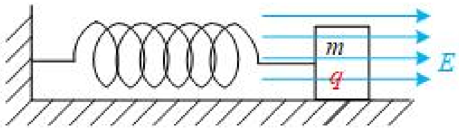
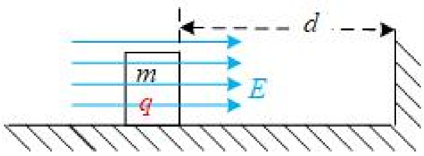
Code: Phy/ EM-1/EFnP/002

Electromagnetism: Electrostatics-Electric Fiend and Potential

(Selected Questions: Set2)

Important Note: Determination of Electric Field using Coulomb's Law basic concept based on which concept of electric potential has been developed. Yet, electric field at a point due to multiple charges or distributed charges becomes simple by determining electric potential. Extension of Coulomb's Law into Gauss's Law is a further simplification that is helpful in determining electric field at a point. It, however, require to define Gaussian surface with due consideration to geometrical symmetry of charge distribution. Illustrations here will lead to better understanding of the concepts and their applications.

Abbreviations: Q- Question, A- Answer, I – Illustration to the solution

Q-1	A block of mass m having a charge q is placed on a smooth horizontal able and is connected to a wall through an upstretched spring of spring constant k as shown in the figure. A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block. 
A-1	$\frac{qE}{k}$
I-1	The mass m carrying a charge q in presence of electric field would experience a force $F_E = qE$….(1) This would cause an elongation l in the spring whose spring constant is k with one end fixed such that restraining force as per spring law $F_s = kl$…(2) .This is the position of equilibrium of the mass such that $F_s = F_E$…(3). Combining (1) and (2) in (3) we have $kl = qE \Rightarrow l = \frac{qE}{k}$. But, in the process, the mass will have acquired kinetic energy and this cause an overshoot of the mass by a distance x, where spring creates an additional restraining force $f = -kx$, as per spring law. This force f is proportional to displacement from mean position and against direction of motion i.e. towards mean position. So also on achieving maximum displacement. This restraining force f will cause retardation on the mass until its velocity becomes zero. At this position, existence of f would accelerate the mass towards the mean position. But due to kinetic energy gained by the mass, as per Law of Conservation of Energy, it would again overshoot to compress the spring until it reaches its natural length. Thus, motion of the mass satisfies criteria of Simple Harmonic Motion (SHM) whose magnitude is $l = \frac{qE}{k}$. is the answer. **N.B.:** In the problem natural length of spring has no role. And spring is assumed to be an ideal spring following law of linear restraining force.
Q-2	A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure. The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion? 
A-2	$\sqrt{\frac{8md}{qE}}$, No

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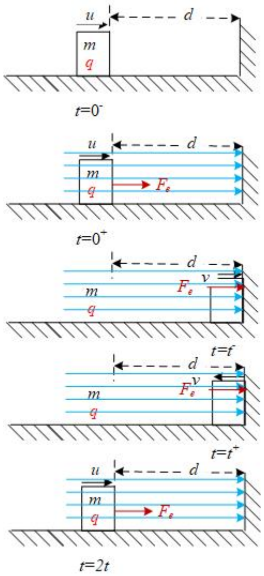
The mass m carrying a charge q, placed on a smooth table would, in absence of electric field, would continue to be in state of rest as per Newton's Third and First Law of motion. As soon electric field is switched in would experience a force $F_E = qE$….(1) and thereby an acceleration a, as per Newton's Second Law of Motion, $F_E = ma \Rightarrow a = \frac{F_E}{m} = \frac{qE}{m}$…(2). This question has two parts – **Part 1:** time period of oscillation, **Part II:** Is the oscillation a simple harmonic motion (SHM). Analysis is facilitated with five instance diagrams, and each stage is being analyzed-

At $t = 0^-$: is initial position when electric field is not switched on.

At $t = 0^+$: under influence of F_E , the mass at rest $u = 0$, and it will move toward wall with acceleration a.

At $t = t^-$: while covering a distance d, before it collides with the wall, will acquire velocity, as per Third Equation of Motion $v^2 = u^2 + 2ad \Rightarrow v = \sqrt{2ad}$. Time t taken by the mass to reach the wall, as per First equation of motion, $v = u + at \Rightarrow t = \frac{v}{a} = \frac{\sqrt{2ad}}{a} = \sqrt{\frac{2d}{a}}$…(3). Combining (2) and (3), $t = \sqrt{\frac{2d}{\left(\frac{qE}{m}\right)}} = \sqrt{\frac{2md}{qE}}$.



At $t = t^+$: time: after the elastic collision , as stated in the problem, the mass will return with the same velocity v, but in a direction opposite to the electric field as shown in the figure.

At $t = 2t$: Thus electric force, due to charge on the mass, during $t = t^+$to $t = 2t$ will cause retardation a of magnitude, but in direction opposite to velocity of the mass. Thus, as per equations of motion in another time t it will reach the original position, with velocity Zero to complete one cycle of oscillation .

A Thus total period of oscillation would be $T = 2t = 2\sqrt{\frac{2md}{qE}} \Rightarrow T = \sqrt{\frac{8md}{qE}}$ **is the answer of part I.**

This kind of motion with time period T would continue, between the wall and initial position of the mass, as long as electric field exists. In this case force on the particle is constant and unidirectional irrespective of the fact whether mass is approaching the wall or moving away from the wall. This is against basic parameters of a simple harmonic motion (SHM), which requires force to be proportional to displacement a from mean position. Hence, **motion of the mass is not SHM, is the answer of part II.**

N.B.: Infact this question can be framed into option question. Yet framing such question in an illustrative problem requies that necessary considerations are brought out, rather than just straight answer in few steps.

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<tr><td>Q-3</td><td>

An electric field of 20 NC^{-1} exits along X-axis on space. Calculate the potential difference $V_B - V_A$, such that points A and B are given by

(a) A = (0, 0); B = (4 m, 2 m)
(b) A = (4 m, 2 m); B = (6 m, 5 m)
(c) A = (0,0); B = (6 m, 5 m)

Find relation between the answers of part (a), (b) and (c).

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<tr><td>A-3</td><td>-80 V (b) -40 V (c) -120 V</td></tr>
<tr><td>I-3</td><td>

Given that $\vec{E} = 20\hat{\imath}$ N/C …(1), is uniform and pair of coordinates of A and B in three sets. For each set $\Delta V = V_B - V_A$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_B - V_A = -\left(\vec{E}.\vec{r}_B - \vec{E}.\vec{r}_A\right) \Rightarrow V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_{AB})$…(2). Using the given data in (2), and taking each part separately –

