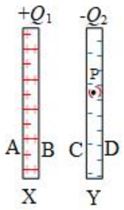
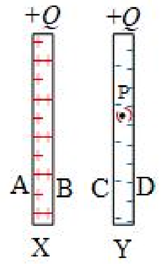
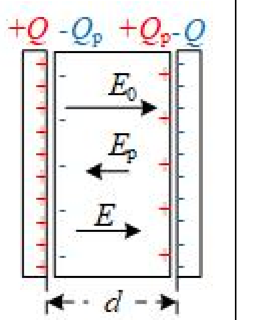
Electromagnetism: Electrostatics -Capacitors

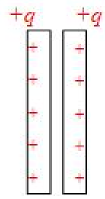
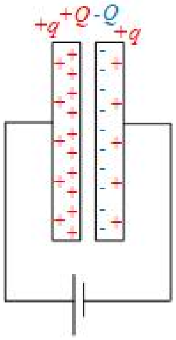
(Selected Questions)

Important Note: Capacitors are implementation aspect of concepts of electrostatics. The capacitors are integral part of any electrical system or circuit and any kind of application of electricity.

Abbreviations: Q- Question, A- Answer, I – Illustration to the solution

Q-1	Suppose a charge $+Q_1$ is given to the positive plate and $-Q_2$ is given to the negative plate of a capacitor. What is the charge on the capacitor.
A-1	$\frac{Q_1+Q_2}{2}$
I-1	Given system is conceptualized in the figure. The plate X containing charge $+Q_1$ has two faces, outer surface is A and inner surface is B. Likewise, the plate Y containing charge $-Q_2$ has two faces, outer surface is C and inner surface is D. Charge on the capacitor is the charge on the two surfaces facing each other.  In the instant case these surfaces are B and C. Let, surface B has charge $Q_{\mathrm{B}} = +q \Rightarrow \rho_{\mathrm{B}} = \frac{Q_{\mathrm{B}}}{A} = \frac{q}{A}$ an accordingly corresponding complementary charge on surface C is $Q_{\mathrm{C}} = -q \Rightarrow \rho_{\mathrm{C}} = \frac{Q_{\mathrm{C}}}{A} = -\frac{q}{A}$. The system is standalone i.e. not connected to either a battery or any other material, therefore, after charge distribution on plates charge on the surfaces A and D is $Q_{\mathrm{A}} = +Q_1 - (+q) = Q_1 - q \Rightarrow \rho_{\mathrm{A}} = \frac{Q_{\mathrm{A}}}{A} = \frac{Q_1-q}{A}$, and $Q_{\mathrm{D}} = -Q_2 - (-q) = -(Q_2 - q) \Rightarrow \rho_{\mathrm{D}} = \frac{Q_{\mathrm{D}}}{A} = \frac{-(Q_2-q)}{A}$. Now, that capacitor plates are of conductive materials and hence charges reside on the surfaces of the plates and inside the plates electric potential is Zero. Taking a point P inside any of the plate say Y of the capacitor, between surfaces C and D. Therefore, primarily electric field at the point $\vec{E}_{\mathrm{P}} = 0 \ldots(1)$. This is can be equated to electric field by charges on the four surfaces. As per Gauss's law electric field vector due to a charged surface is $\vec{E} = \frac{\rho}{\varepsilon_0}\hat{z}$, here $\hat{z}$ is direction vector perpendicular to the surface joining the point under consideration. Accordingly, $\vec{E}_{\mathrm{A}} = \frac{\rho_{\mathrm{A}}}{\varepsilon_0}\hat{z} = \frac{Q_1-q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{B}} = \frac{\rho_{\mathrm{B}}}{\varepsilon_0}\hat{z} = \frac{q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{C}} = \frac{\rho_{\mathrm{C}}}{\varepsilon_0}\hat{z} = -\frac{q}{\varepsilon_0 A}\hat{z}$ and $\vec{E}_{\mathrm{D}} = \frac{\rho_{\mathrm{D}}}{\varepsilon_0}(-\hat{z}) = -\frac{Q_2-q}{\varepsilon_0 A}(-\hat{z}) = \frac{Q_2-q}{\varepsilon_0 A}\hat{z}$. Thus, analytically net electric field at P is $\vec{E}_{\mathrm{P}} = \vec{E}_{\mathrm{A}} + \vec{E}_{\mathrm{B}} + \vec{E}_{\mathrm{C}} + \vec{E}_{\mathrm{D}} \Rightarrow \vec{E}_{\mathrm{P}} = \frac{Q_1-q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + -\frac{q}{\varepsilon_0 A}\hat{z} + \frac{Q_2-q}{\varepsilon_0 A}\hat{z} \ldots(2)$. Combining (1) and (2), $\frac{Q_1-q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + \left(-\frac{q}{\varepsilon_0 A}\hat{z}\right) + \frac{Q_2-q}{\varepsilon_0 A}\hat{z} = 0 \Rightarrow (Q_1 - q) + (Q_2 - q) = 0 \Rightarrow q = \mathbf{\frac{Q_1+Q_2}{2}}$, **is the answer.** **N.B.:** It is an interesting application of Gauss's law in capacitors.
Q-2	The plates of a parallel plate capacitor are given positive charges. What will be the potential difference between the plates? What will be the charge on the facing surfaces and on the outer surfaces?
A-2	Zero, Q
I-2	Given system is conceptualized in the figure. The plate X and Y are given equal charge say $+Q$. The two faces of X, outer surface is A and inner surface is B. Let, B the inner surface has charge $Q_{\mathrm{B}} = q$ and therefore the outer surface A will have balance of the given charge $Q_{\mathrm{A}} = Q - q$. Thus, by electrostatic induction the inner surface C of the plate Y will have charge $Q_{\mathrm{C}} = -q$ and the surface D shall have balance charge $Q_{\mathrm{D}} = Q - (-q) = Q + q$.

	The system is standalone i.e. not connected to either a battery or any other material, therefore,, surface charge densities platers of the capacitor, having each surface area A, which are essentially conductive will be $\rho_{\mathrm{B}} = \frac{Q_{\mathrm{B}}}{A} = \frac{q}{A}$, $\rho_{\mathrm{A}} = \frac{Q_{\mathrm{A}}}{A} = \frac{Q-q}{A}$, $\rho_{\mathrm{C}} = \frac{Q_{\mathrm{C}}}{A} = -\frac{q}{A}$ and $\rho_{\mathrm{D}} = \frac{Q_{\mathrm{D}}}{A} = \frac{Q+q}{A}$.  Now, that capacitor plates are of conductive materials and hence charges reside on the surfaces of the plates and inside the plates electric potential is Zero. Taking a point P inside any of the plate say Y of the capacitor, between surfaces C and D. Therefore, primarily electric field at the point $\vec{E}_{\mathrm{P}} = 0 \ldots(1)$. This is can be equated to electric field by charges on the four surfaces. As per Gauss's law electric field vector due to a charged surface is $\vec{E} = \frac{\rho}{\varepsilon_0}\hat{z}$, here $\hat{z}$ is direction vector perpendicular to the surface joining the point under consideration. Accordingly, $\vec{E}_{\mathrm{A}} = \frac{\rho_{\mathrm{A}}}{\varepsilon_0}\hat{z} = \frac{Q-q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{B}} = \frac{\rho_{\mathrm{B}}}{\varepsilon_0}\hat{z} = \frac{q}{\varepsilon_0 A}\hat{z}$, $\vec{E}_{\mathrm{C}} = \frac{\rho_{\mathrm{C}}}{\varepsilon_0}\hat{z} = -\frac{q}{\varepsilon_0 A}\hat{z}$ and $\vec{E}_{\mathrm{D}} = \frac{\rho_{\mathrm{D}}}{\varepsilon_0}(-\hat{z}) = \frac{Q+q}{\varepsilon_0 A}(-\hat{z}) = -\frac{Q+q}{\varepsilon_0 A}\hat{z}$. Thus, analytically net electric field at P is $\vec{E}_{\mathrm{P}} = \vec{E}_{\mathrm{A}} + \vec{E}_{\mathrm{B}} + \vec{E}_{\mathrm{C}} + \vec{E}_{\mathrm{D}} \Rightarrow \vec{E}_{\mathrm{P}} = \frac{Q-q}{\varepsilon_0 A}\hat{z} + \frac{q}{\varepsilon_0 A}\hat{z} + \left(-\frac{q}{\varepsilon_0 A}\hat{z}\right) + \frac{Q+q}{\varepsilon_0 A}(-\hat{z}) \ldots(2)$. Combining (1) and (2), $(Q-q)+q-q-(Q+q)=0 \Rightarrow 2q=0 \Rightarrow q=0$**,** is the charge on the inner plates of the capacitor (i.e.) charge on the capacitor is **Zero is the answer.** Accordingly, the parts is charge on the outer surfaces of the capacitors is ***Q*****, is the answer.** **Hence, answers are Zero, *Q*.** **N.B.:** It is an interesting application of Gauss's law in capacitors. But, qualitatively the like charges would repel each other leading to net charge appearing on the outer surfaces. Thus, this can be framed as an object question also.
Q-3	A dielectric slab is inserted between the plates of a capacitor. The charge on the capacitor is Q and magnitude of the induced charge on the each surface of the dielectric is Q_p (a) Q_p may be larger than Q (b) Q_p must be larger than Q (c) Q_p may be equal to Q (d) Q_p must be smaller than Q
A-3	(d)
I-3	Given is a capacitor of capacitance $C = \frac{\varepsilon_0 A}{d}$, having charge Q on it. Here, A is the area of plates of the capacitor and d is separation between the plates. Therefore, voltage across the capacitor will be $Q = CV \Rightarrow V = \frac{Q}{C} = \frac{Q}{\frac{\varepsilon_0 A}{d}} \Rightarrow V = \frac{Qd}{\varepsilon_0 A} \Rightarrow E_0 = \frac{V}{d} \Rightarrow E_0 = \frac{Q}{\varepsilon_0 A}$..(1).  Now a dielectric slab of dielectric constant $K = \varepsilon_r \ldots(2)$, here ε_r is the relative permittivity of the dielectric slab inserted between plates of the capacitor. Let this electric field would cause polarization in dielectric inducing a charge Q_p as shown in the figure. Accordingly, an electric field $E_p = \frac{Q_p}{\varepsilon_0 A} \ldots(3)$, develops within the dielectric and it is in a direction opposite to E_0 as shown in the figure. Thus combined effect of Q charge on plates of the capacitor an induced charge in dielectric Q_p is the electric field within the dielectric $E = E_0 - E_p \ldots(4)$. Combining (1) and (3) in (4), $E = \frac{Q}{\varepsilon_0 A} - \frac{Q_p}{\varepsilon_0 A} \Rightarrow E = \frac{Q-Q_p}{\varepsilon_0 A} \ldots(5)$. Net electric fields between the plates, in presence of dielectric filling the gap between the plates, is $E = \frac{Q}{\varepsilon_0 \varepsilon_r A} \ldots(6)$.

