $\vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0\left(\frac{L}{\pi}\right)^2} \Rightarrow \vec{E}_R = (-\hat{\jmath})\frac{Q}{2\pi^2\varepsilon_0\left(\frac{L}{\pi}\right)^2} = (-\hat{\jmath})\frac{Q}{2\varepsilon_0 L^2}$. . Thus magnitude of electric field at the center is $E_R = \boldsymbol{\frac{Q}{2\varepsilon_0 L^2}}$ **is the answer**.
Q-17	A 10 cm long rod varies a charge +50 μC is distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both ends of the rod.
A-17	5.2×10^7 N/C
I-17	Given is rod of length $L = 0.10$ carries a charge $Q = 50\times10^{-6}$C. It is required to find electric field which is at a distance $R = 0.10$ m from both the ends of the rod. As per Coulomb’s Law force between two charges Q and q is $\vec{F} = \frac{Q\times q}{4\pi\varepsilon_0 r^2}\hat{r}$ …(1). And electric field due to charge is force caused by it on unit positive charge. Accordingly, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$. Here, $\frac{1}{4\pi\varepsilon_0} = (9\times10^9)$ Nm^{2}C^{-2}.  The system, as given, is shown in the figure with detailing for the purpose of analysis. Charge on the element of length $\Delta x \to 0$ is $\Delta Q = \frac{Q}{L}\times\Delta x$…(2). Thus electric field at point P is force $\Delta\vec{E} = \Delta\vec{E}_x + \Delta\vec{E}_y = E\cos\theta\,\hat{\imath} + E\sin\theta\,\hat{\jmath}$…(3). Geometrically, distance of point P, equidistant from both ends of the rod. from the midpoint of the rod O is $y = \sqrt{R^2 - \left(\frac{L}{2}\right)^2} = \frac{\sqrt{4R^2-L^2}}{2}$…(4) And field at point P due an element at A is $\Delta\vec{E} = \frac{\Delta Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\Delta Q}{4\pi\varepsilon_0(x^2+y^2)}\hat{r}$, since $r = \sqrt{x^2+y^2}$. And as per (3) it is $\Delta\vec{E} = \frac{\left(\frac{Q}{L}\times\Delta x\right)\cos\theta}{4\pi\varepsilon_0(x^2+y^2)}\hat{\imath} + \frac{\left(\frac{Q}{L}\times\Delta x\right)\sin\theta}{4\pi\varepsilon_0(x^2+y^2)}\hat{\jmath}$…(5). Net field at P requires integration of (4), but here $\Delta E = f(x,\theta)$ where $\theta < 90^0$ appears in the form of trigonometric ratios such that $\cos\theta =$

$\frac{-x}{r} = -\frac{x}{\sqrt{x^2+y^2}}$ and $\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$. Accordingly, $\Delta\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left(-\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{\imath} + \frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{\jmath}\right)\Delta x$ is in one variable x. Therefore, $\vec{E} = \int d\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left[\left\{\int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{y}{(x^2+y^2)^{\frac{3}{2}}}dx\right\}\hat{\jmath} - \left\{\int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx\right\}\hat{\imath}\right] = \frac{Q}{4\pi\varepsilon_0 L}[I_1\hat{\jmath} + I_2\hat{\imath}]_{-\frac{L}{2}}^{\frac{L}{2}}$. …(6)

Here, *it involves a bit of calculus*, which is first solved as indefinite integral and limits of definite integration shall be used in its final form.

(a) $I_1 = y\int\frac{1}{(x^2+y^2)^{\frac{3}{2}}}dx$. Substitute $x = y\cot(\pi-\theta) = -\cot\theta \Rightarrow dx = -(-y\,\mathrm{cosec}^2\,\theta\, d\theta) = y\,\mathrm{cosec}^2\,\theta\, d\theta$. It leads to $I_1 = y\int\frac{y\,\mathrm{cosec}^2\,\theta}{(y^2\cot^2\theta+y^2)^{\frac{3}{2}}}d\theta = \frac{1}{y}\int\frac{\mathrm{cosec}^2\,\theta}{\mathrm{cosec}^3\,\theta}d\theta = \frac{1}{y}\int\sin\theta\, d\theta = -\frac{1}{y}\cos\theta = -\frac{1}{y}\times\frac{\cot\theta}{\sqrt{1+\cot^2\theta}} = -\frac{1}{y}\times\frac{\frac{x}{y}}{\sqrt{1+\left(\frac{x}{y}\right)^2}} = -\frac{x}{yr}$. Accordingly, $I_1' = [I_1]_{\frac{L}{2}}^{-\frac{L}{2}} = \frac{1}{yr}[-x]_{\frac{L}{2}}^{-\frac{L}{2}} = \frac{1}{yr}[x]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{1}{yr}\left[\frac{L}{2} - \left(-\frac{L}{2}\right)\right] = \frac{L}{yr}$…(7)

(b) $I_2 = \int\frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx$. This an integration of an even function and its definite integral within limit $-\frac{L}{2} \le x \le \frac{L}{2}$ is zero. On its definite integral within limit $-\frac{L}{2} \le x \le \frac{L}{2}$ is zero. Accordingly, $I_2' = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x}{(x^2+y^2)^{\frac{3}{2}}}dx = 0$…(8)

Combining (4),(6), (7) and (8) we have, $\vec{E} = \frac{Q}{4\pi\varepsilon_0 L}\left(\frac{L}{yr}\right)\hat{\jmath} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{\left(\sqrt{r^2-\left(\frac{L}{2}\right)^2}\right)r}$…(9). Using (4)On substituting the available data, magnitude of electric field at point P is $E = \left(9\times10^9\right)\left(\frac{50\times10^{-6}}{0.1\times\sqrt{0.75}}\right) = 4.5\times10^5$N/C is the answer. 5.2×10^7 N/C

N.B.: This problem being a little simple, electric field is determined without simplification of analysis using symmetry of the geometry. It is good case study for students to practice for gaining analytical proficiency. However, using geometrical symmetry for simplification of illustration and ease of understanding is generally used.

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