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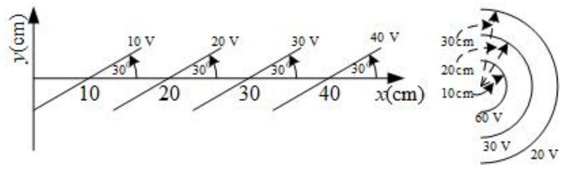
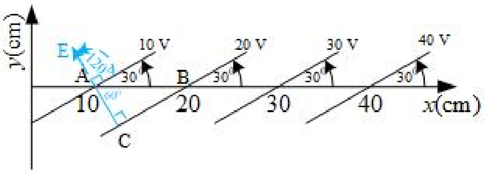
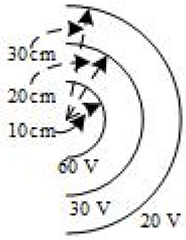
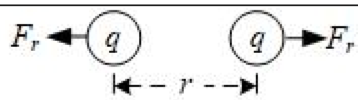
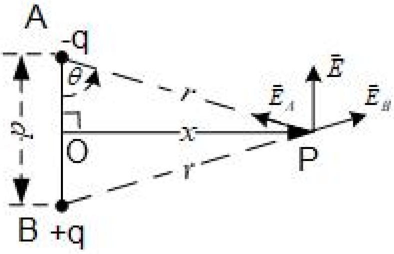
	Part (a): $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 4\hat{\imath} + 2\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (4\hat{\imath} + 2\hat{\jmath})\right) = -20 \times 4 = \mathbf{-80V}$ **Part (b):** $\vec{r}_A = 4\hat{\imath} + 2\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((4\hat{\imath} + 2\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right) = -20 \times 2 = \mathbf{-40V}$ **Part (x):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $V_B - V_A = 20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right) = -20 \times 6 = \mathbf{-120V}$ **Hence answers are (a) -80 V (b) -40 V (c) -120 V** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath}\cdot\hat{\imath} = 1; \hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = 0$. This is done to encourage mathematical ability.
Q-4	An electric field of 20 NC^{-1} exits along X-axis on space. A charge of -2.0×10^{-4} C is moved from point A to the point B. Find change in potential energy $U_B - U_A$, such that points A and B are given by (a) A = (0, 0); B = (4 m, 2 m) (b) A = (4 m, 2 m); B = (6 m, 5 m) (c) A = (0,0); B = (6 m, 5 m)
A-4	0.016 J, (b) 0.008 J, (c) 0.024 J
I-4	Given that $\vec{E} = 20\hat{\imath}$ N/C …(1), is uniform and pair of coordinates of A and B in three sets. For each set $\Delta V = V_B - V_A$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_B - V_A = -\left(\vec{E}.\vec{r}_B - \vec{E}.\vec{r}_A\right) \Rightarrow V_B - V_A = \vec{E}.(\vec{r}_A - \vec{r}_{AB})$…(2). Further, change in potential energy is $U_B - U_A = q(V_B - V_A)$ and given that $q = -2.0 \times 10^{-4}$C. Using the given data in (2), and taking each part separately – **Part (a):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 4\hat{\imath} + 2\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (4\hat{\imath} + 2\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 4) = \mathbf{0.016\ J.}$ **Part (b):** $\vec{r}_A = 4\hat{\imath} + 2\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((4\hat{\imath} + 2\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 2) = \mathbf{0.008\ J.}$ **Part (x):** $\vec{r}_A = 0\hat{\imath} + 0\hat{\jmath}$ and $\vec{r}_B = 6\hat{\imath} + 5\hat{\jmath}$, $U_B - U_A = (-2.0 \times 10^{-4})\left(20\hat{\imath}.\left((0\hat{\imath} + 0\hat{\jmath}) - (6\hat{\imath} + 5\hat{\jmath})\right)\right) = (-2.0 \times 10^{-4})(-20 \times 6) = \mathbf{0.024\ J}$ **Hence answers are (a) 0.016 J, (b) 0.008 J, (c) 0.024 J** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath}\cdot\hat{\imath} = 1; \hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = 0$. This is done to encourage mathematical ability.
Q-5	An electric field $\vec{E} = (20\hat{\imath} + 30\hat{\jmath})$NC^{-1} exists in the space. If potential at the origin is taken to be zero, find the potential at (2m, 2m).
A-5	-100 V
I-5	Given that electric field in pace is $\vec{E} = (20\hat{\imath} + 30\hat{\jmath})$ N/C …(1), and potential at origin $\vec{r}_O = 0\hat{\imath} + 0\hat{\jmath}$ is $V_O = 0$. It is required to find potential at a point P defined by $\vec{r}_P = 2\hat{\imath} + 2\hat{\jmath}$. Potential at point P ($\vec{r}_P$) w.r.t origin ($\vec{r}_O$) is $\Delta V = V_P - 0 \Rightarrow V_P = \Delta V$ and $V = -\vec{E}.\vec{r}$. Therefore, $V_P = -\left(\vec{E}.\vec{r}_P - \vec{E}.\vec{r}_O\right) \Rightarrow V_P = \vec{E}.(\vec{r}_O - \vec{r}_P)$…(2). Using the given data in (2), $V_P = \left(20\hat{\imath} + 30\hat{\jmath}\right).\left((0\hat{\imath} + 0\hat{\jmath}) - (2\hat{\imath} + 2\hat{\jmath})\right) \Rightarrow V_P = -\left(20\hat{\imath} + 30\hat{\jmath}\right)\cdot\left(2\hat{\imath} + 2\hat{\jmath}\right)$. It leads to $V_P = -\left(20\hat{\imath}\cdot 2\hat{\imath} + 30\hat{\jmath}\cdot 2\hat{\jmath}\right) \Rightarrow V_P = -(40 + 60) = \mathbf{-100}$ **V is the answer.** **N.B.:** This problem has been solved in pure mathematical manner using concept of **Dot product** of vectors which is scalar quantity such that $\hat{\imath}\cdot\hat{\imath} = 1$; $\hat{\jmath}\cdot\hat{\jmath} = 1$ and $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{\imath} = 0$. This is done to encourage

	mathematical ability. At the same time it easier to solve the problem mathematically rather than graphically.
Q-6	The electric potential existing in space is $V(x, y, x) = A(xy + yz + zx)$. (a) Write the dimensional formula of A (b) Find expression for the electric field (c) If A is 10 SI units, find the magnitude of electric field at (1m, 1m, 1m).
A-6	$MT^{-3}I^{-1}$ (b) $-A\{\hat{\imath}(y+z) + \hat{\jmath}(z+x) + \hat{k}(x+y)\}$ (c) 35 N/C
I-6	Taking each part separately – **Part (a):** Potential at a point is defined as work done in moving a unit (+) charge from infinity to the point. Accordingly, as per dimensional analysis $[\text{Potential}] = \frac{[\text{Work}]}{[\text{Charge}]} \Rightarrow [V] = \frac{ML^2T^{-2}}{IT} = ML^2T^{-3}I^{-1}$…(1). Dimensional equivalent of the given expression is $[V] = [A]L^2$…(2). Combining (1) and (2), $[A]L^2 = ML^2T^{-3}I^{-1} \Rightarrow [A] = \frac{ML^2T^{-3}I^{-1}}{L^2} = \mathbf{MT^{-3}I^{-1}}$ **is the answer**. **Part (b):** Electric potential is $\Delta V = -\vec{E}\cdot\Delta\vec{r} \Rightarrow \vec{E} = -\frac{\partial V}{\partial r}$…(3). It is a partial derivative. Given that $V(x, y, x) = A(xy + yz + zx)$. Taking partial derivative $\vec{E} = \frac{\partial}{\partial r}A(xy + yz + zx)$. It is to be noted that $\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$. Therefore, $\vec{E} = A\left(\hat{\imath}\frac{d}{dx}(xy + zx) + \hat{\jmath}\frac{d}{dy}(xy + yz) + \hat{k}\frac{d}{dz}(yz + zx)\right)$. It leads to $\vec{E} = A\left((y+z)\hat{\imath}\frac{d}{dx}x + (x+z)\hat{\jmath}\frac{d}{dy}y + (x+y)\hat{k}\frac{d}{dz}z\right)$, and finally $\boldsymbol{\vec{E} = A\left((y+z)\hat{\imath} + (x+z)\hat{\jmath} + (x+y)\hat{k}\right)}$. **Is the answer.** **Part (c):** Given $A = 10$ SI Unit and a point P is (1m, 1m, 1m). i.e. $x = 1\text{m}$, $y = 1\text{m}$ and $z = 1\text{m}$. Therefore electric field at P, with the given data $\vec{E}_P = 10\left((1+1)\hat{\imath} + (1+1)\hat{\jmath} + (1+1)\hat{k}\right) \Rightarrow \vec{E}_P = 20(\hat{\imath} + \hat{\jmath} + \hat{k}) = 20 \times \sqrt{1^2 + 1^2 + 1^2} \times \hat{r} \Rightarrow \vec{E}_P = 20\sqrt{3}\hat{r}$.Thus magnitude of electric field using principle of SDs is $20\sqrt{3} = $ **35 N/C is the answer.** **Thus, answers are (a) $MT^{-3}I^{-1}$ (b) $-A\{\hat{\imath}(y+z) + \hat{\jmath}(z+x) + \hat{k}(x+y)\}$ (c) 35 N/C.** **N.B.:** Part (b) of the illustration requires clarity of partial derivative of a vector, a part of Vector Calculus. This being an exclusive topic is being skipped in illustration
Q-7	Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field? 
A-7	(a) 200 V/m making an angle 120^0 with the X-axis (b) Radially outward, decreasing with distance as $E = \frac{6}{r^2}\ V/m$
I-7	In the given figure two sets of equipotential surfaces are shown – A) Plane surfaces and (b) spherical surfaces. Using the basics two important considerations are (a) relation between electric potential and electric field is $\Delta V = -\vec{E}\cdot\Delta\vec{r}$ …(1), and (b) electric field is always perpendicular to the equipotential surface in a direction from higher potential to lower potential. Applying these two consideration to each of the two given sets of surfaces –