	Combining (5) and (6) $\frac{Q-Q_p}{\varepsilon_0 A}=\frac{Q}{\varepsilon_0\varepsilon_r A}\Rightarrow\frac{Q_p}{\varepsilon_0 A}=\frac{Q}{\varepsilon_0 A}-\frac{Q}{\varepsilon_0\varepsilon_r A}\Rightarrow Q_p=Q\left(1-\frac{1}{\varepsilon_r}\right)$...(7). For dielectrics $\varepsilon_r>1$ and hence $\left(1-\frac{1}{\varepsilon_r}\right)<1$. Accordingly, $Q_p<Q$ is must as provided in **option (d), is the answer**.
Q-4	Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now (a) The facing surfaces of the capacitor have equal and opposite charges (b) The two plates of the capacitor have equal and opposite charges (c) The battery supplies equal and opposite charges to the two plates (d) The outer surfaces of the plates have equal charges
A-4	(a), (c), (d)
I-4	Given that each plate of a parallel-plate capacitor is given a charge q. The like charges would repel each other and would get uniformly distributed on outer surface of metallic plates of the capacitor.   Now the capacitor is connected to a battery as shown in the figure. Battery imparts say $+Q$ charge to one plate and $-Q$ on the other plate. These charges would appear on inner surfaces of the plates as shown in the figure in accordance with the principle of electrical neutraliy. It is observed that – • Inner surfaces of the plates of capacitor have equal and opposite charges as given in **option (a), is correct**. • Charge on left plate of the capacitor is $Q_L=Q+q$ while charge on the right plate is $Q_R=-Q+q$, thus $Q_L\neq Q_R$, thus, **option (b), is incorrect**. • Battery would supply equal and opposite charges $+Q$ and $-Q$ to the two plates of the capacitor, as per the principle of electrical neutrality. Thus, **option (c) is correct**. • Outer surfaces of the plates of the capacitor would continue to carry equal charge q while complying with the electrical neutrality of the battery. Thus, **option (d) is correct.** **Thus, answer is option (a), (c) and (d).**
Q-5	Following operations can be performed on a capacitor. X—Connected the capacitor to a battery of emf E Y- Disconnect the battery Z – Connect the battery with polarity reversed W- Insert a dielectric slab in the capacitor (a) In operations X, Y and Z sequentially the electric energy stored in the capacitor remains unchanged. (b) The charge appearing on the capacitor is greater after X,W and Y than after X,Y and W (c) The electric energy stored in the capacitor is greater after W, X and Y than after X, Y and W. (d) The electric field in the capacitor after X and W is same as that after W and X.
A-5	(b), (c), (d)
I-5	Each of the given option is being analyzed in context of sequence of operation – **Option (a):** A capacitor of capacitance say C prior to connection to a battery of emf E, has zero potential difference across it i.e. $V=0$. Energy stored in the capacitor is $U=\frac{1}{2}CV^2$...(1). Thus initial energy stored in the capacitor is $U_0=0$.(2). Next in operation X, $V\rightarrow E$ and thus $U_1=\frac{1}{2}CE^2$...(3). When operation Y is performed there no change in voltage across the capacitor and hence $U_3=\frac{1}{2}CE^2$...(4). Now, when operation Z is performed, due to reversal of polarity of the battery, first capacitor will discharge and then will be recharged corresponding to reverse polarity

at $U_4 = \frac{1}{2}C(-E)^2 \Rightarrow U_4 = \frac{1}{2}CE^2 \ldots(5)$. Thus, in sequence of operations it is seen that U changes-

- **(i)** from 0 to $U_1 = \frac{1}{2}CE^2$;
- **(ii)** Remains at U_1
- **(iii)** Changes from U_1 to zero and then recovers to U_1.

Thus, changes in energy stored in capacitor makes **option (a) incorrect**

Option (b): Charge on capacitor after operation X is $Q_X = CE \ldots(6)$. In the following operation W, battery remains connected and capacitance increases to $C' = \varepsilon_r C$ and thus charge becomes $Q_W = C'E = \varepsilon_r CE \ldots(7)$. On action Y following W increased charge Q_W continues to be held by the capacitor.

While, operation Y after X, disconnection of battery, retains charge Q_X as per (6) on the capacitor. And operation W following it, changes capacitance $C \to C'$ which in turn affects only voltage across the capacitor. But, on account of disconnected battery, the charge on the capacitor remains at $Q_X = CE$.

Comparing remnant charge $Q_W = \varepsilon_r CE$ in X-W-Y sequence of operations the remnant charge $Q_X = CE$ in X-Y-W sequence of operations and that $\varepsilon_r > 1$, in all certainty $Q_W > Q_X$.

Thus, **option (b) is correct**.

Option (c): In sequence of operations W-X-Y capacitor in presence of dielectric, in operation W, increases to $C' = \varepsilon_r C$. Then in action X capacitor is charged by battery and as a result it stores energy $U' = \frac{1}{2}C'E^2$. Following it action Y retains energy $U' = \frac{1}{2}(\varepsilon_r C)E^2 = \frac{1}{2}\varepsilon_r CE^2$.

Whereas in sequence X-Y-W capacitor stores energy in operation X , $U = \frac{1}{2}CE^2$, and in following operation, battery is disconnected. Therefore, the last action W, does not lead to any change in energy.

Comparing $U' = \frac{1}{2}\varepsilon_r CE^2$ with $U = \frac{1}{2}CE^2$,dielectric constant $\varepsilon_r > 1$, we have $U' > U$. Thus, **option (c) is correct.**

Option (d): Electric field inside capacitor is $E = \frac{V}{d}$, here V is potential difference applied across the capacitor, in the instant case it is battery voltage, and d is the separation between plates of capacitor. In this expression dielectric constant does not appear as long as battery emf remains the same, this is ensured by action X. Thus, sequence of actions X and W does not change electric field in the capacitor. Thus, **option (d) is correct.**

Accordingly, **answer is options (b), (c) and (d)**

N.B.: This is a multiple choice objective question and can be solved with clarity of concepts. A detailed illustration has been made to bridge gaps, if any, in understanding of concepts.

Q-6 A parallel-plate capacitor having plate area 25 cm^2 and separation 1.0 mm is connected to a battery of 6.0 V. Calculate the charge flown through the battery. How much work has been done by the battery during the process?

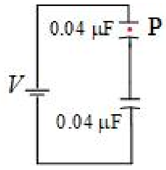
A-6 1.33×10^{-10} C, 8.0×10^{-10} J

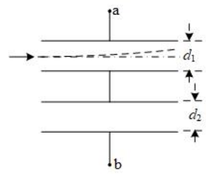
I-6 Capacitance of a parallel plate capacitors is $C = \frac{\varepsilon_0 A}{d} \ldots(1)$, where area of the plates is $A = 25 \times 10^{-4}$ m^2, with a separation $d = 1.00 \times 10^{-3}$m. It is also given that voltage across the capacitor is connected to a battery is $V = 6.0$ V.

Thus, as desired charge on a capacitor is $Q = CV \ldots(2)$. and work done by battery in charging the capacitor is analyzed a little later.

In first part combining (1) and (2), $Q = \left(\frac{\varepsilon_0 A}{d}\right)V = \frac{\varepsilon_0 AV}{d}$. Using the available data with $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $Q = \frac{(8.85\times10^{-12})(25\times10^{-4})6}{1.00\times10^{-3}} = 1.33 \times \mathbf{10^{-10}}$ C, say $\mathbf{1.3 \times 10^{-10}}$ **C is the answer.**

And in second part energy of the capacitor result of part (1) is used. In a capacitor when Δq charge is transferred when potential difference across the capacitor is V' then work done transferring the charge is $\Delta W =$

	$\Delta q \times V'$. Using (2) this transforms into $\Delta W = \Delta q \times \frac{q}{C}$, this equation variable V' is substituted in terms of constant C and the variable charge q under consideration. Accordingly, $W = \int dw = \int_0^Q \left(\frac{q}{C}\right) dq \Rightarrow W = \frac{1}{C}\int_0^Q qdq \Rightarrow W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C} = \frac{1}{2}QV$…(3). It is a general expression of energy stored in a capacitor. But, the problem statement is that charge is flown through the battery of 6.0 V into a capacitor and charge in the instant case is $Q = 1.33 \times 10^{-10}$ C. Therefore, as per principle of electrostatics work done in flowing a unit charge through a potential difference V is $W' = V$. Therefore, for flowing a charge Q through a potential difference V work done by the battery is $W = QV$…(4). This flow of charge is equivalent to time integral of flow of current $Q = \int_{t_1}^{t_2} Idt$. Thus, work done in the instant case is not as per general expression (3), instead it is as per case specific expression (4). Accordingly, W= $(1.33 \times 10^{-10}) \times 6 = \mathbf{8.0 \times 10^{-10}}$ **J, is the answer**. Thus, **answers are $\mathbf{1.33 \times 10^{-10}}$ C, $\mathbf{8.0 \times 10^{-10}}$ J.** **N.B.: 1.** Precision of the problem statement is extremely important, and makes difference in solution. In high level competitions such linguistic twists are used to create a difference. It is excellently demonstrated in this problem. This made it essential to bring out a detailed illustration. **2.** As per principles of significant digits' intermediate results are retained until end and rounding to SDs is done at the last stage. This is becoming clear while using value of C in calculating U, while rounding to SDs is don for reporting both the results.
Q-7	The particle P shown in the figure has a mass of 10 mg and a charge -0.01 μC. Each plate has a surface area 100 cm^2 on one side. What potential difference *V* should be applied to the combination to hold the particle in equilibrium? 
A-7	44 mV
I-7	An observation of the figure shows that a potential difference V is applied across the combination of two capacitors, connected in series, each of capacitance $C = 0.04 \times 10^{-6} = 4.0 \times 10^{-8}$F having surface area of one side of the plates $A = 100 \times 10^{-4}\text{m}^2$. The potential difference would divide equally across the two capacitors such that potential difference across each $V' = \frac{V}{2}$…(1) The particle of mass $m = 10 \times 10^{-6}$kg having a charge $q = -0.01 \times 10^{-6} = -1.0 \times 10^{-8}$ C is to be kept in equilibrium in one of the capacitors as shown in the figure. It is possible only when $\vec{F_e} + \vec{F_g} = 0$…(2). Here, electric force is $\vec{F_e} = \vec{E}q$…(3), and gravitational force $\vec{F_g} = m\vec{g} \Rightarrow \vec{F_g} = -mg\hat{\jmath}$…(4). Here, $\hat{\jmath}$ is unit vector in upward direction. Combining (2), (3) and (4), $\vec{E}q = mg\hat{\jmath}$. Using the available data and taking $g = 10$ m/s², $\vec{E} = \frac{mg\hat{\jmath}}{q} \Rightarrow \vec{E} = -\frac{(10\times10^{-6})\times10}{-1.0\times10^{-8}}\hat{\jmath} \Rightarrow E\hat{E} = 1.0 \times 10^4(-\hat{\jmath}) \Rightarrow E = 1.0 \times 10^4$…(5) Here, negative sign signifies electric field in downward direction and is in agreement with the positive polarity of the battery connected to upper plate of the capacitor. Now, electric field across a capacitor is $E = \frac{V'}{d}$…(6), while capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d} \Rightarrow d = \frac{\varepsilon_0 A}{C}$…(7). Combining (1), (6) and (7), $E = \frac{\frac{V}{2}}{\frac{\varepsilon_0 A}{C}} \Rightarrow E = \frac{VC}{2\varepsilon_0 A}$ …(8). Using the available data with $\varepsilon_0 = 8.85 \times 10^{-12}$,we have $E = \frac{V(4.0\times10^{-8})}{2(8.85\times10^{-12})(100\times10^{-4})} \Rightarrow E = (2.26 \times 10^5)V$…(9). Comparing (5) and (9), $(2.26 \times 10^5)V = 1.0 \times 10^4 \Rightarrow V = 44 \times 10^{-3}$V or **44 mV is the answer**. **N.B.:** Choice of value of g slightly affects numerical value of answer.

