

LET US DO SOME PROBLEMS IN Mathematics –XXVII

SB Dhar

Some beautiful questions for students of all standards are selected here from different sources with gratitude. Some questions are from the world-wide-mathematics competitive examinations too. These questions will need some extra efforts to reach their solutions. The solutions are not being written here except the answer. If some student wants the solution, the request may be sent to the coordinator’s desk for that.

QUESTIONS

Q1. What is the value of M in the following numbers sum?

MAM
MAT
MTT
1416

Ans. 4

Q2. The numbers like 121, 1221, 9889,...etc., that are same on reading forward or backward, are called Palindrome Numbers. If you have a 24-hour digital clock, then how many Palindrome Numbers will be there in a day when there are only Hour and Minute Indicators (i.e. 00.00)?

Ans. 16

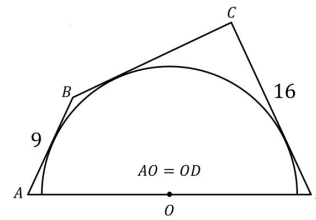
Q3. There are row-houses in a street. They are numbered as 1, 2, 3, 4, ... consecutively. You live in a certain house. The number of your house is such that the sum of all the house numbers to the left side of your house is equal to the sum of all the house numbers to the right of your house. What is the number of your house and the total number of houses if the number of total houses is between 1 and 9?

Ans.6

Q4. If two days before yesterday was Friday, what day will be the day after tomorrow?

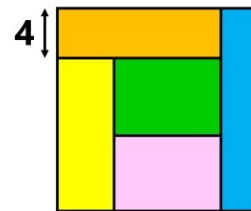
Ans. Thursday

Q5. ABCD is a quadrilateral. Point O is along AD. A semicircle has a diameter along AD and is tangent to the sides AB, BC, and CD. If AB = 9, CD = 16, and AO = OD. Find the length of AD.



Ans. 24

Q6. A square is divided into 5 rectangles. Each rectangle has the same area. The top left rectangle has a width equal to 4, as shown. What is the area of the square?



Ans.256

Q7. A father said to his son, “Two years ago I was three times as old as you; but in fourteen years I shall be only twice as old as you.” What were the ages of each?

Ans. The father is 50 and the son is 18.

Q8. Two farmers brought a total of 100 eggs to market. Each farmer set a different price to sell his eggs. Farmer A is selling each egg at a price of a and the Farmer B is selling each egg at a price of b . After selling all eggs, each farmer received as much money as the other.

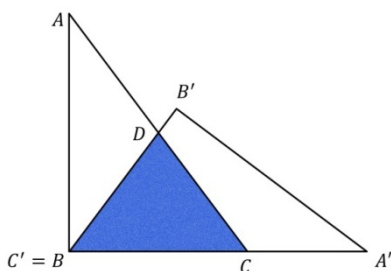
The Farmer A remarked, "If I had sold your eggs at my price, I would have gotten 15 coins."

The Farmer B replied, "If I had sold your eggs at my price, I would have gotten $20/3$ coins."

How many eggs did each farmer sell?

Ans. Farmer A had 40 eggs and Farmer B had 60 eggs. Each farmer would have received 10 coins from selling the eggs.

Q9. Right triangle ABC is rotated and translated to $A'B'C'$, as shown below, with B' exterior to ABC . What is the area of the shaded region BDC divided by the area of the entire shape $BADB'A'B'$?



Ans. $1/3$

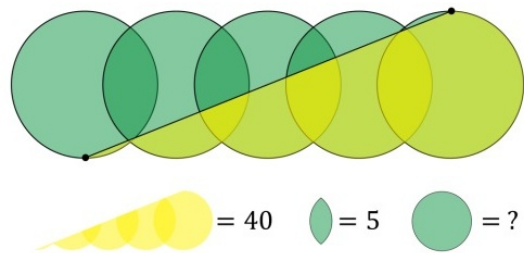
Q10. Solve: $2\sqrt[3]{2x+1} = x^3 - 1$

Ans. $x = -1, x = \frac{1+\sqrt{5}}{2}, x = \frac{1-\sqrt{5}}{2}$

Q11. I bought 72 identical items. Each item had the same cost, and the cost was a whole number of Rupees. The total cost was *679* (we do not know the first or last digit). How much did each item cost?

Ans. 511

Q12. Five congruent circles are placed with their centers equally spaced and collinear as shown. A line connects the bottom of the first circle and the top of the fifth circle. The area under the line enclosed by the circles, shown in yellow, is equal to 40. The overlapping area between two circles is 5. What is the area of a single circle?



Ans. 20

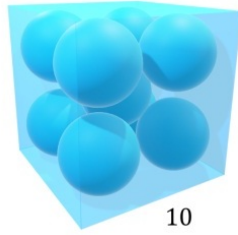
Q13. I am thinking of a two-digit number. If I write 3 to the left of my number, and double this three-digit number, the result is 27 times my original number. What is my number?

Ans. 24

Q14. If $3 + 5 = 16, 4 + 7 = 30$, then find the value of $1 + 1$.

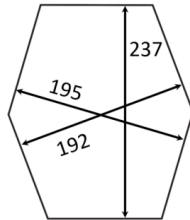
Ans. -12

Q15. A cube contains 9 identical spheres, as shown below. There is one sphere in the center of the cube. Above (and below) the center sphere are 4 spheres tangent to the center sphere and the 4 corners of the cube. What is the radius of each sphere?



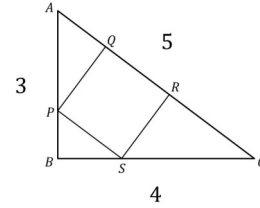
Ans. Approx 2.32

Q16. A hexagon has six equal sides. The opposite sides are parallel. The distances between the parallel sides are 237, 195, and 192, as shown. Find the area of the hexagon.



Ans. 39000

Q17. A triangle with sides $AB = 3$, $BC = 4$, and $AC = 5$ has an inscribed square $PQRS$ with side QR along the side AC . Find the area of the square.



Ans. Approx 2.63

Q18. Evaluate:

$$\frac{e}{\sqrt{e}} \cdot \frac{\sqrt[3]{e}}{\sqrt[4]{e}} \cdot \frac{\sqrt[5]{e}}{\sqrt[6]{e}} \cdot \frac{\sqrt[7]{e}}{\sqrt[8]{e}} \dots \text{upto infinity.}$$

Ans. 2

Q19. Let Z be the set of integers. Determine all functions $f : Z \rightarrow Z$ such that, for all integers a and b , $f(2a) + 2f(b) = f(f(a+b))$.

Ans. So the function is either $f(x)=0$ or $f(x)=2x+n$ for all constant n .

Q20. Simplify:

$$(x - 1)(x - 2)(x - 3) \dots (x - n)$$

Ans. $\prod_{k=0}^n (x - k)$ or $\frac{x!}{(x-n-1)!}$