	Set A: Equipotential surfaces are uniformly spaced. Therefore, taking points A and B on surfaces of potential 10 V and 20 V respectively $\Delta V = (20-10) = -E(r_b - r_a)\cos 120^0 \Rightarrow 10 = -E(0.2-0.1)(-0.5) \Rightarrow E = \frac{10}{0.1\times 0.5} = \mathbf{200V/m}$ **inclined at 120⁰ to X-axis is the answer.**  **Set B:** In this case surfaces are though parallel but they are spherical and therefore electric field will not be uniform in the space between two spherical surfaces. Further observation of the figure reveals that with the increase in the radius of the spherical surface potential is decreasing. Therefore, electric field is radially outward with charge causing the electric field at the center of the sphere. In such cases $V = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow Vr = \text{Cont}\ldots(2)$. Using (2), spacing of the electric field is to be verified. Three surfaces $V_1 = 60$ V, $V_2 = 30$ and $V_3 = 20$ V whose radius of curvature are $rV_1 = 0.10$ m, $r_2 = 0.20$ m and $r_3 = 0.30$ m. And we see that $V_1r_1 = V_2r_2 = V_3r_3 = 6$ Vm...(3) Hence, using (2) again, the premise that the equipotential spherical surfaces are caused by a charge $Q = Vr \times 4\pi\varepsilon_0$ ... (4). Accordingly, electric field at a distance r from O, the center of curvature of the concentric spherical surface, as per Coulomb's Law is $E = \frac{Q}{4\pi\varepsilon_0 r^2}\ldots(5)$. Combining (4) and (5), $E = \frac{Vr\times 4\pi\varepsilon_0}{4\pi\varepsilon_0 r^2} = \frac{Vr}{r^2}\ldots(6)$. Combining (3) and (6), $E = \frac{6}{r}$ **V/m, is the answer, radially outward.**  **Thus answers are (a) 200V/m inclined at 120⁰ to X-axis is the answer (b)** $\frac{6}{r}$ **V/m, is the answer, radially outward.** **N.B.:** This questions has mix of two problems of same nature, but with different nature of equipotential surfaces. But, approach to solution requires identical basic concepts.
Q-8	Two identical particles each having a charge 2.0×10^{-4} C and mass of 10 g are kept at a separation of 10 cm and then released. What would be the speed of the particle when separation becomes large.
A-8	600 m/s
I-8	Given that equal charges are of magnitude $q = 2.0\times 10^{-4}$ C, having mass $m = 10\times 10^{-3}$kg are separated by $r_i = 0.10$m. Potential energy of two charges at a separation r is $PE_r = V_r q \Rightarrow PE_r = \left(\frac{q}{4\pi\varepsilon_0 r}\right)q$. Therefore, initial potential energy would be $PE_{r_i} = \frac{q^2}{4\pi\varepsilon_0 r_i}$. When charges are separated at $r \gg$ then $PE_r = \left.\frac{q^2}{4\pi\varepsilon_0 r}\right\|_{r\gg} = 0$. Therefore, $\Delta PE = PE_r - PE_{r_i} \Rightarrow \Delta PE = 0 - \frac{q^2}{4\pi\varepsilon_0 r_i} = -\frac{q^2}{4\pi\varepsilon_0 r_i}\ldots(1)$  As per principle of conservation of energy, as system undergoing changes under internal forces $\Delta U = 0 = \Delta W + \Delta KE \Rightarrow \Delta KE = -\Delta W$. On its integration $KE = -W\ldots(2)$. Both the charges, having equal mass, are at initially at rest, and are experiencing equal and opposite forces. Therefore, will acquire equal velocities in direction of forces acting upon them, their total kinetic energy would be $KE = 2\times\left(\frac{1}{2}mv^2\right)\ldots(3)$. Combing (1), (2) and (3) $2\times\left(\frac{1}{2}mv^2\right) = -\left(-\left(\frac{q\times q}{4\pi\varepsilon_0}\right)\frac{1}{r_i}\right) \Rightarrow v = q\left(\sqrt{\frac{1}{4\pi\varepsilon_0}\times\frac{1}{mr_i}}\right)$ Using the data given in the problem $v = (2.0\times 10^{-4})\left(\sqrt{(9\times 10^9)\times\frac{1}{(10\times 10^{-3})\times 0.1}}\right) \Rightarrow v = (2.0\times 10^{-4})(3\times 10^6) \Rightarrow v = 6.0\times 10^2$m/s **or 600 m/s is the answer.**

	N.B.: This problem can also be solved by using general expression of electric force, which is function of separation r, determining work done electric field, instead of direct use of general expression of potential energy of charges. But, that ignores concept of PE, gained and is less correct.
Q-9	Two particles have equal masses of 5.0 g each and opposite charges $+4.0 \times 10^{-5}$ C and -4.0×10^{-5}C. They are released from rest with a separation of 1.0 m between them. What would be the speed of the particles when separation is reduced to 50 cm?
A-9	54 m/s for each particle
I-9	Given that two charges are of magnitude $q_1 = 4.0 \times 10^{-5}$ C and $q_2 = -4.0 \times 10^{-5}$ C having mass $m = 5 \times 10^{-3}$kg are separated by $r_i = 1.0$m. Potential energy of two charges at a separation r is $PE_r = V_r q \Rightarrow PE_r = \left(\frac{q_1}{4\pi\varepsilon_0 r}\right) q_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$. Therefore, initial potential energy would be $PE_{r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_i}$. When charges are separated at $r_f = 0.50$ m, then $PE_{r_f} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_f}$. Therefore,  $\Delta PE = PE_{r_f} - PE_{r_i} \Rightarrow \Delta PE = \frac{q_1 q_2}{4\pi\varepsilon_0 r_f} - \frac{q_1 q_2}{4\pi\varepsilon_0 r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$...(1) As per principle of conservation of energy, as system undergoing changes under internal forces $\Delta U = 0 = \Delta W + \Delta KE \Rightarrow \Delta KE = -\Delta PE$. On its integration $KE = -PE$...(2). Both the charges, having equal mass, are at initially at rest, and are experiencing equal and opposite forces. Therefore, will acquire equal velocities in direction of forces acting upon them, their total kinetic energy would be $KE = 2 \times \left(\frac{1}{2}mv^2\right)$...(3). Combing (1), (2) and (3) $2 \times \left(\frac{1}{2}mv^2\right) = -\left(\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{m}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right) \Rightarrow v = \left(\sqrt{\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{m} \times \left(\frac{1}{r_i} - \frac{1}{r_f}\right)}\right)$ Using the data given in the problem $v = \left(\sqrt{(9 \times 10^9) \times \frac{(4.0\times10^{-5})(-4.0\times10^{-5})}{5\times10^{-3}} \times \left(\frac{1}{1} - \frac{1}{0.5}\right)}\right)$. It leads to $v = \frac{(120)}{\sqrt{5}} \Rightarrow v =$ **54m/s is the answer.** **N.B.:** This problem can also be solved by using general expression of electric force, which is function of separation r, determining work done electric field, instead of direct use of general expression of potential energy of charges. But, that ignores concept of PE, gained and is less correct.
Q-10	Two particles A and B having opposite charges $+2.0 \times 10^{-6}$ C and -2.0×10^{-6}C are placed at a separation of 1.0 cm. (a) Write down the electric dipole moment of the pair (b) Calculate the electric field at a point on the axis of the dipole at 1.0 cm away from center. Calculate the electric field at a point on the perpendicular bisector of the dipole and 1.0 m away from the center.
A-10	(a) 2.0×10^{-8}Cm (b) 1.3×10^{8} N/C (c) 180 N/C
I-10	The two opposite charges of magnitude $q = 2.0 \times 10^{-6}$ C placed at A and B are separated by $d = 0.01$m as shown in the figure. Taking each part separately – **Part (a):** Electric dipole moment is $\vec{P} = q d\vec{\imath}$...(1) Therefore, its magnitude using the available data in (1) we have $P = (2.0 \times 10^{-6})(1.0 \times 10^{-2}) = 2.0 \times 10^{-8}$ Cm is the answer.