Q-8	Both the capacitors shown in the figure are made of square plates of edge a. The separation between the plates of the capacitors are d_1 and d_2 as shown in the figure. A potential difference V is applied between points a and b. An electron is projected between the plates of the upper capacitor along its central line. With what minimum speed should the electron be projected so that it does not collide with any plate? Consider only the electric forces. 
A-8	$\sqrt{\frac{Vq_e a^2}{m_e d_1(d_1+d_2)}}$
I-8	The problem involves multiple concepts- (a) Division of potential difference V applied across the series combination into potential difference $V_1 = \left(\frac{C_2}{C_1+C_2}\right)V$ …(1) is applied across the upper capacitor; (b) Electric field within upper capacitor $E_1 = \frac{V_1}{d_1} \Rightarrow E_1 = \frac{\left(\frac{C_2}{C_1+C_2}\right)V}{d_1} \Rightarrow E_1 = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right)$ …(2); (c) Electric force on the electron when inside the capacitor $F_1 = E_1 q_e \Rightarrow F_1 = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right) q_e$…(3); (d) Acceleration of the electron perpendicular to its projected velocity $a' = \frac{F_1}{m_e} \Rightarrow a' = \left(\frac{C_2}{C_1+C_2}\right)\left(\frac{V}{d_1}\right)\left(\frac{q_e}{m_e}\right)$…(4); it is given that only electric force is to be considered and hence gravitational force is ignored; (e) Time (t) taken by electron to sweep across the side of capacitor plate a , given in the problem, with its horizontal projected velocity v, along the central line of the capacitor shown in the figure, is $t = \frac{a}{v}$…(5); (f) Time taken by the projected electron to just reach the upper plate as per second equation of motion $\frac{d_1}{2} = 0 \times t + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{d_1}{a'}} \Rightarrow t = \sqrt{d_1\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{d_1}{V}\right)\left(\frac{m_e}{q_e}\right)} \Rightarrow t = d_1 \times \sqrt{\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{1}{V}\right)\left(\frac{m_e}{q_e}\right)}$… (6) (g) In order to satisfy the condition that electron projected along the central line of the capacitor does not collide with the upper plate, time in (5) and (6) must be equal, such that- $\frac{a}{v} = d_1 \times \sqrt{\left(\frac{C_1+C_2}{C_2}\right)\left(\frac{1}{V}\right)\left(\frac{m_e}{q_e}\right)} \Rightarrow v = \sqrt{\left(\frac{a}{d_1}\right)^2\left(\frac{C_2}{C_1+C_2}\right)} \times \sqrt{V\left(\frac{q_e}{m_e}\right)}$…(7). (h) Here, $C_1 = \frac{\varepsilon_0 a^2}{d_1}$ and $C_2 = \frac{\varepsilon_0 a^2}{d_2}$ hence, $\frac{C_2}{C_1+C_2} = \frac{\frac{\varepsilon_0 a^2}{d_2}}{\frac{\varepsilon_0 a^2}{d_1}+\frac{\varepsilon_0 a^2}{d_2}} = \frac{d_1}{d_1+d_2}$…(8). Combining (7) and (8), $v = \sqrt{\left(\frac{a}{d_1}\right)^2\left(\frac{d_1}{d_1+d_2}\right)} \times \sqrt{V\left(\frac{q_e}{m_e}\right)} \Rightarrow v = \sqrt{\frac{Vq_e a^2}{m_e d_1(d_1+d_2)}}$ **is the answer**. **N.B.:** This is a good example of integration of multiple concepts.
Q-9	The plates of a capacitor are 2.00 cm apart. An electron-proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as electron reaches the positive plate. At what distance from the negative plate was the pair released?
A-9	1.09×10^{-3} cm
I-9	When a pair of electron and proton are released from some point between the plates of a capacitor having separation $d = 2.00 \times 10^{-2}$m , at a distance say x from a negative plate. Therefore, distance of the pair from positive plates is $x' = d - x$…(1) Charge of electron and proton are equal and opposite say $(-e)$ and $(+e)$respectively. Say potential difference between the plates of the capacitor be V then electric field in the inter-spacing between plates of the capacitor would be $E = \frac{V}{d}$…(2) Therefore, both electron and proton will experience equal and opposite forces of

magnitude $F = eE \Rightarrow F = \frac{eV}{d}$ ...(3), but their accelerations would be $a_e = \frac{F}{m_e} = \frac{\frac{eV}{d}}{m_e} \Rightarrow a_e = \frac{eV}{dm_e}$...(4) and likewise $a_p = \frac{eV}{dm_p}$...(5), respectively.

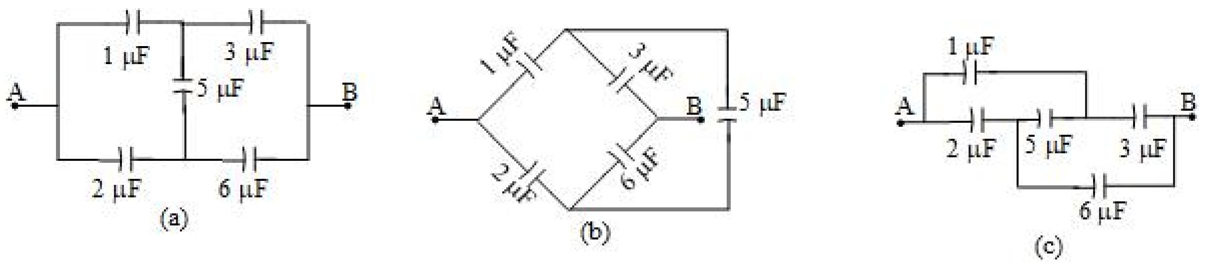
Due to opposite charges, proton would travel distance x towards negative plate in time t while, as per problem statement, in the same time electron would travel a distance x'reach positive plate.

As per Second Equation of Motion, for electron, $x' = d - x = \frac{1}{2}a_e t^2 \Rightarrow t = \sqrt{\frac{2(d-x)}{a_e}} \Rightarrow t = \sqrt{\frac{2(d-x)}{\frac{eV}{dm_e}}} \Rightarrow t = \sqrt{\frac{2(d-x)dm_e}{eV}}$...(6). Likewise, for proton $t = \sqrt{\frac{2xdm_p}{eV}} \Rightarrow t = \sqrt{\frac{2xdm_p}{eV}}$...(7).

Comparing (6) and (7), $\sqrt{\frac{2xdm_p}{eV}} = \sqrt{\frac{2(d-x)dm_e}{eV}} \Rightarrow xm_p = (d-x)m_e \Rightarrow \frac{d-x}{x} = \frac{m_p}{m_e} \Rightarrow \frac{d}{x} = \frac{m_p}{m_e} + 1 \Rightarrow x = \frac{d}{\frac{m_p}{m_e}+1}$...(8). We know that $\frac{m_p}{m_e} = 1836$, accordingly using the available data $x = \frac{2.00\times10^{-2}}{1+1836} \Rightarrow x = \frac{2.00\times10^{-2}}{1837} =$ 1.09×10^{-5}m or $\mathbf{1.09 \times 10^{-3}}$ **cm is the answer**.

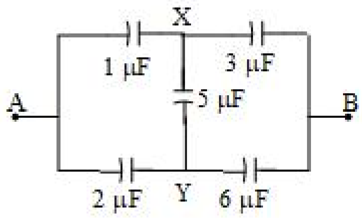
N.B.: This a very good example of delaying numerical calculations till last stage where problem reduces to simple division. Mostly problems in physics are of this type and more require proficiency in handling algebraic formulations.

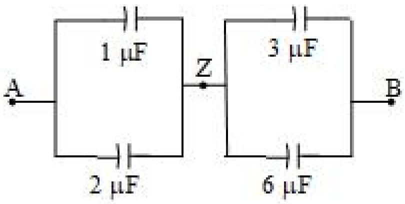
Q-10 Convince yourself that parts (a), (b) an (c) of figure are electrically identical. Find capacitance between points A and B of the assembly.



A-10 2 μF

I-10 Topologically each of the three combinations are identical. And therefore any of the topology can be used for determine capacitance between points A and B. Taking (a), simpler to analyze, ratio of capacitance in upper and lower series combinations $\frac{1}{3} = \frac{2}{6} = k$. Therefore, if potential difference V is applied at A w.r.t. B, then potential at X and Y w.r.t. B would be $V_X = \frac{1}{1+k}V \Rightarrow V_X = \frac{1}{1+\frac{1}{3}}V \Rightarrow V_X = \frac{3}{4}V$. Likewise, potential at Y would be $V_Y = \frac{1}{1+\frac{1}{3}}V \Rightarrow V_Y = \frac{3}{4}V$. Thus, potential difference across points X and Y would be $\Delta V_{X-Y} = V_X - V_Y \Rightarrow \Delta V_{X-Y} = \frac{3}{4}V - \frac{3}{4}V = 0$. Thus, eventually capacitor is a shorted with its equivalent combination as shown in the figure.

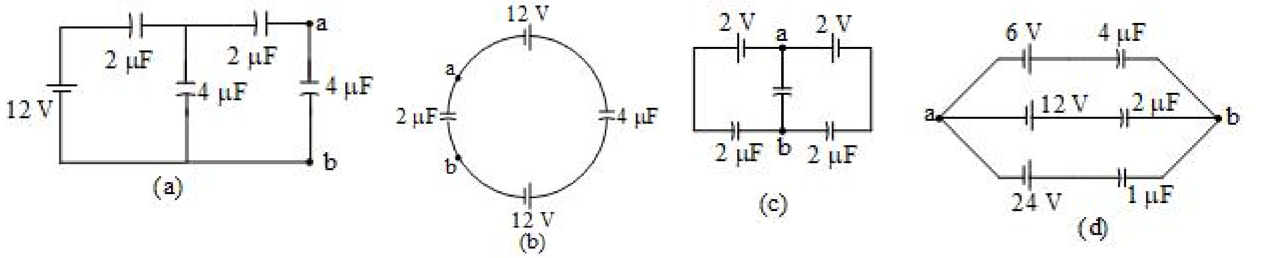




This is a pair of parallel combination between A-Z such that $C_1 = 1 \times 10^{-6} + 1 \times 10^{-6} \Rightarrow C_1 = 3 \times 10^{-6}$ and between Z and B such that $C_2 = 3 \times 10^{-6} + 6 \times 10^{-6} \Rightarrow C_2 = 9 \times 10^{-6}$.

This pair in turn is connected in series with an equivalent capacitance $C = \frac{C_1C_2}{C_1+C_2}$. Using the available data $C = \frac{(3\times10^{-6})(9\times10^{-6})}{3\times10^{-6}+9\times10^{-6}} \Rightarrow C =$ 2.25×10^{-6}F or 2.25 μF. Using the principle of significant digits $C =$ **2 μF is the answer.**

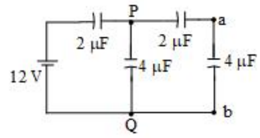
Q-11 Find the potential difference $V_{\mathrm{a}} - V_{\mathrm{b}}$, between the points a and b shown in each part of the figure.



