Let Us Do Some Problems In Mathematics -XXVI

SB Dhar

Junior Science Talent Search Examination (JSTSE) is an Indian scholarship examination. It is conducted by the Science Branch of the Directorate of Education in Delhi every year in the month of January or December. It is open for the students of recognized schools in Delhi. 150 students every year are awarded scholarships.

The students who are studying in the class IX of a recognized school in Government or Government added/Public/KV/Navoday/NDMC and have earned at least 65% marks in class VIII are eligible to appear in the examination. There is no fee for the examination. The examination has two papers (a) General Knowledge of 50 Questions of 50 marks of 50 minutes and (b) General Science and Mathematics containing 150 Questions 150 marks 150 minutes.

The result is declared in the month of February or March.

Some of the questions from the examination 2019-2020 held on December 15, 2019 from Mathematics segment are given here for the readers to understand its standard. The solutions are not being written here. If some reader needs the detail solution of any problem, he or she may request the Coordinator Desk, Gyan Vigyan Sarita, for it.

QUESTIONS

Q1. If $\left(\frac{x+1}{x+3}\right)^3 = \frac{x-1}{x+5}$, then the value of x is

(a)2   (b)-2

(c)1   (d)-1

Ans.(b)

Q2. Value of $\frac{1}{2+\sqrt{3}-2\sqrt{2}} + \frac{3}{2+\sqrt{3}+2\sqrt{2}}$ is

(a)$\frac{4}{47}\left(9\sqrt{3} - 4\sqrt{6} - \sqrt{2} + 14\right)$

(b)$\frac{4}{47}\left(9\sqrt{3} + 4\sqrt{6} - \sqrt{2} + 14\right)$

(c)$\frac{4}{47}\left(9\sqrt{3} - 4\sqrt{6} - \sqrt{2} - 14\right)$

(d)$\frac{4}{47}\left(9\sqrt{3} + 4\sqrt{6} + \sqrt{2} + 14\right)$

Ans.(a)

Q3. If $x = \frac{5\sqrt{21}}{\sqrt{3}+\sqrt{7}}$, then the value of

$\frac{x+5\sqrt{7}}{x-5\sqrt{7}} - \frac{x+5\sqrt{3}}{x-5\sqrt{3}}$ is

(a)2   (b) $\sqrt{21}$

(c) $\frac{8}{\sqrt{21}}$ (d) $\frac{4}{\sqrt{21}}$

Ans.(c)

Q4. If the polynomials $p(x)=4x^3-ax^2+2x-1$ and $q(x)=3x^3-7x^2-8x+a$ leave the same remainder, when divided by *(x-1)*, then the value of *a* is

(a)1   (b)$\frac{1}{2}$

(c)$\frac{3}{2}$ (d)$-\frac{3}{2}$

Ans. $\frac{17}{2}$

Q5. Factors of $6x^2-5xy-4y^2+x+17y-15$ are

(a)*(2x+y-3)(3x-4y+5)*   (b) *(2x-y-3)(3x-4y-5)*

(c) *(2x-y-3)(3x+4y+5)*   (d) *(2x+y+3)(3x+4y-5)*

Ans.(a)

Q6. If $x = \sqrt[3]{28}$ and $y = \sqrt[3]{27}$ then the value of $x + y - \frac{1}{x^2+xy+y^2}$ is

(a)8   (b)7

(c)6   (d)5

Ans.(c)

Q7. The value of $0.\overline{2} + 0.2\overline{3}$ is

(a) $0.4\overline{3}$ (b) $0.\overline{43}$

(c) $0.4\overline{5}$ (d) $0.\overline{45}$

Ans.(c)

Q8. If *x, y* and *z* are real and $(x-2)^2+(y-3)^2+(z-4)^2=0$, then the value of

xy + yz + zx is
(a)24   (b)26
(c)28   (d)30
Ans.(b)

Q9. If *p²-3p-1=0*, then the value of $p^2 + \dfrac{1}{p^2}$ is

(a)7   (b)9
(c)11   (d)13
Ans.(c)

Q10. If *m+n=7* and *m³+n³=133*, then the value of *m²+n²* is
(a)29   (b)49
(c)69   (d)59
Ans.(a)

Q11. If $x + y = \sqrt{3}$, $x - y = \sqrt{2}$, then the expression *8xy(x²+y²)* has the value
(a) $5\sqrt{2}$ (b) $10\sqrt{2}$
(c)20   (d)5
Ans.(d)