Part (b): Electric field at a point P on the axis of the dipole, which passes from its midpoint O and is perpendicular to AB, as per Coulomb's Law is $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$. Here, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$. Accordingly, two charges of the dipole would produce $\vec{E}_A = \frac{+q}{4\pi\varepsilon_0 r^2}\widehat{AP}$ and $\vec{E}_B = \frac{-q}{4\pi\varepsilon_0 r^2}\widehat{BP}$ as shown in the figure and both the field are symmetrical about perpendicular to axis OP. Therefore, their components along $\hat{j}$ would be equal and opposite cancelling each other. Whereas, their component along axis P would be $\vec{E} = \vec{E}_A + \vec{E}_B = \frac{2q}{4\pi\varepsilon_0 r^2}\cos\theta\,\hat{j} \Rightarrow E = \frac{2q}{4\pi\varepsilon_0 r^2}\cos\theta$…(2)



Here, from given data $r = \sqrt{x^2 + \left(\frac{d}{2}\right)^2}$ and $\sin\theta = \frac{\frac{d}{2}}{r} = \frac{d}{2\times\sqrt{x^2+\left(\frac{d}{2}\right)^2}}$. Thus final form of (2) is $E = \frac{2q}{4\pi\varepsilon_0} \times \frac{d}{2\left(\sqrt{x^2+\left(\frac{d}{2}\right)^2}\right)^3} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \times \frac{qd}{\left(\sqrt{x^2+\left(\frac{d}{2}\right)^2}\right)^3}$ …(3). Using the available data $E = (9 \times 10^9) \times \frac{(2.0\times10^{-6})(1.0\times10^{-2})}{\left(\sqrt{(1.0\times10^{-2})^2+\left(\frac{1.0\times10^{-2}}{2}\right)^2}\right)^3} = \frac{18}{\left(\frac{\sqrt{5}\times10^{-2}}{2}\right)^3} = \mathbf{1.3 \times 10^8}$ **N/C is answer of part (b).**

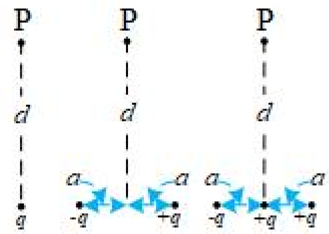
Part (c): In this part all other parameters remain the same as in part (b) except that $x = 1.0\ m$ or $x \gg d \Rightarrow$ $x^2 + \left(\frac{d}{2}\right)^2 \approx x^2 \Rightarrow \left(\sqrt{x^2 + \left(\frac{d}{2}\right)^2}\right)^3 = x^3$. Accordingly, (3) approximates to $E = \frac{1}{4\pi\varepsilon_0} \times \frac{qd}{x^3}$.

Thus using the available data $E = (9 \times 10^9) \times \frac{(2.0\times10^{-6})(1.0\times10^{-2})}{1^3} = 180$ **N/C is answer of part (c).**

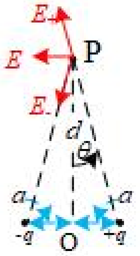
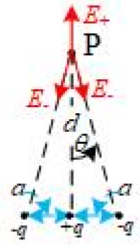
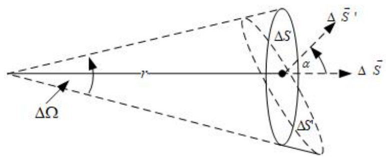
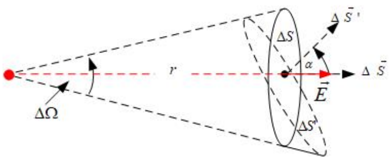
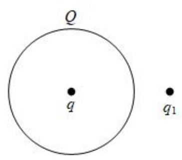
Thus answers are (a) 2.0×10^{-8}Cm (b) 1.3×10^7 N/C (c) 180 N/C.

N.B.: Part (b) and (c) are identical except value of x, in part (b) $x = d$ and hence approximation is not done. But, in part (c) $x \gg d$ and leads to approximation. It is a very explicit case which emphasizes that conceptual approach is more error proof, while use of direct formula may incorporate approximation which may not be valid in specific problem, and thus lead to wrong answer.

Q-11 Find the magnitude of the electric field at the point P in the configuration shown in the figure for $d \gg a$. Take $2qa = p$.



A-11 (a) $\frac{q}{4\pi\varepsilon_0 d^2}$ (b) $\frac{p}{4\pi\varepsilon_0 d^3}$ (c) $-\frac{q}{4\pi\varepsilon_0 d^3}$

I-11	Taking each case separately-  **Case (a):** It has a single charge and, therefore, as per Coulomb's Law, magnitude of the electric field is $E = \frac{q}{4\pi\varepsilon_0 d^2}$ **is answer of part (a).** **Case (b):** It is case of a dipole where $d \gg a$, and distance of point P from charges is $r = \sqrt{d^2 + a^2} \Rightarrow r \approx d$. Here, as shown in the figure, as per geometrical symmetry resultant of fields E_+ and E_- along OP is zero, however, electric field at P parallel to line q_+ and q_- is $E = E_+ \sin\theta + E_+ \sin\theta = \frac{q}{4\pi\varepsilon_0 d^2} \times 2\sin\theta$. It leads to $E = \frac{q}{4\pi\varepsilon_0 d^2} \times \frac{2a}{d} \Rightarrow E = \frac{2aq}{4\pi\varepsilon_0 d^3}$. It is given that $2qa = p$, therefore. $E = \frac{p}{4\pi\varepsilon_0 d^3}$ **is answer of part (b).**  **Case (c):** Net electric field at P due system of charges as shown in the figure $E = E_+ - 2E_- \cos\theta$. Here, $E_- = \frac{q}{4\pi\varepsilon_0 (d^2+a^2)} = \frac{q}{4\pi\varepsilon_0 d^2}\Big\|_{d\gg a}$, $E_+ = \frac{q}{4\pi\varepsilon_0 d^2}$ and $\cos\theta = \frac{d}{\sqrt{d^2+a^2}} \Rightarrow \cos\theta = 1\|_{d\gg a}$. Accordingly, $E = \frac{q}{4\pi\varepsilon_0 d^2} - 2 \times \frac{q}{4\pi\varepsilon_0 d^2} \times 1 = -\frac{q}{4\pi\varepsilon_0 d^2}$ **is the answer of part (c).**
Q-12	A small plane area is rotated in an electric field. In which orientation of the area is the flux of the electric field through the area maximum? In which orientation is it zero?
A-12	**(a) $\Delta\phi\|_{\text{Max}}$ at $\alpha = 0$ and (b) $\Delta\phi = 0$ at $\alpha = \pm\frac{\pi}{2}$**
I-12	 As per Gauss's Law flux out of a surface area s per Gauss's Law flux out of a surface area $\Delta\phi = \vec{E}.\Delta\vec{S}$. Here, $\Delta\vec{S}$ is small surface-element, it is vector quantity of magnitude ΔS and direction perpendicular to the the elemental surface area as shown in the figure and $\vec{E}$ is the electric field at ΔS, while total flux $\Delta\phi$ is a scalar quantity of magnitude $\Delta\phi = E\Delta S\cos\alpha$, where $\alpha$is the angle between vectors $\vec{E}$ and $\Delta\vec{S}$.  In the instant case a small plane of the area ΔS is being rotated in an electric field $\vec{E}$, which are constant, and it is required orientation α when flux through $\Delta\phi$ is maximum and minimum. **Maximum value of $\Delta\phi$ is when $\cos\alpha\|_{\alpha=0} = 1$ and minimum value of $\Delta\phi$ is when $\cos\alpha\|_{\alpha=\pm\frac{\pi}{2}} = 0$ or $\Delta\phi = 0$** **Thus answers are (a) $\Delta\phi\|_{\text{Max}}$ at $\alpha = 0$ and (b) $\Delta\phi = 0$ at $\alpha = \pm\frac{\pi}{2}$**
Q-13	A thin, metallic shell contains a charge Q on it. A point charge q is placed at the center of the shell and another charge q_1 is placed outside it as shown in the figure. All the three charges are positive. The force on the central charge due to the shell is –  Towards left (b) Towards right (c) Upward (d) Zero
A-13	(b)
I-13	It is required to determine force experienced on charge q placed at the center of thin spherical shell. All the charges are given to be positive. The problem is solved in two phases an then direction of the net force on q is determined using superimposition principle as under-