A-11 (a) 1.09 V (b) Zero (c) Zero (d) -10.3 V

I-11 Solving each part separately to find $\Delta V = V_{\mathrm{a}} - V_{\mathrm{b}}$.

Part (a): The given connection of capacitors can be taken as a series combination of capacitance of 2 μF and the combination between P-Q.



The combination between P-Q is a parallel combination whose equivalent capacitance is $C' = 4 \times 10^{-6} + \frac{(2\times10^{-6})(4\times10^{-6})}{(2\times10^{-6})+(4\times10^{-6})}$

$\Rightarrow C' = 4 \times 10^{-6} + \frac{4}{3} \times 10^{-6} \Rightarrow C' = \frac{16}{3} \times 10^{-6} \Rightarrow C' = 5.33 \times 10^{-6}$F.

Thus voltage across P-Q would be $V' = \frac{2\times10^{-6}}{2\times10^{-6}+C'} \times 12 \Rightarrow V' = \frac{2\times10^{-6}}{2\times10^{-6}+5.33\times10^{-6}} \times 12 = 3.27\frac{9}{2}$Volts.

This voltage V' would further divide in series combination of capacitor containing points a-b. Therefore, voltage across a-b as desired would be $V_{ab} = \frac{2\times10^{-6}}{2\times10^{-6}+4\times10^{-6}} \times 3.27 \Rightarrow V_{ab} = \mathbf{1.09}$ **V.**

Part (b): In this circuit two batteries of equal emf are connected with their polarities reversed and thus they would cancel each other. Thus it is equivalent to both capacitors connected in parallel between points a and b, but without a battery. Therefore, the desired potential difference $V_{ab} = \mathbf{0}$**.**

Part (c): Symmetry of the circuit the circuit provides a clue that point **a** between two batteries of equal emf will be at mid potential of the potential difference across combination of batteries i.e. $V_a = 2$V.

Likewise, point **b** in the combination of the two equal capacitors connected in series across the combination of batteries and hence $V_b = \frac{2+2}{2} \Rightarrow V_b = 2$ V. Thus, potential difference between points a and b would be $V_{ab} = V_a - V_b = 2 - 2 \Rightarrow V_{ab} = \mathbf{0}$

Part (d): This combination of capacitors and batteries is a network having three branches connected in parallel between points a and b. Let potential difference between the points is V_{ab} and potential difference across a capacitor is $V = \frac{Q}{C}$. Accordingly, equations for three parallel branches are as under –

Upper Branch: $V_{ab} = 6 - \frac{Q_1}{4\times10^{-6}} \Rightarrow Q_1 = (6 - V_{ab})4 \times 10^{-6}$ …(1)

Middle Branch: $V_{ab} = 12 - \frac{Q_2}{2\times10^{-6}} \Rightarrow Q_2 = (12 - V_{ab})2 \times 10^{-6}$ …(2)

Lower Branch: $V_{ab} = 24 - \frac{Q_3}{1\times10^{-6}} \Rightarrow Q_3 = (24 - V_{ab})1 \times 10^{-6}$ …(3)

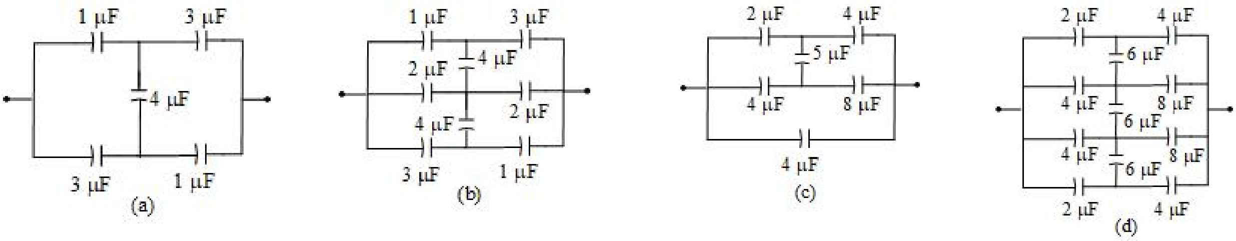
Further, it is an isolated system and hence, as per principle of electrical neutrality, net charge at on set of capacitors would remain zero such that $Q_1 + Q_2 + 4Q_3 = 0$…(4). Now we have four variables $V_{ab}, Q_1, Q_2,$ and Q_3 and a set of four independent linear equations.

Combining (1), (2), (3) and (4), $Q_1 + Q_2 + 4Q_3 = 0 = \left((6 - V_{ab})4 + (12 - V_{ab})2 + (24 - V_{ab})\right) \times 10^{-6}$

$\Rightarrow 72 - 7V_{ab} = 0 \Rightarrow V_{ab} = \frac{72}{7} = 10.3$V. Since all batteries have their positive polarity towards point b, hence, $V_a - V_b = -V_{ab} \Rightarrow V_a - V_b = \mathbf{-10.3}$ **V is the answer**.

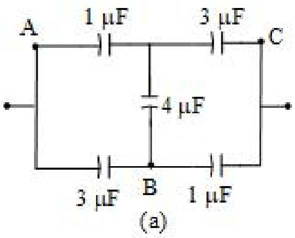
Thus, **answers are (a) $\mathbf{1.09}$ V (b) Zero (c) Zero (d) -10.3 V**

Q-12 Find the equivalent capacitance of the combination shown in each part of the figures between the indicated points.

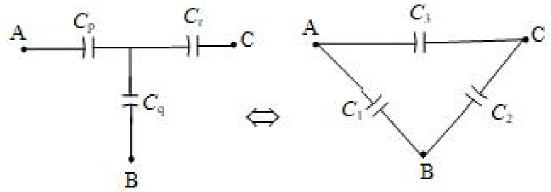


A-12 (a) $\frac{11}{6}$ μF (b) $\frac{11}{6}$ μF (c) 8 μF (d) 8 μF

I-12 This set of network involve star-delta connection of capacitances. Such questions can be easily solved with star-delta equivalent to be used interchangeably to simplify given network into series-parallel combination of capacitors. Taking each network separately –



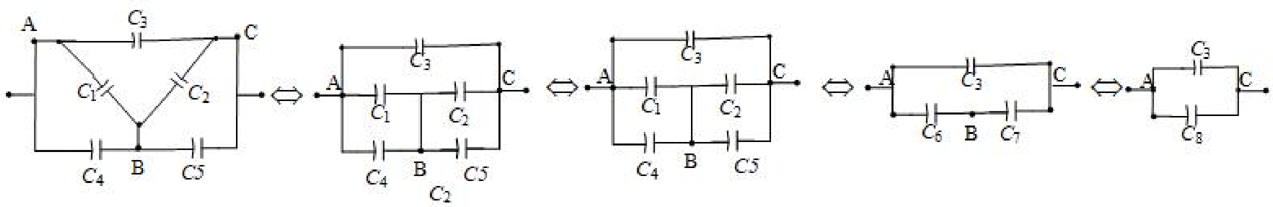
Network (a): In this case A, B, C nodes marked in the figure is seen as star connected capacitor where $C_p = 1 \times 10^{-6}$ F. $C_q = 4 \times 10^{-6}$ and $C_r = 3 \times 10^{-6}$.



This star connection is shown in the figure and is required to be converted into an equivalent delta connection of capacitors C_1, C_2, and C_3. As derived in Appendix $C_1 = \frac{C_pC_q}{C_p+C_q+C_r}$….(1), $C_2 = \frac{C_qC_r}{C_p+C_q+C_r}$ …(2), $C_3 = \frac{C_rC_p}{C_p+C_q+C_r}$ …(3).

Using the data, $C_1 = \frac{(1\times10^{-6})(4\times10^{-6})}{(1\times10^{-6})+(4\times10^{-6})+(3\times10^{-6})} \Rightarrow C_1 = \frac{(1\times10^{-6})(4\times10^{-6})}{(8\times10^{-6})} \Rightarrow, C_1 = 0.5 \times 10^{-6}$F….(4). Likewise, $C_2 = \frac{(4\times10^{-6})(3\times10^{-6})}{(8\times10^{-6})} \Rightarrow C_2 = 1.5 \times 10^{-6}$F…(5), and $C_3 = \frac{(3\times10^{-6})(1\times10^{-6})}{(8\times10^{-6})} \Rightarrow C_3 = 0.375 \times 10^{-6}$F…(6).

Using values in (4), (5) and (6), the network is reconstructed and reduced in stages to a simple parallel combination $C_2||C_3$, as shown in the figure. In this simplification algebraic values, from the given data are $C_4 = 3 \times 10^{-6}$ and $C_5 = 1 \times 10^{-6}$/



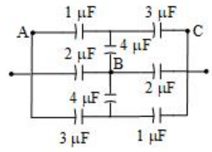
Accordingly, $C_6 = C_1||C_4 \Rightarrow C_6 = 0.5 \times 10^{-6} + 3 \times 10^{-6} \Rightarrow C_6 = 3.5 \times 10^{-6}$F.

Likewise, $C_7 = C_2||C_5 \Rightarrow C_7 = 1.5 \times 10^{-6} + 1 \times 10^{-6} \Rightarrow C_7 = 2.5 \times 10^{-6}$F.

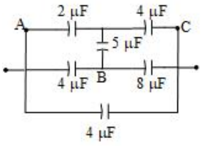
And series combination of C_6 and C_7 is $C_8 = \frac{C_6\times C_7}{C_6+C_7} \Rightarrow C_8 = \frac{(3.5\times10^{-6})\times(2.5\times10^{-6})}{(3.5\times10^{-6})+(2.5\times10^{-6})} \Rightarrow C_8 = 1.46 \times 10^{-6}$.

The last stage is , $C = C_3 || C_8 \Rightarrow C = C_3 + C_8 \Rightarrow C = C_3 + C_8 \Rightarrow C = 0.375 \times 10^{-6} + 1.46 \times 10^{-6} \Rightarrow C = 1.83 \times 10^{-6}$ F or $1.83 = \frac{11}{6}$ **μF is the answer of part (a).**

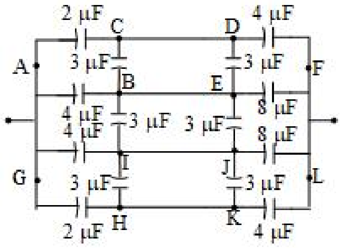
In a similar way networks in (b), (c) and (d) can be analyzed with clues as given below -



Network (b): In first stage determine $Y - \Delta$ equivalent of capacitors connected between A, B, C nodes. Then follow stage-wise simplification as done in network (a).



Network (c):): In first stage determine $Y - \Delta$ equivalent of capacitors connected between A, B, C nodes. Then follow stage-wise simplification as done in network (a).



Network (d): In first stage split each of the 6 **μF** into a parallel combination of two capacitors of 3 **μF.** In next stage determine $\Delta - Y$ equivalent of capacitors connected across group of nodes (A, B, C), (D, E, F), (G, H, I) and (J, K, L), nodes. Then follow stage-wise simplification as done in network (a).

N.B.: 1. A generic conversion of a delta-star conversion of capacitor connections has been derived in Appendix, It may add clarity, and has been used in the problem as per need. These formula need not be derived in solution, instead. they can be used directly with care to designate nodes correctly. This simplifies the solution in a stage wise manner.

2. In such problems, determining equivalent capacitance at each stage, simplifies solving of long algebraic expressions, rather leaving them for last stage, as generally advised.