Q12. Factors of *(3x²-2x)(6-3x²+2x)-5* are
(a)*(x-1)(x+1)(1+3x)(5-3x)*
(b) *(x-1)(x+1)(1+3x)(5+3x)*
(c) *(x-1)(x+1)(1-3x)(3+5x)*
(d) *(x-1)(x+1)(3-x)(5-3x)*
Ans.(a)

Q13. If $m = 2p + \sqrt{p^2 + k}$, then *k* in terms of *p* and *m* is
(a)*(m+p)(m+3p)*   (b)*(m+p)(m-3p)*
(c)*(m-2p)(m-3p)*   (d)*(m-p)(m-3p)*
Ans.(d)

Q14. If *p-x=1* and $\dfrac{3x+2}{5} + \dfrac{3}{2} = \dfrac{4p-3}{2}$ then the value of *x* is
(a)1   (b)-1
(c)0   (d)2
Ans.(a)

Q15. If *5²ᵐ⁻¹=25ᵐ⁻¹+100*, then the value of *6⁻ᵐ* is
(a)6   (b)36
(c)$\dfrac{1}{6}$ (d)$\dfrac{1}{36}$
Ans.(4)

Q16. If $x = 3 + 3^{1/3} + 3^{2/3}$, then the value of *x³-9x²+18x-10* is
(a)-1   (b)0
(c)1   (d)2

Ans.(d)

Q17. If *a+b+c=2, ab+bc+ca=-1* and *abc=-2* then the value of *a³+b³+c³* is
(a)-8   (b)0
(c)8   (d)6
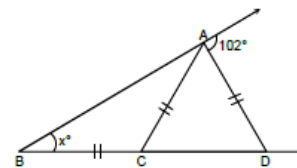*Ans.(c)*

Q18. The coefficient of *x²* in *(x+3)(x-5)(x+7)* is
(a)28   (b)-28
(c)-5   (d)5
Ans.(d)

Q19. In figure below, AD=AC=CB, then the value of *x* is

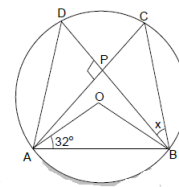(a)51⁰   (b)78⁰
(c)34⁰   (d)43⁰
Ans.(c)

Q20. If $\left(\sqrt{32}\right)^{m} \div 2^{n+1} = 1$ and $16^{4-\frac{m}{2}} - 8^n = 0$, then the value of *m* and *n* are
(a)*m=2, n=4*   (b)*m=2, n=3*
(c)*m=4, n=2*   (d)*m=3, n=2*
Ans.(a)

Q21.In the figure below, O is the centre of the circle, $\angle OAB=32^0$, $\angle APD=90^0$, then the value of *x* is

(a)30^0 (b)32^0
(c)34^0 (d)36^0
Ans.(b)

Q22. If volume of a cube is L cubic units, its surface area is M square units and the length of the diagonal is N unit, then
(a)6L=MN   (b)$6\sqrt{3}$L=MN
(c) $\sqrt{3}$M=LN   (d)6N=LM
Ans.(b)

Q23. The area of circular ring enclosed between two concentric circles is 286cm². If the difference of their radii is 7cm, then the radii of these circles are
(a)2cm, 9cm   (b)5cm, 12cm
(c)4cm, 11cm   (d)3cm, 10cm
Ans.(d)

Q24. If 49^x - 49^{x-1} = *16464*, then which of the following is equivalent to $(2x)^x$?
(a)$(5)^{5/2}$ (b)$(7)^{7/2}$
(c)$(3)^{3/2}$ (d)None of these
Ans.(a)

Q25. The value of
$\sqrt[3]{20 + 14\sqrt{2}} + \sqrt[3]{20 - 14\sqrt{2}}$ is
(a)4   (b)6
(c)8   (d)10
Ans.(a)

Q26. If $m + \dfrac{1}{m} = 5$, then the value of
$\dfrac{m^4+3m^3+5m^2+3m+1}{m^4+1}$ is
(a)$\dfrac{47}{21}$ (b) $\dfrac{45}{21}$
(c) $\dfrac{43}{23}$ (d) $\dfrac{41}{23}$
Ans.(c)

Q27. If *x : y : z = 4 : 3 : 2* and
$x^2 + y^2 + z^2$ = *11600*, then the value of
$\sqrt{x + y - z}$ is
(a)10   (b)100
(c)180   (d)60
Ans.(a)

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