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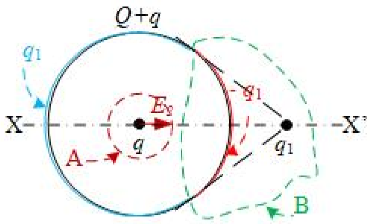
Phase I - Electric field at q due to charge Q: Consider a Gaussian surface A around the charge q. Since the charge The Q is outside A hence electric field at A would be $E_1 = 0$. Therefore, force on charge q would be $\boldsymbol{F_1 = E_1 q}$**…(1)**

Phase II – Electric field due to spherical shell due to charge q_1: The charge $q_1$would electrostatically induce charge $(-q_1)$ on the part of surface of the shell facing the charge q_1, this area would be enclosed in conical cap tangential to the spherical shell whose boundary is depicted by tangents on sphere drawn from q_1. While the remaining part of the spherical shell will have $(+q_1)$ charge. Thus, based on induced charge on the spherical shell a Gaussian surface B is y=taken enclosing the charge q_1 and induced charge $(-q_1)$. Net charge inside B is $q_1 + (-q_1) = 0$, hence electric field at B caused by charges inside B is zero. But, the induced charge $(+q_1)$ on the remaining part of the shell is on the left of charge q and is symmetrical about the line X-X' passing through charge q. Therefore, by geometrical symmetry it will produce electric field E_2 at q directed towards right. Therefore, force on charge q due to shell, eventually charges on the shell will be $\boldsymbol{F_2 = E_2 q}$ **…(2), will be directed towards right.**



Superimposing analysis in phase I and phase II ne force on charge q due to charge on shell would be obtained by combining (1) and (2) $F = F_1 + F_2 = 0 + E_2 q = \boldsymbol{E_2 q}$ in **option (b) is the answer.**

N.B.: It is a good case of superimposition of electric field.

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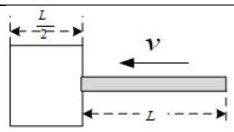
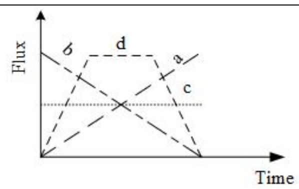
<tr><td>Q-14</td><td>

Figure shows an imaginary cube of edge $\frac{L}{2}$. A uniformly charged rod of length L moves toward left at a small but constant speed v. At $t = 0$, the left end touches the center of the face of the cube opposite it. Which graph shown in the figure represents the flux of the electric field through the cube as the rod goes through it?





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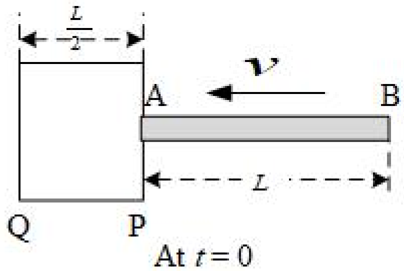
<tr><td>A-14</td><td>(d)</td></tr>
<tr><td>I-14</td><td>

In the given figure the charged rtod is passing through an imaginary cube withat a constant speed.. The cube in this case is like a Gaussian surface. As per Gauss's Law $\phi = \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$…(1), here Q is the amount of charge inside the Gaussian surface. It is given that charge on the rod is uniformly distributed and therefore charge per unit length of the rod is $q = \frac{Q}{L}$…(2). The problem is analyzed in Five states as under –



State 1 at $t = 0$: End A of the rod, while being outside the cube, is touching the center of its surface P as shown in the figure. Since there is no charge inside the Gaussian Surface i.e. cube, hence the flox through surface of the cube is $\phi_0 = \frac{\Delta q}{\varepsilon_0} = 0$.