3. Proficiency in solving such networks grows patience, which increases with practice.

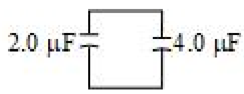
Q-13 A capacitor with stored energy 4.0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in two capacitors.

A-13 2.0 J

I-13 Let two equal capacitors are of capacitance C. One of them has energy stored $E' = 4.0$J. Energy stored in capacitor is $E' = \frac{1}{2}QV$. Since $Q = CV \Rightarrow V = \frac{Q}{C} \Rightarrow E' = \frac{Q^2}{2C}$…(1) When this charged capacitor is connected to another uncharged capacitor makes sense when both are in parallel. Thus charge on each capacitor in new formation would be equally shared such that $Q' = \frac{Q}{2}$…(2). Thus, combining (1) and (2), energy stored in each capacitor would be $E = \frac{\left(\frac{Q}{2}\right)^2}{2C} = \frac{1}{4}\left(\frac{Q^2}{2C}\right) = \frac{E'}{4} = \frac{4.0}{4} \Rightarrow E = 1.0$J.

Accordingly energy stored in the two capacitors is $= 2E'' = 2 \times 1.0 = $ **2.0 J is the answer**.

Q-14 A capacitor of capacitance 2.0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4.0 μF as shown in the figure. Find –



(a) The charge on each of the capacitors after the connection.
(b) The electrostatic energy stored in each of the two capacitors
(c) The heat produced during the charge transfer from one capacitor to the other.

A-14 (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ

I-14 Charge on capacitor of capacitance $C_1 = 2.0 \times 10^{-6}$ F when charge to a potential difference $V = 12$ V is $Q_1 = C_1 V$…(1)

$\Rightarrow Q_1 = (2.0 \times 10^{-6}) \times 12 = 24 \times 10^{-6}$ F.

When it is connected to an uncharged capacitor of capacitance $C_2 = 4.0 \times 10^{-6}$, there will be charge sharing across the two capacitor until both the capacitors are at a common potential difference V' such that charges on the two capacitors C_1 and C_2 would be $Q_1' = C_1 V'$ and $Q_2' = C_2 V'$, respectively. As shown in the figure, in the problem, it is an isolated system, and therefore it would conserve charge such that $Q_1 = Q_1' + Q_1'$...(2). Accordingly, combining (1) and (2) with the available data $(2.0 \times 10^{-6}) \times 12 = (2.0 \times 10^{-6}) \times V' + (4.0 \times 10^{-6}) \times V'$, we have $V' = \frac{(2.0 \times 10^{-6}) \times 12}{2.0 \times 10^{-6} + 4.0 \times 10^{-6}} = 4\text{V}$.

With this basics taking each part separately –

Part (a): Charge on capacitor C_1 is $Q_1' = C_1 V' \Rightarrow Q_1' = (2.0 \times 10^{-6}) \times 4 = 8.0 \times 10^{-6}$ or **8.0 μC.**

Likewise, charge on capacitor C_1 is $Q_1' = (4.0 \times 10^{-6}) \times 4 = 16.0 \times 10^{-6}$ or **16.0 μC**.

Part (b): Energy stored in capacitor in capacitor C_1 is $E_1' = \frac{1}{2} C_1 V'^2 \Rightarrow E_1' = \frac{1}{2} \times (2.0 \times 10^{-6}) \times 4^2 = 16.0 \times 10^{-6}$ or 16 **μJ.**

Likewise, charge on capacitor C_2 is $E_1' = \frac{1}{2} C_2 V'^2 \Rightarrow E_1' = \frac{1}{2} \times (4.0 \times 10^{-6}) \times 4^2 = 32.0 \times 10^{-6}$ or **32 μJ**.

Part (c): Energy initially stored in the capacitor C_1 is $E_1'' = \frac{1}{2} C_1 V^2 \Rightarrow E_1' = \frac{1}{2} \times (2.0 \times 10^{-6}) \times 12^2 = 144.0 \times 10^{-6}$ or 144 **μJ.** While energy stored in the combination of the two capacitors, using analysis in part (b) is $E' = E_1' + E_2' \Rightarrow E' = 16 + 32 = 48$ **μJ**. Therefore, The heat produced during the charge transfer from one capacitor to the other is the difference of energy $E_H = E_1' - E' \Rightarrow 144 - 48 = 96$ **μJ.**

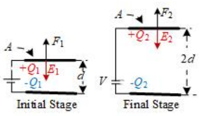
Thus, answers are (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ.

Q-15 A parallel-plate capacitor having plate area 20 cm^2 and separation between the plates 1.00 mm is connected to a battery of 12.0 V. The plates are pulled apart to increase the separation to 2.00 mm.

(a) Calculate the charge flown through the circuit during the process.
(b) How much energy is absorbed by the battery during the process?
(c) Calculate the stored energy in the electric field before and after the process.
(d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart.
(e) Show and justify that no heat is produced during the transfer of charge as separation is increased.

A-15 (a) 1.06×10^{-10} C (b) 12.7×10^{-10} J (c) 12.7×10^{-10} J, 6.35×10^{-10}J,
(d) 12.7×10^{-10} J (e) True

I-15 Given system of parallel-plate capacitors having plate area $A = 20 \times 10^{-4}\text{m}^2$ having a separation $d = 1.00 \times 10^{-3}$ m. Therefore, initial capacitance of the capacitor is $C_i = \frac{\varepsilon_0 A}{d}$ ...(1). Electric field between the plates is $E_i = \frac{V}{d}$...(2) Holding the capacitor in this initial positon requires a restraining force $F_i = E_i \times Q_i$ ...(3), as shown in the figure, here $Q_i = C_i \times V$...(4).



Combing (1), (2), (3) and (4) initially energy stored in the capacitor is $E_i' = \frac{1}{2} \times C_i V_i^2 = \frac{1}{2} C_i V^2$, here $V_i = V$...(5)

Likewise, energy stored in the capacitor to take it to final stage, is $E_f' = \frac{1}{2} \times C_f V_f^2 = \frac{1}{2} C_f V^2$, here $V_f = V$....(6).

And Using these basic, taking each part separately.

Part (a): Charge flown though the process of initial to final stage is $\Delta Q = Q_i - Q_f \Rightarrow \Delta Q = C_i \times V - C_f \times V \Rightarrow \Delta Q = (C_i - C_f)V \Rightarrow \Delta Q = \left(\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{2d}\right) V \Rightarrow \Delta Q = \frac{\varepsilon_0 A V}{2d}$. Using the available data and $\varepsilon_0 = 8.85 \times 10^{-12}$ we have $\Delta Q = \frac{(8.85 \times 10^{-12})(20 \times 10^{-4})12.0}{2 \times (1.00 \times 10^{-3})} \Rightarrow \Delta Q = \mathbf{1.06 \times 10^{-10}}$**C.**

Part (b): How much energy is absorbed by the battery during the process is $E' = \frac{Q}{2}V$, here $\Delta Q = \frac{Q}{2}$ is the charge returned by the capacitor at a constant voltage V of the battery. Accordingly, using result of part (a) $E' = (1.06 \times 10^{-10}) \times 12 = \mathbf{12.7 \times 10^{-10}}$J.

Part (c): Energy stored in the capacitor is initially is $E_i' = \frac{1}{2} \times C_i V_i^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2 = \mathbf{12.7 \times 10^{-10}}$ J, is same as that in part (b). But energy stored in the capacitor after the process is $E_f' = \frac{1}{2} \times C_f V_f^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{2d}\right)V^2 = \frac{12.7\times10^{-10}}{2} = \mathbf{6.35 \times 10^{-10}}$**J.**

Part (d): It is desired to use the expression for the force between the plates to find the work done by the person pulling the plates apart. Accordingly, work done by the person in increasing the separation from d to $2d$ such that for every change in separation $\Delta W = F_x \Delta x = (E_x Q_x)\Delta x = \left(\left(\frac{V}{x}\right)(C_x V)\right)\Delta x \Rightarrow \Delta W = V^2\left(\frac{C_x}{x}\right)\Delta x \Rightarrow \Delta W = V^2\left(\frac{\frac{\varepsilon_0 A}{x}}{x}\right)\Delta x \Rightarrow \Delta W = \varepsilon_0 A V^2 \frac{1}{x^2}\Delta x$. It is to be noted charge on the capacitor and potential difference both are function of separation between the plates and eventually $\Delta W = f(x)\Delta x$. Accordingly, it would require integration of the function such that $W = \varepsilon_0 A V^2 \int_d^{2d} \frac{1}{x^2}dx \Rightarrow W = -\varepsilon_0 A V^2 \left[\frac{1}{x}\right]_d^{2d} \Rightarrow W = -\varepsilon_0 A V^2\left[\frac{1}{2d} - \frac{1}{d}\right] \Rightarrow W = \frac{\varepsilon_0 A V^2}{2d}$.
Using the available data $W = \frac{(8.85\times10^{-12})(20\times10^{-4})12^2}{2\times(1.00\times10^{-3})} = \mathbf{12.7 \times 10^{-10}}$ J

Part (e): The amount of work done in the process [Answer of part (d)] is equal to amount of energy absorbed by the battery in the process [Answer of part (b)]. It is therefore concluded that no heat is produced in the process. **Answer is True.**

Thus, answers are (a) $\mathbf{1.06 \times 10^{-10}}$ C (b) $\mathbf{12.7 \times 10^{-10}}$ J (c) $\mathbf{12.7 \times 10^{-10}}$ J, $\mathbf{6.35 \times 10^{-10}}$J, (d) $\mathbf{6.35 \times 10^{-10}}$ J (e) True

Q-16

A capacitor of capacitance 5.00 μF is charged to 24.0 V and another capacitor of capacitance 6.00 μF is charged to 12.0 V.

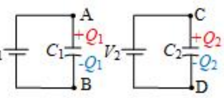
(a) Find the energy stored in each capacitor.
(b) The positive plate of the first capacitor is connected to the negative plate of the second and vice versa. Find the new charges on the capacitors.
(c) Find the loss of electrostatic energy during the process.
(d) Where does this energy go?