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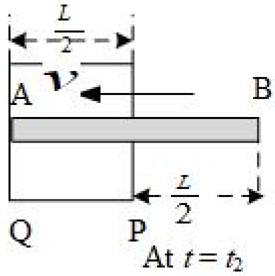
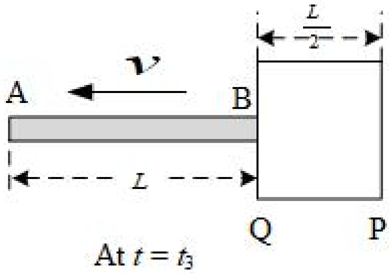
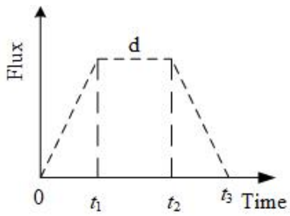
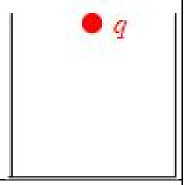
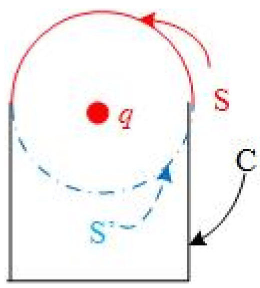
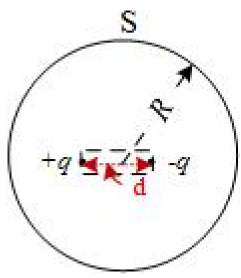
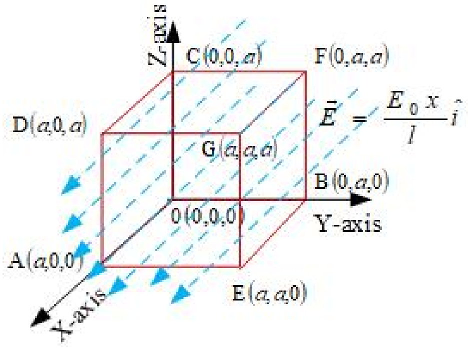
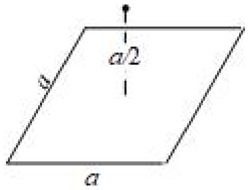
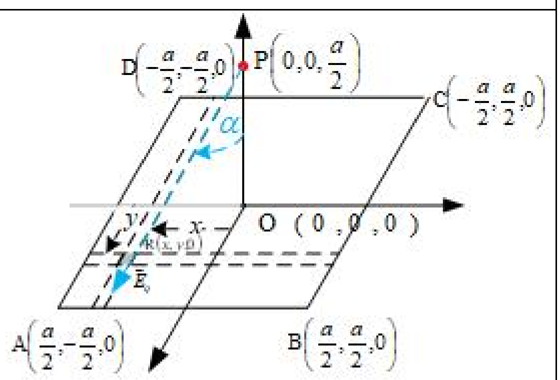
	Stage 2 during $0 < t < t_2$: Rod is moving through the cube (i.e. entering it) and is short of touching surface Q such that end A of the rod is ion-between face P and Q. Since, size of the cubical box is $\frac{L}{2}$, for the rod moving at a constant speed $t_2 = \frac{\frac{L}{2}}{v} \Rightarrow t_2 = \frac{L}{2v}$…(3)Further, charge at any time inside the Gaussian surface is $q_t = (vt)q$…(4). Combining (1), (2) and (4), flux through the box will be $\phi_t = \frac{q_t}{\varepsilon_0} \Rightarrow \phi_t = \frac{1\times(vt)q}{\varepsilon_0} \Rightarrow \phi_t = \frac{Qv}{\varepsilon_0 L}t \Rightarrow \phi_t \propto t$, since, $Q.v, L$ and ε_0 are constant. It implies that during this period flox throgh the cube is increasing linearly.  **Stage 3 during $t = t_2$:** At his instant End A of the rod touches face Q of the cube and at P is the mid point of the rod. Thus half of the length of the rod is inside the cube. Therefore, combining (1), (2) and (3) flux through the cube would be $\phi_t = \frac{q_2}{\varepsilon_0} \Rightarrow \phi_t = \frac{1\times(vt_2)q}{\varepsilon_0} \Rightarrow \phi_t = \frac{1}{\varepsilon_0}\times v\times\frac{L}{2v}\times\frac{Q}{L} \Rightarrow \phi_t = \frac{Q}{2}$. **Stage 4 during $t_2 < t < t_3$:** Rod is moving through the cube (i.e. coming out of it). During this period end B of the rod moves from face P towards face Q and is short of leaving the face Q. Using the discussions in stage 2, length of the rod inside the cube reduces linearly. Therefore, $\phi_t = -\frac{Qv}{\varepsilon_0 L}t \Rightarrow \phi_t \propto -t$.  **Stage 5 during $t = t_3$:** At his instant End B of the rod will be touching face Q of the cube charge inside the cube $q_3 = 0$. Hence at this instant electric flux throgh the cube as per (1) would be $\phi_3 = \frac{q_3}{\varepsilon_0} \Rightarrow \phi_3 = \frac{0}{\varepsilon_0} = 0$.  This kind of variation of flux through the cube is characterized only in **graph (d) the answer.**
Q-15	A charge q is placed at the center of the open end of a cylindrical vessel as shown in the figure. The flux of the electric field through the surface of the vessel is  Zero (b) $\frac{q}{\varepsilon_0}$ (c) $\frac{q}{2\varepsilon_0}$ (d) $\frac{2q}{\varepsilon_0}$
A-15	(c)
I-15	Charge q is placed at the center o the opening of a cylinder C as shown in the figure. A Gaussian surface combining S and S' encloses charge q and is symmetrical about it. Therefore, electric field at the two surfaces. would be radial and equal in magnitude at every point on them $\lvert\vec{E}_S\rvert = \lvert\vec{E}_{S'}\rvert$. Moreover their areas $A_S = A_{S'}$  Thus, as per Gauss's Law $\phi = \oint \vec{E}\cdot d\vec{s} = \int \vec{E}_S\cdot d\vec{s} + \int \vec{E}_{S'}\cdot d\vec{s'} = \frac{q}{\varepsilon_0}$. According to the discussions above, $\phi = \int \vec{E}_S\cdot d\vec{s} + \int \vec{E}_{S'}\cdot d\vec{s'} \Rightarrow \phi = \phi_S + \phi_{S'} \Rightarrow 2\phi_{S'} = \frac{q}{\varepsilon_0} \Rightarrow \phi_{S'} = \frac{q}{2\varepsilon_0}$. The surface S' together with the surface C form a closed Gaussian surface within which no charge. Therefore as per Gauss's Law $\phi_{S'} + \phi_C = \frac{0}{2\varepsilon_0} \Rightarrow \phi_C = -\phi_{S'}$For this combined new Gaussian surface (-)ve sign is assigned to flux entering it and therefore conversely flux leaving the surface would be (+)ve. Thus taking $\phi_{S'}$, as per sign convention $\phi_C = -\left(-\frac{q}{2\varepsilon_0}\right) \Rightarrow \phi_C = \frac{q}{2\varepsilon_0}$**, is the answer as per option (c).**

Q-16	An electric dipole is placed at the center of a sphere. Mark the correct options – (a) The flux of the electric field through the sphere is zero (b) The electric field is zero at every point on the sphere (c) The electric field is zero anywhere on the sphere The electric field is zero on a circle on the sphere
A-16	(a), (c)
I-16	 The given problem is conceptualized in the figure shown. As per Gauss's law, $\phi = \oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$ where is the flux through a closed surface, $\oint \vec{E}.d\vec{s}$ is the surface integral of electric field $\vec{E}$ along the surface vector $d\vec{s}$, and q is the net electric charge enclosed inside the surface. Here, sphere acts like a Gaussian surface, and it is enclosing a dipole. whose net charge is $Q = +q + (-q) = 0$, while surface area of any sphere is non-zero. Therefore, $\phi = 0$ as provided in **option (a), and it is correct.** Further, assuming any other Gaussian surface inside the sphere, enclosing the dipole, electric field at any point on it would be zero as per Gauss's Law. **It makes option (c) as correct.** Hence, **answer is options (a) and (c).** **N.B.:** In the figure distance d between charges of dipole is small as compared to the geometry under consideration. But, it could not be shown in the figure due to visual clarity.
Q-17	The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}$ with $E_0 = 2.0 \times 10^3$N/C. find the flux of this field through a rectangular surface of area 0.2 m^2 parallel to Y-Z plane.
A-17	240 Nm^2C^{-1}
I-17	Equation of a surface area $A = 0.2$ m^2 parallel to Y-Z plane is $\vec{A} = A\hat{\imath}$. Given that $\vec{E} = \frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}$ where $E_0 = 2.0 \times 10^3$. Then as per Gauss's Law flux through the area is $\phi = \oint \vec{E}.d\vec{s} = \left(\frac{3}{5}E_0\hat{\imath} + \frac{4}{5}E_0\hat{\jmath}\right) \cdot A\hat{\imath} \Rightarrow$ $\phi = \frac{3}{5}E_0A(\hat{\imath}\cdot\hat{\imath}) + \frac{3}{5}E_0A(\hat{\imath}\cdot\hat{\jmath}) = \frac{3}{5}E_0A$. Thus using the given data $\phi = \frac{3}{5}(2.0 \times 10^3)(0.2) =$ **240 Nm^2C^{-1} is the answer.** **N.B.:** Illustration has been developed in a pure mathematical manner without a supporting figue to catalyze visualization of concept among reader.
Q-18	The electric field in a region is given by $\vec{E} = \frac{E_0x}{l}\hat{\imath}$. Find the charge contained inside a cubical volume bounded by surface $x = 0, x = a. y = a. z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N/C, $l = 2$ cm and $a = 1$ cm.
A-18	2.2×10^{-12}C

I-18	Charge inside a region as per Gauss's Law is $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$…(1) The region has been specified by a cubical surface and as shown in the figure. Area of the surfaces enclosing the region are $A_1 = (\mathrm{AEGD})_{ar} = a^2$, $A_1 = (\mathrm{AEGF})_{ar} = a^2$, $A_2 = (\mathrm{COBF})_{ar} = a^2$, $A_3 = (\mathrm{AOCD})_{ar} = a^2$, $A_4 = (\mathrm{EBFG})_{ar} = a^2$, $A_5 = (\mathrm{DGFC})_{ar} = a^2$ and $A_6 = (\mathrm{AEB0F})_{ar} = a^2$.  It is given that $\vec{E} = \frac{E_0 x}{l}\hat{\imath}$. Thus, using (1) $\frac{q}{\varepsilon_0} = \left[\frac{E_0 x}{l}\hat{\imath}.\left((A_1 + A_2)\hat{\imath} + (A_3 + A_4)\hat{\jmath} + (A_5 + A_2)\hat{k}\right)\right]$….(2) Using the magnitudes of each surface area of the cubical volume in (2) with the corresponding DOT products $\frac{q}{\varepsilon_0} = \left[\frac{E_0 x}{l}(A_1 + A_2)\right]$…(3). Using values of x corresponding to A_1 and A_2 as a and 0 respectively. Thus, (3) gets transformed into $\frac{q}{\varepsilon_0} = \left[\frac{E_0}{l}(a \times a^2 + 0 \times a^2)\right] \Rightarrow q = \frac{\varepsilon_0 E_0 a^3}{l}$. Using the available data $q = \frac{(8.85\times10^{-12})\times(5\times10^{3})\times(1\times10^{-2})^3}{(2\times10^{-2})} \Rightarrow q = \mathbf{2.2 \times 10^{-12}}$ **C is the answer.**
Q-19	A charge Q is placed at a distance $\frac{a}{2}$ above the center of a horizontal, square surface of edge a as shown in the figure. Find the flux of the electric field through the square surface. 
A-19	$\frac{Q}{6\varepsilon_0}$
I-19	Given system is shown in the figure. Where, charge Q is placed at $\mathrm{P}\left(0,0,\frac{a}{2}\right)$ and the given surface ABCD is along $\hat{\jmath}$. Consider an elemental area at $\mathrm{R}(x,y,0)$ of size $\Delta\vec{A} = (\Delta x \times \Delta y)(-\hat{\jmath})$. ..(1). Electric field at the point shall be along PQ such that $\vec{E}_R = \frac{Q}{4\pi\varepsilon_o r^2}\hat{r}$…(2) Here,$\vec{r} = r\hat{r} \Rightarrow r = \sqrt{x^2 + y^2 + \left(\frac{a}{2}\right)^2}$..(3). Therefore, flux through the surface ABCD as per Gauss's Law is $\phi = \int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}} \vec{E}_R \cdot \vec{A}dx\right)dy$…(4).  Using (1), (2) and (3) in (4) $\phi = \frac{Q}{4\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{\cos\alpha}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)}dx\right)dy$…(5). Here, $\alpha = \angle\mathrm{OPR}$ and $\cos\alpha = \frac{\mathrm{OP}}{\mathrm{OR}} = \frac{\frac{a}{2}}{\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}} \Rightarrow \cos\alpha = \frac{a}{2\times\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}}$…(6). Combining (5) and (6) $\phi = \frac{Q}{4\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)} \times \frac{a}{2\times\sqrt{x^2+y^2+\left(\frac{a}{2}\right)^2}}dx\right)dy \Rightarrow \phi = \frac{aQ}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx\right)dy \Rightarrow \frac{aQ}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}(I)\,dy$…(7). Here, $I = \int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx$ is the inner integral.

For solving $I=\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(x^2+y^2+\left(\frac{a}{2}\right)^2\right)^{\frac{3}{2}}}dx$ a substitution is made $p^2=y^2+\left(\frac{a}{2}\right)^2$ and $x=p\tan\theta \Rightarrow x = p\sec^2\theta\, d\theta$ We have $I=\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\left(p^2\tan^2\theta+p^2\right)^{\frac{3}{2}}}p\sec^2\theta\, d\theta=\frac{1}{p^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{\sec^3\theta}\sec^2\theta\, d\theta=\frac{1}{p^2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\theta\, d\theta=\frac{1}{p^2}[\sin\theta]_{-\frac{a}{2}}^{\frac{a}{2}}$. We know that $\sin\theta=\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}=\frac{x}{\left(\sqrt{x^2+p^2}\right)}$. This leads to $I=\frac{1}{p^2}\left[\frac{x}{\sqrt{x^2+p^2}}\right]_{-\frac{a}{2}}^{\frac{a}{2}}=\frac{1}{p^2}\left[\frac{\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2+p^2}}-\frac{\left(-\frac{a}{2}\right)}{\sqrt{\left(-\frac{a}{2}\right)+p^2}}\right]\Rightarrow I=\frac{2a}{p^2\times\sqrt{a^2+4p^2}}$. Substituting p we have $I=\frac{2a}{\frac{(4y^2+a^2)}{4}\times\sqrt{4y^2+2a^2}}\Rightarrow I=\frac{8a}{(4y^2+a^2)\times\sqrt{4y^2+2a^2}}\Rightarrow I=\frac{4\sqrt{2}a}{(4y^2+a^2)\times\sqrt{2y^2+a^2}}$.

Using expression of I in (7) we have $\phi=\frac{4\sqrt{2}a^2Q}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{1}{(4y^2+a^2)\times\sqrt{2y^2+a^2}}\right)dy$. Instead of taking this expression as an integral of product of two functions a substitution is made $\sqrt{2}y=a\tan\varphi\Rightarrow\sqrt{2}dy=a\sec^2\varphi\, d\varphi$, we have $\phi=\frac{4\sqrt{2}a^2Q}{8\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{1}{a^2(2\tan^2\varphi+1)\times a\sec\varphi}\right)\frac{a\sec^2\varphi}{\sqrt{2}}d\varphi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\sec\varphi}{2\, sec^2\varphi-1}\right)d\varphi$. We know that $sec^2\varphi=1+tan^2\varphi\Rightarrow 2\tan^2\varphi+1=2\, sec^2\varphi-1$. Thus, $\phi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\sec\varphi}{2\, sec^2\varphi-1}\right)d\varphi\Rightarrow\phi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\frac{1}{\cos\varphi}}{\frac{2}{\cos^2\varphi}-1}\right)d\varphi$. It solves to $\phi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\cos\varphi}{2-\cos^2\varphi}\right)d\varphi\Rightarrow\phi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\left(\frac{\cos\varphi}{1+\sin^2\varphi}\right)d\varphi$.

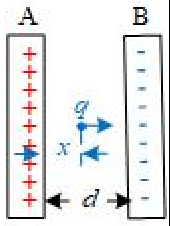
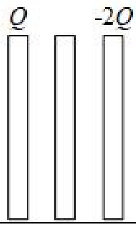
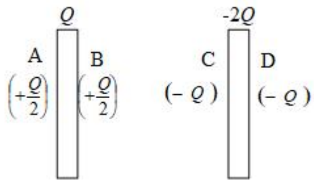
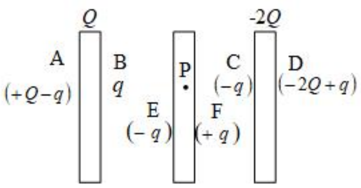
Substituting, $\sin\varphi=t\Rightarrow\cos\varphi\, d\varphi=dt$. Accordingly, $\phi=\frac{Q}{2\pi\varepsilon_o}\int_{-\frac{a}{2}}^{\frac{a}{2}}\frac{1}{1+t^2}dt$. At this stage standard integration $\int\frac{1}{1+t^2}dt=\tan^{-1}t$ is used. It leads to , $\phi=\frac{Q}{2\pi\varepsilon_o}[\tan^{-1}t]_{-\frac{a}{2}}^{\frac{a}{2}}$. Here, limits are for initial variable y in (5).

Hence, doing reverse substitution $\tan^{-1}t=\tan^{-1}(\sin\varphi)=\tan^{-1}\left(\frac{\tan\varphi}{\sqrt{1+\tan^2\varphi}}\right)=\tan^{-1}\left(\frac{\frac{\sqrt{2}y}{a}}{\sqrt{1+\frac{2y^2}{a^2}}}\right)=\tan^{-1}\left(\frac{\sqrt{2}y}{\sqrt{a^2+2y^2}}\right)$. Accordingly, $\phi=\frac{Q}{2\pi\varepsilon_o}\left[\tan^{-1}\left(\frac{\sqrt{2}y}{\sqrt{a^2+2y^2}}\right)\right]_{-\frac{a}{2}}^{\frac{a}{2}}\Rightarrow\phi=\frac{Q}{2\pi\varepsilon_o}\left[\tan^{-1}\left(\frac{\sqrt{2}\frac{a}{2}}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\left(-\frac{a}{2}\right)}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)\right]$.