A-16

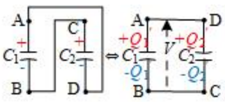
(a) 1.44 mJ, 0.432 mJ (b) 21.8 μC, 26.2 μC (c) 1.76 mJ (d) Dissipated as heat

I-16

Given are two capacitors of capacitance $C_1 = 5.00 \times 10^{-6}$F and $C_2 = 6.00 \times 10^{-6}$ F charged to $V_1 = 24.0$ V and $V_2 = 12.0$ V are initially independent. With this each of the part is being taken up progressively –

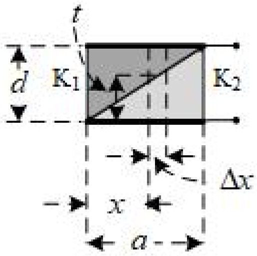
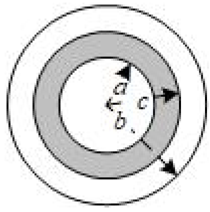


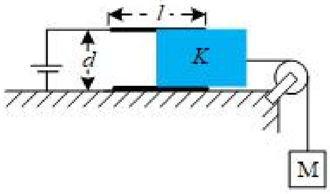
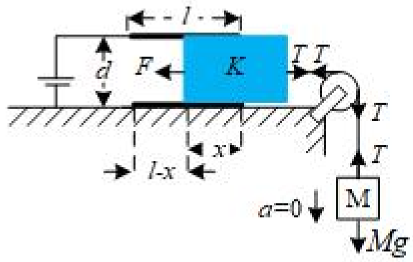
Part (a): Initially, energy stored in capacitor C_1 is $E_1' = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times (5.00 \times 10^{-6})(24.0)^2 \Rightarrow E_1' = 1.44 \times 10^{-3}$J or **1.44 mJ**. Likewise, $E_2' = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times (6.00 \times 10^{-6})(12.0)^2 \Rightarrow E_1' = 0.432 \times 10^{-3}$ or **0.432 mJ.**

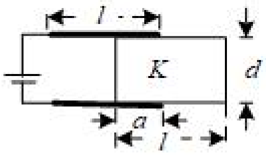
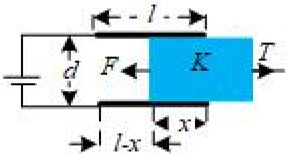


Part (b): Capacitors charged independently at $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$ as shown in figure part (a) are now connected as shown in the figure here. It becomes a closed loop electrically a parallel combination of the capacitors. Thus charge Q on the combination is $Q = Q_1 - Q_2 = (5.00 \times 10^{-6}) \times 24 - (6.00 \times 10^{-6}) \times 12 = 48.0 \times 10^{-6}$ C. Let voltage across the parallel combination $C' = C_1 + C_2 = 11 \times 10^{-6}$ F is $V' = Q \times C' \Rightarrow V' = \frac{Q}{C'} = \frac{48.0\times10^{-6}}{11\times10^{-6}} = \frac{48}{11}$V.
Accordingly, charge on capacitor C_1 is $Q_1' = C_1V' = (5.00 \times 10^{-6}) \times \frac{48}{11} = 21.8 \times 10^{-6}$C or $\mathbf{21.8}$ **μC.** Likewise, charge on capacitor C_2 is $Q_2' = C_2V' = (6.00 \times 10^{-6}) \times \frac{48}{11} = 26.2 \times 10^{-6}$C or $\mathbf{26.2}$ **μC.**

	Part (c): Initial energy stored in both the capacitors, based on part (a) is $E' = E_1' + E_2' = 1.44 + 0.432 = 1.87$mJ. But, in new configuration $E" = E_1^{"} + E_2^{"} = \frac{1}{2} \times Q_1'V' + \frac{1}{2} \times Q_2'V' = \frac{1}{2} \times QV$. Thus using the quantities arrived at above $E" = \frac{1}{2} \times (48.0 \times 10^{-6}) \times \frac{48}{11} = 0.105$ mJ. Accordingly, , loss of electrostatic energy is $\Delta E = E' - E" = 1.87 - 0.105 = \mathbf{1.76}$ **mJ**. **Part (d):** The loss of energy in part (c) is **dissipated as heat**. **Thus, answers are (a) 1.44 mJ, 0.432 mJ (b) 21.8 μC, 26.2 μC (c) 1.76 mJ (d) Dissipated as heat** **N.B.:** Decimal points in the given data needs to watched carefully and used in reporting answers on the principle of SDs.
Q-17	A parallel-plate capacitor of 5 μF is connected to a battery of emf 6 V. The separation between the plates is 2 mm. (a) Find charge on the positive plate of the capacitor. (b) Find the electric field between the plates. (c) A dielectric slab of thickness 1mm and dielectric constant 5 is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted?
A-17	(a) 30 μC, (b) 3×10^3 V/m (c) 8.3 μF, (d)20 μC,
I-17	Given is a parallel plate capacitor of capacitance $C = 5 \times 10^{-6}$F is connected to a battery with $V = 6$ V. Separation between the plates is $d = 2 \times 10^{-3}$m. With this each part of the problem is solved progressively- **Part (a):** Charge on positive plate of the capacitor is $Q = CV \Rightarrow Q = (5 \times 10^{-6}) \times 6 = 30 \times 10^{-6}$ C=**30 μC.** **Part (b):** Electric field between the plates is $E = \frac{V}{d} \Rightarrow E = \frac{6}{2\times10^{-3}} = \mathbf{3 \times 10^3}$ **V/m.** **Part (c):** Gap between plates of the capacitor is filled with a dielectric of thickness $t = 1 \times 10^{-3}$m the capacitor $C = \frac{\varepsilon_0 A}{d} \Rightarrow \varepsilon_0 A = Cd$ is equivalent to series combination of two capacitor one with air occupying free space of thickness $t' = d - t$ having capacitance $C_1 = \frac{\varepsilon_0 A}{t'}$ and the other with dielectric of dielectric constant $k = 5$ having thickness t such that $C_2 = \frac{\varepsilon_0 KA}{t}$. Thus equivalent capacitance of the new configuration is $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C' = \frac{\left(\frac{\varepsilon_0 A}{d-t}\right)\left(\frac{\varepsilon_0 KA}{t}\right)}{\frac{\varepsilon_0 A}{d-t} + \frac{\varepsilon_0 KA}{t}}$. It solves into $C' = \frac{\varepsilon_0 AK}{t(1-K)+dK} \Rightarrow C' = \frac{CdK}{t(1-K)+dK}$. Using the available data $C' = \frac{(5\times10^{-6})(2\times10^{-3})5}{(1\times10^{-3})(1-5)+(2\times10^{-3})5} = \frac{50\times10^{-6}}{6} \Rightarrow C' = 8.3 \times 10^{-6}$F or **8.3 μF.** **Part (d):** Charge held by the capacitor in this new configuration is $Q' = C'V \Rightarrow Q' = (8.3 \times 10^{-6}) \times 6 = 49.8 \times 10^{-6}$F,or 49.8 μC . Thus additional charge supplied by the battery is $\Delta Q = Q' - Q = 49.8 - 30 = 19.8$ μC. Using principle of SDs answer is **20 μC.** **Thus answers are (a) 30 μC, (b) 3×10^3 V/m (c) 8.3 μF, (d)20 μC.**
Q-18	A capacitor is formed by two square metal-plates of edge a, separated by a distance d. Dielectric of dielectric constant K_1 and K_2 are filled in the gaps as shown in the figure. Find the capacitance. 
A-18	$\frac{\varepsilon_0 K_1 K_2 a^2 \ln\frac{K_1}{K_2}}{(K_1-K_2)d}$ $\left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right) \ln\left(\frac{K_1}{K_2}\right)$.

I-18	 The given system having linear variation in thickness of the two dielectrics filling the gap between the square plates, having edge a, of the capacitor can be considered as a parallel combination of thin slits of dielectric at distance x from left edge. The slit has width a and a thickness Δx. In this slit, geometrically, separation between plates filled by the dielectric K_2 is $t_2 = \left(\frac{d}{a}\right)x = \frac{dx}{a}$ and that filled by dielectric K_2 is $t_1 = d - t_2 \Rightarrow t_1 = d - \frac{dx}{a} \Rightarrow t_1 = \frac{d(a-x)}{a}$. Thus capacitance of these two elemental capacitors is $\Delta C_{1x} = \frac{\varepsilon_0 K_1 A}{t_1} = \frac{\varepsilon_0 K_1(a\Delta x)}{\frac{d(a-x)}{a}} \Rightarrow \Delta C_{1x} = \left(\frac{\varepsilon_0 K_1 a^2}{d}\right)\frac{\Delta x}{a-x}$. Likewise, $\Delta C_{2x} = \frac{\varepsilon_0 K_2 A}{t_2} = \frac{\varepsilon_0 K_2(a\Delta x)}{\frac{dx}{a}} \Rightarrow \Delta C_{2x} = \left(\frac{\varepsilon_0 K_2 a^2}{d}\right)\frac{\Delta x}{x}$. These two capacitors forming series combination its capacitance2is $\Delta C_x = \frac{\Delta C_{1x}\Delta C_{2x}}{\Delta C_{1x}+\Delta C_{2x}} = \frac{\left(\left(\frac{\varepsilon_0 K_1 a^2}{d}\right)\frac{\Delta x}{a-x}\right)\left(\left(\frac{\varepsilon_0 K_2 a^2}{d}\right)\frac{\Delta x}{x}\right)}{\left(\frac{\varepsilon_0 K_1 a^2}{d}\right)\frac{\Delta x}{a-x}+\frac{2\varepsilon_0 K_2 a^2}{d}\frac{\Delta x}{x}} \Rightarrow \Delta C_x = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1 x+K_2(a-x))}\right)\Delta x \Rightarrow \Delta C_x = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_2 a+(K_1-K_2)x)}\right)\Delta x$. Thus, equivalent capacitance of the system is $C = \sum\left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_2 a+(K_1-K_2)x)}\right)\Delta x = \int_{x=0}^{a}\left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_2 a+(K_1-K_2)x)}\right)dx$. Here, it involves little integral calculus wherein a substitution $u = K_2 a + (K_1 - K_2)x \Rightarrow du = (K_1 - K_2)dx \Rightarrow dx = \frac{du}{(K_1-K_2)}$ is made. Using these substitutions, $C = \int_{x=0}^{a}\left(\frac{\varepsilon_0 K_1 K_2 a^2}{u}\right)\frac{du}{(K_1-K_2)} = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)\int_{x=0}^{a}\frac{1}{u}du \Rightarrow C = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)[\ln u]_{x=0}^{a}$. Making reverse substation to arrive results $C = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)[\ln(K_2 a + (K_1 - K_2)x)]_{x=0}^{a}$. This expression using the limits becomes $C = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)[\ln(K_2 a + (K_1 - K_2)a) - \ln(K_2 a)] \Rightarrow C = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)[\ln(K_1 a) - \ln(K_2 a)]$. In its final form $\boldsymbol{C = \left(\frac{\varepsilon_0 K_1 K_2 a^2}{d(K_1-K_2)}\right)\ln\left(\frac{K_1}{K_2}\right)}$. **N.B.:** This problem is an excellent example of integration of concepts and higher mathematics. It needs a systematic mathematical formulation of the problem and solving it with patience.
Q-19	A spherical capacitor is made of two conducting spherical shells of radii a and b. The space between the shells is filled with a dielectric of dielectric constant K upto radius c as shown in the figure. Calculate the capacitance. 
A-19	$\frac{4\pi\varepsilon_0 Kabc}{Ka(b-c)+b(c-a)}$
I-19	The given assembly is equivalent to series combination of two spherical capacitors – One of capacitance C_1 is of inner radius $r_{i1} = a$ and outer radius $r_{o1} = c$ filled with a dielectric of dielectric constant K. And the other of capacitance C_2 is of inner radius $r_{i2} = c$ and outer radius $r_{o2} = b$ filled with air. Capacitance of a spherical capacitor is $C = \frac{4\pi\varepsilon_0 K r_i r_o}{r_o - r_i}$. In the given problem inner capacitor with the dielectric $C_1 = \frac{4\pi\varepsilon_0 K r_{i1} r_{o1}}{r_{o1}-r_{i1}} \Rightarrow C_1 = \frac{4\pi\varepsilon_0 acK}{c-a}$ and for the outer without dielectric $C_2 = \frac{4\pi\varepsilon_0 r_{i2} r_{o2}}{r_{o2}-r_{i2}} \Rightarrow C_2 = \frac{4\pi\varepsilon_0 bc}{b-c}$. Therefore net capacitance of the series combination is $C = \frac{C_1 C_2}{C_1+C_2} \Rightarrow C = \frac{\left(\frac{4\pi\varepsilon_0 acK}{c-a}\right)\left(\frac{4\pi\varepsilon_0 bc}{b-c}\right)}{\frac{4\pi\varepsilon_0 acK}{c-a}+\frac{4\pi\varepsilon_0 bc}{b-c}}$. It further solves to $C = \frac{4\pi\varepsilon_0 abc^2 K}{c(Ka(b-c)+b(c-a))} \Rightarrow C = \boldsymbol{\frac{4\pi\varepsilon_0 abcK}{Ka(b-c)+b(c-a)}}$ **is the answer.**
Q-20	A parallel-plate capacitor with the plates area 100 cm^2 and separation between the plates is 1.0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.
A-20	0.25×10^{-6} N