It leads to $\phi=\frac{Q}{2\pi\varepsilon_o}\left[\tan^{-1}\left(\frac{\sqrt{2}\frac{a}{2}}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)+\tan^{-1}\left(\frac{\sqrt{2}\left(-\frac{a}{2}\right)}{\sqrt{a^2+2\left(\frac{a}{2}\right)^2}}\right)\right]\Rightarrow\phi=\frac{Q}{2\pi\varepsilon_o}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\Rightarrow\phi=\frac{Q}{2\pi\varepsilon_o}\times\frac{\pi}{3}\Rightarrow\phi=$ **$\frac{Q}{6\varepsilon_o}$, is the answer.**

N.B.: This has become an intensely mathematical solution. It could have been solved using Gauss's Law and make it worth an objective problem. Therefore, for the problem being subjective full length analytical illustration has been made. Thus, indispensability of mathematical proficiency in handling problems on physics I unique problems has been emphasized.

Q-20 Two large conducting plates are placed parallel to each other with a separation 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces.

A-20	5.03×10^{-13}C/m^2
I-20	 Given the large conducting parallel plates plate having charge on their inner surface is shown in the figure. Thus electric field at a point between the two plates due to plate A having positive charge, as per Gauss's Law, is $\vec{E}_A = \frac{\rho}{2\varepsilon_0}\hat{z}$, here ρ C/m^2 is the charge density on the plate. Charge on A would induce negative charge density $(-\rho)$ C/m^2 and as a result electric field at the point $\vec{E}_B = \frac{(-\rho)}{2\varepsilon_0}(-\hat{z}) = \frac{\rho}{2\varepsilon_0}\hat{z}$. Therefore, force on a electron carrying charge $q = -1.6 \times 10^{-19}$ is $\vec{F} = q \times \left(\vec{E}_A + \vec{E}_B\right) = \frac{q\rho}{\varepsilon_0}\hat{z}$. Accordingly, as per Newton's Second law of motion the particle will experience acceleration $\vec{a} = \frac{\vec{F}}{m} = \frac{q\rho}{m\varepsilon_0}\hat{z}$ …(1), here $m = 9.1 \times 10^{-31}$kg and $\varepsilon_0 = 8.85 \times 10^{-12}$. Further, given that an electron is projected from a plate towards another plate at a separation $d = 2.00 \times 10^{-2}$ m, with a velocity u m/s such that it reaches the another plate i.e. $v = 0$ in $t = 2.00 \times 10^{-6}$. With this system shown in the figure, it can be only when electron is project with velocity $\vec{u} = u\hat{z}$ and $\vec{a}$ acts as retardation. Thus, as per Third equation of motion $v^2 = u^2 + 2ad \Rightarrow u = \sqrt{(-2ad)}$. Since, $\vec{u} = u\hat{z}$ and $\vec{a} = a(-\vec{z})$, algebraically, $u = \sqrt{2ad}$. Further, as per First Equation of Motion, $v = u + at \Rightarrow 0 = u + at \Rightarrow \sqrt{2ad} = -at \Rightarrow a = \frac{2d}{t^2}$…(2). Combining (1) and (2), $\frac{q\rho}{m\varepsilon_0} = \frac{2d}{t^2} \Rightarrow \rho = \frac{2dm\varepsilon_0}{qt^2}$. Using the available data $\rho = \frac{2\times(2.00\times10^{-2})(9.1\times10^{-31})(8.85\times10^{-12})}{(1.6\times10^{-19})(2.00\times10^{-6})^2}$ $= \mathbf{5.03 \times 10^{-13}}$ **C/m^2 is the answer.** **N.B.:** This problem is a good example of integration of multiple concepts.
Q-21	Three identical metal plates with large surface area are kept parallel to each other as shown in the figure. The leftmost plate is given a charge Q, the rightmost a charge -2Q and the middle remains neutral. Find the charge appearing on the outer surface of the rightmost plate. 
A-21	$-\frac{Q}{2}$
I-21	 The given system of identical metal plates is shown in figure two stages. Taking first stage with the two charged plates and excluding the middle plates which is electrically neutral. The plates being metallic charge on it will get distributed on their surfaces. The two surfaces A and B of the left plates carrying charge will have charge densities $\rho_A = \frac{+Q}{2A}$ and $\rho_B = \frac{+Q}{2A}$ and the two surface C and D of right plate will have charge densities $\rho_C = \frac{-2Q}{2A} = \frac{-Q}{A}$ and $\rho_D = \frac{-2Q}{2A} = \frac{-Q}{A}$. Now, when middle, electrically neutral plate is introduced as shown in the figure. It will have induced charges say $-q$ and $+q$ on its each of the two surfaces E and F, to maintain its electrically neutrality,  which are electrically opposite to the charge on the inner surfaces B and C such that charge densities are $\rho_E = \frac{-q}{A}$ and $\rho_F = \frac{+q}{A}$ respectively. This will result in redistribution of charges on surfaces A leading to surface charge densities $\rho'_A = \frac{+(Q-q)}{A}$ and $\rho'_B = \frac{+q}{A}$ such that net charge on the plate remains at $+Q$. Likewise, surfaces C and D will be densities $\rho'_C = \frac{-q}{A}$ and $\rho'_D = \frac{(-2Q+q)}{2A}$ leading to net charge on it to $-2Q$.

	Next step is to determine electric field at point P, due to each of the surface, which as per general equation of Gauss's law is $E = \frac{\rho}{\varepsilon_0}$, where ρ is the charge density of the corresponding surface. It is independent of its distance from the surface. Accordingly, $\vec{E}_{\text{A}} = \frac{+(Q-q)}{A\varepsilon_0}\hat{z}$, $\vec{E}_{\text{B}} = \frac{+q}{A\varepsilon_0}\hat{z}$, $\vec{E}_{\text{C}} = \frac{-q}{A\varepsilon_0}(-\hat{z}) = \frac{q}{A\varepsilon_0}\hat{z}$, $E_{\text{D}} = \frac{(-2Q+q)}{A\varepsilon_0}(-\hat{z}) = \frac{(2Q-q)}{A\varepsilon_0}\hat{z}$, $E_{\text{E}} = \frac{-q}{A\varepsilon_0}\hat{z}$ and $E_{\text{F}} = \frac{+q}{A\varepsilon_0}(-\hat{z}) = \frac{-q}{A\varepsilon_0}\hat{z}$, a set of equations (2). In this negative sign with unit vector $\hat{z}$ in E_{C}, E_{D} and E_{F} is indicative of direction of field at point P, due to respective charge distribution. Likewise, E_{A}, E_{B} and E_{E} are towards right i.e. along $+\hat{z}$. Now a point P is taken inside the middle metal plate electrical field $E_{\text{P}} = 0 \ldots(3)$ it is characteristic to the metals. Thus using superimposition of electric fields due to each charged resultant electric field at point P is Combing (2) and (3) we have $E_{\text{A}} + E_{\text{B}} + E_{\text{C}} + E_{\text{D}} + E_{\text{E}} + E_{\text{F}} = E_{\text{P}} \ldots(4)$. Combining (2) and (3) in (4) we have $\frac{(Q-q)}{A\varepsilon_0} + \frac{q}{A\varepsilon_0} + \frac{q}{A\varepsilon_0} + \frac{(2Q-q)}{A\varepsilon_0} + \frac{-q}{A\varepsilon_0} - \frac{q}{A\varepsilon_0} = 0 \Rightarrow Q + 2Q - -2q = 0 \Rightarrow q = \frac{3}{2}Q \ldots(5)$ Accordingly, the charge appearing on the outer surface of the rightmost plate, as required to be determined, using (5) is $Q_{\text{D}} = (-2Q + q) = \left(-2Q + \frac{3}{2}Q\right) = -\boldsymbol{\frac{Q}{2}}$ **is the answer**. **N.B.:** Surfaces B and E, and F and C are acting like pair of capacitor plates having equal and opposite charges remaining charge on the two charged plates appear on surfaces A and D.

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