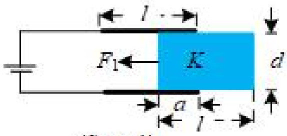
I-20	Given is a parallel-plate capacitor having plate area $A = 100 \times 10^{-4}\text{m}^2$ and separation between plates $d = 1.0 \times 10^{-2}$ m. Capacitance of the capacitor is $C = \frac{\epsilon_0 A}{d} \Rightarrow C = \frac{(8.85\times10^{-12})(100\times10^{-4})}{1.0\times10^{-2}} \Rightarrow C = 8.85 \times 10^{-12}$ F. The capacitor is connected across battery $V = 24$ V. Thus, charge on the capacitor, using the available data is $Q = CV \Rightarrow Q = (8.85 \times 10^{-12}) \times 24 \Rightarrow Q = 0.21 \times 10^{-9}$C. A close examination of Coulomb's Law reveals that force experienced by a charge q is due to electric field E produced by charge(s) other than itself and is expressed as $F = Eq$. Accordingly, force experienced by charge on a plate of the capacitor is due to field produced by charge on the other plate, and it is symmetrical to both the plates. In parallel plate capacitor uniform electric field $E = \frac{\rho}{2\epsilon_0}$is produced by charge of $(+Q)$ on the positive plate of the capacitor. Thus, force experienced by negative plate carrying charge $(-Q)$ is $F = \frac{\rho}{2\epsilon_0}(-Q) \Rightarrow F = \frac{\rho A}{2\epsilon_0 A}(-Q) \Rightarrow F = \frac{(+Q)}{2\epsilon_0 A}(-Q) \Rightarrow F = -\frac{Q^2}{2\epsilon_0 A}$. The (-) ve sign signifies that the force is attractive having magnitude $\lvert F \rvert = \frac{Q^2}{2\epsilon_0 A}$. Accordingly attractive force between the two plates, using the available data is $\lvert F \rvert = \frac{(0.21\times10^{-9})^2}{2\times(8.85\times10^{-12})\times(100\times10^{-4})} \Rightarrow \lvert F \rvert = 0.25 \times 10^{-6}$ N. **N.B.: (1)** The problem involves clarity of concept of electric field in capacitor and electrostatic force between the plates of the capacitors. (2) Mutual force on plates of a charged capacitor is different from that on a charge q in between the plates of a charged capacitor. In this case force on the charge is combined effect of external field due to positively charged plate $\vec{E}_+ = \frac{\rho}{2\epsilon_0}\hat{r}$ and that due to negatively charged plate$\vec{E}_- = \frac{-\rho}{2\epsilon_0}(-\hat{r}) = \frac{\rho}{2\epsilon_0}\hat{r}$. Thus net electric field influencing force on the charge q is $\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho}{2\epsilon_0}\hat{r} + \frac{\rho}{2\epsilon_0}\hat{r} \Rightarrow \vec{E} = \frac{\rho}{\epsilon_0}\hat{r}$. This is double of that considered in illustration of the solution. Moreover, magnitude of the electric field at charge q is $E = \frac{\rho}{\epsilon_0} = \frac{\rho A}{\epsilon_0 A} \Rightarrow E = \frac{Q}{\frac{\epsilon_0 A d}{d}} \Rightarrow E = \frac{Q}{\left(\frac{\epsilon_0 A}{d}\right)d} \Rightarrow E = \frac{Q}{Cd} \Rightarrow E = \left(\frac{Q}{C}\right) \times \frac{1}{d} \Rightarrow E = V \times \frac{1}{d} \Rightarrow E = \frac{V}{d}$.
Q-21	Consider the situation shown in the figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf E. All surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium. 
A-21	$\frac{\epsilon_0 b E^2 (K-1)}{2dg}$
I-21	Capacitance C of a capacitor change in absence of dielectric $C' = \frac{\epsilon_0 A}{d}$ changes to $C'' = \frac{\epsilon_0 K A}{d}$ and hence energy stored in capacitor without dielectric $E' = \frac{1}{2}QV \Rightarrow E' = \frac{C'V^2}{2} \Rightarrow E' = \frac{\epsilon_0 A V^2}{2d}$ changes to $E'' = \frac{\epsilon_0 K A V^2}{2d}$.  In the given system say with a width x of the dielectric inside the gap, it is equivalent to two capacitors as under – (a) With Dielectric Filled: $C_1 = \frac{\epsilon_0 K A\left(\frac{x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 K A x}{ld}$ (b) Without dielectric: $C_2 = \frac{\epsilon_0 A\left(\frac{l-x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 A(l-x)}{ld}$. This system is equivalent to a parallel combination of capacitors C_1 and C_2 having an equivalent capacitance $C = C_1 + C_2 \Rightarrow C = \frac{\epsilon_0 K A x}{ld} + \frac{\epsilon_0 A(l-x)}{ld} \Rightarrow C = \frac{\epsilon_0 A}{ld}(Kx + l - x) \Rightarrow C = \frac{\epsilon_0 A}{ld}(l + (K-1)x)$. Accordingly,

	energy stored in the system is $E' = \frac{\left(\frac{\epsilon_0 A}{ld}(l+(K-1)x)\right)V^2}{2} \Rightarrow E' = \frac{\epsilon_0 AV^2}{2ld}(l+(K-1)x)$. Now, when dielectric is moved $\Delta E' = \frac{d}{dx}\left(\frac{\epsilon_0 AV^2}{2ld}(l+(K-1)x)\right)\Delta x \Rightarrow \Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. If Δx decreases it becomes negative and hence $\Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}(-\Delta x) \Rightarrow \Delta E' = \frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. Further, $\Delta E' = \vec{F}.\Delta\vec{x}$ and hence magnitude of inward force on dielectric being pulled out is $F = \frac{\epsilon_0 AV^2(K-1)}{2ld}$. Here, an element of concepts mechanics is involved. Natural tendency of the mass M is to lower down under gravity. But, the system is stated to be in equilibrium and therefore $a = 0$ such that $M\vec{g} + \vec{T} = 0 \Rightarrow M\vec{g} = -\vec{T}$. With the sequence of forces in the string are shown in the figure eventually for dielectric slab to be in equilibrium $\vec{F} = -\vec{T}$. Ignoring intermediate forces in the string connecting its final form $F = Mg$. It implies that both the forces are equal in magnitude and opposite in direction. Accordingly, $Mg = \frac{\epsilon_0 AV^2(K-1)}{2ld}$, it leads to $M = \frac{\epsilon_0 AV^2(K-1)}{2ldg}$. It is given in the problem that width of each plates of capacitors is b and their length as per given figure is l and, therefore, $A = bl$. It leads to $M = \frac{\epsilon_0 (bl)V^2(K-1)}{2ldg} \Rightarrow$ $M = \frac{\epsilon_0 bV^2(K-1)}{2dg}$ **is the answer**. **N.B.: (1)** This problem is a good example of integration of concepts as one either moves forward in physics or application of concepts of physics. **(2)** Work or energy being scalar, what is important is change in energy as per relevant formulae and not the direction of mechanical force or electric field.
Q-22	Consider the situation shown in the figure. The plates of the capacitor have plate area A and are clamped in the laboratory. The dielectric slab is released from rest with a length a inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period. 
A-22	$8 \times \sqrt{\frac{(l-a)lmd}{\varepsilon_0 AE^2(K-1)}}$
I-22	Any object which experiences force and executes periodic motion it must have a mass and let mass of the dielectric slab in the system is m. It is given that a dielectric slab of dielectric constant K is held in a state of rest such that it partially fills the gap to a length a. Length and width of the slab and dielectric plates are l and b respectively. The dielectric slab is released is released from a state of rest and it is required to demonstrate that it performs a periodic motion and to find periodic motion. In the first go it is required to determine force on a dielectric slab when it is partially filling the gap and it is analyzed as under. Capacitance C of a capacitor change in absence of dielectric $C' = \frac{\epsilon_0 A}{d}$ changes to $C'' = \frac{\epsilon_0 KA}{d}$ and hence energy stored in capacitor without dielectric $E' = \frac{1}{2}QV \Rightarrow E' = \frac{C'V^2}{2} \Rightarrow E' = \frac{\epsilon_0 AV^2}{2d}$ changes to $E'' = \frac{\epsilon_0 KAV^2}{2d}$. In the given system say with a width x of the dielectric inside the gap, it is equivalent to two capacitors as under –  (a) With Dielectric Filled: $C_1 = \frac{\epsilon_0 KA\left(\frac{x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 KAx}{ld}$ (b) Without dielectric: $C_2 = \frac{\epsilon_0 A\left(\frac{l-x}{l}\right)}{d} \Rightarrow C_1 = \frac{\epsilon_0 A(l-x)}{ld}$. This system is equivalent to a parallel combination of capacitors C_1 and C_2 having an equivalent capacitance $C = C_1 + C_2 \Rightarrow C = \frac{\epsilon_0 KAx}{ld} + \frac{\epsilon_0 A(l-x)}{ld} \Rightarrow C = \frac{\epsilon_0 A}{ld}(Kx + l - x) \Rightarrow C = \frac{\epsilon_0 A}{ld}(l + (K-1)x)$. Accordingly,

energy stored in the system is $E' = \frac{\left(\frac{\epsilon_0 A}{ld}(l+(K-1)x)\right)V^2}{2} \Rightarrow E' = \frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)$. Now, when dielectric is moved $\Delta E' = \frac{d}{dx}\left(\frac{\epsilon_0 AV^2}{2ld}(l + (K-1)x)\right)\Delta x \Rightarrow \Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. If Δx decreases it becomes negative and hence $\Delta E' = -\frac{\epsilon_0 AV^2(K-1)}{2ld}(-\Delta x) \Rightarrow \Delta E' = \frac{\epsilon_0 AV^2(K-1)}{2ld}\Delta x$. Further, $\Delta E' = \vec{F}.\Delta\vec{x}$ and hence magnitude of inward force on dielectric being pulled out is $F = \frac{\epsilon_0 AV^2(K-1)}{2ld} = \frac{\epsilon_0 b(lb)V^2(K-1)}{2ld} \Rightarrow F = \frac{\epsilon_0 bV^2(K-1)}{2d}$. It is to be noted that inward electrostatic force on dielectric caused is independent of depth of the slab inside the capacitor plates and length of the plates of the capacitors.

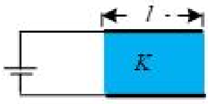
Now an element of mechanics is involved and there are three states of the block as under –

State 1: In the state of rest initial velocity of the block $u = 0$. But, due to the uniform dielectric force $F_1 = \frac{\epsilon_0 bV^2(K-1)}{2d}$ the dielectric slab would experience uniform acceleration, as per Newton's Second Law of Motion $k_1 = \frac{F_1}{m} \Rightarrow k_1 = \frac{\epsilon_0 bV^2(K-1)}{2dm}$, as shown in the figure (i).



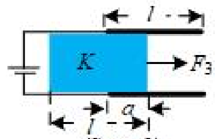
(State 1)

State 2: The slab remains accelerated until it fill gap completely as shown if figure (ii), and by then acquires velocity as per Third Equation of Motion $v^2 = 2ks \Rightarrow v = \sqrt{2ks} \Rightarrow v_{max} = \sqrt{2\left(\frac{\epsilon_0 bV^2(K-1)}{2dm}\right)(l-a)} = \sqrt{\left(\frac{\epsilon_0 bV^2(K-1)}{dm}\right)(l-a)}$. At this state when gap is completely filled force on the dielectric becomes zero.

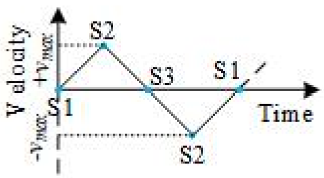


(State 2)

State 3: In state 2, under inertia as per Newton's First Law of Motion would continue with the instantaneous velocity to create gap on right side and experience electrostatic force $F_3 = F_1$ i.e. of same magnitude but in opposite direction which would retard the slab until it reaches state 3.



(State 3)



Considering the three states the slab would continue perform a periodic motion in the system where effect of friction and gravity is ignored. Thus, transition of slab from one state to the other is shown in the figure where at $t = 0$ the slab is in state 1 (S1), at $t = \frac{T}{4}$ the slab is in state 2 (S2). at $t = \frac{T}{2}$ the slab is in state 3 (S3). at $t = \frac{3T}{4}$ the slab is in state 2 (S2) and at $t = T$ the slab is back in state 1 (S1). Like this periodic motion continues. Accordingly from First Equation of motion $v_{max} = 0 + k_1 \times \frac{T}{4} \Rightarrow T = \frac{4v_{max}}{k_1}$. Using results in stage 1 and 2, $T = \frac{4\sqrt{\left(\frac{\epsilon_0 bV^2(K-1)}{2dm}\right)(l-a)}}{\frac{\epsilon_0 bV^2(K-1)}{2dm}} \Rightarrow T = 8 \times \sqrt{\frac{(l-a)md}{\epsilon_0 bV^2(K-1)}} \Rightarrow T = 8 \times \sqrt{\frac{(l-a)mdl}{\epsilon_0(bl)V^2(K-1)}} = \mathbf{8} \times \sqrt{\frac{(l-a)mdl}{\epsilon_0 AV^2(K-1)}}$ **is the answer.**

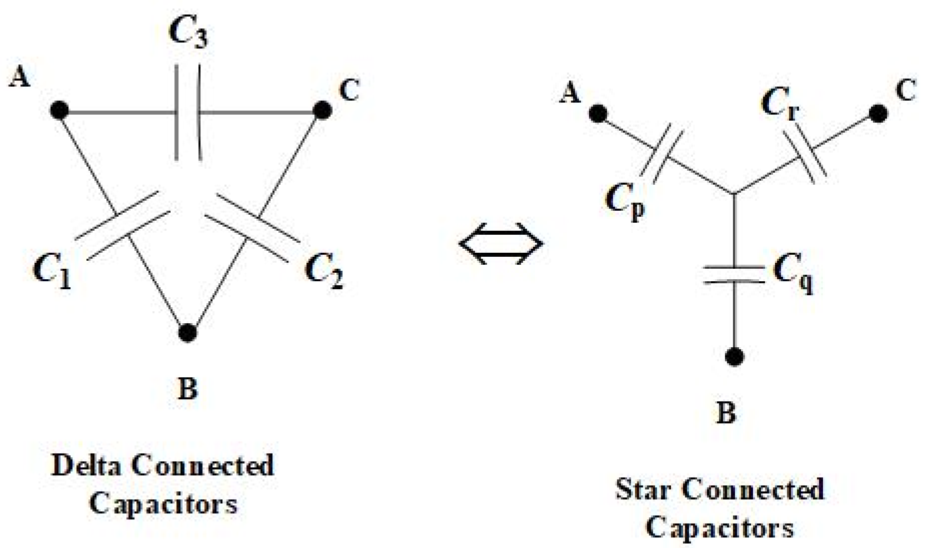
N.B.: (1) This problem is a good example of integration of concepts as one either moves forward in physics or application of concepts of physics.

(2) Work or energy being scalar, what is important is change in energy as per relevant formulae and not the direction of mechanical force or electric field.

(3) Step by step application of concepts makes solution of a problem, apparently difficult, very simple and adds to clarity of concepts.

APPENDIX

Star-Delta $(Y - \Delta)$ Equivalent of Capacitor Connection



Operation	Delta Connection	Star Connection	Eqn.
Equivalent capacitance across nodes a-b	$C_{ab} = C_1 + \frac{C_2C_3}{C_2+C_3}$ $\Rightarrow \frac{C_1C_2 + C_2C_3+C_3C_1}{C_2+C_3}$ $\frac{1}{C_{ab}} = \frac{C_2+C_3}{C_1C_2 + C_2C_3+C_3C_1}$	$C_{ab} = \frac{C_pC_q}{C_p + C_q}$ $\frac{1}{C_{ab}} = \frac{C_p + C_q}{C_pC_q}$	(1)
Equivalent capacitance across nodes b-c	$C_{bc} = C_2 + \frac{C_3C_1}{C_3+C_1}$ $\Rightarrow \frac{C_1C_2 + C_2C_3+C_3C_1}{C_3+C_1}$ $\frac{1}{C_{bc}} = \frac{C_3+C_1}{C_1C_2 + C_2C_3+C_3C_1}$	$C_{bc} = \frac{C_qC_r}{C_q+C_r}$ $\frac{1}{C_{bc}} = \frac{C_q + C_r}{C_qC_r}$	(2)
Equivalent capacitance across nodes c-a	$C_{ca} = C_3 + \frac{C_1C_2}{C_1+C_2}$ $\Rightarrow \frac{C_1C_2 + C_2C_3+C_3C_1}{C_1+C_2}$ $\frac{1}{C_{ca}} = \frac{C_1+C_2}{C_1C_2 + C_2C_3+C_3C_1}$	$C_{ca} = \frac{C_rC_p}{C_r+C_p}$ $\frac{1}{C_{bc}} = \frac{C_r + C_p}{C_rC_p}$	(3)
(2)-(1):	$\frac{C_1-C_2}{C_1C_2 + C_2C_3+C_3C_1}$	$\frac{C_q + C_r}{C_qC_r} - \frac{C_p + C_q}{C_pC_q}$	(4)

$\frac{1}{C_{\mathrm{bc}}}-\frac{1}{C_{\mathrm{ab}}}$		$\Rightarrow \frac{C_{\mathrm{p}}C_{\mathrm{q}}+C_{\mathrm{p}}C_{\mathrm{r}}-C_{\mathrm{r}}C_{\mathrm{p}}-C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$ $\Rightarrow \frac{C_{\mathrm{p}}C_{\mathrm{q}}-C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$	
Delta to Star Conversion			
(3)+(4)	$\frac{2C_1}{C_1C_2+C_2C_3+C_3C_1}$	$\left(\frac{C_{\mathrm{r}}+C_{\mathrm{p}}}{C_{\mathrm{r}}C_{\mathrm{p}}}\right)\times\frac{C_{\mathrm{q}}}{C_{\mathrm{q}}}+\frac{C_{\mathrm{p}}C_{\mathrm{q}}-C_{\mathrm{r}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}$ $\Rightarrow \frac{2C_{\mathrm{p}}C_{\mathrm{q}}}{C_{\mathrm{p}}C_{\mathrm{q}}C_{\mathrm{r}}}=\frac{2}{C_{\mathrm{r}}}$	
	$\frac{C_1}{C_1C_2+C_2C_3+C_3C_1}=\frac{1}{C_{\mathrm{r}}}$ $\boldsymbol{C_{\mathrm{r}}=\frac{C_1C_2+C_2C_3+C_3C_1}{C_1}}$		(5)
Applying analogy of (5)	$\boldsymbol{C_{\mathrm{p}}=\frac{C_1C_2+C_2C_3+C_3C_1}{C_2}}$		(6)
	$\boldsymbol{C_{\mathrm{q}}=\frac{C_1C_2+C_2C_3+C_3C_1}{C_1}}$		(7)
Star to Delta Conversion			
(5) × (6)	$C_{\mathrm{r}}C_{\mathrm{p}}=\frac{(C_1C_2+C_2C_3+C_3C_1)^2}{C_1C_2}$ $\Rightarrow \frac{1}{C_{\mathrm{r}}C_{\mathrm{p}}}=\frac{C_1C_2}{(C_1C_2+C_2C_3+C_3C_1)^2}$		(8)
(6) × (7)	$C_{\mathrm{p}}C_{\mathrm{q}}=\frac{(C_1C_2+C_2C_3+C_3C_1)^2}{C_2C_3}$ $\Rightarrow \frac{1}{C_{\mathrm{p}}C_{\mathrm{q}}}=\frac{C_2C_3}{(C_1C_2+C_2C_3+C_3C_1)^2}$		(9)
(5) × (7)	$C_{\mathrm{q}}C_{\mathrm{r}}=\frac{(C_1C_2+C_2C_3+C_3C_1)^2}{C_3C_1}$		(10)

	$$\Rightarrow \frac{1}{C_q C_r} = \frac{C_3 C_1}{(C_1C_2 + C_2C_3 + C_3C_1)^2}$$	
(8)+(9)+(10)	$$\frac{1}{C_r C_p} + \frac{1}{C_p C_q} + \frac{1}{C_q C_r} = \frac{C_1C_2 + C_2C_3 + C_3C_1}{(C_1C_2 + C_2C_3 + C_3C_1)^2}$$ $$\Rightarrow \frac{1}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{1}{C_r C_p} + \frac{1}{C_p C_q} + \frac{1}{C_q C_r}$$ $$\Rightarrow \frac{1}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{C_p + C_q + C_r}{C_p C_q C_r}$$	(11)
(5) × (11)	$$\left(\frac{C_1C_2 + C_2C_3 + C_3C_1}{C_1}\right) \times \left(\frac{1}{C_1C_2 + C_2C_3 + C_3C_1}\right) = C_r \times \left(\frac{C_p + C_q + C_r}{C_p C_q C_r}\right)$$ $$\Rightarrow \frac{1}{C_1} = \frac{C_p + C_q + C_r}{C_p C_q}$$ $$\boldsymbol{C_1 = \frac{C_p C_q}{C_p + C_q + C_r}}$$	(12)
Applying analogy of (125)	$$C_2 = \frac{\boldsymbol{C_q C_r}}{\boldsymbol{C_p + C_q + C_r}}$$	(13)
	$$C_3 = \frac{\boldsymbol{C_r C_p}}{\boldsymbol{C_p + C_q + C_r}}$$	(14)

N.B.: 1. Derivation of $(Y - \Delta)$ is based on series-parallel equivalents capacitances with algebraic manipulations.

2.Derivation of equivalent capacitance in A close observation of formulae in relation to the figure will help to develop easy way to remember it.

2. A similar, but with a difference formulation for star-delta network of resistance in DC circuit analysis and impedances in AC circuit analysis will be supplemented at a later stage.

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